Federal Reserve Bank of Minneapolis
Research Department

Putty-Clay Capital and Energy

Andrew Atkeson and Patrick J. Kehoe*

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ABSTRACT

We evaluate the ability of models with putty-clay capital and stochastic energy prices to account for the dynamics of energy use and output. Economists have noted a close relationship between changes in the price of energy and changes in output. Moreover, they have documents that this relationship is asymmetric: energy price increases are associated with large output charges while energy prices decreases are associated with small output changes. Finally, following energy price changes, energy use adjusts slowly over time. Standard models with putty-putty capital fail to reproduce the features of the data. In our study of putty-clay models, we first develop a simple characterization of equilibrium. We apply these results to solve a prototype model. Preliminary results suggest that models with putty-clay capital improve on putty-putty models in accounting for the data.

*Atkeson, University of Pennsylvania and National Bureau of Economic Research; Kehoe, Federal Reserve Bank of Minneapolis, University of Minnesota, and University of Pennsylvania.

Introduction

In 1959, Leif Johansen noted that production technologies in the existing growth literature were based either on the assumption that capital and labor are used in fixed proportions or on the assumption of smooth substitutability between factors. He proposed (p. 157) a synthesis of these extremes in which "any *increment* in production can be obtained by different combinations of *increments* in labor and capital inputs, whereas any piece of capital which is already installed will continue to be operated by a constant amount of labor throughout its lifespan." Subsequently, Solow (1962), Sheshinski (1967), Calvo (1967, 1976), Cass and Stiglitz (1969), and numerous others investigated the properties of this so-called putty-clay model.

In most quantitative work to date, economists have used the now standard model of production with smooth substitution between factors. In that model, the capital stock of the economy can be aggregated into a single state variable. In contrast, in the putty-clay model, capital goods come in a wide variety of types indexed by the proportions in which they can be combined with other factors, and, in general, no single capital aggregate can be formed. The concern that this feature of the putty-clay model might give rise to an intractable "curse of dimensionality" may have hindered its application.

In this paper, we build a version of the putty-clay model in which there is a large variety of types of capital goods which are combined with energy in different fixed proportions. Our principal contribution is to establish easily checked conditions under which the problem of solving for the equilibrium of the model economy reduces to a dynamic programming problem with only two endogenous state variables, regardless of the number of different types of capital goods that are allowed. In appropriate applications, this result allows us to avoid the "curse of dimensionality" that typically plagues attempts to analyze the dynamics of economies with a wide variety of capital goods and binding non-negativity constraints on investment. We apply these results to study the equilibrium dynamics of value-added, investment, wages, and energy use in a simple model of energy use with putty-clay capital.

Several applied economists have suggested that putty-clay models may be useful in modeling the relationships between energy prices, output, and other aggregate variables. For example, in interpreting differing estimates of the elasticity of energy use in time series and cross section data, Griffin and Gregory (1976) and Pindyck and Rotemberg (1983) argue that the elasticity of energy use is low in the short run and high in the long run. Griffin and Gregory go on to suggest that the putty-clay capital model may provide a framework that is capable of generating a gradual adjustment of energy use in response to a persistent change in energy prices. Pindyck and Rotemberg develop an alternative model of gradual adjustment of energy use based on costs of adjustment in capital and labor. In another example, in the business cycle literature, Mork (1989), Tatom (1988), and others document an asymmetric relationship between energy price changes and output. Specifically, they observe a large negative relationship between energy price increases and output but only an insignificant relationship between energy price decreases and output. Tatom suggests that these observations may be explained in a model in which the energy intensity of existing capital goods is fixed.

In the business cycle literature more broadly, several economists have noted a close relationship between changes in the price of energy and changes in output in postwar data. For example, in an often cited article, Hamilton (1983) notes that all but one of the U.S. recessions since World War II have been preceded by a large increase in the price of oil. Others have noted that the correlation between output and the real price of energy is significantly negative. Motivated by these observations, Kim and Loungani (1992), Finn (1992), and Ratti and Raymon (1992) have used models with putty-putty capital to assess the role of energy price shocks in generating business cycles. They conclude that energy price shocks can play a significant role, potentially accounting for between 1/6 and 1/3 of the variability of output. However, the predictions these models give for energy use differ from the data in a key respect: in these models, energy price changes have a large immediate effect on energy use, while in the data, energy use adjusts slowly over time. In related work, Rotemberg and Woodford (1993) develop a model of energy use with fixed costs and imperfect competition. We use a simple calibrated model to obtain some preliminary answers to the question of whether introducing putty-clay capital would in fact be useful in addressing these questions.

We present three main propositions regarding the solution of the putty-clay model. First, it is immediate that given any wage, energy price, and vector of existing capital goods, there is a cutoff energy intensity such that all capital goods with lower energy intensities are fully utilized and those with higher intensities are left idle. Second, our main result is that, when all existing capital goods are always fully utilized, the equilibrium of the model can be found as the solution of a dynamic programming problem with only two aggregate endogenous state variables. Finally, we show that in this dynamic programming problem there is at most one type of capital with positive investment, even when energy prices are stochastic. These results give rise to a simple algorithm for computing equilibrium in applications: first solve the simplified dynamic programming problem to obtain a candidate solution, and then calculate the cutoff rule corresponding to this solution to verify that the solution indeed satisfies the assumption of full utilization. Verification of this condition confirms the candidate solution as the equilibrium of the original model.

We use this algorithm to analyze the impacts of energy price changes in a calibrated model. It turns out in this application that the full utilization condition is always met. Intuitively this is because, in the data, energy costs as a share of total costs are typically low — on the order of from 5 to 15 percent of total costs. We analyze the properties of the model economy under the assumption that energy prices follow a Markov process with persistence similar to that estimated by Kim and Loungani (1992) and Finn (1992). We relate these properties to the applied issues mentioned above.

1. The Economy

Index time by $t = 0, 1, 2, \ldots$. At each date, a random event $s_t \in S$ is realized, where S is a finite set. Let $s^t = (s_0, s_1, s_2, \ldots, s_t)$ be the history of realizations of the events up through date t, and let $\pi(s^t)$ denote the probability of s^t. Output is produced with inputs of capital, energy, and labor. Energy is imported from abroad at an exogenous world price $p_t(s^t)$, and energy imports are paid for with exports of output, with trade being balanced at every date. There exists a variety of differentiated capital goods with types indexed by $v \in V$, where V is a finite set contained in $[0, \infty)$. A unit of capital of type v provides capital services in production only in combination with $1/v$ units of energy. If k units of capital of type v are combined with e units of energy where $e > k/v$, then the energy in excess of k/v is wasted. If $e < k/v$, then the capital in excess of ev is left idle. Capital services are then combined with labor to produce output. Use of k units of capital of type v, together with e units of energy and n units of labor yields

$$(\min(k/v, e))^{\theta} f(v)^{\theta} n^{(1-\theta)} \tag{1.1}$$

units of output, where $f(v), f'(v) \geq 0$ and $f''(v) < 0$.

Heuristically, the relationship between this production function and more typical putty-putty production functions can be understood as follows. Consider the production function $Q = F(k, e)^{\theta} n^{(1-\theta)}$, where k is the capital stock, e is energy use, n is labor, F is a constant returns to scale production function, and Q is gross output. Production may be written $Q = e^{\theta} f(v)^{\theta} n^{(1-\theta)}$, where $v = k/e$ and $f(v) = F(v, 1)$. Thus, production can be expressed as a function of the energy intensity of the capital stock v, energy use e, and labor n. To obtain the putty-clay model, suppose v, the energy intensity of the existing capital stock, is fixed, and let output be the same function of energy use and labor given above. Since at most k/v units of energy e can be used productively with k units of capital of type v, equation (1.1) follows.

The stock of capital in this economy at date t in state s^t is represented as a function $k_t : V \times S^{t-1} \to [0, \infty)$ where $k_t(v, s^{t-1})$ is the stock of capital of type v. Let the functions $e_t : V \times S^t \to [0, \infty)$ and $n_t : V \times S^t \to [0, \infty)$ represent the quantities of energy and labor used in production, where $e_t(v, s^t), n_t(v, s^t)$ are the quantities of energy and labor used in

As stated here, problem (1.4) has endogenous state variables ($k_t(v, s^{t-1})$) of dimension equal to the number of elements in V. To allow smooth substitution between energy and other inputs in the long run, it is necessary to make the number of elements in V large. Thus, the "curse of dimensionality" prevents a direct attack on this problem. In what follows, we show that if all existing capital goods are always fully utilized in equilibrium, then the vector of state variables k can be reduced to two aggregated state variables, regardless of the number of elements in V.

We begin by analyzing the decision to utilize existing capital goods. Observe that this decision is static. Consider problem (1.4). Clearly, given a realization of s^t and $k_t(v, s^{t-1})$ and $p_t(s^t)$, the energy use $e_t(v, s^t)$ and labor allocations $n_t(v, s^t)$ that maximize value-added at t and s^t, maximize (1.5) subject to constraints (1.7)-(1.9). Analysis of this problem yields

Proposition 1 (A Cutoff Rule): Given capital stock vector k and energy price p, there is a cutoff type of capital $v^*(k, p)$ such that all capital of types $v > v^*(k, p)$ is fully utilized and capital of types $v < v^*(k, p)$ is not utilized at all. The cutoff intensity level v^* is increasing in p.

Proof: Consider the Lagrangian

$$\max \sum_v [e(v)^\theta f(v)^\theta n(v)^{(1-\theta)} - pe(v)]$$

$$+\sum_v \mu(v)[k(v) - e(v)v] + \sum_v \xi^e(v)e(v)$$

$$+w[1 - \sum_v n(v)] + \sum_v \xi^n(v)n(v).$$

Here, the multiplier $\mu(v)$ is the marginal product of capital goods of type v and the multiplier w corresponds to the wage rate that clears the labor market when capital stock k and energy price p are given. The first order conditions of this problem include

$$\theta f(v)^\theta (\frac{n(v)}{e(v)})^{(1-\theta)} - p = \mu(v)v - \xi^e(v)$$

$$(1-\theta)(\frac{e(v)}{n(v)})^\theta f(v)^\theta = w - \xi^n(v).$$

From these first order conditions, we get the result that

$$\mu(v)v = \max[\theta f(v)(\frac{(1-\theta)}{w})^{(1-\theta)/\theta} - p,\ 0].$$

Thus, capital of type v is utilized, in the sense that $e(v)$ and $n(v)$ are positive, only if

$$\theta f(v)(\frac{1-\theta}{w})^{(1-\theta)/\theta} - p \geq 0, \tag{1.10}$$

and it is utilized fully if this is a strict inequality. Since the first term in (1.10) is strictly increasing in v, we see that the decision to utilize capital is determined by a cutoff rule, with energy saving capital (high v) being used fully and energy intensive capital (low v) being left idle. The cutoff energy intensity, denoted v^*, is increasing in both the energy price and the wage rate, where the wage rate is determined by the existing stock of capital goods k.∎

For any given energy price p and capital stock $k(v)$, we can check whether the whole capital stock is fully utilized as follows. Since production is Cobb-Douglas in capital services and labor, wages are $w = (1-\theta)Q$ where Q is gross output. If all capital is fully utilized, $e(v) = k(v)/v$ for all v, $n(v) = k(v)f(v)v^{-1}/\sum_v k(v)f(v)v^{-1}$ for all v, gross output is $Q = (\sum_v k(v)f(v)/v)^{\theta}$, and value-added is

$$Y = (\sum_v k(v)f(v)/v)^{\theta} - p\sum_v k(v)/v. \tag{1.11}$$

Given capital stock k, substitute the expression for wages under the assumption of full utilization into (1.10) and then check the condition that $v > v^*(k,p)$ for all v such that $k(v) > 0$.

2. A Simple Algorithm

We now turn to our main result. We present a simple programming problem with two state variables (Z and M) which we refer to as *aggregate capital services* and *aggregate energy use*. We then show that, if the solution to this problem satisfies our full utilization condition, then we can use it to construct the solution to the original problem (1.4).

To that end, consider the problem of choosing sequences $\{Z_{t+1}(s^t), M_{t+1}(s^t), x_t(v, s_t), c_t\}_{t=0}^{\infty}$ to maximize

$$\sum_{t=0}^{\infty}\sum_{s^t}\beta^t\pi(s^t)u(c_t(s^t)) \tag{2.1}$$

subject to

$$c_t(s^t) + \sum_v x_t(v, s^t) \leq Z_t(s^{t-1})^{\theta} - p_t(s^t)M_t(s^{t-1}) \tag{2.2}$$

$$Z_{t+1}(s^t) = (1-\delta)Z_t(s^{t-1}) + \sum_v x_t(v, s^t)f(v)/v \tag{2.3}$$

$$M_{t+1}(s^t) = (1-\delta)M_t(s^{t-1}) + \sum_v x_t(v, s^t)/v \tag{2.4}$$

$$x(v, s^t) \geq 0 \;\; \forall\, v, s^t \tag{2.5}$$

with Z_0, M_0 given.

Given any choice of sequences $\{\hat{Z}_{t+1}, \hat{M}_{t+1}, \hat{x}_t, \hat{c}_t\}_{t=0}^{\infty}$ and initial Z_0, M_0 that satisfy constraints (2.2)-(2.5) and have $Z_0(s^0) = \sum_v k_0(v, s^0)f(v)/v$ and $M_0(s^0) = \sum_v k_0(v, s^0)/v$, we can construct an allocation $\{\hat{k}_{t+1}, \hat{x}_t, \hat{e}_t, \hat{n}_t, \hat{c}_t\}_{t=0}^{\infty}$ which satisfies (1.3) and (1.5)-(1.9) as follows. Let

$$\hat{k}_{t+1}(v, s^t) = (1-\delta)\hat{k}_t(v, s^{t-1}) + \hat{x}_t(v, s^t)$$

$$\hat{e}_t(v, s^t) = \hat{k}_t(v, s^{t-1})/v$$

$$\hat{n}_t(v, s^t) = \hat{k}_t(v, s^{t-1})f(v)v^{-1}/\sum_v \hat{k}_t(v, s^{t-1})f(v)v^{-1},$$

and let $\hat{x}_t$ and $\hat{c}_t$ be the same. We then have the following:

Proposition 2 (An Equivalent Problem): Given initial capital stock k_0, let $\{\hat{Z}_{t+1}$, $\hat{M}_{t+1}$, $\hat{x}_t$, $\hat{c}_t\}_{t=0}^{\infty}$ solve problem (2.1) with $Z_0 = \sum_v k_0(v)f(v)/v$ and $M_0 = \sum_v k_0(v)/v$. Let the sequences $\{\hat{k}_{t+1},\ \hat{x}_t,\ \hat{e}_t,\ \hat{n}_t,\ \hat{c}_t\}_{t=0}^{\infty}$ be the allocation derived from that solution. If this allocation satisfies the condition that $v > v^*(\hat{k}_t(s^{t-1}), p_t(s^t))$ for all dates t, states s^t, and capital types v such that $\hat{k}_t(v, s^{t-1}) > 0$, then this allocation solves problem (1.4).

Proof: If $v > v^*(\hat{k}_t(s^{t-1}), p_t(s^t))$ for all dates t, states s^t, and capital types v such that $\hat{k}_t(v, s^{t-1}) > 0$, then constraint (1.7) always binds. To see that the constructed sequences $\{\hat{k}_{t+1},\ \hat{x}_t,\ \hat{e}_t,\ \hat{n}_t,\ \hat{c}_t\}_{t=0}^{\infty}$ solve problem (1.4), it is easiest to rewrite (1.4) with the assumption that (1.7) always binds. To do this, use (1.7) to substitute out for energy use. Then note that the labor allocation problem is static and that for a fixed capital stock its solution takes the form given by the expression for $\hat{n}_t(v, s^t)$. After substitution for energy use and labor into (1.5) combining (1.5) with (1.6) gives

$$c_t(s^t) + \sum_v x_t(v, s^t) \leq (\sum_v k_t(v, s^{t-1}) f(v)/v)^{\theta} - p_t(s^t)(\sum_v k_t(v, s^{t-1})/v). \tag{2.6}$$

Keeping (1.3) as before, this simplified problem is then one of choosing sequences for $\{k_{t+1}(v, s^t), x_t(v, s^t), c_t\}_{t=0}^{\infty}$ to maximize (1.4) subject to (2.6) and (1.3). Final substitution of $Z_{t+1}(s^t)$ for $\sum_v k_{t+1}(v, s^t) f(v)/v$ and $M_{t+1}(s^t)$ for $\sum_v k_{t+1}(v, s^t)/v$ makes it easy to see that the constructed sequences derived from the solution to (2.1) solve this simplified version of (1.4). ∎

We now show that in the solution to problem (2.1), in each period and each state of nature, there is positive investment in at most one type of capital.

Proposition 3 (One Type of Investment): Let $\{\hat{Z}_{t+1}, \hat{M}_{t+1}, \hat{x}_t, \hat{c}_t\}_{t=0}^{\infty}$ solve problem (2.1). Then, at each date t and in each state s^t, there is at most one $v \in V$ such that $\hat{x}_t(v, s_t) > 0$.

Proof: In the case that all capital is always fully utilized, we may examine the properties of the solutions of problem (2.1). Let $\lambda_t^c(s^t), \lambda_t^Z(s^t), \lambda_t^M(s^t)$ and $\lambda_t^x(v, s^t)$ be Lagrange multipliers on constraints (2.2),(2.3),(2.4), and (2.5), respectively. The first order conditions of the problem (2.1) with respect to $x_t(v, s^t)$ are then

$$\lambda_t^x(v, s^t) = \lambda_t^c(s^t) - \lambda_t^Z(s^t) f(v)/v - \lambda_t^M(s^t)/v. \tag{2.7}$$

Investment $x_t(v, s^t)$ is positive only if the multiplier $\lambda_t^x(v, s^t) = 0$. Furthermore, this multiplier is non-negative, so that zero is its minimum value. Let $\tilde{v}$ be a type of capital that receives positive investment. Since zero is the minimum value of $\lambda_t^x(v, s^t)$, not only is $\lambda_t^x(\tilde{v}, s^t)$ equal to zero, but so is the derivative of $\lambda_t^x(\tilde{v}, s^t)$ with respect to v. The derivative of $\lambda_t^x(v, s^t)$ with respect to v is given by

$$[-\lambda_t^Z(s^t)(f'(v)v - f(v)) + \lambda_t^M(s^t)]/v^2.$$

The sign of this derivative depends on the sign of the numerator of this expression. The multiplier λ^Z is positive (since Z increase welfare), and the multiplier λ^M is negative (since M decreases welfare). Since $f(v)$ is strictly concave, this numerator is strictly increasing in v. Thus, the derivative of $\lambda_t^x(v, s^t)$ is strictly negative for $v < \tilde{v}$ and strictly positive for $v > \tilde{v}$. Hence, if it exists, $\tilde{v}$ is the unique minimizer of $\lambda_t^x(v, s^t)$. ∎

While we find that Proposition 3 is useful in simplifying computation of equilibrium, we also find that it sheds some light on the workings of the model. At first glance, one might think that the problem of choosing which types of capital goods to invest in would be similar to a portfolio allocation problem, with multiple types of capital receiving positive investment in a given period for reasons of diversification. This proposition shows that this analogy is flawed: under the condition that all capital is always fully utilized, in this model, at each date and in each state of nature, there is always at most one optimal type of capital for investment. To see why this is true, it is useful to consider the problem of choosing investment in this model as a portfolio selection problem.

Given the description of the production technology, at each date t and in each state of nature s^t, it takes one unit of consumption to produce a unit of new investment in capital of type v. Thus the one period return, in terms of the consumption good, of investing in one unit of new capital of type v is given by

$$R_{t+1}(v, s^{t+1}) = (1 - \delta)q_{t+1}(v, s^{t+1}) + r_{t+1}(v, s^{t+1}),$$

where $q_{t+1}(v, s^{t+1})$ is the price of a unit of capital of type v at date $t+1$ in state s^{t+1} and $r_{t+1}(v, s^{t+1})$ is the rental rate for a unit of capital of type v at date $t+1$ in state s^{t+1}. A consumer will be willing to make a positive new investment in any type of capital v such that

$$\sum_{s^{t+1}} \beta \frac{\pi(s^{t+1}) u'(c_{t+1}(s^{t+1}))}{\pi(s^t) u'(c_t(s^t))} R_{t+1}(v, s^{t+1}) \geq 1. \tag{2.8}$$

In equilibrium, aggregate investment adjusts so that the left-hand side of (2.8) is less than or equal to 1 for all values of v and equal to 1 for all values of v that receive positive new investment. Thus, if the returns on investment of capital of different types v are sufficiently correlated in the sense that the expression on the left-hand side of (2.8) is single peaked for any portfolio of investments, then there is investment in at most one type of capital at each date and state t, s^t.

We can obtain an expression for the return $R_{t+1}(v, s^{t+1})$ from the firm's profit maximization problem. The firm purchases investment goods and produces output in order to maximize

$$\sum_{t, s^t} \lambda_t^c(s^t)[\theta Z_t(s^{t-1})^\theta - p_t(s^t) M_t(s^{t-1}) - \sum_v q_t(v, s^t) x_t(v, s^t)] \tag{2.9}$$

subject to the constraints (2.3) and (2.4). In (2.9), $\lambda_t^c(s^t)$ is the date zero price of consumption at date t, state s^t, and $\theta Z_t(s^{t-1})^\theta$ is gross output less payments to labor. The first order condition for investment from this problem is

$$q_t(v, s^t) = \frac{\lambda_t^Z(s^t)}{\lambda_t^c(s^t)} f(v)/v + \frac{\lambda_t^M(s^t)}{\lambda_t^c(s^t)} 1/v. \tag{2.10}$$

If capital of type v is fully utilized, then at the margin one unit of new investment contributes

$$r_{t+1}(v, s^{t+1}) = \theta Z_{t+1}(s^t)^{\theta-1} f(v)/v - p_{t+1}(s^{t+1})/v \tag{2.11}$$

to value-added. From these two expressions, it is clear that the return to investment in new capital of type v takes a simple form:

$$R_{t+1}(v, s^{t+1}) = A_{t+1}(s^{t+1}) f(v)/v + B_{t+1}(s^{t+1})/v, \tag{2.12}$$

where $A_{t+1}(s^{t+1}) = [(1-\delta)\lambda^Z_{t+1}(s^{t+1}) + \theta Z_{t+1}(s^{t+1})^{\theta-1}]$ and $B_{t+1}(s^{t+1}) = [(1-\delta)\lambda^M_{t+1}(s^{t+1}) + p_{t+1}(s^{t+1})]$. The term $A_{t+1}(s^{t+1})$ is positive and $B_{t+1}(s^{t+1})$ is negative. As we have seen in the proof to Proposition 3, functions of this form are single peaked in v. The first order condition (2.8) is formed from a weighted sum of these returns and thus takes the form

$$G_t(s^t)f(v)/v + H_t(s^t)/v = 1 \tag{2.13}$$

where $G_t(s^t) = \sum_{s^{t+1}} \beta \frac{\pi(s^{t+1})u'(c_{t+1}(s^{t+1}))}{\pi(s^t)u'(c_t(s^t))} A_{t+1}(s^{t+1})$, $H_t(s^t) = \sum_{s^{t+1}} \beta \frac{\pi(s^{t+1})u'(c_{t+1}(s^{t+1}))}{\pi(s^t)u'(c_t(s^t))} B_{t+1}(s^{t+1})$. Again, $G(s^t)$ is positive and $H(s^t)$ is negative, so this weighted sum is also single peaked in v. Intuitively, the fact that the returns to investment in capital of type v separate into terms dependent on the state of nature and terms dependent on v in the manner indicated in (2.12) forces these returns to be sufficiently well lined up that there is at most one optimal choice of type of capital for investment. This choice, of course, varies at each date with the current state.

This result does not generalize to cases in which some types of capital are left idle in some states of nature. To see why not, consider a two period example, with dates $t = 0, 1$, with uncertainty represented by two states of nature s_1, s_2 in the second period. Let there be two types of capital v_1, v_2, and assume that capital of type 1 is left idle in state s_2. The return to investing in capital of type v in this case is simply the marginal product of capital of that type at $t = 1$. Thus $R_1(v_1, s_2) = 0$. Consumers will invest in both types of capital at $t = 0$ if

$$\beta \frac{\pi(s_1)u'(c_1(s_1))}{u'(c_0)} [\theta Z^{\theta-1} f(v_1)/v_1 - p(s_1)/v_1] = 1$$

and

$$\sum_s \beta \frac{\pi(s)u'(c_1(s))}{u'(c_0)} [\theta Z^{\theta-1} f(v_2)/v_2 - p(s)/v_2] = 1.$$

Here, these two expressions do not separate as in (2.13) in the same way for both types of capital. It is a straightforward exercise to construct an example in which two types of capital receive positive investment in a case such as this.

3. An Application

In this section we present an application of these results in which energy prices follow a Markov process. The solution to the model can be found as the solution to a Bellman equation with two endogenous state variables and two controls. Let energy prices follow a Markov process with a finite set of states $\{p\}$. Let $\pi(p', p)$ be the probability that the energy price is p' at date $t+1$, given that the energy price is p at date t. We solve for equilibrium under the assumption that for every realization of the Markov process for energy prices, the corresponding equilibrium capital stock is always fully utilized. We then verify this assumption after calculation. Given this assumption, we can treat the problem (2.1) as a dynamic program with endogenous state variables: $Z_t = \sum_v k_t(v) f(v)/v$ and $M_t = \sum_v k_t(v)/v$. We assume that $F(k, e) = k^\alpha e^{(1-\alpha)}$, so $f(v) = v^\alpha$. We write the Bellman equation as follows:

$$W(Z, M, p) = \max_{c, x, v, Z', M'} u(c) + \beta \sum_{p'} W(Z', M', p') \pi(p', p) \tag{3.1}$$

subject to the constraints

$$c + x = Z^\theta - pM \tag{3.2}$$

$$Z' = (1 - \delta) Z + x f(v)/v \tag{3.3}$$

$$M' = (1 - \delta) M + x/v. \tag{3.4}$$

Note that we have used the result of Proposition 3 restricting investment to one type of capital good. We use a method outlined by Judd (1992) to solve this problem — we approximate the decision rules $c(Z, M, p)$ and $v(Z, M, p)$ with Chebyshev polynomials, choosing the approximation that minimizes the error induced when these approximate decision rules are inserted in the two intertemporal Euler equations derived from this Bellman equation. Presumably, a wide variety of alternative solution methods would work as well.

Before presenting the findings from the stochastic simulation of the model, it is useful to look at some static calculations to get a sense of the size of the immediate impact of energy price shocks of various sizes on value-added in the model economy and to examine the range of price changes for which all capital remains fully utilized. We present these calculations in Table 1. We assume that the capital stock that exists at date 0 is the capital stock that holds in the steady state of the model when energy prices are permanently fixed at $p = 1$. We then calculate value-added in period 0 for various values of p. For comparison, we include the same calculation for the case in which capital is putty-putty, so that there is a single capital stock k and gross output is given by $Q = (k^{\alpha}e^{(1-\alpha)})^{\theta}n^{(1-\theta)}$. We also calculate the steady state level of value-added that will obtain if the new energy price lasts forever. In the calculations in Table 1, we assume that $\beta = .96$; labor's share $(1-\theta)$ in total costs due to capital, labor, and energy is 2/3; and energy's share $\theta(1-\alpha)$ in these costs in the steady state when energy prices are constant is 1/20. We set $\delta = .08$. These figures are roughly consistent with cost shares for the U.S. economy as a whole.

Three main regularities emerge from these calculations. First, the immediate impact of a large energy price increase is larger in the putty-clay economy than in the putty-putty economy. Second, the impact of energy price changes is asymmetric in the putty-clay economy in comparison with the putty-putty economy: energy prices increases have a greater impact on value-added than energy price decreases. Third, when the cost share for energy is low, there is a wide range of energy prices for which all capital remains fully utilized.

In comparing the asymmetric relationship between energy prices and output reported in Table 1 to that found by Mork (1989) and Tatom (1988), recall that these authors ran regressions of log changes in output on log changes in energy prices. The price columns in Table 1 correspond to equal positive and negative changes in the log of the energy price while the percentage changes in output reported in the table approximate the log of the change in output. The figures in the table indicate that with an energy cost share of 5 percent, approximating the energy cost share for the economy as a whole, capital is not left idle until energy prices increase by more than a factor of 6. If we set the energy cost share to 15 percent, approximating numbers reported by Griffin and Gregory (1976) for the manufacturing sector of the economy alone, capital is not left idle until energy prices more than double.

We now consider an economy in which energy prices follow a Markov process. We compare the properties of models with putty-clay and putty-putty technologies. Let the parameters of preferences and technology be as before. Let the energy price process take on two states, a high state p_h and a low state p_l. Let the mean energy price equal 1, let the variance of energy prices be .1, and let the serial correlation be .95. This serial correlation and variance are similar to those found by Finn (1992), Kim and Loungani (1992), and Ratti and Raymon (1992). These statistics give energy prices $p_h = 1.3162$ and $p_l = .6838$ and transition probabilities $\pi(p_h, p_h) = \pi(p_l, p_l) = .975$.

In Table 2, we report statistics from these economies assuming first putty-clay and then putty-putty capital. The mean and the standard deviation of output are nearly the same in the two models. Mean energy use in the putty-clay economy is about 5 percent lower than in the putty-putty economy. The standard deviation of energy use in the putty-clay economy is less than 60 percent of its level in the putty-putty economy. The standard deviation of wages in the putty-clay economy is less than 75 percent of that in the putty-putty economy, while the standard deviation of investment is 15 percent higher in the putty-clay economy. Energy use, wages, and investment are highly correlated with output in both economies, but slightly less so in the putty-clay economy. Finally, in the putty-clay economy, the response of output to energy price changes is asymmetric, while in the putty-putty model it is symmetric.

To get some intuition for the workings of the economy, we plot segments of realized energy prices and major aggregates from simulations of both economies. These plots are presented in Figures 1-7. Figures 1 and 2 plot the path of energy prices and the corresponding paths of value-added. Note that when the energy price rises, value-added falls more initially and stays lower in the putty-clay case. When the energy price falls, value-added rises further initially in the putty-putty case. In these figures, we also see the asymmetric initial response of value-added to energy prices changes in the putty-clay model.

The asymmetric response of output to energy price changes in the putty-clay economy can be understood as follows. During a long spell of low energy prices, the energy intensity of the putty-clay economy builds up over time. When this long spell is followed by an energy price increase, the impact of this price increase on value-added is large. On the other hand, if a long spell of high energy prices is followed by an energy price decline, the energy intensity of the economy will be low so that the price decrease will not have much effect on value-added.

Figure 3 plots the transition paths of energy use in the two models. Here the difference between the two models is stark: in the putty-clay model, energy use adjusts slowly, while in the putty-putty model, the adjustment is instantaneous. Figure 4 shows that when the price of energy rises, investment falls in both the putty-clay and putty-putty models, but the drop is larger in the putty-clay model. Figure 5 shows that wages adjust more gradually following energy price changes. Figure 6 shows the fraction of the payments to energy and capital that go to energy. This fraction almost doubles initially when the energy price doubles and then falls gradually as there is investment in less energy intensive types of capital. Note that when the energy price rises, the higher payments to energy come out of the returns to capital.

Figure 7 shows the energy intensity of new investment. The choice of the type of new investment is quite responsive to energy price changes because these changes are persistent. In addition, there are small changes in the type of new investment that occur as the energy intensity of the existing capital stock gradually adjusts.

Conclusion

Despite the early theoretical attention given to putty-clay models of capital, they have not found frequent use in stochastic applications. We show that, in fact, the putty-clay model is quite tractable, even in stochastic applications, as long as a certain condition is met. We present an application of the model to energy use and find that this condition is met in practice.

Simulation of this simple model produces several observations. First, this model produces a negative correlation between energy prices and output. Second, this relationship between energy prices and output is asymmetric. Third, energy use responds gradually to persistent changes in energy prices. While it is true that the putty-putty model produces a negative correlation between output and energy prices, it is not consistent with these latter two observations. In this sense, the putty-clay model is an improvement over the putty-putty model in terms of capturing salient features of the data. While these results seem promising, more detailed work will be needed to see if this putty-clay model will prove useful in modeling energy use and business cycles.

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Table 1

Immediate Impact of Oil Price Changes

$\theta = 1/3$, $(1-\alpha)\theta = .05$, $\beta = .96$, $\delta = .08$, $\bar{p} = 1$

Price p_0	Clay $Y_0/\bar{Y}-1$	Putty $Y_0/\bar{Y}-1$	S.S. $Y_0/\bar{Y}-1$	Price p_0	Clay $Y_0/\bar{Y}-1$	Putty $Y_0/\bar{Y}-1$	S.S. $Y_0/\bar{Y}-1$
1.01*	-.0005	-.0005	-.0007	1/1.01*	.0005	.0005	.0007
1.05*	-.0026	-.0026	-.0037	1/1.05*	.0025	.0026	.0037
1.10*	-.0053	-.0050	-.0071	1/1.10*	.0048	.0050	.0072
1.25*	-.0132	-.0117	-.0166	1/1.25*	.0105	.0118	.0169
1.5 *	-.0263	-.0211	-.0300	1/1.5 *	.0175	.0216	.0309
2*	-.0526	-.0358	-.0507	1/2*	.0263	.0372	.0534
3*	-.1053	-.0562	-.0791	1/3*	.0351	.0595	.0859
4*	-.1579	-.0704	-.0987	1/4*	.0395	.0757	.1096
5*	-.2105	-.0812	-.1137	1/5*	.0421	.0884	.1283
6*	-.2632	-.0900	-.1257	1/6*	.0439	.0989	.1438
7	-.3152	-.0973	-.1358	1/7*	.0451	.1078	.1571
8	-.3594	-.1037	-.1444	1/8*	.0461	.1157	.1688
9	-.3960	-.1092	-.1519	1/9*	.0468	.1226	.1791
10	-.4270	-.1141	-.1586	1/10*	.0474	.1288	.1885

* indicates that the capacity constraint is binding in this case

Table 2

Statistics from Putty-Clay and Putty-Putty Economies with Markov Energy Prices

$\theta = 1/3$, $(1-\alpha)\theta = .05$, $\beta = .96$, $\delta = .08$

Y = value-added, M = energy use, w = wages, x = investment

Statistic	Putty-Clay	Putty-Putty
mean(Y)	1.09	1.09
mean(M)	.061	.064
std(Y)/mean(Y)	.027	.027
std(M)/mean(M)	.200	.341
std(w)/std(Y)	.562	.766
std(x)/std(Y)	.434	.377
corr(M,Y)	.874	.979
corr(w,Y)	.926	1.00
corr(x,Y)	.773	.887
corr(p,Y)	-.961	-.985
mean($\Delta \log(Y) \| \Delta p > 0$)	-.040	-.034
mean($\Delta \log(Y) \| \Delta p < 0$)	.029	.034

Figure 1: Energy Price

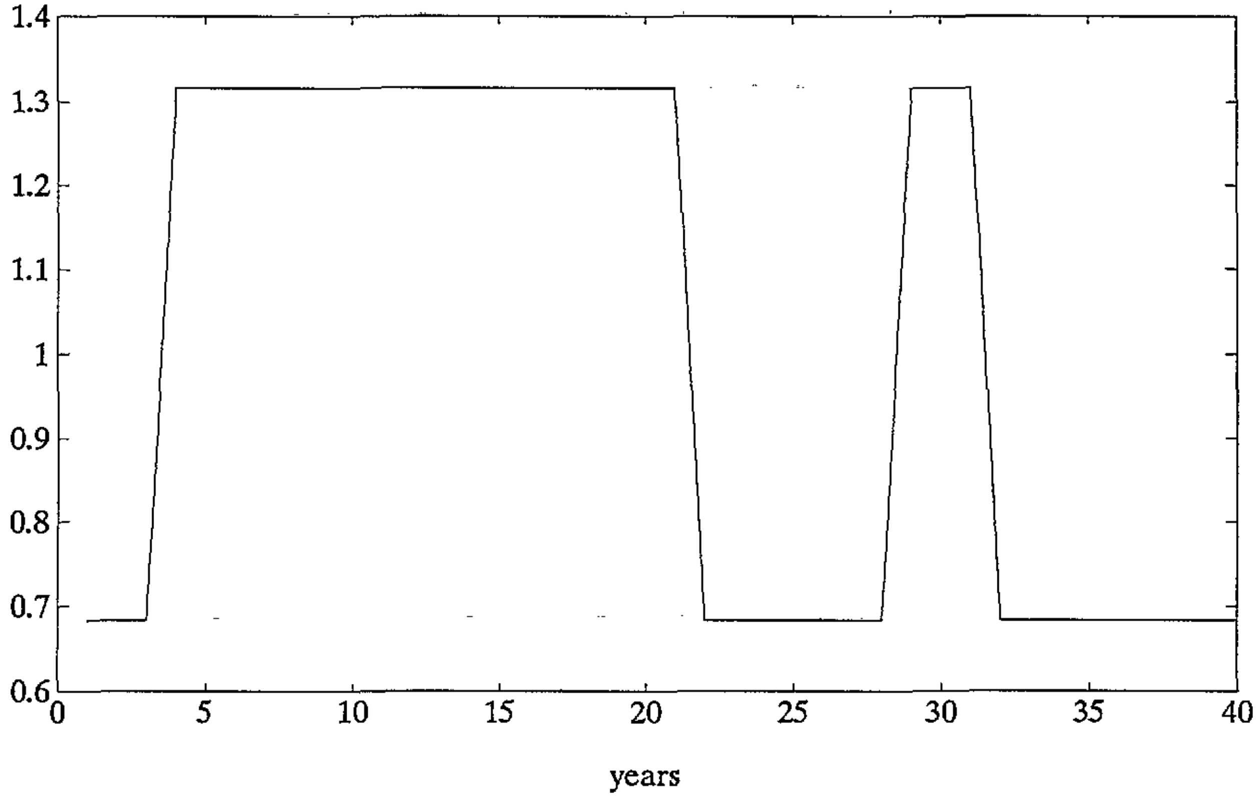

Figure 2: Value Added

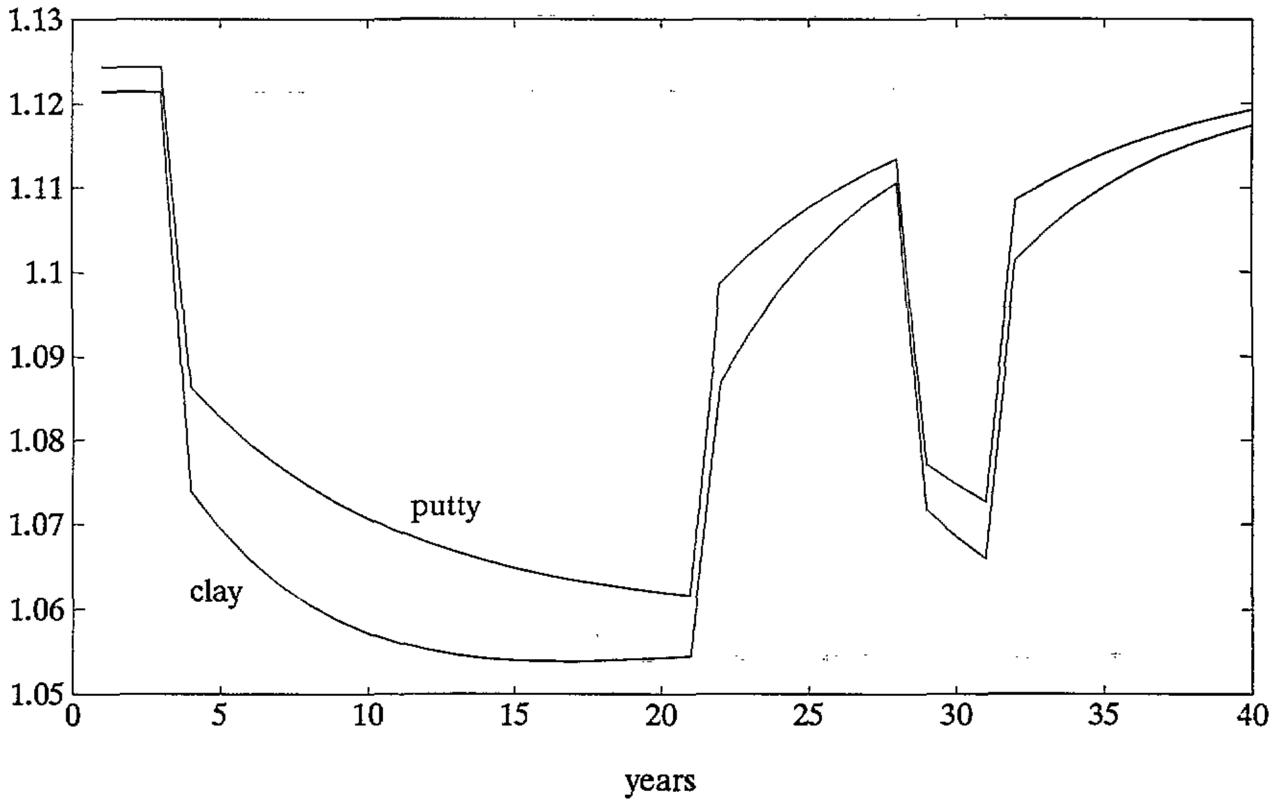

Figure 3: Energy Use

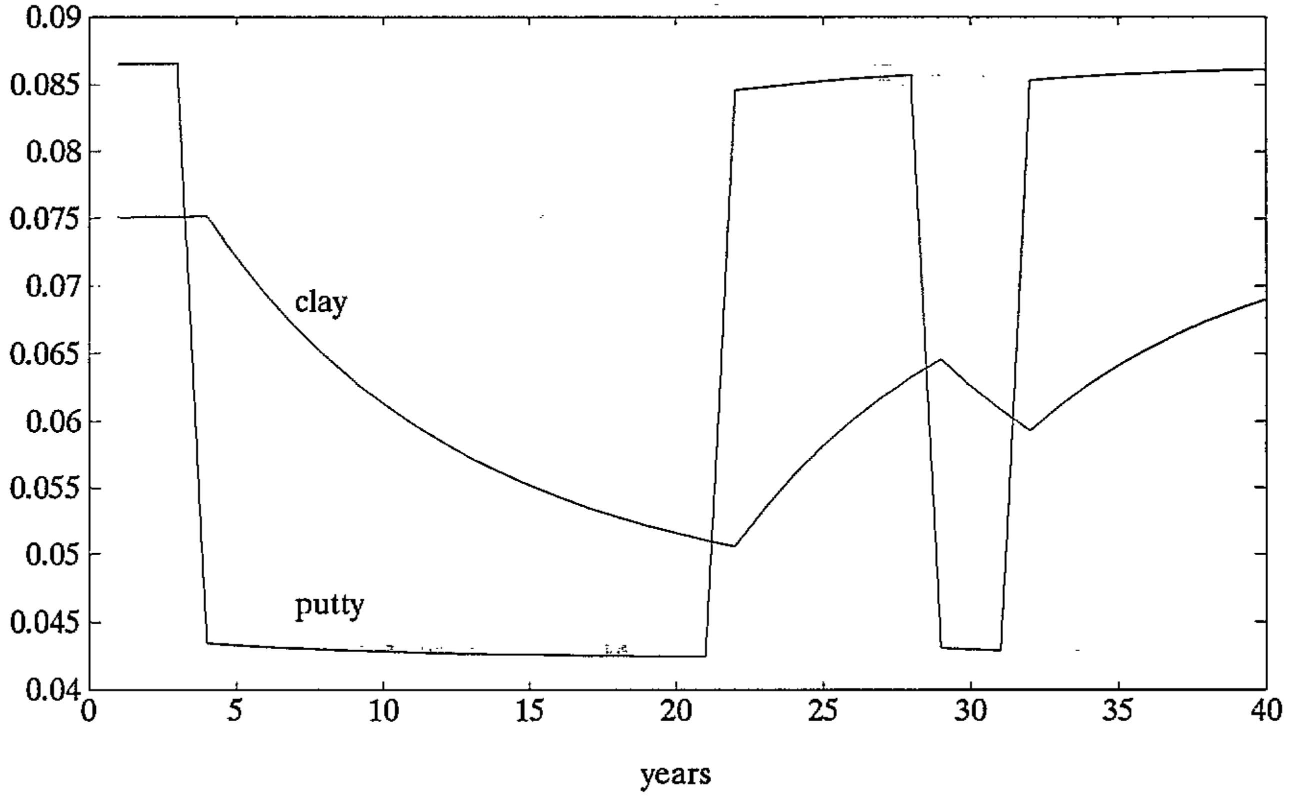

Figure 4: Investment Over Value Added

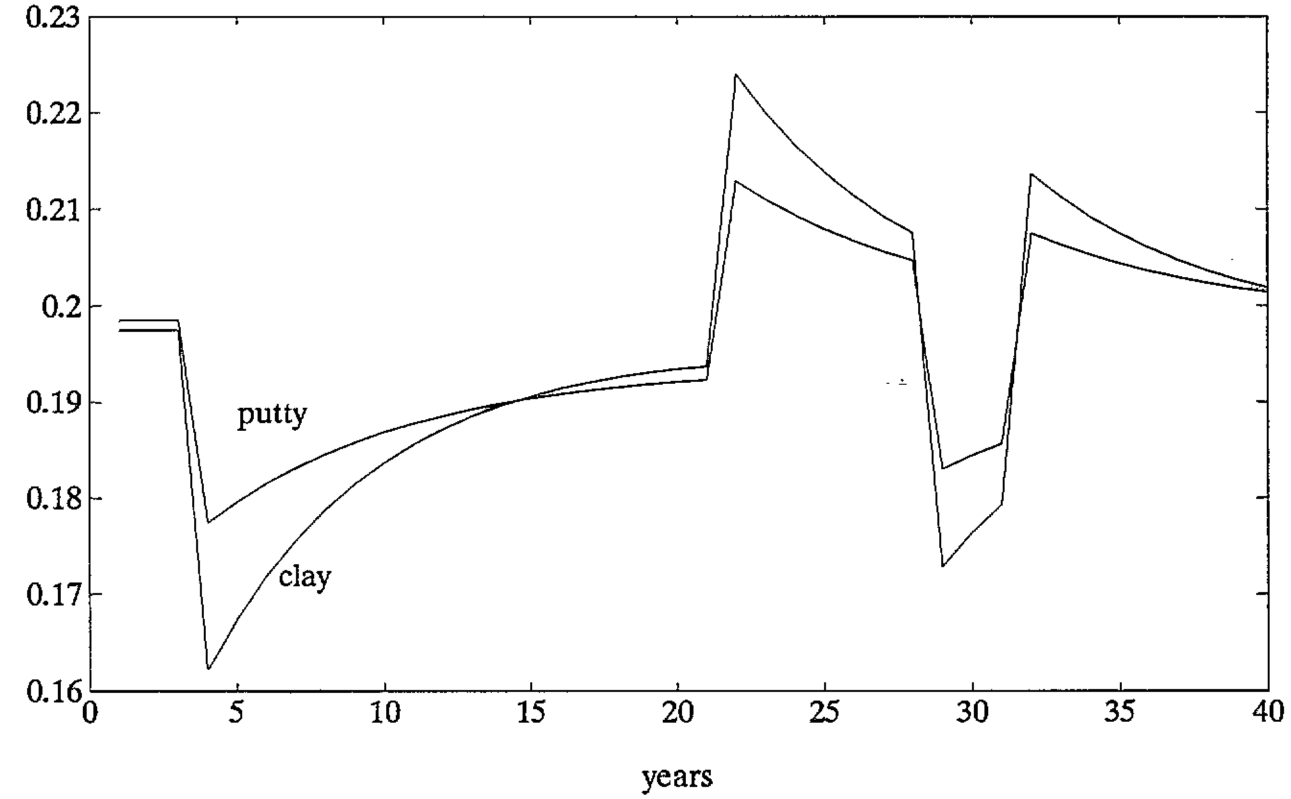

Figure 5: Wages

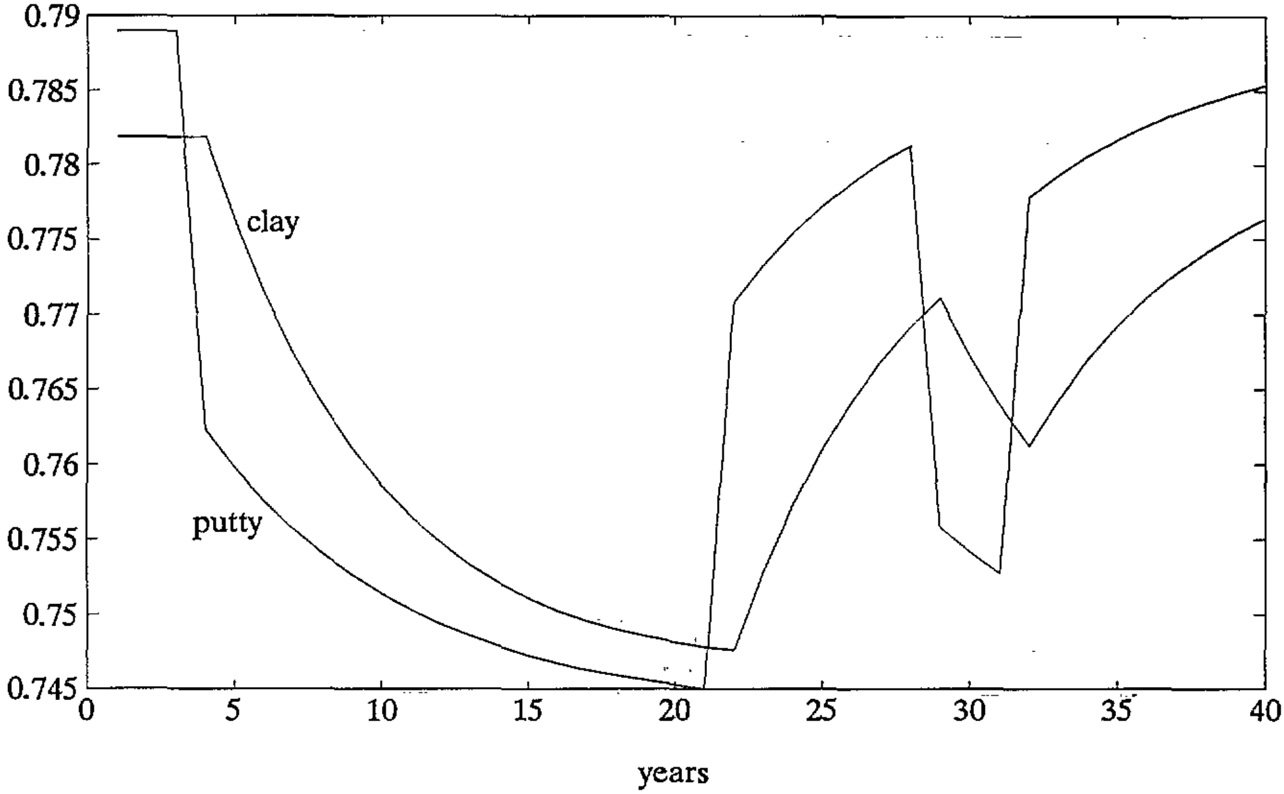

Figure 6: Payments to Energy Over Total Payments to Capital and Energy

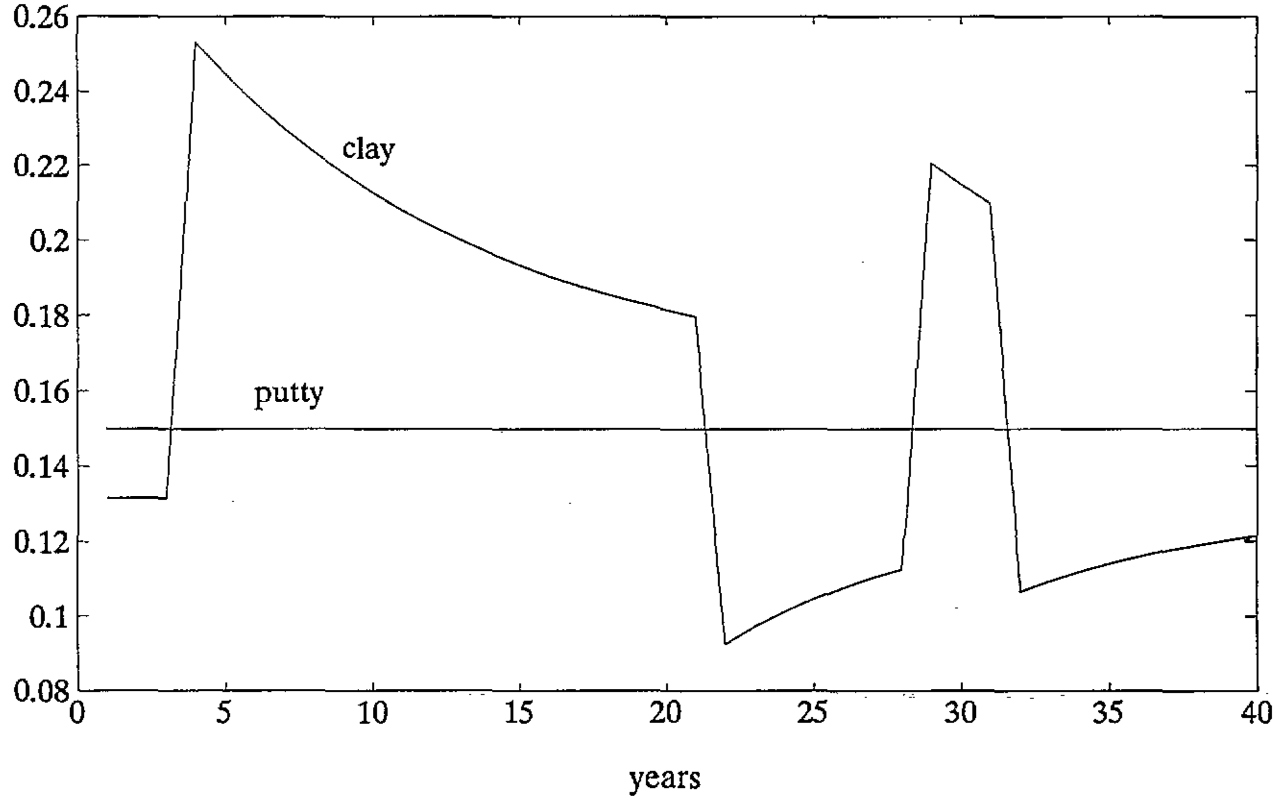

Figure 7: Energy Intensity of New Investment