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FINANCIAL DEVELOPMENT, GROWTH, AND THE DISTRIBUTION OF INCOME

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ABSTRACT

A paradigm is presented where both the extent of financial intermediation and the rate of economic growth are endogenously determined. Financial intermediation promotes growth because it allows a higher rate of return to be earned on capital, and growth in turn provides the means to implement costly financial structures. Thus, financial intermediation and economic growth are inextricably linked in accord with the Goldsmith–McKinnon–Shaw view on economic development. The model also generates a development cycle reminiscent of the Kuznets hypothesis. In particular, in the transition from a primitive slow–growing economy to a developed fast–growing one, a nation passes through a stage where the distribution of wealth across the rich and poor widens.

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I. Introduction

Two themes pervade the growth and development literature. The first is Kuznets' (1955) hypothesis on the relationship between economic growth and the distribution of income. On the basis of somewhat slender evidence, Kuznets (1955) cautiously offered the proposition that during the course of an economy's lifetime, income inequality rises during the childhood stage of development, tapers off during the juvenile stage, and finally declines as adulthood is reached. *While far from being incontrovertible*, other researchers have found evidence in support of this hypothesis. For example, Lindert and Williamson (1985) suggest that "British experience since 1688 looks like an excellent advertisement for the Kuznets Curve, with income inequality rising across the Industrial Revolution, followed by a prolonged leveling in the last quarter of the nineteenth century" (p. 344). Using cross–country data, Paukert (1973) finds evidence of intra–country income inequality rising and then declining with economic development. Finally, inter–country inequality is examined by Summers, Kravis, and Heston (1984). They discover that income inequality fell sharply across industrialized countries from 1950 to 1980, declined somewhat for middle income ones, and rose slightly for low income nations.[1] Of related interest is their finding that between 1950 and 1980 real per capita income grew at about half the rate for low income countries as it did for high and middle income nations.

The second major strand of thought prevalent in the growth and development literature, often associated with the work of Goldsmith (1968), McKinnon (1973), and Shaw (1973), stresses the connection between "a country's financial superstructure and its real infrastructure." Simply put by Goldsmith (1968), the financial superstructure of an economy "accelerates economic growth and improves economic performance to the extent that it facilitates the migration of funds to the best user, i.e., to the place in the economic system where the funds will yield the highest social return" (p. 400). Further evidence, *again not decisive*, establishes a link between financial structure and economic development. For instance, Goldsmith (1968) presents data showing a well-defined upward secular drift in the ratio of financial institutions' assets to GNP for both developed and less developed countries for the 1860–1963 period. As he notes, though, it is difficult to establish "with confidence the direction of the causal mechanism, i.e., of deciding whether financial factors were responsible for the acceleration of economic development or whether financial development reflected economic growth whose mainsprings must be sought elsewhere" (p. 48). And indeed Jung (1986) provides postwar econometric evidence for a group of 56 countries of causality (in the Granger sense) running in either and both ways. Finally, historical case studies such as those undertaken in Cameron (1967) have stressed the key importance of financial factors in the economic development of several European countries.

The current analysis focuses on economic growth, institutional development, and the distribution of income. Economic growth fosters investment in organizational capital which in turn promotes further growth. In the model, institutions arise endogenously to facilitate trade in the economy, and they do so in two ways: First, trading organizations allow for a higher expected rate of return on investment to be earned. In particular, in the environment modeled, information is valuable since it allows investors to learn about the aggregate state of technology. Through a research-type process, intermediaries collect and analyze information that allows investors' resources to flow to their most profitable use. By investing through an intermediary, individuals gain access, so to speak, to a wealth of experience of others. While Boyd and Prescott (1986) also stress the role that intermediaries can play in overcoming information frictions, the nature of these frictions is different. Second, trading organizations also play the traditional role of pooling risks across large numbers of investors. Townsend (1978) highlights the insurance role of intermediaries, but not their role in allowing a more efficient allocation of resources for production. Thus, by investing through intermediated structures individuals obtain both a higher and safer return.

As in Townsend (1978, 1983a), investment in organizational capital is costly. Consequently, high income economies are better disposed to undertake such financial superstructure building than are ones with low income levels. The development of financial superstructure, since it allows a higher return to be earned on capital investment, in turn feeds back on economic growth and income levels. In this latter regard, the current analysis is a close cousin of Townsend (1983b) which also examines the relationship between financial structure and economic activity, although within the context of a framework where the extent of financial markets is exogenously imposed and that abstracts from the issue of growth. Also, in the spirit of recent work by Lucas (1988), Rebelo (1987), and Romer (1986), growth is modeled as an endogenous process, i.e., it does not depend on exogenous technological change.

The dynamics of the development process resemble the Kuznets (1955) hypothesis. In the early stages of development an economy's financial markets are virtually nonexistent and it grows slowly. Financial superstructure begins to form as the economy approaches the intermediate stage of the growth cycle. Here the economy's growth and savings rates both increase, and the distribution of income across the rich and poor widens. By maturity, the economy has developed an extensive structure for financial intermediation. In the final stage of development the distribution of income across agents stabilizes, the savings rate falls, and the economy's growth rate converges (although perhaps nonmonotonically) to a higher level than that prevailing during its infancy. According to Lindert and Williamson (1986), "it is exactly this kind of correlation—rising inequality coinciding with rising savings and accumulation rates during Industrial Revolutions—that encouraged the trade-off belief (between growth and inequality) among classical economists who developed their growth models while the process was underway in England" (pp. 342–43, material in parenthesis added).

II. The Economic Environment

Consider an economy populated by a continuum of agents distributed over the interval [0,1] with Lebesgue measure λ. An agent's goal in life is to maximize his expected lifetime utility as given by

$$E_0\left[\sum_{t=0}^{\infty}\beta^t \ln c_t\right] \quad \text{with } 0 < \beta < 1,$$

where c_t is his period–t consumption flow and β the discount factor.

Each agent is entitled to operate one or both of two linear production technologies. The first offers a safe but relatively low return on investment. Here i_{t-1} units of capital invested at the end of period t – 1 yields δi_{t-1} units of output in period t, or y_t. Thus, more formally

$$y_t = \delta i_{t-1},$$

where δ is a technological constant. The second investment opportunity yields a higher (unconditional) expected return but is more risky. Specifically, with this technology production is governed by the following process:

$$y_t = (\theta_t + \epsilon_t) i_{t-1},$$

where $(\theta_t + \epsilon_t)$ represents a *composite* technology shock. Each technology can only be operated once by the individual in a period. Now, at the beginning of each period t an agent will have a certain amount of wealth, k_t, at his disposal. This wealth can be used either for current consumption or it can be invested in capital for use in production next period. Individuals are heterogeneous in the sense that their stocks of capital in any given period may differ. At the start of time, each agent is endowed with a certain amount of goods or capital, k_0. The initial distribution of wealth in the society is represented by the cumulative distribution function $\hat{H}_0 \colon \mathbb{R}_{++} \to [0,1]$.

The period–t technological shock has two components. The first component, θ_t, represents an aggregate disturbance and thus is common across technologies while the second, ϵ_t, portrays an individual (or project) specific shock. All that an agent can costlessly observe is the *realized* composite rate of return $(\theta_t+\epsilon_t)$ on his *own* project. The stochastic structure of the economic environment will be delimited in the following way:

(A) The aggregate shock θ_t is governed by the time–invariant distribution function $F(\theta_t)$. Let $\Theta = [\underline{\theta},\overline{\theta}] \subset \mathbb{R}_{++}$ and $F\colon \Theta \to [0,1]$. Furthermore, suppose that $E[\ell n(\phi\theta + (1-\phi)\delta)] = \int[\ell n(\phi\theta + (1-\phi)\delta)]dF(\theta) > \ell n\delta > -\ell n\beta$ for all $\phi \in [0,1]$; by Jensen's inequality this implies $E[\theta] > \delta > 1/\beta$.

(B) For each individual $j \in [0,1]$ the idiosyncratic shocks $\epsilon_t(j)$ are drawn from the distribution function $G(\epsilon_t(j))$. Let $\Lambda = [\underline{\epsilon},\overline{\epsilon}] \subset \mathbb{R}$ and $G\colon \Lambda\to [0,1]$. Additionally, assume that $E[\epsilon] = \int \epsilon\, dG(\epsilon) = 0$ and $\underline{\theta} + \underline{\epsilon} > 0$.

Following Townsend (1978, 1983a), it will be assumed that trading arrangements are costly to establish. Given that setting up organizational structures is costly, institution formation will be economized on. Imagine some collections of agents forming a coalition among themselves to collect and process information, coordinate production activity, and spread risk across projects. Specifically, let A denote the set of $j \in [0,1]$ constituting the intermediary structure. First, it will be assumed that there is a once–and–for–all lump sum cost of α associated with incorporating each agent j into the trading syndicate.[2] Thus, on this account the total fixed cost associated with building the trading network would be $\alpha\int_A d\lambda(j)$. Second, suppose that each period there are costs incurred in proportion $(1-\gamma)$ to the amount of funds each agent invests in the syndicate. Consequently, if in a given period agent j invests $i(j)$ units of capital in the co–operative, the total variable cost associated with running the financial structure would be $(1-\gamma)\int_A i(j)d\lambda(j)$. Clearly, if trading arrangements are ever to emerge, these proportional costs cannot be too high. To ensure that in the subsequent analysis they will not be prohibitively large the following assumption is made:

(C) Let γ, δ, and $F(\cdot)$ be specified such that

$$\int \theta dF(\theta) < \int \gamma \max(\delta,\theta) dF(\theta).$$

Note that this assumption implies the random variable $\gamma\max(\delta,\theta)$ stochastically dominates θ in the second-order sense, and is automatically satisfied when $\gamma = 1$ (no proportional transactions costs).[3]

The potential benefits from establishing networks are threefold. First, information has a public good aspect to it. Each entrepreneur desires information on the realized project returns of others. This would allow his production decisions to be better made since such realized returns contain useful information about the magnitude of the aggregate shock. Even if such information was public knowledge no individual entrepreneur would want to produce first since by waiting he would gain the experience of others. Thus, there is a coordination problem inherent in individual entrepreneurs' production planning which trading agreements may be able to overcome. Second, trading mechanisms could potentially be used to diversify away the idiosyncratic risk associated with individual production projects. Third, they may allow an agent better opportunities for transferring consumption across time through arrangements for borrowing and lending. The emergence of such trading arrangements is the subject of the next section.

III. Competitive Equilibrium

Financial Intermediation

Many organizational structures can be decentralized with a subset of agents acting as go-betweens who intermediate economic activity for some larger set of individuals. They charge competitively determined fees for this service. Suppose that in period $t-1$ some individual in the economy has assumed (at a cost of α) the role of being an intermediary for a set of agents A_{t-1} with positive measure. This go–between offers the following service: In exchange for a once–and–for–all fee of q, plus the rights to operate an individual's project, the intermediary promises a return of $r(\theta_{t+j})$ per unit of capital invested in any period $t + j - 1$, with the go–between absorbing all costs associated with trading. Needless to say, since the go–between's goal is to maximize profits, he will adopt the most efficient scheme possible for intermediation. In pursuit of this end, let the intermediary follow in every period the investment plan outlined below for period t.

To begin with, suppose person j invests $i_{t-1}(j)$ units of capital with the intermediary at the end of period $t - 1$. Then the aggregate amount of capital (net of the proportional transactions costs) that the intermediary has to invest in t from these deposits is $\gamma \int_{A_{t-1}} i_{t-1}(j) d\lambda(j)$, where again λ is Lebesgue measure. Now, let the intermediary randomly select some finite number of high risk/return projects, say τ, from the set A_{t-1}; denote this set of projects by A^e_{t-1}. Each of the "trial" projects selected are funded with the amount $\gamma K_t = [\gamma \int_{A_{t-1}} i_{t-1}(j) d\lambda(j)]/[\int_{A_{t-1}} d\lambda(j)]$. The intermediary then calculates the average net realized rate of return, $\hat{\theta}_{t\tau}$, on these projects where formally[4]

$$\hat{\theta}_{t\tau} = \frac{\gamma}{\tau}\left[\theta_t \tau + \sum_{m=1}^{\tau} \epsilon_{tm}\right].$$

Now, if the "test statistic" $\hat{\theta}_{t\tau}$ is greater than $\gamma\delta$, then the remaining high risk/return projects operated by the intermediary are each funded with γK_t units of capital, otherwise the go–between invests its resources in safe projects.[5]

Note that relative to the size of intermediary's portfolio of projects, the number of production technologies chosen for research purposes is negligible. More precisely, the set of experimental projects, A^e_{t-1}, being countable has (Lebesgue) measure zero. Consequently, other than the important informational role these test projects play, they have a negligible impact on the profits earned by the intermediary. Thus, the net rate of return on the intermediary's production activities, or $z(\theta_t,\hat{\theta}_{t\tau})$, will be given by

$$z(\theta_t,\hat{\theta}_{t\tau}) = \begin{cases} \left\{\gamma\int_{A_{t-1}-A^e_{t-1}}[\theta_t+\epsilon_t(j)]d\lambda(j)+\hat{\theta}_{t\tau}\int_{A^e_{t-1}} d\lambda(j)\right\}/\left[\int_{A_{t-1}} d\lambda(j)\right] = \gamma\theta_t, \\ \text{if } \hat{\theta}_{t\tau} > \gamma\delta, \text{ or} \\ \left\{\gamma\int_{A_{t-1}-A^e_{t-1}} d\lambda(j) + \hat{\theta}_{t\tau}\int_{A^e_{t-1}} d\lambda(j)\right\}/\left[\int_{A_{t-1}} d\lambda(j)\right] = \gamma\delta, \\ \text{if } \hat{\theta}_{t\tau} \leq \gamma\delta. \end{cases}$$

The following lemma can now be stated:

<u>Lemma 1</u>: As $\tau \to \infty$,

$$z(\theta_t,\hat{\theta}_{t\tau}) \overset{a.s.}{\to} \gamma\max(\delta,\theta_t).$$

<u>Proof</u>: For $x \in (-\gamma\delta,\infty)$, let $I(x) = 1$ if $x > 0$ and $I(x) = 0$ otherwise. Then $z(\theta_t,\hat{\theta}_{t\tau})$ can be expressed as

$$z(\theta_t,\hat{\theta}_{t\tau}) = I(\hat{\theta}_{t\tau}-\gamma\delta)\gamma\theta_t + \left[1 - I(\hat{\theta}_{t\tau}-\gamma\delta)\right]\gamma\delta.$$

Clearly, if $\theta_t = \delta$ then $z(\theta_t,\hat{\theta}_{t\tau}) = \gamma\delta$, *regardless* of the value of $\hat{\theta}_{t\tau}$. Therefore trivially here $z(\theta_t,\hat{\theta}_{t\tau}) = \gamma\max(\delta,\theta_t)$. Suppose alternatively that $\theta_t \neq \delta$. Now as $\tau \to \infty$, $\hat{\theta}_{t\tau} \overset{a.s.}{\to} \gamma\theta_t$ by assumption (B) and the Strong Law of Large Numbers. This, though, implies that $I(\hat{\theta}_{t\tau}-\gamma\delta) \overset{a.s.}{\to} I(\theta_t-\gamma\delta)$, since $I(\cdot)$ is a continuous function on $(-\gamma\delta,0) \cup (0,\infty)$. Hence in the case where $\theta_t \neq \delta$, it follows that $z(\theta_t,\hat{\theta}_{t\tau}) \overset{a.s.}{\to} \gamma\max(\delta,\theta_t)$ as $\tau \to \infty$. □

In competitive equilibrium the profits realized from financial intermediation must be zero. This transpires since any agent in the economy (willing to incur the cost of

α) can establish himself as an intermediary. The zero profit condition for intermediation is

$$(1) \qquad [\gamma\max(\delta,\theta_t) - r(\theta_t)]\int_{A_{t-1}} i_{t-1}(j)d\lambda(j) + \gamma\max(\delta,\theta_t)[q - \alpha]\int_{A'_{t-1}} d\lambda(j) = 0,$$

where $A'_{t-1} \subseteq A_{t-1}$ represents the set of agents entering into an agreement with the go–between for the first time at $t - 1$. This condition necessitates that $r(\theta_t) = \gamma\max(\delta,\theta_t)$ and $q = \alpha$, since it must hold for arbitrary $\int_{A_{t-1}} i_{t-1}(j)d\lambda(j) \geq 0$ and $\int_{A'_{t-1}} d\lambda(j) \geq 0$.[6] Note that the intermediary offers agents a rate of return on their investments that is (i) completely devoid of idiosyncratic production risk and (ii) safeguarded from the potential losses that could occur when the aggregate return on the risky technology falls below the opportunity cost of the resources committed. Also, investors are only charged a lump–sum fee which exactly compensates the go–between for the once–and–for–all cost of establishing a business arrangement with them. Finally, in line with Goldsmith (1968), intermediaries allocate resources to the place in the economic system where they earn the highest return.

Discussion: The exact story of intermediation told is not crucial for the subsequent analysis. What is necessary is that intermediaries provide customers with a distribution of returns on their investments that is both preferred and has a higher mean. For instance, following Freeman (1986) it could simply be assumed that there exists a technology which yields a superior return on investment, but requires large minimum amounts of capital.[7] This nonconvexity in project size would provide a rationale for individuals to pool funds. Alternatively, financial intermediaries may arise to service the liquidity needs of agents. Specifically, along the lines of Diamond and Dybvig (1983), suppose that agents face two investment opportunities: an illiquid investment which yields a high rate of return, or a liquid one with a low yield. In a world with idiosyncratic risk agents may be reluctant to save substantial parts of their wealth in an illiquid asset for fear that they may need to use these funds before the investment matures. Large financial intermediaries can calculate the average demand for early withdrawal due to idiosyncratic events and adjust their investment portfolios to accommodate this better than an individual saver can. Bencivenga and Smith (1988) model the effect that intermediaries can have on an economy's growth rate by encouraging a switch in savings from unproductive liquid assets into productive illiquid ones. Other work stresses the role that intermediaries play in overcoming informational frictions. For example, Boyd and Prescott (1986) focus on financial intermediary coalitions as an incentive compatible mechanism for allocating resources to their most productive use in a world where borrowers have private information about the potential worthiness of their investment projects. In principle, their framework could be incorporated into a growth model. Finally, Diamond (1983) and Williamson (1986) stress the importance of large intermediary structures for minimizing the costs to lenders (depositors) of monitoring the behavior of both borrowers and intermediary managers.

The current paper stresses the role that intermediaries play in collecting and analyzing information, thereby facilitating the migration of funds to the place in the economy where they have the highest social return. The model of intermediation presented above could undoubtedly be generalized to capture reality better. For instance, industry specific shocks could be introduced. Suppose that the risky technology now operates in several sectors. Let the risky technology be formulated as $y_t = (\theta_t+\nu_t(\ell) + \epsilon_t(j))i_{t-1}$, where $\nu_t(\ell)$ is a disturbance specific to industry ℓ. Now through a sampling process analogous to that analyzed above, intermediaries could uncover $(\theta_t+\nu_t(\ell))$ for each industry ℓ. If the aggregate state of the economy warranted—i.e., if $(\theta_t + \nu_t(\ell)) > \gamma\delta$ for some ℓ—the funds available would be directed to the sector(s) with the highest ν_t. Otherwise, the resources would be invested in the safe technology (which would perhaps be better labeled in the current context as an "industry").

Market Participation

Not all agents may find the terms of the investment contract offered currently attractive. In particular, for some agents it may not be worthwhile now to pay a lump–sum fee of q in order to gain permanent access to the intermediation technology paying a random return of $r(\theta_t)$ in each t. Thus, it is natural at this point to examine the determination of participation in the exchange network. To do this, consider the decision–making of an individual in period t who is currently outside of the intermediated sector. His actions in this period are summarized by the outcome of the following dynamic–programming problem:

$$\text{(P1)}\qquad w(k_t) = \max_{s_t, \phi_t} \{\ln(k_t - s_t) + \beta \int \max[w(s_t(\phi_t(\theta_{t+1}+\epsilon_{t+1}) + (1-\phi_t)\delta)),$$

$$v(s_t(\phi_t(\theta_{t+1}+\epsilon_{t+1}) + (1-\phi_t)\delta) - q)]dF(\theta_{t+1})dG(\epsilon_{t+1})\},$$

where s_t is the agent's period–t saving level, ϕ_t the fraction of his portfolio invested in the high risk/return technology, and $v(s_t(\phi_t(\theta_{t+1}+\epsilon_{t+1}) + (1-\phi_t)\delta) - q)$ represents the expected lifetime utility the agent would realize in t+1 if he then entered the intermediated sector with $s_t(\phi_t(\theta_{t+1}+\epsilon_{t+1}) + (1-\phi_t)\delta) - q$ units of capital at his disposal.[8] It can be demonstrated that w is a continuous and increasing function for any function v sharing these properties; it will be uniquely determined as well—see Stokey and Lucas with Prescott (1989). Note that the above programming problem presumes that in t + 1 the agent will enter or remain outside of the intermediated sector depending upon which choice then yields the highest expected utility. Hence, $w(k_t)$ gives the maximum lifetime utility an individual with k_t units of capital can expect in period t if he chooses not to participate in the exchange network just then.

Likewise, the dynamic–programming problem for any agent currently within the intermediated sector is given by[9]

$$\text{(P2)} \qquad v(k_t) = \max_{s_t}\{\ell n(k_t - s_t) + \beta \int \max[w(s_t r(\theta_{t+1})), v(s_t r(\theta_{t+1}))] dF(\theta_{t+1})\}.$$

If w is a continuous and increasing function then v inherits these traits as well. Thus, (P1) and (P2) jointly define the pair of functions w and v. Specifically, consider the vector function (w,v). Then (P1) and (P2) define a mapping Ω such that $(w,v) = \Omega(w,v)$. It is easy to establish that the operator Ω is a contraction in the space of continuous vector functions with norm $\max[\sup_x |w(x)|, \sup_x |v(x)|]$, and consequently has a *unique* fixed point.

Presumably, in any period t a given endowment of capital, k_t, is worth more to an agent operating within the intermediated sector than to one outside of it; that is $v(k_t) > w(k_t)$. This should transpire since exchange with the go–between yields a better *distribution* of returns per unit of capital invested than autarky does. If this is so, then once an individual enters the intermediated sector he will never leave it. This conjecture will now be tested.

If it is true, the functional equation (P2) could be simplified to allow v to be defined without reference to w. Specifically, (P2) would now read

$$\text{(P3)} \qquad v(k_t) = \max_{s_t}\{\ell n(k_t - s_t) + \beta \int v(s_t r(\theta_{t+1})) dF(\theta_{t+1})\}.$$

Furthermore, given the logarithmic form of the utility function it is straightforward to establish that the value function $v(k_t)$ and the policy–rule $s_t = s(k_t)$ would have the following simple forms:

$$\text{(2)} \qquad v(k_t) = \frac{1}{1-\beta}\ell n(1-\beta) + \frac{\beta}{(1-\beta)^2}\ell n\, \beta + \frac{\beta}{(1-\beta)^2}\int \ell n\, r(\theta) dF(\theta) + \frac{1}{1-\beta}\ell n\, k_t$$

and

$$\text{(3)} \qquad s(k_t) = \beta k_t.$$

Thus, agents within the intermediated sector would save a constant fraction of their wealth each period. Given the above solution for v, problem (P1) then implies a solution for w. If it can be established that the implied solution for w is such that $w \leq v$, then the solution for (w,v) has been found. Toward this end, assume v is given by (2).

<u>Lemma 2</u>: $v(k) > w(k)$.

<u>Proof</u>: Let $w^{j+1} \equiv Tw^j$, where the operator T is defined by

$$Tw^j = \max_{s^j, \phi^j} \{\ell n(k-s^j) + \beta \int \max[w^j(s^j(\phi^j(\theta+\epsilon) + (1-\phi^j)\delta)),$$

$$v(s^j(\phi^j(\theta+\epsilon) + (1-\phi^j)\delta)-q)]dF(\theta)dG(\epsilon)\}.$$

Now consider the sequence of functions $\{w^j\}_{j=0}^{\infty}$. Denote the optimal policy functions associated with the above mapping by $\tilde{s}^j$ and $\tilde{\phi}^j$. The proof will proceed by induction. First, it will be demonstrated that if $w^j \leq v$ then $w^{j+1} < v$. Second, to start the induction hypothesis, a w^0 will be chosen such that $w^0 < v$. Thus, $w = \lim_{j\to\infty} w^j < v$.

Suppose $w^j \leq v$. Then

$$v - w^{j+1} \geq \{\ell n(k-\tilde{s}^j) + \beta \int v(\tilde{s}^j r(\theta))dF(\theta)\}$$

$$- \{\ell n(k-\tilde{s}^j) + \beta \int \max[w^j(\tilde{s}^j(\tilde{\phi}^j(\theta+\epsilon) + (1-\tilde{\phi}^j)\delta)),$$

$$v(\tilde{s}^j(\tilde{\phi}^j(\theta+\epsilon) + (1-\tilde{\phi}^j)\delta)-q)]dF(\theta)dG(\epsilon)\},$$

since the savings rule $\tilde{s}^j$ is suboptimal for the program (P3). Next, by the induction hypothesis, $w^j \leq v$ so that

$$v - w^{j+1} > \beta \int v(\tilde{s}^j r(\theta))dF(\theta) - \beta \int v(\tilde{s}^j(\tilde{\phi}^j(\theta+\epsilon) + (1-\tilde{\phi}^j)\delta))dF(\theta)dG(\epsilon).$$

This can be rewritten in light of (2) and Jensen's inequality as

$$v - w^{j+1} > \beta \int v(\tilde{s}^j r(\theta))dF(\theta) - \beta \int v(\tilde{s}^j(\tilde{\phi}^j\theta + (1-\tilde{\phi}^j)\delta))dF(\theta) \geq 0.$$

The nonnegative sign of the above expression obtains since the random variable $r(\theta) = \gamma\max(\delta,\theta)$ stochastically dominates θ in the second-order sense by assumption (C).[10] Finally, to start the induction hypothesis, let $w^0(k) = 1/(1-\beta)\ \ell n(1-\beta) + \beta/(1-\beta)^2\ \ell n\ \beta + 1/(1-\beta)\ \ell n\ k$. Then, $v(k) - w^0(k) = \beta\gamma/(1-\beta)^2 \int \ell n \max(\delta,\theta)dF(\theta) > 0$. □

The extent of participation in the exchange network is now easily characterized. Consider some arbitrary set of agents for whom it was not in their individual interests to engage in trade with the intermediary up until the current period t. (This set of agents could be all or none of the actors in the economy.) Each of these individuals must now decide on whether or not to join the market sector. Given that the cost of accessing the intermediary is lump–sum, it seems likely that agents with a capital stock falling below some minimal level $\underline{k} > 0$ will remain outside of the exchange network while those having an endowment exceeding some upper threshold level $\bar{k} \geq \underline{k}$ will join.

<u>Lemma 3</u>: There exist $\underline{k}$ and $\bar{k}$, with $0 < \underline{k} \leq \bar{k}$, such that

$$v(k_t-q) < w(k_t) \quad \text{for } 0 < k_t < \underline{k}, \quad \text{and } v(k_t-q) > w(k_t) \quad \text{for } k_t > \bar{k}.$$

<u>Proof</u>: Since both w(k) and v(k) are continuous functions in k, it is enough to demonstrate that (i) $\lim_{k\to q} [w(k) - v(k-q)] > 0$ and (ii) $\lim_{k\to\infty}[w(k) - v(k-q)] < 0$. To show (i), note on the one hand that from equation (2) $\lim_{k\to q} v(k-q) = -\infty$. On the other hand, though, it is feasible never to join the coalition and pursue the dynamic–program shown below:

$$\text{(P4)} \qquad w^o(k) = \max_{s,\phi}\{\ell n(k-s) + \beta\int w^o(s(\phi(\theta+\epsilon) + (1-\phi)\delta))dF(\theta)dG(\epsilon)\}.$$

It is easy to show that the value function $w^o(k)$ and the policy–rules $s = s(k)$ and $\phi = \phi(k)$ have the following simple forms:

(4) $$w^{o}(k) = \frac{1}{1-\beta}\ln(1-\beta) + \frac{\beta}{(1-\beta)^2}\ln\beta + \frac{\beta}{(1-\beta)^2}\int \ln(c(\theta+\epsilon) + (1-c)\delta)dF(\theta)dG(\epsilon) + \frac{1}{1-\beta}\ln k$$

(5) $s(k) = \beta k$ [cf. (2) and (3)]

and

(6) $\phi(k) = c > 0,$

with the constant $c \in (0,1]$ solving the condition $\int[\theta+\epsilon-\delta]/[c(\theta+\epsilon) + (1-c)\delta] dF(\theta)dG(\epsilon) \geq 0$.[11] Clearly, $w(k) \geq w^{o}(k) > -\infty$ [by assumptions (A) and (B)] for all $k > 0$.

To establish (ii) observe that equation (P1) and Lemma 2 imply

$$w(k) \leq \max_{s,\phi}\{\ln(k-s) + \beta\int v(s(\phi(\theta+\epsilon) + (1-\phi)\delta))dF(\theta)dG(\epsilon)\},$$

which together with (P3) yields that

$$w(k) - v(k-q) \leq \max_{s,\phi}\{\ln(k-s) + \beta\int v(s(\phi(\theta+\epsilon) + (1-\phi)\delta))dF(\theta)dG(\epsilon)\} - \max_{s}\{\ln(k-q-s) + \beta\int v(sr(\theta))dF(\theta)\}.$$

Next, given the logarithmic form of the value function, $v(\cdot)$, the first term in braces is maximized by setting $s = \beta k$ and $\phi = c$ [cf (5) and (6)]. These policy–rules are also feasible choices for the second term in braces providing that $k > q/(1-\beta)$. Thus, for $k > q/(1-\beta)$,

$$w(k) - v(k-q) \leq \ln(k(1-\beta)) + \beta\int v(\beta k(c(\theta+\epsilon) + (1-c)\delta))dF(\theta)dG(\epsilon) - \ln(k(1-\beta)-q) - \beta\int v(\beta k(cr(\theta) + (1-c)\delta))dF(\theta).$$

Since $v(\cdot)$ is concave, by Jensen's inequality

$$w(k) - v(k-q) \leq \ln[k(1-\beta)/(k(1-\beta) - q)]$$
$$+ \beta\int[v(\beta k(c\theta + (1-c)\delta)) - v(\beta k(cr(\theta) + (1-c)\delta))]dF(\theta).$$

Consequently, $\lim_{k\to\infty} [w(k) - v(k-q)] < 0$ since first, $\lim_{k\to\infty} \ln[k(1-\beta)/(k(1-\beta) - q)] = 0$ and second, $\beta\int[v(\beta k(c\theta + (1-c)\delta)) - v(\beta k(cr(\theta) + (1-c)\delta))]dF(\theta) < 0$ because $r(\theta)$ stochastically dominates θ by assumption (C) and the expression behind the integral sign does not depend on k. □

Remark: If $v(k-q) - w(k)$ is strictly increasing in k, then $\underline{k} = \overline{k}$. In general, though, this result doesn't appear to transpire.

Now define the sets B^c and B in the following manner:

(7) $$B^c = \{k_t: v(k_t-q) < w(k_t)\} \text{ and } B = \{k_t: v(k_t-q) \geq w(k_t)\}.$$

By Lemma 3, the sets B^c and B are nonempty. Also, $\underline{k} = \inf B$ and $\overline{k} = \sup B^c$. Clearly, it is in the interest of those individuals having a capital stock $k_t \in B$ to establish a trading link with the go-between, and likewise not so for those agents with an endowment $k_t \in B^c$. Equally as evident, it is possible to have a competitive equilibrium prevailing in period t where some agents choose to participate in the market sector and others pick to remain outside; this will depend on the distribution of capital across individuals who were outside of the trading network in $t - 1$.

Equilibrium

To summarize the discussion so far, it has been shown that there exists a competitive equilibrium of the form defined below:

Definition: A competitive equilibrium is a set of value functions, $v(k_t)$ and $w(k_t)$, savings rules, $s(k_t)$ and $\phi(k_t)$, and pricing functions, $r(\theta_{t+1})$ and q, such that:

(i) For agents participating in the market sector the functions $v(k_t)$ and $s(k_t)$ solve problem (P2), given $w(k_t)$, and $r(\theta_{t+1})$. Individuals choose to remain or not in this sector in period t depending on whether $v(k_t) \gtrless w(k_t)$. [It was demonstrated in Lemma 2 that $v(k_t) > w(k_t)$, which resulted in entry into the market sector being permanent. This lemma also implied that problem (P2) could be reduced to (P3) which has the solution $s(k_t) = \beta k_t$].

(ii) For individuals in the nonmarket sector the functions $w(k_t)$, $s(k_t)$ and $\phi(k_t)$ solve problem (P1), given $v(k_t)$, $r(\theta_{t+1})$ and q. Agents choose to transact or not with an intermediary in t depending on whether $v(k_t - q) \gtrless w(k_t)$. [Lemma 3 establishes the existence of two nonempty sets B and B^C such that for $k \in B$, $v(k-q) \geq w(k)$, while for $k \in B^C$, $w(k) > v(k-q)$. Thus, it is possible to have a competitive equilibrium where some agents participate in the market sector and others do not.]

(iii) All intermediaries earn zero profits in accord with (1), paying a rate of return $r(\theta_{t+1}) = \gamma\max(\delta, \theta_{t+1})$ and charging a membership fee $q = \alpha$. [Any intermediary with positive measure can effectively access the technology $y_{t+1} = \gamma\max(\delta, \theta_{t+1})i_t$, as was shown in Lemma 1.][12]

The following proposition concludes this section:

Proposition 1: The allocations generated by the above competitive equilibrium are Pareto-optimal.

Proof: See Appendix.

Remark: Note that in the competitive equilibrium modeled, members of the nonmarket sector can effectively borrow from intermediaries of the market rate of return of $r(\theta_{t+1})$ in order to finance entry into the market sector at time t; this opportunity affords no benefits, however.

IV. Savings, Growth, Development, and Income Distribution

Some of the model's predictions about savings, growth, development, and income distribution will now be presented. To begin with, it will be demonstrated that economies in phases of development where institutional infrastructure building is occurring will tend to have high rates of savings. This occurs since the construction of economic organization is expensive; specifically, it costs α to incorporate each individual into an institutional arrangement. Recall that those agents transacting in the intermediated sector save the amount $s_t = \beta k_t$. Individuals outside the trading network save in accord with the following dynamic-program [see (P1) and (7)]:

$$\text{(P5)} \qquad w(k_t) = \max_{s_t, \phi_t} \{\ln(k_t - s_t) + \beta\int_{D^c(s_t,\phi_t)} w(s_t(\phi_t(\theta_{t+1}+\epsilon_{t+1}) + (1-\phi_t)\delta))dF(\theta_{t+1})dG(\epsilon_{t+1}) + \beta\int_{D(s_t,\phi_t)} v(s_t(\phi_t(\theta_{t+1}+\epsilon_{t+1}) + (1-\phi_t)\delta) - q)dF(\theta_{t+1})dG(\epsilon_{t+1})\},$$

where $D^c(s_t,\phi_t) = \{(\theta_{t+1},\epsilon_{t+1})\colon s_t(\phi_t(\theta_{t+1}+\epsilon_{t+1}) + (1-\phi_t)\delta) \in B^c\}$ and $D(s_t,\phi_t) = \{(\theta_{t+1},\epsilon_{t+1})\colon s_t(\phi_t(\theta_{t+1}+\epsilon_{t+1}) + (1-\phi_t)\delta)\in B\}$. Now, denote the decision-rules governing optimal savings portfolio allocation in the above problem by $s_t = s(k_t)$ and $\phi_t = \phi(k_t)$. These individuals will save an amount $s(k_t)$ which is greater than βk_t, since they expect at some future date to incur the lump–sum cost q of developing a link with the exchange system.

Proposition 2: $s(k_t) > \beta k_t$.

Proof: The proof proceeds by induction. Consider the sequence of functions $\{w^j\}_{j=0}^{\infty}$ and $\{s^j\}_{j=0}^{\infty}$ generated from the mapping $w^j = Tw^{j-1}$, with the operator T defined by

$$(8) \qquad Tw^{j-1} = \max_{s^{j-1}} \Big\{ \ell n(k-s^{j-1}) + \beta\int_{D^c(s,\phi)} w^{j-1}(s^{j-1}(\phi(\theta+\epsilon)+(1-\phi)\delta))dF(\theta)dG(\epsilon)$$
$$+\beta\int_{D(s,\phi)} v(s^{j-1}(\phi(\theta+\epsilon)+(1-\phi)\delta)-q)dF(\theta)dG(\epsilon)\Big\}.$$

Observe that the mapping T depends upon the values for s and ϕ specified by (P5), that is, here s and ϕ are being taken as exogenously given constants invariant with the value of k in (8). *Given the fixity of the sets* $D^c(s,\phi)$ and $D(s,\phi)$, the operator T maps concave functions into strictly concave ones. The efficiency condition governing the optimal choice of s^{j-1} in the above mapping is shown below:

$$(9) \qquad \frac{1}{k-s^{j-1}} = \beta\int_{D^c(s,\phi)} (\phi(\theta+\epsilon)+(1-\phi)\delta)w_k^{j-1}(s^{j-1}(\phi(\theta+\epsilon)+(1-\phi)\delta))dF(\theta)dG(\epsilon)$$
$$+\beta\int_{D(s,\phi)}(\phi(\theta+\epsilon)+(1-\phi)\delta)v_k(s^{j-1}(\phi(\theta+\epsilon)+(1-\phi)\delta)-q)dF(\theta)dG(\epsilon).$$

It is easy to show that the operator T is a contraction whose fixed point defined by $w = Tw$ is characterized by (P5).[13] Thus given any initial function w^o, $\lim_{j\to\infty} w^j = w$ and $\lim_{j\to\infty} s^j = s$. Now, first it will be demonstrated that if $w_k^j > w_k^{j-1}$ then $s^j > s^{j-1}$ and $w_k^{j+1} > w_k^j$. Second, to start the induction hypothesis, a concave w^o will be chosen so that $w_k^1 > w_k^o$ and $s^o(k) > \beta k$. Consequently, $s(k) = \lim_{j\to\infty} s^j(k) > \beta k$ since $s^j(k)$ is an increasing sequence.

Assume that $w_k^j > w_k^{j-1}$. From (8), the first–order condition governing the optimal choice of s^j is

$$\text{(10)}\qquad \frac{1}{k - s^j} = \beta\int_{D^c(s,\phi)} (\phi(\theta+\epsilon) + (1-\phi)\delta) w_k^j(s^j(\phi(\theta+\epsilon) + (1-\phi)\delta)) dF(\theta)dG(\epsilon)$$
$$+ \beta\int_{D(s,\phi)} (\phi(\theta+\epsilon) + (1-\phi)\delta) v_k(s^j(\phi(\theta+\epsilon) + (1-\phi)\delta) - q) dF(\theta)dG(\epsilon).$$

By comparing (10) with (9), observe that if $s^j = s^{j-1}$ then the right–hand side of the above expression would exceed the left–hand side since $w_k^j > w_k^{j-1}$. To restore equality s^j must be increased, since the right–hand side is decreasing in s^j while the left–hand side is increasing given that w^j and v are both strictly concave. Next, note that by the envelope theorem

$$w_k^{j+1} = \frac{1}{k - s^j},$$

(recall that s and ϕ are being held constant). Thus, if $s^j > s^{j-1}$ then $w_k^{j+1} > w_k^j$.

Finally, let w^o be specified as in (4) and consequently be concave. Then using (2), (4), and (9) the efficiency condition governing the optimal choice of s^o can be written as

$$\frac{1}{k - s^o} = \beta\int_{D^c(s,\phi)} \frac{\phi(\theta+\epsilon) + (1-\phi)\delta}{(1-\beta)[s^o(\phi(\theta+\epsilon) + (1-\phi)\delta)]} dF(\theta)dG(\epsilon)$$
$$+ \beta\int_{D(s,\phi)} \frac{\phi(\theta+\epsilon) + (1-\phi)\delta}{[(1-\beta)[s^o(\phi(\theta+\epsilon) + (1-\phi)\delta) - q]} dF(\theta)dG(\epsilon).$$

It is easy to see that $s^o(k) > \beta k$, since when $s^o(k) = \beta k$ the right–hand side of this expression (which is decreasing in s^o) exceeds the left–hand side (which is increasing in s^o). Last, it immediately follows that $w_k^1 > w_k^o$ as

$$w_k^1 = \frac{1}{k - s^o} > \frac{1}{(1-\beta)k} = w_k^o. \square$$

Agents transacting with an intermediary save the amount $s_t = \beta k_t$ and earn a per unit rate of return of $r(\theta_{t+1}) = \gamma \max(\delta,\theta_{t+1})$ on this savings. Consequently, their wealth grows at the expected rate $\underset{t}{E}[k_{t+1}/k_t] = \beta\gamma\int \max(\delta,\theta_{t+1})dF(\theta_{t+1}) > 1$ [by assumptions (A) and (C)]. Individuals outside of the exchange network save $s_t = s(k_t) > \beta k_t$, earning a rate of return of $\phi(k_t)(\theta_{t+1}+\epsilon_{t+1}) + (1-\phi(k_t))\delta$. Thus, they accumulate wealth at the expected rate $\underset{t}{E}[k_{t+1}/k_t] = (s(k_t)/k_t)[\phi(k_t)\int \theta_{t+1}dF(\theta_{t+1}) + (1-\phi(k_t))\delta] > 1$ [by (A), and Proposition 2]. It's unclear whose wealth is growing faster on average. While on the one hand agents in autarky face an inferior distribution of returns on their investments, on the other they tend to save more.

It seems reasonable to suspect, though, that very poor agents have a low savings rate. That is, for the very poor $s(k_t) \simeq \beta k_t$. If so, then poor individuals will accumulate wealth at approximately the expected rate $\beta[\phi_t\int \theta_{t+1}dF(\theta_{t+1}) + (1-\phi_t)\delta] < \beta\int \gamma \max(\delta,\theta_{t+1})\, dF(\theta_{t+1})$. Consequently, there will be an increase in inequality across the very rich and very poor segments of the population. The rationale underlying this conjecture is that very poor agents are likely to remain outside of the intermediated sector for some time to come and consequently are heavily discounting the future cost of developing a link with the exchange network. Additionally, from (P4) it is known that in circumstances where an agent will never transact with the go-between the amount $s_t = \beta k_t$ is saved.

<u>Proposition 3</u>: For all $\epsilon > 0$ there exists a $\breve{k}$ such that

(a) $\sup_{k\in[0,\breve{k}]} |s(k)/k - \beta| < \epsilon$ and (b) $\sup_{k\in[0,\breve{k}]} |\phi(k) - c| < \epsilon$.

Proof: Consider the dynamic–programs (P5) and (P4) defining the value functions w(k) and $w^o(k)$, respectively, and the associated policy–rules s(k), $\phi(k)$, and βk, c. Since the value function connected with problem (P4) is strictly concave, it suffices to demonstrate that

$$(11)\qquad \lim_{\bar{k}\to 0}\left\{\sup_{k\in[0,\bar{k}]} |w(k) - w^o(k)|\right\} = 0.$$

It will be shown first that (11) holds and second that this condition implies the assertion made in the proposition.

Note that under program (P5) the minimal capital stock for which it is potentially profitable to join the exchange network is $\underline{k}$. Let the current period be t and consider an individual who has an initial endowment of capital $k_t = k$ and is saving in line with this program. Now define $P_{t+j}(k';k)$ as the probability that under the savings plan $s_t = s(k_t)$ and portfolio rule $\phi_t = \phi(k_t)$ the agent's capital stock will exceed $\underline{k}$ for the first time at t + j but then have a value less k′; that is, more formally, $P_{t+j}(k';k) \equiv \text{prob}[k_{t+j} \leq k', k_{t+j} \geq \underline{k}$, and $k_{t+i} < \underline{k}$ for $0 < i < j-1]$ with k_{t+j} being generated by the law of motion $k_{t+j} = [\phi(k_{t+j-1})(\theta_{t+j}+\epsilon_{t+j}) + (1-\phi(k_{t+j-1}))\delta]s(k_{t+j-1})$. The savings plan $s_t = s(k_t)$ and portfolio rule $\phi_t = \phi(k_t)$ are also *feasible* for an individual following the other program (P4). Note that while implementing this scheme is clearly *suboptimal* for (P4) it will yield the same time path of momentary utility as (P5) for the duration of time that the agent remains outside of the intermediated sector under the latter program. Thus,

$$(12)\qquad w(k) - w^o(k) \leq \sum_{j=1}^{\infty}\beta^j\int_{\underline{k}}^{\infty}[v(k') - w^o(k')]dP_{t+j}(k';k)$$

[recalling that $v(k') \geq w(k')$ by Lemma 2].

Next, from (2) and (4) it is known that

$$v(k') - w^{o}(k') = \frac{\beta}{(1-\beta)^2}\,[\int \ln\gamma \max(\delta,\theta)dF(\theta) - \int \ln[c(\theta+\epsilon)$$

$$+ (1-c)\delta]dF(\theta)dG(\epsilon)] \equiv \Upsilon > 0,$$

implying

$$w(k) - w^{o}(k) \le \Upsilon \sum_{j=1}^{\infty} \beta^j \int_{\underline{k}}^{\infty} dP_{t+j}(k';k) = \Upsilon \sum_{j=1}^{\infty} \beta^j P_{t+j}(k),$$

where $P_{t+j}(k) = \int_{\underline{k}}^{\infty} dP_{t+j}(k';k)$ is the marginal probability of crossing the threshold level of capital $\underline{k}$ for the first time at t + j. Alternatively, consider the situation where capital evolves according to the law of motion $k_{t+j} = \prod_{i=1}^{j} \max[\delta,(\theta_{t+i}+\epsilon_{t+i})]k_t$ and define $Q_{t+j}(k',k)$ as the probability the threshold level of capital k will be crossed for the first time at t + j and have a value no greater than k′. Therefore, $Q_{t+j}(k) \equiv \int_{\underline{k}}^{\infty} dQ_{t+j}(k',k)$ represents the marginal probability of crossing $\underline{k}$ for the first time at t + j. Clearly, this alternative generating process leads to the threshold level of the capital stock being passed for the first time at an earlier date. It follows that the distribution of the $P_{t+j}(k)$'s *stochastically dominates* the distribution of the $Q_{t+j}(k)$'s in the first order sense, or that $\sum_{j=1}^{m} P_{t+j}(k) \le \sum_{j=1}^{m} Q_{t+j}(k)$ for all m. Since β^j is a decreasing function in j this implies[14]

$$w(k) - w^{o}(k) \le \Upsilon \sum_{j=1}^{\infty} \beta^j P_{t+j}(k) \le \Upsilon \sum_{j=1}^{\infty} \beta^j Q_{t+j}(k).$$

Now given any ϵ, T > 0 a sufficiently small value for $\tilde{k}$, denoted by $\tilde{k}(\epsilon,T)$, can be chosen so that $\sum_{j=1}^{T} Q_{t+j}(k) \le \epsilon$ for all $k \in [0,\tilde{k}(\epsilon,T)]$. Therefore

$$(13) \qquad \sup_{k\in[0,\tilde{k}(\epsilon,T)]} |w(k) - w^{o}(k)| \le \Upsilon\epsilon \sum_{j=1}^{T} \beta^j + \Upsilon \sum_{j=T+1}^{\infty} \beta^j \le \frac{\beta\Upsilon[\epsilon(1-\beta^T) + \beta^T]}{(1-\beta)}.$$

Since this can be done for *any* ϵ and T, the right–hand side of (13) can be made arbitrarily tiny by choosing a small ϵ and large T. The desired result (11) now immediately obtains by letting $\epsilon \to 0$ and $T \to \infty$ in a manner such that $\bar{k}(\epsilon,T) \to 0$.

It remains to establish that (11) implies the assertion made in the proposition. Suppose that (a) is false. Then there exists some $\epsilon > 0$ such that for all δ, $\tilde{k}(\delta) > 0$ there is a $k \in (0,\tilde{k}(\delta)]$ for which $|s(k)/k - \beta| > \epsilon$ but $\sup_{x\in(0,\tilde{k}(\delta)]} |w(x) - w^{o}(x)| < \delta$. Let $\tilde{s}(k) \equiv s(k)/k$ and $A \equiv \min_{x\in[0,1]} [1/(1-x)^2 + (\beta/(1-\beta))/x^2] > 0$. Now choose $\tilde{k}$ such that (i) $\sup_{k\epsilon(0,\tilde{k}]}|w(k)-w^{o}(k)| < A/4\ \epsilon^2$ and (ii) $\tilde{k} < \underline{k}/(\bar{\theta}+\bar{\epsilon})$. Note (ii) implies that $[\phi(\theta+\epsilon) + (1-\phi)\delta]\tilde{s}k \in B^c$ with probability one. Therefore, for $k \in (0,\tilde{k}]$,

$$\begin{aligned} w(k) &= \max_{\tilde{s},\phi}\{\ell n((1-\tilde{s})k) + \beta \int w(\tilde{s}k(\phi(\theta+\epsilon) + (1-\phi)\delta))dF(\theta)dG(\epsilon)\} \\ &\le \max_{\tilde{s}}\{\ell n((1-\tilde{s})k) + \beta \int w^{o}(\tilde{s}k(c(\theta+\epsilon) + (1-c)\delta))dF(\theta)dG(\epsilon)\} + \frac{A}{4}\epsilon^2, \end{aligned}$$

using (i) and the fact that setting $\phi = c$ is optimal for w^{o}. Taking a second-order Taylor expansion of the term in braces [using (4)] around the point $\tilde{s} = \beta$, while noting first that at $\tilde{s} = \beta$ this term equals $w^{o}(k)$ and second that its first derivative at $\tilde{s} = \beta$ is zero, yields the result

$$w(k) \le w^{o}(k) - \frac{A}{2}(\tilde{s}-\beta)^2 + \frac{A}{4}\epsilon^2.$$

The constant A represents the lower bound on the absolute value of the second derivative of the expression in braces with respect to $\tilde{s}$. Suppose for some $k \in (0,\tilde{k}]$ that $|\tilde{s}(k) - \beta| > \epsilon$. Then for this k, $w(k) < w^{o}(k)$. This is the desired contradiction, since by construction $w(k) \ge w^{o}(k)$ for all $k > 0$. Finally, (b) can be proved by similar argument. □

To reiterate, Proposition 3 implies that the difference in relative wealth levels between members of the intermediated sector and the very poor will widen over time. This result obtains since both groups have the same savings rate while the former face a better distribution of returns on their investments.

Some of the long–run properties of the developed model will now be presented. To begin with, agents in the less–developed sector of the economy accumulate wealth according to

$$k_{t+1} = [\phi(k_t)(\theta_{t+1}+\epsilon_{t+1}) + (1-\phi(k_t))\delta]s(k_t).$$

Now define $\hat{\psi}(k';k)$ as the law of motion, in cumulative distribution function form, governing the evolution of the capital stock that is implied by the above equation. Thus, $\hat{\psi}(k';k) \equiv \text{prob}[k_{t+1} \leq k' \mid k_t = k]$. Note that those agents entering t with a $k_t \in B$ will join the intermediated sector, it not being worthwhile for the rest ($k_t \in B^c$) to establish a link at that time. Therefore, $\psi(k';k) \equiv \int_{B^c \cap (0,k']} d\hat{\psi}(z;k)$ represents the probability that an agent residing in period t in the less–developed sector of the economy with k units of capital will remain in this sector in t + 1 with a capital stock in value no greater than k'.

Next, let $\hat{H}_0(k)$ represent the economy's initial time zero distribution of capital over people so that $\hat{H}_0\colon \mathbb{R}_{++} \to [0,1]$. The initial sizes of the developed and less–developed sectors of the economy will therefore be $\int_B d\hat{H}_0(k)$ and $1 - \int_B d\hat{H}_0(k)$. Consequently, the distribution function governing the allocation of capital across individuals in the less–developed sector of the economy in period one will be given by $H_1(k') = \int_{B^c} \psi(k';k)d\hat{H}_0(k)$. In general,

$$H_{t+1}(k') = \int_0 \psi(k';k)dH_t(k),$$

where $H_{t+1}(k')$ measures the expected size of the population in period t who are outside of the intermediated sector and have a capital stock $k_{t+1} \leq k'$.[15] Note that by construction the H_{t+1}'s have all of their mass on B^c. Since in any given period t+1 no agent outside of the developed sector has a capital stock $k_{t+1} \geq \bar{k}$ (Lemma 3), it follows that the expected t + 1 size of the less–developed sector is $H_{t+1}(\bar{k})$. Given the assumed growth in the economy, $\lim_{t\to\infty} H_{t+1}(\bar{k}) = 0$ (i.e., the less–developed sector fades away).[16]

Finally, in any given period t those agents in the less–developed sector of the economy realize a rate of return of $[\phi(k_{t-1})(\theta_t+\epsilon_t) + (1-\phi(k_{t-1}))\delta]$ on their investments while those in the developed part obtain the yield $\gamma\max(\delta,\theta_t)$. Therefore, for any given realization of the aggregate shock $\theta_t = \theta$ the expected return earned across individuals, denoted by $R_t(\theta)$, is

$$R_t(\theta) \equiv \int_0^{\bar{k}}[\phi(k)\theta + (1-\phi(k))\delta]dH_{t-1}(k) + [1 - H_{t-1}(\bar{k})]\ \gamma\max(\delta,\theta).$$

Clearly, as the future time horizon is extended, $R_t(\theta)$ converges (although perhaps nonmonotonically) to the best technologically feasible expected return possible, $\gamma\max(\delta,\theta)$, conditional on the aggregate state–of–the–world.

<u>Proposition 4</u>: $\lim_{t\to\infty} \sup_{\theta} |R_t(\theta) - \gamma\max(\delta,\theta)| = 0.$

Furthermore, note that individuals outside and inside of the organized sector of the economy save the amounts $s_t = s(k_t)$ and $s_t = \beta k_t$, respectively. Consequently, as the less–developed sector atrophies, larger numbers of agents are accumulating wealth at the expected rate $\beta\gamma E[\max(\delta,\theta)]$. Thus, asymptotically all agents' wealth will be growing at the same rate and a stable distribution of relative wealth levels, say as measured by a Lorenz curve, will attain.[17] The economy's expected growth rate converges (though nonmonotonically) to $\beta\gamma E[\max(\delta,\theta)]$ with variance $(\beta\gamma)^2 var(\max(\delta,\theta))$.

Finally, observe if a nation's initial distribution of capital, $\hat{H}_0$, is concentrated sufficiently close to the origin then its growth factor $R_1(\theta)$ approaches $c\theta + (1-c)\delta$ with mean and variance $cE[\theta] + (1-c)\delta$ and $c^2 var(\theta)$, respectively. Thus, the relationship between a nation's per capita income and its (subsequent) growth is likely to be positive on average, and if $c^2 var(\theta) > \gamma^2 var(\max(\delta,\theta))$, the relation between per capita income and the variance of growth will be negative.[18] This is illustrated in Figure 1. These two predictions of the model carry over to cross sections of countries' per capita incomes and their growth rates.[19]

V. Conclusions

Two themes have been prominent in the growth and development literature: the link between economic growth and the distribution of income, and the connection between financial structure and economic development. Both of these issues were addressed here within the context of a single model. Growth and financial structure were inextricably linked. Growth provided the wherewithall to develop financial structure, while financial structure in turn allowed for higher growth since investment could be more efficiently undertaken. The model yields a development process consistent, at least, with casual observation. In the early stages of development where exchange is largely unorganized, growth is slow. As income levels rise financial structure becomes more extensive, economic growth more rapid, and income inequality across the rich and poor widens. In maturity an economy has a fully developed financial structure, attains a stable distribution of income across people, and has a higher growth rate than in its infancy.

Appendix

It will now be demonstrated that the competitive equilibrium constructed in Section III is Pareto–optimal. The discussion is brief, drawing on material presented in Lucas and Stokey (1983) and Stokey and Lucas with Prescott (1989). The environment modeled here is more general than that presented in the text. In particular, imagine that a contingent claims market operates in the developed sector, with additional separate contingent claims markets functioning at each undeveloped autarkic location. Each individual in the developed sector has access to three production technologies: $y_t = \gamma\max(\delta,\theta_t)i_{t-1}$, $y_t = (\theta_t+\epsilon_t)i_{t-1}$, and $y_t = \delta i_{t-1}$. Here, as in the text, θ_t represents an aggregate shock which is common across individual production processes while ϵ_t portrays an idiosyncratic (or project) specific shock. For subsequent use, denote a person's investment in these technologies by i_t^1, i_t^2, and i_t^3. Agents in the undeveloped sector can only access the latter two processes. Finally, an individual can move (so to speak) to the developed sector from an undeveloped one at a fixed cost of α. Note that this structure allows an agent living in an undeveloped sector to finance a move by issuing contingent claims in the developed sector to cover the incurred costs. Individuals in either sector are also free to insure themselves against both aggregate and idiosyncratic shocks to production. For subsequent use, let $\theta^t \equiv (\theta_0,\theta_1,\ldots,\theta_t)$, $\theta_t^{t+j} \equiv (\theta_t,\theta_{t+1},\ldots,\theta_{t+j})$, $\epsilon^t \equiv (\epsilon_0,\epsilon_1,\ldots,\epsilon_t)$, and $\epsilon_t^{t+j} \equiv (\epsilon_t,\epsilon_{t+1},\ldots,\epsilon_{t+j})$, and f and g represent the density functions associated with F and G. (Note that at this stage of the analysis there is no need to identify any particular individual in the economy. Therefore, for the time being, let agents remain anonymous.)

Developed Sector

As was just mentioned, agents in the developed sector are free to participate on a sector–wide contingent claims market. Define $b_t^{t+j} = b_t^{t+j}(\theta^{t-1},\epsilon^{t-1};\theta_t^{t+j},\epsilon_t^{t+j})$ as the amount of contingent claims purchased by an individual at the end of period t–1, given that the event $(\theta^{t-1},\epsilon^{t-1})$ has occurred, for consumption in period t+j contingent on the realization of $(\theta_t^{t+j},\epsilon_t^{t+j})$. The market price of a claim to period–t consumption, conditional on the event (θ^t,ϵ^t), will be denoted by $p_t = p_t(\theta^t,\epsilon^t)$. Now, suppose an agent in the developed sector enters period t with k_t units of wealth. (Let period–zero consumption be the numeraire.) This wealth can be used to purchase current consumption, a portfolio of contingent claims, or to finance physical investments in any or all of the three production technologies. Thus, the individual's period–t budget constraint is

$$\text{(A.1)}\quad p_t c_t + \left[\sum_{j=t+1}^{\infty} \int p_j b_{t+1}^j d\theta_{t+1}^j d\epsilon_{t+1}^j\right] + [p_t - \int p_{t+1}\gamma \max(\delta,\theta_{t+1}) d\theta_{t+1} d\epsilon_{t+1}] i_t^1$$

$$+ [p_t - \int p_{t+1}(\theta_{t+1}+\epsilon_{t+1}) d\theta_{t+1} d\epsilon_{t+1}] i_t^2 + [p_t - \int p_{t+1}\delta d\theta_{t+1} d\epsilon_{t+1}] i_t^3 \leqq p_t k_t.$$

Observe that the individual sells forward the proceeds he earns on any period–t investment in physical capital. An agent's savings and investment in period t, s_t and i_t, are given by $p_t s_t = \sum_{j=t+1}^{\infty} \int p_j b_{t+1}^j d\theta_{t+1}^j d\epsilon_{t+1}^j$ and $i_t = i_t^1 + i_t^2 + i_t^3$. The individual will then enter into t + 1 with k_{t+1} units of wealth, where

$$\text{(A.2)}\qquad k_{t+1} = b_{t+1}^{t+1} + (1/p_{t+1}) \sum_{j=t+2}^{\infty} \int p_j\, b_{t+1}^j\, d\theta_{t+1}^j\, d\epsilon_{t+1}^j.$$

It is now easy to see that agents in the market sector solve the dynamic programming problem shown below:

$$\text{(P6)}\qquad v(k_t) = \max\{\ell n(c_t) + \beta \int v(k_{t+1}) dF(\theta_{t+1}) dG(\epsilon_{t+1})\},$$

subject to the constraint (A.1), with the choice variables being c_t, b_{t+1}^j, i_t^1, i_t^2, and i_t^3, and where k_{t+1} is given by (A.2).[20] The upshot of the implied maximization routine is the following set of efficiency conditions:

(A.3) $\quad \frac{1}{c_t} = \beta v'(k_{t+1})p_t/p_{t+1}f(\theta_{t+1})g(\epsilon_{t+1})$

(A.4) $\quad p_t \geqq \int p_{t+1}\ \gamma \max(\delta,\theta_{t+1})d\theta_{t+1}\ d\epsilon_{t+1}$ (with equality if $i_t^1 > 0$)

(A.5) $\quad p_t \geqq \int p_{t+1}(\theta_{t+1}+\epsilon_{t+1})d\theta_{t+1}\ d\epsilon_{t+1}$ (with equality if $i_t^2 > 0$)

and

(A.6) $\quad p_t \geqq \int p_{t+1}\ \delta d\theta_{t+1}\ d\epsilon_{t+1}$ (with equality if $i_t^3 > 0$).

Less–Developed Sectors

Agents residing in less–developed sectors have access to *local* contingent claims markets. Let $\hat{b}_t^{t+j} = \hat{b}_t^{t+j}(\theta^{t-1},\epsilon^{t-1};\theta_t^{t+j},\epsilon_t^{t+j})$ represent the amount of contingent claims purchased by an individual at the end of $t-1$, given that the event $(\theta^{t-1},\epsilon^{t-1})$ has occurred, for consumption in period $t+j$ contingent on the realization of $(\theta_t^{t+j},\epsilon_t^{t+j})$. Also, define $\hat{p}_t = \hat{p}_t(\theta^t,\epsilon^t)$ to be the market price to a claim of period–t consumption conditional on the event (θ^t,ϵ^t) occurring. Now suppose that an agent enters period t with k_t units of wealth. In line with the earlier discussion his decision–making is represented by the dynamic–programming problem shown below, with the choice variables being c_t, $\hat{b}_{t+1}^j$, i_t^2, and i_t^3:

(P7) $\quad w(k_t) = \max\{\ell n\ c_t + \beta\int \max[w(k_{t+1}),v(k_{t+1}-\alpha)]dF(\theta_{t+1})dG(\epsilon_{t+1})$

s.t.

(A.7) $$\hat{p}_t c_t + \left[\sum_{j=t+1}^{\infty}\int \hat{p}_j\ \hat{b}_{t+1}^j\ d\theta_{t+1}^j\ d\epsilon_{t+1}^j\right] + [\hat{p}_t - \int \hat{p}_{t+1}(\theta_{t+1}+\epsilon_{t+1})d\theta_{t+1}d\epsilon_{t+1}]i_t^2$$

$$+ [\hat{p}_t - \int \hat{p}_{t+1}\delta d\theta_{t+1}d\epsilon_{t+1}]i_t^3 \leqq \hat{p}_t k_t,$$

and where $k_{t+1} = \hat{b}_{t+1}^{t+1} + (1/\hat{p}_{t+1})\sum_{j=t+2}^{\infty}\int \hat{p}_j\hat{b}_{t+1}^j d\theta_{t+1}^j d\epsilon_{t+1}^j$. In the above problem, an agent's period–t savings, or s_t, is given by $\hat{p}_t s_t = \sum_{j=t+1}^{\infty} \int \hat{p}_j \hat{b}^j_{t+1} d\theta^j_{t+1} d\epsilon^j_{t+1}$ and his investment, i_t, in this period by $i_t = i^2_t + i^3_t$.

The following efficiency conditions summarize the solution to problem (P7):

$$\text{(A.8)} \qquad \frac{1}{c_t} = \beta \frac{d\max[w(k_{t+1}), v(k_{t+1}-\alpha)]}{dk_{t+1}} \frac{\hat{p}_t}{\hat{p}_{t+1}} f(\theta_{t+1}) g(\epsilon_{t+1})$$

$$\text{(A.9)} \qquad \hat{p}_t \geqq \int \hat{p}_{t+1}(\theta_{t+1}+\epsilon_{t+1}) d\theta_{t+1}\, d\epsilon_{t+1} \quad \text{(with equality if } i^2_t > 0)$$

$$\text{(A.10)} \qquad \hat{p}_t \geqq \int \hat{p}_{t+1}\, \delta d\theta_{t+1}\, d\epsilon_{t+1} \quad \text{(with equality if } i^3_t > 0).$$

Competitive Equilibrium

The equilibrium allocations generated by the economy modeled in the text also constitute a competitive equilibrium for the economy being studied here.

Lemma 4: The following is a competitive equilibrium for the economy under study.

(i) Given a level of wealth k_t and the function $v(k_t)$, agents in less–developed sectors in any period t realize the expected utility level $w(k_t)$, save the amount $s_t = s(k_t)$, and invest physical capital in the high risk/return and safe technologies in the ratio $\phi(k_t)/[1 - \phi(k_t)]$ as determined by problem (P1). For all individuals savings equals investment in physical capital so that $i_t = s_t$, $i^2_t = \phi_t s_t$, and $i^3_t = (1-\phi_t)s_t$. This savings and investment plan is supported by each person purchasing in t *any* portfolio of contingent claims satisfying $\hat{b}^{t+1}_{t+1} + (1/\hat{p}_{t+1}) \sum_{j=t+2}^{\infty} \int \hat{p}_j \hat{b}^j_{t+1} d\theta^j_{t+1} d\epsilon^j_{t+1} = [\phi_t(\theta_{t+1} + \epsilon_{t+1}) + (1-\phi_t)\delta]s_t$. Finally, agents migrate to the market sector or not in period t depending on whether $v(k_t-\alpha) \gtrless w(k_t)$.

(ii) Given a level of wealth k_t, individuals in the developed sector in t realize the expected utility level $v(k_t)$ and save the amount $s_t = \beta k_t$ in accord with (P3). For all individuals savings equals investment in physical capital all of which is channeled through the intermediated technology implying $i_t = s_t = i_t^1$ and $i_t^2 = i_t^3 = 0$.[21] This savings and investment plan is supported by each person purchasing in t *any* portfolio of contingent claims satisfying $b_{t+1}^{t+1} + (1/p_{t+1}) \sum_{j=t+2}^{\infty} \int p_j b_{t+1}^j d\theta_{t+1}^j d\epsilon_{t+1}^j = \gamma \max(\delta, \theta_{t+1}) s_t$. Migration to this sector is permanent; $v(k_t) > w(k_t)$ for all k_t.

(iii) Asset prices in the developed and undeveloped sectors are given, respectively, by

(a) $$p_{t+1} = \frac{f(\theta_{t+1})g(\epsilon_{t+1})}{\gamma \max(\delta, \theta_{t+1})} p_t,$$

and

(b) $\hat{p}_{t+1} =$

$$\frac{d\max[w(k_{t+1}), v(k_{t+1}-\alpha)]/dk_{t+1}\, f(\theta_{t+1})g(\epsilon_{t+1})\hat{p}_t}{\int(\phi_t(\theta_{t+1}+\epsilon_{t+1})+(1-\phi_t)\delta)d\max[w(k_{t+1}), v(k_{t+1}-\alpha)]/dk_{t+1}\, dF(\theta_{t+1})dG(\epsilon_{t+1})},$$

with $p_0 = \hat{p}_0 = 1$.

Proof: It will be demonstrated that the price system described in (iiib) supports the equilibrium outlined in (i), given the function $v(\cdot)$ postulated in (ii). The proof that (iiia) implies (ii) is similar. To begin with, substitute (iiib) into (A.8) to obtain the condition

$$1/c_t = \beta \int (\phi_t(\theta_{t+1}+\epsilon_{t+1}) + (1-\phi_t)\delta) d\max[w(k_{t+1}), v(k_{t+1}-\alpha)]/dk_{t+1}\, dF(\theta_{t+1})dG(\epsilon_{t+1}). \tag{A.11}$$

Now from the allocation rules specified in (i), together with (A.9), (A.10), and the definition for k_{t+1}, it follows that $c_t = k_t - s_t$ and $k_{t+1} = (\phi_t(\theta_{t+1}+\epsilon_{t+1}) + (1-\phi_t)\delta)s_t$. These results, in conjunction with the envelope theorem, allow (A.11) to be rewritten as

$$\frac{1}{k-s(k)} = \beta\int_{D^c(s(k),\phi(k))} \frac{[\phi(k)(\theta+\epsilon) + (1-\phi(k))\delta]\,dF(\theta)\,dG(\epsilon)}{[s(k)(\phi(k)(\theta+\epsilon)+(1-\phi(k))\delta)-s(s(k)(\phi(k)(\theta+\epsilon)+(1-\phi(k))\delta))]}$$

$$\text{(A.12)} \qquad + \beta\int_{D(s(k),\phi(k))} \frac{\phi(k)(\theta+\epsilon) + (1-\phi(k))\delta}{(1-\beta)[s(k)(\phi(k)(\theta+\epsilon)+(1-\phi(k))\delta)-\alpha]}\, dF(\theta)dG(\epsilon),$$

where the sets $D^c(s(k),\phi(k))$ and $D(s(k),\phi(k))$ are as defined in the text, and time subscripts have been dropped for convenience.

Next, note from (i) that when $s_t > 0$, $i_t^2 = 0$ if and only if $\phi_t = 0$ and similarly $i_t^3 = 0$ if and only if $\phi_t = 1$. Subtracting (A.10) from (A.9) while making use of (iiib) yields the result that $\int((\theta_{t+1}+\epsilon_{t+1}) - \delta)d\max[w(k_{t+1}),v(k_{t+1}-\alpha)]/dk_{t+1}dF(\theta_{t+1})$ $dG(\epsilon_{t+1}) \lesseqgtr 0$, as $\phi_t = 0$, $0 \leq \phi_t \leq 1$, and $\phi_t = 1$, respectively.[22] The envelope theorem allows this expression to take the form

$$\text{(A.13)} \qquad \int_{D^c(s(k),\phi(k))} \frac{[(\theta+\epsilon) - \delta]dF(\theta)dG(\epsilon)}{[s(k)(\phi(k)(\theta+\epsilon)+(1-\phi(k))\delta)-s(s(k)(\phi(k)(\theta+\epsilon)+(1-\phi(k))\delta))]}$$

$$+ \int_{D(s(k),\phi(k))} \frac{(\theta+\epsilon) - \delta}{(1-\beta)[s(k)(\phi(k)(\theta+\epsilon)+(1-\phi(k))\delta)-\alpha]}\, dF(\theta)dG(\epsilon) \lesseqgtr 0,$$

as $\phi_t = 0$, $0 \leq \phi_t \leq 1$, and $\phi_t = 1$. Finally, observe that (A.12) and (A.13) are nothing but the efficiency conditions for problem (P1) defining solutions for s(k) and $\phi(k)$ and the associated one for w(k) [see problem (P5)]. □

Pareto–Optimality

It remains to establish that the valuation equilibrium modeled above is Pareto–optimal. To do this, a bit more notation must be developed. To begin with, observe that under *any* interesting allocation rule for the economy, entry into the developed sector will be permanent. This must be so since it is feasible for an agent in the developed sector to duplicate the returns he could realize in autarky by simply operating the high risk/return and safe technologies in isolation from others in the sector. Therefore to conserve notation, attention will be limited to situations where entry into the developed sector is permanent. Note that when an individual moves in period t from a less–developed sector to the developed one, he takes a certain stock of wealth in terms of goods, to be denoted by $\hat{k}_t$, with him. Thus, let agent j's period–t allocation in a less–developed location be represented by his consumption there, $\hat{c}_t(j)$, investment in physical capital, $(\hat{i}_t^1(j),\hat{i}_t^2(j),\hat{i}_t^3(j))$, and his transfer of physical goods to the developed sector, $\hat{k}_t(j)$. Similarly, j's allocation in the developed sector t is specified by his consumption $\tilde{c}_t(j)$, investment in physical capital, $(\tilde{i}_t^1(j),\tilde{i}_t^2(j),\tilde{i}_t^3(j))$, and transfer of wealth to *his* autarkic island, $\tilde{k}_t(j)$. While this notation has been defined at a general level, it should be understood that (i) since an individual cannot consume and invest at two locations simultaneously at least one of the vectors $[\hat{c}_t(j),\hat{i}_t^1(j),\hat{i}_t^2(j),\hat{i}_t^3(j)]$ or $[\tilde{c}_t^1(j),\tilde{i}_t^1(j),\tilde{i}_t^2(j),\tilde{i}_t^3(j)]$ must be identically zero, (ii) if any one of $\tilde{c}_t(j)$, $\tilde{i}_t^1(j)$, $\tilde{i}_t^2(j)$, $\tilde{i}_t^3(j)$ or $\hat{k}_t(j)$ is nonzero then the vector $[\hat{c}_h(j),\hat{i}_h^1(j),\hat{i}_h^2(j),\hat{i}_h^3(j),\hat{k}_h(j)]$ is identically zero for all $h > t$ since membership in the developed sector is permanent, and (iii) $\hat{i}_t^1(j) = \tilde{k}_t(j) = 0$ for all t. Consequently, using the above notation, agent j's allocation in an economy can be summarized by

$$\left\{c_t(j),i_t^1(j),i_t^2(j),i_t^3(j),k_t(j)\right\}_{t=0}^{\infty}$$

where

$$c_t(j) \equiv \max\left[\hat{c}_t(j),\tilde{c}_t(j)\right],\ i_t^1(j) \equiv \max\left[\hat{i}_t^1(j),\tilde{i}_t^1(j)\right], \text{etc., etc.}$$

Next, for an allocation to be feasible for *any* less–developed sector (say sector j) in period t, the following must hold:

$$\text{(A.14)} \qquad \hat{c}_t(j) + \hat{i}_t^2(j) + \hat{i}_t^3(j) + \hat{k}_t(j) \leq (\theta_t+\epsilon_t(j))\hat{i}_{t-1}^2(j) + \delta\hat{i}_{t-1}^3(j).$$

Similarly, for an allocation to be feasible for the developed sector in t, it must transpire that

$$\text{(A.15)} \qquad \int\left[\tilde{c}_t(j) + \tilde{i}_t^1(j) + \tilde{i}_t{}^2(j) + \tilde{i}_t^3(j)\right]d\lambda(j)$$

$$\leq \int\left[\gamma\max(\delta,\theta_t)\tilde{i}_{t-1}^1(j)+(\theta_t+\epsilon_t(j))\tilde{i}_{t-1}^2(j)+\delta\tilde{i}_{t-1}^3(j)+\hat{k}_t(j)-\alpha I(\hat{k}_t(j))\right]d\lambda(j),$$

where $I(x) = 1$ if $x > 0$ and $I(x) = 0$ when $x = 0$.[23] Finally, let an asterisk be attached to a quantity variable to denote its value in the competitive equilibrium modeled above, which will be dubbed the *star–allocation* system.

Proposition 1: The star–allocation system is Pareto–optimal.

Proof: Consider some alternative allocation system distributing to agent j the assignment of goods $\{c_t(j),i_t^1(j),i_t^2(j),i_t^3(j),k_t(j)\}_{t=0}^{\infty}$. It will be shown that it is impossible for this allocation scheme to make some set of agents (with positive measure) in the economy better off without making others worse off. To begin with, if agent j strictly prefers the plan $\{c_t(j),i_t^1(j),i_t^2(j),i_t^3(j),k_t(j)\}_{t=0}^{\infty}$ to $\{c_t^*(j),i_t^{*1}(j),i_t^{*2}(j),i_t^{*3}(j),k_t^*(j)\}_{t=0}^{\infty}$ then the first plan must be more expensive; otherwise j would purchase it under the star–allocation system. This implies that at least one of the following inequalities must hold strictly:

$$\text{(A.16)} \qquad \hat{p}_0\left[\hat{c}_0(j)+\hat{i}_0^2(j)+\hat{i}_0^3(j)+\hat{k}_0(j)\right] + \sum_{t=1}^{\infty}\int\hat{p}_t\left[\hat{c}_t(j)+\hat{i}_t^2(j)+\hat{i}_t^3(j)+\hat{k}_t(j)\right]d\theta_1^t d\epsilon_1^t(j)$$

$$\geq \hat{p}_0\hat{a}_0(j) + \sum_{t=1}^{\infty}\int\hat{p}_t\left[(\theta_t+\epsilon_t(j))\hat{i}_{t-1}^2(j)+\delta\hat{i}_{t-1}^3(j)\right]d\theta_1^t d\epsilon_1^t(j)$$

or

$$\text{(A.17)}\quad p_0\Big[\tilde{c}_0(j)+\tilde{i}^1_0(j)+\tilde{i}^2_0(j)+\tilde{i}^3_0(j)\Big] + \sum_{t=1}^{\infty}\int p_t\Big[\tilde{c}_t(j)+\tilde{i}^1_t(j)+\tilde{i}^2_t(j)+\tilde{i}^3_t(j)\Big]d\theta^t_1 d\epsilon^t_1(j)$$

$$\geq p_0\Big[\tilde{a}_0(j)+\hat{k}_0(j)-\alpha I(\hat{k}_0(j))\Big] + \sum_{t=1}^{\infty}\int p_t\Big[\gamma\max(\delta,\theta_t)\tilde{i}^1_{t-1}(j)$$

$$+(\theta_t+\epsilon_t(j))\tilde{i}^2_{t-1}(j)+\delta\tilde{i}^3_{t-1}(j)+\hat{k}_t(j)-\alpha I(\hat{k}_t(j))\Big]d\theta^t_1\epsilon^t_1(j),$$

where $\hat{a}_0(j)$ and $\tilde{a}_0(j)$ denote agent j's time–zero endowments of goods in the less–developed and developed sectors, respectively (only one of which may be positive).[24] If the agent is indifferent between the two plans then both of these inequalities may hold weakly.

Now suppose (A.17) holds strictly for some set of agents with positive measure. Then integrating both sides of (A.17) over all agents in the economy yields

$$p_0\int\Big[\tilde{c}_0(j) + \tilde{i}^1_0(j) + \tilde{i}^2_0(j) + \tilde{i}^3_0(j) - \tilde{a}_0(j) - \hat{k}_0(j) + \alpha I(\hat{k}_0(j))\Big]d\lambda(j)$$

$$+ \sum_{t=1}^{\infty}\int p_t\int\Big[[\tilde{c}_t(j) + \tilde{i}^1_t(j) + \tilde{i}^2_t(j) + \tilde{i}^3_t(j) - \gamma\max(\delta,\theta_t)\tilde{i}^1_{t-1}(j)$$

$$-(\theta_t+\epsilon_t(j))\tilde{i}^2_{t-1}(j) - \delta\tilde{i}^3_{t-1}(j) - \hat{k}_t(j) + \alpha I(\hat{k}_t(j))\Big]d\lambda(j)d\theta^t_1\, d\epsilon^t_1(j) > 0.$$

But this leads to the feasibility condition (A.15) being violated at some date t. Similarly, assume that (A.16) holds strictly for *any* individual j. This would lead to the feasibility condition (A.14) being violated at some date. Therefore, it is not possible for the proposed allocation to make some set of agents in the economy better off without making others worse off. □

<u>Remark</u>: As in discussed in Stokey and Lucas with Prescott (1989), the first welfare theorem does not depend on *any* assumptions about technology (such as the absence of fixed costs).

Footnotes

[1]The evidence that early stages of growth are accompanied by a worsening of the income distribution is by no means clear cut. Korea, for example, grew very fast over the 1965–85 period, but while income inequality did worsen slightly among rural households over this period, a bigger improvement took place in the distribution of income among urban households (Dornbush and Park 1987, Table 8).

[2]Following Townsend (1983a), "the idea that trade links are costly, per se, seems to be a useful formalism, presumably capturing the cost of bookkeeping, the cost of enforcement, the cost of monitoring when there is imperfect information, the physical cost of exchange (transportation), the difficulties of communication, and so on" (p. 259). For instance, *each* party to an agreement may have to hire lawyers or accountants to advise on its details, or pay the cost of installing communication devices (computers, liaisons, or transportation terminals), or simply incur the educational expenses involved with learning new business procedures, etc., etc.

[3]To see this, let $\xi \equiv \gamma\max(\delta,\theta)$ and $H\colon \mathbb{R}_+ \to [0,1]$ represent the distribution function governing ξ. Note that assumption (C) can now be written as

$$(C') \qquad \int_0^\infty [1 - H(\xi)]d\xi = \int_0^\infty \gamma\max(\delta,\theta)dF(\theta) > \int_0^\infty \theta dF(\theta) = \int_0^\infty [1 - F(\theta)]d\theta,$$

where F has been extended to $\mathbb{R}_+$ by defining $F(\theta) = 0$ for $\theta \in [0,\underline{\theta})$ and $F(\theta) = 1$ for $\theta \in [\overline{\theta},\infty)$. For ξ to be larger than θ in the sense of second–order stochastic dominance, it must happen that $\int_0^x [F(t) - H(t)]dt \geq 0$ for all $x \in \mathbb{R}_+$ with strict equality obtaining for some x (Hadar and Russell 1971, Definition 2). To show this, observe the following: (i) $H(t) < F(t)$ for $0 < t < \gamma\delta$ since $H(t) = 0 < F(t)$ and (ii) $H(t) > F(t)$ for all $t \geq \gamma\delta$ since $H(t) = F(t/\gamma) > F(t)$. Now consider the expression $\int_0^x [F(t) - H(t)]dt$. For $x \leq \gamma\delta$ this is clearly positive since $F(t) > H(t)$ by (i). For $x \geq \gamma\delta$ rewrite this expression as $\int_0^{\gamma\delta} [F(t) - H(t)]dt + \int_{\gamma\delta}^x [F(t) - H(t)]dt$. Here, by (i) and (ii), the first term is positive while the second is negative. But (C′) guarantees that the first term always dominates since $\int_0^\infty[F(t) - H(t)]dt > 0$ so that $\int_0^{\gamma\delta}[F(t) - H(t)]dt > - \int_{\gamma\delta}^\infty[F(t) - H(t)]dt > - \int_{\gamma\delta}^x[F(t) - H(t)]dt$.

[4]For the purpose of taking sums, reindex the (countable) collection of agents in the set A^e by the natural numbers.

[5]Envision each period as consisting of two subintervals. In the first subinterval production is undertaken. Production can be done at anytime within this subinterval—some projects can be undertaken early, others can be done late. The intermediary's trial projects are run early, the rest late. Agents who choose not to transact with an intermediary are indifferent about when to operate their projects within this subinterval, since they cannot observe at that time what is happening to production elsewhere in the economy. In the second subinterval the output from production is distributed and agents decide how much to consume currently out of their proceeds and how much to invest for future consumption.

[6]Note here that it is being presumed the intermediary commits himself forever to the policy of paying the return $r(\theta_t)$ in each period t on any and all deposits made in $t - 1$, subject only to the stipulation that the depositor has paid at some time the once–and–for–all fee of q. The possibility of default is precluded by assumption. Now, suppose that for some $\int_{A_{t-1}} i_{t-1}(j)d\lambda(j) \geq 0$ and $\int_{A'_{t-1}} d\lambda(j) \geq 0$ condition (1) could become negative with positive probability. Then, with positive probability, any intermediary could go bankrupt in the first period of its operation and would have to default on its obligations to depositors (since the intermediary would owe an infinitely large amount relative to his start up wealth). This, though, is prohibited. Any agent can become an intermediary, rather than transact with one, if it is in his own best interest to do so. Alternatively then suppose that $r(\theta_t)$ and q are such that (1) never becomes negative, and is strictly positive with nonzero probability. Here the intermediary could

realize infinite profits in any particular period with positive probability, and never realize any losses, a situation ruled out by the assumption of free entry into the industry.

[7]In a similar vein, Gertler and Rogoff (1989) assume that the probability of an investment project attaining a good return is an increasing function of the amount of funds invested. This again could provide a rationale for agents to pool funds.

[8]Throughout the analysis, it will be implicitly assumed that the constraints $s_t \in [0,k_t]$ and $\phi_t \in [0,1]$ apply, as relevant, to the optimization problems formulated.

[9]Problem (P2) assumes that agents participating in the intermediated sector will invest all their savings with the go-between. Strictly speaking, the intermediary requires the use of only one safe technology and a countable infinity of the high risk/return ones. Thus it may seem reasonable to allow some agents to make individual isolated use of the unneeded technologies so as to economize on the proportional transactions costs associated with intermediated activity. It is easily demonstrated, using equations (A5) and (A6) in the Appendix, that the following assumption ensures such options, even if available, would never be executed: (D) For all $\phi \in [0,1]$, $\int[\phi\theta + (1-\phi)\delta]/[\gamma\max(\theta,\delta)]dF(\theta) < 1$. Note that this assumption holds automatically when $\gamma = 1$ (i.e., no proportional transactions costs). Finally, assumption (C) and (D) can be guaranteed by imposing the single restriction that $E[\theta]E[1/(\gamma\max(\delta,\theta))] < 1$.

[10]See Hadar and Russell (1977), Theorem 2, for more detail.

[11]Some more detail on the derivation of the constant c may be warranted. Suppose that (4) specifies w^o. Then the following conditions govern the solution for ϕ in (P4): (i) $\int[\theta+\epsilon-\delta]/[\phi(\theta+\epsilon) + (1-\phi)\delta]dF(\theta)dG(\epsilon) \leq 0$ if $\phi = 0$, (ii) if $\phi \in (0,1)$, $\int[\theta+\epsilon-\delta]/[\phi(\theta+\epsilon) + (1-\phi)\delta]dF(\theta)dG(\epsilon) = 0$, and (iii) $\int[\theta+\epsilon-\delta]/[\phi(\theta+\epsilon) + (1-\phi)\delta]dF(\theta)dG(\epsilon) \geq 0$ if $\phi = 1$. Clearly, the solution for ϕ does not depend on k, i.e., $\phi = c$ where c is a constant. It is easy to deduce that $c \neq 0$. Evaluating (i) at $\phi = 0$ yields $\int[\theta+\epsilon-\delta]/\delta dF(\theta)dG(\epsilon) > 0$, which contradicts assumptions (A) and (B).

[12]In competitive equilibrium the goods markets always clear since, for each agent, consumption plus physical investment in capital (inclusive of transactions costs) equals his endowment of output.

[13]Briefly, the Euler equation connected with problem (P5) is

$$\frac{1}{k-s(k)} = \beta\int_{D^c(s(k),\phi(k))} \frac{[\phi(k)(\theta+\epsilon) + (1-\phi(k))\delta]\,dF(\theta)dG(\epsilon)}{[s(k)(\phi(k)(\theta+\epsilon)+(1-\phi(k))\delta)-s(s(k)(\phi(k)(\theta+\epsilon)+(1-\phi(k))\delta))]}$$

$$+ \beta\int_{D(s(k),\phi(k))} \frac{\phi(k)(\theta+\epsilon) + (1-\phi(k))\delta}{(1-\beta)[s(k)(\phi(k)(\theta+\epsilon)+(1-\phi(k))\delta)-q]}\, dF(\theta)dG(\epsilon),$$

with s(k) denoting the optimal policy function. Now consider the fixed point associated with the mapping shown by (8). Here the sets $D^c(s(k),\phi(k))$ and $D(s(k),\phi(k))$ are *fixed*, as far as the implied maximization is concerned. The choice problem underlying this mapping has the following Euler equation:

$$\frac{1}{k-\tilde{s}(k)} = \beta\int_{D^c(s(k),\phi(k))} \frac{[\phi(k)(\theta+\epsilon)+(1-\phi(k))\delta]\,dF(\theta)dG(\epsilon)}{[\tilde{s}(k)(\phi(k)(\theta+\epsilon)+(1-\phi(k))\delta)-s(\tilde{s}(k)(\phi(k)(\theta+\epsilon)+(1-\phi(k))\delta))]}$$

$$+ \beta\int_{D(s(k),\phi(k))} \frac{\phi(k)(\theta+\epsilon) + (1-\phi(k))\delta}{(1-\beta)[\tilde{s}(k)(\phi(k)(\theta+\epsilon) + (1-\phi(k))\delta) - q]}\, dF(\theta)dG(\epsilon),$$

where $\tilde{s}(k)$ denotes the optimal policy function. Next, examine the solutions for the policy functions to each of these Euler equations; they are the same, implying $\tilde{s}(k) = s(k)$. Thus, the fixed point to (8) must be represented by (P5).

[14]See Hadar and Russell (1971), Definition 1 and Theorem 1.

[15]Strictly speaking, the H_t functions are not proper cumulative distribution functions as in general $H_t(\infty) < 1$. Also, the distribution H_t is an expectation conditional on period–zero information. The actual distribution of capital, $\tilde{H}_t$, evolves randomly in response to the realization of θ_t. The period–t state of the less–developed sector is $(\theta_t,\tilde{H}_t)$ while the economy–wide state is a triple made up of θ_t, $\tilde{H}_t$, and the distribution of capital at t in the developed sector.

[16]By Proposition 3, $s(k_t) > \beta k_t$. Consequently, it follows that $\ln k_t > \ln k_o + \sum_{j=1}^{t} \{\ln[\phi_j(\theta_j+\epsilon_j) + (1-\phi_j)\delta] + \ln \beta\}$. The right–hand side of this expression is a random walk with positive drift, since $E[\ln(\phi_j(\theta_j+\epsilon_j) + (1-\phi_j)\delta)] + \ln \beta > 0$ for all $\phi_j \in [0,1]$ by assumptions (A) and (B). Thus, k_t must become absorbed into the set $[\bar{k},\infty)$ with probability one. For more detail see Feller (1971).

[17]In a somewhat different context, Hart and Prais (1956) present evidence on the tendency for Lorenz curves to first worsen and then improve over time.

[18]It is easy to construct examples where $c^2\mathrm{var}(\theta) > \gamma^2\mathrm{var}(\max(\delta,\theta))$. As a case in point, suppose θ is drawn from the set $\Theta \equiv \{a_1\delta, a_2\delta\}$ with $a_1 < 1 < a_2$. Now let $\pi \equiv \mathrm{prob}[\theta = a_1\delta]$ so that $1 - \pi = \mathrm{prob}[\theta = a_2\delta]$. Recall that $E[\theta] > \delta$, by assumption (A), which translates here to requiring $\pi a_1 + (1-\pi)a_2 > 1$. Finally, set $\epsilon = 0$. Given that $E[\theta] > \delta$, it was demonstrated in footnote (11) that $c \in (0,1]$. First, consider the case where $c = 1$. Trivially, here $c^2\mathrm{var}(\theta) > \gamma^2\mathrm{var}(\max(\delta,\theta))$ since $\gamma < 1$ and $\mathrm{var}(\theta) > \mathrm{var}(\max(\delta,\theta))$. Second, consider the situation where $c \in (0,1)$. Here the constant c is determined by condition (ii) in footnote (11), which now reads $\pi(a_1-1)/[ca_1+(1-c)] + (1-\pi)(a_2-1)/[ca_2+(1-c)] = 0$, implying $c = [\pi a_1+(1-\pi)a_2-1]/[(a_2-1)(1-a_1)]$. Clearly, a_1 and a_2 can be chosen in a manner which makes c sufficiently close to γ so that $c^2\mathrm{var}(\theta) > \gamma^2\mathrm{var}(\max(\delta,\theta))$.

[19]Some interesting evidence that countries' growth rates have actually tended to increase over time is reported in Romer (1986). Also, Baumol (1986) presents (graphically on p. 1080) some data in which it appears that *dispersion* in growth rates across countries with similar per capita incomes declines as per capita income rises.

[20]The standard solvency conditions apply to the optimalization problems presented in the Appendix. The solvency condition associated with (P6) is $\lim_{t\to\infty} \int p_{t+1} k_{t+1}\, d\theta^{t+1}\, d\epsilon^{t+1} \geq 0$.

[21]Assumption (D), footnote 9, is relevant here.

[22]For instance, consider the case when $\phi_t = 0$. Here by subtracting (A.10) from (A.9) one obtains $\int \hat{p}_{t+1}(\theta_{t+1}+\epsilon_{t+1}-\delta)d\theta_{t+1}\ d\epsilon_{t+1} \leq 0$. The formula for $\hat{p}_{t+1}$ given in (iiib) then allows this expression to be rewritten as $\int((\theta_{t+1}+\epsilon_{t+1}) - \delta)$ $d\max[w(k_{t+1}),v(k_{t+1}-\alpha)]/dk_{t+1}\ dF(\theta_{t+1})dG(\epsilon_{t+1}) \leq 0$, as stated in the text. The other two cases, $0 < \phi_t < 1$, and $\phi_t = 1$, can be analyzed in similar fashion.

[23]Strictly speaking, the feasibility condition (A.15) is too generous. An agent could be transferred to the market sector in t with zero capital, yet the resource cost of moving would still have to be absorbed. To avoid this problem, one could alternatively use the indicator function $I_t(j)$ which is defined to be unity for those states in period t when agent j moves, and to be zero otherwise. Note that (A.15) still holds for the original formulation of the indicator variable, though, since $I(\hat{k}_t(j)) \leq I_t(j)$.

[24]The left–hand sides of equations (A.16) and (A.17) represent the present–value of expenditure in the developed and less–developed sectors from the proposed plan, while the right–hand sides show the present–value of income.

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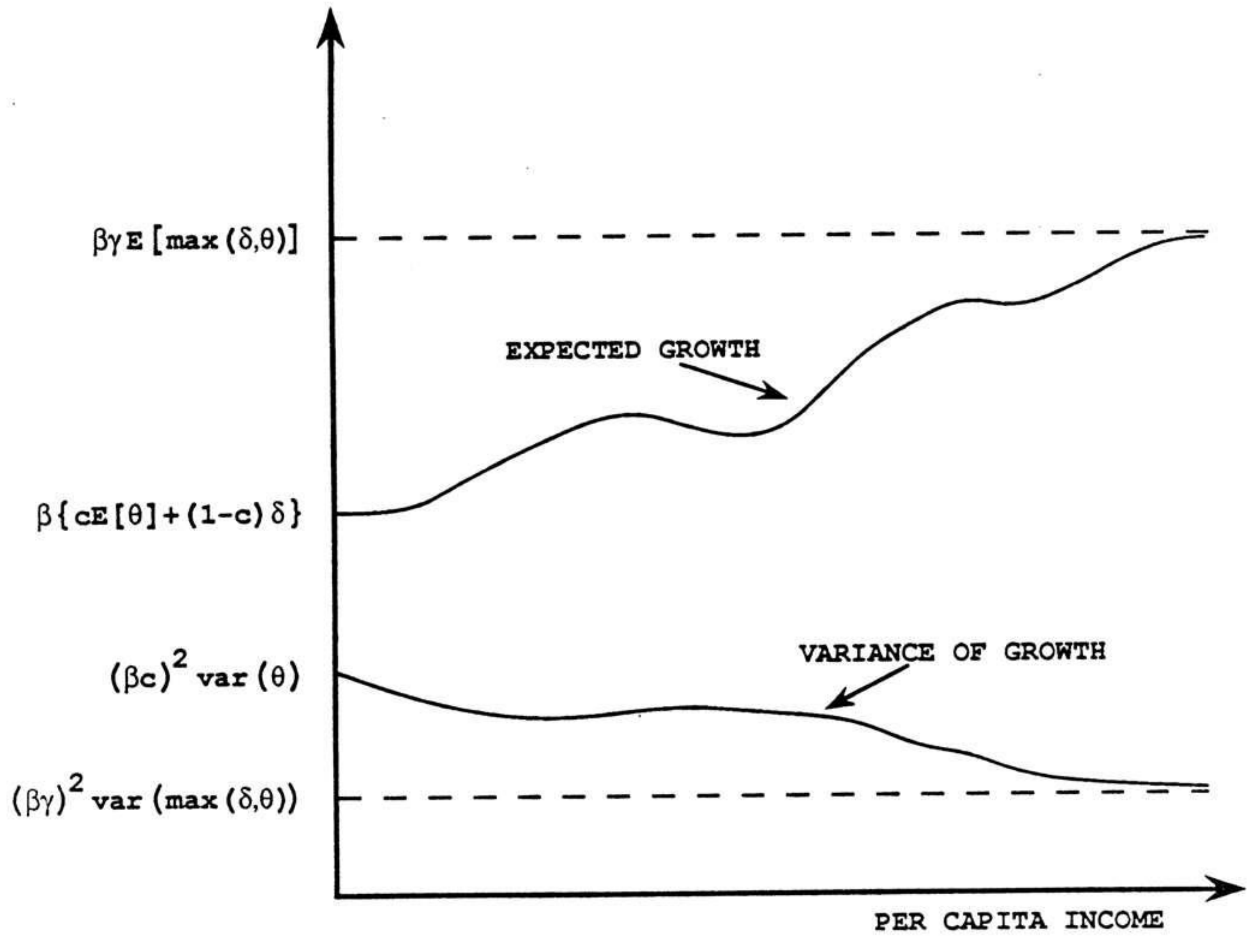

Figure 1: Empirical Implications