"Testing for Neutrality and Rationality"

Tom Sargent

March 1976

Working Paper #: 54

Rsch. File #: 287.1

Testing for Neutrality and Rationality

by Thomas J. Sargent

This is a revised version of a paper initially prepared for a briefing on November 25, 1975, for Dr. Bruce MacLaury, president of the Federal Reserve Bank of Minneapolis. The views expressed herein are solely those of the author and do not necessarily represent the views of the Federal Reserve Bank of Minneapolis or the Federal Reserve System.

The natural unemployment rate hypothesis first made its first appearance as the provocative but somewhat vague statement that "in the long run" a higher rate of inflation would not result in a lower unemployment rate, because agents' expectations would eventually adjust to eliminate any money illusions. Under the autoregressive or "adaptive" expectations schemes initially used to fill it out, the natural rate hypothesis didn't seem to pose any threat to conventional "activist" Keynesian policy strategies incorporating feedback from current economic conditions to future policy settings. That was because "long run" could be taken to mean "in the distant future." A meaningful tradeoff between inflation and unemployment, one with an interesting dynamic structure, still existed under the natural rate hypothesis with adaptive expectations. The feedback rules that resulted from solving the dynamic optimization problem posed by that tradeoff were of the usual activist Keynesian form.[1/]

It was left for Robert E. Lucas[2/] to show that when combined with the hypothesis of rational expectations, the natural unemployment rate hypothesis has very unconventional policy implications. In particular, there obtains a class of stochastic neutrality propositions that imply severely limited possibilities for engaging in successful activist countercyclical policy. These neutrality propositions emerge in models that, potentially at least, seem to be capable of generating the correlations between

---

[1/] Phelps discussed the optimal control problem in adaptive expectations, natural rate models.

[2/] See Lucas [1972]. A neutrality theorem that obtains for a simple macroeconomic model is discussed by Sargent and Wallace.

policy variables and real economic variables that form the empirical basis for Keynesian models. One of the virtues of models like Lucas's is that they do not involve vague concepts such as "short-run" and "long-run." Instead, the models are equilibrium models that (like Arrow-Debreu state preference models) determine the probability distributions of all the endogenous variables as functions of the probability distributions of the exogenous variables and random shocks. The models restrict data and are thus refutable.

This paper summarizes the evidence that has been adduced in attempts to refute the natural rate hypothesis. There is widespread agreement that the value of the natural unemployment rate hypothesis--or for that matter the value of any other hypothesis about the world--ought to be measured by how well the hypothesis accords with the data. While participants on both sides of the controversy pitting the natural rate against various unnatural rate theories accept that measure of value, judging how well one hypothesis fares vis à vis another in fitting the data is often a very subtle matter. One reason it is so subtle is that there are always alternative models that can fit the data equally well. Philosophers have known this for a long time; econometricians discovered it after World War II and called it the "identification problem." I will argue that this identification problem--or multiplicity of models fitting the data equally well--is at the heart of the difficulty in definitively testing the natural rate hypothesis. The data certainly do not support the emphatic rejection of the rational expectations natural rate hypothesis made by members of the "Brookings Panel" School of

Macroeconomics.[3/] On the other hand, it is only with some hesitation and circumspection that I would claim that the data support the rational expectations version of the natural rate hypothesis. The econometric evidence is not spectacular in ruling against either the natural rate hypothesis or its potential competitors.

This paper proceeds by outlining four layers of increasingly sophisticated ways of writing down and testing the natural unemployment rate hypothesis. Describing these "layers" provides a way of summarizing the empirical evidence as it has gradually been produced and interpreted by economists over the last eight years. In the course of discussing the third and fourth layers, I will discuss the structure of the stochastic neutrality theorems that characterize the new classical models pioneered by Lucas.

A main point of this review is that the natural rate hypothesis delivers severe policy implications and stringent restrictions on data only when it is combined with the hypothesis of rational expectations (or some other equally restrictive hypothesis about expectations). We see this immediately as we turn to our first layer, the famous Solow-Tobin test, which cannot be implemented without some a priori restrictions on expectations.

[3/] Commenting on R. J. Gordon's paper in 1970, Robert Solow wrote, "My comment number zero is that the paper demonstrates that the accelerationist idea of inflation gets essentially no support from the data-confirming my work and that of others. I would suggest that we leave that theoretical question out of our discussion unless somebody has something new to offer." Commenting on Robert E. Hall's paper in 1975, Solow wrote, "I thought Hall convincingly demolished the view that unemployment is a disequilibrium phenomenon in the sense of rational expectations." In his presidential address to the American Economic Association, R. A. Gordon said: "Another related recent development in which theory proceeds with impeccable logic from unrealistic assumptions to conclusions that contradict the historical record, is the recent work on rational expectations."

Layer 1: The Solow-Tobin Test

In 1967, Robert Solow and James Tobin independently formulated a version of the natural rate hypothesis that seemed to permit a straightforward test. They wrote a Phillips curve in the form

$$(1) \qquad Un_t = \beta_1(p_t - {}_tp^*_{t-1}) + \beta_2 {}_tp^*_{t-1} + \sum_{t=1}^{n} \lambda_i Un_{t-i} + \varepsilon_t$$

where ε_t is a well-behaved random term, Un_t is the unemployment rate, p_t is the rate of inflation at time t, and ${}_tp^*_{t-1}$ is the rate of inflation expected by the public at time t-1 to prevail at time t. On the natural unemployment rate hypothesis, only unexpected inflation, i.e., $p_t - {}_tp^*_{t-1}$, affects unemployment. An increase in expected inflation ${}_tp^*_{t-1}$ by itself would leave unemployment unaffected. Therefore, the natural unemployment rate hypothesis asserts that $\beta_2 = 0$ in (1), which is the implication of the natural rate hypothesis that Solow and Tobin proposed to test. Of course, if there is a "Phillips curve," then $\beta_1 < 0$, indicating that unexpected inflation causes unemployment to decrease. The expectation ${}_tp^*_{t-1}$ is unobservable, so Solow and Tobin posited that the expected inflation rate is a weighted sum of past observed actual rates of inflation,

$$(2) \qquad {}_tp^*_{t-1} = \sum_{i=1}^{m} v_i p_{t-i}$$

Substituting (2) into (1) gives the equation that Solow estimated

$$(3) \qquad Un_t = \beta_1 p_t + (\beta_2 - \beta_1) \sum_{i=1}^{m} v_i p_{t-i} + \sum_{i=1}^{n} \lambda_i Un_{t-i} + \varepsilon_t .$$

Unfortunately, using the technique of regression to estimate equation (3) will <u>not</u> permit one to estimate the critical parameter β_2. Applying regression to (3) permits one to recover only β_1, the λ_i's, and the products $(\beta_2 - \beta_1) \cdot v_i$ for i=1, ..., m. But there is no way to unscramble the critical β_2 parameter from estimates of (3) alone. This is not surprising since equation (3) contains n+m+2 parameters but only m+n+1 variables on the right-hand side.

Solow and Tobin recognized this problem and proposed solving it by imposing the *a priori* restriction

$$\sum_{i=1}^{m} v_i = 1.$$

They justified this restriction, interestingly enough, by making an appeal, albeit a faulty one, to the hypothesis of rational expectations. They considered the following experiment. Suppose inflation had been zero forever. Expected inflation given by (2) would be zero. Then suppose that in period $\bar{t}$ inflation suddenly jumped to .01 per year and stayed there forever. If people are eventually to catch on to this fact, and if they form their expectations according to (2), we want that eventually (after m periods have passed)

$${}_tp^*_{t-1} = \sum_{i=1}^{m} v_i p_{t-i}$$

$$.01 = \sum_{i=1}^{m} v_i \cdot .01$$

or

$$1 = \sum_{i=1}^{m} v_i. \tag{4}$$

So Solow and Tobin deduced the restriction (4), which they used to recover an estimate of β_2, from assuming the particular path for inflation characterized by a single once-and-for-all jump in inflation and by requiring that people eventually catch on to what has occurred to inflation.

Imposing (4), Solow and Gordon, who used Solow's test, found that β_2 was not zero, and so rejected the natural rate hypothesis. But Gordon's estimates of β_2 have been approaching zero as he has used more and more of the data from the post war.[4/] Others who have imposed (4) have recently not been able to reject $\beta_2 = 0$, which is the natural rate hypothesis.

The estimates of β_2 obtained by the preceding procedure are only as valid as the restriction (4) which is imposed _a priori_. The restriction (4) was derived from an experiment involving a very special pattern of behavior for the inflation rate. The restriction $\sum_{i=1}^{m} v_i = 1$ would _not_ be an implication of the basic assumption that people eventually catch on to what the inflation process is, were the inflation process assumed to be some process other than the particular one Solow and Tobin imagined. For example, suppose that from period $\bar{t}$ on, inflation followed the process

$$p_t = .3p_{t-1} + U_t, \text{ where } U_t \text{ is an unpredictable residual.}$$

The assumption that people eventually catch on to how inflation is behaving implies that eventually people would forecast inflation according to

$${}_tp^*_{t-1} = .3p_{t-1}.$$

This is a version of (4) in which the weights in (4) add up to .3, not 1. If inflation had actually behaved according to the process $p_t = .3p_{t-1}+U_t$, and if people had caught on to what was going on during the estimation period, Solow and Tobin's test would erroneously reject the

---

[4/]For example, compare Gordon's estimates in Gordon [1970], Gordon [1971, p. 137] and Gordon [1972].

natural rate hypothesis even if the natural rate were correct.[5/] For suppose the natural rate hypothesis is correct so that $\beta_2 = 0$ implying that

$$Un_t = \beta_1 p_t - \beta_1 \sum_{i=1}^{m} v_i p_{t-i} + \sum \lambda_i Un_{t-i} + \varepsilon_t =$$

$$\beta_1 p_t - \beta_1 \cdot .3 p_{t-1} + \sum \lambda_i Un_{t-i} + \varepsilon_t$$

The regression coefficient on p_t estimates β_1 while the coefficient on p_{t-1} estimates $.3\beta_1$. Solow and Tobin take the coefficient on p_{t-1} to be an estimate of $v_1(\beta_2-\beta_1)$. Because in this context they would impose the assumption that $v_1 = 1$, they misestimate β_2. They get their estimate of β_2 from

$$(5) \qquad v_1(\hat{\beta}_2-\hat{\beta}_1) = -.3\hat{\beta}_1$$

Wrongly imposing $v_1 = 1$ and adding $\hat{\beta}_1$ (the coefficient on p_t) to both sides gives

$$\hat{\beta}_2 = .7\hat{\beta}_1,$$

so that if $\hat{\beta}_1 \neq 0$, they erroneously take $\hat{\beta}_2 \neq 0$. Notice that if they had used the true value of $v_1 = .3$, then they would have obtained

$$.3\hat{\beta}_2 - .3\hat{\beta}_1 = -.3\hat{\beta}_1$$

or

[5/] The rest of this paragraph is a numerical illustration which can be skipped with little cost.

$$\hat{\beta}_2 = 0$$

from their calculations.

In summary, Solow and Tobin's test hinges critically on the validity of their a priori restriction on the sum of the lag weights. Solow and Tobin deduced their prior restriction $\sum v_i = 1$ by assuming a particular kind of inflation process and assuming that eventually people catch on to the inflation process (i.e., they are "rational"). Their test is vulnerable to the criticism that the behavior they assume for the inflation process is incredibly simple and does not resemble the actual pattern of inflation during any historical period. Furthermore, assuming that people actually had caught on to the process apparently governing inflation during the sample period used in their test regressions would not, in general, imply $\sum v_i = 1$, but would imply some other restrictions on the v_i's. Properly incorporating this observation gives rise to rational expectations tests, which brings us to our second layer.

Layer 2: Rational Expectations Tests

Rational expectations tests of the natural rate hypothesis should be viewed merely as extensions of the Solow-Tobin test that correct a technical error in the Solow-Tobin test, but embody exactly the same strategy. In particular, rational expectations tests retain Solow and Tobin's equation (1) but more carefully work out the implications of the assumption that people catch on to the process governing inflation. As an illustration, suppose that during the sample period inflation was well modeled by the autoregression

$$(6) \qquad p_t = \sum_{i=1}^{m} w_i p_{t-i} + u_t$$

where u_t is an unpredictable residual with mean zero. Then the rational expectations test would suppose that people used the above autoregression to forecast inflation, and would set

$$(7) \qquad {}_t p^*_{t-1} = \sum_{i=1}^{m} w_i p_{t-i}.$$

Restricting ${}_t p^*_{t-1}$ in this way provides the rational expectations counterpart to Solow and Tobin's a priori restriction that $\sum_{i=1}^{m} v_i = 1$. What recommends (7) over and above the Solow and Tobin's $\sum v_i = 1$ is that (7) requires that the way expectations are assumed to be formed be compatible with the actual inflation process during the sample period. The restriction (7) thus embodies the notion that people were aware of the process generating inflation in the sample period, and permits that process to vary from one sample period to another.

The rational expectations tests essentially involve two steps. First, estimate the autoregression (6) for the actual p_t process. Second, take the ${}_t p^*_{t-1}$ implied by that autoregression via formula (7) and estimate

$$(8) \qquad Un_t = \beta_1 (p_t - {}_t p^*_{t-1}) + \beta_2 \, {}_t p^*_{t-1} + \sum_{i=1}^{n} \lambda_i Un_{t-i} + \varepsilon_t$$

On the natural rate hypotheses, β_2 should be zero.[6/]

This test is distinguished from Solow and Tobin's only in that it examines the actual p_t process during the sample period to figure out a reasonable way for expectations to have been formed. The Solow-Tobin test, on the other hand, only imposes a restriction on the process assumed to govern expectations a priori and without examining the data.

[6/] In practice, the tests can also be implemented in two alternative ways. The first way is to write (6) and (8) as

$$p_t = \sum_{i=1}^{m} w_i p_{t-i} + u_t$$

$$Un_t = \beta_1 p_t + (\beta_2 - \beta_1) \sum_{i=1}^{m} w_i p_{t-i} + \sum_{i=1}^{m} \lambda_i Un_{t-i} + \varepsilon_t,$$

to notice that these equations share some common parameters, the w_i's, and then to test the cross-equation restrictions on the parameters of these two equations that obtain under the natural rate hypothesis $\beta_2 = 0$. The second way is to note that under suitable restrictions on ε_t, rational expectations with $\beta_2 = 0$ in (8) implies that unemployment is econometrically exogenous with respect to p. The first of these ways is the one proposed by Lucas [ ]. The second way is the one implemented by Sargent [1976]. The method in the text was implemented by Sargent [1973]. Other recent studies testing the natural rate hypothesis under the hypothesis of rational expectations are those of McCallum [ ] and Barro [ ]. Those studies do not turn up evidence that would strongly require rejecting the natural rate, rational expectations hypothesis.

The rational expectations test is thus truer to the notion that people catch on--which Solow and Tobin also attempted to take into account--because it explicitly recognizes that the manner in which expectations about inflation ought reasonably to be formed depends on how the inflation process is actually evolving.

The natural rate hypothesis can be interpreted broadly as asserting that predictable changes in either wages, prices, the money supply, or the government deficit have no effect on the unemployment rate. The hypothesis does permit unexpected changes in any of those variables to affect the unemployment rate. This broad version of the natural rate hypothesis can be tested using the same techniques outlined above. In particular, equations (6) and (8) are fit, only now p_t and ${}_tp^*_{t-1}$ are defined to be, for example, the money supply and expected money supply, respectively.

The test can be modified to permit expectations of the rate of inflation and the money supply, for example, to depend on more than just their own lagged values. This is accomplished by entering lagged values of other variables in the counterpart of equation (6), the equation used to define the public's expectation of the variable in question.

The rational expectations test of the natural rate hypothesis was originally proposed by Lucas (1972) and Sargent (1971) and was implemented for post-war U.S. data by Sargent (1975). These tests suggest that it is difficult to reject the narrow natural rate hypothesis for unemployment vis-à-vis prices: the hypothesis that expected changes in price inflation do not affect unemployment can't be rejected. Neither can the hypothesis be rejected that the expected deficit or level of government expenditures has no effects on the unemployment rate. On the other hand, the evidence on whether expected changes in the money supply or the rate of wage inflation influence the unemployment rate is ambiguous and marginal. This latter evidence is not strong enough to overrule the prejudices of either a true believer or a strong doubter of the natural rate.

The natural rate hypothesis fared much better in these tests than I thought it would. The tests suggest that a model combining rational expectations with the natural unemployment rate hypothesis is not spectacularly inconsistent with the post-war U.S. data.[7/] This conclusion is a weak one that is purposefully cast in statistical language. The tests do turn up some evidence that seems to call for rejection of the natural rate (the money wage and money supply results mentioned above), but this evidence is not spectacular in calling for rejection of the natural rate hypothesis, and its positive implications are not necessarily of comfort to advocates of any particular alternative to the natural unemployment rate hypothesis. That is, the tests performed in effect pit the natural rate hypothesis against a wide composite hypothesis; namely, that the foreseen part of inflation or other variables helps to explain the unemployment rate. To have confidence that a particular macroeconomic policy (feedback rule) would be effective, one would want evidence that a particular complete structural macroeconomic model embodying an "unnatural rate" thesis could outperform a model embodying the natural rate hypothesis. The preceding tests, even if they call for rejection of the natural rate hypothesis, don't establish that any now-

---

[7/] A recent paper by a Minnesota graduate student, George Patten, suggests that this statement also summarizes the evidence for the U.S. from 1900-1940.

existing macroeconomic model can outperform the natural rate model--the tests only indicate that with enough work one could find such a model. To know that, given our current state of knowledge, one can do better than to use the no-feedback rules seemingly indicated by the rational expectations-natural rate theory, it is not enough simply to believe that with enough further work one could, perhaps, eventually find a model that would deliver feedback rules that outperform no-feedback rules. One has to have the superior alternative model already in hand. It has never been claimed that the existing macroeconometric models (Wharton, MPS, DRI) predict unemployment better than the naivest of natural unemployment rate theories.[8/]

I conclude this section by noting that the tests discussed so far all accept the Solow-Tobin ground rules in that the natural rate hypothesis is taken to assert that only the <u>currently</u> unexpected part of inflation (or of any other variable) affects unemployment. Neither the expected part of inflation nor any lagged unexpected rate of inflation is permitted to affect unemployment. Now this seems to be a much too stringent interpretation of the natural unemployment rate hypothesis, in the sense that one can produce a model delivering all of the "neutrality" results associated with the natural rate hypothesis, but that would be rejected according to the tests described so far. Such models are arrived at by considering the implications of permitting <u>lagged</u> unexpected parts of inflation (or another variable) to affect unemployment and investigating optimal policy in such a system. This brings us to the

---

[8/] The MPS model predicts unemployment considerably worse than does a very naive natural rate model. This is the result of a study by C. R. Nelson.

third layer of ways of stating and testing the natural unemployment rate hypothesis.

Layer 3: The Identification Problem

I begin by stating a pair of bald statistical facts. Let (y_t, m_t) be any pair of time series. I will think of y as real GNP or unemployment and m as the money supply, but they could be any other two variables. (To compare this section with the last one, think of y_t as unemployment, and m_t as the inflation rate.) Then under very general conditions the following two facts hold for (y_t, m_t) series generated by an economy performing under a given policy regime:[9]

Fact (1): There exists a model describing y and m of the form

(9a) $$y_t = \sum_{i=0}^{\infty} a_i m_{t-i} + \sum_{i=1}^{\infty} b_i y_{t-i} + u_t$$

(9b) $$m_t = \sum_{i=1}^{\infty} c_i m_{t-i} + \sum_{i=1}^{\infty} d_i y_{t-i} + \varepsilon_t$$

where the a_i, b_i, c_i, and d_i's are fixed numbers, and u_t and ε_t are random terms with means of zero that are serially uncorrelated and mutually uncorrelated at all lags ($Eu_t\varepsilon_s = 0$ for all t, s; $E\varepsilon_t\varepsilon_s = Eu_tu_s = 0$ for $t \neq s$).

Fact (2): There exists a model describing y and m of the form

(10a) $$y_t = \sum_{i=0}^{\infty} a_i'(m_{t-i} - E_{t-i-1}m_{t-i}) + \sum_{i=1}^{\infty} b_i' y_{t-i} + u_t$$

(10b) $$m_t = \sum_{i=1}^{\infty} c_i m_{t-i} + \sum_{i=1}^{\infty} d_i y_{t-i} + \varepsilon_t$$

---

[9] These facts are stated carefully and proved in Sargent [1975, b]. It is assumed that (y_t, m_t) is a wide-sense stationary, indeterministic (or "linearly regular") stochastic process that possesses an autoregressive representation. I think of y and m in the text as being detrended and as having their means extracted.

where the a_i', b_i''s are fixed numbers, the c_i and d_i's are the same as in (9b), and the u_t's and ε_t's are exactly the same as in (9a) and (9b). Here $E_{t-i-1}m_{t-i}$ is the linear least squares prediction of m_{t-i} using information available at time t-i-1. The fact that the disturbance u_t is exactly the same for all t's in (9a) and (10a) means that (9a) and (10a) fit the data equally well and therefore can never be distinguished by observing a single economy moving along under a single policy rule (9b). The two models generate data that look exactly identical, so the data can't be used to distinguish between the two models. This is an example of the "identification problem" that Koopmans wrestled with, as did Berkeley and Hume before him.

For any economy moving along in a stationary fashion under a single, policy regime, there is available a pair of models, one of the form (9) and one of the form (10). The models work equally well in the sense of fitting the sampled data. However, under the usual way of manipulating macroeconomic models in deriving optimal feedback central rules, the models imply radically different policy conclusions. To see this, suppose that model (9) is "correct" in the sense that if the policy authority changes the regime, equation (9b), equation (9a) remains unchanged. That is, suppose that (9a) is invariant across policy regimes. Suppose that the authority wants to keep y_t close to some constant desired level y*, e.g., it wants to minimize the variance of y_t around y*. To find the feedback rule accomplishing this, set (9a) equal to y*, and set the residual u_t to zero, which is its expected value conditional on information available before time t:

$$y^* = \sum_{i=0}^{\infty} a_i m_{t-i} + \sum_{i=1}^{\infty} b_i y_{t-i}$$

or

$$(11) \qquad m_t = \frac{1}{a_0} y^* - \frac{1}{a_0} \sum_{i=1}^{\infty} a_i m_{t-i} - \frac{1}{a_0} \sum_{i=1}^{\infty} b_i y_{t-i}.$$

Rule (11) minimizes the variance of y around y*. It is a rule incorporating feedback from lagged y's to current m, and delivers a smaller variance of y than does Friedman's rule without feedback. The critical assumption in showing that (11) is the optimal rule is that (9a) is invariant across monetary regimes.

But now consider the different assumption that it is equation (10a) that is invariant across regime changes. Consider how y would behave across different deterministic feedback rules for m of the form

$$(12) \qquad m_t = \sum_{i=1}^{\infty} \gamma_i m_{t-i} + \sum_{i=1}^{\infty} \delta_i y_{t-i}.$$

For any rule of the form (12), money is perfectly forecastable, so that under any such rule

$$E_{t-1} m_t = m_t$$

for all t. Substituting this equation into (10a) gives

$$y_t = \sum_{i=1}^{\infty} b_i' y_{t-i} + u_t,$$

which summarizes the behavior of y under any rule of the form (12) and in which the parameters γ_i and δ_i don't appear. Therefore, the behavior of y is independent of the values of the γ's and δ's so that one rule is as good as another given the assumption that (10a) is invariant across regimes. Thus, we have a strong, stochastic neutrality proposition.

Given an economy operating under a single regime, equations (9a) and (10a) cannot be distinguished: this is what facts 1 and 2 mean. To draw policy implications from a model, it is critical to know that the model is invariant across regimes, or at a minimum to know how it will vary across regimes. Empirical observations alone from an economy operating under a single regime can never provide any evidence about whether a model like (9) or like (10) or of some other form is the one that is invariant across policy regimes.

It bears mentioning that it is possible for a model of form (10) to be the one that is invariant across policy regimes, and yet for tests of the natural rate hypothesis along the lines of section 2 to reject the natural rate hypothesis--this in spite of the strong neutrality implications of the assumption that model (10) is invariant across regimes. The reason is that the section 2 formulation of the natural rate hypothesis prohibits lagged unexpected inflation or money creation from influencing unemployment, which is much stronger than what is needed to deliver neutrality implications. For this reason, the section 2 tests can't be regarded as definitive.

The preceding argument indicates that to test the neutrality proposition which is at the heart of the natural rate-rational expectations hypothesis, one needs evidence about which models are stable across breaks in policy regimes. If there aren't any differences in regimes across countries or across time, there is no hope of bringing empirical evidence to bear. One procedure is to find periods with different regimes and to test for whether (9) is constant across regimes, and then for whether (10) is constant across regimes. (It is, of course, possible that neither one is.) Salih Neftci and I have done this for the U.S., using quarterly money and real GNP as m and y, respectively, for the post-war U.S. We found a break in regime in 1965, that is, we had to reject the hypothesis that the feedback rule for money was the same before and after 1965. We then tested the hypothesis that equation (9a) was stable across this break in regime, being forced to reject the hypothesis (the marginal significance level of the test was .002.) On the other hand, at the conventional significance level of .05, we could not reject the hypothesis that (10a) was stable across the two regimes (the marginal significance level was .062). For monthly data on industrial production and money, for y and m, respectively, from 1919-1940, we obtained similar results. We detected a change in monetary regime after 1929, and had to reject the hypothesis that (9a) was stable across regimes. At the 95 percent confidence level, however, we couldn't reject the hypothesis that (10a) was stable across regimes.[10/] This evidence is fragmentary and unspectacular, but is kinder to the hypothesis of invariance of (10a) than to that of (9a). As such, the tests are consistent with the rational expectations natural rate hypothesis.

Lucas has tried to test for invariance across different countries. However, he faced severe data limitations, and the results of his tests are subject to varying interpretations. He could not reject the natural rate hypothesis as he formulated it, though I regard his results as tentative as are Neftci and mine. In particular, plenty of "unnatural" rate hypotheses are probably compatible with the observations Lucas collected. This is not to quarrel, however, with the insight, originally in Lucas's work, that to test the natural rate-rational expectations hypothesis it is essential to acquire evidence about the invariance of

---

[10/] The marginal significance levels for testing stability of (9a) and (10a) were .008 and .076, respectively.

alternative models across different regimes. The tests by Lucas and by Neftci and myself, crude as they are, are the only efforts to assemble such evidence of which I am aware.

Layer 4--The Persistence of Unemployment and Unobserved Components Models of the Business Cycle

Some economists have interpreted the observed high degree of serial correlation in unemployment and in the deviation of real GNP from trend as calling for rejecting the natural-rate, rational expectations theory. Their argument seemingly runs as follows. Write the natural-rate, rational expectations version of the Phillips curve (or aggregate supply schedule in the special form[11/]

$$(8') \qquad y_t = \beta(p_t - {}_tp^*_{t-1}) + \sum_{i=0}^{\infty} w_i \varepsilon_{t-i}$$

where y_t is, say, the deviation of GNP from its trend, ε_t is a serially uncorrelated random process, $\beta > 0$, and the w_i's are fixed numbers satisfying $\sum_{i=0}^{\infty} w_i^2 < \infty$. By choosing the variance of ε and the w_i's suitably, any arbitrary pattern of serial correlation in y can be modeled. The term $\sum w_i \varepsilon_{t-i}$ represents shocks to aggregate supply that persist in a fashion determined by the w_i's. Equation (8') embodies the natural rate hypothesis since it asserts that only unexpected price changes cause GNP to move relative to its trend. The only way that movements in aggregate demand can exert an influence on output is by inducing forecast errors $p_t - {}_tp^*_{t-1}$ which act upon y through equation (8').

[11/] Notice that (8') is <u>not</u> the same statistical model the tests of which were discussed in Sections 2 and 3. It is noteworthy that (8') is not the only possible representation of a natural-rate, rational expectations aggregate supply schedule, and that the argument that Hall and others have developed on the basis of (8') does not apply, for example, to the formulations that appear in Sections 2 and 3 in the text.

If one maintains the hypothesis of rational expectations and assumes that the information set conditioning the forecast ${}_tp^*_{t-1}$ includes the past values of the price level p_{t-1}, p_{t-2}, ..., then it follows that the forecast errors $p_t - {}_tp^*_{t-1}$ are themselves serially uncorrelated. This follows because if the conditioning set includes lagged forecasts and lagged values of the variable being forecast, p, then that set in effect includes lagged forecast errors. Linear least squares forecast errors are by construction orthogonal to (uncorrelated with) each variable in the set conditioning the forecasts. Thus, it follows that the forecast error is uncorrelated with its own past values.

If the price forecast errors must be serially uncorrelated, then since aggregate demand can influence y in the context of (8') only by inducing price forecast errors, it follows that aggregate demand fluctuations are capable of contributing only a component $\beta(p_t - {}_tp^*_{t-1})$ to fluctuations in y, a component that must be serially uncorrelated. In the context of (8'), any serial correlation that there is in y cannot be accounted for by aggregate demand fluctuations, but must instead by attributed to the term $\sum w_i \varepsilon_{t-i}$ that represents shifts of the aggregate supply schedule in the $(y_t,\ p_t - {}_tp^*_{t-1})$ plane.

Now since y is known to be highly serially correlated, it is evident that most of the variance in y must, in the context of (8'), be attributed to the term $\sum w_i \varepsilon_{t-i}$. According to Robert E. Hall, this indicates that the theory embodied in (8') is faulty.[12/] Apparently

---

[12/] Sims' comment on Hall's paper is well worth reading.

Hall is arguing that it is his a priori belief that fluctuations in aggregate demand account for a large proportion of the variance in measures of real economic activity, such as y, and that (8') must be rejected because it is inconsistent with that prior belief.[13/] Perhaps this is an appealing argument, though it is entirely an a priori one and is not based on any empirical tests designed to refute the implications of the statistical hypothesis formed by (8') as completed under a given specification of the information conditioning ${}_tP^*_{t-1}$.[14/] Despite its entirely a priori nature, I suspect that many economists might subscribe to this argument, if only because the Keynesian macroeconomic model that most of us teach assigns to aggregate demand a dominant role in generating fluctuations in real activity.

Lucas has recently constructed a natural-rate, rational expectations model in which fluctuations in nominal aggregate demand do generate persistent fluctuations in real economic activity. He accomplishes this by restricting agents' information sets so that rationality does not necessarily imply that agents' forecasting errors are serially uncorrelated. Lucas posits a particular setup that effects this restriction on agents' information sets, but it is easy to imagine a variety of

---

[13/] Notice that the statistical representations of the natural-rate, rational expectations hypothesis in Sections 2 and 3 imply that aggregate demand fluctuations do generate movements in real output that persist.

[14/] Hall reports a decomposition of the variance in the quarterly unemployment rate that in effect substantiates the claim that the unemployment rate is highly serially correlated, as is well known. The casual reader of Hall's paper may misinterpret those calculations as constituting an econometric test of the model (8'), which they clearly do not. To refute (8'), some econometric evidence would have to be adduced to show that statistical measures of aggregate demand do contribute to explaining the persistence of unemployment.

other setups in which such a structure of information will emerge. The key idea of Lucas's model is the appealing notion that nominal aggregate demand is never directly observed.

In this section, I describe Lucas's model in general terms and show how the presence of demand-induced, serially correlated movements in aggregate demand does not in itself invalidate the neutrality propositions that characterize earlier rational expectations business cycle models. Just as in those earlier models, neutrality theorems hold under the assumption that the public and the government share the same information about aggregate economic variables. In this section, I will also briefly indicate how models of this kind can be tested.

I utilize the following two-equation version of Lucas's rational expectations model of the business cycle:

$$(13)\qquad y_t = \gamma(n_t - E_{t-1}n_t) + \varepsilon_t \qquad\qquad \gamma > 0$$

$$(14)\qquad p_t = E_{t-1}n_t + c(n_t - E_{t-1}n_t) + v_t, \qquad c > 0$$

where y_t is the log of real GNP, p_t is the log of the GNP deflator, ε_t and v_t are stationary random variables, and n_t is nominal aggregate demand; $E_{t-1}n_t$ is the least-squares linear forecast of n_t conditional on information assumed to be available at time t-1. According to the model, fully expected increases in nominal aggregate demand (those for which $E_{t-1}n_t = n_t$) cause the price level to jump, but have no effect on real GNP. However, unexpected movements in nominal aggregate demand cause sympathetic movements in real output.

To complete the model requires positing a statistical model of nominal aggregate demand together with a specification of the information set assumed to be used in forming $E_{t-1}n_t$. Here I consider the specification

$$(15)\qquad n_t = \rho_t + m_t,$$

where ρ_t is private nominal aggregate demand and m_t is a component of nominal aggregate demand perfectly under the control of the government (e.g., the money supply). I assume that ρ_t is a stationary random process that may or may not be correlated with values of m at various lags. The public's information set consists of lagged values of the observable variables y, p, and m, but excludes observations on n. That is, nominal aggregate demand n_t is an unobservable variable that the public never directly sees. However, the public is assumed to know the first and second moments of the probability distribution of n_t, so that it does have the information needed to solve the classic linear least squares signal-extraction problem in the optimal way. Let x_t be the (3x1) vector (y_t, p_t, m_t). The public forms its forecast of n_t conditioned on $x_{t-1}, x_{t-2}, \ldots$ as

$$E_{t-1}n_t = \sum_{j=1}^{\infty} h_j x_{t-j}$$

where h_j is a (1x3) vector, and the h_j's are uniquely determined by the least squares orthogonality conditions

$$(16) \qquad E\{(n_t - \sum_{j=1}^{\infty} h_j x_{t-j})x_{t-\tau}\} = 0_{1x3}, \qquad \tau \geq 1.$$

By virtue of the orthogonality conditions (16), we have the decomposition

$$(17) \qquad n_t = E_{t-1}n_t + u_t$$

$$(18) \qquad E(u_t \cdot x_{t-\tau}) = 0 \text{ for all } \tau \geq 1.$$

Here u_t is the residual from the (population) regression of n_t on past values of (y, p, m). In some earlier versions of Lucas's models, the information set x_t was assumed to include the variable n_t and so also, by implication, the forecasting error u_t. Under that assumption, the orthogonality condition (6) implies that the forecast errors u_t are serially uncorrelated, i.e., (18) implies $E[u_t u_{t-\tau}] = 0$ for all $\tau \geq 1$. However, in the present setup, n is never observed, so that u can never be observed either. Since $x_{t-\tau}$ does not include $u_{t-\tau}$--that is, because n is not observable--condition (18) does not imply that the forecasting error u_t is serially uncorrelated. The public does not observe its own forecasting errors, and so is unable to eliminate serial correlation in them. Consequently, the unobservable aggregate demand model is compatible with serially correlated movements in real GNP that are induced by disturbances in nominal aggregate demand. That is, (13) can be written

$$y_t = \gamma u_t + \varepsilon_t,$$

where u_t is unexpected nominal aggregate demand, which is, in general, serially correlated despite expectations having been formed "rationally" as linear-least squares forecasts.

At first glance it is perhaps tempting to guess that the existence of serially correlated fluctuations in real income that are induced by nominal aggregate demand disturbances would set up possibilities for engaging in systematic, stabilizing counter-cyclical policy. However, this guess turns out to be wrong, as the following argument shows.[15/] Suppose that the policy authority considers linear feedback rules of the form

$$(19) \qquad m_t = \sum_{j=1}^{\infty} g_j x_{t-j}$$

where g_j is a (1x3) vector of parameters, and as before $x_t = (y_t, p_t, m_t)'$. Equation (19) is a standard linear feedback rule setting m_t as a function of past values of things that the authority observes. Here I am assuming that the public and the authority share a common information set, namely, observations on past x's.

[15/] Lucas briefly points out the neutrality proposition characterizing his model in the first paragraph of Section 15, p. 1139 in Lucas [1975].

Under (19), (15) becomes

$$n_t = \rho_t + \sum_{j=1}^{\infty} g_j x_{t-j}$$

Our decomposition (17) is

$$n_t = E_{t-1}\rho_t + E_{t-1}(\sum_{j=1}^{\infty} g_j x_{t-j}) + (\rho_t - E_{t-1}\rho_t) +$$

$$(\sum_{j=1}^{\infty} g_j x_{t-j} - E_{t-1}\sum_{j=1}^{\infty} g_j x_{t-j}).$$

Now since the conditioning information includes past values of all x's, we have

$$E_{t-1}\sum_{j=1}^{\infty} g_j x_{t-j} = \sum_{j=1}^{\infty} g_j x_{t-j},$$

so that our decomposition becomes

$$n_t = E_{t-1}\rho_t + \sum_{j=1}^{\infty} g_j x_{t-j} + (\rho_t - E_{t-1}\rho_t)$$

so that in (17) we have

$$u_t = (\rho_t - E_{t-1}\rho_t).$$

By virtue of the orthogonality condition (18), we have that

$$E_{t-1}\ \{u_t\} = 0,$$

regardless of the value of the feedback coefficients g_j. This establishes the following neutrality theorem:

> There is no choice of g_j's, i.e., no feedback rule, that permits the authority to offset expected movements in real output.

This neutrality theorem about conditional predictions has been obtained without restricting the ρ process in any way, beyond assuming that it is a wide sense stationary process. If we add the restriction that the parameters of the probability distribution of ρ are not functions of the parameters g_j of the authority's feedback rule, then a stronger neutrality proposition holds, namely:

> The variance of u, and so
> the variance of y, is
> independent of the choice of g_j's.

This follows because under the added restriction the variance of $u_t = (\rho_t - E_{t-1}\rho_t)$ is independent of the choice of the g_j's. Notice that the restriction permits ρ to respond to lagged values of the m's, and only requires that the form of the dependence not vary systematically with the g_j's.

Lucas's model can be shown to place testable restrictions upon time series of macroeconomic variables. However, they are restrictions that do not seem to be susceptible to testing by standard econometric techniques because of the underlying assumption that the variable nominal aggregate demand that accounts for most of the covariation among measures of real activity is unobservable. Testing the model requires resorting to techniques for treating "unobservable" variables. Sims and I have explored the prospect of testing Lucas's model by using statistical techniques that blend spectral analysis with factor analysis, the latter being a technique that psychologists developed to study unobservable factors such as "intelligence." A detailed discussion of those techniques and the test results is beyond the scope of this paper. In my own view, however, the statistical results we have obtained thus far on the basis of a study of post-war U.S. data are quite favorable to Lucas's models. The interested reader is referred to Sargent and Sims [ ].

Conclusions

The empirical work done to data does not support out-of-hand rejection of the natural rate-rational expectations hypothesis. This is a weak statement, however, since, as I hope the above review convinces the reader, many of the purported tests of the hypothesis performed to date possess conceptual flaws that render them invalid. The very few tests of the hypothesis that seem clean technically do not strongly call for rejection of the natural rate hypothesis.

The various forms of the tests described in this review raise a question of on which side of the argument lies the burden of proof. Many (though not all) of the tests described above require the natural rate hypothesis to be rejected unless the data are consistent with the notion that no structural model even potentially exists which would permit improving on rules without feedback. But for the policy maker what matters is whether there now exists an estimated "nonneutral" macroeconomic model that works better than a natural rate model, works better in the sense that there is reason for believing the invariance assumption that will be imposed in deriving optimal policy feedback rules from the model. The record is that existing macroeconometric models have not held up well across breaks in regimes.[16/]

[16/] E.g., see Muench, Rolnick, Weiler, and Wallace [ ].

References

Gordon, Robert J., "The Recent Acceleration of Inflation and Its Lessons for the Future," Brookings Papers on Economic Activity, ed. by Arthur Okun and George Perry, 1970(1).

Gordon, Robert J., "Inflation in Recession and Recovery," Brookings Papers on Economic Activity, ed. by Arthur Okun and George Perry, 1971(1).

Gordon, Robert J., "Wage-Price Controls and the Shifting Phillips Curve," Brookings Papers on Economic Activity, ed. by Arthur Okun and George Perry, 1972(2).

Hall, Robert E., "The Rigidity of Wages and the Persistence of Unemployment," Brookings Papers on Economic Activity, ed. by Arthur Okun and George Perry, 1975(2).

Lucas, Robert E., Jr., "An Equilibrium Model of the Business Cycle," Journal of Political Economy, December 1975.

Lucas, Robert E., Jr., "Expectations and the Neutrality of Money," Journal of Economic Theory, Vol. 4, (April 1973), pp. 103-24.

Lucas, Robert E., Jr., "Econometric Policy Evaluation: A Critique," Journal of Monetary Economics, 1976.

Lucas, R. E., "Econometric Testing of the Natural Rate Hypothesis," in Otto Eckstein (ed.), The Econometrics of Price Determination Conference, (Federal Reserve Board, 1972).

Lucas, R. E., "Some International Evidence on Output-Inflation Trade-offs," American Economic Review, (June 1973).

Phelps, Edmund S., Inflation Policy and Unemployment Theory: The Cost-Benefit Approach to Monetary Planning (W. W. Norton, 1972).

Proceedings of a Symposium on Inflation: Its Causes, Consequences, and Control (Wilton, Conn: Kazanjian Economics Foundation, Inc. 1968). "Comment" by James Tobin. Paper by Robert Solow.

Nelson, Charles R., "The Prediction Performance of the FRB-MIT-PENN Model of the U.S. Economy," American Economic Review, Vol. 62 (December 1972).

Sargent, T. J., "A Classical Macroeconometric Model for the U.S.," Journal of Political Economy, Federal Reserve Bank of Minneapolis, manuscript, 1975, forthcoming.

Sargent, T. J. "A Note on the Accelerationist Controversy," Journal of Money, Credit, and Banking, (August 1971).

Sargent, T. "The Observational Equivalence of Natural and Unnatural Rate Theories of Macroeconomics," Journal of Political Economy, Federal Reserve Bank of Minneapolis, manuscript, 1975, forthcoming.

Sargent, Thomas J. and Neil Wallace, "Rational Expectations, the Optimal Monetary Instrument and the Optimal Money Supply Rule," Journal of Political Economy, March/April 1975.

Sargent, Thomas J. and Christopher A. Sims, "Business Cycle Modeling Without Much A Priori Economic Theory," in New Methods in Business Cycle Research, Federal Reserve Bank of Minneapolis, (forthcoming 1976).

Sims, C. A., Comment on Hall in Brookings Papers on Economic Activity, 1975(2).

McCallum, B. T., "Rational Expectations and the Natural Rate Hypothesis: Some Consistent Estimates," Econometrica, Vol. 44, No. 1, January 1976, pp. 43-52.

Muench, T., A. Rolnick, N. Wallace, and W. Weiler, "Tests for Structural Change and Prediction Intervals for the Reduced Forms of Two Structural Models of the U.S.: the FRB-MIT and Michigan Quarterly Models," Annals of Economic and Serial Measurement, March 31, 1974.

Neftci, Salih, and T. Sargent, "A Scrap of Evidence on the Natural Rate Hypothesis," manuscript, 1976.

Barro, Robert J., "Unemployment and Unanticipated Money Growth in the U.S.," manuscript, University of Rochester, 1975.