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# A Locational Model of Human Capital Acquisition

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## 1 Introduction

It has become common practice to include human capital formation in models of economic growth. Romer and Lucas have offered two different ways that human capital formation can take place: Romer as a result of the country's level of already existing human capital and Lucas as a function of work done in the industrialized sector. In both cases, the level of human capital is aggregated at the country level in each period.

What is implicit in Romer's model is that individuals learn from the people around them, their neighbors. Not all neighbors are alike. Some neighborhoods are made up of highly skilled individuals and others of lower skilled individuals. The higher the level of human capital of those neighbors with whom the young come in contact, the easier it is for the young to acquire human capital. In this paper, I look explicitly at these neighborhood effects and assume that rate at which individuals can learn is not simply a function of the aggregate level of skills in the country, but rather a function of the aggregate skill level of their closest neighbors.

By concentrating on individuals and their neighbors, one can explore a number of issues. These models permit persistent income inequality along the lines of Durlauf (1992 and 1994) and an exploration the relationship of market sturcture and income inequality. The effects of migration on income inequality both between and inside a country can be considered. In addition, certain neighborhood effects can work as a propagation mechanism and generate fairly large, long run cycles out of very small individual shocks.

To study these issues, a model must have many agents with well defined neighbors. I construct a very simple OLG model populated by two period lived agents with labor endowments. Agents are born on a location on a grid and live there all their life (although in some version of the model they can trade locations with other agents). The aggregate human capital of the old living on neighboring locations on the grid determine the ability of each young person to acquire human capital. The young use some or all of their labor endowment to acquire skills. Initially, individuals are identical except for the human capital endowments of their old neighbors. The exact rate at which they can acquire human capital mimics the functional forms found in Romer. :In a later version of the model that explores issues of migration, individuals are differentiated by family specific talent levels. Individuals with higher talent accumulate more skills for the same effort

The structure of the model is similar to that of a two dimensional cellular automata.[1] As with cellular automata, the state of each cell (location on the grid) at any period is determined by the states of the neighborhood cells in the previous period. Research in cellular automata have concentrated on exploring the rich set of outcomes that are possible with very simply defined neighborhoods and with very few possible states (often only two) for each cell. Following on the cellular automata tradition, the set of possible outcomes for the OLG model discribed above is explored by simulations. The initial state of the machine[2] for each simulation is a uniformly random distribution of skill levels of the initial old and, when required, a uniformly random distribution of familial talent. One important consideration of this paper is the speed at which economies arrive at persistant states.

Of course, this is not the first paper to consider neighbors and neighborhoods. Interesting recent work in which local effects are important has been done by Bell (1994),Blume (1993), Ellison (19XX), Ellison and Fudenberg (1993), Kandori, Mailath, and Rob (1993), and Young (1993). Most of these papers have looked at games with local information and relyed little on the cellular automata approach. Bell's paper is an exception to this and uses a cellular automata approach to study interdependent preferences.

## 2 The model

### 2.1 The environment

The model is a variant on the standard OLG model with agents who live for two periods. There are $I \times J$ members of each generation. Each individual is located in position $ij$ of a matrix, where, for early versions of the model, they stay. Each person has a labor endowment over the two periods of life: The labor endowment of person $ij$ of generation $t$ is $\omega_t^{ij} = \left[\omega_t^{ij}(t), \omega_t^{ij}(t+1)\right]$. For examples used here, the labor endowment is $\omega_t^{ij} = [2, 1]$.

When young, person $ij$ can allocate labor away from working to gain skills. The amount of skills gained are determined by the amount of labor the young person $ij$ of generation $t$ allocates to learning, $b_t^{ij}$, and by the amount of skills of a set of old who live in matrix positions near that of young person $ij$. For

[1] See Wolfram (1995) for an extended discussion of cellular automata. John von Neumann (1965) is responsible for the initial development of the concept of cellular automata.

[2] A cellular automata with specific states and rules for changing states is refered to as a *machine*.

the examples used here, the set of old whose skill levels are relevant are those in the locations

$$\begin{bmatrix} \ddots & \vdots & \vdots & \vdots & \ddots \\ \cdots & i-1, j-1 & i-1, j & i-1, j+1 & \cdots \\ \cdots & i, j-1 & i, j & i, j+1 & \cdots \\ \cdots & i+1, j-1 & i+1, j & i+1, j+1 & \cdots \\ \ddots & \vdots & \vdots & \vdots & \ddots \end{bmatrix}.$$

One can think of the old person $ij$ ~~and~~ as young person $ij$'s parent and the rest of those indicated as the neighboring old people. For those on the edge of the $I \times J$ matrix, the otherwise missing neighbors are found by wrapping in the appropriate direction[3]. The young of column $I$ have neighbors in columns $I-1$, $I$ and 1. The young of row 1, for example, have neighbors in rows 1, 2, and $J$. We will refer to the set of neighbors (and parents) of young person $ij$ of generation $t$ as $N_t^{ij}$. The skills of this group are the set, $S(N_t^{ij})$. The skill level of young person $ij$ of generation $t$ is found by the function

$$s_t^{ij} = b_t^{ij} \cdot \beta(S(N_t^{ij})),$$

where $\beta(\cdot)$ is a function of the set of skills of the neighboring old. In order to make things simple, skills are denoted in units of labor. The $\beta(\cdot)$ function used in this paper is shown in Figure 1. The argument of the version of the function is the average skill level of the parent and eight closest neighbors.[4]

There is one good each period. It is produced by labor and skills. (Capital could be introduced but would only complicate the model.) The production technology is linear in labor and skills. Total output at time $t, Y(t)$, is simply

$$Y(t) = \alpha \cdot \left( \sum_{i=1}^{I} \sum_{j=1}^{J} (\omega_t^{ij}(t) - b_t^{ij}) + \sum_{i=1}^{I} \sum_{j=1}^{J} (\omega_{t-1}^{ij}(t) + s_{t-1}^{ij}) \right),$$

where $\alpha$ is some constant.

Individuals' preferences are given by the utility function, $u_t^{ij}\left(c_t^{ij}(t), c_t^{ij}(t+1)\right)$, where $c_t^{ij}(s)$ is the consumption of person $ij$ of generation $t$ at dates $s = t, t+1$. To keep things simple, the utility function used in the example economies is a product utility,

$$u_t^{ij}\left(c_t^{ij}(t), c_t^{ij}(t+1)\right) = c_t^{ij}(t) \cdot c_t^{ij}(t+1).$$

[3]This is a common wrapping technique in two dimensional cellular automata simulations. The resulting surface has a torus or donut shape.

[4]The functional form of the equation shown in Figure 1 is $\beta(S(N_t^{ij})) = .5 + \frac{2}{1+e^{3-3*avg(S(N_t^{ij}))}}$.

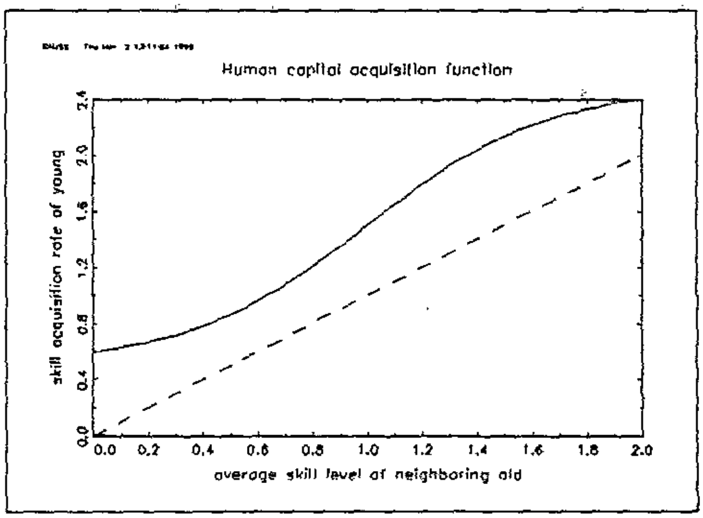


Figure 1: Graph of $\beta(S(N_t^{ij}))$ for example economy

### 2.2 Skill acquisition with no markets

In an economy with no markets, the only way to increase consumption when old is by taking time from work and invest in skills. In this economy, individuals maximize their utility functions subject to the budget constraints,

$$c_t^{ij}(t) = wage(t) \cdot \left(\omega_t^{ij}(t) - b_t^{ij}\right),$$

and

$$c_t^{ij}(t+1) = wage(t+1) \cdot \left(\omega_t^{ij}(t) + s_t^{ij}\right).$$

The gross rate of return on time spent acquiring human capital, $r_h(t)$, is simply

$$r_h(t) = \frac{wage(t+1) \cdot s_t^{ij}}{wage(t) \cdot b_t^{ij}}.$$

Given the linear production function, wages at each date are $wage(t) = wage(t+1) = \alpha$, the constant marginal product of labor. Solving the optimization problem for the product utility function, we get the desired time spent acquiring skills

$$b_t^{ij} = 1 - \frac{1}{2 \cdot r_h(t)}.$$

That , along with the condition, $r_h(t) = s_t^{ij}/b_t^{ij} = \beta(S(N_t^{ij}))$, gives the desired time spend acquiring skills as

$$b_t^{ij} = 1 - \frac{1}{2 \cdot \beta(S(N_t^{ij}))}.$$

Each young person's desired time spent acquiring skills is simply a function of the skill levels of the neighboring old.

The skill level of person $ij$ of generation $t$ is $s_t^{ij} = b_t^{ij} \cdot \beta(S(N_t^{ij}))$. After substituting in the optimal choice for $b_t^{ij}$, the competitive equilibrium level of skills for this young person at date $t$ is

$$s_t^{ij} = \beta(S(N_t^{ij})) - \frac{1}{2}.$$

The skill level to skill level relationship for this model, using the $\beta(\cdot)$ function shown in Figure 1, is quite similar to the one found by Romer (19xx) and others.

### 2.3 A time path of the model: endogenous ghettos

Let $I, J = 59$. Each period, the economy is a $59 \times 59$ matrix of young and old. The old of generation 0 are given a random endowment of skills drawn from a uniform distribution between zero and two. The young of generation one and each continuing generation choose how much time to allocate to human capital acquisition based on the average skill level of their parent and their eight closest neighbors (wrapping in the appropriate way at the edges of the matrix). Initially, the parent and each of the eight closest neighbors are given the same weight in finding the average skill level.

Figure 2 shows the initial distribution of skill levels. The figure shows the surface of the skill levels on a $59 \times 59$ population grid for generation 0. After 5 periods have passed, the skill levels begin to smooth out. Figure 3 shows the skill surface for members of generation 5. By generation 10, the economy is dividing into rich (high skill) regions and poor (low skill regions). Figure 4 shows the utility levels for members of generation 5. Notice that the utility surface is very similar to that of the skill surface. With no markets, skill levels map directly into utility levels. The areas of high and low skill levels become more cohesive as time passes. Figures 5 and 6 show the skill level surfaces for the members of generations 20 and 100 respectively. By generation 100, the skill surface has smoothed to the point where there is only one high skill and one low skill region (although of somewhat twisted shapes). Given the one to one mapping that occurs in this model between the skill levels and utility levels, the utility surface at generation 100 is of the same shape as the skill level surface and for that reason is not shown.

For the example economy shown above 873 individuals (out of a total of 3481 individuals in the economy) end up with low skill levels (with $s_{100}^{ij}$ of less than .3). With only one example economy, it is impossible to know how to interpret this number of poor. Figure 7 shows the distribution of the number of poor in generation 100 from 1000 repetitions of identical economies with different uniformly random initial distributions of skills. The bar graph shows the actual distribution generated by the 1000 repetitions divided into 40 segments. The solid line drawn in is a normal curve with the same mean and standard distribution as the sample of 1000 repetitions. These economies seem to settle

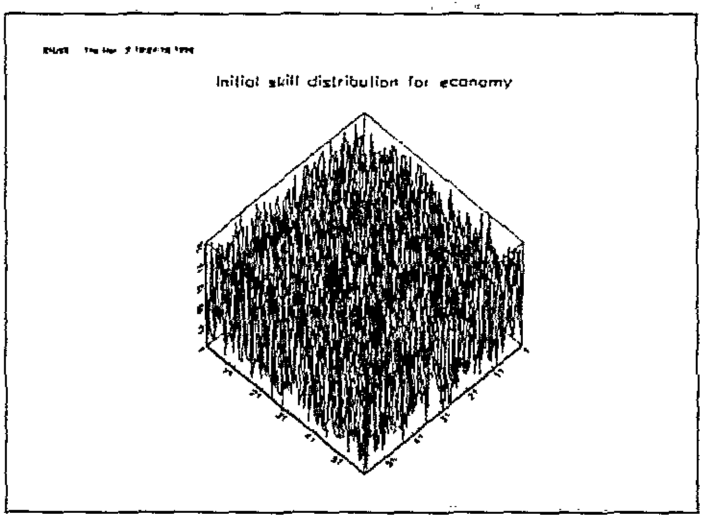


Figure 2: Skill distribution for generation 0

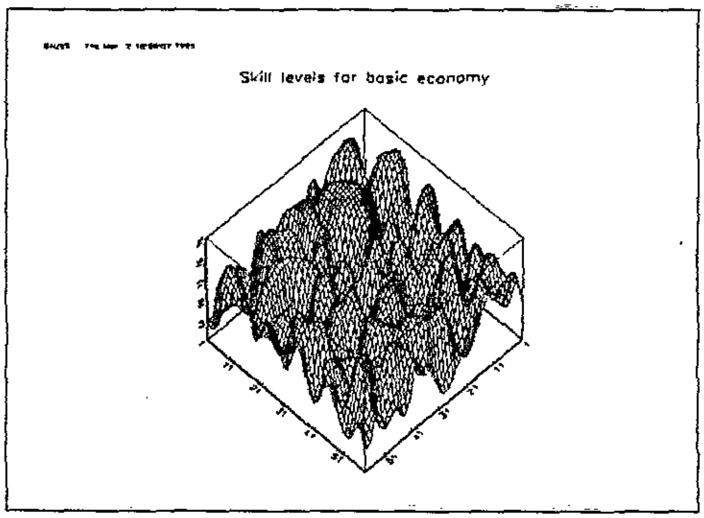


Figure 3: Skill distribution for generation 5

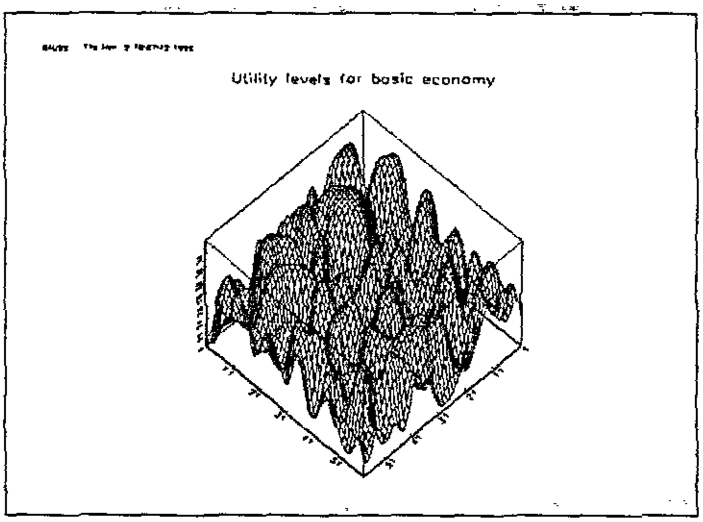


Figure 4: Utility levels for generation 5

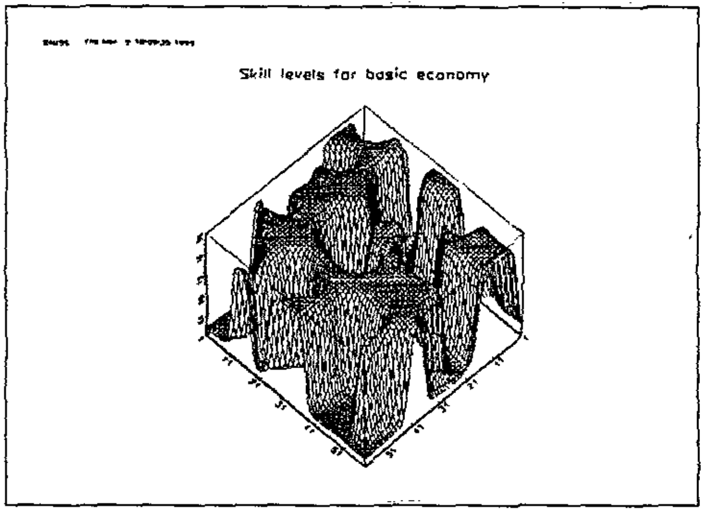


Figure 5: Skill levels for generation 20

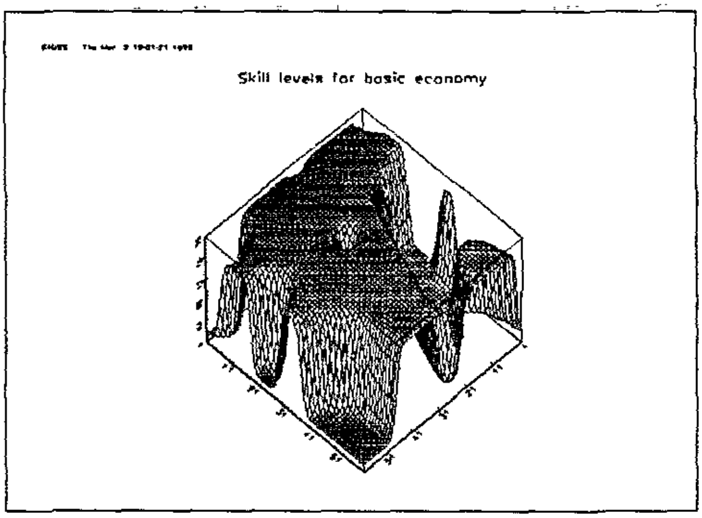


Figure 6: Skill levels for generation 100

down to equilibria where the number of poor follow a distribution that is close to normal.

Increasing the relative influence of the parent (as compared to that of the neighbors) ends up changing the shapes of the high and low skill surfaces and slowing up the smoothing process, but the pattern is similar to the one where all nine members of the older generation get the same weight. There is a small difference in the number of poor. Figure 8 shows skill surface for generation 100, beginning with the same skill distribution as in the earlier case, when the parent is given 4 times the weight of each of the eight neighbors in the averaging process. After 100 periods in this economy, there are 1126 individuals with skill levels less than .3. Figure 9 shows the skill surface for generation 100 when the parent is given 8 times the weight of each of the neighbors. In that figure, 1205 members of generation 100 have skill levels less than .3.

## 2.4 Economy with student loan market

The above version of the model allows no markets. One potential market is for loans between members of the same generation. Since different individuals inhabit locations with different opprotunities for acquiring human capital, those young who live in low skill areas could lend to those in high skill areas and get a higher rate of return than they might from acquiring their own human capital.

If the economy is expanded to allow borrowing and lending among members

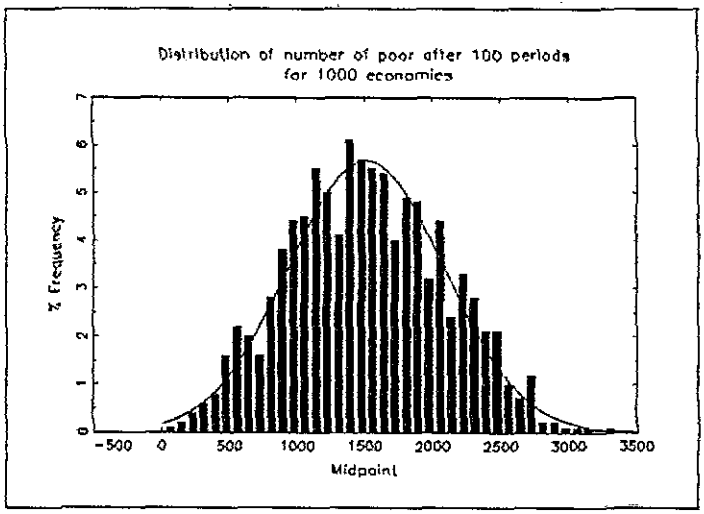


Figure 7: Number of poor in example economies

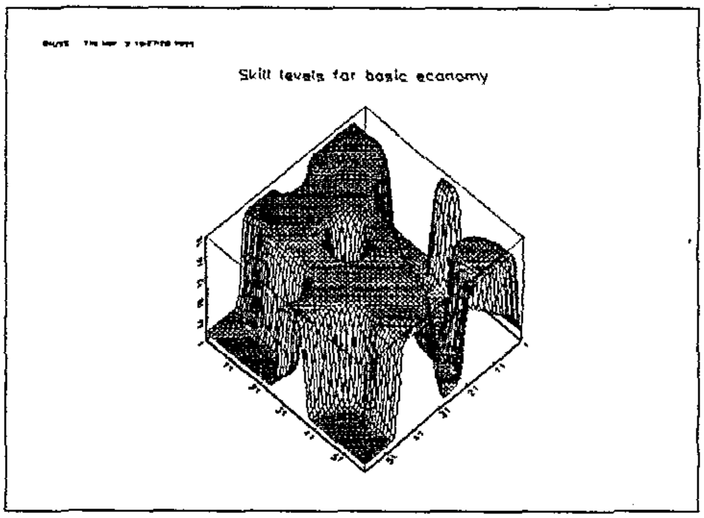


Figure 8: Skill level for generation 100 when parent counts 4 times neighbors

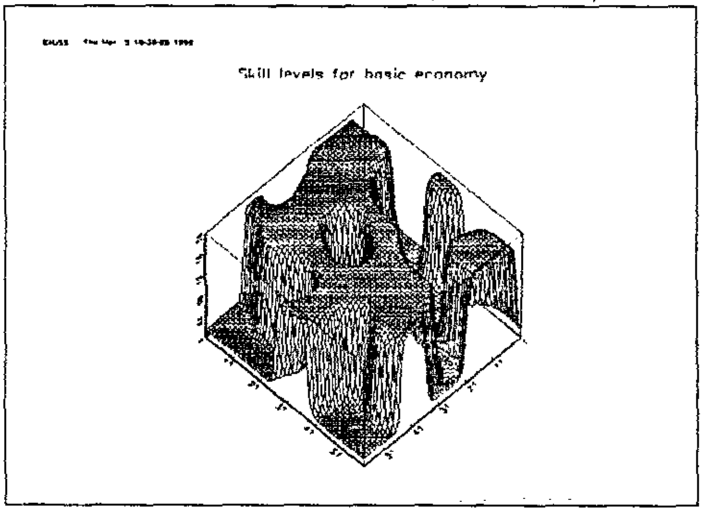


Figure 9: Skill level for generation 100 when parents count 8 times neighbors

of the same generation, the budget constraints change to become

$$c_t^{ij}(t) = wage(t) \cdot \left(\omega_t^{ij}(t) - b_t^{ij}\right) + \lambda^{ij}(t),$$

and

$$c_t^{ij}(t+1) = wage(t+1) \cdot \left(\omega_t^{ij}(t) + s_t^{ij}\right) - r_b(t)\lambda^{ij}(t),$$

where $\lambda^{ij}(t)$ is the amount of loans that person $ij$ of generation $t$ want to make and $r_b(t)$ is the gross rate of return on lending. The existence of this market along with the linear technology for converting skills into goods in the second period of life means that the young will either work full time or study full time. When young, individuals compare the gross rate of return they can get in the loan market with the gross rate of return they can get on skills, $r_b(t)$ versus $r_h(t) = s_t^{ij}/b_t^{ij}$. For those for whom $r_b(t) > r_h(t)$,all savings is done in the lending market. For those for whom $r_b(t) < r_h(t)$,all of their labor when young is allocated to study and they borrow for consumption in that period. For those in neighborhoods where the human capital level is low enough to make them lenders, the total amount lent is

$$\lambda^{ij}(t) = 1 - \frac{1}{2 \cdot r_b(t)}.$$

For those in neighborhoods where human capital level is high enough to make their return on human capital higher than the interest rate on borrowing, their

Figure 10: Skill levels for generation 3 with student loans

borrowings are equal to

$$\lambda^{ij}(t) = -\frac{1 + 2 \cdot s_t^{ij}}{2 \cdot r_b(t)}.$$

The final condition for each period's competitive equilibrium is that the borrowing and lending market clears. Clearing of the loan market in each period requires that

$$\sum_{i=1}^{I} \sum_{j=1}^{J} \lambda^{ij}(t) = 0.$$

An equilibrium sequence for the same example economy as the one used above was found each period by using a routine for solving nonlinear equations as the Walrasian auctioneer in the market for student loans. The initial distribution of skills was the one shown in Figure 2. With the loan market and the linear technology for converting labor and skills into goods, skill levels separate into high and low skill regions very quickly, by period 3 the basic regions are delineated. Figure 10 shows the skill levels of generation 3. Notice that these regions are substantially different from those that formed in the no market version of the model. As additional time passes, the only real changes that occur are the firming up of the edges of the high and low skill regions and the disappearance of some very small high or low skill zones. Figure ?? shows the skill levels of generation 100.

Because of the market in loans, those born in low skill level regions work full

Figure 11: Utility levels for generation 3 with student loans

time when young and lend to those born in high skill regions. The low skilled are able to end up with substantially higher utility levels with the loan market than they could in the no market economy. In addition, as the regions of high and low skills become better defined, the interest rate rises and the utility levels of the high and low skilled become more alike. Figure 11 shows the utility levels for generation 3 and Figure ?? shows them for generation 100.

Repeating the loan market economy with 1000 different initial distribution results in economies with quite similar numbers of poor (those with low skill levels) after ten periods. Figure 12 shows the distribution of the number of poor for the example economy. Especially when compared to the distribution that was found for the economy without a student loan market, the distribution of the number of poor is very tight. A student loan market seems to separate the skilled and unskilled very quickly. The distribution of the gross interest rate in period 10 for the 1000 thousand economies is not shown, but it is equally tight, ranging between 2.45 and 2.49.

## 2.5 Economy with a market for locations

Suppose that the young of each period were allowed to trade places. In the model as it currently stands, there is no way that such trades could occur. While those who reside in regions where the old have low skill levels would like to move to regions where the old have high skill levels, there is no way they can induce those already residing in those locations to leave. To make location

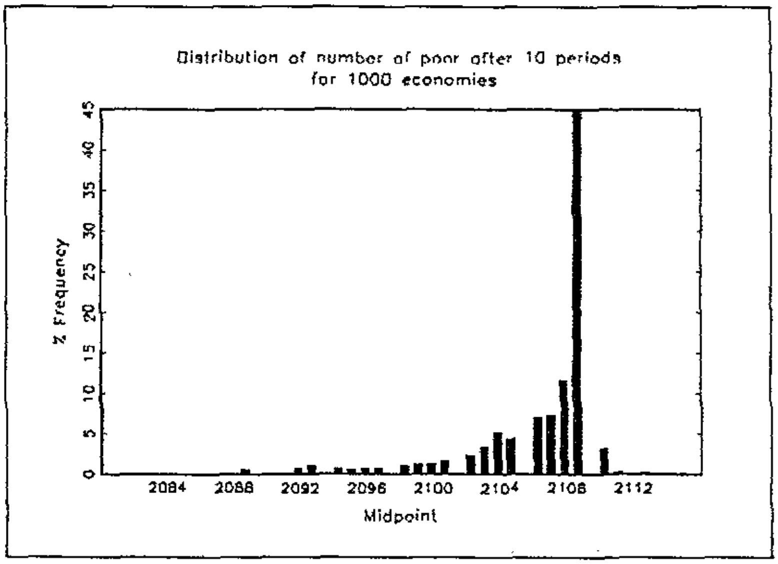


Figure 12: Distribution of number of poor for student loan economy

trades possible, we need to have differences among individuals that would allow them to benefit in different amounts from residing in high or low skill level regions.

Families (a sequence of individuals) differ in something we could call natural ability or talent. In this version of the model, greater natural ability would allow the individual to gain extra skills for each unit of time spent studying from the same local skill level of the old. Suppose that there is a random (uniform) distribution of ability over the population. Let $\phi_t^{ij}$ be the talent level of person $ij$ of generation $t$ and let $\phi_t^{ij}$ take on positive values with mean equal to 1. The skill level that person $ij$ of generation t gains is equal to

$$s_t^{ij} = b_t^{ij} \cdot \phi_t^{ij} \cdot \beta(S(N_t^{ij})),$$

where, as before, $b_t^{ij}$ is the amount of time that person $ij$ studies and $\beta(S(N_t^{ij}))$ is the rate at which each unit of time spent studying becomes skills for the average of the population (when $\phi_t^{ij}$ is equal to 1). Clearly, individuals with higher natural ability can obtain higher skills and, therefore, higher income when old, from any given location.

Suppose that there is no market for loans between members of the same generation. Then one can find the maximum that any individual is willing to pay to move to each of the other locations or the minimum that individual is willing to accept to leave the location they currently inhabit to go to each of the other locations. Figure ?? shows how to find this maximum price. Point

$\omega_t^{ij} = [\omega_t^{ij}(t), \omega_t^{ij}(t+1)]$ is the labor endowment. Assume that one unit of labor or skill produces one unit of the consumption good. In the position the young person currently is at, there is a rate at which study can be converted to skills: $\phi_t^{ij} \cdot \beta(S(N_t^{ij}))$. Finding the point of tangency of the line with that slope out of the endowment point and the indifference curve set gives the consumption pair (shown as point $c_1$and utility obtainable. At some other location $\widehat{ij}$, (the example in the figure is one with higher skill levels for the old) a young person with talent level $\phi_t^{ij}$ can obtain a rate of return from study of $\phi_t^{ij} \cdot \beta(S(N_t^{\widehat{ij}}))$. If this move could be obtained costlessly, the young person would consume at point $c_2$. The maximum amount of labor that the individual would be willing to exchange (in the form of the consumption good) for being able to move to the new, higher skill position is the amount $p$, such that maximizing utility with endowment point $\hat{\omega}_t^{ij} = [\omega_t^{ij}(t) \quad p, \omega_t^{ij}(t+1)]$ and a rate of return on study of $\phi_t^{ij} \cdot \beta(S(N_t^{\widehat{ij}}))$ results in the same utility as with the initial endowment and rate of return. In Figure ?? the maximum price is shown by the distance $p$ and the resulting consumption point by point $c_3$. For the sample economy with the product utility function and with labor endowments of $\omega_t^{ij} = [2, 1]$, the function for finding the reservation offer or price of the person residing at location $ij$ for location $\widehat{ij}$ is

$$p = 2 \cdot \left\{ \left(1 + \frac{1}{2 \cdot \phi_t^{ij} \cdot \beta(S(N_t^{\widehat{ij}}))}\right) - \left(1 + \frac{1}{2 \cdot \phi_t^{ij} \cdot \beta(S(N_t^{ij}))}\right) \cdot \left(\frac{\beta(S(N_t^{ij}))}{\beta(S(N_t^{\widehat{ij}}))}\right)^{1/2} \right\}.$$

Two individuals will be willing to trade places if the maximum that one is willing to offer for the second's position, number one's reservation offer, is more that the minimum the second is willing to take to move to the first's position, number two's reservation price. If there is a difference between these two prices, then the price at which the trade actually takes place is a result of a two person bargaining problem. Following Osborne and Rubinstein (1990)[5], assume that individuals are paired by a random matching procedure and will bargain if the reservation offer is equal to or greater than the reservation price. Bargaining is assumed to take time and both involved in the bargaining take utility losses the longer the negotiations continue, determined by a discount factor. The standard solution to this problem is that agreement will be reached on the first offer and if both have the same discounting factor, the price will exactly split the difference between the reservation offer and reservation price.

Once a pair of individuals switch locations, the descendents of these individuals, who have the same skill level as the parent, continue to reside in the new location unless they move by a welfare improving trade. We can observe how the ability to make locational trades affects the locational distribution of talent.

The model begins with the same distribution of skills as the earlier models (shown in Figure 2). A draw from a uniform random distribution of talent with $.9 < \phi_t^{ij} < 1.1$ is also assigned to each location and this assigned talent

[5]A discussion fo the random matching model used here is found on pages 173 to 180.

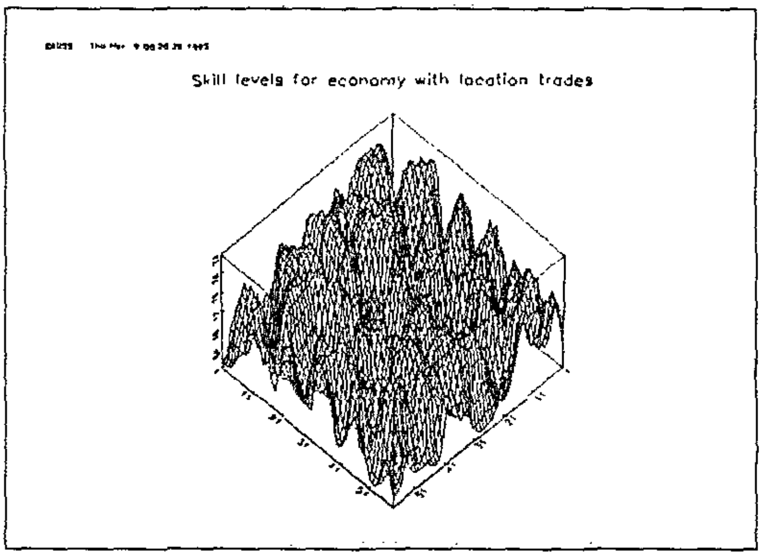


Figure 13: Skill levels for generation 5 for economy with location trade

value stays with the descendents of the individual who starts period one on that location wherever they move. In each period, five random matchings are made and members of a matching trade locations with side payments, if doing so is utility improving for both. Following Osborne and Rubinstein (1990), the difference between the reservation offer and reservation price is divided equally between them. After each trade, the endowment when young of the one moving to a higher skill are is reduced by the amount paid and the endowment of the one moving to the lower skill are is increased by the same amount. By period five, a regular distribution of skills is begins to appear that is quite similar to the one that occurred in the original version of the model, the one without student loans or location trades. Figure 13 shows the distribution of skills for the fifth generation of the sample economy. The distribution of talent for generation five is shown in Figure 14. While some clumping of families with similar talent levels has begun, the talent distribution still appears quite random.

By generation 100, the pattern for this economy is much more developed. Figure 15 shows the skill surface for generation 100. The region of high skills is much larger (it has grown since generation 5) and has coalesced into one large region. Again, this region is quite similar to the one found in Figure5, the distribution for the twentieth generation of the original economy. More interesting is what has happened to the talent levels. Figure 16 shows the talent surface for generation 100. Much of the high talent population has migrated into the high skill region (compare the high regions of Figure 15 to those of Figure

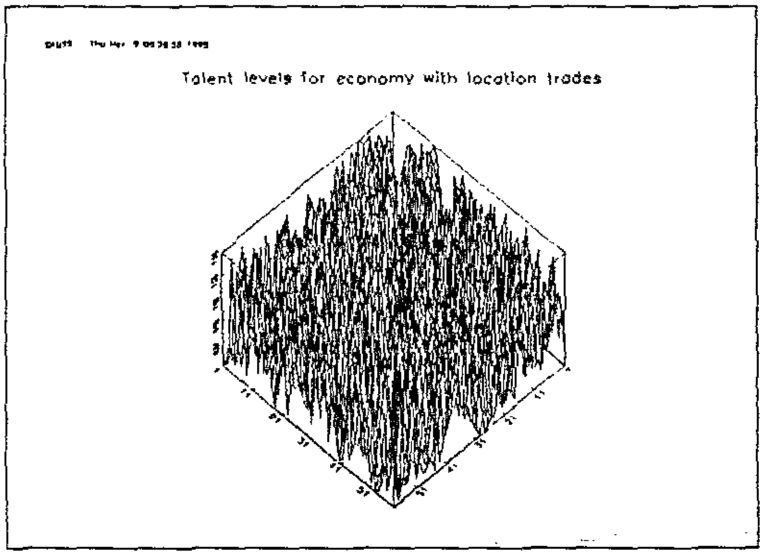


Figure 14: Talent levels for generation 5

16). While it is clear that the migration is not perfect, especially along the sides of the high talent regions where a number of talent spikes can be observed, there is substantial separation of individuals by talent.

The talent separation is even greater when the variance of the original distribution of talent is greater. Figure 17 shows the distribution of talent in generation 100 for an economy with location trades that is identical to the one shown above except that the initial distribution of talent was uniformly random with $.7 < \phi_t^{ij} < 1.3$. Notice how much smoother the surface is. With the greater talent variance, there are greater rewards for the most highly talented to migrate to the high skill areas and more trades are possible. In fact, more trades take place. For the economy where $\phi_t^{ij}$ ranges between .7 and 1.3, the matching model causes 8221 trades to take place during the first five periods. In an otherwise identical economy where $\phi_t^{ij}$ ranges between .9 and 1.1, only 6039 trades take place in the first five periods. The corresponding numbers after 100 periods are closer, 36,288 trades when $\phi_t^{ij}$ ranges between .9 and1.1 and 39,682 trades when it ranges from .7 to 1.3.

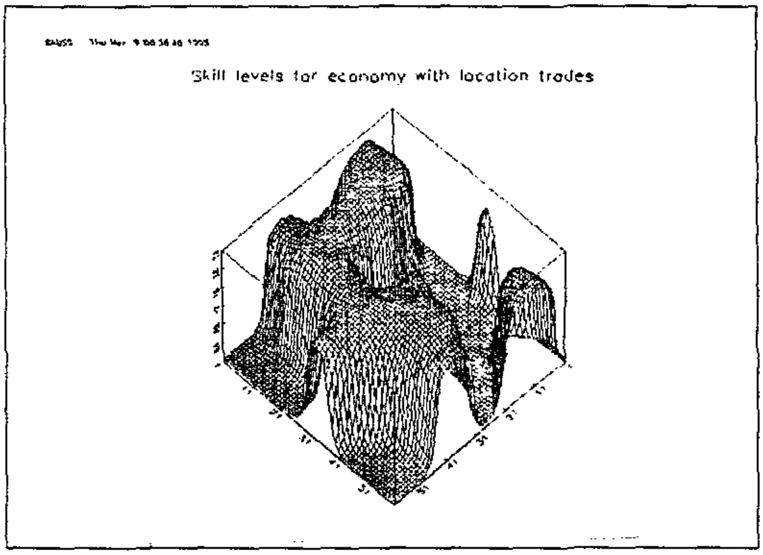


Figure 15: Skill levels of generation 100

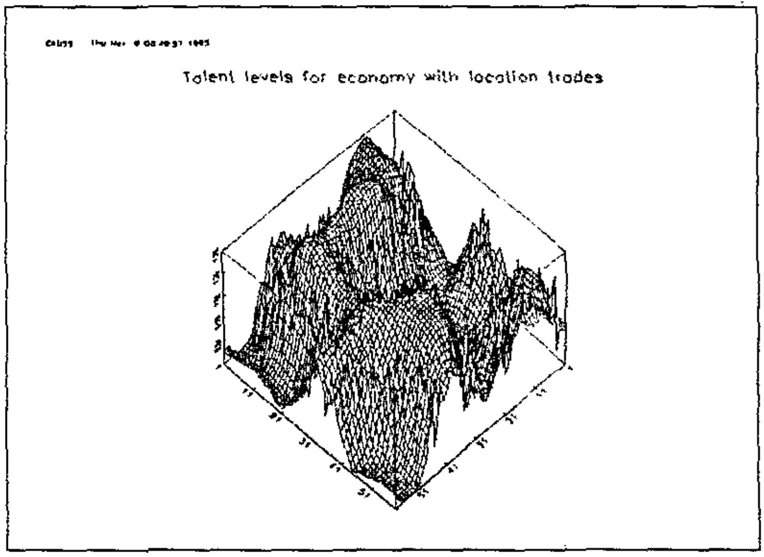


Figure 16: Talent levels for generation 100

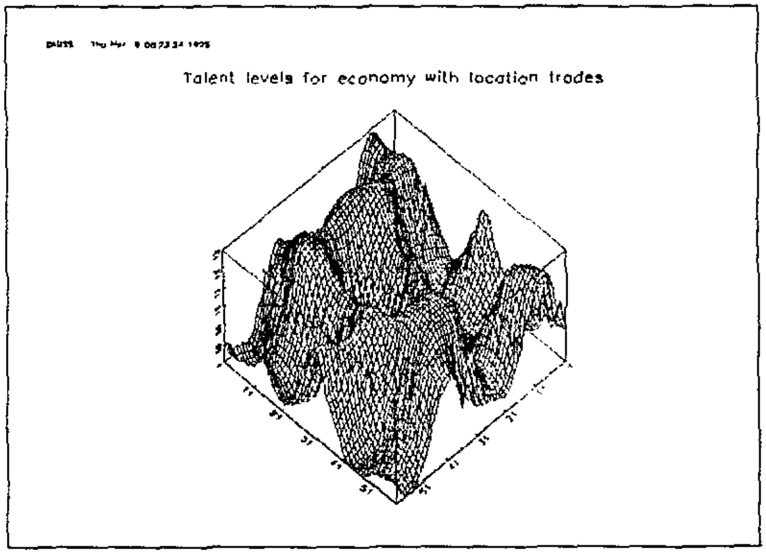


Figure 17: Talent levels for generation 100 for initial distribution of $.7 < \phi_t^{ij} < 1.3$