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Introduction

This study seeks to derive a dynamic theory of demand for money from individual optimization behavior. A model is constructed which includes essential features of both inventory-theoretic and neoclassical time preference models. The model extends the inventory framework developed by Baumol[2/] and Tobin[3/] in two respects:

1. Transactions in both commodity and bond markets are assumed to incur costs; in the Baumol-Tobin model commodity market transactions are assumed costless. Models constructed by Feige-Parkin[4/] and Perlman[5/] also have assumed costs to transacting in both markets.
2. The individual's objective function permits desired consumption in current <u>and</u> future periods to change as initial conditions change; in the Baumol-Tobin model consumption in the current period is taken as fixed.

The individual's objective in this model is to maximize his satisfaction, a function of level consumption during the period ("consumption") and wealth at the end of the period ("terminal wealth"). A period is defined as the time elapsing between noninterest income receipts, and terminal wealth is taken as a proxy for future consumption.

Satisfaction maximization guarantees that the individual will choose a consumption-terminal wealth pair on the boundary of his attainable

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[2/] Baumol [1].

[3/] Tobin [17].

[4/] Feige and Parkin [4].

[5/] Perlman [16].

set. This boundary, dubbed the consumption-terminal wealth frontier, gives the greatest feasible rates of consumption for given stocks of terminal wealth, and vice versa. Attainment of a point on the frontier is a problem in dynamic inventory control.

The model can be described as follows: In a world of perfect certainty an individual is able to hold stocks of commodities, bonds, and money. Commodities diminish from consumption and depreciate physically in storage. The price of commodities is expected to change at a constant rate over the period. Bond holdings earn interest. Exchanges in the bond and commodity markets incur costs which are independent of transaction size. Money can be exchanged for either bonds or commodities, but relative magnitudes of market transaction costs ensure that bonds and commodities are not exchanged directly for one another.

The individual's control variables are the times of transactions and the quantities of goods exchanged; a specific setting for the control variables defines a market strategy. An efficient market strategy is one which allows the greatest rate of consumption for given terminal wealth and vice versa. When an efficient strategy implies nonsynchronization in timing of bond and commodity transactions, positive money holdings result.

The overall satisfaction maximization problem is solved in two steps. First, a consumption-terminal wealth frontier is constructed for given values of six parameters: initial wealth, physical depreciation rate, interest rate, expected rate of commodity price change, and commodity and bond market transaction costs. Second, the individual selects the point on the frontier which maximizes his level of satisfaction.

Average holdings per period of commodities, bonds, and money correspond to each point on the frontier. Asset demand functions indicate average holdings of these assets corresponding to the satisfaction maximization point. Thus, for given preferences over the consumption-terminal wealth space, asset demand functions depend only on the values of the six underlying parameters which determine the location of the frontier in that space.

As the description of the model indicates, the individual will hold bonds only if it is profitable to do so. For low enough values of the interest rate or high enough values of the bond market transaction cost, the individual's asset portfolio will be composed of only commodities and money. This is treated as a special case of the general model, and it bears some resemblance to Harris's model of household behavior.[6/] There seem to be two major differences in the models. First, for given end-of-period wealth, individual objective functions are different. In Harris's model the individual maximizes an accumulated flow of utility, where utility at a point in time depends only on the current rate of consumption. In the present model the individual maximizes a level rate of consumption over the period. Second, transaction cost functions are different. In Harris's model transaction costs are incurred in a flow, and the amount of the costs varies with the rate of real expenditures. In the present model, transaction costs are incurred whenever a transaction is made, and the costs are independent of the amount of goods exchanged.

[6/] Harris [8].

In judging the reasonableness of the alternative assumptions, it is helpful to keep in mind the differences in their implications. Harris's objective function is more general in permitting consumption to vary over time, but it causes the optimal consumption pattern over time to depend on the assumed depreciation process. The type of depreciation process we each assume leads the individual maximizing Harris's objective function to consume most heavily immediately after a shopping trip and to steadily reduce consumption until the next trip. This type of pattern allows the individual to reduce the loss from depreciation. In the present model, the individual consumes the same quantity no matter how much time has elapsed from his last shopping trip.

Harris's transaction cost function implies an infinite cost to exchanging stocks at a point in time, while that in the present model implies an infinite cost to making expenditures in a flow. Harris's assumption implies the individual will continually be in the market exchanging money for commodities in a flow, while the present model's assumption implies the individual will make a finite number of trips to the market at discrete points in time to exchange stocks of commodities and money.

Given these major differences in assumptions, it is somewhat surprising that the real money demand functions derived in Harris's model and the two-good version of the present model respond qualitatively the same to changes in parameter values. In each model it is found that average real money balances desired over the period increase in response to an increase in initial money balances, an increase in the depreciation rate, a fall in anticipated inflation, or a fall in the commodity market transaction cost.[7/]

The general three-good model in the present paper is closely related to models by Perlman and Feige-Parkin. All three models introduce a physical good into the Baumol-Tobin money and bonds inventory framework. Perlman's third good corresponds to a consumption durable; it does not diminish from consumption. The third asset in the Feige-Parkin model is a nondurable consumption good, making their model more closely related to the one in this paper.[8/] There are important differences, however, between the model in this paper and its two immediate precursors:

1. Control variables. In the present model the control variables are the times of transactions and the amounts of commodities or bonds purchased or sold in each transaction. In the previous two models it was assumed at the outset that:

    a. transactions in each market are equally spaced in time,

    b. bond market transactions are made at times of commodity market transactions,

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[7/]These results follow from the present model when it is assumed that zero wealth is desired at the end of the period.

[8/]In their model the individual is restricted to hold end-of-period wealth in the form of physical capital. If it were possible in their model to buy and sell physical capital and the individual were not forced to hold this asset, he would either hold bonds or physical capital; he would not hold both.

c. the same quantity of bonds is sold in each bond market transaction, and

d. the same quantity of commodities is purchased in each commodity market transaction.

Given these assumptions, the control variables in the Feige-Parkin and Perlman models are just the number of transactions to be made in each market. A priori, there is no reason to believe that their assumptions are consistent with optimizing behavior, and, as this paper shows, they aren't. Although properties of efficient market strategies and the frontier are derived over the set of feasible strategies in this paper, it is necessary to restrict analysis to equal spacing strategies in order to derive properties of asset demand functions.

2. Objective functions. While the Perlman and Feige-Parkin models assume the individual maximizes either current consumption for given future consumption or vice versa, the present model assumes he maximizes utility, a function of both variables. This generalization leads to a division of the model into its objective and subjective parts, e.g., the frontier and asset demand functions, respectively. The division of the model permits implications for market behavior which depend only on efficiency criteria to be separated from those which depend also on individual preferences.

3. Solution methods. Assuming 1a-1d makes it possible to write consumption c as a function of the number of

transactions in the commodity and bond markets, m and n respectively, for given terminal wealth W and parameter vector γ:

$c = \xi(m,n;W;\gamma)$. If 1a and 1b are to be satisfied, maximizers of this function $\hat{m}$ and $\hat{n}$ must satisfy three integer constraints (assuming $\hat{n} > 0$):

a. $\hat{m} \varepsilon \mathbb{N}$

b. $\hat{n} \varepsilon \mathbb{N}$

c. $\hat{m}/\hat{n} \varepsilon \mathbb{N}$, where $\mathbb{N}$ denotes the set of positive integers.

Feige-Parkin attempt to solve this problem using calculus maximization techniques, but this approach clearly ignores the third integer constraint. Perlman, on the other hand, transforms the consumption function to $c = \xi^*(v,n;W;\gamma) \equiv \xi(vn,n;W;\gamma)$, where $v = \frac{m}{n}$, and uses calculus maximization techniques to solve for $\hat{v}$ and $\hat{n}$.[9/] It follows, however, that ξ^* is not necessarily concave in v and n for all values of W and γ. Even when ξ^* is concave, calculus maximization can provide very poor approximations to the integer solutions.[10/] Hence, Perlman's approach is not appropriate either. In this paper parameter

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[9/] Perlman actually maximizes $W = \xi^{*-1}(v, n; c; \gamma)$ with respect to v and n for given c.

[10/] In general, when the function being maximized depends on a single variable, concavity implies that the integer maximizer is one of the two integers which bracket the real number maximizer. When the maximand is a function of two variables, concavity does not imply that the integer maximizers are one of the four corners of the unit box which encloses the real number maximizers. This point is discussed further in the next section.

values are selected and specific solutions which satisfy the three integer constraints are found to the maximization problem.

4. Inflation rate. Both previous models assume prices are constant over the period. The present model permits nonzero price growth and is applied to examine the effects of a change in anticipated inflation or deflation on efficient market behavior and asset demand functions.

Some important implications of the present model are:

1. The individual's attainable set in the consumption-terminal wealth space lacks convexity. Thus, preferences must be strongly convex if, in equilibrium, an individual desires both consumption and savings.
2. Regular timing of market transactions, which is an efficiency condition for two-asset inventory models, does not generalize to inventory models with more than two assets. Hence, the assumption of equal spacing of market transactions in the Feige-Parkin model can imply inefficient market strategies.
3. Both commodities and bonds may be held although their rates of return differ; hence, they are not perfect substitutes. This implies that the nominal rate of interest <u>and</u> the expected rate of inflation are arguments in the individual's asset demand functions.
4. Asset demand functions depend importantly on individual time preferences. Preferences between present and future consumption in some cases can even determine the sign of

asset demand changes due to changes in the values of parameters.

5. Changes in desired holdings of an asset, which result from a given parameter change, may vary in sign depending on initial parameter values. Corresponding changes in aggregate asset demand functions might be expected, therefore, to depend on values of certain variables (e.g., interest rate) and on the distribution over individuals of other variables (e.g., wealth).

6. The signs of changes in asset demands due to changes in the expected rate of inflation π depend on both individual time preferences and initial parameter values. Although an increase in π increases the yield on commodities relative to the yields on money and bonds, it has other effects in the model:

   a. A rise in π requires more initial wealth to be allocated to investment purposes in order to attain the same real terminal wealth.

   b. Since nominal transaction costs grow at the same rate as commodity prices, an increase in π raises the cost of transactions over time.

   c. A rise in π alters the distribution of commodity payments for a fixed number of commodity market transactions. Under the assumption of equal spacing of market transactions, purchases of commodities over time have the same real value. Hence, a rise in π increases the nominal value of a purchase made late in the period relative to one made earlier.

Because of the pervasive effects of inflation in this model, the change in asset holdings to a change in π is generally unpredictable in sign.

THE THREE-ASSET INVENTORY MODEL: A MATHEMATICAL STATEMENT

I. Notation

$[0,1]$ = time period, where 0 = current point in time and 1 = end of period.

$M(t)$ = stock of money held at time $t\varepsilon[0,1]$.

$B(t)$ = dollar value of bonds held at time $t\varepsilon[0,1]$.

$C(t)$ = stock of commodities measured in physical units held at time $t\varepsilon[0,1]$.

$p(t)$ = expected and actual price of commodities at time $t\varepsilon[0,1]$; $p(0) = 1$.

$W(t) = \frac{M(t)}{p(t)} + \frac{B(t)}{p(t)} + C(t)$ = constant-dollar wealth at time $t\varepsilon[0,1]$; $W \equiv W(1)$.

$M(0^-)$ = initial money holdings.

$B(0^-)$ = initial bond holdings.

$C(0^-)$ = initial commodity holdings.

The individual's initial endowment is assumed to be in the form of money:

$$\langle M(0^-),\ B(0^-),\ C(0^-)\rangle = \langle M(0^-),\ 0,\ 0\rangle \geq \langle 0,\ 0,\ 0\rangle .$$

No income other than flows from wealth are received during the period.

$\overline{M} = \int_0^1 M(t)dt$ = average money holdings per period.

$\overline{B} = \int_0^1 B(t)dt$ = average bond holdings per period.

$\bar{C} = \int_0^1 p(t) \cdot C(t)dt$ = average dollar value of commodity holdings per period.

$P(t)$ = dollar amount of bonds purchased at time $t\varepsilon[0,1]$.

$S(t)$ = dollar amount of bonds sold at time $t\varepsilon[0,1]$.

$G(t)$ = physical amount of commodities purchased at time $t\varepsilon[0,1]$.

$H(t)$ = physical amount of commodities sold at time $t\varepsilon[0,1]$.

$c(t)$ = physical rate of consumption per period at time $t\varepsilon[0,1]$. The physical rate of consumption is assumed to be constant over the period and is denoted by c: $c(t_1) = c(t_2)$ $(t_1, t_2\varepsilon[0,1])$.

δ = rate of physical depreciation per period [0,1], $0 < \delta < 1$.

r = rate of interest per period [0,1] paid on bonds, $r \geq 0$.

$\pi(t) = \frac{\frac{dp}{dt}}{p(t)}$, rate of expected and actual price change per period at time $t\varepsilon[0,1]$. The rate of price change is constant over the period and is denoted by π: $\pi(t_1) = \pi(t_2)$ $(t_1, t_2\varepsilon[0,1])$. Thus, $p(t) = e^{\pi t}$ $(t\varepsilon[0,1])$, and it is assumed $-\frac{1}{2} < \pi < \delta$.11/

a = constant-dollar cost per transaction in the commodity market, $a > 0$.

b = constant-dollar cost per transaction in the bond market, $b > 0$.

II. Market Constraints and the Individual's Objective

Definition: A market strategy α is a vector of functions $\langle G,H,P,S \rangle$, each function having domain [0,1] and range $[0,\infty)$.

11/ The case of $\pi \geq \delta$ might be considered hyperinflation, because the individual would never want to hold money. This could be treated as a special case in the model, but it is not examined due to space limitations. The restriction $\pi > -1/2$ is a sufficient condition to rule out bond purchases after t=0.

For a given α the following characteristic functions are defined:

$$\chi_C(t) = \begin{cases} 0 \text{ if } G(t) = 0 \text{ and } H(t) = 0 \\ 1 \text{ if } G(t) > 0 \text{ or } H(t) > 0 \end{cases} \qquad (t\varepsilon[0,1])$$

$$\chi_B(t) = \begin{cases} 0 \text{ if } P(t) = 0 \text{ and } S(t) = 0 \\ 1 \text{ if } P(t) > 0 \text{ or } S(t) > 0 \end{cases} \qquad (t\varepsilon[0,1])$$

For a given market strategy α the following rules on market transactions must be satisfied:

T.1) $G(t)\cdot H(t) = 0$ and $P(t)\cdot S(t) = 0$ $(t\varepsilon[0,1])$; commodities are not both purchased and sold at the same time, and bonds are not both purchased and sold at the same time.

T.2) $\chi_B(t) = 1 \Rightarrow B(t) = B(t^-)$ and $M(t) = M(t^-)$ $(t\varepsilon[0,1])$

$\chi_C(t) = 1 \Rightarrow C(t) = C(t^-)$ and $M(t) = M(t^-)$ $(t\varepsilon[0,1])$,

the stock of any asset held at the time of a transaction is the amount held immediately before the transaction.

T.3) $B(t^+) = B(t) + P(t) - S(t)$ $(t\varepsilon[0,1])$; bond holdings change at a point in time from a purchase or a sale of bonds.

T.4) $C(t^+) = C(t) + G(t) - H(t)$ $(t\varepsilon[0,1])$; commodity holdings change at a point in time from a purchase or a sale of commodities.

T.5) $M(t^+) = M(t) - [P(t)-S(t)] - p(t) \cdot \{\chi_B(t)\cdot b + [G(t)-H(t)] + \chi_C(t)\cdot a\}$ $(t\varepsilon[0,1])$; money holdings are diminished by net purchases of bonds and commodities and by payments of transaction costs.

A traditional budget constraint is implied by T.5 and nonnegativity of M:

$M(t) \geqq [P(t)-S(t)] + p(t)\cdot\{\chi_B(t)\cdot b + [G(t)-H(t)] + \chi_C(t)\cdot a\}$ $(t\varepsilon[0,1])$;

money on hand at a point in time must be enough to cover net purchases of commodities and bonds made at the time plus transaction costs incurred.

T.6) $\left.\frac{dB(t)}{dt}\right|_{t=t_0} = rB(t_0) \quad (t_0\varepsilon(0,1])$; interest is compounded continuously

and payable at any point in time. (For arbitrary $t\varepsilon(0,1)$ $\chi_B(t) = 0 \Rightarrow B$ is differentiable at t. If $\chi_B(t) = 1$, the derivative of B at t is defined to be the left-hand derivative).

T.7) $\left.\frac{dC(t)}{dt}\right|_{t=t_0} = -\delta C(t_0) - c \quad (t_0\varepsilon(0,1])$; the stock of goods held at a

point in time diminishes from depreciation at a percentage rate independent of the stock held and from consumption at a percentage rate which is the ratio of consumption per period over the stock of goods held. (For arbitrary $t\varepsilon(0,1)$ $\chi_C(t)=0 \Rightarrow C$ is differentiable at t. If $\chi_C(t)=1$, the derivative of C at t is defined to be the left-hand derivative.)

The consumer's objective is to maximize $U(c,W)$ subject to given parameter values and market constraints. The utility function U is assumed to be strictly concave.

A necessary condition for $\langle\hat{c},\hat{W}\rangle$ to be a maximizer is that it be a point on the consumption-terminal wealth frontier (i.e., $\hat{c}$ maximizes c for $W = \hat{W}$ and $\hat{W}$ maximizes W for $c = \hat{c}$). Moreover, if it is a point on the frontier, it must have been generated by an efficient strategy. Thus, solution to the overall optimization problem and analysis of asset demand functions can be executed in sequential stages:

1. Determine properties of efficient strategies. In this stage principles of inventory management are derived which are required to attain a point on the frontier.

2. Determine the nature of the frontier. Given the results from the first stage, properties of the frontier as a whole are investigated in this stage.
3. Derive properties of asset demand functions. Given the results from the second stage, asset demand functions which depend only on underlying parameters and consumption-terminal wealth preferences, are examined in this stage.

Properties of Efficient Strategies

Definition:

The six parameters of the model are written in vector notation as $\gamma = \langle M(0^-), \delta, r, \pi, a, b \rangle$. For given γ and $c \geqq 0$, the set of feasible market strategies is denoted by $A[c]$. Then, a strategy $\alpha = \langle G, H, P, S \rangle$ is feasible if and only if each function in the vector $\langle M, B, C \rangle$ implied by α is nonnegative at every point in $[0,1]$, i.e., $\alpha \varepsilon A[c] \iff \alpha \Rightarrow \langle M, B, C \rangle \geq 0$.

Definition:

For given α the sets of times of transactions in the commodity and bond markets, T and $\sum$ respectively, are:

$$T = \{t \varepsilon [0,1] : \chi_C(t) = 1\}$$

$$\sum = \{t \varepsilon [0,1] : \chi_B(t) = 1\} .$$

With fixed market transaction costs, an infinite number of transactions in either market would imply infinite costs over the period and is clearly not feasible.

Lemma 1:[12]

For arbitrary γ and $c \geqq 0$, $\alpha \varepsilon A[c] \Rightarrow T$ and $\sum$ are finite sets; α is feasible only if a finite number of transactions are made in each market.

In view of Lemma 1 and the propositions which follow, it becomes convenient to provide notation for the total number of trips and times of trips to each market.

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[12] Proofs of mathematical propositions can be found in Miller [15].

Definitions:

For given c and $\alpha\varepsilon A[c]$ let:

m = number of "transactions in" (or equivalently "trips to") the commodity market in [0,1]

n = number of "transactions in" (or equivalently "trips to") the bond market in [0,1]

$\tau(k)$ = the point in time when the kth trip is made to the commodity market, $\tau(k)\varepsilon[0,1]$ $k=1, \ldots, m$ (defined for $m \geqq 1$)

$\sigma(k)$ = the point in time when the kth trip is made to the bond market, $\sigma(k)\varepsilon[0,1]$ $k = 1, \ldots, n$ (defined for $n \geqq 1$)

Given a value for the depreciation rate, the amount of commodities held after a commodity market trip must be enough to support consumption until the next trip. This amount can be computed directly from T.7 and is

$$C[\tau(i)^{+}] \geqq c/\rho_i \qquad i=1, \ldots, m,$$

where $\rho_i = \dfrac{\delta}{e^{\delta[\tau(i+1)-\tau(i)]}-1}$ and $\tau(m+1) \equiv 1$.

While the previous lemma and definitions refer to all feasible strategies, discussion now turns to properties of efficient strategies.

Definition:

A strategy $\hat{\alpha}\varepsilon A[c]$ is efficient if there does not exist a strategy $\beta\varepsilon A[c]$ which implies greater end-of-period wealth, i.e., $W_{\hat{\alpha}} \geqq W_\beta$ $(\beta\varepsilon A[c])$.

It is apparent that for an efficient strategy $\hat{\alpha}$, $\tau(m) < 1$ and $\sigma(n) < 1$; no transactions are made at $t = 1$. If a transaction were made at $t = 1$, a transaction cost would be incurred with no offsetting increase in asset return. For efficient strategies we can define $\tau(m+1) = 1$ and $\sigma(n+1) = 1$.

Since commodities earn a negative rate of return, only the minimal amounts necessary to support consumption are ever held. Thus, commodities are never sold.

Lemma 2:

For arbitrary γ and $c \geqq 0$, $\hat{\alpha}\varepsilon A[c] \Rightarrow H(t) = 0 \quad (t\varepsilon[0,1])$.

Definition:

$$A_1[c] = \{\alpha\varepsilon A[c] : n \leqq 1\}$$

$$A_2[c] = \{\alpha\varepsilon A[c] : n \geqq 2\}$$

Restricted to a feasible strategy subset $A_1[c]$ or $A_2[c]$, market transactions required for efficient consumption (e.g., those which minimize amount of money which must be set aside at t=0 to finance c--call this M_0) are independent of market transactions required for maximum investment return on $M(0^-)-M_0$. The investment decision is simply a choice between holding money or holding bonds over the entire period. The independence between consumption and investment decisions in a given strategy subset permits derivation of relationships between consumption rates, terminal wealth, and parameter values.

If $c = 0$, end-of-period wealth is maximized by investing the entire initial money holdings at the beginning of the period in the asset with the highest return. Thus, an efficient strategy in A[0] is to:

purchase bonds at $t = 0$ if $(M(0^-)-b)e^r > M(0^-)$

hold money over the period if $(M(0^-)-b)e^r \leqq M(0^-)$.

Hence, if $c = 0$, maximal end-of-period real wealth $\hat{W}$ is simply: $\hat{W} = \max\{(M(0^-)-b)e^{r-\pi}, M(0^-)e^{-\pi}\}$, and $\langle c, W\rangle = \langle 0, \hat{W}\rangle$ is one point on the consumption-terminal wealth frontier. For the remainder of this section it is supposed that $c > 0$. The following three lemmas state properties of efficient strategies in $A[c]$ for arbitrary α and $c > 0$.

Lemma 3:

$G(t) > 0 \iff C(t) = 0$ $(t\varepsilon[0,1))$; commodities are purchased if and only if the stock of commodities on hand is zero.

Lemma 4:

$$\tau(j+1)-\tau(j) = 1/\delta\cdot\ln\left(1 + \frac{G[\tau(j)]}{c/\delta}\right) \qquad j=1, \ldots, m ;$$

given a consumption rate and depreciation rate, there is a one-to-one correspondence between the amount of commodities purchased at any time and the length of time which elapses until the next purchase.

Lemma 5:

$\hat{\alpha}\varepsilon A_2[c] \Rightarrow$

a) $\sigma(1) = 0$; the first dealing in bonds is a purchase made at $t = 0$.

b) $P(t) = 0$ $(t\varepsilon(0,1])$; no bonds are purchased after $t = 0$.

c) $\sum \subseteq T$; all bond market transactions occur at times of commodity purchases (note $\sum = T$ is not being excluded).

Corollary 1:

$$W = \frac{P(0)e^{r} - \sum_{j=2}^{n} S[\sigma(j)]e^{r[1-\sigma(j)]}}{p(1)} ;$$ real end-of-period wealth is the price deflated value of bonds held at $t = 1$.

Corollary 2:

$P(0) = M(0^-)-G(0)-a-b-M(0^+)$; the value of bonds purchased at the beginning of the period is by T.5 equal to the initial money endowment, less the value of commodities purchased, less the transaction costs incurred, and less any money still held.

Corollary 3:

$S[\sigma(i)] = M[\sigma(i)^+] + p[\sigma(i)]\ \{G[\sigma(i)] + a + b\} \qquad i=2, .., n;$

bonds are sold only when holdings of money are depleted to zero and a commodity purchase is immediately forthcoming.

Corollary 4:

$$M[\sigma(i)^+] = \textstyle\sum_{E_i} p(t)\cdot[G(t) + a] \qquad\qquad i=1, \ldots, n,$$

where $E_i = \{t\varepsilon(\sigma(i),\sigma(i+1)):\ \chi_C(t) = 1\}$;

money on hand following a bond dealing is precisely enough to cover planned commodity purchases and commodity market transaction costs until the next bond sale.

From the definitions and results developed thus far, it follows that given a consumption rate c and parameter vector γ, an efficient strategy can be identified by times of commodity market transactions $\{\tau(1), \ldots, \tau(m)\}$ and the subset of commodity market transactions which are accompanied by bond market dealings $\{m_1, \ldots, m_n\}$ (defined to be the null set if n=0, where $\tau(m_i) = \sigma(i)$ and $m_i = \tau^{-1}[\sigma(i)]$ $i=1, \ldots, n$). For strategies with n=0, it follows from T.5 and Lemmas 2, 3, and 4 that

$$(1) \qquad W = [M(0^-)-a\sum_{i=1}^{m} e^{\pi\tau(i)}]\cdot e^{-\pi}- [\sum_{i=1}^{m} e^{\pi\tau(i)}\cdot(e^{\delta[\tau(i+1)-\tau(i)]}-1)]\cdot e^{-\pi}\cdot\frac{c}{\delta}$$

$$\equiv f_m^N(\tau(1), \ldots, \tau(m); c; \gamma).$$

For strategies with $n \geq 1$, it follows from T.5, Lemmas 2, 3, and 4 and Corollaries 1-4 of Lemma 5 that

$$\text{(2)} \qquad W = \{M(0^-)-\sum_{i=1}^{n}[a(\sum_{j=m_i}^{m_{i+1}-1} e^{\pi\tau(j)})-be^{\pi\tau(m_i)}]e^{-r\tau(m_i)}\}e^{-(\pi-r)}$$

$$-\frac{c}{\delta}\{\sum_{i=1}^{n}\{\sum_{j=m_i}^{m_{i+1}-1} e^{\pi\tau(j)}[e^{\delta[\tau(j+1)-\tau(j)]}-1\}e^{-r\tau(m_i)}\}e^{-(\pi-r)}$$

$$\equiv f^{B}_{m,n}(\tau(1), \ldots, \tau(m), m_1, \ldots, m_n; c; \gamma).$$

A straightforward approach to solve for an efficient strategy seemingly would be to apply maximization theory three times in succession (essentially Tobin's technique):

First, for fixed values of m and n maximize f^N_m and $f^B_{m,n}$ with respect to the control variables $\tau(1), \ldots, \tau(m)$ and $m_1, \ldots, m_n$. Express the maximizers $\hat{\tau}(1), \ldots, \hat{\tau}(m)$ and $\hat{m}_1, \ldots, \hat{m}_n$ in terms of c, γ, m and n. This would enable a single function to replace each countably infinite family of functions:

$$f^N(m; c; \gamma) \equiv f^N_m(\hat{\tau}(1), \ldots, \hat{\tau}(m); c; \gamma) \text{ and}$$

$$f^B(m, n; c; \gamma) \equiv f^B_{m,n}(\hat{\tau}(1), \ldots, \hat{\tau}(m), \hat{m}_1, \ldots, \hat{m}_n; c; \gamma).$$

Second, maximize each of the two resulting functions with respect to m and n. Let W_N and W_B be the values of the functions at their maxima:

$$W_N = f^N(\hat{m}; c; \gamma)$$

$$W_B = f^B(\hat{m}, \hat{n}\ c; \gamma).$$

Third, determine the maximum W for given c and γ by $W = \max \{W_N, W_B\}$. An efficient strategy for given c and γ can then be determined from the maximizing values of m, n, $\tau(1), \ldots, \tau(m), m_1, \ldots, m_n$.

If equal spacing of trips to each market were an implication from the first stage of maximization, the proposed solution routine would be greatly simplified. For arbitrary c and γ and for strageties with n=0, equal space requires that for any given m the maximizers of f_m^N are given by:

$$\hat{\tau}(i) = \frac{i-1}{m}, \qquad i=1, \ldots, m.$$

For arbitrary c and γ and for strategies with $n \geq 1$, equal spacing requires that two conditions be satisfied:

(a) Given m and n such that $n|m$ (m is an integer multiple of n), the maximizers of $f_{m,n}^B$ are given by:

i. $\hat{\tau}(i) = \frac{i-1}{m}$ $\qquad i=1, \ldots, m$

ii. $\hat{m}_j = 1 + (j-1)\cdot\frac{m}{n}$ $\qquad j=1, \ldots, n$

(i. and ii. $\Rightarrow \sigma(j) = \frac{j-1}{n}$ $\qquad j=1, \ldots, n$)

(b) Given m and n such that $n \nmid m$, the maximum of $f_{m,n}^B$ is no greater than that provided by a strategy having properties described in (a).

The following result is a sufficient condition for equal spacing to be a property of efficient strategies given the assumptions of this model.[13/]

[13/] This result should not be confused with Tobin's. Tobin's result states that bond market trips should be equally spaced in time to finance a steady expenditure stream. This result states that commodity market trips should be equally spaced in time to minimize the cost of financing a level flow of consumption. Tobin assumes a simple interest rate and transaction costs which are payable at the end of the period. This result assumes a depreciation rate with continuous compounding and transaction costs which are payable when incurred.

<u>Theorem 1</u>. Let c > 0 be arbitrarily given and let γ be given such that $\pi=0$. For arbitrary m the maximizers of $f^N_m(\tau(1), \ldots, \tau(m); c; \gamma)$ and $f^B_{m,1}(\tau(1), \ldots, \tau(m), m_1; c; \gamma)$ are given by:

$$\hat{\tau}(i) = \frac{i-1}{m} \qquad i=1, \ldots, m$$

($\hat{m}_1=1$ if n=1 by Lemma 5).

It is straightforward to show by means of a counterexample that Theorem 1 need not hold when $\pi \neq 0$. It seems likely, however, that equal spacing is "close" to efficient for strategies with $n \leqq 1$ as long as π is "close" to zero.

Neither requirement of equal spacing for strategies with $n \geqq 2$ ((a) nor (b)) need be an efficiency property given any value of π. Again, by use of counterexamples it can be shown first that if n divides m ($n|m$) and n is greater than one, equal spacing <u>is not</u> efficient and second that strategies for which $n|m$ <u>do not</u> dominate strategies for which $n\nmid m$. The first finding is not very damaging, because when $n|m$ the deviation of efficient spacing from equal spacing can be expected to be minor. If it were assumed, as Feige and Parkin did, that interest, depreciation charges, and transaction costs are paid at the end of the period, that the rate of interest and depreciation involve no compounding, and that $\pi=0$, equal spacing for strategies with $n|m$ would be efficient.

The second finding is important, and it applies both to the present model and the Feige-Parkin model. There are values of parameters and consumption rates for which strategies with $n\nmid m$ imply greater end-of-period wealth than do strategies with $n|m$.[14/] Because Lemma 5 indicates that for any efficient strategy times of bond market transactions will coincide with times of commodity market transactions, an efficient strategy with $n\nmid m$ cannot have equal spacing in both markets. This finding implies that for some parameter values the equal spacing assumption $n|m$ is overly restrictive and that the integer maximizers to $\bar{f}^B(v, n; c; \gamma) \equiv f^B(vn, n; c; \gamma)$ can be very distant from the real number maximizers. These results are explained intuitively as follows:

Suppose the depreciation rate and commodity market transaction cost are relatively high and the interest rate and bond market transaction cost are relatively low. In such a case the timing of commodity market transactions is relatively more important in terms of efficiency than is the timing of bond market transactions.[15/] Suppose that a prime number of trips to the commodity market, say m=17, is best for managing commodity inventories. Restricted to equal spacing strategies, the only feasible values of n are then n=0, 1, or 17. With m=17 the constraint $n|m$ might imply the efficient equal spacing strategy for the assumed parameter values is $\langle m, n\rangle = \langle 17, 1\rangle$. However, assuming it is profitable to deal in bonds, a strategy having m=17, n equal to say 5, $\tau(i) = \frac{i-1}{17}$ i=1, .., 17, and $\sigma(1) = 0$, $\sigma(2) = \frac{3}{17}$, $\sigma(3) = \frac{7}{17}$, $\sigma(4) = \frac{10}{17}$,

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[14/] For $\langle M(0^-), \delta, r, \pi, a, b\rangle = \langle 10{,}000, .250, .004, .005, 20.0, 1.0\rangle$ and W = 2000, the efficient equal spacing strategy is m=7, n=7, and implies c=7708.27. However the nonequal spacing strategy with m=7, n=3, $\tau(i) = \frac{i-1}{7}$ i=1, ..., m, $\sigma(1) = 0$, $\sigma(2) = \frac{2}{7}$, $\sigma(3) = \frac{4}{7}$ implies c = 7709.00. In this example, c is maximized subject to W = 2000. The superiority of a nonequal spacing strategy would still hold if W were maximized subject to c = 7709.0.

[15/] That is, a change in the timing of commodity market trips has a larger impact on end-of-period wealth than does a similar change in the timing of bond market trips.

$\sigma(5) = \frac{14}{17}$, deviates only slightly from equal spacing and can be expected to do better than the efficient equal spacing strategy. Now, if $\overrightarrow{f}^B(v, n; c; \gamma)$ were maximized with respect to v and n for these parameter values, the real number calculus maximizers should turn out to be close to $\hat{v}=3.4$, $\hat{n}=5$ (implying $\hat{m}=17$). Supposing for concreteness that $\hat{n}$ turns out to be marginally greater than 5, it would seem that the integer maximizers should be one of the four corners on the unit box which encloses $\langle \hat{v}, \hat{n} \rangle$, i.e., $\langle v, n \rangle \in \{\langle 3, 5 \rangle, \langle 4, 5 \rangle, \langle 3, 6 \rangle, \langle 4, 6 \rangle\}$. However, none of these pairs imply m=17, which was by assumption optimal. The function $\overrightarrow{f}^B$ has a steep ridge along $v \cdot n=17$, and these pairs lie off of the ridge. Since the efficient equal spacing strategy $\langle 17, 1 \rangle$ is not even close to one of the four corners around $\langle \hat{v}, \hat{n} \rangle$, calculus maximization would in this case provide a very poor approximation to the integer solution.

Because equal spacing is not a necessary property of efficient strategies, the proposed solution routine proves unworkable. For arbitrary m and n, it is not possible to solve explicitly for $\hat{\tau}(1), \ldots, \hat{\tau}(m)$ and $\hat{m}_1, \ldots, \hat{m}_n$ in terms of m, n, c, and γ, and thus it is not possible to determine for $\langle m', n' \rangle \neq \langle m, n \rangle$ whether $f^B_{m,n}(\hat{\tau}(1), \ldots, \hat{\tau}(m), \hat{m}_1, \ldots, \hat{m}_n; c; \gamma) > f^B_{m',n'}(\hat{\tau}(1), \ldots, \hat{\tau}(m'), \hat{m}_1, \ldots, \hat{m}_{n'}; c; \gamma)$. In the next section explicit expressions for efficient strategies are not needed to prove that the individual's opportunity set lacks convexity. In the final section, however, the search for efficient strategies is restricted to the class of equal spacing strategies in order to derive properties of asset demand functions for specific sets of parameter values. The constraint $n|m$ proves not to be effective for many sets of parameter values, and, thus, the computed asset demand functions for these sets are consistent with individual optimizing behavior.

Properties of The Consumption - Terminal Wealth Frontier

By results of the previous section, terminal wealth can be written as a function of feasible strategies, consumption, and parameters: $W = g(\alpha, c, \gamma)$. Let $\hat{g}(c, \gamma) = \max_{\hat{\alpha}\varepsilon A[c]} g(\alpha, c, \gamma)$. The set of efficient strategies which generates $\hat{g}(c, \gamma)$ need not be unique. The consumption-terminal wealth frontier can be written as the set $\{\langle c, W\rangle \geq 0: \; W = \hat{g}(c, \gamma)\}$. The location of the frontier in the c-W space depends only on values of the parameter vector γ.

The first result of this section states for arbitrary parameter values $\hat{g}$ is strictly monotonically decreasing with respect to consumption. This implies an efficient strategy not only provides maximal W for given c, it must also provide maximal c for given W. Thus, the same frontier results whether W is maximized subject to a c constraint or c is maximized subject to a W constraint.

Lemma 8:

Let γ be arbitrarily given, let $c_1 > 0$, and let c_2 be chosen such that $c_1 > c_2 \geqq 0$. Suppose $A[c_1] \neq \emptyset$. Then,

$$\hat{g}(c_1, \gamma) < \hat{g}(c_2, \gamma).$$

The final result of this section states that the individual's opportunity set lacks convexity, a result implied by the assumption of fixed market transaction costs. It follows if positive amounts of both c and W are desired at the utility maximum, the individual's preferences must be strongly convex. Lack of convexity of the individual's opportunity set would seem to have implications also for general equilibrium theory in an economy with transaction costs, but these implications are not explored here.[16]

Theorem 2:

Let γ be arbitrarily given, $0 < \lambda < 1$, $c_1 > 0$, and let c_2 be chosen such that $c_1 > c_2 \geqq 0$. Suppose $A[c_1] \neq \emptyset$. Then,

$$\hat{g}(\lambda c_1+(1-\lambda)c_2,\gamma) \leqq \lambda\hat{g}(c_1,\gamma) + (1-\lambda)\hat{g}(c_2,\gamma), \text{ equality} \iff$$

there exist efficient strategies $\alpha\varepsilon A[c_1]$ and $\beta\varepsilon A[c_2]$ for which times of transactions are identical.

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[16] Implications of transaction costs in general equilibrium models can be found in Hahn [7], Heller [9], and Heller and Starr [10].

Properties of Asset Demand Functions

Because the location of the consumption-terminal wealth frontier in the c-W space depends only on γ, the utility maximizing c-W pair will, for given U, also depend only on γ. Average holdings per period of money, bonds, and commodities correspond to the utility maximizing point on the frontier and can be expressed as functions of γ for given U:

$$\overline{M} = F^1(M(0^-), \ \delta, \ r, \ \pi, \ a, \ b),$$
$$\overline{B} = F^2(M(0^-), \ \delta, \ r, \ \pi, \ a, \ b), \text{ and}$$
$$\overline{C} = F^3(M(0^-), \ \delta, \ r, \ \pi, \ a, \ b).$$

The asset demand functions F^1, F^2, and F^3 are well defined only over values of γ for which the utility maximizing c-W pair is generated by precisely one strategy.

Partial asset demand functions are constructed in this section which describe how desired asset holdings change due to a change in the value of a single parameter for given values of the other parameters. To generate points on a partial asset demand function an initial, or "basic," parameter vector γ^o is specified. Next, a point $\langle c_0, W_0 \rangle$ on the frontier restricted to equal spacing strategies is designated as the utility maximizing consumption terminal wealth pair. Finally, a single parameter is varied, and points on the shifted consumption-terminal wealth frontiers are chosen by assuming that the individual either maximizes c for $W=W_0$ or maximizes W for $c=c_0$. Average asset holdings per period are computed at each point.

Given an arbitrary utility function, an "equilibrium path for γ_i" can be generated in the c-W space by varying a parameter $\gamma_i (i \varepsilon \{1, \ldots, 6\})$

over a continuum of values. For given values of the other parameters $\gamma_{(i)}^{o} = (\gamma_1^{o}, \ldots, \gamma_{i-1}^{o}, \gamma_{i+1}^{o}, \ldots, \gamma_6^{o})$, this path defines the locus of utility maximizing pairs $\langle c^*, W^* \rangle$ traced out by varying γ_i: $W^* = \omega_i(c^*, \gamma_{(i)}^{o})$ $i=1, \ldots, 6$. (See figures 1 and 2, p. 28a.) Equilibrium paths can be considered short-hand representations for individual preferences. Given initial parameter values, the intersection of an equilibrium path with the frontier is the individual's utility maximizing consumption-terminal wealth pair.

An equilibrium path constructed from a general utility function and going through the point $\langle c_0, W_0 \rangle$ can be approximated by the linear function: $(1-\theta)(c^*-c_0) - \theta(W^*-W_0) = 0$. It seems reasonable to assume $0 \leqq \theta \leqq 1$; that is, a change in parameter values which permits the individual to have both more consumption and more end-of-period wealth will not cause him to choose less of either at the new equilibrium. Thus, assuming the individual maximizes c for $W=W_0$ (the objective function assumed by Johnson [12] and Feige-Parkin) is equivalent to assuming the individual moves along the equilibrium path with $\theta=1$. Assuming the individual maximizes W for $c=c_0$ (the objective function assumed by Baumol and Tobin) is equivalent to assuming the individual moves along the equilibrium path with $\theta=0$. In general, θ represents the individual's relative preference for consumption over terminal wealth; if he receives an extra dollar at the beginning of the period, he will desire approximately $\theta \cdot (\$1)$ extra consumption and $(1-\theta) \cdot (\$1)$ extra terminal wealth.

Partial asset demands are constructed and plotted in this section only for the two extreme equilibrium paths $\theta=0$ and $\theta=1$. It is claimed that, in general, partial asset demand functions are monotonic in θ; that for any asset and given $0 \leqq \theta_1 < \theta_2 < \theta_3 \leqq 1$, the partial

Figure 1

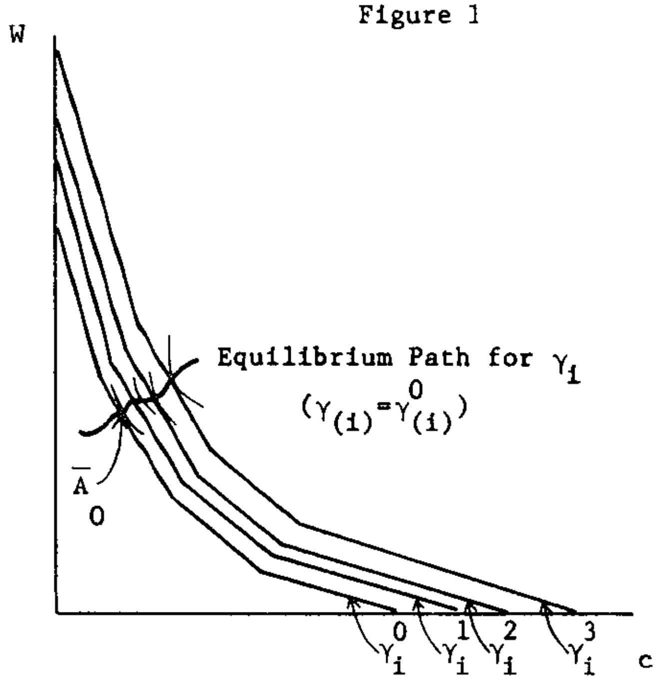
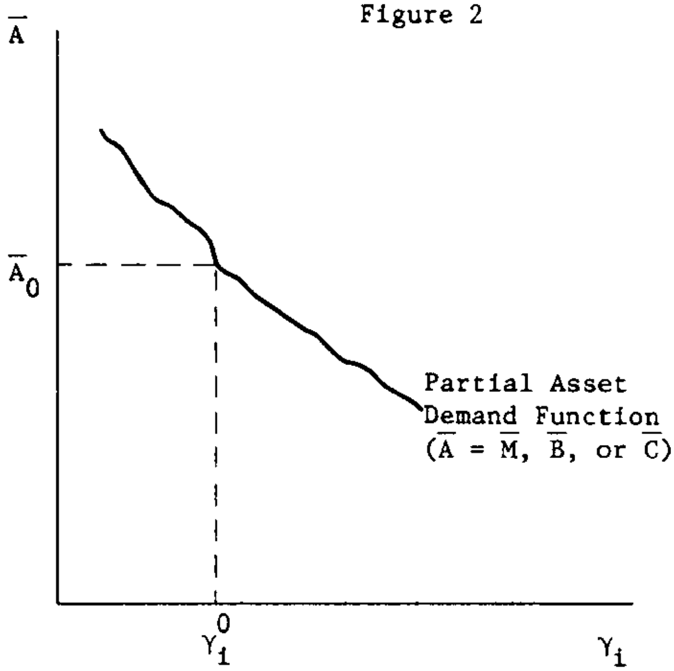


Figure 2



asset demand function for the θ_2 equilibrium path will fall between the demand functions for the θ_1 and θ_3 paths. Thus, partial asset demand functions for general utility functions will in general fall in between the demand functions for the θ=0 and θ=1 equilibrium paths. In addition, the change in partial asset demand functions as θ increases from 0 to 1 gives the direction of change in desired asset holdings as the individual's relative preference for consumption over end-of-period wealth increases.

The assumption of equal spacing makes it possible to derive algebraic expressions for end-of-period wealth and average asset holdings per period as functions of m, n, c and γ. For each set of parameter values, an exhaustive sampling technique is employed to solve for the values of m and n which maximize W for c = c_0 (when θ=0), or which maximize c for W = W_0 (when θ=1).[17/]

For each basic parameter vector γ^o and for each initial utility maximization point on the frontier for γ^o, one set of partial asset demand functions is generated for θ=0 and another for θ=1. This process is repeated for two basic parameter vectors and two initial utility maximization points on a frontier for a basic parameter vector. Values of the two parameter vectors are specified in the table below.

Basic Parameter Set 1	Basic Parameter Set 2
$M(0^-) = 1,000$	$M(0^-) = 10,000$
$\delta = .250$	$\delta = .250$
$r = .004$	$r = .004$
$\pi = .005$	$\pi = .005$
$a = 2.0$	$a = 2.0$
$b = 1.0$	$b = 1.0$

Figure 3

17/ A detailed description of the sampling technique can be found in Miller [15].

The values of parameters in the first set were chosen to resemble real-world data. The time period is implicitly assumed to be a month. The initial money endowment of \$1,000 is interpreted as a monthly paycheck of the like amount. The nominal rate of interest and rate of inflation translate to annual rates of roughly 5 percent and 6 percent, respectively. The depreciation rate of 25 percent per month may be of the right magnitude if commodities are construed to be groceries. Values of transaction costs were specified somewhat arbitrarily, but together with the other specifications they produce reasonable results. One implication of this parameter set, for instance, is that with nominal consumption expenditures of roughly \$150 to \$450 per month the optimal spacing of commodity market trips is about once a week. Another implication is that between \$200 and \$250 has to be deposited in a savings account (a bond purchase) before a trip to the bank (bond market) becomes profitable. It also follows that it is never profitable to deposit money in a savings account (purchase bonds) with the intention of withdrawing money (selling bonds) in the same month.

Given basic parameter set 1, most points which were sampled on equilibrium paths for γ_j (j=1, ..., 6) imply $\hat{n} \leqq 1$. Thus, partial asset demand functions corresponding to this parameter set are close to optimal: they are numerically close to functions which would have been derived had equal spacing not been assumed.

When $\hat{n} \leqq 1$, the integer constraint $\frac{\hat{m}}{\hat{n}} \epsilon \mathbb{N}$ is not operative. In order to explore implications of the model when there are both purchases and sales of bonds, the first parameter set was modified by increasing the money endowment to \$10,000 while leaving the other parameter values unchanged, and this modified set constitutes basic parameter set 2.

Since the three rates seem to be of the right magnitude, a change in the value of the scale variable $M(0^-)$ seemed appropriate for the stated purpose. While \$10,000 is too high to be considered a monthly paycheck, the model can be reinterpreted to be one of a firm, so that the \$10,000 could represent receipts from payments on account, for instance.

Different points on a frontier correspond to different saving rates. Defining the saving rate SR* by $SR^* = \frac{W}{M(0^-)}$, the two initial utility maximization points on a frontier for each basic parameter set correspond to saving rates of .10 and .30. This choice of saving rates is especially important for partial asset demand functions constructed from basic parameter set 1, because at the initial utility maximization point no dealings in bonds take place when $SR^* = .10$, but a bond purchase is made when $SR^* = .30$.

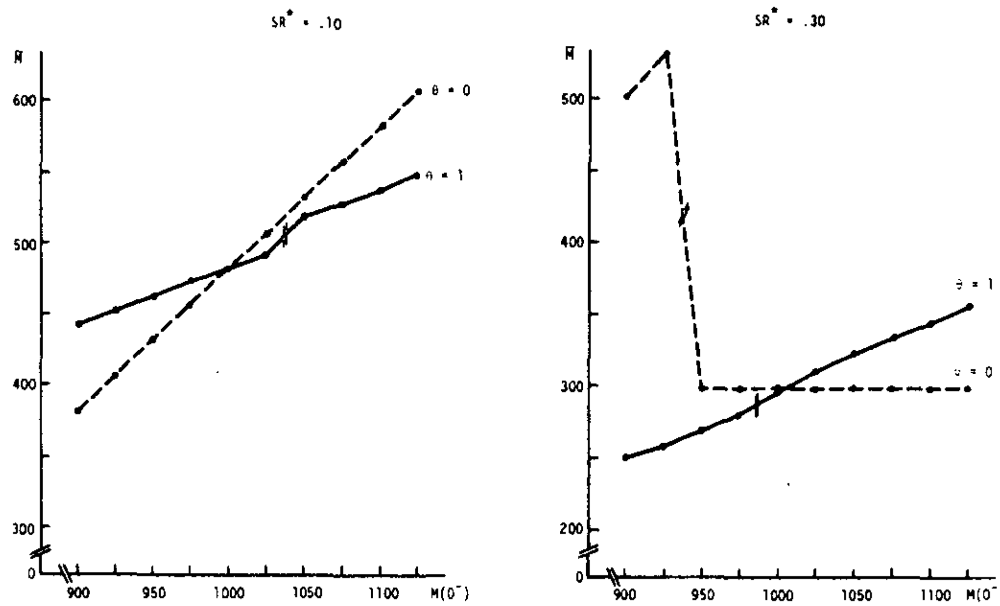
IV. Results of Investigation

Graphs of partial asset demand functions were plotted for each of the three assets with respect to each of the six parameters. Two of the more interesting sets of graphs are displayed on pages 32-33. Each page contains four sets of graphs, each set having a graph of a partial asset demand function for $\theta=0$ and one for $\theta=1$. Two sets of graphs in a row are partial asset demand functions for the same basic parameter set but for different initial saving rates. Two sets of graphs in a column are for different basic parameter sets and the same initial saving rate. A hatch mark on a line segment indicates a change in $\hat{m}$ or $\hat{n}$ occurred from one computed point to the next, implying a discontinuous jump in average asset holdings (not graphed) occurred between the two points. Finally, it is noted that except on equilibrium paths for $M(0^-)$, $\hat{m}$ and $\hat{n}$

Figure 4

PARTIAL MONEY DEMAND FUNCTION WRT INITIAL MONEY HOLDINGS

BASIC PARAMETER SET 1



BASIC PARAMETER SET 2

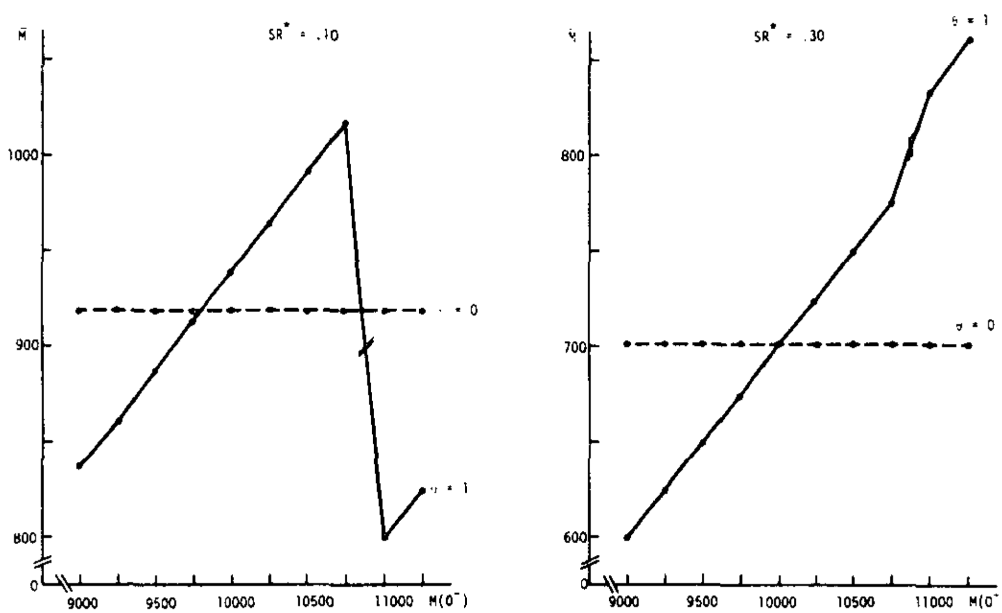
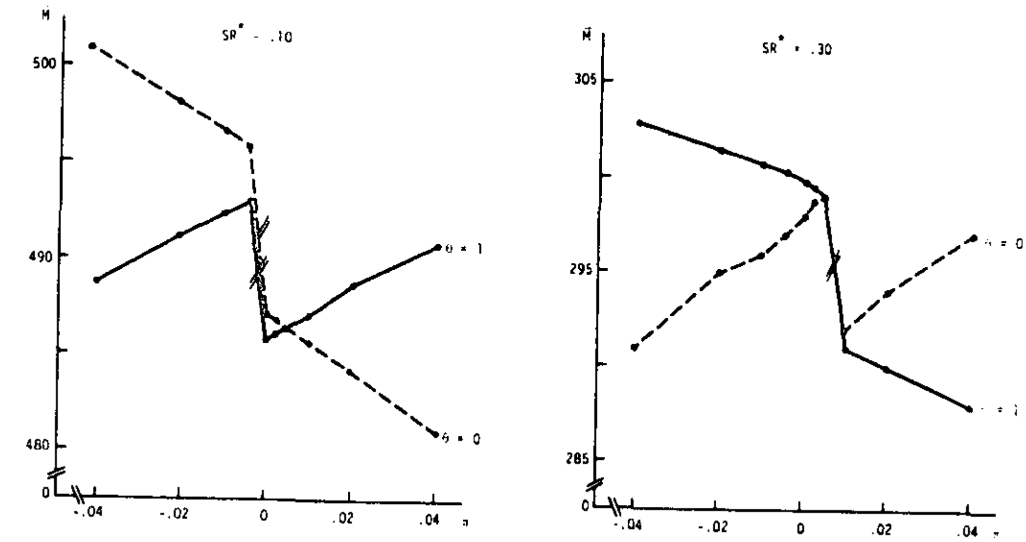


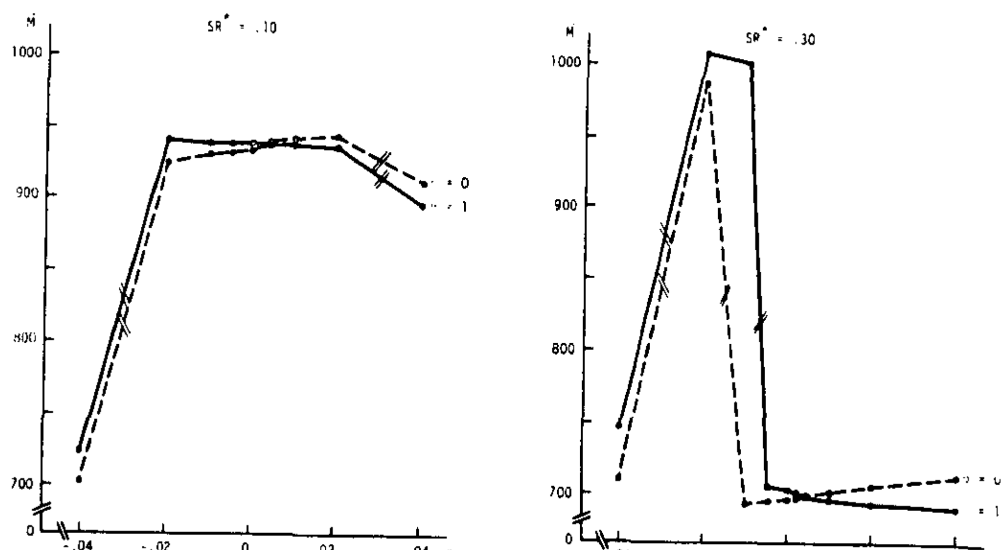
Figure 5

PARTIAL MONEY DEMAND FUNCTION WRT INFLATION RATE

BASIC PARAMETER SET 1



BASIC PARAMETER SET 2



are almost always independent of θ, which implies that partial asset demand functions for arbitrary equilibrium paths almost always lie between the functions for $\theta=0$ and $\theta=1$.

The importance of time preferences in determining the elasticities of asset demands is clearly exhibited in the set of graphs on the top row of Figure 4, p. 32. Given basic parameter set 1, the individual's savings rate determines whether or not it is profitable to deal in bonds. If the individual has a low savings rate and does not deal in bonds, and his initial money holdings increase, average money holdings over the period increase by a larger amount the <u>greater</u> are his relative preferences for end-of-period wealth over current consumption. This is because an increase in initial money holdings allocated to end-of-period wealth raises average money holdings one-for-one, while an increase in initial money holdings allocated to consumption raises average money holdings less than one-for-one (in fact, in the limit when commodity purchases are made in a flow, another dollar allocated to consumption raises average money holdings by only fifty cents). However, if the individual has a high savings rate and does deal in bonds, and his initial money holdings increase, average money holdings over the period increase more the <u>less</u> are his relative preferences for end-of-period wealth over current consumption. This follows because an increase in initial money holdings allocated to end-of-period wealth goes into bonds and does not raise average money holdings at all.

The sets of graphs in Figure 5 on p. 33 are perhaps most interesting because they are so counter-intuitive. Slopes of partial money demand functions with respect to the inflation rate in the top row of Figure 5 are of opposite signs for different values of θ and the same savings rate and of opposite signs for same values of θ and different savings rates. If the individual initially is not dealing in bonds and the inflation rate rises, average money holdings given the new inflation rate will be higher the greater the attempt to maintain real wealth at its previous value (closer θ is to 1). In order to maintain the same end-of-period real wealth with a rise in the inflation rate requires more initial wealth to be allocated to W and less to c, and as the previous paragraph explains that increases average money holdings. Also, for reasons given in the previous paragraph, the results reverse when the individual initially deals in bonds.

Signs of partial derivatives of asset demand functions correspond to the slopes of partial asset demand functions. Although the partial asset demand functions which were plotted are not always monotonic, many of them tend to be with only few and minor exceptions. They also tend to have the same sign of slope for $\theta=0$ or $\theta=1$. The tendencies of the signs of slopes of the plotted functions are summarized in Figures 6 and 7 on p. 36. The partial derivatives are for nominal asset demands, but the tables would not differ if they were for real asset demands (except the partial of real commodity holdings with respect to π would become indeterminate).

Signs of Partial Derivatives Taken From Tendencies on Graphs

Figure 6

From Graphs for Basic Parameter Set 1

Nominal Asset Holdings	Parameter					
	$M(0^-)$	δ	r	π	a	b
$\overline{M}$	+	+	-	?	-	+
$\overline{B}$	+	-	+	?	-	-
$\overline{C}$	+	-	?	+	+	?

Figure 7

From Graphs for Both Basic Parameter Sets

Nominal Asset Holdings	Parameter					
	$M(0^-)$	δ	r	π	a	b
$\overline{M}$	+	?	-	?	-	+
$\overline{B}$	+	?	+	?	?	-
$\overline{C}$	+	-	?	+	+	?

Key

+ => ≥ 0

- => ≤ 0

? => no apparent tendency

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