Federal Reserve Bank of Minneapolis
Research Department

Predicting the Effects of Federal Reserve Policy in a Sticky-Price Model: An Analytical Approach*

Ellen R. McGrattan

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ABSTRACT

In this paper, I characterize equilibria for a sticky-price model in which Federal Reserve policy is an interest-rate rule similar to that described in Taylor (1993). For standard preferences and technologies used in the literature, the model predicts that the nominal interest rate is negatively serially correlated, and that shocks to interest rates imply a potentially large but short-lived response in output. Shocks to government spending and technology lead to persistent changes in output but the percentage change in output is predicted to be smaller than the percentage changes in spending or technology. I compare the model's predictions to data using innovations backed out from estimated processes for interest rates, government spending, and technology shocks. These comparisons confirm the theoretical findings. In response to observed changes in government spending and technology, the model predicts a path for output that is much smoother than the data and much smoother than that predicted by non-sticky price models.

*McGrattan, Federal Reserve Bank of Minneapolis. The views expressed herein are those of the author and not necessarily those of the Federal Reserve Bank of Minneapolis or the Federal Reserve System.

1. Introduction

What are the effects of Federal Reserve policy? For many years, macroeconomists both inside and outside of the Federal Reserve have grappled with this question but have not yet reached a consensus. The main difficulty has been to develop models that can generate the salient features of aggregate time series, which is a first step to reliable policy analysis.

One class of models that has dominated much of the recent work in this area assumes that monetary policy can have an important effect on real economic activity because prices cannot be costlessly adjusted in response to monetary shocks.[1] Recent versions of these "sticky price" models also assume that monetary policy is central to the study of aggregate fluctuations because the policy rule followed by the Federal Reserve plays an important role in amplifying and propagating shocks to the economy. Following Taylor (1993), monetary policy is typically modeled as a rule for setting interest rates which has the Federal Funds rate respond positively to changes in inflation and output. Taylor (1993) showed that a rule of this form fits the data well. And others have argued that models with sticky prices and a Taylor-like rule for the Federal Funds rate can generate the salient features of output, inflation, and interest rates.[2]

In this paper, I characterize equilibria for a sticky-price model in which Federal Reserve policy is an interest-rate rule similar to that described in Taylor (1993). In particular, I characterize the responses of output, inflation, and nominal interest rates to innovations in interest rates, government spending, and technology. I first derive qualitative predic-

[1] See, for example, Ohanian and Stockman (1994), Kimball (1995), King and Watson (1996), Rotemberg (1996), Woodford (1996), Yun (1996), Chari et al. (1999). Earlier work that had an important impact on the literature includes Taylor (1980), Blanchard and Kiyotaki (1987), Ball and Romer (1989, 1990), and Blanchard (1991).

[2] See, for example, Brayton and Tinsley (1996), Rotemberg and Woodford (1997,1998) and Clarida et al. (1998).

tions of the model for various special cases that have been considered in the literature. I then explore the quantitative predictions of the model using innovations in interest rates, government spending, and technology estimated from U.S. data. In response to observed changes in the nominal interest rate the model predicts a potentially volatile path for output with little persistence in its growth rate. In response to observed changes in government spending and technology, the model predicts a path for output that is persistent but much smoother than the data. Thus, from these exercises, I reach a different conclusion about the reliability of sticky price models for policy analysis than that of earlier work.

The model that I work with has a continuum of monopolistically competitive firms that produce differentiated products using capital and labor. These firms set nominal prices for a fixed number of periods and do so in a staggered fashion. The rigidity in prices implies that monetary policy through changes in nominal interest rates can have a real effect. For example, a fall in the nominal interest rate with prices sticky leads to a fall in the real interest rate, a rise in demand for goods, and a rise in output. Monetary policy also affects the equilibrium responses of other shocks through its feedback rule on inflation and output. In fact, some have argued that this is the main role played by the Fed because empirical studies attribute only a small fraction of the variance of output to unanticipated changes in money or nominal interest rates. (See, for example, Christiano, Eichenbaum, and Evans (1998).)

A general characterization of equilibria is obtained for cases in which the utility function for a typical consumer is separable in consumption, leisure, and real money balances. Separable utility is commonly assumed in the literature. In these cases, the theoretical findings run counter to conventional wisdom. First, I find that for standard preferences and technologies used in the literature, the nominal interest rate is negatively serially correlated – not positively serially correlated as we see in U.S. data. Thus, after a fall in interest rates, output initially rises but is below trend when all monopolists have had a chance to reset prices. Second, I find that unanticipated changes in interest rates typically have a large impact on output – not small as many empirical studies conclude. Third, I find that while the interest rate rule does play a role for business cycles, the results are much more sensitive to choices of preferences and technologies. Fourth, I find that shocks to government spending and technology lead to persistent but small changes in output – too small to account for observed business cycles. To illustrate these results more directly, I compare the model's predictions to data using innovations backed out from estimated processes for interest rates, government spending, and technology shocks. The numerical results are dramatic and confirm the theoretical findings.

The results stand in sharp contrast with those of Rotemberg and Woodford (1997) who show that a sticky price model with a Taylor-like interest rate rule generates time series that are remarkably close to the U.S. data. The main difference in the analysis is the choice of shocks. Rotemberg and Woodford (1997) allow for taste shocks and fiscal shocks that are unobserved. These shocks are treated as residuals; they are set so that the Euler equations of the model will be satisfied. Here, I do not allow for unobserved shocks, and I match up innovations of the model with their counterparts in the data.

This paper builds on the work of Chari et al. (1999) who analyze a sticky price model in which monetary policy is a process for the growth rate of the money supply. Chari et al. (1999) show that it is difficult to generate persistent patterns of output in response to changes in monetary growth rates. The problem is especially acute when their model includes capital and interest-sensitive money demand. The results here show that the persistence problem can be solved if we consider non-monetary shocks. However, there seems to be a trade-off between the amplitude and persistence of output that is difficult to overcome.

2. The Model Economy

The model economy has a continuum of infinitely-lived consumers, producers of final and intermediate goods, and a government. There are three sources of uncertainty. There may be unanticipated shocks to interest rates, to government spending, and to technology. I use $\epsilon_{r,t}$ to denote the shock to the nominal interest rate in period t, $\epsilon_{g,t}$ to denote the shock to government spending, and $\epsilon_{z,t}$ to denote the shock to technology. The history of shocks is denoted by $\epsilon^t = \{\epsilon_0, \ldots, \epsilon_t\}'$ where ϵ_t is the vector $[\epsilon_{r,t}, \epsilon_{g,t}, \epsilon_{z,t}]'$. These shocks are the only sources of cyclical movements in the model economy.

Consumers have preferences defined over consumption, labor, and real money balances. These are given by

$$E\left[\sum_{t=0}^{\infty} \beta^t\, U\left(C_t, L_t, M_t/\bar{P}_t\right)\, | M_{-1}, B_0\right] \tag{2.1}$$

where C_t, L_t, M_t, and $\bar{P}_t$ are consumption, labor, nominal money balances, and the aggregate price level, respectively, t is an index for time, $0 < \beta < 1$ is the discount factor, and expectations are conditioned on initial nominal money balances M_{-1} and initial nominal bond holdings B_0. In each period $t = 0, 1, \ldots$, consumers choose their period t allocations after the realization of the event $(\epsilon_{r,t}, \epsilon_{g,t}, \epsilon_{z,t})$. The problem of consumers is to choose rules for consumption C_t, labor L_t, nominal money balances M_t, and one-period nominal bonds B_{t+1} based on the history of these shocks up to t subject to the sequence of budget constraints:

$$\bar{P}_t C_t + M_t + \sum_{\epsilon_{t+1}} Q(\epsilon^{t+1}|\epsilon^t) B(\epsilon^{t+1}) \\ \leq \bar{P}_t W_t L_t + M_{t-1} + B(\epsilon^t) + \Pi_t + T_t \qquad t = 0, 1, \ldots$$

and borrowing constraints $B(\epsilon^{t+1}) \geq \bar{B}$ for some large negative number $\bar{B}$. The term Π_t is the nominal profits of the intermediate goods producers, and the term T_t is nominal transfers. Each of the nominal bonds $B(\epsilon^{t+1})$ is a claim to one dollar in state ϵ^{t+1} and costs $Q(\epsilon^{t+1}|\epsilon^t)$ dollars in state ϵ^t.

Final goods to the consumer are produced by firms who behave competitively. Each period, these firms choose inputs $Y(i)$, $i \in [0, 1]$, and output Y to maximize profits, that is

$$\max_{\{Y(i)\}_{i \in [0,1]}} \bar{P} - \int P(i) Y(i)/Y \, di \tag{2.2}$$

subject to the technology for producing final goods from intermediate goods given by

$$\int g\left(\frac{Y(i)}{Y}\right) di = 1 \tag{2.3}$$

where $\bar{P}$ is the price of the final good and $P(i)$ is the price of intermediate good i. From this problem, I can derive a demand function for each intermediate good. This demand function takes the form

$$Y(i) = D\left(\frac{P(i)}{\bar{P}} \int g'\left(\frac{Y(j)}{Y}\right) \frac{Y(j)}{Y} dj\right) Y \tag{2.4}$$

where $D \equiv (g')^{-1}$. From the zero-profit condition, I can also derive an expression for aggregate prices in terms of the intermediate goods prices.

Intermediate goods producers behave as imperfect competitors. They set prices for N periods and do so in a staggered fashion. In particular, in each period t, a fraction $1/N$ of these producers choose new prices $P_t(i)$ before the realization of the policy shocks and the technology shock $(\epsilon_{r,t}, \epsilon_{g,t}, \epsilon_{z,t})$. These prices are set for N periods, so for this group of intermediate goods producers, $P_{t+s}(i) = P_t(i)$ for $s = 0, \ldots, N-1$. The intermediate goods producers are indexed so that producers indexed $i \in [0, 1/N]$ set new prices in 0, N, $2N$, and so on, while producers indexed $i \in [1/N, 2/N]$ set new prices in 1, $N+1$, $2N+1$, and so on, for the N cohorts of intermediate goods producers. The problem solved by the monopolist adjusting his price is to choose prices $P_s(i)$, capital stocks $K_s(i)$, investments $X_s(i)$, and labor inputs $L_s(i)$ $s \geq t$ to maximize

$$\sum_{s=t}^{\infty} \sum_{\epsilon^s} Q(\epsilon^s | \epsilon^{t-1}) \left[P_s(i) Y_s(i) - \bar{P}_s W_s L_s(i) - \bar{P}_s X_s(i) \right] \tag{2.5}$$

subject to $P_s(i) = P_t(i)$ for $s = t, \ldots t+N-1$, $P_s(i) = P_{t+N}(i)$ for $s = t+N, \ldots t+2N-1$, and so on, demand for good i in (2.4), the production technology:

$$Y_t(i) = F(K_{t-1}(i), Z_t L_t(i)) \tag{2.6}$$

with labor-augmenting technical change governed by

$$\log Z_t = \rho_z \log Z_{t-1} + \epsilon_{z,t},$$

and the law of motion for capital used in producing i given by:

$$K_t(i) = (1-\delta) K_{t-1}(i) + X_t(i) - \phi\left(\frac{X_t(i)}{K_{t-1}(i)}\right) K_{t-1}(i). \tag{2.7}$$

The last term in (2.7) captures costs associated with adjusting the capital stock.

I now describe the actions of the government. The government's monetary policy is a rule for setting interest rates. Let R_t be the gross nominal interest rate which satisfies

$$
\begin{aligned}
1/R_t &= \sum_{\epsilon_{t+1}} Q(\epsilon^{t+1}|\epsilon^t) \\
&= \beta E_t \frac{U_c(C_{t+1}, L_{t+1}, M_{t+1}/\bar{P}_{t+1})}{U_c(C_t, L_t, M_t/\bar{P}_t)} \frac{\bar{P}_t}{\bar{P}_{t+1}}.
\end{aligned}
$$

Let $r_t = R_t - 1$. The interest rate rule is then given by:[3]

$$
r_t = \rho_r r_{t-1} + (1-\rho_r)\left[a(\log \bar{P}_{t+1} - \log \bar{P}_t) + b(\log Y_t - \log Y) + r\right] + \epsilon_{r,t}, \qquad (2.8)
$$

where ρ_r, a, and b are assumed to be positive constants, and variables without a time index are assumed to be steady state levels. Taylor argued that such a relationship between nominal interest rates, inflation, and output roughly fits the U.S. time series after 1987. Federal Reserve documents also specify rules of this form. (See, for example, Brayton and Tinsley (1996), Brayton et al. (1997), and Reifschneider et al. (1999).) To get Taylor's (1993) original rule, set a equal to 3/2 and b equal to 1/2. In Figure 1, I plot the Federal funds rate and what Taylor's policy rule implies. The series show similar patterns – both rise dramatically near the end of the 1980s and fall dramatically at the beginning of the 1990s. Clarida et al. (1998) allow for lagged nominal interest rates in estimating the interest rate rule and improve the fit. In Figure 2, I plot their estimated policy rule with $\rho_r = 0.66$, $a = 1.8$ and $b = 0.12$ over the sample 1979:3-1996:4. A comparison between

[3] I misuse the term "rule" here. I am assuming that the FOMC sets interest rates as a function of all available information at time t. Therefore, they need to compute the equilibrium. In a separate appendix, I characterize equilibria for an alternative specification, namely

$$
r_t = \rho_r r_{t-1} + (1-\rho_r)\left[a(\log \bar{P}_t - \log \bar{P}_{t-1}) + b(\log Y_{t-1} - \log Y) + r\right] + \epsilon_{r,t},
$$

which assumes that the Federal Reserve uses information on past inflation and past output. It turns out that the results are very similar in the two cases.

Figures 1 and 2 for Taylor's sample shows that there is indeed better agreement with the data when lagged nominal interest rates are included.

The government's fiscal policy is assumed to be an exogenous process for government purchases which evolves according to

$$\log G_t = (1 - \rho_g) \log G + \rho_g \log G_{t-1} + \epsilon_{g,t}$$

where $\epsilon_{g,t}$ is a serially uncorrelated and normally distributed error which is intended to capture unanticipated changes in government purchases. Any revenues not used for spending are assumed to be transferred back to consumers. The budget constraint for the government is, therefore, given by

$$\bar{P}_t G_t + T_t = M_t - M_{t-1}.$$

An equilibrium for this economy is a collection of allocations for consumers C_t, L_t, M_t, B_{t+1}; allocations for intermediate goods producers $K_t(i)$, $L_t(i)$ for $i \in [0,1]$; allocations for final goods producers Y_t and $Y_t(i)$ for $i \in [0,1]$; together with prices W_t, $\bar{P}_t$, $P_t(i)$, for $i \in [0,1]$, and $Q(\epsilon^s|\epsilon^t)$ for $s = t, \ldots, t+N-1$, that satisfy the following conditions: (i) taking prices as given, consumer allocations solve the consumer's problem; (ii) taking all prices but his own as given, each intermediate goods producer's price and factor choices solve the intermediate goods producer's problem; (iii) taking the prices as given, the final goods producer's allocations solve the final goods producer's problem; (iv) the labor market clears, i.e.,

$$L_t = \int L_t(i)\, di \tag{2.9}$$

(v) the resource constraint is satisfied, i.e.,

$$Y_t = C_t + \int X_t(i)\, di + G_t \tag{2.10}$$

and the bond market clears.

In Appendix A, first order conditions for the optimization problems described above are provided. In Appendix B, I report linearized versions of the first order conditions. The linearized conditions are analyzed in the next section.

3. Characterization of Equilibria

In this section, I characterize equilibria of the model economy for certain specifications of the model. In particular, I derive analytical results for cases with two cohorts of monopolists ($N = 2$), preferences that are separable in all arguments, a production function with Cobb-Douglas form ($F(K, ZL) = K^{\alpha}(ZL)^{1-\alpha}$), and costs of adjusting capital equal to zero ($\phi = 0$). These assumptions are standard in the literature. Furthermore, I later argue by way of numerical simulations that such restrictions do not change the general nature of the results. For simplicity, I start first with the case in which capital is fixed (i.e., $Y = (ZL)^{1-\alpha}$) and then allow the capital stock to vary.

3.1. The Case with Fixed Capital

The system of equations determining inflation, output, and nominal interest rates (ignoring constant terms) in this case is given by:

$$\Delta p_t = E_{t-1}\left\{\beta\Delta p_{t+1} + 2\gamma_y(y_t + \beta y_{t+1}) - 2\gamma_g(g_t + \beta g_{t+1}) - 2\gamma_z(z_t + \beta z_{t+1})\right\} \quad (3.1)$$

$$y_t = s_g g_t + E_t\left\{y_{t+1} - s_g g_{t+1}\right\} - \frac{s_c}{\kappa}\left\{\beta r_t - \frac{1}{2}(\Delta p_{t+1} + \Delta p_t)\right\} \quad (3.2)$$

$$r_t = \rho_r r_{t-1} + (1-\rho_r)\left[\frac{a}{2}(\Delta p_{t+1} + \Delta p_t) + b y_t\right] + \epsilon_{r,t} \quad (3.3)$$

where $\Delta p_t = p_t - p_{t-1}$, $\kappa = -U_{cc}C/U_c$ is the parameter governing risk aversion, $\xi = U_{ll}L/U_l$ is the inverse of the labor elasticity, and $s_c = C/Y$ and $s_g = G/Y$ are shares of private and public consumption in output. The coefficients on output, spending, and technology in the pricing equation are given by

$$\gamma_y = \left(\frac{\varphi(1-\alpha)}{1-\alpha+\alpha\varphi\varepsilon}\right)\left(\frac{\kappa}{s_c}+\frac{\xi+\alpha}{1-\alpha}\right)$$
$$\gamma_g = \left(\frac{\varphi(1-\alpha)}{1-\alpha+\alpha\varphi\varepsilon}\right)\left(\frac{\kappa s_g}{s_c}\right)$$
$$\gamma_z = \left(\frac{\varphi(1-\alpha)}{1-\alpha+\alpha\varphi\varepsilon}\right)(1+\xi)$$

where $\varphi = (1-1/\varepsilon)/(2-\chi/\varepsilon)$, $\varepsilon = -D'(g'(1))g'(1)$ is the elasticity of demand and $\chi = -D''(g'(1))g'(1)/D'(g'(1))$ is the parameter governing the curvature of the demand function. The notational convention used here assumes that lowercase letters are the logarithms of the variables denoted by uppercase letters with the exception of the interest rate which is not logged. Variables not indexed by time are assumed to be the steady state levels.

I am interested in solutions of the form:

$$\begin{bmatrix} p_t \\ y_t \\ r_t \end{bmatrix} = \mathcal{A}\begin{bmatrix} p_{t-1} \\ r_{t-1} \end{bmatrix} + \mathcal{B}\begin{bmatrix} \epsilon_{r,t} \\ g_t \\ z_t \\ g_{t-1} \\ z_{t-1} \end{bmatrix}$$

$$\begin{bmatrix} g_t \\ z_t \end{bmatrix} = \mathcal{P}\begin{bmatrix} g_{t-1} \\ z_{t-1} \end{bmatrix} + \begin{bmatrix} \epsilon_{g,t} \\ \epsilon_{z,t} \end{bmatrix} \tag{3.4}$$

that are unique and bounded, where p_{t-1} and r_{t-1} are state variables. Note that I could also write the form of the solution with $\Delta p_t = p_t - p_{t-1}$ in place of p_t and only one state variable, r_{t-1}, since only the difference in prices appears in the system of equations (3.1)-(3.3).

To derive conditions on the parameters that ensure a unique and bounded solution, I need to check the eigenvalues of a certain matrix. Consider the system of equations (3.1)-(3.3) with stochastic variables set equal to their means for all t. Rearranging terms, I can write it succinctly as

$$\begin{bmatrix} \Delta p_{t+1} \\ y_{t+1} \\ r_t \end{bmatrix} = \Psi \begin{bmatrix} \Delta p_t \\ y_t \\ r_{t-1} \end{bmatrix} \tag{3.5}$$

where again I am ignoring constant terms. If Ψ has two eigenvalues outside the unit circle and one inside the unit circle, then there is a unique and bounded solution. I want one inside the unit circle and two outside the unit circle because there is one state variable, r_{t-1}, and two choice variables, Δp_t and y_t. (See Blanchard and Kahn 1980.)

For the simple Taylor rule with $\rho_r = 0$, it is easy to derive conditions on the parameters because, in this case, prices and output next period can be written in terms of prices and output this period. The problem simplifies to checking the roots of a quadratic polynomial. This yields a restriction on how the Fed reacts to increases in inflation.

Proposition 1. If $\rho_r = 0$ and $a > 1/\beta$, then there is a unique and bounded solution to the system of equations in (3.1)-(3.3).

Proof: See Appendix C.

This condition on the Fed's rule essentially says that *real* interest rates $(\beta r_t - 1/2(\Delta p_{t+1} + \Delta p_t))$ respond positively to an increase in inflation, where the coefficient on inflation is $a\beta - 1$. Empirical results show that this condition is easily satisfied.

More generally, if the Fed reacts to past interest rates (so $\rho_r > 0$), then checking uniqueness requires checking the roots of a complicated cubic polynomial. If I compute eigenvalues numerically, varying all parameters in their feasible ranges, then I find that there is only one eigenvalue inside the unit circle as long as $a > 1/\beta$. In such cases, Δp_t and y_t are functions of the state variable r_{t-1} and the shocks. The following proposition shows that there is only one solution that has this form as long as $a > 1/\beta$.

Proposition 2. If $a > 1/\beta$, then

i. there exists one solution where $\mathcal{A}$ in (3.4) is of the form

$$\mathcal{A} = \begin{bmatrix} 1 & \mathcal{A}_{pr} \\ 0 & \mathcal{A}_{yr} \\ 0 & \mathcal{A}_{rr} \end{bmatrix} \tag{3.6}$$

with $\mathcal{A}_{pr} \leq 0$, $\mathcal{A}_{yr} \leq 0$, and $-1 < \mathcal{A}_{rr} \leq \rho_r$;

ii. if $\gamma_y < \kappa/s_c$, then $\mathcal{A}_{rr} > 0$ and otherwise $\mathcal{A}_{rr} < 0$;

iii. the maximum value for $\mathcal{A}_{rr}$ can be achieved when $b = 0$ and $\gamma_y = 0$.

Proof: See Appendix C.

This proposition gives a characterization of the solution in the case that monetary policy is sufficiently responsive to movements in inflation. The restriction $a > 1/\beta$ is the typical one used to get a unique, bounded solution. (See, for example, Bernanke and Woodford (1997), Clarida et al. (1998), and King and Kerr (1996)).

The signs on $\mathcal{A}_{pr}$ and $\mathcal{A}_{yr}$ coincide with what we would intuitively expect. When there is an increase in the interest rate, demand falls off, and prices and output drop. The magnitude of $\mathcal{A}_{rr}$ is important since it determines how persistent the response of interest rates and hence output is to unexpected changes in monetary policy. According to the proposition, its magnitude depends on the relative magnitudes of γ_y and κ/s_c.

The magnitude of γ_y governs how responsive monopolists are to current and future changes in output. (See equation (3.1).) Monopolists set their prices using expectations of their current and future marginal costs. Marginal costs, in turn, are related to aggregate output. The coefficient γ_y determines how prices change in response to expected changes in current and future output. Its magnitude depends on assumptions about the elasticity of demand, ε, and its curvature, φ, parameters of preferences, κ and ξ, and labor's share in production, $1-\alpha$. The smaller is γ_y, the more rigid are prices.

The magnitude of κ/s_c depends on the curvature of utility and on the share of consumption in output. If κ is small, then consumer's respond a lot to changes in real interest rates. If κ is large, they do not. (See equation (3.2).) Suppose that κ is large and γ_y is small (with $\gamma_y < \kappa/s_c$). Then neither the monopolists nor the consumers are sensitive to price changes. If there is a negative shock to interest rates, then the increase in output is small in response to the change in real interest rates, and the increase in prices is small because γ_y is small and output has not changed much. Increases in prices and output do lead to an increase in interest rates but small increases imply that the interest rate returns to its steady state level only gradually. Suppose on the other hand that κ is small and γ_y is large (with $\gamma_y > \kappa/s_c$). In this case, the responses of monopolists and consumers are large, so large that the reaction of the Fed leads the interest rate to overshoot its steady state level after one period.

In Figure 3, I illustrate how the magnitudes of γ_y and κ/s_c can affect the results by showing impulse responses in four different cases. In all cases, I assume that firms must hold prices fixed for one-half of a year. All other decisions are made quarterly. In all cases, I set $\kappa = 1$ which implies logarithmic utility, and $s_c = 1$ which implies no government spending. The discount factor is 0.97 at an annual frequency. The policy rule for nominal interest rates is assumed to be that estimated by Clarida et al. (1998) with the shock to interest rates equal to an annualized 25 basis point decline.[4]

Consider the first case in Figure 3 with $\gamma_y = 0$. This is a case in which prices are extremely rigid – monopolists do not respond at all to changes in marginal costs. One rationale for such a choice is given by Kimball (1995) who assumes that monopolists are reluctant to increase their prices much above the average in response to a shock if demand for their own good falls sharply as a result. This could happen if the demand for intermediate goods was not constant as is typically assumed. (See for example Dixit and Stiglitz (1977) who work with $g(x) = x^{(\varepsilon-1)/\varepsilon}$.) In particular, Kimball works with functions in which χ is very negative and φ is small. Taken to the extreme, we could assume $\varphi = 0$ and therefore $\gamma_y = 0$. In this case, prices are fixed, the nominal interest rate is a persistent autoregressive process, and output is given by

$$y_t = -\left(\frac{s_c\beta}{\kappa}\right)\left(\frac{1}{1-\mathcal{A}_{rr}}\right) r_t.$$

With the estimates of Clarida et al. (1998), the Fed's response to output (b) is small and $\mathcal{A}_{rr}$ is only slightly less than ρ_r (=0.66). With $\kappa = s_c = 1$ and β close to 1, the increase in output on impact is roughly 3 times as large as the change in interest rates. (See Figure 3.)

The second impulse response displayed in Figure 3, with $\gamma_y = 1/2$, shows how convex demand has to be in order to get persistence in output. In the case where monetary policy is a given process for the growth rate of money, Chari et al. (1999) show that extreme assumptions on demand are required for monetary shocks to lead to persistent movements in output. The same is true here where monetary policy is described by an interest rate

[4] Throughout the paper, I report parameters in annualized terms. Thus, for the quarterly model, I am using a discount factor of $0.97^{1/4}$, a value of b in (3.3) equal to 1/4 of the estimate of Clarida et al. (1998), and a quarterly interest rate change of $6\frac{1}{4}$ basis points.

rule. Consider Figure 3 again. The persistence of output is very nonlinearly related to γ_y and in turn φ. For the case with $\gamma_y = 0$, I set the $\varphi = 0$ so that demand was very convex. For the case with $\gamma_y = 1/2$, I set $\varphi = 1/2$ (with $\xi = \alpha = 0$). Notice that the response of output in the case of $\gamma_y = \varphi = 1/2$ is much less persistent. However, demand in the case with $\varphi = 1/2$ is still very convex. In this case, a 2% increase in relative prices implies a 20% decline in demand.

If γ_y is equal to κ/s_c, then the nominal interest rate is not serially correlated (that is, $\mathcal{A}_{rr}$ is equal to 0). The interest rate falls on impact and then is back at its steady state level in the next period because $r_t = \mathcal{B}_{rr}\epsilon_{rt}$. In this case, output and prices evolve according to:

$$y_t = -\frac{s_c}{a\kappa + bs_c}\left(\frac{\rho_r}{1-\rho_r}\right) r_{t-1} - \left(\frac{2s_c}{a\kappa + bs_c}\frac{\rho_r}{1-\rho_r} + \frac{\beta s_c}{\kappa}\right)\mathcal{B}_{rr}\epsilon_{r,t}$$

$$\Delta p_t = -\frac{2\kappa}{a\kappa + bs_c}\left(\frac{\rho_r}{1-\rho_r}\right) r_{t-1}.$$

Since the interest rate is back to the steady state in one period, output and prices must be back to their steady state levels in two periods – coincidentally with the period in which all monopolists are allowed to respond to the interest rate shock and adjust prices. This is what we see in Figure 3.

The last example displayed in Figure 3 illustrates the key result from Proposition 2. When $\gamma_y > \kappa/s_c$, the nominal interest rate is negatively serially correlated. Thus, in response to a negative shock to the interest rate output rises on impact but is below its steady state level two periods after the shock. Standard assumptions about technologies such as constant returns to scale, constant elasticities of demand, and finite labor elasticities necessarily lead to a picture like this. With constant returns ($\alpha = 0$), a constant elasticity of demand ($\varphi = 1$), and a finite labor elasticity ($\xi > 0$), Proposition 2 states that nominal interest rates are negatively serially correlated ($\mathcal{A}_{rr} < 0$) since $\gamma_y = \kappa/s_c + \xi$ must be greater than κ/s_c. Given interest rates are not negatively serially correlated in the data (See Figures 1 and 2), this is a negative result.

The mechanism at work here is slightly different than that in Chari et al. (1999). Suppose that I replace the interest rate rule in (3.3) with a simple money demand relation: $m_t - \bar{p}_t = y_t$. To determine what happens to output and inflation I only need to consider the money demand relation and the pricing rule for the monopolists in (3.1). If γ_y is small, then the monopolist's price is not sensitive to changes in current and future marginal costs (and hence current and future output). In response to a monetary shock, the price would not change very much. If prices do not change very much then the path of output tracks the path of the money supply because $y = m - \bar{p}$. Persistent changes in m would imply persistent changes in y. Nowhere does this logic rely on (3.2) which residually determines the path of nominal interest rates. But in the case with monetary policy described by the interest rate rule (3.3), the Euler equation (3.2) is an integral part of the solution for output and inflation and it is the money demand equation (in Appendix A) which residually determines the path of the money supply. This difference can have a big effect in some cases. For example, Chari et al. (1999) show that having $\kappa + \xi$ very small can lead to very persistent responses in output. Here, having $\kappa + \xi$ small is not sufficient to guarantee that. In fact, if anything having κ small has the opposite effect.

In Figure 4, I display responses of output to an interest rate shock for different values of κ (and $s_c = 1$, $\beta = 0.97$, $\varphi = 1$, $\xi = 0.1$, $\alpha = 0$, and parameter estimates of Clarida et al. (1998) for the Fed's rule). Again, these responses follow a fall in the nominal interest rate equal to 25 basis points when annualized. If consumers are very risk averse and κ is large, then consumption and output do not change much in response to the shock. On the other hand, risk-loving households with $\kappa = 0.1$ would alter consumption dramatically, roughly 2 percent, in response to a shock in the interest rate. Changes of this magnitude would imply wildly erratic business cycles in response to observed innovations in interest rates.

Changing the coefficients in the Fed's interest rate rule can also affect the impact of shocks on output and prices. Consider, for example, the impulse responses of output displayed in Figure 5. Here, I use the same baseline parameters used in Figure 4 with $\kappa = 1$ and vary the coefficients of the Taylor rule. If $a = 1/\beta$, then the real interest rate changes little when prices change. If the Fed's reaction to output is small (b small) then the Taylor rule looks roughly like an autoregressive process for real interest rates. A negative shock to ϵ_r then has a potentially large effect on output because price increases do not dampen the change in the real interest rate as it does in the case with Clarida et al.'s (1998) estimates (the dashed line). The third example in Figure 5 shows that changing b has a small quantitative effect on the results. Finally, for the sake of comparison, I include the parameters originally used by Taylor (1993).

Thus far, I have considered only nominal interest rate shocks. Many have argued that only a small fraction of the variance of output is due to unanticipated shocks to money growth or to interest rates. Rather, policy is relevant because the Federal Reserve's policy rule affects how other shocks influence output. For example, Rotemberg and Woodford (1997) attribute a larger fraction of the variance of output to unanticipated changes in government spending than to unanticipated changes in interest rates. The next proposition shows that output can take on the characteristics of the spending process.

Proposition 3. If $b = 0$ and $\gamma_y s_g = \gamma_g$, then the nominal interest rate and the prices of monopolists do not change with a change in government spending. Output, in this case, follows the same path as government spending since $y_t = s_g g_t$ in equilibrium.

Proof: See Appendix C.

Proposition 3 can be interpreted to mean that persistent output paths are possible if fiscal variables are persistent. However, by the proposition, a one percent change in government spending leads to a s_g percent change in output, where s_g is the share of government spending in GDP. If this share is on the order of 20 percent, then government spending will lead to little overall variation in output.

I now consider changes in technology. From the system of equations in (3.1)-(3.3), we see that the level of technology only enters into the pricing equation for the monopolists because it directly affects marginal costs. Thus, if prices were very sticky then technology shocks would have only a small effect on the three endogenous variables. A positive shift in z would imply a fall in labor because firms could meet the same amount of demand with fewer workers.

If prices are not perfectly sticky, then the response in output is hump-shaped and persistent. The following proposition makes this precise.

Proposition 4. Let $b = 0$ and $\gamma_y = \kappa/s_c$. Then the solution to the dynamical system is

$$\Delta p_t = -\frac{2\rho_r}{a(1-\rho_r)} r_{t-1} + \mathcal{B}_{pz} z_{t-1}$$

$$y_t = -\frac{s_c \rho_r}{a\kappa(1-\rho_r)} r_{t-1} + \frac{\theta}{1-\rho_z} \mathcal{B}_{pz}(z_t + (1-\rho_z)/2\, z_{t-1})$$

$$r_t = \frac{a(1-\rho_r)}{2(1+\rho_r)} \mathcal{B}_{pz}(z_t + z_{t-1})$$

where $\mathcal{B}_{pz}$ is the initial response of prices to technology (i.e., the coefficient on z_{t-1}) and $\theta = (1-\rho_r)/(1+\rho_r) \times (1-a\beta)s_c/\kappa$.

Proof: See Appendix C.

In Figure 6, I display two impulse responses of output after a one-percent shock in technology which is expected to persist ($\rho_z = 0.91$). The first assumes no feedback on output in the Fed's policy rule and parameters of preferences and technology are set so that $\gamma_y = \kappa/s_c$. These are the assumptions made in Proposition 4. The other coefficients in the policy rule, ρ_r and a, are set to the values estimated by Clarida et al. (1998). The second impulse response assumes a value for γ_y that is two times κ/s_c and uses Clarida et al.'s (1998) exact policy rule. This is included to show the affect of the restrictions on γ_y and b. As we can see from the figure, the response of output is hump-shaped and persistent. If I use the solution above to trace out the impulse response function, we would see that the Fed's rule has the largest effect on the initial response. As the Fed becomes less sensitive to changes in inflation (i.e., lower a), output responds less to technology shocks on impact. On the other hand, as the Fed becomes less sensitive to changes in output (i.e., lower b), output responds more to technology shocks on impact. The pattern after all firms have gotten to adjust prices, however, depends little on the policy rule of the Fed.

3.2. The Case with Capital

I turn now to the case with capital. Assuming as before that there are two cohorts of monopolists ($N = 2$), we have two additional equations for solving an equilibrium which are the two dynamic Euler equations for the capital stocks.

In the case of the simple Taylor rule (with $\rho_r = 0$ in (3.3)), the dynamics of output and inflation depend not on lagged nominal interest rates but on lagged capital. Let $k_t = 1/2(k_{1,t} + k_{2,t})$ be the aggregate capital stock. Then the response of output to interest rate shocks in this case depends on the relative sizes of the coefficients $\mathcal{A}$ and $\mathcal{B}$ of:

$$k_t = \mathcal{A}_{kk} k_{t-1} + \mathcal{B}_{kr} \epsilon_{r,t}$$

$$y_t = \mathcal{A}_{yk} k_{t-1} + \mathcal{B}_{yr} \epsilon_{r,t} \tag{3.7}$$

which is the form of the solution when $\rho_r = 0$.

Proposition 5. If there are no adjustment costs incurred when changing capital ($\phi(X/K) = 0$) and the current nominal interest rate does not depend on last period's rate ($\rho_r = 0$), then capital and output evolve according to (3.7) and

$$\frac{\mathcal{A}_{kk}}{\mathcal{B}_{kr}} = \frac{\mathcal{A}_{yk} + K/Y(1-\delta)}{\mathcal{B}_{yr}}. \tag{3.8}$$

Proof: See Appendix C.

Although it may not be obvious, this proposition can be used to show that output displays similar patterns in the cases with fixed capital and variable capital. To see this, consider the form of the solution in (3.7). If the nominal interest rate rises by 1 percentage point, the change in logged output is $\mathcal{B}_{yr}$ on impact. In the next period the deviation in output from trend is $\mathcal{A}_{yk}\mathcal{B}_{kr}$. By the proposition, the difference between $\mathcal{B}_{yr}$ and $\mathcal{A}_{yk}\mathcal{B}_{kr}$ is at least as great as the term $K/Y(1-\delta)\mathcal{B}_{kr}$ because $\mathcal{A}_{kk}$ is less than 1. If this term is large, the model predicts that output will be much higher on impact than in the next period and therefore will display little persistence. For example, assuming a capital/output ratio around 10 and a depreciation rate around 2 percent for quarterly data, we have that $K/Y(1-\delta)\mathcal{B}_{kr}$ is roughly 10 times the size of the impact of a change in the nominal interest rate on the capital stock – which is typically much smaller than the impact on output or investment.

Another way to see the effect on output is by way of the following dynamic first-order conditions for the consumer and monopolist i:

$$1 = \beta E_t \frac{U_{ct+1}}{U_{ct}} \frac{\bar{P}_t}{\bar{P}_{t+1}} (1 + r_t)$$

$$1 = \beta E_t \frac{U_{ct+1}}{U_{ct}} \Big[\left(\frac{W_{t+1}}{F_{lt+1}(i)} \right) F_{kt+1}(i) + 1 - \delta \Big].$$

Suppose that prices are very sticky and that the Fed does not respond to output ($b = 0$). In this case the rule for the Fed implies that the nominal interest rate is a simple autoregressive process $r_t = \rho_r r_{t-1} + \epsilon_{r,t}$. Consider a negative shock to the interest rate. With interest rates down, demand goes up. To meet demand, labor goes up – induced by higher wages. The equations above show however that in the period following the shock, the return on capital should be equated to the interest rate. Assuming a Cobb-Douglas form for F, this implies that $WL/K + 1 - \delta$ is equal to $1 + r$ in the period following the shock. The wage in equilibrium is equal to $-U_l/U_c$ and therefore depends on consumption and labor. By the consumer's Euler equation, we know that a persistent drop in interest rates leads to a persistent rise in consumption. If capital does not respond very much to the change in the interest rate and if $WL/K + 1 - \delta$ is equal to $1 + r$ in the period following the shock, then it must be the case that labor is below trend in the period following the shock. Hence, in the case that prices are very sticky, output falls below trend in the period following the shock.

What happens more generally? In Figure 7, I compare the case with no capital and a coefficient on lagged nominal interest rates of 0.66 as in Clarida et al. (1998) with the case with capital ($\alpha = 1/3$) and no weight on lagged nominal interest rates in the monetary rule ($\rho_r = 0$). For both cases, the shock is a 25 basis point decline in the nominal interest rate when annualized. In the case with capital, adjustment costs on investment are set so that investment is around three times as variable as output. Without adjustment costs, the model predicts a much larger increase in output on impact. Notice that the response in output is similar for the two cases. Output increases when the shock hits and quickly returns to its trend level. I also include the case, not covered by the propositions above, with capital ($\alpha = 1/3$) and with lagged interest rates in the monetary rule ($\rho_r = .66$). For the same size fall in interest rates, the rise in output is higher. Output is also more persistent with the dynamics from both capital and interest rates – but not significantly.

4. Numerical experiments

While an analytical characterization is useful for understanding the possible outcomes of the model, one would like to know how well the model mimics the aggregate data using actual observations of policy variables and technology shocks as inputs. In this section, I simulate the model of Section 2 with innovations to monetary policy backed out from estimates for the Fed's interest rate policy rule and innovations to spending and technology found from simple autoregressive models for G and Z. I then compare the model's predicted output with actual GDP.

Innovations to monetary policy are taken to be the residuals from Clarida et al.'s (1998) estimated policy rule. This rule is plotted in Figure 2.[5] Innovations to government spending and to technology are found by regressing the logged and detrended series on their lagged values. To detrend the series I regress logged values on a linear and a quadratic trend. This procedure yields the following processes for spending and technology:

$$g_t = 0.97 g_{t-1} + \epsilon_{g,t}$$

$$z_t = 0.91 z_{t-1} + \epsilon_{z,t}$$

where g is the logarithm of government spending minus its mean and z is the logarithm of the technology shock.

Additional parameters are needed to simulate the economy. I assume that utility is separable and that the risk aversion parameter κ is equal to 1. I use a labor elasticity of 10 so that ξ is equal to 0.1. The discount factor is 0.97 annually. The depreciation rate is 8 percent annually. The capital share is 1/3. The demand elasticity is 10 and is assumed

[5] Annualized rates are plotted in Figure 2. The innovations used in the quarterly model are roughly 1/4 of the size of the annual innovations.

to be constant (implying that $\varphi = 1$). Adjustment costs are set so that investment is about 3 times as volatile as output. Government spending is approximately 20% of GDP on average.

In Figure 8, I report the responses of output to innovations in nominal interest rates only. The model predictions and actual data have dramatically different characteristics. As the propositions of Section 3 make clear, changes in interest rates have a potentially large but short-lived affect on output. By varying the key parameters, we can affect the magnitude of the standard deviation of output, but we will not radically change the erratic nature of the responses. For example, if I decrease the risk-aversion parameter κ, I can amplify the changes in output; the standard deviation of the logarithm of output increases more than three-fold as I change κ from 1 to 1/10.

In Figure 9, I report the responses of output to innovations in government spending only. The spending shocks have a small impact on output although the changes in output are persistent. In fact, this model predicts that very little of the variance in output is attributable to spending shocks on the order of those observed after 1979.

In Figure 10, I report the responses of output to innovations in technology only. As in the case of government spending, I find a persistent response in output. However, the standard deviation in output is roughly half of what it is in the data. This is due in part to the fact that the labor input falls with an increase in technology. Furthermore, the standard deviation of the logarithm of the labor input is roughly half of the standard deviation of the logarithm of output – which is too low relative to the data. The correlation between the predicted and actual output series in Figure 10 is high; it is roughly 0.65. But, the correlation between predicted and actual inflation is only 0.11.

5. Discussion

For those familiar with the results of Rotemberg and Woodford (1997), Figures 8 through 10 should seem surprising. For a model very similar to that of Section 2, Rotemberg and Woodford (1997) display predicted time series for output, inflation, and nominal interest rates that look almost exactly like the data. Why is there such a difference in results?

The main difference is our assumptions about the shocks. Rotemberg and Woodford (1997) have three sources of uncertainty: nominal interest rate shocks, government spending shocks, and taste shocks. As in Clarida et al. (1998), Rotemberg and Woodford estimate a Taylor-like interest rate rule using data on the Federal Funds Rate, inflation, and logged output. This estimated rule represents monetary policy in their model. The other two shocks are set as residuals – they are whatever they have to be to have the model's time series match the data for their benchline parameterization. What the above results indicate is that Rotemberg and Woodford's taste shocks must be an important driving force in their model.

A. Equilibrium Conditions

In this section, I report all of the equilibrium conditions for the model economy of Section 2. First-order conditions derived from the consumer's problem are as follows:

$$W_t = -U_{lt}/U_{ct} \tag{A.1}$$

$$\begin{aligned} U_{ct} &= U_{mt} + \beta E_t U_{ct+1}\bar{P}_t/\bar{P}_{t+1} \\ &= U_{mt}(1+r_t)/r_t \end{aligned} \tag{A.2}$$

$$Q(\epsilon^s|\epsilon^t) = \beta^{s-t}\pi(\epsilon^s|\epsilon^t)U_{cs}\bar{P}_t/(U_{ct}\bar{P}_s) \quad \text{for all s > t.} \tag{A.3}$$

where U_{ct} and U_{mt} are used in place of $U_c(C_t, L_t, M_t/\bar{P}_t)$ and $U_m(C_t, L_t, M_t/\bar{P}_t)$, respectively, to simplify the expressions. Condition $(A.1)$ is the standard static first order condition relating wages to marginal utilities on leisure and consumption. The condition in $(A.2)$ is the consumer's money demand equation where I have used the definition of the nominal interest rate r. Equation $(A.3)$ is the price of a bond in equilibrium.

Equations derived from the problem of the final goods producers are as follows:

$$Y_t(i) = D\left(\frac{P_t(i)}{\bar{P}_t}\left[\sum_j g'\left(\frac{Y_t(j)}{Y_t}\right)\frac{Y_t(j)}{Y_t}\right]\right)Y_t \tag{A.4}$$

$$\bar{P}_t = \int P_t(i)Y_t(i)/Y_t\, di, \tag{A.5}$$

which is the demand function for the ith intermediate good and the aggregate price level, respectively.

From the monopolists' problem, I derive the following conditions

$$\sum_s\sum_{\epsilon^s} Q(\epsilon^s|\epsilon^t)\Big\{Y_s(i) + Y_s\left[1 - \bar{P}_s MC_s(i)/P_t(i)\right] \tag{A.6}$$

$$D'\left(\frac{P_t(i)}{\bar{P}_s}\int g'\left(\frac{Y_s(j)}{Y_s}\right)\frac{Y_s(j)}{Y_s}\,dj\right)g'\left(\frac{Y_s(i)}{Y_s}\right)\Bigg\} = 0$$

$$U_{ct} = \beta E_t U_{ct+1}\Bigg[(1-\phi_t'(i))MC_{t+1}(i)F_{kt+1}(i) \qquad (A.7)$$

$$+\frac{1-\phi_t'(i)}{1-\phi_{t+1}'(i)}\left\{1-\delta-\phi_{t+1}(i)+\phi_{t+1}'(i)\frac{X_{t+1}(i)}{K_t(i)}\right\}\Bigg]$$

$$MC_t(i) = W_t/F_{lt}(i) \qquad (A.8)$$

$$K_t(i) = (1-\delta)K_{t-1}(i) + X_t(i) - \phi_t(i)K_{t-1}(i) \qquad (A.9)$$

where the arguments of the utility function, the production function, the adjustment cost function and all of their derivatives have been dropped to simplify the notation. An index of t on $F(i)$ implies that the arguments are $(K_{t-1}(i), Z_t L_t(i))$. An index of t on $\phi(i)$ implies that the argument is $X_t(i)/K_{t-1}(i)$.

B. Linearized Equilibrium Conditions

Each of the equations in Appendix A have been linearized around the economy's steady state. With the exception of the nominal interest rate, variables are assumed to be logged with their means subtracted (for example, $w_t = \log W_t - \log W$).

$$\begin{aligned} w_t &= \left(\frac{U_{cl}}{U_l} - \frac{U_{cc}}{U_c}\right) Cc_t + \left(\frac{U_{ll}}{U_l} - \frac{U_{cl}}{U_c}\right) Ll_t \\ &+ \left(\frac{U_{lm}}{U_l} - \frac{U_{cm}}{U_c}\right) M/\bar{P}(m_t - \bar{p}_t) \end{aligned} \quad (B.1)$$

$$\begin{aligned} \frac{\beta r_t}{r} &= \left(\frac{U_{cm}}{U_m} - \frac{U_{cc}}{U_c}\right) Cc_t + \left(\frac{U_{lm}}{U_m} - \frac{U_{cl}}{U_c}\right) Ll_t \\ &+ \left(\frac{U_{mm}}{U_m} - \frac{U_{cm}}{U_c}\right) M/\bar{P}(m_t - \bar{p}_t) + \beta \end{aligned} \quad (B.2)$$

$$\begin{aligned} -\beta r_t &= E_t\Bigg(\frac{U_{cc}C}{U_c}(c_{t+1} - c_t) + \frac{U_{cl}L}{U_c}(l_{t+1} - l_t) \\ &+ \frac{U_{cm}M/\bar{P}}{U_c}(m_{t+1} - \bar{p}_{t+1} - m_t + \bar{p}_t) + \bar{p}_t - \bar{p}_{t+1}\Bigg) \end{aligned} \quad (B.3)$$

$$y_{i,t} = y_t - \varepsilon(p_{i,t} - \bar{p}_t) \quad (B.4)$$

$$\bar{p}_t = \sum_{i=1}^{N} p_{i,t}/N \quad (B.5)$$

$$p_{i,t} = E_{t-1} \sum_{j=0}^{N-1} \beta^j \left(\bar{p}_{t+j} + \varphi N mc_{i,t+j}\right) / \sum_{j=0}^{N-1} \beta^j \quad (B.6)$$

$$y_{i,t} = \left(F_k K k_{i,t-1} + F_l L(l_{i,t} + z_t)\right)/Y \quad (B.7)$$

$$mc_{i,t} = w_t + l_{i,t} - F_l L/Y(l_{i,t} + z_t) - F_k K/Y k_{i,t-1} \quad (B.8)$$

$$0 = E_t\Bigg(\frac{U_{cc}C}{U_c}(c_{t+1} - c_t) + \frac{U_{cl}L}{U_c}(l_{t+1} - l_t)$$

$$+\frac{U_{cm}M/\bar{P}}{U_c}(m_{t+1}-\bar{p}_{t+1}-m_t+\bar{p}_t)$$

$$+(1-\beta(1-\delta))(w_{t+1}+l_{i,t+1}-k_{i,t})$$

$$+\phi''(\delta)\delta(\beta x_{t+1}(i)-\beta k_t(i)-x_t(i)+k_{t-1}(i)) \tag{B.9}$$

$$k_{i,t}=(1-\delta)k_{i,t-1}+\delta x_{i,t} \tag{B.10}$$

$$r_t=\rho_r r_{t-1}+(1-\rho_r)[a(\bar{p}_{t+1}-\bar{p}_t)+by_t+(1-\rho_r)r]+\epsilon_{r,t} \tag{B.11}$$

$$l_t=\frac{1}{N}\sum_i l_{i,t} \tag{B.12}$$

$$c_t=\left(Yy_t-Gg_t-X\sum_i x_{i,t}/N\right)/C \tag{B.13}$$

C. Proofs of Propositions

Proof of Proposition 1. Substitute (3.3) into (3.2). If $\rho_r = 0$, then (3.1) and (3.2) can be written

$$\begin{bmatrix} \Delta p_{t+1} \\ y_{t+1} \end{bmatrix} = \begin{bmatrix} \beta & 2\gamma_y\beta \\ \frac{s_c(1-a\beta)}{2\kappa} & 1 \end{bmatrix}^{-1} \begin{bmatrix} 1 & -2\gamma_y \\ -\frac{s_c(1-a\beta)}{2\kappa} & 1+\frac{s_c\beta b}{\kappa} \end{bmatrix} \begin{bmatrix} \Delta p_t \\ y_t \end{bmatrix}$$

where Ψ in (3.5) is the coefficient matrix here. The eigenvalues of Ψ satisfy the quadratic equation

$$\lambda^2 - (\theta+\psi)\lambda + \frac{1}{\beta}(1+\psi) = 0$$

where

$$\theta = \left(\frac{1+\beta}{\beta}\right)\left(\frac{1+\gamma_y s_c(1-a\beta)/\kappa}{1-\gamma_y s_c(1-a\beta)/\kappa}\right), \quad \psi = \frac{\beta s_c b/\kappa}{1-\gamma_y s_c(1-a\beta)/\kappa}.$$

The product of the eigenvalues is equal to $(1+\psi)/\beta$. Assuming $b > 0$ as I am throughout the analysis, then $\psi > 0$ for all $a > 1/\beta$. Therefore, if the roots are imaginary, then they are equal in absolute value and lie outside the unit circle. If the roots are real, then one must show that the one with the smallest absolute value is outside the unit circle. There are two cases to consider. First, assume that $\theta > 0$. Then, it is easy to show that

$$\frac{1}{2}(\theta+\psi) - \frac{1}{2}\sqrt{(\theta+\psi)^2 - 4/\beta(1+\psi)} > 1$$

if $a > 1/\beta$ using the definitions of θ and ψ above. If $\theta < 0$ and the roots are real, then

$$\frac{1}{2}(\theta+\psi) + \frac{1}{2}\sqrt{(\theta+\psi)^2 - 4/\beta(1+\psi)} < -1$$

if $a > 1/\beta$. Thus, if $a > 1/\beta$, Ψ has both eigenvalues outside the unit circle, and conditions of Blanchard and Kahn (1980) are met that guarantee a unique and bounded solution to the system of equations in (3.1)-(3.3).

Proof of Proposition 2. I start by assuming that $\mathcal{A}_{pr} \leq 0$. Applying the method of undetermined coefficients, I want to find $\mathcal{A}_{pr}$, $\mathcal{A}_{yr}$, and $\mathcal{A}_{rr}$ that set the coefficients on r_{t-1} to zero in the first order conditions. Substituting our guess (3.4) with (3.6) into the first order equations, the coefficients to be set equal to zero can be summarized as follows

$$z = f(x, \mathcal{A}_{yr}) = a\gamma_y \mathcal{A}_{yr}(1+\beta x)(1+x)/(1-\beta x) + b\mathcal{A}_{yr} \tag{C.1}$$

$$z = g(x, \mathcal{A}_{yr}) = a\kappa/s_c \mathcal{A}_{yr}(1-x) + a\beta x + b\mathcal{A}_{yr} \tag{C.2}$$

$$z = h(x, \mathcal{A}_{yr}) = (x-\rho_r)/(1-\rho_r) \tag{C.3}$$

where $z = a/2\mathcal{A}_{pr}(1+\mathcal{A}_{rr}) + b\mathcal{A}_{yr}$ and $x = \mathcal{A}_{rr}$. I have done a change of variables to transform three nonlinear equations into two linear equations and one nonlinear equation. Thus, I can state the problem as: find z, x, and $\mathcal{A}_{yr}$ that satisfy $(C.1)$-$(C.3)$. Because I have assumed $\mathcal{A}_{pr} \leq 0$, it follows from $(C.1)$ and the definition of z that $\mathcal{A}_{yr} \leq 0$ for all stationary equilibria (i.e., for all values $x \in [-1, 1]$). With $\mathcal{A}_{yr} \leq 0$, it is easy to see that $f(x, \cdot)$ and $g(x, \cdot)$ have positive slopes. On the other hand, $h(x, \cdot)$ is monotonically decreasing in x. Therefore, if a fixed point exists, it must be unique.

To prove existence, I start with $\mathcal{A}_{yr}$ equal to zero and show that as I decrease $\mathcal{A}_{yr}$ the three curves have to intersect at some point with $x \in [-1, \rho_r]$. Consider the diagram in Figure C.1. First note that the line $z = h(x, \mathcal{A}_{yr})$ is independent of $\mathcal{A}_{yr}$ so it stays fixed as I vary $\mathcal{A}_{yr}$. If $\mathcal{A}_{yr}$ is zero, then the intersection of curves f and g are at the origin which is point A in Figure C.1. If $\gamma_y > \kappa/s_c$, then decreasing $\mathcal{A}_{yr}$ leads one ultimately from point A to a point like B, with $x < 0$ and hence $\mathcal{A}_{rr} < 0$. At $x = -1$, the curve f is always above g, so the intersection occurs for some $x \geq -1$. If $\gamma_y < \kappa/s_c$, then decreasing $\mathcal{A}_{yr}$ leads one ultimately from point A to some point with $x > 0$ and $z < 0$. If $z < 0$, then $x < \rho_r$ and hence $\mathcal{A}_{rr} < \rho_r$. Thus, the claims of the proposition are proved conditional on

$\mathcal{A}_{pr} \leq 0$.

Next I show that the condition on a implies that $\mathcal{A}_{pr} \leq 0$. I do this by way of contradiction. Suppose that the model can be parameterized in such a way as to achieve a stationary solution with $\mathcal{A}_{pr}$ positive. As before, we can use equation $(C.1)$ and the definition of z to show that that $\mathcal{A}_{pr} > 0$ implies $\mathcal{A}_{yr} \geq 0$. If these coefficients are positive and $a\beta > 1$, then curve h cannot intersect g at any point $x \in [0, 1]$. If $\rho_r \geq 0$, then curve h cannot intersect f at any point $x \in [-1, 0]$. Therefore, there can be no fixed point, and we have a contradiction.

Proof of Proposition 3. Substitute the form of the solution (3.4) into the first order conditions. If $b = 0$, then it is easy to show that the coefficient on g_t is equal to the coefficient on g_{t-1} in the equation for nominal interest rates. If additionally I assume that $\gamma_y s_g = \gamma_g$, it is easy to see that $y_t = s_g g_t$ implies that government spending has no effect on the monopolist's price. (See equation (3.1).) If the monopolist's prices do not change and $b = 0$ then interest rates do not change. (See equation (3.3).) If interest rates do not change it is easy to see from the consumer's Euler equation (3.2) that $y_t = s_g g_t$ is the solution.

Proof of Proposition 4. Substitute the form of the solution (3.4) into the first order conditions. Let $\mathcal{B}_{pz}$ be the coefficient on z_{t-1} in the equilibrium equation for prices. After some algebraic manipulation, it is easy to show that

$$\mathcal{B}_{pz} = -\left[1 + \frac{2\rho_r}{1+\rho_r}\frac{\beta(1+\rho_z)}{1-\beta\rho_z} - \theta\frac{(1+\beta\rho_z)(1+\rho_z)}{(1-\beta\rho_z)(1-\rho_z)}\right]^{-1}\left(\frac{2\rho_z(1+\beta\rho_z)}{1-\beta\rho_z}\right).$$

where $\theta = (1-\rho_r)/(1+\rho_r) \times (1-a\beta)s_c/\kappa$. All other coefficients can then be defined in terms of this coefficient.

Proof of Proposition 5. The general form of the solution in this case is $X_t = \mathcal{A}X_{t-1} + \mathcal{B}S_t$ with the states X_{t-1} given by $[\Delta p_{t-1}, y_{t-1}, r_{t-1}, k_{1,t-1}, k_{2,t-1}]'$ and the shocks S_t given by $[\epsilon_{r,t}, g_t, z_t, g_{t-1}, z_{t-1}]'$. With $\rho_r = 0$, the interest rate depends only on contemporaneous output and inflation. Therefore, r_{t-1}, Δp_{t-1}, and y_{t-1} appear in none of the first-order conditions, and the coefficients in the dynamical system multiplying these terms are equal to zero. I can further simplify the final solution by noting the following. If $\phi(X/K) = 0$, then subtracting the two Euler equations for capital (equation $(B.9)$ with i=1 and i=2) yields the following result

$$p_{t+1} = p_t + \frac{1}{\varepsilon}(k_{1t} - k_{2t}) \tag{$C.4$}$$

where group i=1 is assumed to be changing prices in t. This result gives us a restriction for the coefficients in $\mathcal{A}$ on k_1 and k_2: they are proportional to each other. Therefore, I can reduce the state vector by one by recording only the aggregate capital $k_t = 1/2(k_{1,t} + k_{2,t})$. This reduces the problem to finding the coefficients of:

$$\begin{bmatrix} \Delta p_t \\ y_t \\ r_t \\ k_t \end{bmatrix} = \begin{bmatrix} \mathcal{A}_{pk} \\ \mathcal{A}_{yk} \\ \mathcal{A}_{rk} \\ \mathcal{A}_{kk} \end{bmatrix} k_{t-1} + \mathcal{B} \begin{bmatrix} \epsilon_{r,t} \\ g_t \\ z_t \\ g_{t-1} \\ z_{t-1} \end{bmatrix}$$

Substituting this simpler guess into our first order conditions and manipulating terms, it is easy to show that (3.8) must hold where $\mathcal{B}_{yr}$ and $\mathcal{B}_{kr}$ are the coefficients on $\epsilon_{r,t}$ in the equations for y_t and k_t, respectively.

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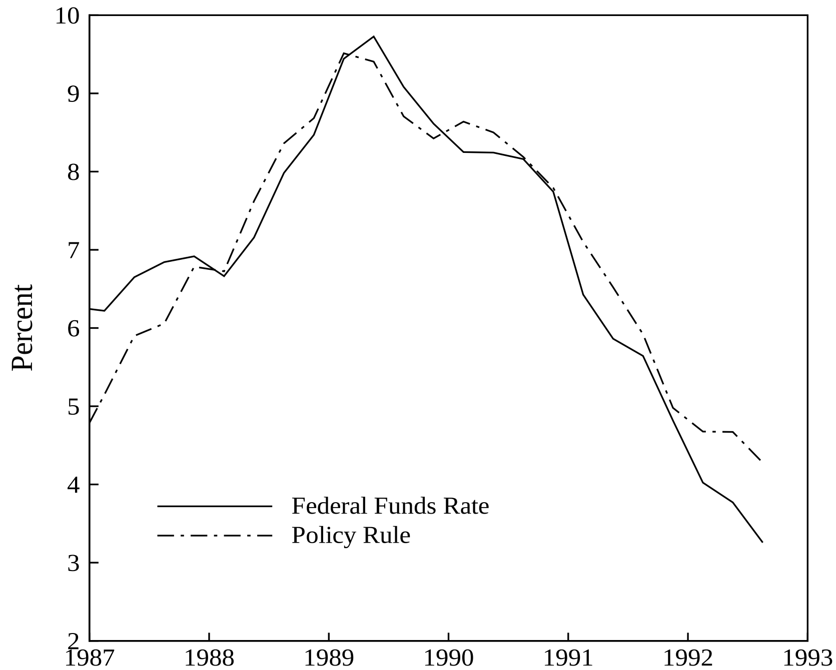

Figure 1. Federal Funds Rate and Estimated Policy Rule of Taylor (1993).

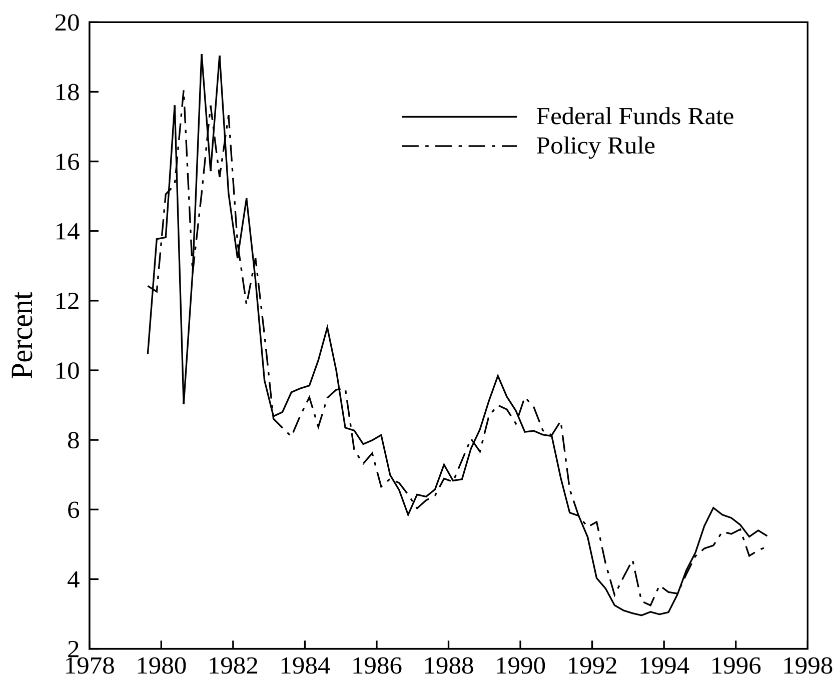

Figure 2. Federal Funds Rate and Estimated Policy Rule of Clarida et al. (1998).

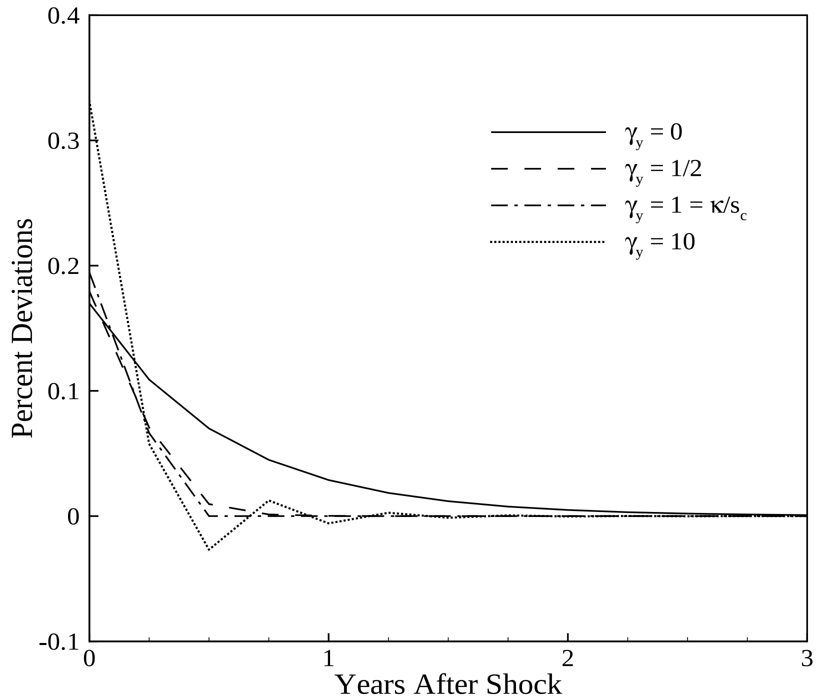

Figure 3. Response of Output to Interest Rate Shock Varying the Sensitivity of Monopolists to Changes in Output.

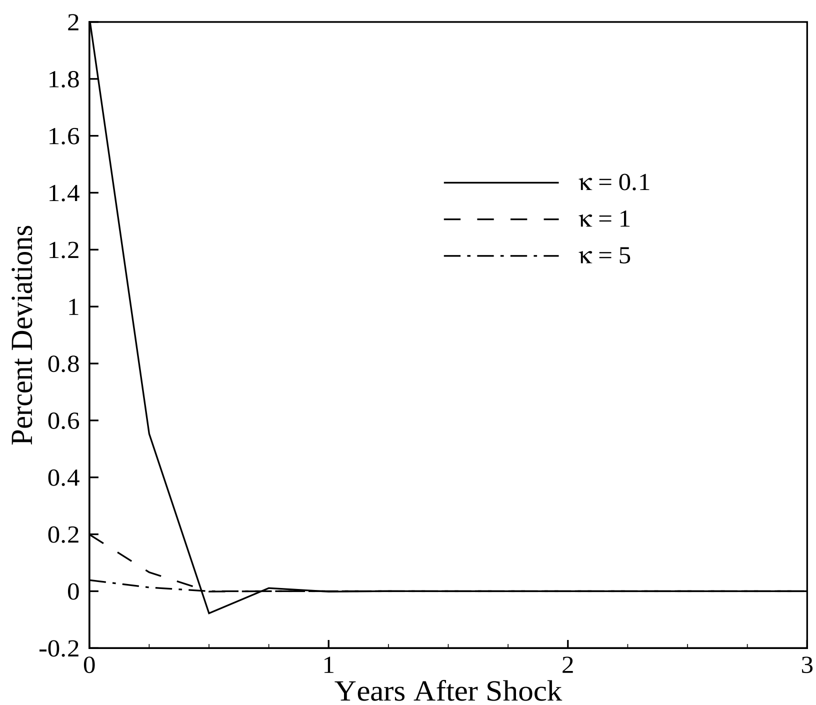

Figure 4. Response of Output to Interest Rate Shock Varying Risk Aversion.

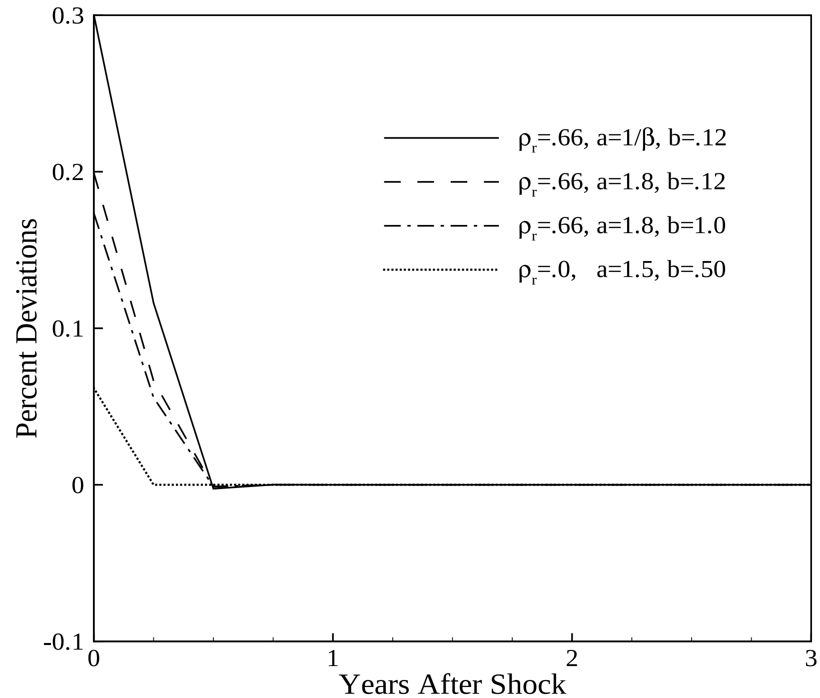

Figure 5. Response of Output to Interest Rate Shock Varying Taylor's Rule.

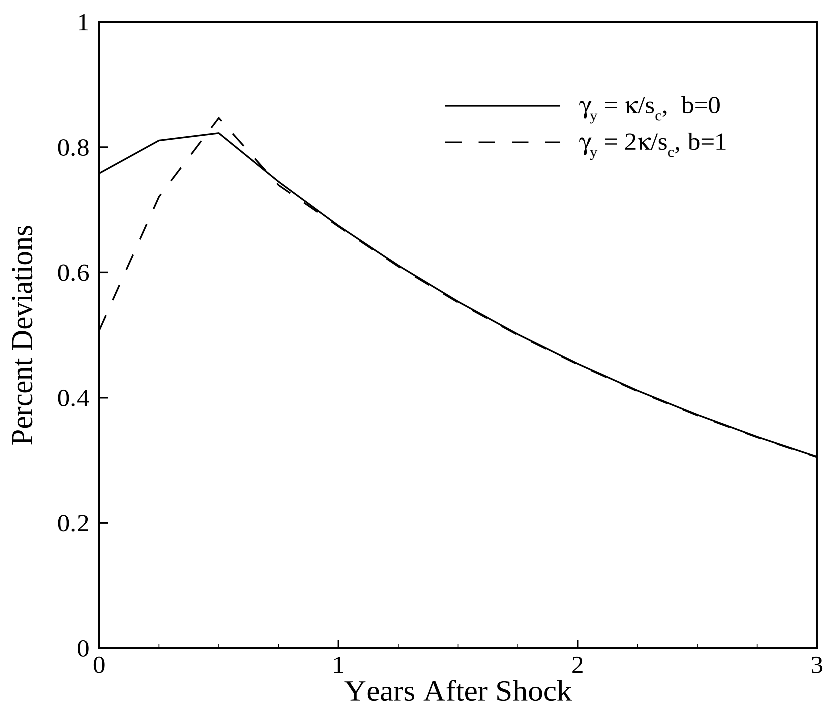

Figure 6. Response of Output to a Technology Shock Varying ξ and b.

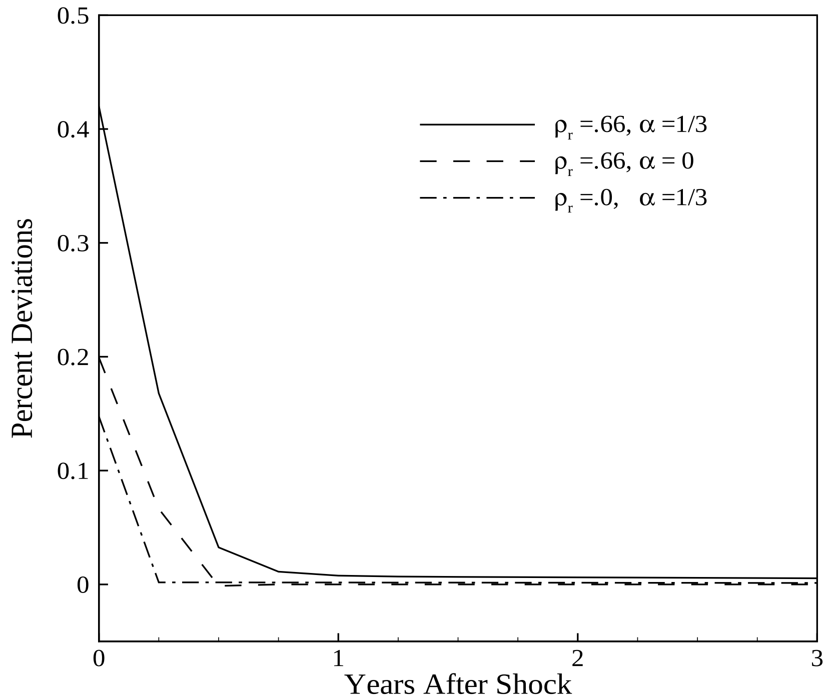

Figure 7. Response of Output to Spending Shock With and Without Capital.

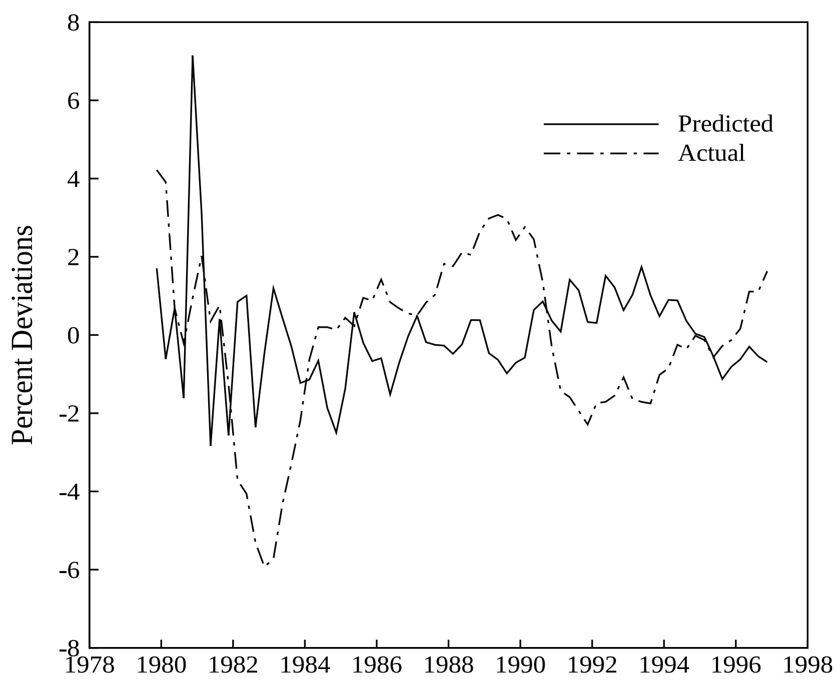

Figure 8. Model's Prediction for Logged and Detrended GDP
Using Interest Rate Innovations, 1979:3-1996:4.

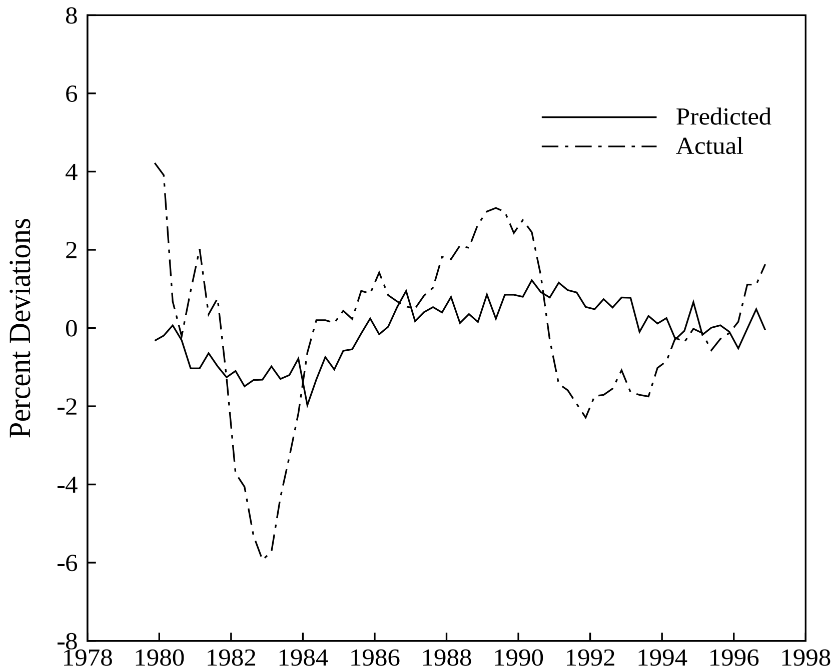

Figure 9. Model's Prediction for Logged and Detrended GDP Using Innovations in Government Spending, 1979:3-1996:4.

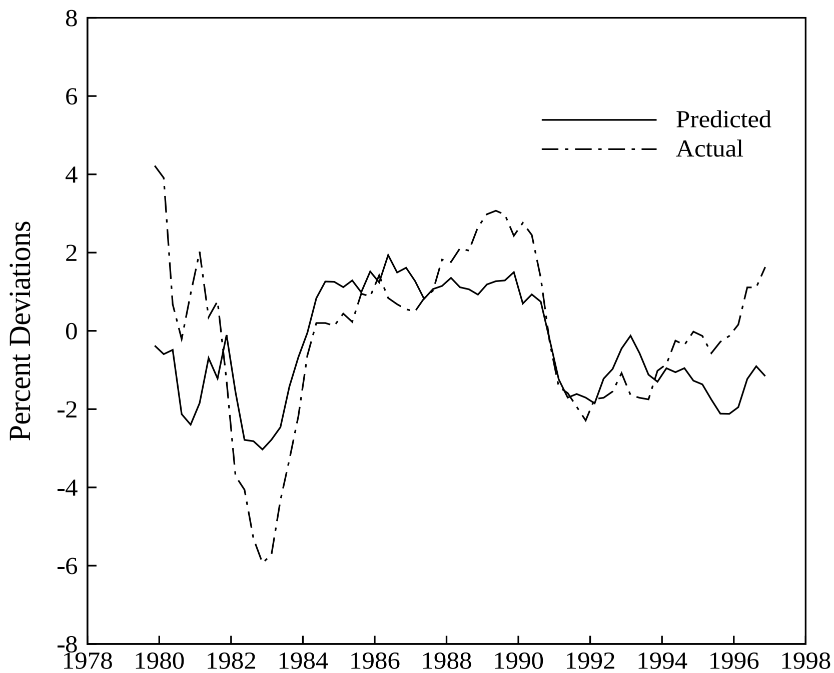

Figure 10. Model's Prediction for Logged and Detrended GDP Using Innovations in Technology, 1979:3-1996:4.

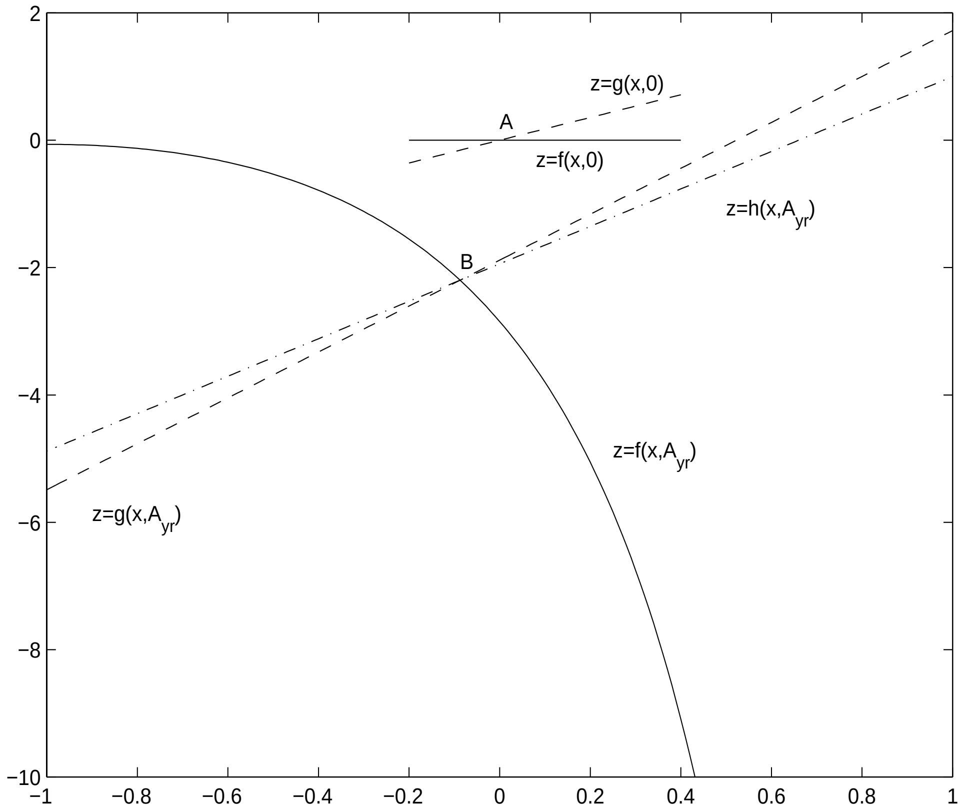

Figure C.1. An Equilibrium in the Fixed Capital Case.