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INTERNATIONAL LENDING WITH MORAL HAZARD
AND RISK OF REPUDIATION

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ABSTRACT

This paper examines the optimal debt contract between lenders and
a sovereign borrower when the borrower is free to repudiate the
debt and when his decision to invest or consume borrowed funds is
unobservable. We show that recurrent debt crises are a necessary
part of the incentive structure which supports the optimal pattern
of lending.

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1. Introduction

International lending over the last one hundred and fifty years has occurred in a succession of waves of lending followed by a lack of lending. [1] A simple theory of international capital flows through a credit market with full information and full enforcement of contracts predicts that as countries borrow to smooth domestic consumption over variations in domestic output, each country should receive a net inflow of capital when output is low relative to aggregate world output and a net outflow of capital when the reverse is true. This predicted pattern of international capital flows is not the pattern that is typically observed. Debt crises in which countries suffer a net outflow of capital when domestic output is low relative to aggregate world output recur throughout this period. These capital outflows occur despite evidence of efficient investments that go unexploited in the borrowing country. During these debt crises, borrowing countries face credit rationing and bear a substantial debt service burden without the benefit of much new lending. Most recently, through the mid 1980's, many Third World debtors were net creditors to the developed world as they continued to make at least partial debt service without net new lending at a time in which their output fell dramatically relative to trend relative to output in the developed world relative to trend.

In this paper I examine the optimal pattern of international capital flows in an environment in which sovereign borrowers are able to repudiate their debts and in which the decision of borrowers to invest or consume borrowed funds is unobservable. I find that recurrent debt crises are a necessary part of the optimal pattern of international lending with repudiation risk when lenders must design contracts to provide incentives for borrowers to maintain high investment. Under the optimal debt contract, a country which has a low realization of output is punished under the suspicion that the low realization of output was a result of underinvestment by being compelled to make a large debt service payment relative

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[1] For a discussion of the pattern of international lending observed over the long run, see among others, Lindert and Morton [1987] and Eichengreen and Portes [1986]

1

to the new lending that it receives. When the country is being punished, it is indifferent between continuing with debt service and repudiation of the debt. Net new lending does not resume and the borrowing country does not begin to enjoy gains from its access to the credit markets again until high output is again realized.

Lending across national borders is distinct from lending within a single country in that a sovereign borrower cannot be compelled to service his debt once he perceives that he no longer benefits from being held in good standing by his creditors. Whether he services his debt is determined by his willingness, and not his ability, to pay. In their seminal paper studying debt with risk of repudiation, Eaton and Gersovitz [1981] find that a sovereign borrower may have his access to the credit markets restricted according to a credit limit on overall indebtedness because of the risk of repudiation, and that debt crises occur when that credit limit is binding. A borrower who accumulated debt because of a string of low realizations of domestic output may be forced to suffer a net capital outflow with low output if that string of low realizations of output continues longer than can be accomodated under the credit limit. But the environment that Eaton and Gersovitz analyze is simplified in an important respect: even when the borrower's output is stochastic, the borrower and lenders are restricted to write debt contracts in which required debt service cannot be made contingent on output. In a full information credit market, a borrower with stochastic output does not accumulate debt over a string of bad realizations of output because it is more efficient from the standpoint of risk sharing to write down the debt service before the debt ceiling binds than it is to ration the borrower's credit as a result of a string of bad luck. While it is true that the standard international debt contract does not explicitly specify debt service to be state contingent, the realized net flow of capital between borrower and lender is made state contingent in a variety of ways. First, when the flow of direct investment, while relatively small, and the flow of new lending net of repayments is considered in the aggregate flow, the pattern of capital flows can be made state contingent. More importantly, the recent observation of frequent reschedulings and the setting aside by the major banks of

substantial international loan loss reserves indicates that the debt service currently expected is less than what was initially specified in the lending contracts. Worrall [1987] obtains a different set of results from those of Eaton and Gersovitz by relaxing the assumption that debt service is not state contingent.

Worrall [1987] examines a partial equilibrium environment with lending with repudiation risk. He finds that when the borrower's output is stochastic and has finite support and when debt service can be made state contingent, that the borrower achieves complete consumption smoothing in finite time under the optimal contract. This result implies that while repudiation risk is an important constraint on international lending in the short run, this risk itself does not satisfactorily explain the recurrence of debt crises over the long run.

One reason that there is not more risk sharing observed across countries is that moral hazard is an important constraint on the insurance that can be achieved through international lending. The decision of the borrower to invest or consume borrowed funds is difficult to observe reliably. Lending has to be structured to provide the borrower with the incentive to invest when efficient investments cannot be distinguished from those which contain a disguided component of consumption. Consumption may be disguised as investment in a number of ways, including through the overemployment of labor when shadow values are obscured in a state dominated economy, or through outright fraud when there is imperfect monitoring of the execution of investment projects. In modelling this constraint on international lending contracts imposed by moral hazard, I formalize an incentive compatibility constraint on the set of allocations which can be supported by contracts in this environment.

The technical approach I take to solving for the optimal allocation that can be supported by contracts which satisfy the incentie compatibility constraint in this environment is similar to the approach taken by Abreu, Pearce, and Stacchetti [1986a,b] for solving the repeated Cournot oligopoly problem and by Spear and Srivastava [1987] for solving the repeated priciple–agent incentive problem. The net capital flow may be interpreted as the agent's compensation and the investment level as the agent's effort. The environment ana-

lyzed here is different from the repeated principle–agent problem in two important respects: the first is that the borrower (who might be likened to the agent) retains possession of his output and pays the lender only if he chooses, and the second is that the borrower's output net of repayments is a physical state variable which is passed from period to period changing the feasible set of actions and payoffs that can be attained. My technical results are new in that I generalize the theorems of self generation and factorization of Abreu, Pearce, and Stacchetti [1986a] which characterize the set of payoffs that can be supported by allocations which are incentive compatible to handle the incentive compatibility constraint in this environment with a state variable and in that I must reinterpret the standard bang–bang results on optimal punishments for this case where the choice of an output contingent repayments schedule together with the choice of investment determines the distribution of the state. I find that current lending always proceeds under the optimal contract for the current value of the state, so that the provision of incentives for investment is achieved entirely through the design of the repayments schedule and not through a cutoff of lending. There is no alternation between sets of actions defined in punishment phases and reward phases as in these other two models. Punishment is achieved through the imposition of large net repayments that render the value of the optimal lending contract at that new value of the state equal to the value that the borrower could receive by repudiating his debt. As a result, I can characterize the payoff to the borrower of the optimal debt contract in a simple dynamic program which can be solved in the usual manner.

In section 2, I describe the environment. In section 3, I state the Pareto problem when allocations are constrained both by moral hazard and the risk of repudiation. In section 4, I characterize the set of payoffs for the borrower that can be supported by allocations in the constraint set of the Pareto problem using generalizations of the concepts of self–generation and factorization developed in Abreu, Pearce, and Stacchetti [1986a]. In section 5, I characterize the continuation value function necessarily used to support the constrained optimal payoff for the borrower. I use this characterization to construct a straightforward dynamic

4

programming problem which necessarily characterizes both the constrained optimal equilibrium payoff for the borrower and the constrained optimal allocations. I also show that any constrained optimal lending program in which the expected value of debt service contingent on output is an incresing function of investment specifies net flows from borrower to lender when the borrower's income is low as is typical of the empirical observations called debt crises. In section 6, I use this characterization to solve for the constrained optimal lending program in an example economy which displays debt crises (Section 6 forthcoming). In the appendix, I review the Pareto problem with full information and full enforcement of contracts and I review Worrall's results on the Pareto problem with risk of repudiation.

2. The Environment

In this environment there is an infinite horizon and a single good which may be either consumed or invested. There is one infinitely lived risk averse agent, whom I call the borrower, who owns a stochastic returns storage technology. The considerations of moral hazard constrain contracting in this environment when the borrower's investment in his storage technology is unobservable by the other agents. The other agents in this environment are a sequence of short lived, risk neutral agents, whom I call lenders, arranged in overlapping generations. The lenders are assumed to have access to perfect capital markets. The borrower is assumed to not have such access. An allocation in this environment is supported by contracts through which the lenders gain access to the potentially superior returns offered by the borrower's stochastic storage technology and the borrower gains access to the risk sharing opportunities offered by the lenders' position vis-a-vis the capital markets. For the purposes of defining optimal contracting problem, the reservation utility of an agent in this environment of entering into contracts is naturally defined by the utility that agent can obtain if he does not enter into any contracts.

The enviroment is described more specifically as follows. Time is discrete and denoted by $t = 0, 1, 2, \ldots$ The borrower is alive in all periods $t \geq 0$. The borrower is endowed

5

with Y_0 of the single good at $t = 0$. The single good may be consumed or invested. The consumption of the borrower is denoted c_t. The borrower's stochastic returns storage technology transforms I_t units of good t into Y_{t+1} units of good $t + 1$. In any period, the choice of investment I in the storage technology from the set of feasible investments $\mathbf{I}$ induces a distribution over next period's output Y' received as a product of the storage technology, where this distribution has support $\mathbf{Y}$. I assume that the cumulative distribution function specifying the probability of realizing output tommorow of at most Y' given a fixed level of investment today of I, written $G(Y'; I)$, has density $g(Y'; I)$.

There is one lender born each period and each lender lives two periods. The lender born in t is alive in t and $t + 1$. Each lender has no endowment of the good. Because of the access each lender has to perfect capital markets, we define the preferences of the lenders over allocations which may specify negative consumption. The consumption of the young lender born in t is denoted b_t, and the consumption of that lender while old is denoted d_{t+1}. The lenders are introduced into this environment in a manner intended to render the analysis of their side of the optimal contracting problem particularly simple. The assumption that the lenders are short–lived forces lending to occur through short–term contracts only. The motivation for this construction is the desire to prevent any individual lender from having monopoly power over the borrower. The assumption that lenders have access to perfect capital markets is to ensure that I do not need to worry about an aggregate resource constraint binding the set of feasible allocations. As the lenders have no endowment of their own, their reservation utility will naturally be zero.

In order to focus attention on the borrower's side of the optimal contracting problem, I impose two additional assumptions about the lenders in this environment. The first assumption is that there is a commitment technology by which the lenders can bind themselves when young to carry out the terms of a contract when they are old. The weight of this assumption is to ensure that the borrower can costlessly enforce his right to make withdrawals on deposits. The second assumption is that an old lender who has suffered

6

a breach of contract may costlessly seize any deposits the borrower might make with the lender who is young as compensation towards his loss. This right is passed down from one generation of lenders to the next until the loss has been entirely compensated. The weight of this assumption is to prevent the borrower from playing one lender off against another through a strategy of repudiating a repayment to the current old lender and then using the funds intended for repayment to establish a deposit with a future young lender against which he might make withdrawals to smooth consumption on his own.

An allocation in this enviroment is a plan which specifies the disposition of the current output Y_t between current consumption for the borrower and for the old and the young lender and investment in the storage technology. The plan for the disposition of current output may depend on the entire history of realizations of output $Y^t = \{Y_0, ..., Y_t\}$ and on the entire history of consumption and investment. Clearly, this complex history dependence can be summarized by the dependence of the current consumption and investment decisions on the history Y^t. For convenience, I choose to use a state variable $Q_t = Y_t - d_t(Y^t)$ to summarize the history dependence of the consumption and investment decisions of everyone except the old lender. Thus, an allocation specifies the consumption of the borrower, c, investment, I, the negative of the consumption of the young lender, b, and the consumption of the old lender, d, written:

$$\sigma = \{c_t(Q^t), I_t(Q^t), b_t(Q^t), d_{t+1}(Y^{t+1})\}_{t=0}^{\infty}$$

An allocation is associated with initial conditions Y_0, Q_0, d_0 with $Q_0 = Y_0 - d_0$. Let

$$\sigma_t = (c_t(Q^t), I_t(Q^t), b_t(Q^t), d_{t+1}(Y^{t+1}))$$

and $\sigma = (\sigma^c, \sigma^I, \sigma^b, \sigma^d)$. The components $-\sigma^b, \sigma^d$ of an allocation are not constrained to be positive. The negative of the consumption of the young lender will sometimes be referred to as lending, and the consumption of the old lender referred to as repayment. These terms are used only to facilitate discussion and do not imply constraints on the signs of these values.

An allocation is feasible in this environment if the current consumption of all agents and the current investment is always less than the current quantity of the good available, that is, for $t \geq 0$, $Y^t \in \mathbf{Y}^t$:

$$c_t(Q^t) - b_t(Q^t) + d_t(Y^t) + I_t(Q^t) \leq Y_t$$

with $c_t(Q^t), I_t(Q^t) \geq 0$ and Y_0, Q_0, d_0 given. Remember that the allocation may specify negative consumption for the lenders, i.e. $(-b, d) < 0$. By convention $d_0(Y_0) = 0$.

The borrower has preferences over allocations denoted by $U^B(\sigma)$ and characterized by

$$U^B(\sigma) = (1 - \delta)E_0^\sigma \sum_{t=0}^{\infty} \delta^t u(c_t(Q^t))$$

with u bounded, $u' > 0$ and $u'' < 0$. E_0^σ denotes the mathematical expectation conditional on the information available at time 0, taken with respect to the probability measure induced by the allocation σ. The lender born in t, $t \geq 0$, has preferences over the expected value of his consumption where the expectation is taken conditional upon the realization of Y^t at the time of his birth. These preferences are denoted by $U^L_{Y^t}(\sigma)$ and characterized by

$$U^L_{Y^t}(\sigma) = (-b_t(Q^t) + \delta \int_{Y' \in Y} d_{t+1}(Y_{t+1}; Y^t)g(Y_{t+1}; I_t(Q^t))dY')$$

where $d_{t+1}(Y_{t+1}; Y^t)$ is a function of a single variable contingent upon the realization Y^t given.

Because I am analyzing optimal allocations supportable by incentive compatible contracts in this environment, I confine myself to examining allocations which are both feasible and provide each party to the contract supporting the allocation with at least as much expected utility as could be obtained by not contracting at all. The reservation utility of the lenders in this environment is zero as they can always receive consumption of zero by not entering into any contracts. The reservation utility of the borrower is defined by the expected utility he can receive by consuming and investing in the storage technology on his own having signed no contracts at all. Such an allocation, needing no contracts to support

8

it, is autarkic and satisfies $b_t(Q^t) = d_t(Y^t) = 0$ for all t, $Y^t \in \mathbf{Y}^t$. The expected utility that the borrower can obtain in autarky is the solution to a programming problem given no trade and an initial stock of the good Y. That utility as the solution of the problem

$$U^B_{aut}(Y) = max_\sigma U^B(\sigma)$$

subject to $Y_0 = Y$ and σ autarkic. The borrower's reservation utility, $U^B_{aut}(Y)$, is a function which is continuous, differentiable, strictly increasing, and strictly concave in Y. This function is characterized by the optimality equation:

$$U^B_{aut}(Y) = \max_{c,I}(1 - \delta)u(c) + \delta \int_{Y' \in \mathbf{Y}} U^B_{aut}(Y')g(Y'; I)dY'$$

where the maximization is subject to the constraint: $c + I \leq Y$.

An allocation is individually rational if it provides each agent with expected utility at least as great as his reservation utility, that is if, $U^B(\sigma) \geq U^B_{aut}(Q_0)$ and $U^L_{Y^t}(\sigma) \geq 0$ for all Y^t, $t \geq 0$. Clearly, only individually rational allocations can be supported by contracts in equilibrium. Note that

$$b_t(Q^t) \leq \delta \int_{Y' \in \mathbf{Y}} d_{t+1}(Y_{t+1}; Y^t)g(Y_{t+1}, I_t(Q^t))dY_{t+1}$$

for all $t \geq 0$, $Y^t \in \mathbf{Y}^t$, is a necessary condition of individual rationality for the lenders.

I use several assumptions about the stochastic storage technology in analyzing this environment. The first assumption is that the support $\mathbf{Y}$ of the distribution of output tommorow given investment today is an interval $[Y_{min}, Y_{max}]$ of the real line $\mathbf{R}$ with $Y_{min} > 0$. This interval is independent of the choice of I, i.e.:

$g(Y'; I) > 0$ for all $Y' \in \mathbf{Y}$ and $I \in \mathbf{I}$.

This assumption guarantees that there are no observations of Y' that allow the lenders to infer with certainty that the borrower did not invest at some level I.

The next two assumptions allow me to summarize the borrower's problem of privately choosing an optimal investment level by the first order condition of that problem. First

9

assume that the distribution of Y' given I satisfies the monotone likelihood ratio property, that is:

$\frac{g_I(Y';I)}{g(Y';I)}$ is monotone in Y' for all $I \in \mathbf{I}$.

where I have assumed that the family of densities is differentiable with respect to I and I let $g_I(Y';I)$ denote $\frac{\partial g(Y';I)}{\partial I}$ and $g_{II}(Y';I)$ denote $\frac{\partial^2 g(Y';I)}{\partial I^2}$.

Next assume that the cumulative distribution of Y' as a function of I satisfies the convexity of the distribution function condition, namely

$G_{II}(Y';I) \geq 0$ for all Y' and I.

These assumptions imply that we may interpret the storage technology as a standard diminishing marginal returns production technology. In particular, the monotone likelihood ratio property implies that the distribution of Y' as a function of I can be ordered by first order stochastic dominance, i.e.

$\int_{Y' \in \mathbf{Y}} f(Y') g_I(Y';I) dY' \geq 0$ for all $I \in \mathbf{I}$ for all monotone functions $f(Y')$

and the convexity of the distribution function implies that

$\int_{Y' \in \mathbf{Y}} f(Y') g_{II}(Y';I) dY' \leq 0$ for all $I \in \mathbf{I}$ for all monotone functions $f(Y')$

An example of a family of CDF's which satisfy these conditions for some $\epsilon > 0$ is given here: [2]

$$G(Y';I) = \left(\frac{Y' - Y_{min}}{Y_{max} - Y_{min}}\right)^{I+\epsilon}$$

3. The Pareto Problem with Moral Hazard and Risk of Repudiation

In this section I define the incentive compatibility constraints on the investment plans that may be implemented through contracts in equilibrium and set up the pareto problem constrained by risk of repudiation and moral hazard.

When there are no impediments to contracting in this environment, the optimal contract supports an allocation which transfers all risk from the risk averse borrower to risk

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[2] This example is due to Rogerson [1985].

neutral lenders. A simple single–good competitive equilibrium model of international capi-tal flows would suggest that countries would be able to obtain insurance from idiosyncratic risk. The optimal allocation in this environment with full enforcement of contracts and full information specifies net transfers from lenders to borrower when the low output from the storage technology is realized and transfers in the other direction when high output is real-ized. Since the empirical observations of international capital flows do not fit this pattern we are led to consider what impediments to contracting might exist in this environment to interfere with risk sharing and produce a pattern of capital flows in the constrained optimal allocation that seems more in the character of the recurring debt crises that we observe.

Because the full enforcement, full information pareto optimal allocation does not pro-duce a pattern of international capital flows that is descriptive of the debt crises that have recurred over the long run, we turn naturally to an examination of the restrictions that sovereign risk places upon the kinds of risk sharing that can be achieved through interna-tional lending. The seminal paper in this area is Eaton and Gersovitz [1981]. They find that a sovereign borrower may find his access to the credit markets restricted in equilibrium as a result of repudiation risk. With this restriction, a borrower is unable to smooth his consumption over cycles in his output when his credit limit is binding. But the environ-ment which Eaton and Gersovitz analyzed is simplified in an important respect: even when the borrower's output is stochastic, the borrower and lender are restricted to writing debt contracts in which repayments cannot be conditioned on the borrower's output. A different set of results arise when this restriction is relaxed.

Worrall [1987] examines a partial equilibrium environment with lending in which there is repudiation risk. He finds that when the borrower's output is stochastic with finite support and the borrower and lenders can write contracts contingent on the realizations of that output, that the borrower is able to achieve complete consumption smoothing in finite time. The implication of this result is that, while repudiation risk is an important constraint on the risk sharing that can be achieved through international lending, this risk itself is not

enough to explain the pattern of international capital flows that have been observed over the very long term. I present Worrall's results in the context of this environment in the appendix.

We see that the risk of repudiation in itself is not a sufficiently strong impediment to the risk sharing that can be achieved through international lending when the income of the borrower is stochastic and contracts can be made fully contingent on the borrower's output to explain the observed recurrence over the long run of debt crises. But one reason that international debt contracts are not made fully contingent on the borrower's income is the moral hazard problem inherent in the borrower's choice of investment. When the borrower must invest for future income and that investment decision is private, then lenders may not be willing to provide the borrower with insurance through a state contingent loan. When the borrower's investment decision is private, he must be induced to invest by having his future consumption depend sufficiently strongly on the output of his production technology. Of course, it is the fact that the borrower's future consumption must be made to vary with the realizations of his output to overcome the moral hazard problem that interferes with the risk sharing that might be achieved between the borrower and lenders.

To define the set of feasible debt contracts when the borrower cannot bind himself to not repudiate contracts, I must describe explicitly the punishment that the lenders can impose upon the borrower for repudiation. In this environment, the lenders have no direct means of interfering with the borrower's consumption and investment decisions. On the other hand, the lenders can punish the borrower indirectly by refusing to extend further credit. As the borrower can always obtain the autarkic payoff on his own and lenders can ensure the autarkic payoff for the borrower by refusing all contracts, the worst punishment which the lender's can impose must be autarky. It is easy to show that the lenders can deny the borrower credit in a perfect equilibrium. Any individual lender will not extend credit if all future lenders plan to not extend credit as the borrower is sure to repudiate any positive repayments due the last lender who extended credit. Specifically, an allocation can

12

be supported by contracts in equilibrium if and only if the gain to each party from carrying out the payments specified by the contracts in every contingency is superior to the gain from refusing to make the payment and doing as well as one can for the remainder of time in autarky. The constraint that contracts be designed such that the borrower never prefers to repudiate the contract is summarized by the constraints that, not only is the allocation supported by the contracts individually rational, but also the expected utility that the borrower receive from continuing with the allocation in every contingency Y^t, $t > 0$ is also in excess of his reservation utility in that contingency. In other words, I require that the continuation of the allocation also be individually rational in every contingency.

I define the continuation of an allocation as specified by the plan in any contingency and individual rationality of the continuation of the allocation as specified by the plan in the obvious way: For every $t > 0$, $Y^t \in \mathbf{Y}^t$, define the continuation allocation $\sigma|_{Y^t}$ given Y^t element by element, $s \geq 0$, $Y^s \in \mathbf{Y}^s$:

$$\sigma_s|_{Y^t} = (c_s(Q^s), I_s(Q^s), b_s(Q^s), d_{s+1}(Y^{s+1}))|_{Y^t} =$$

$$(c_{s+t}(Q^t, Q^s), I_{s+t}(Q^t, Q^s), b_{s+t}(Q^t, Q^s), d_{s+t+1}(Y^t, Y^{s+1}))$$

The continuation of an allocation is itself an allocation, so let preferences over the continuation of an allocation be defined as before. (Here the initial values of Y and Q in the continuation allocation is understood to be Y_t and $Q_t = Y_t - d_t(Y^t)$). I call an allocation which is ex ante individually rational and in every contingency, ex post individually rational, an allocation which is immune from the threat of repudiation. Formally, an allocation σ is immune from the threat of repudiation if for all $t \geq 0$, $Y^t \in \mathbf{Y}^t$, the continuation allocation $\sigma|_{Y^t}$ satisfies

$$U^B(\sigma|_{Y^t}) \geq U^B_{aut}(Y_t)$$

An allocation is incentive compatible when the investment decision is private if the borrower finds it optimal to carry out the investment plan when he takes the lending and

13

repayment plans as given, that is, an allocation σ is incentive compatible if for all feasible allocations $\sigma' = (\sigma^{c'}, \sigma^{I'}, \sigma^b, \sigma^d)$ (with the components b and d unchanged):

$$U^B(\sigma) \geq U^B(\sigma')$$

An allocation is Fully Constrained Pareto Optimal (FC) if it is a solution to the problem:

$$\max_{(\sigma)} U^B(\sigma) \qquad\qquad (FC)$$

subject to (σ) satisfies (1) feasibility, (2) individual rationality, (3) immunity from the threat of repudiation, and (4) incentive compatibility.

The difficult part of solving this program is understanding how to handle the incentive compatibility constraint. Because zero investment cannot be socially optimal if $\int_{Y' \in \mathbf{Y}} Y' g_I(Y'; 0) dY'$ can be arbitrarily large, the utility that the borrower receives in the continuation of the optimal allocation must vary sufficiently with the results of some statistical test on Y' intended to determine whether the borrower made the right level of investment to induce the borrower to make that correct level of investment. Because the borrower will be choosing investment in anticipation of the test on output that determines his continuation allocation, standard results from hypothesis testing do not apply. When this strategic consideration is taken into account, the optimal manner in which to make the borrower's continuation payoff depend on the realization of output is not obvious: should the borrower be punished for low realizations of output in proportion to how seriously output has fallen short of expectations, or possibly should only the severest punishments be used and used as infrequently as possible. Furthermore, should the punishment be imposed through the requirement of large repayments on old debt, or should new lending also be restricted below what could be lent at that value of the state variable. We know from study of the repeated Cournot oligopoly problem that the optimal incentive scheme puts a positive probability on equilibrium price wars and from study of the repeated principal–agent incentive problem that the optimal incentive scheme places a positive probability on the agent being fired in

14

equilibrium, so that in these other problems, the optimal contract is sustained by threats of non-optimal continuation outcomes for some realizations of the mutually observed signal. Given these results, we must take time to show that in this pareto problem, we can indeed summarize the solution in a Bellman's Equation. In section 4, I characterize the set of payoffs for borrower that can be supported by allocations which satisfy constraints (1)-(4) of the pareto problem (FC) above.

4. The Constrained Set of Payoffs

In this section I generalize the notions of admissability, self generation, and factorization of Abreu, Pearce, and Stacchetti [1986a] to characterize the set of payoffs for the borrower that can be supported by allocations which satisfy the constraints (1)-(4) of the program (FC) above.[3]

The central idea behind the results of this section is to simplify the dynamic problem of finding all the allocations in the constraint set to a static problem of finding all the current actions which are incentive compatible with respect to some continuation value function of the state which stands in for the payoff of the continuation allocation. Where the dynamic problem is in the space of infinite sequences of functions specifying consumption and investment contingent on the history of output, the static problem is in the space of actions for the current period and continuation value functions which render those actions incentive compatible. I show that the set of payoffs for the borrower obtained as solutions to the static problem is equal to the set of payoffs for the borrower supported by allocations in the constraint set. The search for the optimal payoff is then a search for the actions for the current period and the continuation value function that yield the highest expected value for the borrower subject to the constraints that the actions be feasible and incentive compatible

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[3] For those who are familiar with the techniques of Abreu, Pearce, and Stacchetti [1986a], the results of this section are the natural extensions of their propositions self-generation and factorization to this control problem. The definition of admissability has been generalized. Otherwise the proofs proceed along the lines of the proofs of propositions 1 and 2 in their paper.

with respect to the continuation value function and that the continuation value function actually stand in for the payoff of continuation allocations which are in the constraint set defined by equations (1)-(4) of the program above.

Because the continuation of any allocation in the constraint set is also in the constraint set, it is clear that we can factor any payoff for the borrower supported by an allocation in the constraint set into two parts. The first part is the payoff received in the initial period as a result of the actions specified in the allocation. The second part is the payoff received from the continuation allocation arising after the realization of output after the initial period. If we let the payoff of the continuation allocation in each state define the continuation value function of a static program, then the initial levels of consumption and investment are incentive compatible in that they solve that static program. Proposition 2 demonstrates that this property of factorization is a necessary property of payoffs supported by allocations in the constraint set.

Proposition 1 establishes a sufficiency result in terms of these static programs and the continuation value functions which are used to create them. The definition of the admissability of a pair describing controls in the static program and a continuation value function parameterizing that program with respect to a collection of feasible payoffs at a value of the state provides a set of criteria for constructing these static problems and their continuation value functions. These criteria specify that the continuation value function have a range confined to the collection of payoffs being analyzed and the the control be a solution of the static program parameterized by that continuation value function. We say that a member of the collection of payoffs being analyzed is generated if it is the payoff of some solution of some static program which is admissable with respect to that collection of payoffs. We say that a collection of feasible payoffs is self generating if every member of that collection can be generated in some static program admissable with respect to that collection of payoffs. I then show that any self generating collection of payoffs must be a subset of the set of payoffs supported by allocations in the constraint set. This is done by

using these static programs iteratively to construct for each member of that collection of payoffs an allocation in the constraint set which supports that payoff.

Define the correspondence Z with domain $\mathbf{Q}$ to be, for each initial value of Q, the set of payoffs for the borrower that can be attained through allocations which satisfy the constraints (1)-(4) in the Pareto Problem above. That is, for each value of Q,

$$Z(Q) = \{U^B(\sigma) | \sigma \ \ satisfies \ \ (1) - (4) \ \ and \ \ Q = Y_0 - d_0\}$$

LEMMA: $Z(Q)$ is non-empty and bounded for all Q.

PROOF: $Z(Q)$ is bounded because $u(.)$ is bounded and $\delta < 1$. $Z(Q)$ is nonempty because $U_{aut}^B(Q) \in Z(Q)$ for all Q.

Let W be a correspondence defined over domain $\mathbf{Q}$, with $W(Q)$ non-empty and uniformly bounded for all values in the domain.

DEFINITION OF ADMISSABILITY: The pair (A, U), with

$$A = (c, I, b, d')$$

a collection of controls for the current round where c, I, and b are scalars and d' is a function of the form $d' : \mathbf{Y} \to \mathbf{R}$ and U a function from the state variable at the beginning of the next period to sets of the correspondence W

$$U : \mathbf{Q} \to \mathbf{R}$$

with $U(Q') \in W(Q')$ for all values of Q' is

ADMISSABLE WITH RESPECT TO W AT Q

if A satisfies one shot incentive compatibility

$$\max_{(c,I)} (1 - \delta) u(c) + \delta \int_{Y' \in Y} U(Q') g(Y'; I) dY' \tag{A}$$

subject to feasibility of the current control:

$$c + I - b \leq Q \tag{B}$$

17

the transition function for the state:

$$Q' = Y' - d'(Y') \tag{C}$$

individual rationality for the current young lender and the borrower:

$$b \leq \delta \int_{Y' \in \mathbf{Y}} d'(Y')g(Y'; I)dY' \tag{D}$$

$$\max_{(c,I)}(1 - \delta)u(c) + \delta \int_{Y' \in \mathbf{Y}} U(Q')g(Y'; I)dY' \geq U_{aut}^B(Q)$$

and no repudiation in the next round for all Y', $Q' = Y' - d'(Y')$:

$$U(Q') \geq U_{aut}^B(Y') \tag{E}$$

Define the payoff $E(A, U; Q)$ generated by a pair (A, U) admissable with respect to W at Q by:

$$E(A, U; Q) = (1 - \delta)u(c) + \delta \int_{Y' \in \mathbf{Y}} U(Q')g(Y'; I)dY'$$

Denote the set of payoffs that can be generated by pairs (A, U) admissable with respect to W at Q by $B(W, Q)$ where:

$$B(W, Q) = \{E(A, U; Q) \ \ s.t. \ \ (A, U) \ \ admissable \ \ w.r.t \ \ W \ \ at \ \ Q\}$$

DEFINITION OF SELF GENERATION: The correspondence W is
self generating if for all $Q \in \mathbf{Q}$

$$W(Q) \subseteq B(W, Q)$$

PROPOSITION 1:(SELF GENERATION)

If W is self generating, then for all $Q \in \mathbf{Q}$,

$$B(W, Q) \subseteq Z(Q)$$

18

PROOF: The Proof proceeds in two steps. Step 1 constructs an allocation $(\sigma(w_Q))$ for each $w_Q \in B(W, Q)$ such that

$$U^B(\sigma(w_Q)) = w_Q$$

Step 2 verifies that each allocation $\sigma(w_Q)$ satisfies constraints (1)-(4) of the Pareto Problem above.

STEP 1: W self generating implies that for each $w_Q \in W(Q)$ for some Q, there exists a pair $(A(w_Q), U(w_Q))$ such that this pair is admissable w.r.t. W at Q and with

$$E(A(w_Q), U(w_Q); Q) = w_Q$$

Choose a $w_{Q^0} \in W(Q_0)$. Assume $Y_0 = Q_0$, $d_0 = 0$. Define $(\sigma(w_{Q^0}))$ as follows: Let

$$\sigma_0(w_{Q^0}) = A(w_{Q^0})$$

Note that $A(w_{Q^0})$ specifies $d_1(Y^1) = d'(Y')$ and that the specification of $\sigma_0(w_{Q^0})$ for values of $Q \neq Q^0$ is irrelevant.

Given a realization of Y_1 and a new value for the state $Q_1 = Y_1 - d_1(Y^1)$, define

$$w_{Q^1} = U(w_{Q^0})(Q_1)$$

and

$$\sigma_1(w_{Q^0})|_{Y^1} = \sigma_0(w_{Q^1}) = A(w_{Q^1})$$

where this is defined in the way above. Iterating this procedure we define

$$w_{Q^{t+1}} = U(w_{Q^t})(Q_{t+1})$$

and

$$\sigma_{t+1}(w_{Q^0})|_{Y^{t+1}} = \sigma_0(w_{Q^{t+1}}) = A(w_{Q^{t+1}})$$

Now I show that for any $w_Q \in W(Q)$ for some Q, that

$$U^B(\sigma(w_Q)) = w_Q$$

19

We have

$$w_Q = (1 - \delta)u(c) + \delta \int_{Y' \in \mathbf{Y}} U(Q')g(Y'; I)dY'$$

(where $Q' = Y' - d'(Y')$) and by definition of $\sigma(w_Q)$,

$$U^B(\sigma(w_Q)) = (1 - \delta)u(c) + \delta \int_{Y' \in \mathbf{Y}} U^B(\sigma(w_{Q^1}))g(Y'; I)dY'$$

where

$$w_{Q^1} = U(w_Q)(Q')$$

and

$$\sigma(w_{Q^1}) = \sigma(w_{Q^0})|_{Y^1}$$

Subtraction gives

$$w_Q - U^B(\sigma(w_Q)) = \delta \int_{Y' \in \mathbf{Y}} w_{Q^1} - U^B(\sigma(w_{Q^1}))g(Y'; I)dY'$$

which since g is a probability density implies that

$$|w_Q - U^B(\sigma(w_Q))| \le \delta \sup_{w_{(Q')} \in B(W,Q')} |w_{Q'} - U^B(\sigma(w_{Q'}))|$$

and since this holds for all w_Q we have

$$\sup_{w_Q \in B(W,Q)} |w_Q - U^B(\sigma(w_Q))| \le \delta \sup_{w_{Q'} \in B(W,Q')} |w_{Q'} - U^B(\sigma(w_{Q'}))|$$

and since $\delta < 1$ and each set $B(W, Q)$ is uniformly bounded given the uniform bound on $W(Q)$ and the bound on u, we have

$$w_Q = U^B(\sigma(w_Q))$$

for all

$$w_Q \in B(W, Q) \quad for \quad some \quad Q$$

STEP 2: Here I show that $\sigma(w_Q)$ satisfies conditions (1)-(4) of the Pareto Problem above.

Check conditions (1)-(4) one at a time.

(1) Feasibility is satisfied period by period in the definition of admissability, so the constructed allocation $\sigma(w_Q)$ must also be feasible.

(2) Individual Rationality holds by condition (D) of admissability.

(3) $\sigma(w_Q)$ is immune from the threat of repudiation by construction:

$$U^B(\sigma(w_Q)|_{Y^t}) = U^B(\sigma(w_{Q^t})) = w_{Q^t}$$

where

$$w_{Q^t} = U(w_{Q^{t-1}})(Q_t) \geq U^B_{aut}(Y_t)$$

(4) Incentive compatibility of the whole allocation follows from the requirement of one shot incentive compatibility of each control in the definition of admisability. This claim rests on the fact that an unimprovable allocation (that is, an allocation for which there are no one shot deviations which are payoff improving) must be incentive compatible. Specifically, I will show that since, for all w_Q, there are no period zero deviations from $\sigma^c(w_Q), \sigma^I(w_Q)$ that improve the borrower's payoffs, there are no deviations initial t periods from $\sigma^c(w_Q), \sigma^I(w_Q)$ for any finite t that do so, and, because the payoff to the borrower is continuous in the product topology there are no infinite deviations either.

That there are no round zero deviations c', I' that are payoff improving for the borrower follows from condition (A) of admissability of the pair

$$(A(w_Q), U(w_Q))$$

and that for all Q_1

$$U(w_Q)(Q_1) = U^B(\sigma(w_Q)|_{Y^1})$$

Proceed by induction. Assume that for all w_Q, there are no payoff improving deviations from $\sigma^c(w_Q), \sigma^I(w_Q)$ in the initial t periods. After any realization of Y^1, by the construction of $\sigma(w_Q)$ and the inductive hypothesis, the continuation allocation

$$\sigma^B(w_Q)|_{Y^1} = \sigma^B(w_{Q^1})$$

also has the property that there are no payoff improving deviations in the first t periods. so there are thus no payoff improving deviations from $\sigma(w_Q)$ in the initial $t+1$ rounds of play.

Finally, since the set of feasible payoffs for the borrower is bounded, then the maximum gain from deviations in the tail is bounded and must go to zero, as $\delta^t \to 0$ as $t \to \infty$. Thus there are no deviations changing actions in an infinite number of rounds of play that are payoff improving if there are no such deviations in the initial t rounds of play for all $t \geq 1$.

PROPOSITION 2: (FACTORIZATION) $Z(Q) = B(Z, Q)$ for all Q.

PROOF: We show that the correspondence Z is self–generating which by proposition 1 gives us the result.

Let $z_Q \in Z(Q)$ be a payoff supported by an allocation that satisfies (1)-(4) and let $\sigma(z_Q)$ be the allocation which supports it. Define $(A(z_Q), U(z_Q))$ as follows: Let

$$A(z_Q) = \sigma_0(z_Q)$$

and

$$U(z_Q)(Q_1) = U^B(\sigma(z_Q)|_{Y^1})$$

Conditions (A)-(E) of admissability are immediately satisfied as a consequence of the fact that $\sigma(z_Q)$ satisfies (1)-(4).

5. The Constrained Optimal Payoff

The search for the constrained optimal payoff for the borrower is not as yet much simplified by the analysis of the previous section. If we must search over the entire space of continuation value functions afresh each time we search for the optimal payoff for the borrower that can be sustained at a new value of the state variable, then the problem remains quite difficult. In this section I show that if the constrained optimal payoff for the borrower as a function of the state variable exists and is continuous in the state variable,

22

then it is necessarily the solution to a straighforward dynamic program. The continuation value function assigns to every state the constrained optimal payoff that can be attained in that state. The debt contract provides incentives for investment through choice of the repayments schedule to determine the distribution of the state variable. Then I show that if the constrained optimum value function is differentiable, and the expected value of repayments under the optimal repayments schedule is increasing in investment, and the ratio $\frac{g_I(Y';I)}{g(Y';I)}$ at $Y' = Y_{min}$ is sufficiently small, then there necessarily are debt crises under the optimal contract. If the debt repayments schedule is monotonic in output, as it is under risk sharing, then its expected value is increasing in investment because the distribution of output as a function of investment can be ordered according to first order stochastic dominance. The likelihood ratio of the example family of distributions of output given at the end of section 2 has $\frac{g_I(Y';I)}{g(Y';I)} = -\infty$ at $Y' = Y_{min}$.

The correspondence Z denotes for each value of the state the set of payoffs for the borrower that can be supported by allocations in the constraint set. Assume that Z is compact valued. Let $V(Q) = maxZ(Q)$ denote the constrained optimal payoff for the borrower for initial value of the state Q. $V(Q)$ is clearly monotonic. Assume that $V(Q)$ is continuous. Because of our assumption that the family of densities of the distribution of output given a fixed level of investment satisfies the monotone likelihood ratio property and the convexity of the distribution function condition, we can summarize the incentive compatibility constraints on investment in the static problem by the first order condition:[4]

$$(1 - \delta)u'(Q + b - I) = \delta \int_{Y' \in Y} U(Q')g_I(Y'; I)dY'$$

We have seen from the previous section that $V(Q)$ is characterized by the program:

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[4] The validity of the first order approach in principle agent problems is discussed in a number of papers including Rogerson [1985].

$$V(Q) = \max_{U(Q'),b,d'(Y'),I} (1-\delta)u(Q+b-I) + \delta \int_{Y' \in Y} U(Y' - d'(Y'))g(Y';I)dY' \qquad (P)$$

subject to:

$$\delta \int_{Y' \in Y} d'(Y')g(Y';I)dY' - b \geq 0 \qquad (5)$$

$$U(Y' - d'(Y')) - U_{aut}^{B}(Y') \geq 0 \; \forall Y' \qquad (6)$$

$$\delta \int_{Y' \in Y} U(Y' - d'(Y'))g_I(Y';I)dY' - (1-\delta)u'(Q+b-I) = 0 \qquad (7)$$

$$U(Y' - d'(Y')) \in Z(Y' - d'(Y')) \; \forall Y' \qquad (8)$$

That is, the optimal payoff for the borrower given any value of the state is found as a solution to a static maximization program in which we search over the space of pairs (A, U), denoting actions for the current period and continuation value functions, which are admissable with respect to the correspondence Z specifying payoffs for the borrower which can be supported by allocations in the constraint set at that value of the state Q.

In proposition 3 I show that the continuation value function at which that static program is solved is necessarily defined for every new value of the state by the solution of the same program at that new value of the state.

PROPOSITION 3: Any set of arguments which solves the program P satisfies

$$\hat{U}(Y' - \hat{d}'(Y')) = V(Y' - \hat{d}'(Y')) \quad a.e.$$

PROOF: The intuition behind this result is that if ever for any Y' a payoff equal to the continuation payoff $U(Q')$ can be obtained at a lower value of the state Q', then the borrower is left indifferent in continuation between making the repayment $d'(Y')$ and receiving the continuation payoff $U(Y' - d'(Y'))$ and a making a larger repayment and receiving the same continuation payoff at that lower value of the state. By revising the continuation value function and the repayments schedule, we can relax constraint (5) above

24

by increasing the expected value of repayments to the original lender and thus strictly improve the borrower's payoff.

Assume there exists a set of positive measure $\tilde{\mathbf{Y}}$ such that for $Y' \in \tilde{\mathbf{Y}}$,

$$\hat{U}(Y' - \hat{d}'(Y')) < V(Y' - \hat{d}'(Y'))$$

Then we can construct a new continuation value function $\tilde{U}$: $\tilde{U}(Q') = V(Q')$ for $Q' + \hat{d}'(Y') \in \tilde{\mathbf{Y}}$, and equal to $\hat{U}$ otherwise. There exists a new repayments schedule $\tilde{d}'(Y') > \hat{d}'(Y')$ for $Y' \in \tilde{\mathbf{Y}}$ that and equal to the old repayments schedule otherwise that equates

$$\tilde{U}(Y' - \tilde{d}'(Y')) = \hat{U}(Y' - \hat{d}'(Y'))$$

for all Y'. Constraints (6),(7),(8) of the program above all remain satisfied, and we can relax constraint (5) because the expected value of $\tilde{d}'(Y')$ is strictly greater than that of $\hat{d}'(Y')$. This is a contradiction. Therefore, $\hat{U}(Q') = V(Q')$ almost everywhere.

Propostion 3 shows that the provision of incentives for the borrower to invest is achieved completely through the design of the repayment schedule and not through variation in the plan for lending as a function of the current state. This method of punishment differs from the method of punishing a borrower who defaults on his debt. We see in proposition 3 that the optimal punishment of a borrower for suspected deviations from the investment plan as indicated by certain realizations of output is achieved through high repayments, lowering the state so that $V(Y' - d'(Y')) = U_{aut}^B(Y')$. The threatened punishment for these realizations of output has the same value for the borrower as the threatened punishment for repudiation: permanent exclusion from the credit markets. But the continuation allocation which supports these punishments is not the same. Net repayments and gross lending are observed in the first case, while no gross transactions are observed in the second. This result can be seen as follows.

The problem of generating the autarkic payoff $U_{aut}^B(Y)$ through some continuation program of lending with payoffs to the borrower of $U(Q)$ is stated: find $(U(Q'), b, d'(Y'))$

25

such that

$$U_{aut}^B(Y) = \max_I (1-\delta)u(Q+b-I) + \delta\left(\int_{Y'\in \mathbf{Y}} U(Q')g(Y';I)dY'\right)$$

where the maximization is subject to the constraints that

$$\int_{Y'\in \mathbf{Y}} d'(Y')g(Y';I)dY' \geq b$$

and for all Y'

$$U(Y' - d'(Y')) \geq U_{aut}^B(Y')$$

$$U(Y' - d'(Y')) \in Z(Y' - d'(Y'))$$

It is clear that if the borrower repudiates repayment $d(Y)$ to the old lender, so that $Q = Y$, then the only way to generate a payoff equal to $U_{aut}^B(Y)$ is to set lending $b = 0$ and continuation payoffs

$$U(Y' - d'(Y')) = U_{aut}^B(Y')$$

almost everywhere. This is because $U_{aut}^B(Y)$ is originally defined by the optimality equation:

$$U_{aut}^B(Q) = \max_I (1-\delta)u(Q-I) + \delta\left(\int_{Y'\in \mathbf{Y}} U_{aut}^B(Y')g(Y';I)dY'\right)$$

Therefore, the only way to punish the lender optimally after repudiation is permanently to deny him access to the credit markets.

Proposition 3 implies that we can write our program (P) as a dynamic program:

$$V(Q) = \max_{b,d'(Y'),I} (1-\delta)u(Q+b-I) + \delta\int_{Y'\in \mathbf{Y}} V(Y' - d'(Y'))g(Y';I)dY' \qquad (P^*)$$

subject to:

$$\delta\int_{Y'\in \mathbf{Y}} d'(Y')g(Y';I)dY' - b \geq 0 \qquad (5)$$

$$V(Y' - d'(Y')) - U_{aut}^B(Y') \geq 0 \;\forall Y' \qquad (6)$$

$$\delta\int_{Y'\in \mathbf{Y}} V(Y' - d'(Y'))g_I(Y';I)dY' - (1-\delta)u'(Q+b-I) = 0 \qquad (7)$$

Constraint (8) is no longer required.

Assume that the optimal repayments schedule satisfies

$$\delta \int_{Y' \in \mathbf{Y}} d'(Y') g_I(Y'; I) dY' > 0$$

that is, that the expected value of repayments is increasing in investment. This will tend to hold if there is risk sharing embodied in the repayments schedule, that is, if the repayments schedule is upward sloping. It will imply that there is necessarily a divergence of interest between the borrower and the lender over the level of investment at the optimal contract. Assume that $\frac{g_I(Y_{min}, I)}{g(Y_{min}; I)} = -\infty$. This assumption holds for the example family of CDF's given in section 2 above. We see that with these assumption we can replace the equality constraint (7) with an inequality constraint of the form:

$$\delta \int_{Y' \in \mathbf{Y}} V(Y' - d'(Y')) g_I(Y'; I) dY' - \delta u'(Q + b - I) \geq 0 \qquad (7')$$

This is because if (7') does not hold with equality, then one can increase the welfare of the borrower by increasing investment as it both increases the borrower's payoff directly, but also serves to relax constraint (5).

Write the Lagrangian associated with the program P^* substituting inequality constraint (7') for equality constraint (7):

$$(1-\delta)u(Q+b-I)+\delta \int_{Y' \in \mathbf{Y}} V(Y'-d'(Y'))(1+\lambda_2(Y')+\lambda_3 \frac{g_I(Y'; I)}{g(Y'; I)})+\lambda_1 d'(Y')g(Y'; I)dY'$$

$$-\lambda_1 b - \lambda_3(1-\delta)u'(Q+b-I)$$

where I have omitted the constants. We see that all the multipliers are nonnegative and λ_3 is positive. Take the first order condition with respect to $d'(Y')$

$$V'(Y' - d'(Y'))(1 + \lambda_2(Y') + \lambda_3 \frac{g_I(Y'; I)}{g(Y'; I)}) = \lambda_1$$

Because the likelihood ratio is large and negative for low values of Y', then $\lambda_2(Y')$ must be large and positive for those same values for there to be a $d'(Y')$ at which this first

27

order condition is satisfied. This implies that constraint (6) is binding for low Y' at the constrained optimum, so that $V(Y' - d'(Y')) = U_{aut}^B(Y')$ for low values of Y'. We see from the problem of generating the autarkic payoff using some continuation value function that is at least as large as the autarkic value function that to punish the borrower using the optimal lending program in continuation, the variable $Q + b$ output net of repayments plus new loans must be strictly less than gross output Y. This implies that the borrower must make a net transfer of capital to the lender when low income is realized. Thus we must observe a debt crises under the optimal lending program when low income is realized.

6. Solving for the Constrained Optimum Allocation (Forthcoming)

I solve this dynamic program in an example economy in section 6 (forthcoming). Define $\bar{Y}$ to be the smallest level of Y' for which the Lagrange multiplier $\lambda_2(Y')$ is strictly positive. We can interpret $\bar{Y}$ as the critical level of output below which debt crises begin. I am particularly interested in the behavior of the level $\bar{Y}$ as a function of the state. If the optimal $\bar{Y}$ is decreasing in the state Q, then there will be a persistence to debt crises. The first realization of Y below the current $\bar{Y}$ forces the state Q' down and thus increase the new level of $\bar{Y}$ which is applied, increasing the probability that a "low" realization of output will also occur next period. If the reverse is true, then debt crises might be seen as more transient occurrences.

7. Conclusion

We can immediately see, though, that the optimal lending program in this environment with moral hazard necessarily specifies that the borrower must make a net transfer of capital to the lenders when low output is realized. These results suggest that we should interpret the observation of debt crises as an essential feature of the proper functioning of international credit markets. If governments were to intervene in these markets to relieve the burden of debt crises on borrowing countries on a consistent basis, then the incentive structure

that supports international lending in the first place would be undermined. On the other hand, if international agencies such as the IMF can assist in monitoring the investment and consumption decisions that are being made in the major borrowing countries, then their intervention can reduce the burden of debt crises by improving the information private lenders have about debtor country behavior.

8. Appendix

For the sake of completeness, I present the full information, full enforcement pareto problem in this section and solve this problem for the optimal allocation. This result is intended to establish a benchmark by which the effects of certain impediments to contracting may be judged. In particular, we will see in the next section that the optimal allocation when there is a risk of repudiation leads to similar consumption smoothing in the borrowing country in finite time. The pareto problem stated here is that of maximizing the welfare of the borrower subject to the constraints that the allocation be feasible and that it provide the lenders with at least their reservation utility.

I define an allocation to be Fully Pareto Optimal (FPO) if it is a solution to the problem:

$$Max_{(c,I,b,d)} U^B(c, I, b, d) \qquad (FPO)$$

subject to (c, I, b, d) feasible and individually rational.

In the full enforcement, full information pareto optimum allocation, the risk neutral lender rents the investment technology from the risk averse borrower and operates it at the rate that equates the marginal rate of transformation to his marginal rate of substitution. The rental payment is the expected value of output less investment when the technology is operated at the specified investment level. This result is presented as a lemma:

LEMMA 2.1: The allocation

$$c_t(Q^t) = (1 - \delta)Y_0 + \delta\left(\int_{Y' \in Y} Y'g(Y', I^*)dY'\right) - I^*$$

$$I_t(Q^t) = I^*$$

$$b_t(Q^t) = \delta\left(\int_{Y' \in Y} Y'g(Y', I^*)dY' - Y_0\right)$$

$$d_{t+1}(Y') = Y' - Y_0$$

is the full enforcement, full information pareto optimum allocation.

30

PROOF: Denote the solution to this problem with $Y_0 = Y$ by $U_{FPO}(Y)$. Set up the Bellman's equation with state variable $Q = Y - d$ defined as output net of the consumption of the old lender. $U_{FPO}(Q)$ is defined recursively by the principle of optimality:

$$U_{FPO}(Q) = \max_{(d'(Y'),I)} u(Q + \delta \int_{Y' \in \mathbf{Y}} d'(Y')g(Y';I)dY' - I)+$$
$$\delta \int_{Y' \in \mathbf{Y}} U_{FPO}(Y' - d'(Y'))g(Y';I)dY'$$

I have used the feasibility constraint to substitute out for the consumption of the borrower and I have used the condition that the allocation to the lender be individually rational to specify the consumption of the lender t while young in terms of the expected value of his consumption while old, where the expectation is taken with contingent on the realization of Y^t.

The first order conditions with respect to each $d'(Y')$:

$$u'(Q + \delta \int_{Y' \in \mathbf{Y}} d'(Y')g(Y';I)dY' - I) = U'_{FPO}(Y' - d'(Y')) \quad for \ all \ Y' \in \mathbf{Y}$$

imply that $Y' - d'(Y')$ is constant (by the concavity of U_{FPO}) so that the first order condition with respect to I simplifies to:

$$\int_{Y' \in \mathbf{Y}} Y'g_I(Y';I)dY' = \frac{1}{\delta}$$

so $I_t(Q^t) = I^*$. The envelope condition at the optimal controls is written:

$$U'_{FPO}(Q) = u'(Q + \delta \int_{Y' \in \mathbf{Y}} d'(Y')g(Y',I^*)dY' - I^*)$$

Together these imply that the state variable Q is constant through time and must equal $Q_0 = Y_0$ (its initial value) so $d'(Y') = Y' - Y_0$ and consumption each period is

$$c^* = (1 - \delta)Y_0 + \delta \int_{Y' \in \mathbf{Y}} Y'g(Y',I^*)dY' - I^*$$

Note that I have asumed that the optimal level of investment I^* is sufficiently small to ensure that the consumption level c^* is positive. A sufficient condition for this assumption is that $I^* < Y_{min}$.

I now state the pareto problem when allocations are constrained by the risk of repudiation: An allocation is Repudiation Constrained Pareto Optimum (RC) if it is a solution to the problem:

$$\max_{(c,I,b,d)} U^B(c,I,b,d) \qquad (RC)$$

subject to (c,I,b,d) feasible and immune from the threat of repudiation.

The following lemma shows that constraint placed on feasible allocations by the threat of repudiation is not enough to explain the long run recurrence of debt crises. The optimal lending contract leads to complete consumption smoothing in finite time. Although borrowing (b_t) is initially restricted, when the distribution of income given investment is stationary, in finite time, the borrower can establish a sufficiently large deposit with the lenders so as to induce him not to repudiate any contracted payments to the lenders for fear of losing his deposit. It remains to be seen if this result is robust in an economy in which the persistence of the capital stock allows for growth, which would make it expensive for the borrowing country to maintain any deposits. This result is due to Worrall [1987].

LEMMA 2.1: Let $\mathbf{Y} = \{Y^1, ..., Y^K\}$ the support of Y be finite. Let $g(Y'; I)$ denote the probability of realization of output Y' given investment I. The repudiation constrained Pareto optimum allocation has

$$c_t(Q^t) = c^* \;\; constant$$

$$I_t(Q^t) = I^*$$

$$b_t(Q^t) = \delta\Big(\sum_{Y' \in \mathbf{Y}} Y' g(Y', I^*) - Y^* \Big)$$
$$d_t(Y') = Y' - Y^*$$

after the first realization of $Y' = Y^K$, which occurs in finite time.

PROOF: Construct a Lagrangean for the pareto problem with multipliers on the individual rationality constraints that hold for every contingency Y^t, $t \geq 1$: $\{\lambda(Y^t)\}$. The

solution to program (RC) is defined recursively by the optimality equation with state variable $Q = Y - d$ for the Lagrangean:

$$U_{RC}(Q) = \max_{(d'(Y'), I, \lambda(Y'))} u(Q + \delta \sum_{Y' \in Y} d'(Y')g(Y'; I) - I) +$$

$$\delta \sum_{Y' \in Y} U_{RC}(Y' - d'(Y'))g(Y'; I)$$

$$+ \sum_{Y' \in Y} \lambda(Y')(U_{RC}(Y' - d'(Y')) - U^1_{aut}(Y'))$$

The individual rationality constraint for the lenders has been used to substitute out for b. The optimality equation has first order conditions:

$$\delta u'(c) = (\delta + \frac{\lambda(Y')}{g(Y'; I)})U'_{RC}(Y' - d'(Y')) \qquad (d'(Y'))$$

$$u'(...)(\delta \sum_{Y' \in Y} d'(Y')g_I(Y'; I) - 1) + \delta \sum_{Y' \in Y} U_{RC}(Y' - d'(Y'))g_I(Y'; I) \qquad (I)$$

$$\lambda(Y')(U_{RC}(Y' - d'(Y')) - U^1_{aut}(Y')) = 0 \qquad (\lambda(Y'))$$

and the envelope condition:

$$U'_{RC}(Q) = u'(c)$$

We demonstrate the result in two steps:

Step 1: There is a critical value $c(Q)$ such that $\lambda(Y^k) = 0$ and $Y^k - d'(Y^k)$ constant for $k \leq c(Q)$ and $Y^k - d'(Y^k)$ is strictly increasing in k for $c < k \leq K$.

Let $\lambda(Y^{c+1})$ be the smallest non-zero lagrange multiplier. The first order condition for $d'(Y')$ and the strict concavity of U_{RC} imply that for all states Y' such that $\lambda(Y') = 0$, $Q' = Y' - d'$ is constant. With $\lambda(Y^{c+1}) > 0$ by the first order condition on λ we have $U_{RC}(Y^{c+1} - d'(Y^{c+1})) = U^B_{aut}(Y^{c+1})$. Since $U^B_{aut}(Y)$ is strictly increasing in Y, we have $U_{RC}(Y^k - d'(Y^k)) > U_{RC}(Y^{c+1} - d'(Y^{c+1}))$ for $c + 1 < k \leq K$.

Step 2: Under the optimal control, the state (Q) and the critical value $c(Q)$ increase over time until in finite time $c = K$. When $c = K$ all the $\lambda(Y')$ are zero and Q and consumption and investment are completely stabilized.

33

Let $c(Q_t)$ be the critical value at t and Y^s be the realization of output at $t+1$. By the first order and envelope conditions,

$$\delta U'_{RC}(Q_t) = (\delta + \frac{\lambda(Y')}{g(Y'; I)})U'_{RC}(Y' - d'(Y')) \qquad (*)$$

so if $s \leq c(Q_t)$, $Y^s - d'(Y^s) = Y^c - d'(Y^c)$ and if $s > c$, $Y^s - d'(Y^s) > Y^c - d'(Y^c)$.

Let $m = max(c, s)$. Then we will show $c(Y^s - d(Y^s)) \geq m$. By $(*)$ and the fact that $\lambda(Y') \geq 0$, we have $Y' - d'(Y') \geq Y^m - d(Y^m)$ for all $Y' \in \mathbf{Y}$. For all $k < m$,

$$U_{RC}(Y^k - d'(Y^k)) \geq U_{RC}(Y^m - d'(Y^m)) \geq U^B_{aut}(Y^m) > U^B_{aut}(Y^k)$$

and thus $\lambda(Y^k) = 0$. Since gross income Y^K must be realized in finite time, complete net income Q smoothing must also be achieved in finite time. As $d'(Y')$ and I are markov in the state Q, complete consumption smoothing must also be achieved in finite time. Note that when complete consumption smoothing is achieved $I = I^*$ for the remainder of the optimal control. The long run value of consumption c^* is generally different from the level in the fully pareto optimal allocation. See T. Worrell [1987] for discussion of its level.

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