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Unemployment, Migration, and Growth

Valerie R. Bencivenga and Bruce D. Smith*

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ABSTRACT

Economic development is typically accompanied by a very pronounced migration of labor from rural to urban employment. This migration, in turn, is often associated with large scale urban underemployment. Both factors appear to play a very prominent role in the process of development. We consider a model in which rural-urban migration and urban underemployment are integrated into an otherwise conventional neoclassical growth model. Unemployment arises not from any exogenous rigidities, but from an adverse selection problem in labor markets. We demonstrate that, in the most natural case, rural-urban migration—and its associated underemployment—can be source of multiple, asymptotically stable steady state equilibria, and hence of development traps. They also easily give rise to an indeterminacy of perfect foresight equilibrium, as well as to the existence of a large set of periodic equilibria displaying undamped oscillation. Many such equilibria display long periods of uninterrupted growth and rural-urban migration, punctuated by brief but severe recessions associated with net migration from urban to rural employment. Such equilibria are argued to be broadly consistent with historical U.S. experience.

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Two very prominent aspects of actual economic growth are typically omitted from standard versions of neoclassical growth models. First, modern economic development has invariably been accompanied by a pronounced migration from rural to urban sectors of employment. This has been accompanied, of course, by substantial movements of labor out of agricultural production. And second, all economies display some unemployment, which is often quite significant in developing countries. Moreover, many developing economies have a very large portion of their labor force employed in a relatively low wage, "informal" urban production sector. This is often viewed as a form of underemployment.

There are significant reasons to think that these factors play an important role in the process of economic development. One is that, at least in the early stages of development, capital formation in urban manufacturing is far more rapid than that in agriculture. As a result, urban wage rates rise relative to rural wage rates, drawing labor into the city. The result is often dramatic: between 1950 and 1975 the share of urban population in total Third World population rose from 16.7 to 28 percent [Williamson (1988), p. 428]. As a consequence, the composition of employment and output typically change accordingly: in Malaysia, for example, the fraction of the work force in non-agricultural employment approximately doubled between 1957 and 1989 (World Development Report 1995, p. 20). These changes, in turn, permit more labor to be combined with capital, thereby promoting output growth.

Moreover, not only does urbanization change the composition of employment and output - as well, presumably, as the terms of trade between agriculture and other sectors - but it is also generally accompanied by significant urban unemployment and/or underemployment. This underemployment, obviously, can be viewed as a factor which is detrimental to growth, serving as a deterrent to urban migration, and also serving to reduce average income and savings. Our goal in the present paper is to produce a theoretical framework for analyzing the interaction between migration, unemployment, capital formation, and economic growth.

The link between rural-urban migration and unemployment has, of course, attracted enormous attention in the literature on economic development. This is much less the case with the link between these two factors and economic growth.[1] Perhaps the best known analysis of rural-urban migration and urban unemployment originated with Todaro (1969) and Harris and Todaro (1970). That work took the view that labor migrates to wherever its expected income[2] is highest: hence in equilibrium expected incomes - at least for relevant workers - must be equated between urban and rural employment. Since urban wage rates are invariably much higher than rural wage rates,[3] the equilibration of incomes occurs through the existence of unemployed or underemployed urban labor.

What permits this unemployment to persist? In the Harris-Todaro model, and in much subsequent literature, it is an "institutionally fixed" urban real wage. This is often viewed as fixed due to the consequences of minimum wage legislation and/or the power of labor unions [Calvo (1978)]. However, to whatever extent that this is true in modern developing countries,[4] it is not a particularly satisfactory explanation of urban unemployment in historical economic development. Hatton and Williamson (1992) and Williamson (1988) document the existence of significant urban-rural wage differentials - and urban underemployment - in the U.S. and the U.K. of the nineteenth century: these presumably are not easily attributed to the presence of minimum wage legislation or to the power of unions. In view of Williamson's (1988, p. 428) conclusion "that

Third World urbanization experience has been fairly conventional by historical standards," it seems that some other explanation for the existence of urban unemployment is called for.

As a final point of note, Hatton and Williamson (1992) document a significant degree of fluctuation over time in urban-rural wage differentials in U.S. economic history. For example, the ratio of (nominal) rural to urban wage rates rose from slightly above 0.5 in 1890, to above 0.6 in 1915, and then fell to about 0.35 in 1940. And while one mostly observes migration from rural to urban employment in the historical development process; this migration has periodically been interrupted by brief but marked episodes of reverse net migration. This reverse migration is typically associated with sharp economic downturns: Hatton and Williamson (1992, p. 277) show that there was considerable net migration back to agriculture in the U.S. in the early 1930s, and they predict that such migration should have occurred immediately before the pronounced recession of 1920-1921. Thus a reasonable view of historical economic development is that generally sustained economic growth and migration from rural to urban employment are occasionally interrupted by brief, but possibly severe downturns, accompanied by migration *from urban to rural employment.*

In this paper we produce a two period, overlapping generations model which contains an urban and a rural production sector. Following conventional "dual economy" formulations, we assume that production in the "formal" urban sector uses both capital and labor, while production in the rural sector uses labor alone. In addition, we allow for the presence of an "informal" urban sector where production takes place using only labor.

We also assume that the labor force is heterogeneous: some types of workers are intrinsically more skilled than others. We impose the assumption that workers' types are private information, with the consequence that an adverse selection problem arises in urban labor markets. It is this adverse selection problem, rather than any exogenous rigidities, which permits unemployment or underemployment to exist: unemployed - or underemployed - workers operate in the informal urban sector, earning relatively low incomes.

As capital is accumulated in the urban sector, this tends to raise the real wage rate in formal urban manufacturing relative to that in agriculture, ceteris paribus (and, as it turns out, in general equilibrium). As a result, under one technical condition, labor is induced to migrate to the city, and the adverse selection problem there becomes more severe. In response to this increased severity of the informational friction, employers are forced to hire less labor than actually migrates, and the urban unemployment (underemployment) rate rises. It is this increase that equilibrates the migration process, exactly as in the Harris-Todaro (1970) model.

Under one technical assumption, high *capital-labor ratios* in formal urban manufacturing are associated with high urban wage rates, high average incomes, net rural-urban migration, and high savings rates. They thus lead to high future aggregate capital *stocks*. However, in the presence of rural-urban migration this does not necessarily imply that future capital-labor *ratios* in the formal urban sector will be high; indeed if too much migration is induced the capital-labor ratio can actually fall. Thus we may *not* observe the monotonic relationship between the current and future capital-labor *ratio* which arises in so much of neoclassical growth theory.

We describe conditions under which our model possesses either one or two non-trivial steady state equilibria. If there is a unique non-trivial steady state, it is necessarily asymptotically stable, and paths approaching it display (locally) monotone dynamics. If there are two non-trivial steady states, one has a relatively high and one a relatively low capital stock (and capital-labor ratio in formal urban manufacturing). The high-capital-stock steady state is again necessarily asymptotically stable, and dynamical equilibrium paths approaching it display locally monotone dynamics. The low-capital-stock steady state may be either asymptotically stable or unstable; in the former case dynamical equilibrium paths approaching that steady state display damped oscillation. Thus development trap phenomena and endogenous oscillation can easily be observed: these are consequences of the process of rural-urban migration. Indeed, it is possible for two economies with the same initial capital stock to approach different steady states: this is obviously a reflection of an indeterminacy of equilibrium that is also unusual in neoclassical growth models.

When there are two asymptotically stable steady states, we show that the following phenomenon can easily arise. There can be a large set of periodic equilibria displaying undamped oscillation: in such equilibria the aggregate capital stock, the capital-labor ratio in formal manufacturing, urban-rural wage differentials, and net rural-urban migration can fluctuate over time. In order to be consistent with observation, these equilibria must have the feature that long periods of growth and net rural-urban migration are interrupted by *brief*, but possibly sharp economic downturns, accompanied by reverse migration. We state conditions under which there is a large set of oscillatory equilibrium paths displaying exactly this pattern. In such equilibria, occasional pronounced recessions will be observed.

The model we use to analyze these issues is a dual economy version of Diamond's (1965) neoclassical growth model. Its most obvious antecedents in the literature appear to be the models of Drazen and Eckstein (1989) and Rauch (1993). The former paper analyzes a totally different set of issues than those occupying our attention. The latter, while in many respects similar to ours, assumes an institutionally fixed formal urban wage rate to generate unemployment, and considers a small open - as opposed to a closed - economy. Finally, while we focus on a model that cannot generate sustained growth - as is also true of the literature just mentioned - it is straightforward to introduce exogenous technical progress, and to obtain exactly the same results we report.

The remainder of the paper proceeds as follows. Section I outlines the model, while section II states a set of equilibrium conditions that must be satisfied in factor markets. Section III derives the full set of equilibrium conditions of the model, and sections IV and V characterize steady state and dynamical equilibria, respectively. Section VI considers some implications of the existence of development traps for the size of the "informal" urban sector, while section VII concludes.

I. The Model

A. Environment

We consider an economy consisting of an infinite sequence of two period lived, overlapping generations, along with an initial old generation. Let $t = 0,1,2,...$ index time. At each date t a new young generation appears, containing a continuum of agents with mass n^t. Thus n is the gross rate of population growth.

In each period two kinds of goods can be produced, which we can think of as manufactured and agricultural commodities. In addition, we divide young agents into two types: types differ in terms of their intrinsic labor skills. Let $i = 1,2$ index agent's types. Then a fraction θ_i of the population is of type i; clearly we will assume that $\theta_i \in (0,1)$; $i = 1,2$, and that $\theta_1 + \theta_2 = 1$.

As is common in dual economy models, we will assume that the production of agricultural commodities uses only labor as an input.[5] A young type i agent employed in agricultural production can produce π_i ($i = 1,2$) units of the good per unit time. We assume that $\pi_2 > \pi_1 > 0$ holds, so that type 2 agents are relatively more productive than type 1 agents in agricultural employment. We also assume that agents engaged in agriculture either work for themselves, or have output levels that are perfectly observed by their employers. This assumption precludes the existence of a private information problem in the agricultural sector.

Any agent can seek employment in either agriculture or manufacturing, but not in both. We can interpret this as agents must move to the city to work in manufacturing. Thus there is a discrete choice to be made by workers as to their sector of employment.

A firm engaged in manufacturing production, and which employs K_t units of capital and L_t units of young type 2 labor at t, can produce $F(K_t,L_t)$ units of the manufactured good at that date. We assume that type 1 agents are totally unproductive in manufacturing. This assumption is not essential to the analysis, and is obviously meant to represent the limiting case of the situation where type 1 agents are merely relatively less productive than type 2 agents in urban manufacturing. We impose the limiting case-where type 1 agents are completely unproductive in the urban sector - largely because it delivers a standard Harris-Todaro (1970) condition that determines the urban unemployment rate.

Let $f(k_t) \equiv F(k_t, 1)$ denote the intensive production function, where $k_t \equiv K_t/L_t$ is the capital-labor ratio in the formal manufacturing sector. We will assume that f has the constant elasticity of substitution form

(1) $$f(k) = [ak^{\rho}+b]^{1/\rho} \; ; \; \rho < 1.$$

We will refer to production of the manufactured good that takes place using the technology in (1) as "formal" urban manufacturing. We also assume that there is an "informal" urban sector whereby young agents can produce the manufactured good using only labor as an input.[6] With this technology, a young type i agent can produce β_i units of the manufactured good per unit time "at home." We will assume that this informal sector is inefficient in the sense that

$$(2) \qquad F_2(k_t, 1) > \beta_2,$$

for all "relevant" values of k_t. We will also maintain our standard assumption that type 2 agents are more productive than type 1 agents, so that $\beta_2 > \beta_1 \geq 0$ holds. We will often set $\beta_1 = 0$, as that specification yields a Harris-Todaro-like condition that determines the urban unemployment rate.

Finally we assume that each agent knows his own type, but that this is private information ex ante. This assumption leads to the existence of an adverse selection problem in urban labor markets. It is this problem which is the source of equilibrium urban unemployment.

With respect to endowments, we assume that old agents are endowed with an initial aggregate capital stock of $K_o > 0$. Thereafter, no agents are endowed with either capital, or final goods. All young agents are endowed with one unit of labor, which they supply inelastically in either the urban or the rural sector. Agents have no labor endowment when old.

With respect to preferences, we assume that all agents care only about second period consumption. This assumption, which is inessential to the analysis, implies that all young period income is saved. This permits us to economize on notation by ignoring any young period consumption-savings decisions. Let c_{mt} (c_{at}) denote the time t consumption of the manufactured (agricultural) good by a representative old agent. For reasons that will become apparent, it is convenient to have agents be risk neutral, and at the same time to have constant aggregate expenditure shares for the two goods. We therefore assume that all agents have the utility function

(3) $$u(c_{mt},c_{at};\psi) = \psi\, c_{mt} + (1-\psi)c_{at},$$

where ψ is an iid (across agents) random variable with the probability distribution

(4) $$\psi = \begin{cases} 0 \text{ with probability } \gamma \\ 1 \text{ with probability } 1-\gamma \end{cases}$$

Realizations of ψ are known at the beginning of an agent's second period. Obviously, then, a fraction $1-\gamma$ (γ) of old agents purchase only manufactured (agricultural) goods.[7]

Finally, it remains to describe how the capital stock evolves. We assume that one unit of the manufactured good set aside at t becomes one unit of capital at t + 1. We also assume that capital is used in manufacturing production, and then depreciates completely. The latter assumption is inessential to the analysis.

B. Trade

Three kinds of transactions occur in this economy. First, old agents use the proceeds of their young period savings to purchase agricultural and manufactured goods in competitive markets. Second, young agents decide in which sector to seek employment, and urban producers decide how much capital and how much labor to employ. Third, young agents save their income; here all savings ultimately take the form of physical capital.

Throughout we let the manufactured good be the numeraire at each date. In addition, we let p_t denote the relative price of the agricultural good at t, and w_t denote the real wage rate paid in urban manufacturing. r_t denotes the time t capital rental rate; since all savings take the form of capital, the one hundred percent depreciation assumption implies that r_t is also the gross real return on savings between $t-1$ and t.

II. Factor Markets

A. Preliminaries

Each young agent has a choice between seeking employment in formal urban manufacturing - and earning the real wage rate w_t if employed there - or working in agriculture. A young type i agent employed in agriculture at t obviously earns an income of $p_t\pi_i$. If a type i agent moves to the urban sector but fails to find employment in formal manufacturing, then they engage in informal production and earn the income β_i. Agents in the urban sector who fail to find formal employment will be called unemployed - but clearly here this term also denotes that they are employed in the informal sector. We let ϕ_t denote the fraction of type 2 agents who choose to seek employment in the urban sector at t, and we let u_t denote the fraction of the urban labor force that is unemployed (employed in the informal sector) at the same date. Clearly there is no unemployment in agriculture.

In equilibrium, only type 2 agents will choose to work in the urban sector.[8] Then if $\phi_{t+1} >$ ($<$) ϕ_t holds, we will say that there is net migration into (out of) that sector.

We focus here on equilibria where $\phi_t \in (0,1)$ holds $\forall$ t. This focus implies that there is always some remaining potential for further rural-urban migration. In order for $\phi_t < 1$ to hold, it is clearly necessary that $p_t\pi_2 > \beta_2$ be satisfied, as we henceforth assume.

Suppose, as we have indicated, that only type 2 agents choose to seek urban employment, and that the fraction $1 - u_t$ of them are employed in the formal sector. Then the total labor force in formal urban manufacturing at t is given by

(5) $$L_t = \theta_2\phi_t\,(1 - u_t)n^t.$$

If the aggregate available capital stock at t is K_t, then the capital-labor ratio in formal urban manufacturing is given by

(6) $$k_t \equiv K_t/L_t = K_t/\theta_2\phi_t\,(1 - u_t)n^t.$$

Obviously an essential aspect of describing an equilibrium in labor markets is the determination of who seeks employment in which sector. We now turn our attention to this issue.

B. Self-Selection

A young type 2 agent employed in agriculture at t earns the real income level $p_t\pi_2$. This income is saved, yielding $r_{t+1}p_t\pi_2$ at t + 1. In addition, at t + 1 an agent will purchase only manufactured (agricultural) goods with probability $1 - \gamma$ (γ): in the former (latter) case $r_{t+1}p_t\pi_2$ ($r_{t+1}p_t\pi_2/p_{t+1}$) units of manufactured (agricultural) goods can be purchased. Hence the expected utility of a young type 2 agent employed in agriculture at t is given by the expression $[(1-\gamma)r_{t+1} + (\gamma r_{t+1}/p_{t+1})]p_t\pi_1$. Similarly, a young type 2 agent who seeks employment in the urban sector will find urban employment with probability $1 - u_t$, and will then earn the real wage w_t. With probability u_t the same agent will be unemployed (or employed informally): in this case the agent's real income will be β_2. Thus, by the same reasoning as before, a young type 2 agent who chooses to work in the urban sector has an expected utility level of $[(1-\gamma)r_{t+1} + (\gamma r_{t+1}/p_{t+1})]\,[(1-u_t)w_t + u_t\beta_2]$. Clearly $\phi_t \in (0,1)$ can hold iff young type 2 agents are indifferent between working in the two sectors: this requires that

(7) $$p_t\pi_2 = (1-u_t)w_t + u_t\beta_2\,;\ t \geq 0.$$

In view of the assumption that $p_t\pi_2 > \beta_2$, obviously (7) can hold only if

(8) $$w_t > p_t\pi_2\,;\ t \geq 0.$$

Hence type 2 agents prefer formal sector urban employment to rural employment, and they prefer the latter to informal urban employment.

Similarly, a young type 1 agent employed in the rural sector at t has a real income of $p_t\pi_1$, and an expected utility level of $[(1-\gamma)r_{t+1} + (\gamma r_{t+1}/p_{t+1})]p_t\pi_1$. If the same agent were to seek employment in the urban sector, he would be formally (informally) employed with probability $1-u_t$ (u_t), and would earn a real income of w_t (β_1). Hence a search for urban sector employment yields a young type 1 agent the expected utility level $[(1-\gamma)r_{t+1} + (\gamma r_{t+1}/p_{t+1})]\ [(1-u_t)w_t + u_t\beta_1]$. Obviously, then, type 1 agents are deterred from seeking urban employment only if

(9) $$p_t\pi_1 \geq (1-u_t)w_t + u_t\beta_1\,;\ t \geq 0.$$

When (8) holds, clearly $u_t > 0$ must obtain in order for (9) to be satisfied. Thus urban unemployment is required in order to deter type 1 agents from entering the urban labor force.

It remains to describe the behavior of employers in the formal urban sector. Following standard conventions in models of adverse selection,[9] we assume that firms in this sector are Nash competitors in labor markets.[10] In particular, each firm announces a contract consisting of a wage rate (w_t) and an employment probability ($1-u_t$) and, in doing so, takes the contract announcements of other firms as given.[11] Clearly there are some constraints on these contract announcements. First, they must satisfy (7) and (9) unless all workers are to be drawn into the urban sector. Second, firms must earn non-negative profits, given the workforce that their announcements attracts. Then if a firm hires N_t workers, of whom a fraction ε_t are of type 2, the non-negative profit condition requires that

$$(10) \qquad \max_{K_t, N_t} \{F(K_t, \varepsilon_t N_t) - w_t N_t - r_t K_t\} \geq 0.$$

Standard arguments [see, for example, Rothschild and Stiglitz (1976)] establish that any Nash equilibrium contract announcements must maximize the expected utility of type 2 agents, subject to the constraints already stated. Thus, in particular, w_t, u_t, and K_t must be chosen to maximize $(1-u_t)w_t + u_t\beta_2$, subject to (9) and (10). At the same time, standard arguments [Rothschild and Stiglitz (1976)] can be used to show that any Nash equilibrium contracts induce a separating equilibrium or, in other words, an equilibrium where only type 2 workers seek urban employment. Therefore, in any Nash equilibrium (9) must hold and - if (8) is satisfied - it must hold as an equality. In addition, $\varepsilon_t = 0$ holds for all t. Since it is easy to see that (10) must also hold with equality in a Nash equilibrium, the standard factor pricing relationships must obtain. In other words, capital and labor both earn their marginal products in formal urban manufacturing, or

$$(11) \qquad r_t = f'(k_t) = a^{1/\rho}\,[1 + (b/a)\ k_t^{-\rho}]^{(1-\rho)/\rho}$$

$$(12) \qquad w_t = f(k_t) - k_t f'(k_t) \equiv w(k_t) = b^{1/\rho}[(a/b)\,k_t^{\rho} + 1]^{(1-\rho)/\rho};\ t \geq 0.$$

In addition, if we impose $\beta_1 = 0$ in (9), then the equilibrium unemployment rate must satisfy[12]

$$(13) \qquad p_t\pi_1 = (1-u_t)w_t;\ \ t \geq 0.$$

Equation (13) asserts that the unemployment rate must equilibrate the expected incomes type 1 agents obtain in both urban and rural employment. It obviously closely resembles the same condition in the Harris-Todaro (1970) model, except that the urban wage rate is not exogenous here, and will evolve over time.

C. A "No-Pooling" Condition

If a Nash equilibrium with respect to contract announcements exists, it has the features just described. However, in order for a Nash equilibrium to exist, it is necessary that no potential employer has an incentive to announce an alternative contract, given the presence of the contracts just derived. In this section we state conditions under which no such incentives exist: these conditions are derived under the same assumptions made previously. In particular, each potential employer takes the sequences $\{r_t\}$ and $\{p_t\}$ as given, and unaffected by their own contract announcements.

Clearly no employer can offer a contract that is profitable, and attracts only type 2 agents in any period. Thus the only possible incentive for offering an alternative contract would be to attract workers of both types, and to pool them in an effort to evade the necessity of satisfying the self-selection constraint (13). If an incentive to offer such a contract exists, then obviously the contract will attract all workers - and therefore will attract them in their population proportions. An additional requirement of a Nash equilibrium, then, is that there exist no such pooling contract which is perceived to be profitable by employers.

Let a potential pooling contract at t consist of a wage offer, $\tilde{w}_t$, and an employment probability (or hours level) $1 - \tilde{u}_t$ for all workers willing to accept the contract. As before, workers seeking a particular contract who are *not* hired enter into the informal urban sector. Finally, let $\tilde{K}_t$ denote the capital stock employed by the firm offering the pooling contract. There is an obvious constraint on the choice of $\tilde{K}_t$: it cannot exceed the aggregate capital stock available at t.

If a firm announcing a pooling contract is to earn non-negative profits, then $\tilde{w}_t$ must satisfy the condition

(14) $$\tilde{w}_t \leq \max_{\tilde{K}_t \leq K_t} \{F[\tilde{K}_t, \theta_2 n^t (1-\tilde{u}_t)] - r_t \tilde{K}_t\}/n^t (1-\tilde{u}_t),$$

since the contract attracts *all* workers. In addition, in order to attract type 2 workers, a pooling contract must satisfy

(15) $$(1-\tilde{u}_t)\tilde{w}_t + \tilde{u}_t \beta_2 > (1-u_t)\, w(k_t) + u_t\beta_2$$

where u_t represents the prevailing equilibrium value of the unemployment rate.

We are now prepared to state conditions implying that there exists *no* pooling contract satisfying (14) and (15) at any date.

Proposition 1. (a) Suppose that

(i) $$\theta_2 w[k_t\phi_t(1-u_t)] \geq \beta_2$$

holds. Then *no* pooling contract satisfying (14) and (15) exists at t if

(ii) $$(1-u_t)\, w(k_t) \geq (\pi_1/\pi_2)\, \theta_2 \{f[k_t\phi_t(1-u_t)] - f'(k_t)\, k_t\phi_t\, (1-u_t)\}$$

is satisfied. (b) Suppose that condition (i) *fails* to hold. Then *no* pooling contract satisfying (14) and (15) exists at t if

(iii) $$[w(k_t) - \beta_2]/\phi_t k_t + \theta_2\, w(k_t)(a/b)\, k_t^{\rho-1} \geq \beta_2(a/b)[w^{-1}(\beta_2/\theta_2)]^{\rho-1}$$

is satisfied.

Proposition 1 is proved in appendix A. It states conditions that must be satisfied in equilibrium in order to sustain self-selection in the labor market.

III. General Equilibrium

We now wish to characterize the determination of p_t, k_t, u_t, and ϕ_t in a full general equilibrium. Doing so requires an analysis of several conditions. First, self-selection in labor markets must occur as part of a full general equilibrium, as opposed to a partial equilibrium of the type discussed in section II. Second, we must describe the evolution of the aggregate capital stock, as well as of the capital-labor ratio in formal urban manufacturing. And third, the markets for agricultural and manufactured goods must clear. We now describe what is required by each of these equilibrium conditions.

A. Self-Selection

Suppose we write equation (7) as

$$p_t\pi_2 = p_t\pi_1(\pi_2/\pi_1) = (1-u_t)w_t + \beta_2 u_t\,;\ t \geq 0. \tag{7'}$$

If (8) holds, then $\beta_1 = 0$ implies that (13) holds as well. Substituting (13) into (7′) yields

$$(\pi_2/\pi_1)(1-u_t)w_t = (1-u_t)w_t + \beta_2 u_t. \tag{16}$$

Solving (16) for u_t and using (12) we obtain

$$u_t = [(\pi_2-\pi_1)/\beta_2\pi_1]w(k_t)/\{[(\pi_2-\pi_1)/\beta_2\pi_1]w(k_t) + 1\}\ ;\ t \geq 0. \tag{16'}$$

Equation (16′) gives the unemployment rate in formal urban manufacturing as a function of the wage rate (and - by implication - the capital-labor ratio) in the same sector. Clearly the higher the formal urban wage rate (and, since $w'(k) > 0$, the capital-labor ratio in formal manufacturing), the higher the rate of urban unemployment. This is what would be expected from the Harris-Todaro model. Finally, for future reference we note that (16′) implies

$$1-u_t = \{[(\pi_2-\pi_1)/\beta_2\pi_1]w(k_t) + 1\}^{-1};\ t \geq 0. \tag{17}$$

B. Savings Equals Investment, and the Formal Sector Capital-Labor Ratio

Since all young period income is saved in the form of investment in physical capital, the time t + 1 capital stock is given by

$$K_{t+1} = n^t \{\theta_1 p_t\pi_1 + \theta_2(1-\phi_t)p_t\pi_2 + \theta_2\phi_t(1-u_t)w_t + \theta_2\phi_t u_t\beta_2\} = n^t(\theta_1 p_t\pi_1 + \theta_2 p_t\pi_2) = n^t[\theta_1 + \theta_2(\pi_2/\pi_1)]p_t\pi_1 = n^t[\theta_1 + \theta_2(\pi_2/\pi_1)](1-u_t)w(k_t)\ ;\ t \geq 0. \tag{18}$$

The first equality in (18) obtains because $\theta_1 n^t$ type 1 workers all work in agriculture, earning the real income $p_t\pi_1$ each. They then save all of this income. Similarly, a fraction $1-\phi_t$ (ϕ_t) of the $\theta_2 n^t$ type 2 workers are employed in agriculture (manufacturing), where their (expected) income is given by $p_t\pi_2$ $[(1-u_t)w_t + \beta_2 u_t]$. Again, all this income is saved, yielding the first equality in (18). The second equality in (18) is then implied by (7), while the last equality in (18) is a consequence of equation (13).

We now wish to express equation (18) in terms of the capital-labor ratio. Noting that equation (6) can be written in the form $K_{t+1} = \theta_2 k_{t+1} n^{t+1}\phi_{t+1}(1-u_{t+1})$, and using this observation in (18), we have that the capital-labor ratio evolves according to

$$\theta_2 k_{t+1}\phi_{t+1}(1-u_{t+1}) = \{[\theta_1 + \theta_2(\pi_2/\pi_1)]/n\}\ (1-u_t)w(k_t)\ ;\ t \geq 0. \tag{19}$$

It is now clearly necessary to analyze the equilibrium determination of ϕ_t. To do so, we must examine the implications of the requirement that the market in agricultural commodities clears.

C. Goods Market Clearing

The supply of agricultural commodities at t is given by $[\theta_1\pi_1 + \theta_2(1-\phi_t)\pi_2]n^t$, since $\theta_1 n^t$ type 1 workers and $\theta_2(1-\phi_t)n^t$ type 2 workers are engaged in agricultural production at that date. The demand for agricultural products at t is given by $\gamma n^{t-1}\{(r_t/p_t)[\theta_1 p_{t-1}\pi_1 + \theta_2(1-\phi_{t-1})p_{t-1}\pi_2 + \theta_2\phi_{t-1}(1-u_{t-1})\, w_{t-1} + \theta_2\, \phi_{t-1}\, \beta_2\, u_{t-1}]\}$. This is the case since all demand for agricultural goods derives from the γn^{t-1} old agents at t who experience $\psi = 0$. The type 1 agents in this group have an old period income of $r_t p_{t-1}\pi_1$ at t. The $\theta_2 n^{t-1}(1-\phi_{t-1})$ $(\theta_2 n^{t-1}\phi_{t-1})$ type 2 agents in the same group who were employed in the rural (urban) sector when young have an (expected) income when old of $r_t p_{t-1}\pi_2$ $(r_t[(1-u_{t-1})w_{t-1} + \beta_2 u_{t-1}])$. All of this is expended on agricultural commodities at t.

We now observe that

$$(20) \qquad \theta_1 p_{t-1}\pi_1 + \theta_2(1-\phi_{t-1})p_{t-1}\pi_2 + \theta_2\phi_{t-1}[(1-u_{t-1})w_{t-1} + \beta_2 u_{t-1}]$$

$$= \theta_1 p_{t-1}\pi_1 + \theta_2 p_{t-1}\pi_2 = [\theta_1 + \theta_2(\pi_2/\pi_1)]p_{t-1}\pi_1 = K_t/n^{t-1}\,;\ t \geq 1,$$

where the first equality in (20) follows from (7) and the third from (18). It is therefore clear that the supply of and the demand for agricultural goods are equated when

$$(21) \qquad [\theta_1\pi_1 + \theta_2\pi_2(1-\phi_t)]p_t n^t = \gamma r_t K_t\,;\ t \geq 0.$$

Substituting $K_t = \theta_2 k_t n^t \phi_t(1-u_t)$ into (21), we have that the agricultural commodities market clears at t iff

$$(22) \qquad \phi_t \gamma k_t f'(k_t)\, \theta_2(1-u_t) = [\theta_1 + \theta_2(\pi_2/\pi_1)(1-\phi_t)]p_t\pi_1\,;\ t \geq 0.$$

Substituting (13) into (22) and rearranging terms, we obtain the equilibrium value of ϕ_t:

$$(23) \qquad \phi_t = [(\theta_1/\theta_2)(\pi_1/\pi_2) + 1]/\{1 + [(\pi_1/\pi_2)\gamma k_t f'(k_t)]/w(k_t)]\}\,;\ t \geq 0.$$

Evidently, $\phi_t < 1$ can hold iff

$$(24) \qquad \gamma k_t f'(k_t)/w(k_t) > \theta_1/\theta_2\,;\ t \geq 0$$

is satisfied.

An important ingredient of our analysis is the relationship between ϕ_t and k_t. Since the urban labor force at t is simply $\theta_2\phi_t n^t$, this relationship tells us how the size of the urban labor force depends on the capital-labor ratio in formal manufacturing and, by implication - the real wage rate in the same sector. Given the technological specification in (1), $k_t f'(k_t)/w(k_t) = (a/b)\, k_t^{\rho}$. Substituting this result into (23), we obtain,

(23′) $$\phi_t = [(\theta_1/\theta_2)(\pi_1/\pi_2) + 1]/\{1 + \gamma(\pi_1/\pi_2)(a/b)\, k_t^{\rho}\,] \ ; \ t \geq 0.$$

Thus ϕ_t increases (decreases) with the capital-labor ratio - and hence with the real wage rate in the formal urban sector - iff $\rho < (\geq)\ 0$ holds. In particular, when $\rho < 0$ holds, an increase in the formal urban real wage rate draws additional workers into the urban labor force. This is obviously the most interesting case, and it is a case for which there is considerable empirical support. In particular, most empirical estimates of the elasticity of substitution between capital and labor in non-agricultural production are less than unity, implying - in turn - that $\rho < 0$ holds. However, for completeness, we report results both for the cases $\rho < 0$, and $\rho \geq 0$.

D. The Equilibrium Law of Motion for the Capital-Labor Ratio

We are now prepared to describe the evolution of the capital-labor ratio in formal manufacturing. Define the function H(k) by

(25) $$H(k) \equiv k(\pi_1/\pi_2)/\{1 + [(\pi_2-\pi_1)/\beta_2\pi_1]w(k)\}\ \{1 + [\gamma(\pi_1/\pi_2)kf'(k)/w(k)]\}.$$

Then, if we substitute equations (17) and (23) into (19), we obtain the following equilibrium law of motion for k_t:

(26) $$H(k_{t+1}) = w(k_t)/n\ \{[(\pi_2-\pi_1)/\beta_2\pi_1]w(k_t) + 1\}\ ; \ t \geq 0.$$

Given an initial value k_o,[13] equation (26) describes the subsequent possible evolutions of the equilibrium sequence $\{k_t\}$. Having obtained this sequence, $\{\phi_t\}$ can be computed from (23′), while the sequence of unemployment rates $\{u_t\}$ can be calculated from (16′). The equilibrium values $\{p_t\}$ can then be derived from (13).

To summarize, a candidate equilibrium can be characterized as a sequence $\{k_t\}$ that satisfies (26) at each date. In addition, our derivation of an equilibrium was predicated on $w(k_t) > \beta_2$ and $\phi_t < 1$ holding in every period. Hence any legitimate equilibrium sequence must satisfy $k_t > w^{-1}(\beta_2)$ and (24) $\forall\, t \geq 0$. Finally, a genuine equilibrium sequence must satisfy the "no-pooling" conditions stated in proposition 1. We now turn our attention to the analysis of sequences satisfying these conditions in all periods. We begin with a consideration of steady state equilibria.

IV. Steady State Equilibria

Imposing $k_t = k_{t+1}$ in (26), using the definition of H(k), and rearranging terms, we obtain the following condition determining the steady state value(s) of k:

$$n(\pi_1/\pi_2) = f'(k)\ \{\gamma(\pi_1/\pi_2) + [w(k)/kf'(k)]\}. \tag{27}$$

Given the form of the production function in (1), equation (27) can be reduced to

$$n(\pi_1/\pi_2)a^{-1/\rho} = [\gamma(\pi_1/\pi_2) + (b/ak^{\rho})]\ [1 + (b/ak^{\rho})]^{(1-\rho)/\rho}. \tag{27'}$$

Equation (27′) gives the steady state equilibrium capital-labor ratio in formal manufacturing as a function of the rate of population growth (n), the relative productivity of type 1 and 2 workers in agriculture (π_1/π_2), the share of agricultural goods in total expenditure (γ), and parameters of the production function.

It will now be useful to make the following transformation. Define

$$x_t \equiv b/a\,k_t^{\rho} = w(k_t)/k_t f'(k_t). \tag{28}$$

Then x_t is simply the ratio of labor's share in formal manufacturing to capital's share; x_t is increasing (decreasing) in k_t if $\rho < (\geq)\ 0$. If we also define the function $Q(x)$ by

(29) $$Q(x) \equiv [\gamma(\pi_1/\pi_2) + x]\ (1+x)^{(1-\rho)/\rho},$$

then we can rewrite the steady state equilibrium condition (27′) in the form

(30) $$Q(x) = n(\pi_1/\pi_2)a^{-1/\rho}.$$

Equation (30) determines the steady state value of x; the steady state value of k can be recovered from (28).[14]

Evidently, it will be important to know the properties of the function Q. These are summarized in the following lemma. Its proof appears in appendix B.

<u>Lemma 1</u>. (a) $Q(0) = \gamma(\pi_1/\pi_2) < 1$ holds. (b) $Q'(x)$satisfies

(31) $$Q'(x)/Q(x) = [(1-\rho)/\rho]/(1+x) + 1/[\gamma(\pi_1/\pi_2) + x].$$

Thus, if $\rho \geq 0$, $Q'(x) > 0$ holds, $\forall$ x. If $\rho < 0$ holds, $Q'(x) \geq 0$ is satisfied iff $x \leq -\rho - (1-\rho)\,\gamma\,(\pi_1/\pi_2)$. (c) If $\rho < 0$ holds, then

$$\lim_{x\to\infty} Q(x) = 0.$$

If $\rho > 0$, then

$$\lim_{x\to\infty} Q(x) = \infty.$$

Armed with the results in lemma 1, we can now consider two general cases with respect to the determination of the steady state values of x.

<u>Case 1: $\rho < 0$</u>. When $\rho < 0$ holds, the function Q has the configuration depicted in figure 1. There are now three possibilities regarding the existence of a steady state equilibrium. These are summarized in the following proposition.

Proposition 2. (a) Suppose that

(32) $$-\rho > (1-\rho)\gamma(\pi_1/\pi_2)$$

and

(33) $$\gamma(\pi_1/\pi_2) < n(\pi_1/\pi_2)\, a^{-1/\rho} < Q[-\rho - (1-\rho)\gamma(\pi_1/\pi_2)]$$

are both satisfied. Then there are two values of x - denoted by $\underline{x}$ and $\overline{x}$ - satisfying (30). (b) Suppose that either (32) or the first inequality in (33) is violated, while the second inequality in (33) holds. Then there is a unique solution to (30), which we denote by $\overline{x}$. (c) Suppose that the second inequality in (33) is violated. Then there is no steady state equilibrium.

Proof. When (32) and (33) hold, we have the situation depicted in figure 1.a. Evidently (30) has two solutions. If [15]

$$b^{1/\rho}\,[(1+\underline{x})/\underline{x}\,]^{(1-\rho)/\rho} > \beta_2$$

holds, both steady states have $w(k) > \beta_2$.

When either (32) or the first inequality in (33) is violated, Q(x) has a configuration along the lines depicted in figure 1.b. It is then immediate that there exists a unique solution to (30). If [16]

$$b^{1/\rho}\,[(1+\overline{x})/\overline{x}\,]^{(1-\rho)/\rho} > \beta_2$$

holds, $w(k) > \beta_2$ will be satisfied at that steady state.

If the second inequality in (33) is violated, it is immediate from figures 1.a and 1.b that there can be no steady state equilibrium. □

When (32) and (33) both hold, two non-trivial steady state equilibria can exist. The potential existence of multiple steady states derives from the possibility of rural-urban migration. When $x = \underline{x}$ in steady state, the capital-labor ratio is low in formal urban manufacturing. As a result, the real wage rate in that sector is also low. With $\rho < 0$ one consequence of this fact is that the urban (rural) labor force is relatively small (large), and the supply of the agricultural good is correspondingly high. This tends to depress the relative price of the agricultural good, with the consequence that aggregate income is low, as is the aggregate capital stock. Thus the economy is stuck in a development trap where both the urban labor force and the capital stock are relatively small. When $x = \overline{x}$ in steady state, on the other hand, there is a relatively high wage rate in formal manufacturing, the urban labor force is correspondingly large, and average incomes are high. The induced savings then generates a relatively large aggregate capital stock. In this steady state, of course, the high wage rate leads to more unemployment than is observed in the low-capital-stock steady state.

When is there a potential for multiple steady state equilibria to arise? As is apparent from figures 1.a and 1.b, there will be a unique steady state equilibrium if the rate of population growth is sufficiently low. For intermediate levels of the population growth rate there can be multiple steady states, while excessive rates of population growth preclude the existence of any steady state equilibria, as is potentially true in the Diamond (1965) model. Thus it is intermediate rates of population growth that are conducive to the existence of more than one steady state.

Case 2: $\rho > 0$. When $\rho > 0$ holds Q is an increasing function, and hence has the configuration depicted in figure 2. The following proposition is then immediate.

Proposition 3. Suppose that $\rho > 0$ and $\gamma < na^{-1/\rho}$ hold. Then there exists a unique steady state equilibrium value of x, denoted by $\overline{x}$.[17]

We now wish to consider two issues: (i) will any steady state equilibrium necessarily be approached, and if so (ii) what is the nature of the dynamical equilibrium paths approaching that steady state? These questions are the topics of section V.

V. Dynamical Equilibria

Dynamical equilibria consist of sequences $\{k_t\}$ that satisfy equation (26), $w(k_t) > \beta_2$, (24), and the conditions of proposition 1 for all t. In this section we undertake an analysis of the equilibrium law of motion for k_t embodied in equation (26). We then produce a set of examples where all candidate equilibrium sequences obeying (26) also satisfy the remaining conditions at all dates.

As before, the cases $\rho < 0$ and $\rho \geq 0$ differ substantially. Since $\rho < 0$ corresponds to the situation where increasing urban wage rates draw labor into the urban sector, that case is of primary interest, and we focus most of our attention on it. However, we also report results for the case $\rho \geq 0$.

In each case it will be useful to work with a transformed version of equation (26). In particular, that equation can be written in the following form:

(34) $$w(k_t) = nH(k_{t+1})/\{1 - n[(\pi_2-\pi_1)/\beta_2\pi_1]\, H(k_{t+1})\}\ ;\ t \geq 0.$$

Equation (34) gives k_t as a function of k_{t+1}. While this is not the nature of the dynamics we wish to work with, it will be most convenient to analyze (34), and from that analysis to draw inferences about the natural "backward" dynamics in this economy. We now consider equation (34) when $\rho < 0$ and when $\rho \geq 0$ hold, respectively.

A. The Equilibrium Law of Motion: $\rho < 0$.

Since $w'(k) > 0$ holds, the properties of the equilibrium law of motion (34) are primarily dictated by the properties of the function $H(k)$. We now state some of these properties when $\rho < 0$ holds.

Lemma 2. Suppose that $\rho < 0$ holds. Then we have

(a) $H(0) = 0$

(b) $\lim_{k \to \infty} H(k) = \infty$

(c) $H'(k) > 0$ holds for all k satisfying $(b/a)k^{-\rho} \geq -\rho - (1-\rho)\,\gamma(\pi_1/\pi_2)$. In addition, $H'(0) > 0$ holds.

Lemma 2 is proved in appendix C. It has two immediate corollaries. First, part (b) of lemma 1 and part (c) of lemma 2 imply that $H'(k) > 0$ holds whenever $Q'[(b/a)k^{-\rho}] \leq 0$ is satisfied. Thus, if there is a unique non-trivial steady state, that steady state must have $H'(k) > 0$. In addition, if there are two non-trivial steady states, $H'(k) > 0$ must hold at the high-capital-stock steady state. In this case the low-capital-stock steady state can have $H'(k) \gtreqless 0$.

Before proceeding to a characterization of equation (34), it will be useful to have a preliminary result. When $\rho < 0$ holds, $w(k) \leq b^{1/\rho}$ is satisfied for all k. As a consequence, it is easy to verify that (34) has a solution iff $H(k_{t+1}) < \beta_2\pi_1/\{n(\pi_2-\pi_1) + n\beta_2\pi_1\, b^{-1/\rho}\} \equiv \eta$ holds. We then have the following result.

Lemma 3. Suppose that (a) $H(k) > H[w^{-1}(\beta_2)]$ holds for all $k > w^{-1}(\beta_2)$, and (b) $H[w^{-1}(\beta_2)] > \beta_2/n(\pi_2/\pi_1)$ holds. Then equation (34) defines a differentiable function G: $(w^{-1}(\beta_2), \eta) \rightarrow$

$(w^{-1}(\beta_2),\infty)$ such that $k_t = G(k_{t+1})$.

The proof of lemma 3 appears in appendix D. All of the examples reported below satisfy conditions (a) and (b) of the lemma.

We are now prepared to characterize the equilibrium law of motion $k_t = G(k_{t+1})$. This law of motion has several possible configurations; we present four representative ones in figures 2.a-2.d. In each case the nature of the equilibrium law of motion is driven by the shape of the function H(k), which is depicted in the upper panel of each figure.

Figure 2.a represents the situation where $H^{-1}([0,\eta))$ is a single interval, and where there are two non-trivial steady state equilibria - denoted $\underline{k}$ and $\bar{k}$ in the figure.[18] As we have already shown, $H'(\bar{k}) > 0$ – and hence $G'(\bar{k}) > 0$ – must hold. However, $H'(\underline{k}) < 0$ – and hence $G'(\underline{k}) < 0$ – are distinct possibilities; we analyze the consequences of this observation below.

Figure 2.b depicts essentially the same situation, but when $H^{-1}((0,\eta))$ consists of two disjoint intervals. Almost all of the numerical examples we generated that have $H'(\underline{k}) < 0$ actually have this configuration.

Figure 2.c represents the possibility that there are two non-trivial steady state equilibria, and that $H'(\underline{k}) > 0$ – and hence $G'(\underline{k}) > 0$ – obtains. This situation is most likely to arise when $\underline{k}$ and $\bar{k}$ are relatively close together. Finally, figure 2.d depicts the possibility that there is a unique non-trivial steady state: here $G'(\bar{k}) > 0$ necessarily holds.

Figures 2.a - 2.d allow us to state an immediate result.

Proposition 4. Let $\bar{k}$ denote the high-capital-stock steady state (or the unique non-trivial steady state, if there is only one). Then $G'(\bar{k}) > 1$ holds.

Proposition 4 asserts that the steady state with $k_t = \bar{k}$ is unstable in the "forward" dynamics of equation (34), and therefore that it is asymptotically stable in the natural "backward" dynamics of equation (26). Hence this steady state can be approached from some initial values of k_0.

If there are two non-trivial steady state equilibria, and if $G'(\underline{k}) \geq 0$ holds, then it is obviously the case that the low-capital-stock steady state is asymptotically stable (unstable) in the forward (backward) dynamics of equation (34) [(26)]. Hence if $k_0 \neq \underline{k}$, the low-capital-stock steady state can never be approached.

When $G'(\underline{k}) < 0$ holds, the low-capital-stock steady state may be either unstable, or asymptotically stable in the forward dynamics of equation (34). In the former case a number of interesting dynamical phenomena can be observed. We now consider this possibility.

1. Oscillation and Development Traps

Figure 3 represents one possible configuration of the equilibrium law of motion for k_t, and it also depicts the "negative 45° line" passing through the point $(\underline{k}, \underline{k})$. As drawn, clearly $G'(\underline{k}) < -1$ holds, so that the low-capital-stock steady state is unstable (asymptotically stable) in the forward (backward) dynamics of equation (34) [(26)]. Thus this steady state can be approached from some initial values k_0 in the natural backward dynamics. Obviously paths approaching the low-capital-stock steady state will display damped oscillation as they do so. Thus oscillation can easily be observed here.

Moreover, when $G'(\underline{k}) < -1$ holds, both steady state equilibria are asymptotically stable (in the backward dynamics), and hence development trap phenomena can be observed. In particular, two otherwise identical economies with different initial values for k_0 can approach different long-run capital-labor ratios, and hence can fail to converge for purely endogenous reasons. Economies that are "trapped" in low level equilibria will also pay an additional price: they will experience oscillation in all endogenous variables en route to the steady state.

The reason that both oscillations, and development trap phenomena can be observed here derives entirely from the role of rural-urban migration. In particular, suppose that k_t is relatively high, which means that - on average - young agents have high incomes at t. The result is large savings, so that K_{t+1} is correspondingly high. However, if enough labor is drawn from the rural to the urban sector as a result, $k_{t+1} \equiv K_{t+1}/L_{t+1}$ can actually fall - producing low incomes at t+1. The result is a low value of K_{t+2}, out-migration from the city, and a potential increase in k_{t+2}. Thus, when rural-urban migration is a strong enough force, the capital-labor ratio in urban manufacturing can fluctuate along perfect foresight equilibrium paths, inducing sympathetic fluctuations in average income levels, and countercyclical fluctuations in real interest rates.

Along paths that approach the low-capital-stock steady state, these fluctuations dampen over time. However, it is also very easy to observe equilibria that display undamped oscillation. We now turn our attention this possibility.

With reference to figure 3, define $\tilde{k}$ by $G'(\tilde{k}) = 0$.[19] In addition, define k* by $G(k^*) = G(\bar{k})$ *and* $G'(k^*) < 0$.[20] We can then state the following result.

Proposition 5. Suppose that $k^* > G(\tilde{k})$ holds. Then, in every neighborhood of $\bar{k}$, there are infinitely many distinct periodic points. Indeed, there exist equilibrium cycles of periodicity 2^j, for all $j \geq 0$.

The proof of proposition 5 is given in appendix E. The proposition asserts that, whenever $k^* > G(\tilde{k})$ holds, as it does in figure 3, a large number of equilibria exist in which $\{k_t\}$ displays cycles of period 2 or higher. Moreover, while such cycles arise in the forward dynamics of equation (34), if $\{k_1, k_2, \ldots, k_J\}$ is a J-period cycle in the forward dynamics, $\{k_J, k_{J-1}, \ldots, k_1\}$ is a J-period cycle in the normal backward dynamics. Thus such cycles may emerge in solutions to equation (26). Parenthetically, the existence of cyclical perfect foresight paths creates the potential for an additional type of non-convergence phenomenon. In particular, there can exist two intrinsically identical economies, one of which has $k_t \uparrow \bar{k}$. At the same time, the other may have $\{k_t\}$ evolving according to a J-period cycle. The economy experiencing fluctuations may have k_t arbitrarily close to $\bar{k}$ at some dates, but it cannot sustain this high level of real activity.

We now display an example in which $G(k_{t+1})$ has the configuration depicted in figures 2.b and 3, and in particular satisfies $k^* > G(\tilde{k})$. In addition, this economy has the feature that the conditions of proposition 1 are satisfied for all $k \in (w^{-1}(\beta_2), \bar{k}]$, as is the condition of equation (24). Finally, the example satisfies conditions (a) and (b) of lemma 3: hence all potential perfect foresight paths have $w(k_t) > \beta_2$ for all t.

Example 1. Let $\pi_1 = 1$ and $\pi_2 = 9$, while $a = b = 5$ and $\rho = -1.5$ also hold. In addition,

$n = 1.03$, $\gamma = 0.8$, $\theta_1 = 0.2$, $\beta_2 = 0.02$, and $\beta_1 = 0$. Here $w^{-1}(\beta_2) = 0.367273$, while equation (24) is satisfied for all $k \leq 2.17153$. The two-steady state capital-labor-ratios are given by $\underline{k}$ = 0.871821 and $\overline{k} = 1.58989$. $G'(\underline{k}) < -1$ also holds.

As example 1 illustrates, it is straightforward to construct examples giving rise to a large number of perfect foresight equilibrium paths displaying undamped oscillation.

2. The Nature of Cycles

Dynamical equilibria displaying either damped or undamped oscillation have the feature that k_t - and $w(k_t)$ - fluctuate over time. When $\rho < 0$ holds, these fluctuations in $w(k_t)$ necessarily induce fluctuations in the process of rural-urban migration. In particular, when $w(k_t)$ is relatively high (or, more generally, increasing), this will cause net migration from rural to urban employment to occur. When $w(k_t)$ is relatively low (or, more generally, falling), the result will be net migration *from* the urban to the rural sector.

In general, the history of economic development suggests that sustained growth is accompanied by sustained net migration from rural to urban sectors, possibly punctuated by relatively brief episodes of reverse migration (see below). Thus, if equilibrium cycles are to be consistent with observation, they must have the feature that periods of out-migration from urban employment are relatively infrequent.

Figure 4 depicts an economy that has a three period cycle, and also depicts the natural backward dynamics associated with this cycle. Evidently, in such an equilibrium, the economy will experience two periods of sustained - and potentially dramatic - growth, interrupted by one period of fairly dramatic decline. During this "recession," there will be net migration from urban to rural employment.

Some reflection on the homoclinic orbit depicted in figure 3 will suggest - quite correctly - that it is straightforward to construct cyclical equilibria of periodicity j, and which have the feature that j−1 periods of sustained growth are followed by one period of pronounced recession. Only during this single period does net out-migration from urban areas occur.

Hatton and Williamson (1992), using U.S. data from 1920-1940, show that there has been consistent net migration from rural to urban employment, with one exception. The early 1930s experienced a pronounced net migration in the opposite direction. Thus, significant declines in output have historically been accompanied by the reverse migration that our model predicts. And, while Hatton and Williamson do not have migration data before 1920, their statistical model - which fits quite well in sample - predicts that there was net out-migration from urban areas in 1919, immediately preceeding the pronounced recession of 1920-21. Thus there appears to be a significant empirical basis for the notion that cyclical equilibria displaying long expansions - followed by relatively brief but severe recessions - can occur, and are associated with net rural-urban migration, which reverses during sharp downturns.

We now display an example which generates a three period cycle.

Example 2. Let $\pi_1 = 1$, $\pi_2 = 16$, $a = b = 11.03175$, and $\rho = -1.5$ be the technological parameters, while $\beta_2 = 0.02$ and $\beta_1 = 0$ also hold. In addition, set $n = 1$, $\gamma = 0.8$, and $\theta_1 = 0.2$. This economy has the equilibrium law of motion depicted in figure 4. There are two steady states, with $\underline{k} = 0.788357$ and $\bar{k} = 1.95611$. Equation (24) is satisfied for all $k \leq 2.17153$, the conditions of proposition 1 are satisfied for all $k \in [w^{-1}(\beta_2), \bar{k}]$, and so are the conditions of lemma 3. Moreover, $G'(\underline{k}) < -1$ holds, so that the low-capital-stock steady state is asymptotically stable in the normal backward dynamics. It is also the case that $k^* > G(\tilde{k})$ holds, so that this economy generates the kind of homoclinic orbits depicted in figure 3.

Numerical calculations indicate that this economy has two period three points in the interval (1.845, 1.862). It is then straightforward to show that equilibrium cycles of all periods exist [Devaney (1989), p. 60]. Moreover, it is straightforward to construct such cycles having the features noted above.

3. Indeterminacy

There are two potential sources of indeterminacy of equilibrium in this economy when $\rho < 0$ holds. One derives from the fact that - in the normal backward dynamics - equation (36) is a correspondence. Hence, for any initial value $k_0 \in (G(\tilde{k}), \bar{k}]$ in figure 3, there are multiple possible perfect foresight equilibrium paths. For example, in figure 4, if k_0 is as shown, there is a perfect foresight equilibrium path corresponding to the three period cycle, and at the same time there is a monotonic perfect foresight path with $k_t \uparrow \bar{k}$. There are also obviously other equilibrium paths in which a period of oscillation is observed, followed by the convergence of $\{k_t\}$ to one of its steady state values.[21]

There is also another source of indeterminacy of equilibrium here, which is more unique to this particular model. It is a consequence of the fact that, while K_0 is given, k_o is not. We now describe the determination of the initial value k_0.

As already noted, $K_t = \theta_2 k_t \phi_t (1-u_t) n^t$ holds at all dates, including t=0. Thus $K_o = \theta_2 k_o \phi_o (1-u_o)$ holds. Substituting (17) and (23) into this relation, we obtain the condition that determines k_o:

(35) $$H(k_o) = K_o/[\theta_1 + \theta_2(\pi_2/\pi_1)].$$

When the function H has the configuration depicted in figures 2.a and 2.b, there will be an interval of values of K_o for which (35) delivers more than one possible value of k_o. When H has the configuration depicted in figure 2.b, only two of these are consistent with the subsequent sequence $\{k_t\}$ being feasible. Nevertheless, for any given initial condition $K_o > 0$, there may be more than one equilibrium value of k_o, which is an additional source of the indeterminacy of equilibrium.[22]

4. Monotone Local Dynamics

It is quite straightforward to produce examples in which there are two non-trivial steady state equilibria, and in which $G'(\underline{k}) > 0$ holds. Indeed, $G'(k) > 0$ necessarily holds when $Q'[(b/a)k^{-\rho}] \leq 0$ holds, and hence $G'(\underline{k}) > 0$ will be satisfied if $0 < -\rho - (1-\rho)\,\gamma(\pi_1/\pi_2)$, and if $(b/a)\,\underline{k}^{-\rho}$ is sufficiently close to $-\rho - (1-\rho)\,\gamma(\pi_1/\pi_2)$. If Q has the configuration depicted in figure 1.a, we can make $(b/a)\,\underline{k}^{-\rho}$ as close to $-\rho - (1-\rho)\,\gamma(\pi_1/\pi_2)$ as desired by an appropriate choice of n. Indeed, a high enough rate of population growth (but one consistent with two steady states) guarantees that $G'(\underline{k}) > 0$ holds.

This observation has an immediate implication: the kinds of oscillatory equilibria discussed above can only be observed if the rate of population growth is not too high. We now illustrate this point with an example.

Example 3. The example is identical to example 2, except that $n = 1.08825$ holds. As with example 2, the conditions of proposition 1 are satisfied for all "relevant" values of k_t as are equations (24), and the conditions of lemma 3.

For this example, there are two steady state values of k_t given by $\underline{k} = 1.23081$ and $\overline{k} = 1.24226$. Straightforward calculations yield $G'(\underline{k}) > 0$.

5. The Effects of an Increase in the Rate of Population Growth

It is easy to verify that an increase in n shifts the equilibrium law of motion G upwards at each value of k_{t+1}. This effect is depicted in figure 5. As in the standard Diamond (1965) model, the effect of an increase in n is to reduce the (urban) capital-labor ratio at the high-capital-stock steady state, and to increase it at the low-capital-stock steady state. Thus, for economies experiencing development traps, higher rates of population growth can actually increase the per capita capital stock, and lead to higher steady state (urban and rural) wage rates.

In addition, the existence of two steady state equilibria depends on the rate of population growth neither being too small nor too large. Finally, high enough rates of population growth render the low-capital-stock steady state unstable, and hence preclude development trap phenomena, as well as oscillatory equilibria. On the other hand, high population growth rates limit the size of $\overline{k}$. Thus, there is a "tension" regarding high rates of population growth: these preclude development traps and equilibria displaying endogenous fluctuations. However, they also imply that - in per capita terms - the economy will be fairly poor in the high-capital-stock steady state. It is not transparent which situation might be preferable.

B. The Equilibrium Law of Motion: $\rho \geq 0$

When $\rho \geq 0$ holds, $-\rho - (1-\rho)\,\gamma(\pi_1/\pi_2) < 0$ is obviously satisfied. It is then immediate from part (c) of lemma 2 that $H'(k) > 0$ holds $\forall\, k \geq 0$, as does $G'(k) > 0$. In addition proposition 3 implies that there is at most one steady state equilibrium. Hence the equilibrium law of motion for k_t has the general configuration depicted in figure 2.d, except that it is monotone increasing everywhere. The unique non-trivial steady state is asymptotically stable in the backward dynamics, so that the equilibrium law of motion looks much like that in the conventional Diamond (1965) model when production displays an elasticity of substitution no less than unity.

In addition, when $\rho \geq 0$ holds, (35) gives a unique value k_o for any given initial aggregate capital stock. If $k_o \in (w^{-1}(\beta_2), \bar{k}]$ holds, then k_t is a monotone increasing sequence with $k_t \rightarrow \bar{k}$.

As we have already noted, when $\rho \geq 0$ obtains, one consequence is that ϕ_t will be a monotonically decreasing sequence,[23] so that development will *not* be accompanied by net migration to the urban sector. This is, of course, highly contrary to observation, which leads us to deem $\rho < 0$ the interesting case.

VI. Some Additional Implications

Suppose that $\rho < 0$ holds, and that there are two non-trivial steady state equilibria. Development trap phenomena can be observed if and only if the low-capital-stock steady state is asymptotically stable in the normal backward dynamics; or, in other words, iff $G'(\underline{k}) < -1$. In addition, it is easy to show that $G'(\underline{k}) < -1$ implies the existence of at least one perfect foresight equilibrium path displaying undamped oscillation. Hence the emergence of many phenomena of interest occurs when $G'(\underline{k}) < -1$ obtains. We now explore some other implications of the condition $G'(\underline{k}) < -1$.

One implication of this condition is that there must be considerable unemployment and/or informal sector employment. We now make this observation more precise.

Proposition 6. Suppose that there are two non-trivial steady state equilibria, and that $G'(\underline{k}) < -1$ holds. Then, at the low-capital-stock steady state, $u > 0.5$ must be satisfied.

The proof of proposition 6 appears in appendix E.

Proposition 6 asserts that, in order for the low-capital-stock steady state to be asymptotically stable (in the natural backward dynamics), the steady state value of u in the urban sector must exceed 0.5. If u is interpreted as a rate of pure unemployment, this is empirically highly implausible. However, if u is interpreted as the fraction of the urban labor force employed in the informal sector - as is proper here - this result is quite consistent with observations from a number of developing countries. Indeed according to the 1995 World Development Report (p. 35), one-third of the developing countries for which data was collected have over 50 percent of the urban labor force employed in the informal sector. Moreover, Sethuraman (1981) reports that the mean fraction of the urban labor force employed in the informal sector across developing countries in general is about 41 percent. When added to genuine unemployment rates - which often exceed 10 percent - these figures suggest that there is considerable scope in practice for development trap phenomena, and endogenously generated oscillation to arise.

We also conjecture that the very high value of u implied by $G'(\underline{k}) < -1$ is a consequence of the assumption that land-labor ratios play no role in determining agricultural productivity. If the land-labor ratio in agriculture were a consideration, we believe that the following phenomenon can arise. As increases in k_t occur, they raise $w(k_t)$ and - with $\rho < 0$ - draw labor into the urban sector. However, as this process continues, the land-labor ratio in agriculture will rise, as will agricultural productivity. This would appear - in turn - to have the power to induce some reverse rural-urban migration, thereby inducing the kinds of cycles we have described. Thus we believe that such cycles will be more easily observed when land is introduced as a factor of production in agriculture.

VII. Conclusions

We have developed a model consistent with the obvious facts that rural-urban migration, and the existence of underemployment are crucial aspects of the development process. In the most natural case - where higher formal urban wage rates draw labor into the city - the result is that development trap phenomena can easily be observed, as can an indeterminacy of equilibrium, and the existence of endogenous, undamped economic fluctuations. It is also easy to generate equilibria where long periods of sustained growth and sustained rural-urban migration are punctuated by brief - but possibly severe - downturns, as well as reverse net migration. Such equilibria seem broadly consistent with historical experience.

It is also the case that the underemployment that arises in our model is not a consequence of any exogenous rigidities, but instead arises endogenously in response to an adverse selection problem in labor markets. It is exactly the presence of this underemployment that allows urban-rural wage differentials to vary over time, and with the level of economic development. Thus, as in the literature following Harris and Todaro (1970), the process of rural-urban migration is equilibrated by the rate of urban underemployment. And, it is exactly this migration which is at the heart of the most interesting aspects of our analysis.

To be more specific with respect to the last point, high current period capital-labor ratios in formal urban manufacturing are associated with high average income levels, and hence with high levels of saving and of the future aggregate capital *stock*. However, if enough labor is drawn into the city, next period's urban capital-labor ratio may actually fall. By implication, there need not be a monotone relationship between the current and future capital-labor ratio in formal manufacturing. It is the presence of non-monotonicities which is the source of multiple asymptotically stable steady states, indeterminacies, and endogenous fluctuations.

There are, of course, a number of dimensions along which the current analysis could be extended. Two very obvious extensions would be to make land a factor of agricultural production, and to move away from the assumption of risk neutral agents. There are also a number of policy issues that could be analyzed. The most obvious would be the creation of unemployment insurance, which would have strong consequences for the equilibrium rate of unemployment, urban-rural wage differentials, and for the potential equilibrium time paths of k_t. A second would be to allow for agricultural subsidies, or other policies designed to discourage rural-urban migration. And finally, the analysis could be expanded to consider how the provision of urban public services impacts on unemployment, migration, and growth. All of these would be interesting topics for future investigation.

VIII. Appenidx

A. Proof of Proposition 1.

For any given choice of $\tilde{u}_t$, the pooling contract offering type 2 agents the highest expected utility level sets $\tilde{w}_t$ to satisfy (14) with equality; that is

$$\text{(A.1)} \qquad \tilde{w}_t \equiv \max_{\tilde{K}_t \leq K_t} \{F[\tilde{K}_t, \theta_2 n^t(1-\tilde{u}_t)] - r_t\tilde{K}_t\} / n^t(1-\tilde{u}_t).$$

In this contract, then, $\tilde{K}_t$ must be chosen to satisfy the first order condition

$$\text{(A.2)} \qquad f'[\tilde{K}_t/\theta_2 n^t(1-\tilde{u}_t)] \geq r_t.$$

However, since $K_t \equiv k_t\phi_t(1-u_t)\theta_2 n^t$, and since $f'(k_t) = r_t$, we have that

$$\text{(A.3)} \qquad f'[K_t/\theta_2 n^t(1-\tilde{u}_t)] \equiv f'[k_t\phi_t(1-u_t)/(1-\tilde{u}_t)] \geq f'[k_t\phi_t(1-u_t)] > f'(k_t) = r_t.$$

It is then immediate that a firm offering the maximum possible wage rate under a pooling contract employs the entire capital stock. The corresponding wage rate is given by

$$\text{(A.4)} \qquad \tilde{w}_t \equiv \{F[k_t\phi_t(1-u_t)\theta_2 n^t, \theta_2 n^t(1-\tilde{u}_t)] - f'(k_t)k_t\phi_t(1-u_t)\theta_2 n^t\}/n^t(1-\tilde{u}_t).$$

The highest possible expected utility level that a type 2 agent can receive under a pooling contract is given by the solution to the problem

$$\text{(P)} \qquad \max_{(1-\tilde{u}_t)} \{F[k_t\phi_t(1-u_t)\theta_2 n^t, \theta_2 n^t(1-\tilde{u}_t)] - f'(k_t)k_t\phi_t(1-u_t)\theta_2 n^t\}/n^t - (1-\tilde{u}_t)\beta_2.$$

If this expected utility level does not exceed $(1-u_t)[w(k_t) - \beta_2]$, then there is no profitable pooling contract that can attract any type 2 workers. It follows that the candidate separating equilibrium contracts derived in section II.B. constitute a Nash equilibrium. There are now two cases to consider.

1. Case 1.

Suppose that

(A.5) $\theta_2 F_2[k_t\phi_t(1-u_t), 1] \equiv \theta_2 w\,[k_t\phi_t(1-u_t)] \geq \beta_2$

holds. Then the expression (P) is maximized by setting $\tilde{u}_t = 0$. There is then no pooling contract that satisfies (15) iff

(A.6) $(1-u_t)\, w(k_t) + u_t\beta_2 \geq \tilde{w}_t$

is satisfied.

Now note that

$$u_t = [(\pi_2-\pi_1)/\beta_2\pi_1]\, w(k_t)/\{1 + [(\pi_2-\pi_1)/\beta_2\pi_1]\, w(k_t)\}.$$

Substituting this expression into (A.6) and rearranging terms, we obtain that there is no pooling contract satisfying (15) iff

(A.7) $(1-u_t)\, w(k_t) \geq (\pi_1/\pi_2)\,\tilde{w}_t = (\pi_1/\pi_2)\theta_2\, \{f\,[k_t\phi_t(1-u_t)] - f'(k_t)k_t\phi_t\,(1-u_t)\}.$

2. <u>Case 2.</u>

Suppose that (A.5) *fails* to hold. Then the expression in (P) is maximized by setting $1-\tilde{u}_t$ to satisfy

(A.8) $\theta_2 F_2[k_t\phi_t(1-u_t)\theta_2 n^t, \theta_2 n^t(1-\tilde{u}_t)] = \beta_2.$

Clearly (A.8) is equivalent to

(A.8′) $w[k_t\phi_t(1-u_t)/(1-\tilde{u}_t)] = \beta_2/\theta_2.$

Therefore, in the maximal pooling contract

(A.9) $1-\tilde{u}_t = k_t\phi_t(1-u_t)/w^{-1}(\beta_2/\theta_2) < 1.$

It is now immediate that the highest expected utility level that a type 2 agent can obtain under a profitable pooling contract is given by

$$(1-\tilde{u}_t)\,(\tilde{w}_t-\beta_2) + \beta_2 = \beta_2 + k_t\phi_t(1-u_t)(\tilde{w}_t-\beta_2)/w^{-1}(\beta_2/\theta_2).$$

This fails to satisfy (15) iff

(A.10) $\qquad w(k_t) - \beta_2 \geq k_t\phi_t(\tilde{w}_t - \beta_2)/w^{-1}(\beta_2/\theta_2)$

holds.

By substituting $\tilde{K}_t = K_t$ and (A.9) into (A.1), it is easy to verify that

(A.11) $\qquad \tilde{w}_t = \theta_2\{f[w^{-1}(\beta_2/\theta_2)] - f'(k_t)w^{-1}(\beta_2/\theta_2)\}.$

Using (A.11) in (A.10), noting the form of the production function in (1), and rearranging terms, we obtain that there is no profitable pooling contract satisfying (15) iff

(A.12) $\qquad [w(k_t) - \beta_2]/\phi_t k_t + \theta_2 w(k_t)(a/b)\, k_t^{\rho-1} \geq \beta_2(a/b)[w^{-1}(\beta_2/\theta_2)]^{\rho-1}$

holds. But this is exactly condition (iii). □

B. Proof of Lemma 1.

Part (a) is immediate from the definition of Q. For part (b), differentiation of (29) yields (31). From (31), $Q'(x) \geq 0$ holds iff

$$-[(1-\rho)/\rho]/(1+x) \leq [x + \gamma(\pi_1/\pi_2)]^{-1}.$$

This is easily shown to be equivalent to the statement in the lemma. Clearly, if $\rho \geq 0$ holds, it follows that $Q'(x) > 0\ \forall\ x$.

Part (c) is obvious for $\rho > 0$. For $\rho < 0$, an application of L'Hopital's rule yields

$$\lim_{x\to\infty} Q(k) = \lim_{x\to\infty} -[\rho/(1-\rho)]\,(1+x)^{1/\rho} = 0,$$

as was to be shown. □

C. Proof of Lemma 2.

When $\rho < 0$ holds, $w(0) = 0$ and

$$\lim_{k\to 0} kf'(k)/w(k) = \infty.$$

It is then immediate that $H(0) = 0$. For part (b), it is easy to verify that

$$\lim_{k \to \infty} kf'(k)/w(k) = 0.$$

Therefore

$$\lim_{k \to \infty} H(k) = \lim_{k \to \infty} (\pi_1/\pi_2)[\beta_2\pi_1/(\pi_2-\pi_1)]\, k/w(k) = \infty.$$

For parts (c) and (d), straightforward differentiation of (25) yields

$$\text{(A.13)} \qquad kH'(k)/H(k) = 1 - [(\pi_2-\pi_1)/\beta_2\pi_1]kw'(k)/\{1 + [(\pi_2-\pi_1)/\beta_2\pi_1]w(k)\} - \rho\,\gamma\,(\pi_1/\pi_2)(a/b)k^{\rho}/\{1 + \gamma(\pi_1/\pi_2)(a/b)k^{\rho}\}.$$

Since $kw'(k)/w(k) = (1-\rho)[ak^{\rho}/(ak^{\rho}+b)]$, we have that $H'(k) \geq 0$ holds iff

$$\text{(A.14)} \qquad 1 \geq \rho\, [\gamma(\pi_1/\pi_2)(a/b)k^{\rho}/\{1 + \gamma(\pi_1/\pi_2)(a/b)k^{\rho}\}] + (1-\rho)\, [ak^{\rho}/(ak^{\rho}+b)]\, \{[(\pi_2-\pi_1)/\beta_2\pi_1]w(k)/[[(\pi_2-\pi_1)/\beta_2\pi_1]w(k) + 1]\}.$$

To prove part (c) we then note that $H'(k) > 0$ obviously holds if

$$\text{(A.15)} \qquad 1 \geq \rho\,\gamma\,(\pi_1/\pi_2)/[x + \gamma(\pi_1/\pi_2)] + (1-\rho)/(1+x)$$

is satisfied, where $x \equiv (b/a)k^{-\rho}$, as in equation (28). Upon rearranging terms in (A.15), we obtain the equivalent condition that $H'(k) > 0$ holds if

$$\text{(A.15}') \qquad x \geq -\rho - (1-\rho)\gamma(\pi_1/\pi_2).$$

This establishes the first claim in part (c). For the second, we evaluate (A.14) at $k = 0$, obtaining $1 \geq \rho$, which obviously holds. Hence $H'(0) > 0$. □

D. Proof of Lemma 3.

It is readily verified that, whenever $H(k_{t+1}) < \eta$ holds,

$$nH(k_{t+1})/\{1 - n[(\pi_2-\pi_1)/\beta_2\pi_1]H(k_{t+1})\} < b^{1/\rho}$$

is satisfied. Moreover, clearly $n[(\pi_2-\pi_1)/\beta_2\pi_1]\eta < 1$ holds; it is then immediate that (34) has a solution for all $k \in H^{-1}([0,\eta))$. This guarantees the existence of the desired function G.

It remains to show that G: $(w^{-1}(\beta_2),\eta) \rightarrow (w^{-1}(\beta_2), \infty)$. To do so, we note that if $H(k) > H[w^{-1}(\beta_2)]$ holds for all $k > w^{-1}(\beta_2)$, and if

(A.18) $$H[w^{-1}(\beta_2)] > w[w^{-1}(\beta_2)]/n \{[(\pi_2-\pi_1)/\beta_2\pi_1]\, w[w^{-1}(\beta_2)] + 1\} = \beta_2/n(\pi_2/\pi_1),$$

then (34) has a unique solution with $k_t > w^{-1}(\beta_2)$. But this establishes the claim. □

E. Proof of Proposition 5.

Proposition 5 follows from the existence of the homoclinic orbit depicted in figure 3. In this appendix we formalize that observation.

To begin, define the local unstable set at $\overline{k}$ to be the maximal open interval containing $\overline{k}$ such that $|G(k)-\overline{k}| > |k-\overline{k}|$. We denote this set by $w^u_{loc}(\overline{k})$. In addition, we define a homoclinic point as follows.

Definition. [Devaney (1989), p. 122] A point q is homoclinic to $\overline{k}$ if $q \in w^u_{loc}(\overline{k})$ and there exists an integer $j > 0$ such that $G^j(q) = \overline{k}$. A homoclinic point is non-degenerate if $G'[G^j(q)] \neq 0\ \forall j$.

Devaney (1989), p. 125 states the following result.

Theorem. Suppose G admits a non-degenerate homoclinic point to $\overline{k}$. Then, in every neighborhood of $\overline{k}$, there are infinitely many distinct periodic points.

Devaney (p. 125) also notes that, by Sarkovskii's theorem, G must have periodic points of all periods 2^j, $j = 0, 1, 2, \ldots$.

Appealing to these results, the claim follows immediately from the existence of a non-degenerate homoclinic point q. But the existence of such a point is immediate from $k^* > G(\tilde{k})$ (see figure 3). This establishes the proposition. □

F. Proof of Proposition 6.

Straightforward differentiation of equation (34) yields the expression

(A.19) $$(k_{t+1}/k_t)\, G'(k_{t+1}) = [k_{t+1}H'(k_{t+1})/H(k_{t+1})][w(k_t)/nH(k_{t+1})][w(k_t)/k_t\, w'(k_t)].$$

Evaluating (A.19) at $k_t = k_{t+1} = \underline{k}$, we obtain

(A.20) $$G'(\underline{k}) = [\underline{k}H'(\underline{k})/H(\underline{k})][w(\underline{k})/nH(\underline{k})][w(\underline{k})/\underline{k}\, w'(\underline{k})].$$

Moreover, it follows from equation (25) that

(A.21) $$kH'(k)/H(k) = [1 + (1-\rho)\,\gamma(\pi_1/\pi_2)(a/b)k^\rho]/[1 + \gamma(\pi_1/\pi_2)(a/b)k^\rho] - [kw'(k)/w(k)]\{[(\pi_2-\pi_1)/\beta_2\pi_1]w(k)/[[(\pi_2-\pi_1)/\beta_2\pi_1]w(k) + 1]\}.$$

Now equation (17) implies that

(A.22) $$[(\pi_2-\pi_1)/\beta_2\pi_1]w(k)/[1 + [(\pi_2-\pi_1)/\beta_2\pi_1]w(k)] = u,$$

while equations (26) and (17) imply that

(A.23) $$w(k)/nH(k) = 1 + [(\pi_2-\pi_1)/\beta_2\pi_1]w(k) = 1/(1-u).$$

Combining equations (A.20)-(A.23) yields

(A.24) $$G'(\underline{k}) = -\,u/(1-u) + [w(\underline{k})/\underline{k}\, w'(\underline{k})][1 + (1-\rho)\gamma(\pi_1/\pi_2)(a/b)\underline{k}^\rho]/[1 + \gamma(\pi_1/\pi_2)(a/b)\underline{k}^\rho](1-u).$$

It is then immediate that $G'(\underline{k}) < -1$ holds iff

(A.25) $$2u - 1 > [w(\underline{k})/\underline{k}\, w'(\underline{k})]\{1 + (1-\rho)\gamma(\pi_1/\pi_2)(a/b)\underline{k}^\rho\}/[1 + \gamma(\pi_1/\pi_2)(a/b)\underline{k}^\rho] > 0.$$

Obviously, then, a necessary condition for $G'(\underline{k}) < -1$ to hold is that $u > 0.5$, as claimed. □

Footnotes

1. See Rauch (1993) for an important exception to this statement.

2. Or expected utility; see Bhatia (1979).

3. See, for instance, The World Development Report 1995 (p. 76) for modern evidence, or Hatton and Williamson (1992) for historical evidence.

4. There are many doubts that these institutional rigidities are the primary source of urban unemployment even in modern developing countries. Stiglitz (1988, p. 134) for instance, argues that "there are many instances in which wages in excess of the minimum wage are paid, and yet there are queues of unemployed." See also Williamson (1988), p. 446.

5. Drazen and Eckstein (1988) and Rauch (1993) make the same assumption. Ranis (1988) describes the hallmark of dual economy models to be the assumption that agricultural production uses relatively little or no capital. The abstraction from the role of a fixed supply of land as an input into agricultural production is less natural. This is an aspect of the model we hope to pursue in future work.

6. Rauch (1993) also assumes that informal sector production uses no capital. Sethuraman (1981) estimates that, on average, informal sector production technologies have a capital-labor ratio no greater than 15 percent of that prevailing in formal manufacturing.

7. The assumption that agents are risk neutral is attractive because it (a) yields a Harris Todaro (1970) - like determination of the unemployment rate, and (b) prevents employment lotteries from being useful in addressing the adverse selection problem present in labor markets. Given risk neutrality, if we want constant aggregate expenditures shares we must either adopt the specification in the text, or simply assume that a fraction γ $(1-\gamma)$ of agents cares only about consumption of the agricultural (manufactured) good.
   If we rule out employment lotteries, we can replace the specification in the text by the assumption that agents have logarithmic utility. All of our results would remain intact, and γ could then be interpreted as agents' expenditure share on agricultural commodities. Parenthetically, Mas-Colell and Razin (1973) also employ the assumption of constant expenditure shares on manufactured and agricultural goods.

8. There is considerable evidence that it is primarily the most highly skilled workers who migrate. See, for instance, Mazumdar (1987, p. 1119), Rosenzweig (1988, p. 754), and Williamson (1988, p. 432).

9. See, for example, Rothschild and Stiglitz (1976).

10. At the same time, we assume that they behave competitively in product markets.

11. Notice that, since workers are risk-neutral, $1-u_t$ (u_t) could equally well be a fraction of time spent working in the formal (informal) sector, so that u_t would then be an under-employment rate.

12. If $\beta_1 > 0$, then the equilibrium unemployment rate satisfies $u_t = (w_t - p_t\pi_1)/(w_t - \beta_1)$.

13. k_o is *not* given as an initial condition, although K_o is. We describe the determination of k_o in section V.A.

14. This statement is true unless $\rho = 0$, in which case we have a Cobb-Douglas production function. Then equation (28) gives a constant value for x, and there is a unique value of k that satisfies (27).

15. This condition will be satisfied iff $Q[(b/a)w^{-1}(\beta_2)^{-\rho}] < n(\pi_1/\pi_2)a^{-1/\rho}$ holds.

16. When either (32), or the first inequality in (33) is violated, this condition holds iff $Q[(b/a)w^{-1}(\beta_2)^{-\rho}] > n(\pi_1/\pi_2)a^{-1/\rho}$ is satisfied.

17. When $\rho > 0$ holds, the associated steady state will have $w(k) > \beta_2$ iff $Q[(b/a)w^{-1}(\beta_2)^{-\rho}] < n(\pi_1/\pi_2)a^{-1/\rho}$ is satisfied.

18. Clearly $(b/a)\underline{k}^{-\rho} = \underline{x}$ and $(b/a)\,\overline{k}^{-\rho} = \overline{x}$. Situations leading to the existence of two non-trivial steady state equilibria are described in part (a) of proposition 2.

19. In other words, $\widetilde{k}$ satisfies $H'(\widetilde{k}) = 0$, and H attains a local minimum at $k = \widetilde{k}$.

20. If G has the configuration depicted in figure 2.b, we are guaranteed that k^* is well-defined.

21. The situation where the equilibrium law of motion is a correspondence rather than a function clearly arises often in pure exchange overlapping generations models of money [Azariadis (1981), Benhabib and Day (1982), Grandmont (1985)] when income effects are sufficiently strong. This situation does *not* arise in any overlapping generations models with production that we are familiar with.

22. Indeterminacies of equilibrium often arise in overlapping generations models with production and variable labor supply [Reichlin (1986), Azariadis (1993)] because there is an interval of initial values of k_o consistent with convergence to a steady state. This is *not* the situation here, where there are only a finite number of possible initial values k_o corresponding to a given $K_o > 0$.

23. If $\rho = 0$, ϕ_t is a constant sequence.

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Figure 1.a

Steady State Equilibria [ρ < 0]

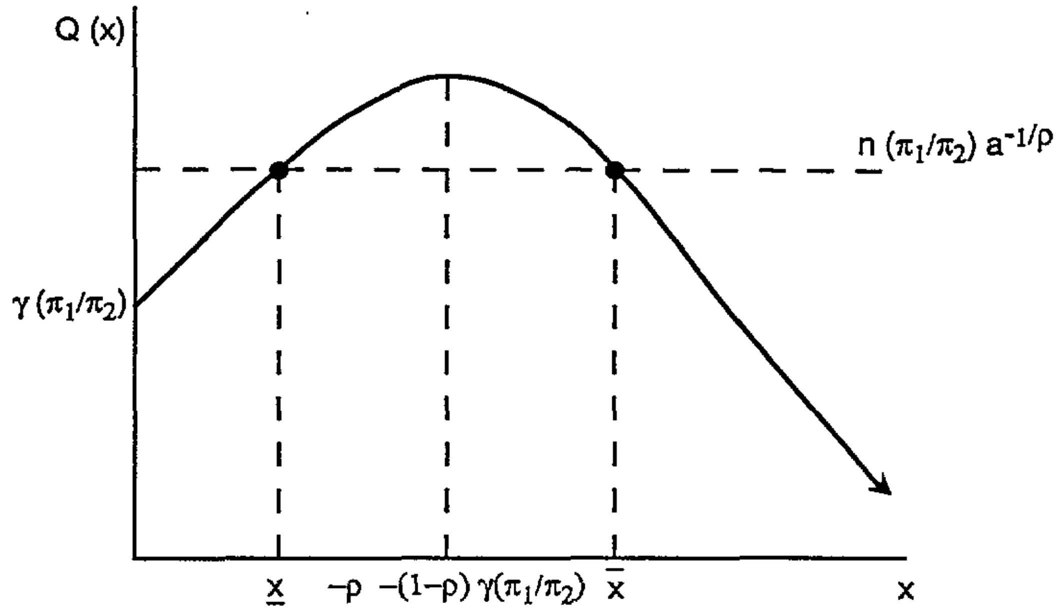


Figure 1.b

Steady State Equilibria [ρ < 0]

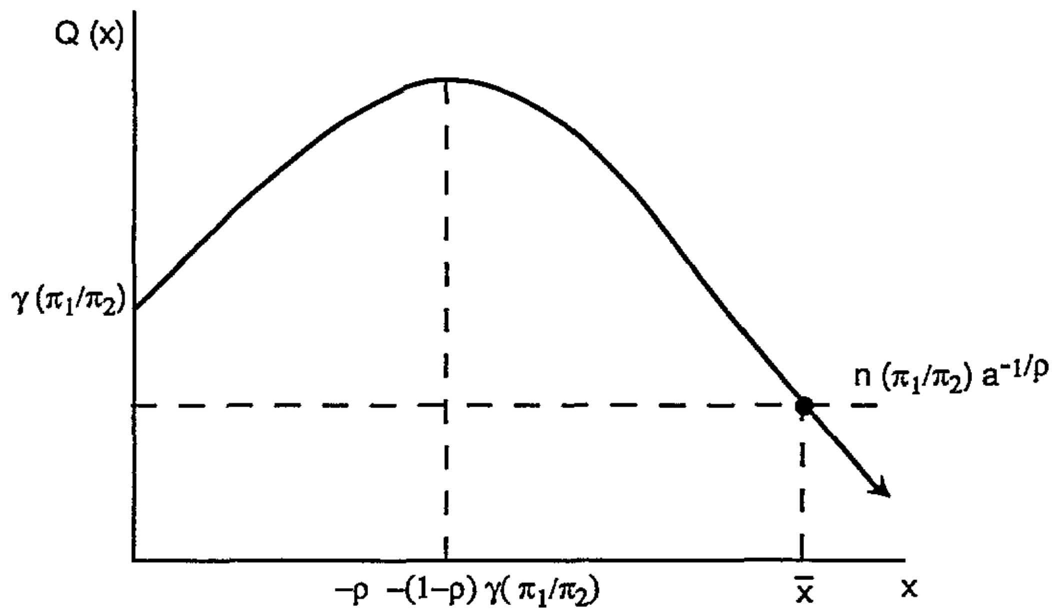


Figure 2.a

Equilibrium Law of Motion for k_t

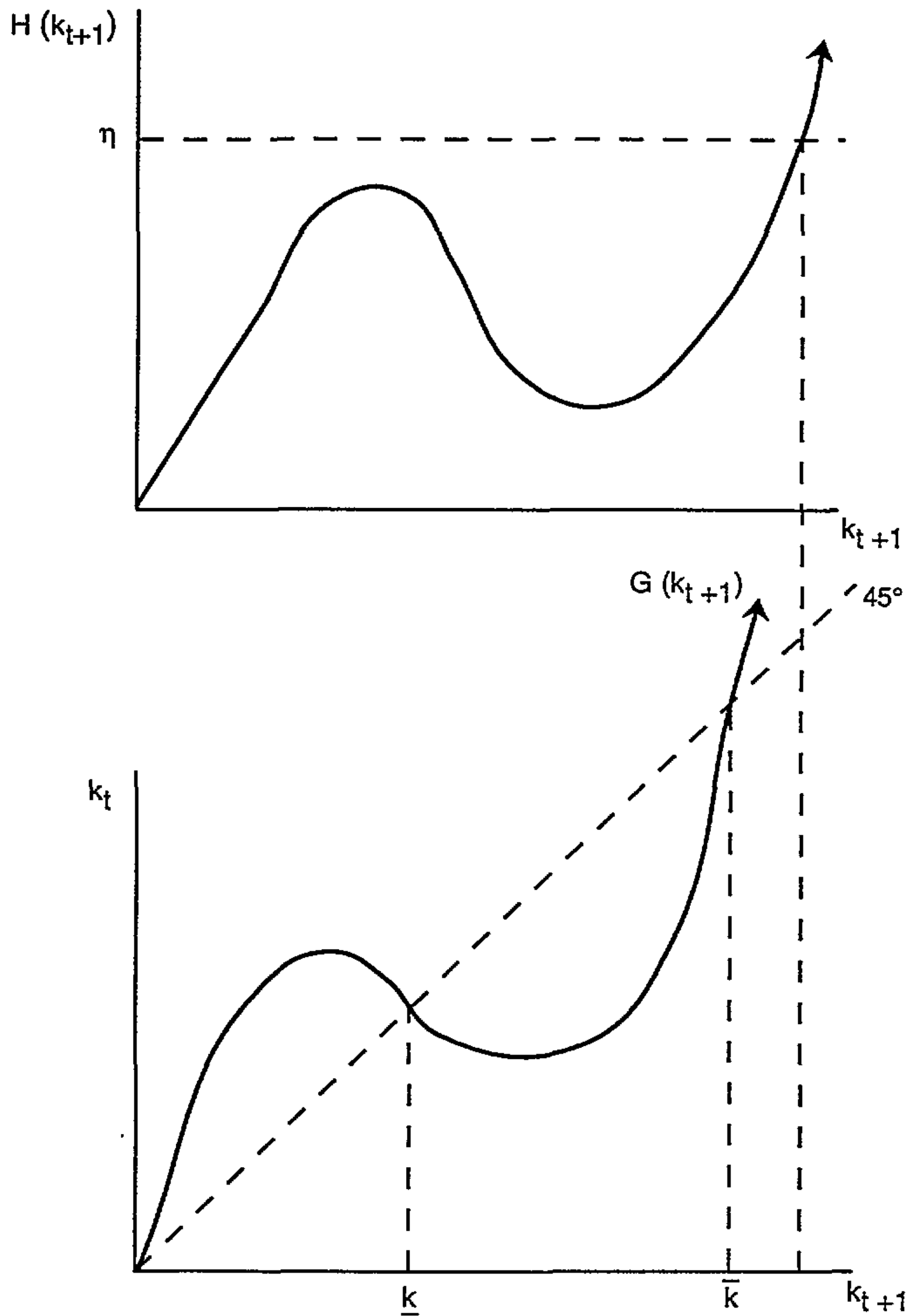


Figure 2.b

Equilibrium Law of Motion for k_t

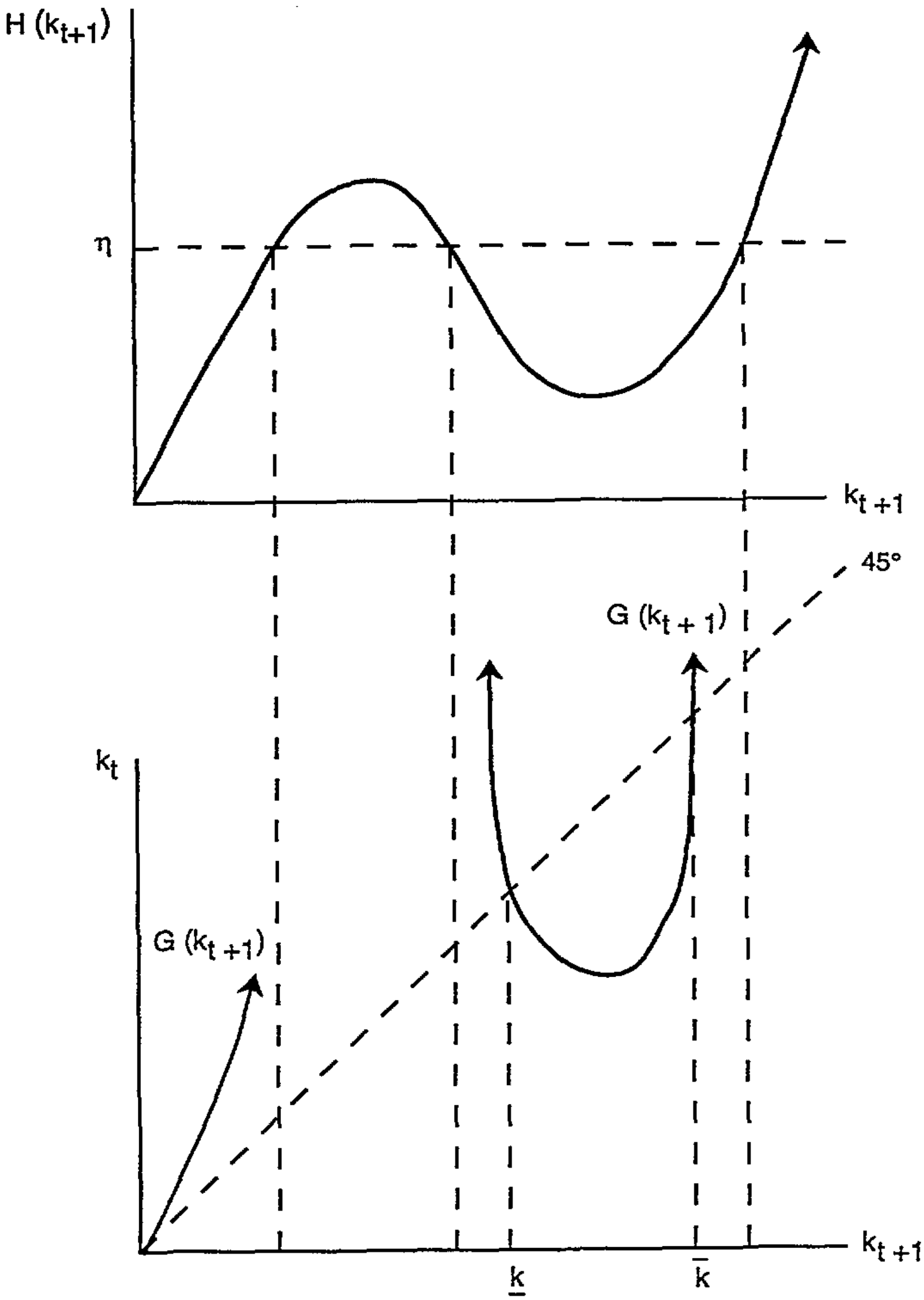


Figure 2.c

Equilibrium Law of Motion for k_t

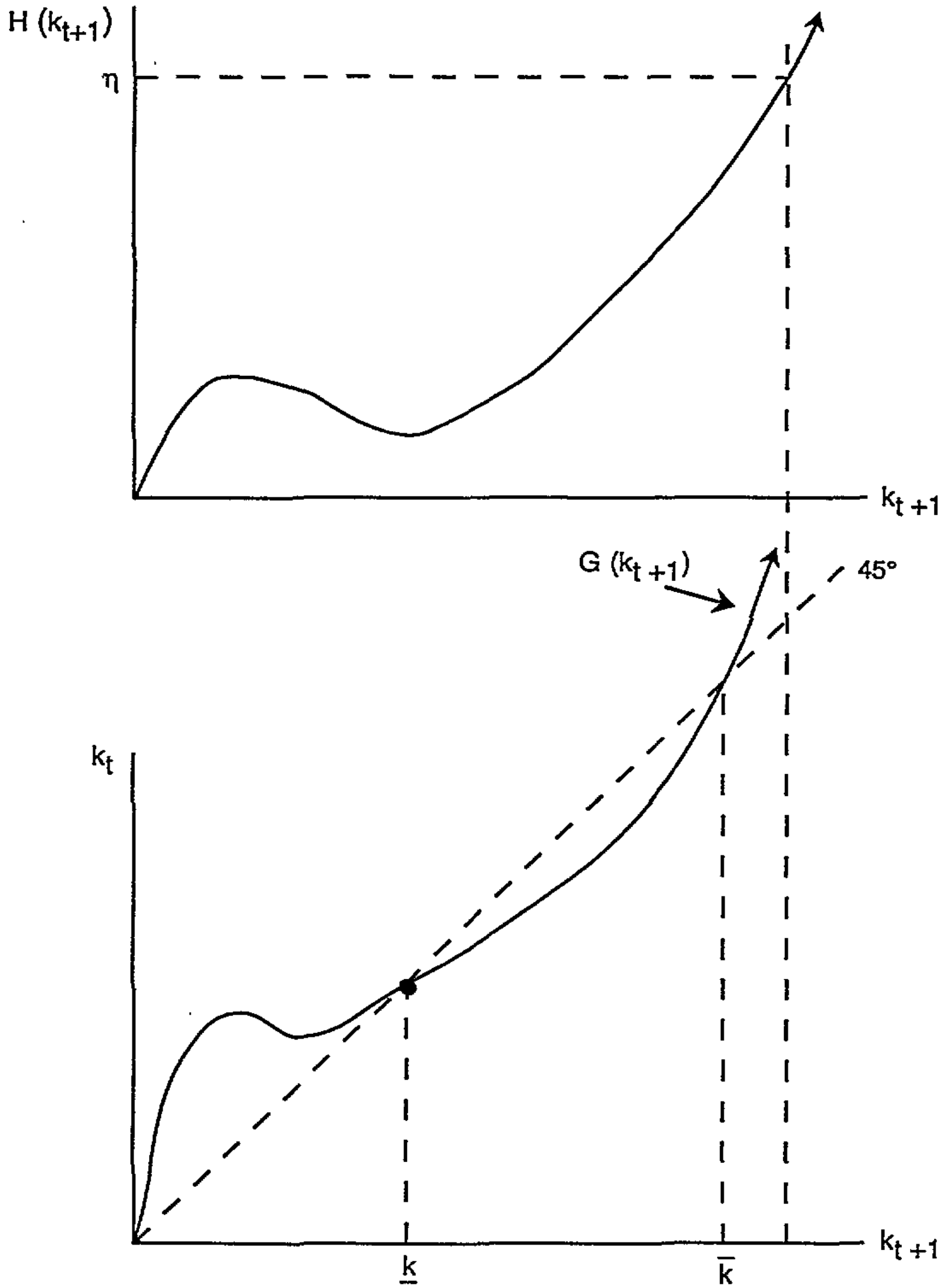


Figure 2.d

Equilibrium Law of Motion for k_t

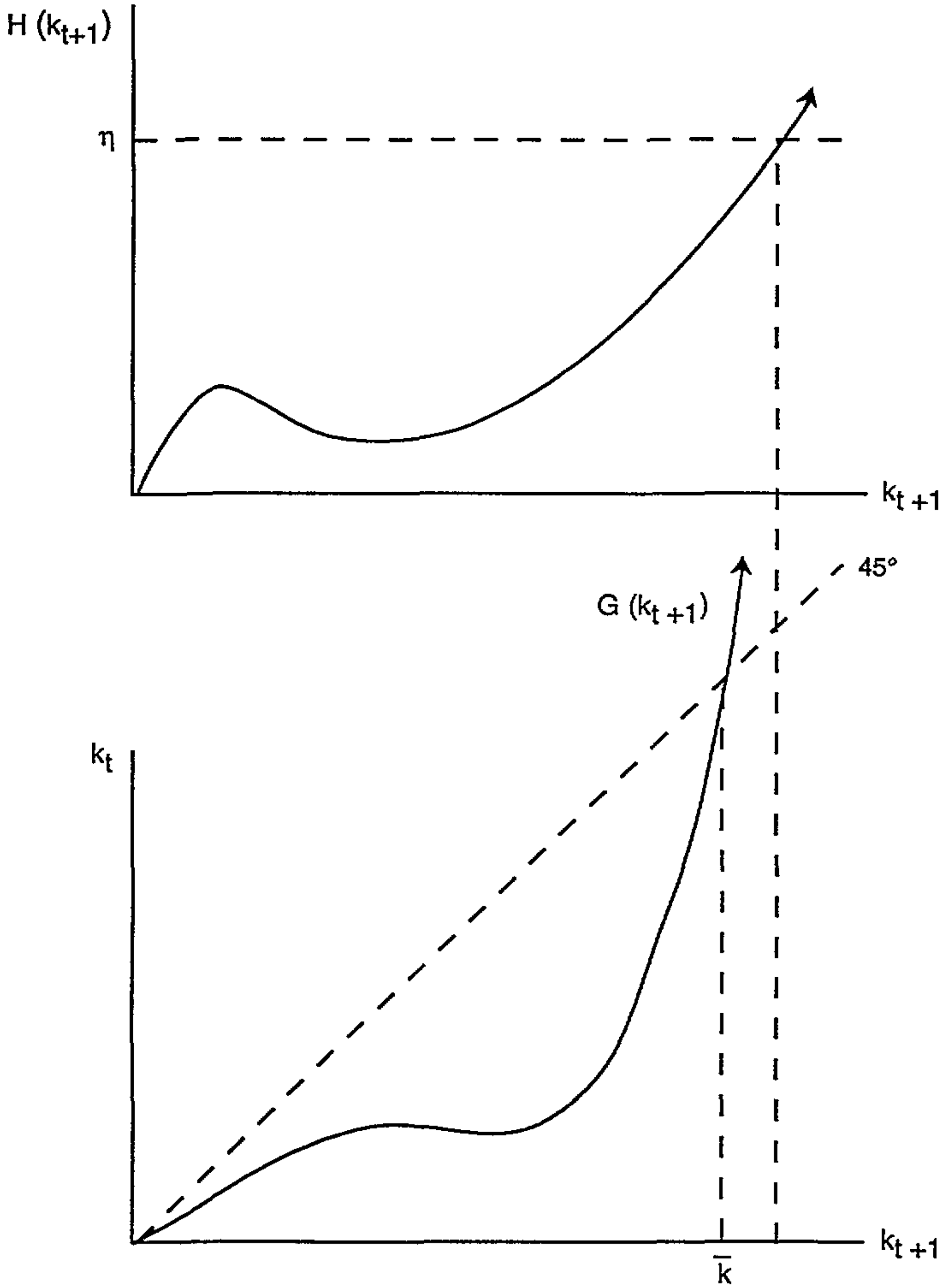
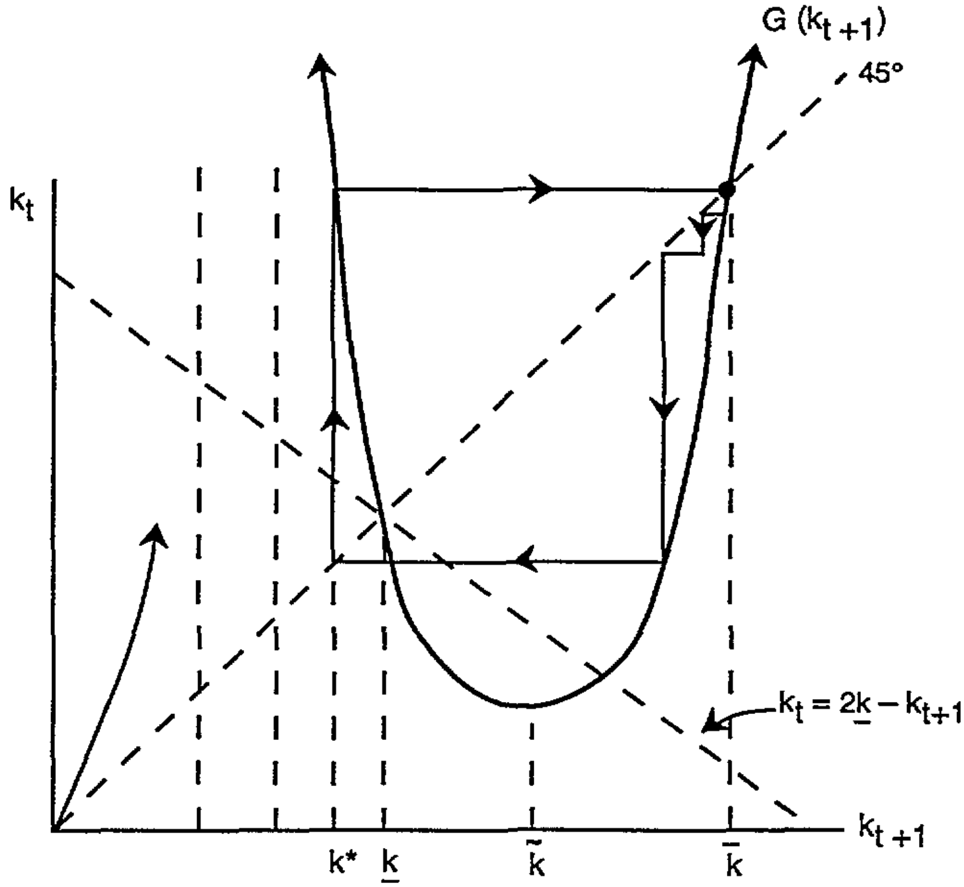


Figure 3

Oscillatory Equilibria



A Homoclinic Orbit

Figure 4

Cyclical Equilibria

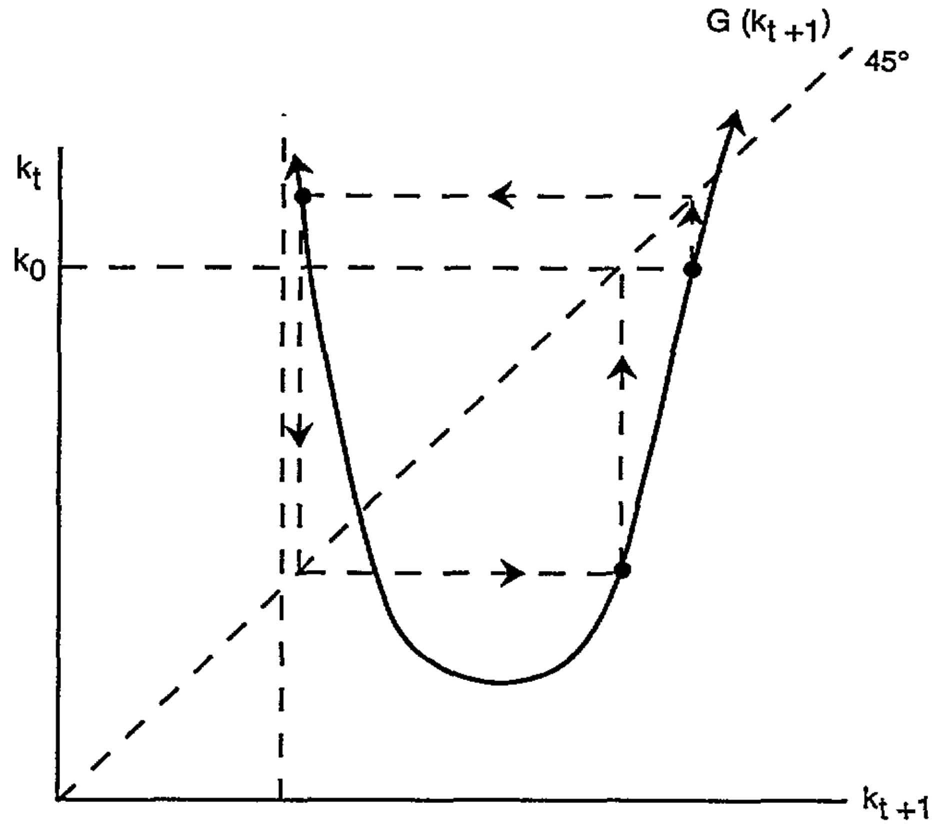


Figure 5

An Increase in the Rate of Population Growth