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ECONOMIC FLUCTUATIONS WITHOUT
SHOCKS TO FUNDAMENTALS;
OR, DOES THE STOCK MARKET DANCE
TO ITS OWN MUSIC?

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I can calculate the motions of the heavenly bodies, but not the madness of people.

Sir Isaac Newton

I. Introduction

The dramatic 23 percent decline in the U.S. stock market in October 1987 sent shock waves through the economy, policymakers, and economists. Non-economists and economists alike scurried to find some previously unforeseen new development that might explain the crash. Could the crash have been caused by the sudden appearance of a comet, by a supernova explosion in a distant galaxy, or by a startling change in sunspot activity? Or perhaps it was caused by psychological factors? Until recently, most economists would have pooh-poohed such ideas as crazy.

To an economist (and also to market analysts on Wall Street) it seems natural to look for changes in technological factors or consumer tastes as possible explanations. After all, one would expect that a new technological development in the computer industry would drive up the stocks of computer firms and that a sudden shift in consumer tastes toward eating out would drive up the stocks of fast-food chains and restaurants. This surely explains why a considerable amount of market research on Wall Street consists of keeping track of technological developments and shifts in consumer trends. It is not easy, however, to see why there should be any relationship between extra-terrestrial happenings and new developments in technology or consumer tastes.

Thus it is that most of the currently popular models of economic fluctuations are based on recurring random shocks to economic fundamentals. These fundamentals consist, of course, of consumer tastes and the technological possibilities available to firms. Shocks to consumer tastes affect the demands for various goods, whereas shocks to technology--by affecting costs of

production--affect the supplies of various goods. In this way, these shocks give rise to fluctuations in prices and quantities. In the absence of such continued random influences on tastes or technology, the currently popular models would predict that the economy would (in a reasonable amount of time) settle down into a steady state, with no fluctuations whatsoever.¹

The stock market crash has revived interest in the possibility of explaining fluctuations without such shocks to fundamentals. One clear reason for this renewed interest has been the inability of economists or market analysts to find any new developments in tastes or technology which could explain a crash of that magnitude. The appeal to psychological factors or, in general, random factors unrelated to fundamentals is, however, not new. In 1936, toward the end of the Great Depression, John Maynard Keynes published his classic General Theory of Employment, Interest, and Money, in which he attributed business fluctuations not to random shocks to tastes or technology, but to the animal spirits of investors. That is, investors may be seized by moods of optimistic or pessimistic expectations which bear no necessary relation to any changes in tastes or technology. Keynes also asserted that such expectations on the part of investors need not necessarily be irrational. The moods of optimism or pessimism can cause investors to either expand or contract investment spending; this, in turn, can lead to either an overall economic expansion or a contraction, thereby justifying the optimistic or pessimistic expectations. Thus, these animal spirits can become self-fulfilling and hence be rational.² This alternative view of business fluctuations may be described as nonfundamental, intrinsic, or endogenous.

In this article I explain how economic fluctuations can occur without shocks to fundamentals. This is not to say that taste or technology shocks do not exist or that they are totally unimportant. Instead, the pur-

pose here is to try and understand whether there exist forces intrinsic to an economic system that tend toward instability; whether such instability is bad from the point of view of economic welfare; and, if so, what sorts of policies or institutions may be set in place to avoid such instability and put the economy on a steady course.³

To explain these issues, I describe a model that illustrates intrinsic fluctuations and the role of animal spirits. The model is a simplified version of existing ones that are part of the burgeoning literature on intrinsic fluctuations. Throughout the paper, the emphasis is on explaining how such fluctuations can arise in an environment in which the economic fundamentals consisting of tastes and technology are unchanging over time. Further, expectations are assumed to be rational. Without this assumption, one can explain anything, given a sufficiently perverse or irrational view of the world. Requiring beliefs to be rational imposes a notion of consistency between beliefs and reality and rules out explanations based on a pathological view of the world.

The model described is a simple model of stock price determination in which consumers may hold many possible sets of beliefs that may be self-fulfilling and hence rational. Some of these beliefs may even be based on random factors totally unrelated to the objective factors of tastes and technology.⁴ Furthermore, some of these beliefs lead the economy to a steady course while many others set the economy on a wildly fluctuating path.⁵

Can such models explain the qualitative and quantitative properties of economic fluctuations in real economies? Perhaps. But I attempt no such explanations here, since the model described is chosen for its expositional simplicity rather than its ability to explain observed business fluctuations. I believe it is much too early to judge the empirical applicability of

these models, for only recently have economists started analyzing such models. Further development and elaboration of such models may prove to be empirically useful, in addition to being theoretically insightful.

Are there any policy implications that emerge from the study of these models? Yes, although these implications are subject to some important qualifications. I show that for the model there exist very simple policies which can eliminate all fluctuations and set the economy on a unique stable course.

The rest of this paper is organized as follows. In Section II, I describe a simple stock price model. In Section III, I illustrate the variety of fluctuations that can emerge from this model. In Section IV, I describe a simple stabilization policy and Section V concludes. The Appendix contains the mathematical details of analyzing the model.

II. A Stock Price Model

Consider an environment that is completely stationary and in which there is one unit of a perfectly divisible asset (a stock, if you like) which pays a constant and known stream of dividends forever. Consumers can purchase shares in this stock with a view to obtaining dividends and capital gains when the shares are sold. The current stock price depends on the current demand, which in turn depends on the capital gains (or losses) that consumers expect. This, in turn, depends on the price at which the stock can be sold, which again depends on the demand for the stock on the part of future buyers. I show by means of examples how, even in a completely stationary environment, the stock price can be subject to wild gyrations. My exposition is based on the models in Grandmont (1985) and Azariadis (1981).⁶

People, Preferences, and Prices

Suppose that at each date t , numbered $1, 2, 3, \dots$, a representative consumer who lives for two periods is born. A consumer born at date t is young at t and old at $t + 1$. Assume that at date 1 , in addition to the young consumer, there is also an old consumer who was born in the previous period. In each period of life, the consumer is endowed with one unit of total time, which may be divided between leisure time and working time. When the consumer is young, each unit of working time results in w_1 units of the consumption good and when old, each unit of working time results in w_2 units of the consumption good. The consumption good is nonstorable and may be either consumed or traded. The old consumer at date 1 is endowed with one unit of a stock which yields a constant dividend stream of d (in units of consumption) each period. The old consumer will, of course, collect the current dividend and then trade the stock for consumption from the young at date 1 . The young consumer, in turn, will hold the shares till period 2 , then collect the dividend and sell the shares to the new young at date 2 . This process then goes on forever.

Let $c_1(t)$ and $c_2(t)$ be the consumptions at date t of the young and the old consumers, respectively, and let $\ell_1(t)$ and $\ell_2(t)$ be the amounts of leisure time enjoyed by the young and the old. The young consumer at each date t maximizes lifetime utility, denoted by u and given by

$$(1) \quad u = U(c_1(t), \ell_1(t)) + V(c_2(t+1), \ell_2(t+1)).$$

In equation (1), the functions $U(\cdot)$ and $V(\cdot)$ represent utility derived in the first and second periods of life. Utility in each period of life depends on consumption and the amount of leisure time enjoyed in that period.

The budget constraints faced by the consumer are

$$(2) \quad c_1(t) = w_1[1 - \ell_1(t)] - p(t)s(t)$$

$$(3) \quad c_2(t+1) = w_2[1 - \ell_2(t+1)] + [p^e(t+1) + d]s(t).$$

In equations (2) and (3), $p(t)$ is the stock price at t , $p^e(t+1)$ is the consumer's expectation (held with certainty) of the stock price at $t + 1$, and $s(t)$ is the quantity of shares purchased by the young at t . Equation (2) states that consumption by the young equals the total output produced when young minus the value of shares purchased. Note that $[1 - \ell_1(t)]$ is the amount of time spent working when young, and hence $w_1[1 - \ell_1(t)]$ is the output produced when young. Equation (3) states that consumption by the old equals the total output produced when old plus the dividends on shares held and the proceeds from the sale of shares. The consumer chooses lifetime consumptions, leisure times, and the demand for shares $s(t)$ in order to maximize lifetime utility given by (1).

The determination of the stock price is shown in Figure 1. It is easy to show that the demand for shares depends on $p(t)$ and $p^e(t+1)$ and that demand is downward sloping in the current price $p(t)$. (See the Appendix for a derivation.) The supply of shares is perfectly inelastic at one unit since there is a fixed amount of one unit of the stock available, all of which is supplied by the old inelastically. Thus, the equilibrium condition for shares is given by

$$(4) \quad s(t) = 1.$$

Since the demand for shares depends on the consumer's expectation of next period's price, it follows that the current equilibrium price of shares also depends on the price expected to prevail next period. Now assume that

the expectations of consumers are rational; that is, the price that consumers at t expect will prevail at $t + 1$ is in fact the actual price at $t + 1$. Therefore, we have

$$(5) \quad p^e(t+1) = p(t+1).$$

It follows that the current equilibrium price $p(t)$ depends on next period's price $p(t+1)$. This relationship is illustrated in Figure 2 for a particular choice of the utility functions $U(\cdot)$ and $V(\cdot)$. These functions have been chosen in such a way as to generate a hump-shaped curve.

It is important to understand the reason for the particular hump-shaped curve shown in Figure 2, since this shape is the source of fluctuations to be described. This shape arises due to the conflict between the substitution effect and the wealth effect of a change in $p(t+1)$ on the demand for shares. These effects may be explained as follows. An increase in $p(t+1)$ increases the rate of return on the stock, thereby making saving for future consumption more attractive. This induces the consumer to reduce current consumption and therefore increases the demand for shares. This is the substitution effect. However, an increase in $p(t+1)$ also increases the value of savings in the form of shares and therefore increases wealth. This perceived increase in wealth causes the consumer to increase current (as well as future) consumption. The increase in current consumption reduces the demand for shares. This is the wealth effect. Consequently, the substitution effect and the wealth effect of an increase in $p(t+1)$ have opposite effects on the demand for shares (as can be seen in Figure 2). At low values of $p(t+1)$ the substitution effect dominates the wealth effect; as a result, an increase in $p(t+1)$ increases the demand for shares thereby increasing the current price $p(t)$. At high values of $p(t+1)$ the wealth effect dominates the substitution effect; as

a result, an increase in $p(t+1)$ reduces the demand for shares and hence also $p(t)$. This conflict between the two effects is the reason for the hump-shaped relationship between $p(t)$ and $p(t+1)$ --a relationship which yields a variety of possibilities for fluctuations.

Since Figure 2 gives a relationship between the stock price today and the stock price tomorrow, it is possible to calculate some equilibrium time paths for the stock price for various parameter values. We can also calculate time paths for the real interest rate and total output by making use of the following relationships. The real interest rate $r(t)$ from t to $t + 1$ is given by

$$(6) \quad r(t) = [p(t+1) - p(t) + d]/p(t).$$

There is a simple linear relationship between total output $y(t)$ and the stock price $p(t)$ for the chosen utility functions $U(\cdot)$ and $V(\cdot)$; that is,

$$(7) \quad y(t) = a + bp(t).$$

Equation (7) is derived in the Appendix.

III. Illustrations of Intrinsic Fluctuations

In this section, I illustrate the variety of fluctuations that can be generated by the model. Each illustration corresponds to a different choice of utility functions.

At this point it is worth emphasizing that each sample economy illustrated is completely stationary in terms of its characteristics over time. Each generation looks exactly the same as any other in terms of its tastes, endowments, and productivities. That is, the fundamentals of each economy are constant over time. In spite of this constancy in the fundamentals, we will see that it is possible for the stock price, real interest rate, and output to exhibit pretty wild behavior.

□ Predictable and Bizarre Paths

In Figure 4 we see that there is indeed a constant time path for the stock price. This price, denoted p^* , corresponds to the intersection in Figure 3 of the forty-five degree line and the hump-shaped curve between $p(t)$ and $p(t+1)$. If all consumers expect that the price next period will be p^* , then it will be p^* today and hence forever. From equations (6) and (7), it follows that the interest rate and output will also be constant over time in this example. However, Figure 4 also shows another time path for the stock price along which it follows an up-and-down cyclical path which repeats every two periods. Therefore, equations (6) and (7) imply that along this alternative path, the interest rate and output will also exhibit a similar pattern. In Figures 5 and 6 we see the generation of a four-period cycle in stock prices and hence also in the interest rate and output. Figures 7 and 8 show how a three period cycle is generated.

The model can also generate some bizarre time paths. Figure 9 depicts a pretty bizarre time path for the stock price in which it is hard to discern any strictly periodic pattern. Figure 10 shows a pattern that is hard to distinguish from a time path that might be generated due to the presence of random shocks, even though such shocks have been explicitly ruled out in constructing these illustrations.

Although we have shown only a few of the possible time paths of the stock price for each example, there are in fact many possible time paths for each set of parameter values. For instance, the example that gives rise to the four-period cycle of Figure 6 can also give rise to a two-period cycle. The example that produces the bizarre path of Figure 9 can also give rise to cycles of two, four, and eight periods as well as periods of some higher powers of two. And the parameter values used in Figure 8 can also give rise

to cycles of every integer period as well as giving rise to the bizarre sorts of time paths in Figure 9, which seem to lack any periodic pattern.⁷ Furthermore, in every example there is an equilibrium path along which the stock price is constant over time. This is because in all of these examples, the nature of the relationship between $p(t)$ and $p(t+1)$ is similar to the hump-shaped curve shown in Figure 2. This constant time path is indicated by the line marked p^* on the figures.

□ Animal Spirits and Hemlines

We now turn to an illustration of the kind of time path that can be generated when consumers are driven by animal spirits. Suppose consumers believe the following maxim:

When hemlines are up, stocks will be up; and when hemlines are down, stocks will be down.

Suppose further that the fashion industry decides randomly when hemlines will be up and when they will be down, perhaps by consulting a different astrologer each period. Even though such randomness has no connection with the tastes, endowments, or productivities of consumers in the model, it turns out that stock prices (and hence interest rates and output) respond to such extraneous randomness.

I now explain how such beliefs, which have no relation to economic fundamentals, can be self-fulfilling. Let the indices i and j indicate the state of hemlines at dates t and $t + 1$, respectively, and suppose that each index takes the value 1 or 2, depending on whether hemlines are high or low. Let p_i be the stock price in state i , s_i the demand for shares, $c_1(i)$ and $c_2(i)$ the consumptions of the young and the old in state i , and $l_1(i)$ and $l_2(i)$ the leisure times of the young and the old in state i . Let π_{ij} be the

probability that the hemline state at $t + 1$ is j , given that the hemline state at t is i . The young consumer at t maximizes expected utility given the state i at t . This is denoted by $E(u|i)$. Using (1), the expression for expected utility can be written as

$$(8) \quad E(u|i) = U(c_1(i), x_1(i)) + \sum_j \pi_{ij} V(c_j(j), x_2(j)).$$

In equation (8), we are simply adding up the utilities in each possible state in the second period of life, weighted by the respective probabilities.

The consumer's budget constraints can be written, by analogy with (2) and (3), as

$$(9) \quad c_1(i) = w_1[1 - x_1(i)] - p_1 s_i$$

$$(10) \quad c_2(j) = w_2[1 - x_2(j)] + (p_j + d)s_i.$$

The interpretation of the constraints (9) and (10) is similar to that for (2) and (3).

It is now possible to solve for the consumer's demand for shares. We can then impose the equilibrium condition (4) and solve for the prices p_1 and p_2 . (Details are provided in the Appendix.) These prices together with the probabilities π_{ij} determine the possible time paths for the stock price. Such an equilibrium is self-fulfilling, or rational, because the distribution of future prices on the basis of which the consumer determines the demand for shares is in fact the actual distribution of prices which lead to equilibrium between the demand and supply of shares. Thus, the consumer's beliefs are consistent with the actual behavior of equilibrium prices.

Figure 11 shows an example in which the stock price fluctuates randomly between two values, marked p_1 and p_2 , with probabilities as noted. The reason for such behavior is the following. If the current state i of hemlines

is different (say 2 instead of 1), then the probabilities π_{ij} for the future state j of hemlines will be different. Given the belief held by consumers about the relationship between hemlines and stock prices, the probabilities π_{ij} affect the consumer's expectation of tomorrow's stock price. This influences the consumer's current demand for the stock and hence its current price.

For this result, it is indeed important that the probabilities π_{ij} vary as i varies. That is, the probability distribution of future hemline states must differ if the current hemline state is different. Otherwise, the consumer's expectation of tomorrow's stock price will be independent of the current state and hence so will be the consumer's demand for shares. Consequently, the current equilibrium price will be the same no matter what the current state is. Rational expectations then imply that the stock price must be constant forever.

□ Summary

So far we have seen many examples in which even though there is always a path along which stock prices and other variables are constant, there are also many other equilibrium paths along which stock prices and other macroeconomic variables can exhibit very interesting fluctuations. Therefore, it follows that the economy can exhibit instability even when there is a stable path that is attainable if only consumers would believe in it.

IV. Policy Implications

What implications does this simple stock price model have for consumer welfare and government policy? It turns out that every one of the equilibrium paths we have studied has the property of being Pareto optimal; that is, none of the paths can make some consumer better off without hurting some other consumer.⁸ Therefore, there is no government policy that will

improve everyone's lot. However, this conclusion depends on how seriously we take the assumption of perfect foresight. Remember that every one of the equilibrium paths was constructed on the assumption that it was perfectly foreseen by all consumers. If consumers make occasional mistakes in expectations, then the welfare properties of the paths discussed may no longer be true. Consequently, there may be a role for government policies that would enhance the welfare of all consumers.

The perfect foresight assumption may not seem unreasonable if the economy has been moving along a constant path or perhaps along a path with an easily discernible cyclical pattern. Then we may reasonably expect that consumers, by looking at the past behavior of stock prices, will be able to form accurate forecasts of their future behavior, somewhat like the chartists on Wall Street. However, some of the paths we have seen (for instance, those in Figures 9 and 10) are so complex that it is hard to imagine how anyone could form an accurate forecast of the future behavior of stock prices based on past observations.⁹ When such forecasting seems difficult, the assumption of rational expectations may be somewhat questionable. At the very least, however, one can argue that the government ought to pursue policies that put the economy on a stable path, thereby making it easier for consumers to form accurate forecasts of the future and thus keeping the economy moving along a stable path. The argument for this approach is simply that mistaken expectations are much more likely when the economy is following a highly unstable path.

Do there exist government policies that can eliminate all the highly fluctuating paths we have seen are possible and push the economy inexorably onto a constant path with no fluctuations whatsoever? For the stock price model, there is in fact a fairly simple policy that can achieve this objec-

tive: Let the government announce a benchmark stock price \hat{p} , which is less than w_1 , and also levy a tax (or subsidy, if negative) at the proportional rate $[1 - \hat{p}/p(t)]$ on the value of shares held by the old at each date t (including the initial old). The proceeds of this tax are handed over to the young at t as a lumpsum rebate (or tax, if negative), denoted $\tau(t)$. This policy will alter the budget constraints (2) and (3) as follows:

$$(11) \quad c_1(t) = w_1[1 - \lambda_1(t)] - p(t)s(t) + \tau(t)$$

$$(12) \quad c_2(t+1) = w_2[1 - \lambda_2(t+1)] + [p(t+1) + d]s(t) \\ - [1 - \hat{p}/p(t+1)]p(t+1)s(t) \\ = w_2[1 - \lambda_2(t+1)] + (\hat{p}+d)s(t).$$

Along an equilibrium path, the rebate $\tau(t)$ must satisfy the following relationship:

$$(13) \quad \tau(t) = p(t) - \hat{p}.$$

Equation (13) follows because in equilibrium the quantity of shares sold is unity, and hence the value of shares sold is $p(t)$. Therefore, taxes paid must be $p(t)[1 - \hat{p}/p(t)]$, which equals $[p(t) - \hat{p}]$.

It is possible to show that under such a policy, the only possible equilibrium path for the stock price (and hence for the interest rate and output) is a constant one. (See the Appendix for details.) The reason for this is as follows. Since the government taxes away any excess of $p(t+1)$ above the benchmark price \hat{p} [or subsidizes the difference if $p(t+1)$ falls short of \hat{p}], the consumer is, in effect, faced with a future price that is always equal to \hat{p} . Consequently, the consumer's current demand for shares depends on \hat{p} but not on $p(t+1)$. Therefore, the current equilibrium price $p(t)$

also depends on \hat{p} only and is hence constant over time. This simple policy, therefore, eliminates the possibility of all fluctuations and leads the economy onto a stable path. In addition, it is possible to choose the benchmark price \hat{p} in order to ensure that the equilibrium path is Pareto optimal.

The policy just described should be viewed with caution, however. Even though it works for the simple stock price model, it may not work for a more complex model with more assets, uncertainty, and capital accumulation. In practice, the policy is likely to be very difficult to define and implement and may also have undesirable side effects on risk taking and investment. To judge the overall desirability of such a policy, these potential ill effects would have to be weighed against the possible benefits from a stabilized economy and improved forecasting.

V. Conclusion

I now summarize what I think economists are learning by studying the type of model I have described in this paper. I should emphasize that this is a tentative report on a relatively new and ongoing research program rather than a definitive judgment of a ripe old one. The important points seem to be the following.

Most business cycle models explain fluctuations in economic variables as resulting from the effects of taste and technology shocks continually impinging on the economy. While some of these models are able to explain some of the qualitative and quantitative features of observed business fluctuations, there are many phenomena that they have difficulty explaining or for which explanations based on taste or technology shocks strain credibility. Some of these phenomena include the high degree of volatility of the financial markets, the great sensitivity of these markets to apparently unrelated events, and deep depressions like the one in 1929.¹⁰

These considerations suggest that perhaps even in the absence of any taste or technology shocks hitting the economy and even when the environment is completely stationary, the economy might be unstable and exhibit fluctuations. As Keynes argued, the economy might be driven by the animal spirits of investors--spirits which need bear no relation to economic fundamentals. I have shown by example that it is not at all difficult to construct simple model economies that exhibit the above properties. Subject to some important qualifications, I have also shown that there exist appropriate government policies that are capable of eliminating fluctuations.

I therefore conclude that there are important advances in understanding to be gained by further study of models of intrinsic fluctuations.

Appendix

I assume the following form for the utility function in equation (1) of the text:

$$(A1) \quad u = c_1(t)^{\alpha_1} \ell_1(t)^{1-\alpha_1} + \beta [c_2(t+1)^{\alpha_2} \ell_2(t+1)^{1-\alpha_2}]^{1-\mu/(1-\mu)}.$$

I assume that $0 < \alpha_1 < 1$, $0 < \alpha_2 < 1$, $\beta > 0$, and $\mu > 0$, but that $\mu \neq 1$. If $\mu = 1$, the second term in (A1) should be replaced by

$$\beta [\alpha_2 \ln c_2(t+1) + (1-\alpha_2) \ln \ell_2(t+1)].$$

Here I characterize some of the differences between my model and the ones of Grandmont (1985) and Azariadis (1981). The main difference is that the asset in their models pays a zero dividend forever, rather than a positive dividend. One may think of their asset as corresponding to cash. In addition, my specification of the utility function is a special case of that of Grandmont (1985). If I set α_1 to zero and α_2 to unity (so that people consume only leisure when young and only the consumption good when old), then my specification of the utility function becomes a special case of that of Azariadis (1981). Grandmont (1985) analyzes only deterministic fluctuations, like the ones in Figures 3-10, where there is no uncertainty about the time path of prices. Azariadis (1981) analyzes fluctuations, like the hemline example in Figure 11, which are generated by extraneous uncertain events that have no connection to tastes or technology.

Consumer Preferences

I now analyze the consumer's choices of lifetime consumptions, leisure times, and the quantity of shares to buy, given the current stock price and the expected future price.

First, the consumer will equate the marginal rate of substitution between leisure time and consumption in each period of life to the corresponding opportunity cost of leisure time. The opportunity cost of leisure time is w_1 when the consumer is young and w_2 when old. This leads to the following relationships:

$$(A2) \quad (1-\alpha_1)c_1(t)/a_1l_1(t) = w_1$$

$$(A3) \quad (1-\alpha_2)c_2(t+1)/\alpha_2l_2(t+1) = w_2.$$

Second, the consumer will equate the marginal rate of substitution between consumption at t and consumption at $t + 1$ to the gross expected rate of return on the stock. This yields

$$(A4) \quad (\alpha_1/\beta\alpha_2)[l_1(t)/c_1(t)]^{1-\alpha_1} [c_2(t+1)^{\alpha_2} l_2(t+1)^{1-\alpha_2}]^\mu \\ \times [c_2(t+1)/l_2(t+1)]^{1-\alpha_2} = [p^e(t+1) + d]/p(t).$$

We may now substitute for $l_1(t)$ and $l_2(t+1)$ from (A2) and (A3) into equations (2) and (3) of the text to obtain the following simplified expressions for the consumer's budget constraints:

$$(A5) \quad c_1(t) = \alpha_1[w_1 - p(t)s(t)]$$

$$(A6) \quad c_2(t+1) = \alpha_2\{w_2 + [p^e(t+1) + d]s(t)\}.$$

Next we may substitute for $l_1(t)$ and $l_2(t+1)$ from (A2) and (A3), and $c_2(t+1)$ from (A6) into (A4) to obtain

$$(A7) \quad \{w_2 + [p^e(t+1) + d]s(t)\}^\mu = A[p^e(t+1) + d]/p(t).$$

Equation (A7) determines the demand for shares in terms of $p(t)$ and $p^e(t+1)$. The coefficient A in (A7) is given by

$$(A8) \quad A = \beta [\alpha_1 w_1 / (1 - \alpha_1)]^{1 - \alpha_1} [\alpha_2 w_2 / (1 - \alpha_2)]^{(1 - \alpha_2)(\mu - 1)} / \alpha_1 \alpha_2^{\mu - 1}.$$

It may be verified from equation (A7) that the demand for shares is decreasing in the current price $p(t)$. Now substitute equations (4) and (5) in (A7) to get the following relationship between $p(t)$ and $p(t+1)$:

$$(A9) \quad p(t) = f(p(t+1)) \equiv A [p(t+1) + d] / [p(t+1) + d + w_2]^\mu.$$

The curve of $p(t)$ against $p(t+1)$ will be hump shaped (as in Figure 2) provided $\mu > 1$ and $w_2 > (\mu - 1)d$. Any time path for $p(t)$ that satisfies (A9) for all t constitutes a perfect foresight or rational expectations equilibrium.

Output and the Stock Price

A simple relationship between total output and the stock price can be obtained as follows. From equations (2), (3), (4), and (5) we have

$$(A10) \quad c_1(t) + c_2(t) = w_1 [1 - \varepsilon_1(t)] + w_2 [1 - \varepsilon_2(t)] + d = y(t).$$

Substituting from equations (A5), (A6), (4), and (5) into equation (A10), we obtain the following linear relationship between $y(t)$ and $p(t)$:

$$(A11) \quad y(t) = \alpha_1 w_1 + \alpha_2 (w_2 + d) + (\alpha_2 - \alpha_1) p(t).$$

Parameter Values and Simulation Method

I now describe the choice of parameter values and the method of simulation used to produce the intrinsic fluctuations shown in Figures 3-8. I chose these parameter values: $\alpha_1 = 1/4$, $\alpha_2 = 1/2$, $w_1 = 50$, and $d = 0.01$. The parameter μ was varied from 2 to 20 in steps of one half. The parameters w_2

and β were chosen indirectly as follows: Let \bar{p} be the maximum value of $f(p)$ and let p_m be the value of p at which $f(\cdot)$ attains its maximum. These values are illustrated in the accompanying figure, which is based on Figure 2 of the text. The value of p_m may be found by setting the derivative of $f(\cdot)$ equal to zero and solving for p . This yields

$$(A12) \quad p_m = [w_2 / (\mu - 1)] - d$$

$$(A13) \quad \bar{p} = A / \mu^\mu (p_m + d)^{\mu - 1}.$$

We may now substitute for w_2 and A from (A12) and (A13) into (A9) and express the function $f(\cdot)$ in terms of the parameters p_m , \bar{p} , μ , and d . I chose $p_m = 1$ and $\bar{p} = 2\mu + 1$. The implied values of w_2 and β may now be found using (A12), (A13), and (A8). Figure 10 was generated using the same parameter values as above, with the following exceptions: $d = 0.001$, $\mu = 15.0$, and $\bar{p} = 10.0$.

Figures 3-9 were generated by iterating backward using the relationship between $p(t)$ and $p(t+1)$ given by equation (A9). That is, I started with a terminal value of the stock price and worked backward to find the values of the stock price at earlier dates. Figure 10, however, was generated by iterating forwards. This procedure has to be used with care. As the appendix figure shows, there are two possible values of $p(t+1)$, p_1 and p_2 , for some values of $p(t)$. Which value of $p(t+1)$ to choose may depend on whether there exists some value of $p(t+2)$ that can follow $p(t+1)$. For instance, if $p(t)$ is too small, then for whichever value of $p(t+1)$ we pick, there will be no value of $p(t+2)$ that can follow it. If $p(t)$ is somewhat larger, then only the larger of the two values of $p(t+1)$ can be chosen. However, if $p(t)$ is sufficiently large, then either of the two values of $p(t+1)$ is a legitimate choice. In generating Figure 10, this type of situation was resolved by selecting randomly between the two values.

Note that the backward iteration time path in Figure 9 can be extended indefinitely into the future by starting with the terminal price and using the forward iteration procedure that generated Figure 10. As noted in the previous paragraph, to do this it is, of course, necessary that the terminal price be not too low. Therefore, the time path in Figure 9 does indeed constitute a legitimate equilibrium time path that satisfies (A9) for all t .

Solving the Hemline Example

I now show how to solve the hemline example presented in the text (and depicted there in Figure 11). Substitute from equations 1 and (A1) into equation (8) to get the following expression for expected utility:

$$(A14) \quad E(u|i) = c_1(i)^{\alpha_1} x_1(i)^{1-\alpha_1} + \beta \sum_{j=1}^2 \pi_{ij} [c_2(j)^{\alpha_2} x_2(j)^{1-\alpha_2}]^{1-\mu} / (1-\mu).$$

In deriving (A14), it is implicitly assumed that the young consumer at date t is born after the current state i is realized. In the contrary case, equation (A14) would have to be modified by also adding up the utilities in each state when young, weighted by the respective probabilities. In addition, we would have to recognize the possibilities for risk sharing between the young and the old, which will alter the budget constraints (9) and (10). By assuming that the young consumer is born after the current state is realized, we rule out such risk-sharing arrangements. This assumption leads to (A14) and the budget constraints (9) and (10). The assumption is indeed very crucial because in the contrary case it can be shown that it is impossible for stock prices to fluctuate in response to extraneous events like hemlines or sunspots. For a demonstration of this statement, see Azariadis (1981).

I now analyze in several steps the consumer's choice problem. As before, the consumer equates the marginal rate of substitution between leisure and consumption in each period and in each state to the corresponding oppor-

tunity cost. This yields the following conditions, analogous to (A2) and (A3):

$$(A15) \quad (1-\alpha_1)c_1(i)/\alpha_1 w_1(i) = w_1$$

$$(A16) \quad (1-\alpha_2)c_2(j)/\alpha_2 w_2(j) = w_2.$$

Now substitute equations (A15) and (A16) into equations (A14), (9), and (10) to simplify them as follows:

$$(A17) \quad E(u|i) = \left[(1-\alpha_1)/\alpha_1 w_1 \right]^{1-\alpha_1} c_1(i) \\ + \left\{ \beta \left[(1-\alpha_2)/\alpha_2 w_2 \right]^{(1-\alpha_2)(1-\mu)} \times \sum_{j=1}^2 \pi_{ij} c_2(j)^{1-\mu} / (1-\mu) \right\}.$$

$$(A18) \quad c_1(i) = \alpha_1 (w_1 - p_i s_i)$$

$$(A19) \quad c_2 = \alpha_2 [w_2 + (p_j + d) s_i].$$

We can now substitute (A18) and (A19) in (A17) and maximize expected utility by choice of s_i . This leads to the following condition:

$$(A20) \quad p_i = A \sum_{j=1}^2 [\pi_{ij} (p_j + d)] / [w_2 + (p_j + d) s_i]^\mu.$$

We may now substitute the equilibrium condition (4) in (A20) to obtain

$$(A21) \quad p_i = A \sum_{j=1}^2 [\pi_{ij} (p_j + d)] / (w_2 + p_j + d)^\mu = \sum_j \pi_{ij} f(p_j), \quad i = 1, 2$$

where $f(\cdot)$ is the same function as in (A9).

We thus have two equations in the two unknowns, p_1 and p_2 . Note that there is always a solution in which p_1 and p_2 both equal p^* . When p_1 equals p_2 , the two equations in (A21) collapse to a single equation because the sum of probabilities ($\pi_{i1} + \pi_{i2}$) must be unity for each i . The resulting equation is the same as equation (A9) with p_t equal to p_{t+1} , and the solution

is p^* . This solution corresponds to the case where the stock price is unaffected by people's belief about hemlines and the stock market. If we can find probabilities π_{ij} such that there is a solution in which p_1 and p_2 are different, then we have an example where the stock price responds to "rational" animal spirits.

Such an example can be constructed as follows. First, substitute $\pi_{12} = 1 - \pi_{11}$ and $\pi_{21} = 1 - \pi_{22}$ in equation (A21) and solve for π_{11} and π_{22} to obtain the following equations:

$$(A22) \quad \pi_{11} = [f(p_2) - p_1] / [f(p_2) - f(p_1)]$$

$$(A23) \quad \pi_{22} = [p_2 - f(p_1)] / [f(p_2) - f(p_1)].$$

I look for a solution such that $p_1 > P^* > p_2$ and such that the points $(p_1, f(p_1))$ and $(p_2, f(p_2))$ lie on the downward sloping branch of the curve $f(\cdot)$. It follows that we must have $f(p_2) > f(p_1)$. (See the appendix figure for an illustration of this.) Since the probabilities π_{11} and π_{22} must each be between zero and one, we require that p_1 and p_2 satisfy the following conditions:

$$(A24) \quad f(p_1) < p_1 < f(p_2)$$

$$(A25) \quad f(p_1) < p_2 < f(p_2).$$

The appendix figure shows two values, p_1 and p_2 , that satisfy the two inequalities. The associated probabilities π_{ij} can be calculated from (A22) and (A23).

For the examples presented here, it is important that the slope of the curve at p^* , shown in the appendix figure, be negative and greater than one in absolute value in order to generate periodic cycles other than the

constant time path corresponding to p^* . This slope condition is also crucial for generating the hemline example of Figure 11. Otherwise the inequalities (A24) and (A25) cannot be met. In fact, it turns out that for the type of model presented here, such a hemline equilibrium will exist if and only if there exists a two-period cycle such as the one generated in Figures 3 and 4 (see Azariadis and Guesnerie 1986). A heuristic argument for the if part of this statement can be made as follows. A two-period cycle corresponds to having π_{11} and π_{22} each equal to zero. Therefore, it will generally be possible to find differing values for p_1 and p_2 if π_{11} and π_{22} are both positive but small. The only if part is not generally true. For example, if the $f(\cdot)$ function has a slope that is positive and greater than one at p^* (this can never happen in the present model), then there cannot be a two-period cycle. However, it is possible to find differing values for p_1 and p_2 and values for the probabilities π_{11} and π_{22} that satisfy equations (A22) and A(23).

As noted in the text, it is also important that the probabilities π_{ij} depend on i . Otherwise, the only solution to equations (A21) is $p_1 = p_2 = p^*$. This follows because the right side of (A21) is then independent of i .

The Tax/Subsidy Policy Implication

I now analyze the tax/subsidy policy described in the text. The consumer's choices lead to the same conditions as before, namely, equations (A2), (A3), and (A4), except that $p^e(t+1)$ is replaced by \hat{p} . This is because the after-tax gross rate of return on the stock is given by $(\hat{p}+d)/(p(t))$. As before, we may substitute for $\lambda_1(t)$ and $\lambda_2(t+1)$ from (A2) and (A3), $s(t)$ from (4), and $\tau(t)$ from (13) into equations (11) and (12) to obtain

$$(A26) \quad c_1(t) = \alpha_1(w_1 - \hat{p})$$

$$(A27) \quad c_2(t+1) = \alpha_2(w_2 + \hat{p} + d).$$

Next, we may substitute for $\ell_1(t)$ and $\ell_2(t+1)$ from (A2) and (A3), and $c_2(t+1)$ from (A27) into equation (A4) and replace $p^e(t+1)$ by \hat{p} to get the following version of equation (A9):

$$(A28) \quad p(t) = A(\hat{p} + d) / (\hat{p} + d + w_2)^\mu.$$

This proves that the equilibrium stock price will be constant over time. The equilibrium price under such a policy need not equal the benchmark price \hat{p} . This will happen only when \hat{p} is the same as p^* , where p^* is the price depicted in the appendix figure. This follows from equations (A9) and (A28), and the figure. Further, if the government announces p^* as the benchmark price, then it can be seen from equation (13) that along the equilibrium path there will be no taxes or rebates.

Footnotes

¹For a recent example of one such model, see Prescott 1986. The fluctuations in Prescott's model are driven by shocks to technology.

²Expectations are said to be rational if beliefs regarding possible future events are (probabilistically) correct, that is, verified by the actual future course of events. In a world without uncertainty, this amounts to having perfect foresight regarding future developments.

³It should be clear that allowing for taste or technology shocks would only magnify the fluctuations.

⁴This may be viewed as capturing Keynes' notion of animal spirits. Fluctuations resulting from such beliefs are often referred to as sunspot fluctuations (see Cass and Shell 1983).

⁵Models exhibiting these features have been studied extensively by many people, among whom the following are prominent: Costas Azariadis (1981), David Cass and Karl Shell (1983), and Jean-Michel Grandmont (1985).

⁶The mathematical details of solving the model are given in the Appendix, where I also note the (very minor) differences between my exposition and the models of Grandmont (1985) and Azariadis (1981).

⁷The variety of different periodic cycles that can exist simultaneously was discovered by the Russian mathematician A. N. Sarkovskii and systematized in a beautiful mathematical theorem. See Grandmont 1985 (pp. 1019-20) for a more detailed explanation.

⁸This property is named after the Italian economist and sociologist Vilfredo Pareto (1848-1923). The converse of this property, that it is possible to improve someone's welfare without hurting anyone else, is known as Pareto nonoptimality. In this case it would generally be possible to find government policies that would make everyone better off.

⁹This is only partially true in the present model because of its very simple structure. For instance, one can use past data on stock prices to plot the current price against the future price, as in Figure 2. In a more complex model such simple procedures will no longer be useful.

¹⁰For instance, Keynesians like James Tobin and Franco Modigliani have ridiculed Neoclassical economists by saying that the only way to explain the Great Depression on the basis of Neoclassical theories is to attribute it to a mass attack of laziness.

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