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Frictions in Asset Pricing and Macroeconomics

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Working Paper 518

September 1993

ABSTRACT

This paper is about a useful way of taking account of frictions in asset pricing and macroeconomics. I start by noting that complete frictionless markets models have a number of empirical deficiencies. Then I suggest an alternative class of models with incomplete markets and heterogenous agents which can also accommodate a variety of other frictions. These models are quantitatively attractive and computationally feasible and have the potential to overcome many or all of the empirical deficiencies of complete frictionless markets models. The incomplete markets model can also differ significantly from the complete frictionless markets model on some important policy questions.

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1. Introduction

This paper is about a useful way of taking account of frictions in asset pricing and macroeconomics. I start by noting that complete frictionless markets models have a number of empirical deficiencies. Then I suggest an alternative class of models with incomplete markets and heterogeneous agents which can also accommodate a variety of other frictions. These models are quantitatively attractive and computationally feasible and have the potential to overcome many or all of the empirical deficiencies of complete frictionless markets models. By quantitatively attractive I mean that these models can be calibrated using macro and micro data in the same way as the standard representative agent growth model of Brock-Mirman is used for studying growth, business cycles with and without money, asset pricing, etc. The incomplete markets model can also differ significantly from the complete frictionless markets model on some important policy questions.

The models are of the Bewley [undated, 1984], Lucas [1980], and Lucas and Prescott [1974] type and contain a large number of infinitely lived agents subject to idiosyncratic shocks to earnings and/or tastes which cannot be insured against. This is the market incompleteness. Even if all agents are identical ex-ante, they will become heterogeneous ex-post. Because the models have heterogeneity they can also accommodate other frictions like borrowing/liquidity constraints. Because of the large number of agents and independent risks there may be little or no aggregate uncertainty. These models capture the notion that uncertainty at the individual level is much more important than uncertainty at the aggregate level.

Because of incomplete insurance markets, there is a self-insurance motive for accumulating and trading assets which has important implications for a variety of empirical issues including the following: (i) variability and comovement of individual consumptions and aggregate consumption, (ii) wealth and income distributions, (iii) portfolio compositions at different wealth levels, (iv) asset returns, (v) importance of precautionary saving in capital accumulation, (vi) taxation.

2. EMPIRICAL DEFICIENCIES OF THE COMPLETE FRICTIONLESS MARKETS MODEL

The following are well known implications of the complete frictionless markets model which are quite counterfactual.

(i) Individual consumptions should be perfectly correlated with each other and with per capita consumption, (ii) each individual's consumption should vary as much as any one else's and as much as per capita consumption (if individuals' risk aversion coefficients are not too different), (iii) an individual's position in wealth distribution should not vary much over time or across states (if individuals' risk aversion coefficients are not too different), (iv) every individual should hold some amount of risky assets with favorable returns. If individuals' risk aversion coefficients are not too different then all individuals should hold roughly similar portfolios, (v) there is no role for asset trading and no predictions regarding transactions volumes and transactions velocities of different assets, (vi) the risk-free rate is too high and the equity premium is too low.

Aiyagari [1993] describes in some detail the derivation of the above implications and the wealth of empirical evidence against these.

3. GROWTH MODEL WITH UNINSURED IDIOSYNCRATIC SHOCKS

Consider an economy with a continuum of infinitely lived agents of measure unity who receive idiosyncratic shocks to their labor endowments. Let l_t denote an individual's labor endowment and suppose that it is i.i.d. across agents, and follows some Markov process over time. We normalize per capita labor endowment to unity so that $E(l_t) = 1$. There are no aggregate shocks. We describe the steady state of such an economy without insurance markets but with trading in risk-free assets - capital, government debt and private debt. In this steady state there will be fluctuations in an individual's consumption, income and wealth but per capita variables and cross section distributions will be constant over time.

Let c_t, a_t, w, r, and τ denote an individual's consumption in period t, an individual's assets at the beginning of period t, the wage rate, return on assets, and the lump sum tax, respectively. The typical consumer maximizes the expected discounted sum of utilities of consumption $E_0 \Sigma_{t=0}^{\infty} (1+\rho)^{-t} U(c_t)$ subject to:

(3.1) $$c_t + a_{t+1} \leq wl_t + (1+r)a_t - \tau, \; a_t \geq -d.$$

In (3.1), d is a borrowing limit.

Let K be the per capita capital stock, N the per capita labor input, and let f(K,N) be a standard neoclassical aggregate production function. Note that N equals unity in equilibrium. Then, from producer profit maximization we have,

(3.2a) $$r = f_1(K,1) - \delta,$$

(3.2b) $$w = f_2(K,1),$$

where δ is the depreciation rate of capital.

Let G denote per capita government consumption and let B denote per capita government debt. Then the government budget constraint is given by

(3.3) $$G + rB = \tau$$

Let A denote per capita assets held by consumers. In a steady state equilibrium of this economy we must have

(3.4) $A = K + B.$

The steady state of this economy is characterized by an interest rate r^* which solves

(3.5) $\alpha^*(r;G,d+B) = \kappa(r),$

where $\alpha^*(r,.)$ is the per capita assets desired by consumers (net of government debt) as a function of the interest rate and $\kappa(r)$ is the per capita capital expressed as a function of the interest rate[1].

The steady state determination under incomplete markets (IM) is shown in figure 1 and is marked IM. The crucial feature of this picture is that $\alpha^*(r;G,d+B)$ tends to infinity as r approaches the time preference rate ρ from below. The intuition behind this is that when r equals ρ the consumer would like to maintain a smooth marginal utility of consumption profile. However, since there is some probability of receiving a long string of low labor endowment shocks, the only way for the consumer to maintain a smooth marginal utility of consumption profile would be to have an infinitely large amount of assets. Note also that under incomplete markets, due to the precautionary motive and borrowing constraints, the consumer will hold

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[1]Equation (3.5) is obtained in the following way. We can use (3.2a) to define K as a function of r which is the right side of (3.5). Also, (3.2a and b) can be used to write w as a function of r. Let this be denoted $\omega(r)$. Then, rewrite the consumer's budget constraint by substituting for taxes τ from (3.3) into (3.1) and defining $a^*_t = a_t - B$. This leads to the following formulation of the consumer's budget constraint. $c_t + a^*_{t+1} \le \omega(r)l_t + (1+r)a^*_t - G$, $a^*_t \ge -d - B$. The steady state equilibrium condition in the asset market is now given by $A^* = K$. The solution to the consumer's problem yields a decision rule for asset accumulation: $a^*_{t+1} = \alpha(a^*_t, l_t, r, G, d+B)$. This decision rule can be used together with the Markov process for the labor endowment shock to calculate the stationary distribution of assets, denoted by $H(a^*;r,G,d+B)$. This stationary distribution then implies an expression for per capita assets (net of government debt), $A^* = \int a^*dH = \alpha^*(r;G,d+B)$. This is the left side of (3.5).

assets over and above the credit limit to buffer earnings shocks even when r is less than ρ. This would not be the case under complete markets.

FIGURE 1 HERE

Under complete markets (CM) the idiosyncratic labor endowment shock would be fully insured against and effectively there would be no uncertainty in individual earnings. The asset demand function in this case is described by the dotted line in figure 1, leading to the steady state marked CM. This is the usual result that the capital stock satisfies the modified golden rule.

Calibration: The above model can be calibrated in the same way as representative agent models are. One can use data on factor shares to calibrate the production function, use data on G, B, and τ, and estimates for δ and preference parameters. A key feature of this model is that in addition to the usual macro data one also needs to use micro data on, say, earnings, in order to calibrate the l_t process.

Implications of the incomplete markets model:

(1) Individual and per capita consumption: The following quantitative example taken from Aiyagari [1992a] indicates the variabilities of individual consumption, income and assets. The example assumes a constant relative risk aversion utility function with risk aversion coefficient denoted μ, and a first order autoregressive process for the logarithm of the labor endowment shock (equivalently, for the logarithm of earnings) with serial correlation denoted by ρ_e and coefficient of variation (c.v.) denoted σ_e. In this example we assume that $(\sigma_e, \rho_e, \mu) = (.2, .6, 3)$.

	consumption	net income	gross income	assets
c.v.	0.12	0.21	0.27	0.62

The key points to note are the following. Individual consumption varies about ten times as much as U. S. per capita consumption. Since the model only has idiosyncratic shocks, individual consumptions are uncorrelated with each other and with per capita consumption (which is constant). If one were to introduce some aggregate shocks into the model then it would be possible to generate the prediction that individual consumptions would be positively but less than perfectly correlated with each other and with per capita consumption. Since the cross-section distributions coincide with the long run distributions the relative variabilites of consumption, income and wealth have obvious implications for wealth and income distributions.

(2) Wealth and income distributions: The model naturally generates greater dispersion in wealth than in income and skewness in the wealth distribution. Both of these features are qualitatively consistent with the data. Aiyagari [1992a] contains some quantitative illustrations of these features.

(3) Precautionary saving and capital accumulation: As can be seen in figure 1, the steady state with incomplete markets is characterized by a lower interest rate and higher capital as compared to the complete markets case. The additional capital accumulation implies a higher saving rate. This increment in the saving rate may be attributed to precautionary saving. The following quantitative example taken from Aiyagari [1992a] shows how important precautionary saving may be.

(σ_e, ρ_e, μ)	reduction in net return to capital	increase in aggregate saving rate
(.2,0,1)	0	0
(.2,.6,3)	.3%	.6%
(.4,.6,3)	1.4%	3%
(.4,.9,5)	4.5%	14%

As can be seen from the above table, persistence in earnings and risk aversion can have a strong impact on the saving rate. Persistence is important because what matters to the individual is variability in permanent income, and more persistence implies greater variability in permanent income.

(4) Asset returns and asset market transactions: In Aiyagari and Gertler [1991] a version of this model was used to study asset returns and asset market transactions. If we assume that there is no capital and interpret d+B as representing liquid assets then the risk-free rate will equal RF (see figure 1) and will be lower than what it would be under complete markets. If we now introduce another asset into the model which is costly to trade then the return on this asset must be higher due to a transaction/liquidity premium. The typical individual uses the low return liquid asset to buffer earnings shocks and only occasionally buys or sells the high return illiquid asset. As a consequence the model naturally generates a higher transactions velocity for the liquid asset relative to the illiquid asset. The quantitative exercises in Aiyagari and Gertler [1991] indicate that the transaction velocity of the liquid asset can be about 10 to 20 times that of the illiquid asset, which is consistent with the data on the relative turnover rates of bank money market funds and stocks. When there are fixed costs of trading the illiquid asset this model also generates portfolio compositions such that people at the high end of the wealth distribution have relatively more illiquid assets compared to people at the low end of the wealth distribution. This is qualitatively consistent with the data.

(5) Capital taxation: An interesting policy implication of the complete markets model is that under Ramsey taxation the optimal capital income tax rate is zero in the long run (Chamley 1986). Lucas [1990a] argued that for the U. S. the welfare gains of switching to zero capital income taxation are quite large. The incomplete markets model suggests otherwise. The optimal

capital income tax rate is always positive (see Aiyagari 1992b). Consequently, switching to zero capital income taxation may well involve welfare losses instead of welfare gains.

The intuition for this is that even under incomplete markets long run optimality under Ramsey taxation requires that the pre-tax return on capital equal the time preference rate. Because of incomplete markets the only way to support this is by a tax on capital income since, in the absence of such a tax, the return to capital will always be less than the time preference rate. The situation is depicted in figure 1 in which $\hat{r}$ is the after-tax return and ρ is the pre-tax return. As can be seen, the amount of the tax will depend on the strength of the precautionary saving motive, which, in turn, depends on the risk aversion and the variability and persistence of earnings.

The following quantitative example taken from Aiyagari [1992b] suggests that it is not too difficult to generate capital income tax rates close to the observed value.

(σ_e, ρ_e, μ)	(.2,0,1)	(.4,.6,3)	(.4,.6,5)	U. S. DATA
capital tax rate	.003	.14	.25	.36

4. Future Developments

There are two important ways in which the model described in this paper needs to be extended. One is by introducing aggregate uncertainty in addition to the idiosyncratic shocks and the other is to take account of private information and the resulting incentive compatibility restrictions properly. These are important for several issues including the following: (i) asset returns, (ii) financial propagation mechanisms, (iii) transmission of monetary shocks.

For a complete resolution of the risk-free rate/equity premium puzzle we need to have aggregate uncertainty so as to simultaneously account for the risk premium and the transaction/liquidity premium. Recent models of financial propagation mechanisms (Williamson 1987, Bernanke and Gertler 1989) are based on optimal contracting in environments with private information. Recent models of the transmission of monetary shocks emphasize the uneven distribution of monetary shocks across households and markets (Grossman and Weiss 1983, Rotemberg 1984, Lucas 1990b).

Obviously, problems of private information are at the root of incomplete risk sharing. In the incomplete markets model described in this paper the absence of risk sharing was simply taken as given, but, it is obviously more desirable to explain this on the basis of some informational frictions. Recent work by Green [1987], Phelan and Townsend [1991] and Atkeson and Lucas [1992] has pursued this approach.

Introducing aggregate uncertainty involves a considerable computational burden. This is because the cross section wealth distribution is an endogenous state variable which evolves stochastically over time in response to aggregate shocks. There is a considerable amount of difficult but exciting work ahead.

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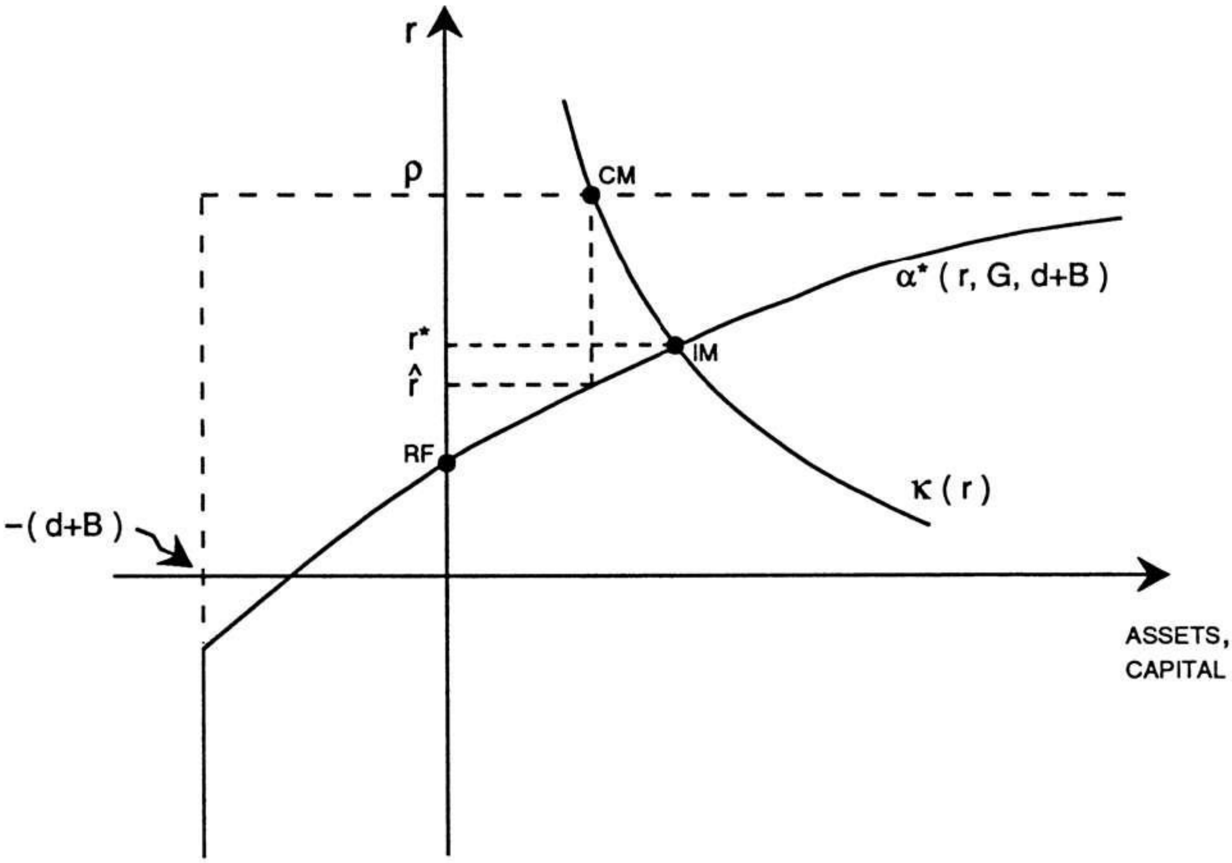

FIGURE 1