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Short-Run and Long-Run Effects
of Changes in Money in a
Random Matching Model

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ABSTRACT

Using an existing random matching model of money, I show that a once-for-all change in the quantity of money has short-run effects that are predominantly real and long-run effects that are in the direction of being predominantly nominal provided (i) the quantity of money is random and (ii) people learn about what happened to it only with a lag. The change in the quantity of money comes about through a random process of discovery that does not permit anyone to deduce the aggregate amount discovered when the change actually occurs.

I show that a random-matching model of money implies the kind of qualitative short-run and long-run effects of changes in the quantity of money that we have often observed; namely, short-run effects that are predominantly real and long-run effects that are in the direction of being predominantly nominal. Those occur in the particular random-matching model studied here, the model in Aiyagari, Wallace, and Wright (1996), provided two conditions are met: (i) the quantity of money is random, and (ii) people learn about what happened to it only with a lag.

Conditions (i) and (ii) are not, of course, new; they are important ingredients in several models consistent with the observed short-run and long-run effects of changes in the quantity of money (see Lucas 1996). Therefore, I should say why it is worthwhile showing that those conditions give rise to similar effects in a random-matching model. Doing so demonstrates that the ingredients of the matching model, ingredients which give outside money a role in overcoming double-coincidence problems, are sufficient to account for those effects. In addition, the random-matching model suggests a new perspective on condition (ii), which is often regarded as implausible for a modern economy. The ingredients which give outside money a role in overcoming double-coincidence problems include the restriction that each person knows only about what happens in meetings in which the person participates. Against the background of that informational restriction, some version of which seems plausible even in a modern economy, condition (ii), which also limits what people know, ought to seem less implausible.

The rest of the paper proceeds as follows. In section 1, I set out the model. In section 2, I describe equilibrium short-run and long-run effects of once-for-all changes in the quantity of money that come about in a way that leads to satisfaction of conditions (i) and (ii). In section 3, I illustrate how to use the model to describe the welfare effects of adopting different objects as money, objects which are subject to different degrees of quantity uncertainty. In section 4, I discuss extensions of the model and suspicions about the robustness of the results to extensions. I conclude in section 5.

1. The Model

Because the model is identical to that in Aiyagari, Wallace, and Wright (1996), I will be brief. Time is discrete and the horizon is infinite. There are N divisible and perishable goods at each date and there is a [0,1] continuum of each of N types of people. Each type

is specialized in consumption and production in the following way: a type i person consumes only good i and produces only good i+1 (modulo N), for i = 1,2,...,N, where N ≥ 3. Each type i person maximizes expected discounted utility with discount factor $\beta \in (0,1)$. Utility in a period is given by u(x) - y, where x is the amount of good consumed and y is the amount of good produced.[1] The function u is defined on $[0,\infty)$, is increasing and twice differentiable, and satisfies u(0) = 0, u" < 0, and u'(0) = ∞.

People meet pairwise at random and each person's trading history is private information to the person. Together, these assumptions rule out all but *quid pro quo* trade for optimizing people. In particular, they rule out private credit. The only storable objects are indivisible units of (fiat) money and each person has a storage capacity of one unit. In a meeting, each person sees the trading partner's type and amount of money held.

The sequence of actions within a period is as follows. Each person begins a period holding either one unit of money or nothing. Then people meet pairwise at random. Because of the upper bound on individual holdings of money and the indivisibility, there is a potential for trade only when a type i person meets a type i+1 person and the type i+1 person, the potential consumer, has money and the type i person, the potential producer, does not. I call such meetings *trade* meetings. People in trade meetings bargain. If the outcome of bargaining implies exchange, then production and consumption occurs. Then people begin the next period. Throughout the paper the following simple bargaining rule is assumed: the potential consumer makes a take-it-or-leave-it offer and the potential producer accepts if made no worse off by accepting. The offer, a scalar, consists of a demand for an amount of production, which, if accepted by the producer, gives rise to the exchange of the consumer's unit of money for that amount of production.

All of the above is as in Aiyagari, Wallace, and Wright (1996), which, in turn, follows closely the models in Shi (1995) and Trejos and Wright (1995). The only addition made here is the following specification of how changes in the quantity of money come about. Let the initial date be date 0 and let $m_0 > 0$ be the initial amount of money per type. At the end of date 0, there is a once-for-all increase in the amount of money. This increase per type, denoted Δ, is a drawing from the following distribution, which is common

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[1] The assumption that the disutility of production is equal to the amount produced is without loss of generality. For details, see Aiyagari, Wallace, and Wright (1996).

knowledge at the beginning of date 0: $\Delta = \Delta_k$ with probability p_k, $k = 1,2, ...,K$, where $p_k > 0$, $K \geq 2$, $\Delta_{k+1} > \Delta_k$, $\Delta_1 \geq 0$, $m_0 + \Delta_K \leq 1/2$, and where the range of Δ, $\Delta_K - \Delta_1$, is sufficiently small in a way to be described later. Conditional on Δ, each person who exits a meeting without money at date 0 discovers a unit of money with probability $\Delta/(1-m_0)$. (This possibility of discovery, which is present only at date 0, was not included in the sequence of actions given above.) At date 1, no one observes Δ, although people use their experience to update the prior given by the p_k, while at date 2, prior to meetings, the realization of Δ is revealed to everyone.

The above model is structured so that there can be equilibria which are symmetric across person types. To permit there to be such equilibria, I assume that the initial money distribution is symmetric across types. Notice that if the money distribution at the beginning of a date is symmetric and trades and discoveries are symmetric, then the money distribution remains symmetric. Given the unit upper bound on holdings of money, at any date there is only one symmetric distribution consistent with all money being held: if m is the per type amount of money, then a fraction m of each type has a unit of money and a fraction 1-m has nothing. In what follows, I limit attention to symmetric equilibria. In such equilibria, it follows that the sequence of money distributions is very simple: the date 0 distribution is the unique symmetric one with $m = m_0$ and the distribution at all other dates is the unique symmetric one with $m = m_0 + \Delta$.

Although most of the special assumptions will be discussed in section 4, the specification of changes in the amount of money deserves some comment now. First, I study a once-for-all change in the quantity of money because it is simple. Second, only those who exit trade without money are eligible to discover a unit of money, because those with money would have to discard a unit if they discovered money.[2] Third, the assumption that $m_0 + \Delta \leq 1/2$ restricts the quantity of money to a range in which the probability of a trade meeting is nondecreasing in the quantity of money. If there were no upper bound on individual holdings, then increases in the quantity of money would never reduce the probability of a trade meeting. Since the upper bound is adopted only for tractability, it seems sensible to restrict the quantity of money to a range in which it does not crowd out trade meetings.

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[2] A version in which everyone could discover would differ only in insignificant details. Alternatively, a version in which, after date 0 trade, people choose whether to expend some small amount of effort in order to be eligible to discover money would not differ at all.

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That range is [0,1/2], because the fraction of all meetings which are trade meetings is $(1-m)m(2/N)$, where m is the fraction of each type with a unit of money. Finally, the assumption that Δ is revealed to everyone at the beginning of date 2 is also made for simplicity. It allows me to easily describe what happens at date 2 and then, by working backwards, describe what happens at dates 1 and 0.

2. A symmetric monetary equilibrium

An equilibrium is a description of what happens in all meetings-- essentially a description of what is produced (and consumed) in trade meetings. The equilibrium concept is the take-it-or-leave-it bargaining described above along with rational expectations. I will construct the simplest kind of monetary equilibrium, one that is constant from date 2 onward. By long-run effects of changes in the quantity of money, I mean the dependence on Δ of what happens in that equilibrium at date 2 and thereafter; by short-run effects, I mean the dependence on Δ of what happens in that equilibrium at date 1. In other words, I will be describing equilibrium cross-section observations at date 2 and thereafter (the long-run) and equilibrium cross-section observations at date 1 (the short-run)-- cross-sections in that they come from economies that are identical except for the realization of Δ. I begin with a summary of those short-run and long-run effects.

Each producer in a trade meeting at date 1 has the same experience; each exited a meeting at date 0 without money, did not discover a unit of money, and met someone with a unit of money (and does not know the source of the consumer's money). Therefore, each has the same posterior. Since that posterior of the producer is known to the producer's trading partner, because the partner knows what happened to the producer, the maximum amount produced in every trade meeting is the same and can be denoted c_1. (An explicit expression for the producer's updated prior and c_1 is given below.) Suppose, as is demonstrated below, that trade occurs in each trade meeting and, therefore, that c_1 is produced in each such meeting. Because all trade at date 1 consists of the exchange of c_1 for one unit of money, the price level at date 1 is $1/c_1$. Therefore, it does not depend on the realization of Δ. Total output can be expressed in terms of c_1 and the realization of Δ. Total output per type is $c_1(m_0 + \Delta)(1 - m_0 - \Delta)(2/N)$. Therefore, total output, denoted $Y_1(\Delta)$, arrived at by summing over types, is given by

$$(1) \qquad Y_1(\Delta) = 2c_1(m_0 + \Delta)(1 - m_0 - \Delta)$$

It follows, from the assumption that $m_0 + \Delta_K \leq 1/2$, that $Y_1(\Delta)$ is increasing in Δ.

The date 2 effects are quite different. At the beginning of date 2, everyone knows Δ. Thus, beginning at date 2, the economy has a constant and known amount of money per type. If $c_2(\Delta)$ denotes the amount produced in exchange for a unit of money when the constant quantity of money is $m_0 + \Delta$, then, as shown below, $c_2(\Delta)$ is decreasing in Δ. Since the price level is $1/c_2(\Delta)$, the price level is increasing in Δ. Total output at date 2, denoted $Y_2(\Delta)$, is given by

$$(2) \qquad Y_2(\Delta) = 2c_2(\Delta)(m_0 + \Delta)(1 - m_0 - \Delta)$$

The assumptions do not imply that $Y_2(\Delta)$ is monotone in Δ or, if monotone, the direction of the monotonicity. Thus, there is no obvious association at date 2 between total output and the realization of Δ.

Notice that the form of the total output function is the same as dates 1 and 2; it is the product of two functions. One function is the probability of a trade meeting. That part, given by $2(m_0 + \Delta)(1 - m_0 - \Delta)$, is identical at dates 1 and 2 and, under my assumption about the range of Δ, is increasing in Δ. The other function is the amount produced in a trade meeting. At date 1, that part is a constant, while at date 2, it is a decreasing function of Δ. That difference between the total output functions captures the sense in which total output varies more strongly with Δ, the realization of the quantity of money, at date 1, the short-run, than at date 2, the long run. [3]

I now show how to construct the equilibrium just described. As noted above, the idea is to work backward from the date 2 constant equilibrium monetary equilibrium that depends on the realization of Δ.

Date 2 and thereafter. For an economy with a constant and known quantity of money per type, let $v(j)$ denote the constant expected discounted value of starting a period with j units of money ($j = 0,1$) and let c denote the amount produced in each trade meeting. The

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[3] If the support of Δ is an interval, then the derivative of $Y_2(\Delta)$, evaluated at the magnitude of Δ at which $c_2(\Delta) = c_1$, is less than the derivative of $Y_1(\Delta)$.

bargaining rule implies that $v(0) = 0$, because all the trading gains go to the consumer and $u(0) = 0$. Therefore, $v(1)$ and c must be a solution to

$$(3) \qquad v(1) = \alpha \max[\max_c u(c), \beta v(1)] + (1-\alpha)\beta v(1)$$

where the maximum over c is subject to

$$(4) \qquad c \leq \beta v(1)$$

and where $\alpha = (1-m)/N$, the probability of meeting a potential producer who has no money. Equation (3) is Bellman's equation, while (4) says that the disutility to the producer cannot exceed the producer's gain. (The result, $v(0) = 0$, has been substituted into (3) and (4).)

Because (4) holds at equality at a solution, if the outer maximum in (3) is $u(c)$, then (3) and (4) imply, by substitution,

$$(5) \qquad [\alpha + (1-\beta)/\beta]c = \alpha u(c)$$

Equation (5) has two solutions for c: 0 and a positive solution, denoted $f(m)$. Because a positive solution to (5) satisfies $u(c) > c$, it follows that $c = f(m)$ and $v(1) = f(m)/\beta$ are such that the outer maximum in (3) is $u(c)$. Therefore, they are a solution to the above problem. Moreover, since differentiation of (5) implies $dc/d\alpha > 0$ at $c = f(m)$, f is decreasing.

The first step in constructing an equilibrium satisfying the claims made above is to let

$$(6) \qquad c_t(\Delta) = f(m_0 + \Delta), \; v_t(0; \Delta) = 0, \; v_t(1; \Delta) = f(m_0 + \Delta)/\beta; \; t \geq 2$$

where $c_t(\Delta)$ denotes production in a trade meeting at t and $v_t(j; \Delta)$ denotes expected discounted utility at the beginning of date t from beginning with j units of money. Equation (6) gives us the long-run effects asserted above. In particular, since f is decreasing, the price level is increasing in the realization of Δ.

Date 1. I now describe c_1. That is done by finding the maximum amount each producer in a trade meeting would be willing to produce in exchange for a unit of money and then

showing that such a trade actually occurs. I begin by computing the posterior of a producer in a trade meeting.

Let I denote information and let I_p denote the specific information of a producer in a trade meeting: I_p consists of not discovering a unit of money and, subsequently, meeting someone with money. Conditional on the realization of Δ, those are independent events. Therefore,

(7) $\quad P(I = I_p \mid \Delta = \Delta_k) = [1 - \Delta_k/(1-m_0)](m_0 + \Delta_k) = (1 - m_0 - \Delta_k)(m_0 + \Delta_k)/(1-m_0)$

Then Bayes rule gives

(8) $\quad P(\Delta = \Delta_k \mid I = I_p) = p_k(1 - m_0 - \Delta_k)(m_0 + \Delta_k)/\Sigma_j[p_j(1 - m_0 - \Delta_j)(m_0 + \Delta_j)]$

It follows that the maximum amount a producer is willing to produce in exchange for a unit of money at date 1 is

(9) $\quad c_1 = \beta \Sigma_k P(\Delta = \Delta_k \mid I = I_p) v_2(1; \Delta_k) = \Sigma_k P(\Delta = \Delta_k \mid I = I_p) f(m_0 + \Delta_k)$

where the first equality follows from noting that the producer's gain is the expected utility of beginning date 2 with a unit of money and the second equality follows from (6), which gives the realized utility at date 2 of beginning with money for each possible Δ.

The next step is to assure that each potential consumer in a trade meeting wants to surrender a unit of money for c_1 as given by (9). That happens if $u(c_1)$ is not less than the discounted expected utility for the consumer of beginning date 2 with a unit of money. If p_k' denotes the posterior of a consumer in a trade meeting, then the condition for trade is [4]

(10) $\quad u[\Sigma_k P(\Delta = \Delta_k \mid I = I_p) f(m_0 + \Delta_k)] \geq \Sigma_k p_k' f(m_0 + \Delta_k)$

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[4] There are two types of consumers; one type exited trade at date 0 with a unit of money, the other did not and discovered a unit of money. They have distinct posteriors, despite my use of a single symbol, p_k'.

If the posterior of the producer and the consumer were the same, than (10) would be an implication of $u[f(m_0 + \Delta_k)] > f(m_0 + \Delta_k)$ for each k, which follows from (5). However, the posteriors are not the same.[5] Therefore, as I now explain, I obtain (10) from the assumption that the range of Δ is sufficiently small.

Because f is decreasing, a sufficient condition for (10) is $u[f(m_0 + \Delta_K)] \geq f(m_0 + \Delta_1)$. Let $\Delta_K - \Delta_1 = r$ (for range) and let $g(r) = u[f(m_0 + \Delta_1 + r)]/f(m_0 + \Delta_1)$. The function g is continuous and decreasing and satisfies $g(0) > 1$ and $g(1 - m_0 - \Delta_1) = 0$. Therefore, there exists a unique and positive r, say r^*, such that $g(r^*) = 1$. Thus, if $\Delta_K - \Delta_1 \leq r^*$, then (10) holds.

Date 0. Although not needed for my claims about short-run and long-run effects, I now complete the description of the equilibrium. The first step is to compute (beginning of) date 1 expected discounted utilities. As at date 2, the expected discounted utility of beginning date 1 without money is zero. There are two distinct expected discounted utilities of beginning date 1 with a unit of money: one is for those who exited trade at date 0 with a unit of money and, therefore, were not in a position to discover a unit of money; the other is for those who exited trade without a unit of money and discovered a unit of money. They are distinct, because such people have different information. Once again, letting I stand for information, both expected discounted utilities can be expressed as

(11) $\quad v_1(1;I) = \Sigma_k P(\Delta = \Delta_k | I)\{[(1 - m_0 - \Delta_k)/N]u(c_1) + [1 - (1 - m_0 - \Delta_k)/N]\beta v_2(1;\Delta_k)\}$

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[5] One way to see why is to consider the consumer who exited trade with a unit of money. Such a consumer updates his or her prior through the experience of having met someone without money. That information leads such a consumer to revise the prior p_i by putting more weight on lower realizations of Δ, which is not same as what the producer does. (If this seems paradoxical, it may help to consider the following. In terms of meetings, the producer draws from a sample space with the following two elements: (i) neither person has money, (ii) one does and one does not; in contrast, the consumer draws from a sample space with the following two elements: (i') both people have money, (ii) one does and one does not. Since (i) and (i') differ, observing (ii) is interpreted differently by the producer and the consumer.

where $v_1(1;I)$ denotes expected discounted utility at date 1 (the subscript) of holding one unit of money in information state I and $P(\Delta = \Delta_k | I)$ denotes the posterior conditional on information I.

The person who exited trade at date 0 with money has no information because the person was not in a position to discover a unit of money. I denote this absence of information by $I = \varnothing$. Obviously, $P(\Delta = \Delta_k | I = \varnothing) = p_k$. I let $I = D$ denote the information at the beginning of date 1 of the person who discovered a unit of money. Because $P(I = D | \Delta = \Delta_k) = \Delta_k/(1-m_0)$, Bayes rule implies

(12) $P(\Delta = \Delta_k | I = D) = p_k \Delta_k/\theta$

where θ denotes the unconditional expected value of Δ, $\Sigma_k p_k \Delta_k$. That completes the description of date 1 discounted expected utilities.

Now I can describe what happens at date 0. Because there is no information about Δ at the beginning of date 0, I let $v_0(j)$ denote the discounted expected utility at the beginning of date 0 of someone with j units of money. At date 0, someone who starts with no money has a chance of discovering a unit. It follows that

(13) $v_0(0) = \beta[\theta/(1-m_0)]v_1(1;D)$

where $v_1(1;D)$ is implied by (11) with $I = D$ and where $\theta/(1-m_0)$ is the unconditional probability of discovering a unit of money. As regards someone who starts with a unit of money,

(14) $v_0(1) = [(1-m_0)/N]\max\{\max_c u(c) + \beta[\theta/(1-m_0)]v_1(1;D), \beta v_1(1;\varnothing)\} +$

$$[1 - (1-m_0)/N]\beta v_1(1;\varnothing)$$

where the maximization over c is subject to

(15) $c \le \beta\{v_1(1;\varnothing) - [\theta/(1-m_0)]v_1(1;D)\} \equiv c_0$

I now show that production in each date 0 trade meeting is equal to c_0. First, by (11) and (12),

(16) $\{v_1(1;\varnothing) - [\theta/(1-m_0)]v_1(1;D)\} =$

$$\Sigma_k p_k[1- \Delta_k/(1-m_0)]\{[(1-m -\Delta_k)/N]u(c_1) + [1- (1-m_0-\Delta_k)/N]\beta v_2(1;\Delta_k)\}$$

which is positive because $\Delta_k < 1-m_0$. Therefore, $c_0 > 0$. Second, since the maximum of $u(c)$ over c is $u(c_0)$, the condition that the outer maximum in (14) involves trade is $u(c_0) \geq c_0$. From (11), we have $v_1(1;\varnothing) < v_2(1; \Delta_1)$. Therefore, from (15), $c_0 < \beta v_1(1;\varnothing) < \beta v_2(1; \Delta_1)= f(m_0+ \Delta_1)$. Since $u[f(m_0+ \Delta_1)] > f(m_0+\Delta_1)$ and $0 < c_0 < f(m_0+ \Delta_1)$, it follows that $u(c_0) > c_0$. Therefore,

(17) $v_0(1) = [(1-m_0)/N]\{u(c_0) + \beta[\theta/(1-m_0)]v_1(1;D)\} + [1 - (1-m_0)/N]\beta v_1(1;\varnothing).$

That completes the construction of an equilibrium. The next section contains a numerical example.

3. The welfare effects of uncertainty about the quantity of money

I now show how to use the model to judge the welfare conseqences of different distributions for the one-time change in the quantity of money, different distributions for Δ. Although the model can be used to make comparisons between arbitrary distributions that satisfy the assumptions and although an analytic analysis of different distributions for Δ seems posssible, I limit comparisons here to an example with distributions that, among other conditions, have the same unconditional mean, the same θ. I hold θ fixed because the effects of different θ's are due primarily to the indivisibility of money and the unit upper bound.

In general, comparisons among different distributions for Δ can be regarded as a policy analysis if we suppose that there are multiple fiat objects and that a policy choice determines which is used as money. If there are multiple fiat objects, then there is an equilibrium in which all but one are valueless. We can regard the equilibrium of section 2 as that equilibrium.

The model lends itself to a representative agent welfare criterion-- namely, the following weighted average of date 0 expected utilities,

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(18) $W_0 = (1- m_0)v_0(0) + m_0v_0(1)$

Here W_0 can be interpreted as the expected discounted utility of each person at date 0 prior to learning whether or not the person starts out with a unit of money-- ($1-m_0$) being the probability of starting without money and m_0 being the probability of starting with money. Since the v's are given in section 2, we have all the ingredients for evaluating W_0 for different probability distributions for Δ.

The example studied here has the following common features: $u(x) = x^{1/2}$, $\beta = .99$, $N = 3$, $m_0 = 1/4$, $K = 2$ and $p_1 = p_2 = 1/2$ and $\theta = 1/16$, a mean change of 25% of the initial quantity of money. I also let $\Delta_1 = \theta - \delta$ and $\Delta_2 = \theta + \delta$, and let δ, the degree of uncertainty in this example, take on the values 0, 1/32, and 1/16. This specification satisfies all the assumptions, including the limitation on the range of Δ. The results are given in the following table, where c_{tk} and Y_{tk} denote consumption and total output at date t in aggregate state k.

	date 2 (long-run)				date 1 (short-run)			date 0		
δ	c_{21}	c_{22}	Y_{21}	Y_{22}	c_1	Y_{11}	Y_{12}	c_0	Y_0	W_0
0	.9173	.9173	.3942	.3942	.9173	.3942	.3942	.8409	.3153	.2914
1/32	.9207	.9137	.3722	.4122	.9170	.3707	.4137	.8409	.3153	.2913
1/16	.9238	.9097	.3464	.4264	.9160	.3435	.4294	.8407	.3153	.2908

As expected, date 0 expected utility, W_0, is decreasing in δ. However, this example, which was chosen for its simplicity (in particular, if $u(x) = x^{1/2}$, then (5) is linear), does not give rise to date 2 effects (long-run effects) that are predominantly nominal. The positive effect on the probability of a trade meeting of larger increases in the quantity of money dwarfs the negative effect on the amount produced in each trade meeting. For each positive δ, the date 2 cross-section elasticity of total output with respect to the quantity of money is between .45 and .50. In accord with the model, the date 1 total output effects are larger, if only slightly so. For each positive δ, the date 1 cross-section elasticity of total

output with respect to the quantity of money is slightly in excess of .50. Finally, notice that date 0 output in each trade meeting is lower than at date 1, which is due primarily to the unconditional probability of discovering a unit of money-- in this example, 1/12.

4. The assumptions and robustness

Although the model contains many extreme assumptions, three deserve special attention: the inability of producers at date 1 to distinguish the source of the consumer's money; the public knowledge at the beginning of date 2 about the realized change in the quantity of money; the indivisibility of money and the upper bound on individual holdings.

As I have specified the form of offers, consumers at date 1 are unable to signal the source of their money holdings. [6] Were they able to, either those who have newly discovered ("new") money or those who have "old" money would want to signal the source; their information is different and if known by the producer would give different producer posteriors, one of which would be consistent with higher production than is implied by pooling. One way to think about the possible consequences of such signalling is to examine an alternative in which new money looks different for one period from old money. Then there is no relevant asymmetric information, but there are different posteriors for producers depending on whether they meet new or old money. That being so, different amounts are produced in the two meetings and we no longer get the implication that the price level at date 1 is independent of Δ. Because different amounts are produced in the different kinds of date 1 single-coincidence meetings, the price level must be computed using an implicit deflator. Also, total output is a weighted sum of the amounts produced in the two meetings. Nevertheless, the result that the date 1 effects are predominantly real and expansionary holds provided the range of Δ is sufficiently small. To see this, let c_{old} be the amount produced in each old-money meeting at date 1 and let c_{new} be the amount produced in each new-money meeting. Neither depends on the realization of Δ, but both depend on all the parameters, including the range of Δ. Total output at date 1 is

(19) $Y_1(\Delta) = 2c_{new}(1 - m_0 - \Delta)\Delta + 2c_{old}(1 - m_0 - \Delta)m_0$

and

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[6] I am indebted to Tom Holmes for discussions that greatly influenced the content of this paragraph. He, however, is not responsible for any errors that I may have made.

(20) $\partial Y_1(\Delta)/\partial\Delta = 2\{c_{new}[1 - 2(m_0 + \Delta)] - (c_{old} - c_{new})m_0\}$

Now consider what happens as the range of Δ, r, gets small. As $r \rightarrow 0$, $c_{old} \rightarrow c_1$ and $c_{new} \rightarrow c_1$. Therefore, the price level becomes independent of Δ. As regards total output, it follows from (20), that its derivative approaches something positive and identical to what is implied by the version examined above. Thus, such a complete information version gives qualitative implications similar to those of the version studied above provided the range of Δ is small enough.

In contrast to the assumption that the realized change in the quantity of money is revealed to everyone with a one-period lag, the natural assumption is that it is never revealed. I see two difficulties in working with that specification or even one that lengthens the lag beyond one period. First, priors get revised in accord with experience (at least experience regarding what the trading partner has). Since experience is diverse, one would have to keep track of groups that are diverse in terms of their posteriors over the realized change in the amount of money. Second, the bargaining would then be between two people who do not know each other's posteriors. Despite those possible difficulties, it is plausible that the qualitative features found for the one-period information lag formulation would continue to hold, but not in the same way. Under the natural specification, because people would learn the realization in the limit, there ought to be an equilibrium that converges to what happens at date 2 under the one-period information lag formulation. Moreover, although the implied "short-run" would then merge smoothly into the "long-run", rather than ending abruptly after one period as under my specification, the effects at date 1 would again be entirely real.

The assumption that money is indivisible and that there is a unit upper bound on individual holdings plays an important role. To consider that role, suppose instead that money is divisible and that there is no bound on individual holdings. Under that alternative, the first issue that arises is how to have the change in the quantity of money come about. In order that people not be able to infer the aggregate change from their own discoveries of money, each person's discovery should not be proportional to the person's initial holdings with a proportionality factor equal to the proportional change in the aggregate quantity of money. In the absence of such proportionality, even if the initial money distribution is a steady state, the money distribution after the change occurs is not a steady state distribution. That will make it difficult to deduce the properties of the equilibrium path.

More interestingly, in my formulation, those who discover money are not producers at date 1; they are either consumers or do not trade-- that being a consequence of the indivisibility and the upper bound. If there is no upper bound, then the process of discovery could be random among everyone. Given such randomness, total output at date 1 may not be increasing in the aggregate discovery of money because producers who have discovered money will tend to produce less.[7] One way to amend the model to restore such dependence is to allow some choice about whether to produce or consume. If there is such choice, then those who discover money would tend to be consumers. Although such a choice appears in some closely related models, they also include indivisible money and a unit upper bound on individual holdings (see Diamond 1984 and Kiyotaki and Wright 1991).

Although the above remarks are necessarily speculative, there are grounds for supposing that the main qualitative finding regarding the effects of once-for-all changes in the quantity of money will survive generalizations of the model in several directions. In addition, it seems clear that the features that produce the distinct short-run and long-run effects in cross-sections in the model of this paper will also produce similar effects in times-series, were we able to both formulate and analyze a version with a stationary process for changes in the quantity of money.

5. Concluding remarks

Lucas's Nobel Lecture (1996), a discussion of facts and theories relating to the short-run and long-run effects of changes in the quantity of money, begins with a detailed and laudatory comment on Hume's discussion of those effects. Here is a summary of Hume's explanation of disparate short-run and long-run effects of changes in the quantity of money.

> ...Accordingly we find, that, in every kingdom, into which money begins to flow in greater abundance than formerly, every thing takes a new face: labour and industry gain life; the merchant becomes more enterprising,... .

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[7] The assumption that new money goes to consumers appears in many other models; see, for example, Lucas (1972), Eden (1994), or Lucas and Woodford (1994). Barro and King (1984) emphasize the important role of the assumption and question the rationale for it.

To account, then, for this phenomenon, we must consider, that though the high price of commodities be a necessary consequence of the encrease of gold and silver, yet it follows not immediately upon that encrease; but some time is required before the money circulates through the whole state, and makes its effect be felt on all ranks of people. At first, no alteration is perceived; by degrees the price rises, first of one commodity, then of another; till the whole at last reaches a just proportion with the new quantity of specie in the kingdom. In my opinion, it is only in this interval or intermediate situation, between the acquisition of money and rise of prices, that the encreasing quantity of gold and silver is favorable to industry. When any quantity of money is imported into a nation, it is not at first dispersed into many hands but is confined to the coffers of a few persons, who immediately seek to employ it to advantage. Here are a set of manufactuers or merchants, we shall suppose, who have received returns of gold and silver for goods which they have sent to Cadiz. They are thereby enabled to employ more workmen than formerly, who never dream of demanding higher wages, but are glad of employment from such good paymasters. [The artisan]...carries his money to the market, where he finds every thing at the same price as formerly, but returns with greater quantity and of better kinds for the use of his family. The farmer and gardener, finding that all their commodities are taken off, apply themselves with alacrity to raising more...It is easy to trace the money in its progress through the whole commonwealth, where we shall find that it must first quicken the diligence of every individual before it encrease the price of labour. (Hume 1752, pages 37, 38)

The model set out above seems to mimic Hume's explanation in this passage much more closely than do the models discussed by Lucas (1996). There seem to be two main ingredients in the above passage: decentralized trade and asymmetric information about the quantity of money. Those are the two main ingredients in the model. [8]

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[8] The common elements in Hume's discussion and the model set out above made me wonder what else I might find in Hume. Because decentralized trade appears prominently in the above passage, I expected to find a general discussion of the role of money in overcoming double-coincidence problems in decentralized trade-- the kind of discussion that appears, for example, in the *Wealth of Nations*. (1776, p.22). And, although not very likely, I was hoping to also find remarks connecting that role of money to Hume's attempt to explain the short-run and long-run effects of changes in the quantity of money. I found neither; Hume does not even allude to double-coincidence problems. Hume's general

I have shown that the features which imply that outside money has a role in overcoming double-coincidence problems also imply the kind of disparate short-run and long-run effects of changes in the quantity of outside money that we have often observed-- provided those changes come about in a way that satisfies two conditions. The changes must be random and people must learn about realizations only with a lag. Since those conditions are not new and since they play the same critical role in the model presented here that they do in other models of the disparate short- and long-run effects of change in money (for example, Lucas 1972), my contribution is not a new theory of disparate short-run and long-run effects. Rather, it is to point out that a random-matching model provides a background setting within which those conditions on changes in money work as they do in other models that explain disparate short- and long-run effects. Pointing that out is worthwhile because the random-matching background setting is one we ought to find appealing on other grounds. It includes features long held to be necessary to give outside money a role in exchange; namely, double-coincidence problems and information restrictions that prevent those problems from being overcome with private credit. No similar claim can be made for the other background settings which have been used to account for the disparate short- and long-run effects of changes in money.

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discussion of the role of money is limited to the following remark: "[Money] is none of the wheels of trade : It is the oil which renders the motion of the wheels more smooth and easy (Hume, 1752, p 33).

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