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Business Cycle Accounting

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ABSTRACT _____

This paper proposes a simple method for guiding researchers in developing quantitative models of economic fluctuations. We show that a large class of models, including models with various frictions, are equivalent to a prototype growth model with time-varying wedges that, at least at face value, look like time-varying productivity, labor taxes, and capital income taxes. We label the time-varying wedges as *efficiency wedges*, *labor wedges*, and *investment wedges*. We use data to measure these wedges and then feed them back into the prototype growth model. We then assess the fraction of fluctuations accounted for by these wedges during the great depressions of the 1930s in the United States, Germany, and Canada. We find that the efficiency and labor wedges in combination account for essentially all of the declines and subsequent recoveries. Investment wedges play, at best, a minor role.

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We propose a simple method for guiding researchers in developing quantitative models of economic fluctuations. Our method has two components: an equivalence result and an accounting procedure. The equivalence result is that a large class of models, including models with various frictions, are equivalent to a prototype growth model with time varying wedges that, at least on face value, look like time-varying productivity, labor taxes, and capital income taxes. For example, we show that an economy in which the technology is constant but input financing frictions vary over time is equivalent to a growth model with time-varying productivity. We show that models with sticky wages or labor unions are equivalent to a growth model with time-varying labor taxes, and a model with investment financing frictions is equivalent to a growth model with time-varying capital income taxes. These examples lead us to label the time varying wedges as *efficiency wedges*, *labor wedges*, and *investment wedges*.

Our accounting procedure begins by using the data together with the equilibrium conditions of a prototype growth model to measure the wedges. We then feed the values of these wedges back into the growth model one at a time and in combinations to assess what fraction of the output movements can be attributed to each wedge separately and in combination. (Of course, in a deterministic model, by construction, all three wedges account for all of the observed movements in output.)

We apply our method to the great depressions in the 1930s in three countries: the United States, Germany, and Canada. In all three countries output declined dramatically in the early 1930's and then recovered to varying extents in the late 1930's. In the United States and Canada the recovery was slow while in Germany the recovery was rapid. Our accounting yields clear results for all three countries. The efficiency wedge alone accounts for roughly half of the decline in output in all three countries. This wedge, however, accounts for at most a third of the decline in labor. The labor wedge alone alone accounts for roughly half of the decline in output and accounts for essentially all of the decline in labor. The efficiency and labor wedges in combination account for essentially all of the fall and subsequent recovery in

output. These wedges in combination also account well for the decline and recovery in labor and investment. These findings lead us to conclude that investment wedges play, at best, a minor role in these depressions.

The goal of this business cycle accounting is to guide researchers in developing detailed models with the kinds of frictions that can deliver the quantitatively relevant types of observed wedges in the prototype economy. For example, both the sticky wage and cartelization theories are promising explanations of the observed labor wedges, while the simplest models of investment financing frictions are not. Theorists attempting to develop models of particular channels through which shocks cause large fluctuations in output will benefit from asking whether those channels are consistent with the fluctuations in wedges that we document.

We emphasize that we view our method as a useful first step in guiding the construction of detailed models. In building detailed models, theorists face hard choices on where to introduce frictions in markets. Our method is intended to help in making those choices. Our method is not a procedure for testing particular detailed models. If a detailed model is already at hand, then presumably it makes sense to confront that model directly with the data.

We also emphasize that our method is not well suited to identifying the source of primitive shocks. It is intended to help us understand the mechanisms through which such shocks lead to economic fluctuations. For example, many economists agree that monetary shocks drove the U.S. Great Depression but disagree about the details of the mechanism. For example, Bernanke (1983) argues that financial frictions play a central role and in the Bernanke and Gertler (1987) model these financial frictions show up as investment wedges. In Bordo, Erceg and Evans (2000) sticky nominal wages play a central role and these frictions show up as labor wedges. In our paper we develop a model entirely consistent with the views of Bernanke (1983) but for which financial frictions show up as efficiency wedges. It is possible to extend our model to have monetary shocks as the primitive source of fluctuations in these

frictions. Our findings for the great depressions suggest that, to the extent that monetary shocks drove the depressions, the mechanisms in Bordo, Erceg and Evans (2000) and our paper are more promising than those in Bernanke and Gertler (1987).

Other economists, like Cole and Ohanian (1999) and Prescott (1999), argue that non-monetary government policies, played an important role in the U.S. Great Depression, especially in the slow recovery. Cole and Ohanian (2001) develop a model in which government-sanctioned increases in the power of unions and cartels lead to labor wedges. It is easy to develop alternative models in which poor government policies lead to efficiency or investment wedges. Our findings suggest that only the models that emphasize the role of efficiency and labor wedges are potentially promising.

The plan of the paper is as follows. We illustrate our equivalence result using several models. We first develop a model with input-financing frictions and show that, in terms of aggregates, it is equivalent to a growth model with productivity shocks. We then show that a sticky wage model, along the lines of Bordo, Erceg, and Evans (2000) and a monopoly union model, along the lines of Cole and Ohanian (2001), are equivalent to a growth model with labor wedges. Finally, we show that a model with investment frictions, along the lines of Bernanke and Gertler (1989) and Carlstrom and Fuerst (1997), is equivalent to a growth model with investment wedges.

We then apply our method to the great depressions in the United States, Germany and Canada. Finally, we examine the sensitivity of our results to alternative specifications of capital utilization. We find that our results are not sensitive.

1. Equivalence Results

Here we show how various models with underlying distortions map into a prototype economy with one or more wedges. We choose simple models to illustrate this mapping. Since many models map into the same configuration of wedges, identifying one particular configuration does not uniquely identify a model; rather, it identifies a whole class of models

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consistent with that configuration. In this sense, our method does not uniquely determine the most promising model; rather it guides researchers to focus on the key margins that need to be distorted.

The prototype economy is a growth model with three stochastic variables: the *efficiency wedge* A_t, the *labor wedge* $1 - \tau_{\ell t}$, and the *investment wedge* $1 - \tau_{kt}$. Consumers maximize expected utility over consumption c_t and labor l_t

$$E_t \sum_t \beta^t U(c_t, l_t)$$

subject to the budget constraint

$$c_t + k_{t+1} - (1 - \delta)k_t = (1 - \tau_{\ell t})w_t l_t + (1 - \tau_{kt})r_t k_t + T_t$$

where k_t denotes the capital stock, w_t the wage rate, r_t the rental rate on capital, β is the discount factor, δ is the depreciation rate, and T_t denotes lump-sum taxes.

Firms maximize $A_t F(k_t, l_t) - r_t k_t - w_t l_t$. The equilibrium is summarized by the resource constraint,

$$c_t + k_{t+1} = y_t + (1 - \delta)k_t$$

together with

$$y_t = A_t F(k_t, l_t), \tag{1}$$

$$-\frac{U_{lt}}{U_{ct}} = (1 - \tau_{\ell t})A_t F_{lt}, \tag{2}$$

$$U_{ct} = \beta E_t U_{ct+1}[(1 - \tau_{kt+1})A_{t+1}F_{kt+1} + 1 - \delta]. \tag{3}$$

Notice that the labor wedge and the investment wedge resemble tax rates on labor income and capital income, respectively. One could consider more elaborate models with other kinds of frictions that look like taxes on consumption or on investment. Consumption taxes induce a wedge between the consumption-leisure marginal rate of substitution and the

marginal product of labor in exactly the same way as do labor taxes. Investment taxes induce a wedge between the intertemporal marginal rate of substitution and the marginal product of capital which is slightly different from that induced by a tax on capital income. In our application of business cycle accounting to depressions, we allow for wedges that look like investment taxes as well as wedges that look like capital income taxes.

2. Efficiency wedges

Here we develop a detailed economy with input financing frictions and show that it maps into a prototype economy with an efficiency wedge. In the detailed economy financing frictions lead to some firms having to finance working capital requirements at higher interest rates than other firms. These frictions lead to a misallocation of inputs across firms. We show that this misallocation of inputs manifests itself in the prototype economy as an efficiency wedge.

A. A detailed economy with input financing frictions

Consider a simple economy with distortions in the allocation of intermediate inputs across two types of firms arising from financing frictions. Both types of firms must borrow in advance of production to pay for an intermediate input. The first type of firm is financially constrained in the sense that it pays a higher price for borrowing than the second type. We think of these frictions as capturing the idea that some firms, namely small firms, find it difficult to finance borrowing. One source of the higher price paid by the financially constrained firms is that moral hazard problems are more severe for small firms.

Specifically, consider the following economy. Aggregate gross output q_t is made from combining the gross output q_{it} from two sectors according to

$$q_t = q_{1t}^{\gamma} q_{2t}^{1-\gamma}. \tag{4}$$

The representative producer of this gross output chooses q_{1t} and q_{2t} to solve

$$\max \ q_t - p_{1t}q_{1t} - p_{2t}q_{2t}$$

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subject to (4).

The resource constraint for gross output is

$$c_t + k_{t+1} + m_{1t} + m_{2t} = q_t + (1 - \delta)k_t \tag{5}$$

where c_t is consumption, k_t is the capital stock, and m_{1t} and m_{2t} are intermediate goods used in sectors 1 and 2, respectively. Final output, given by $y_t = q_t - m_{1t} - m_{2t}$, is gross output less the use of goods as intermediate goods.

The gross output of sector i, q_{it}, is made from intermediate goods m_{it} and a composite value-added good z_{it} according to

$$q_{it} = m_{it}^\theta z_{it}^{1-\theta} \tag{6}$$

where the composite value-added good is produced from capital k_t and labor l_t according to

$$z_{1t} + z_{2t} = z_t = F(k_t, l_t). \tag{7}$$

The producer of gross output of sector i chooses the composite good z_{it} and the intermediate good m_{it} to solve

$$\max \ p_{it}q_{it} - v_t z_{it} - R_{it}m_{it}$$

subject to (6). Here R_{it} is the gross *within-period* interest rate paid on borrowing by firms in sector i. We imagine that firms in sector 1 are more financially constrained that those in sector 2 so that $R_{1t} > R_{2t}$. Let $R_{it} = R_t(1 + \tau_{it})$ where R_t is the rate savers earn within the period t and τ_{it} measures the within-period spread between the rate paid to savers and the rate paid by borrowers in sector i induced by financing constraints. Since consumers do not discount utility within the period, $R_t = 1$.

The producer of the composite good z_t chooses k_t and l_t to solve

$$\max \ v_t z_t - w_t l_t - r_t k_t$$

subject to (7), where v_t is the price of the composite good, w_t is the wage rate and r_t is the rental rate on capital.

Consumers solve

$$\max \sum_{t=0}^{\infty} \beta^t U(c_t, l_t) \tag{8}$$

subject to

$$c_t + k_{t+1} = r_t k_t + w_t l_t + (1 - \delta) k_t + T_t$$

where $l_t = l_{1t} + l_{2t}$ is labor supply and $T_t = R_t \sum_i \tau_{it} m_{it}$ are lump sum payments. Here we assume that the financing frictions act like distorting taxes and the proceeds are rebated to households. If instead we assume that the financing frictions represent, say, lost gross output then we would adjust the resource constraint (5) accordingly.

B. The associated prototype economy with efficiency wedges

Now consider a version of the prototype economy of Section 1 that will have the same aggregate allocations as our input-financing-frictions economy. The prototype is a one sector growth model with the resource constraint

$$c_t + k_{t+1} = y_t + (1 - \delta) k_t$$

where $y_t = A_t F(k_t, l_t)$ is output and the consumer maximizes the utility function in (8) subject to the budget constraint

$$c_t + k_{t+1} = (1 - \tau_{kt}) r_t k_t + (1 - \tau_{lt}) w_t l_t + (1 - \delta) k_t + T_t$$

Here the efficiency wedge is given by

$$A_t = \kappa (a_{1t}^{1-\gamma} a_{2t}^{\gamma})^{\frac{\theta}{1-\theta}} (1 - \frac{\theta}{a_{1t} + a_{2t}}) \tag{9}$$

where $a_{1t} = \gamma/(1 + \tau_{1t})$ and $a_{2t} = (1 - \gamma)/(1 + \tau_{2t})$ and $\kappa = \gamma^{\gamma}(1 - \gamma)^{1-\gamma} \theta^{\frac{\theta}{1-\theta}}$ and τ_{1t} and τ_{2t} are the interest rate spreads in the detailed economy. We prove the following proposition in an appendix which is available upon request.

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Proposition 1. Consider a prototype economy with exogenous processes A_t for productivity given in (9) and

$$\frac{1}{1 - \tau_{lt}} = \frac{1}{1 - \theta} \left[1 - \theta \left(\frac{\gamma}{1 + \tau_{1t}} + \frac{1 - \gamma}{1 + \tau_{2t}} \right) \right] \tag{10}$$

and $\tau_{kt} = \tau_{lt}$. Then the allocations in the prototype economy coincide with those of the detailed economy.

Imagine that in the economy with input financing frictions τ_{1t} and τ_{2t} fluctuate over time but in such a way that the weighted average of the interest rate spreads

$$a_{1t} + a_{2t} = \frac{\gamma}{1 + \tau_{1t}} + \frac{1 - \gamma}{1 + \tau_{2t}}, \tag{11}$$

is constant but $a_{1t}^{1-\gamma} a_{2t}^{\gamma}$ fluctuates. Then from (10) we see that the labor and investment wedges are constant and from (9) we see that the efficiency wedge fluctuates. Thus, on average, financing frictions are unchanged but relative frictions fluctuate. An outside observer who attempted to fit the data generated by the economy with input financing frictions using the prototype economy would identify the fluctuations in relative distortions with fluctuations in technology and would see no fluctuations in either the labor wedge $1 - \tau_{lt}$ or the investment wedge τ_{kt}. In particular, periods in which the relative distortions increase would be misinterpreted as periods of technological regress. This observation leads us to label A_t as the efficiency wedge in the prototype economy.

More generally, fluctuations in the interest rate spreads τ_{1t} and τ_{2t} which lead to fluctuations in τ_{lt} and τ_{kt} show up in the prototype economy as fluctuations in all of the wedges.

3. Labor wedges

We turn now to economies with distortions in the labor market. We will show that they map into prototype economies with labor wedges.

A. Sticky wages

We first describe a sticky-wage economy and then we map it into the prototype economy with labor wedges.

A detailed economy with sticky wages

Consider a monetary economy populated by a large number of identical, infinitely lived consumers. In each period t, the economy experiences one of finitely many events s_t. We denote by $s^t = (s_0, \ldots, s_t)$ the history of events up through and including period t. The probability, as of period 0, of any particular history s^t is $\pi(s^t)$. The initial realization s_0 is given. The economy consists of a competitive final goods producer and a continuum of monopolistically competitive unions that set their nominal wages in advance of the realization of the shocks. Each union represents all consumers with a specific type of labor.

In each period t, the commodities in this economy are a consumption-capital good, money, and a continuum of differentiated types of labor indexed by $j \in [0, 1]$. The technology for producing final goods from capital and a labor aggregate at history s^t is constant returns to scale and is given by

$$y(s^t) = F\big(k(s^{t-1}), l(s^t)\big), \tag{12}$$

where $y(s^t)$ is the final good, $k(s^{t-1})$ is capital and

$$l(s^t) = \left[\int l(j, s^t)^v \, dj \right]^{\frac{1}{v}}. \tag{13}$$

is an aggregate of the differentiated types of labor $l(j, s^t)$.

The final goods producer behaves competitively. This producer has some initial capital stock $k(s^{-1})$ and accumulates capital according to

$$k(s^t) = (1 - \delta)k(s^{t-1}) + x(s^t) \tag{14}$$

where $x(s^t)$ is investment. The present discounted value of profits for this producer are

$$\max \sum_{t=0}^{\infty} Q(s^t) \left[P(s^t) y(s^t) - P(s^t) x(s^t) - W(s^{t-1}) l(s^t) \right] \tag{15}$$

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where $Q(s^t)$ is the price of a dollar at s^t in an abstract unit of account, $P(s^t)$ is the dollar price of final goods at s^t and $W(s^{t-1})$ is the aggregate nominal wage at s^t which only depends on s^{t-1} because of wage stickiness. The producer's problem can be stated in two parts. First, the producer chooses sequences for capital, $k(s^{t-1})$, investment, $x(s^t)$, and aggregate labor, $l(s^t)$, subject to (12) and (14). The first order conditions can be summarized by

$$P(s^t)F_l(s^t) = W(s^{t-1})$$

$$Q(s^t)P(s^t) = \sum_{s_{t+1}} Q(s^{t+1})P(s^{t+1})\left\{F_k(s^{t+1}) + 1 - \delta\right\}$$

Second, for any given amount of aggregate labor $l(s^t)$, the demand for each type of differentiated labor is given by the solution to

$$\min_{\{l(j,s^t)\},j\in[0,1]} \int W(j,s^{t-1})l(j,s^t)\,dj \tag{16}$$

subject to (13) where $W(j,s^{t-1})$ is the nominal wage for differentiated labor of type j. Nominal wages are set by unions before the realization of the shock in period t, and thus they can depend on, at most, s^{t-1}. The demand for labor of type j by the final goods producer is

$$l^d(j,s^t) = \left(\frac{W(s^{t-1})}{W(j,s^{t-1})}\right)^{\frac{1}{1-v}} l(s^t), \tag{17}$$

where $W(s^{t-1}) \equiv \left[\int W(j,s^{t-1})^{\frac{v}{v-1}}\,dj\right]^{\frac{v-1}{v}}$ is the aggregate nominal wage. The minimized value in (16) is thus $W(s^{t-1})l(s^t)$.

Consumers can be thought of as being organized into a continuum of unions indexed by j. Each union consists of all the consumers in the economy with labor of type j. Each union realizes that it faces a downward-sloping demand for its type of labor given by (17). In each period these new wages are set before the realization of the current money shocks.

The preferences of a representative consumer in the jth union is

$$\sum_{t=0}^{\infty}\sum_{s^t} \beta^t \pi(s^t) U\left(c(j,s^t),l(j,s^t),M(j,s^t)/P(s^t)\right) \tag{18}$$

where $c(j, s^t), l(j, s^t), M(j, s^t)/P(s^t)$ are the consumption, labor supply and real money holdings of this consumer. In this economy there are complete markets for state-contingent nominal claims. We represent the asset structure by having complete, contingent, one-period nominal bonds. We let $B(j, s^t, s_{t+1})$ denote the consumers' holdings of such a bond purchased in period t and state s^t with payoffs contingent on some particular state s_{t+1} at $t + 1$. One unit of this bond pays one dollar in period $t + 1$ if the particular state s_{t+1} occurs and 0 otherwise. Let $Q(s^{t+1}|s^t)$ denote the dollar price of this bond in period t and state s^t. Clearly, $Q(s^{t+1}|s^t) = Q(s^{t+1})/Q(s^t)$.

The problem of the jth union is to maximize (18) subject to the budget constraints

$$P(s^t)c(j, s^t) + M(j, s^t) + \sum_{s_{t+1}} Q(s^{t+1}|s^t)B(j, s^{t+1})$$
$$\leq W(j, s^{t-1})l^d(j, s^t) + M(j, s^{t-1}) + B(j, s^t) + T(s^t) + D(s^t)$$

and the borrowing constraint $B(s^{t+1}) \geq -P(s^t)\bar{b}$ where $l^d(j, s^t)$ is given by (17). Here $T(s^t)$ is transfers of home currency and the positive constant $\bar{b}$ constrains the amount of real borrowing of the consumer. Also, $D(s^t) = P(s^t)y(s^t) - P(s^t)x(s^t) - W(s^{t-1})l(s^t)$ are the dividends paid by the firms. The initial conditions $M(j, s^{-1})$, and $B(j, s^0)$ are given and assumed to be the same for all j. Notice that in this problem, the union chooses the wage and agrees to supply whatever is demanded at that wage.

The first order conditions for this problem can be summarized by

$$\frac{U_m(j, s^t)}{P(s^t)} - \frac{U_c(j, s^t)}{P(s^t)} + \beta \sum_{s_{t+1}} \pi(s^{t+1}|s^t)\frac{U_c(j, s^{t+1})}{P(s^{t+1})} = 0, \tag{19}$$

$$Q(s^t|s^{t-1}) = \beta\pi(s^t|s^{t-1})\frac{U_c(j, s^t)}{U_c(j, s^{t-1})}\frac{P(s^{t-1})}{P(s^t)}, \tag{20}$$

$$W(j, s^{t-1}) = -\frac{\sum_{s^t} Q(s^t)P(s^t)U_l(j, s^t)/U_c(j, s^t)l^d(j, s^t)}{v \sum_{s^t} Q(s^t)l^d(j, s^t)}. \tag{21}$$

Here $U_c(s^t)$, $U_l(s^t)$, and $U_m(s^t)$ denote the derivatives of the utility function with respect to its arguments, and $\pi(s^{t+1}|s^t) = \pi(s^{t+1})/\pi(s^t)$ is the conditional probability of s^{t+1} given.

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Notice that in a steady state, this condition reduces to $W/P = (1/v)(-U_l/U_c)$, so that real wages are set as a markup over the marginal rate of substitution between labor and consumption. Given the symmetry among the unions it is clear that all of them choose the same consumption, labor, money balances, bond-holdings and wages which we denote by simply $c(s^t)$, $l(s^t)$, $M(s^t)$, $B(s^{t+1})$ and $W(s^{t-1})$.

Consider next the specification of the money supply process and the market clearing conditions. The nominal money supply process is given by $M(s^t) = \mu(s^t)M(s^{t-1})$, where $\mu(s^t)$ is a stochastic process. New money balances are distributed to consumers in a lump-sum fashion by having nominal transfers satisfy $T(s^t) = M(s^t) - M(s^{t-1})$. The resource constraint for this economy is

$$c(s^t) + k(s^t) = y(s^t) + (1-\delta)k(s^{t-1}). \tag{22}$$

Bond market clearing requires that $B(s^{t+1}) = 0$. An *equilibrium* for this economy is defined in the obvious fashion.

The associated prototype economy with labor wedges

Consider now a prototype economy with money and taxes and a technology given by

$$y(s^t) = F\big(k(s^{t-1}), l(s^t)\big). \tag{23}$$

The representative firm solves the problem

$$\max \sum_{t=0}^{\infty} Q(s^t) \Big[P(s^t)y(s^t) - P(s^t)x(s^t) - W(s^{t-1})l(s^t) \Big] \tag{24}$$

subject to $k(s^t) = (1-\delta)k(s^{t-1}) + x(s^t)$, where $x(s^t)$ is investment and $k(s^{-1})$ is given. The first order conditions can be summarized by

$$P(s^t)F_l(s^t) = W(s^t)$$

$$Q(s^t)P(s^t) = \sum_{s_{t+1}} Q(s^{t+1})P(s^{t+1})\big\{F_k(s^{t+1}) + 1 - \delta\big\}$$

The representative household maximizes

$$\sum_{t=0}^{\infty}\sum_{s^t} \beta^t \pi(s^t) U\left(c(s^t), l(s^t), M(s^t)/P(s^t)\right) \tag{25}$$

subject to the budget constraint

$$P(s^t)c(s^t) + M(s^t) + \sum_{s_{t+1}} Q(s^{t+1}|s^t)B(s^{t+1})$$
$$\leq W^*(s^t)(1 - \tau_l(s^t))l(s^t) + M(s^{t-1}) + B(s^t) + T(s^t) + D(s^t).$$

and a bound on bond holdings, where the lump sum transfer $T(s^t) = M(s^t) - M(s^{t-1}) + \tau_l(s^t)l(s^t)$ and the dividends $D(s^t) = P(s^t)y(s^t) - P(s^t)x(s^t) - W(s^{t-1})l(s^t)$. The first order conditions for this problem are summarized by

$$\frac{U_m(s^t)}{P(s^t)} - \frac{U_c(s^t)}{P(s^t)} + \beta \sum_{s_{t+1}} \pi(s^{t+1}|s^t)\frac{U_c(s^{t+1})}{P(s^{t+1})} = 0, \tag{26}$$

$$Q(s^t|s^{t-1}) = \beta\pi(s^t|s^{t-1})\frac{U_c(s^t)}{U_c(s^{t-1})}\frac{P(s^{t-1})}{P(s^t)}, \tag{27}$$

An equilibrium for the prototype economy is defined in the usual fashion.

Consider an equilibrium of the sticky wage economy for some given stochastic process $M^*(s^t)$ on money growth. Denote all of the allocations and prices in this equilibrium with asterisks.

Proposition 2. Consider a prototype economy with a given stochastic process for money growth $M(s^t) = M^*(s^t)$ and

$$1 - \tau_l(s^t) = -U_l^*(s^t)/U_c^*(s^t)F_l^*(s^t)$$

where $U_l^*(s^t), U_c^*(s^t)$ and $F_l^*(s^t)$ are evaluated at the equilibrium at the sticky wage economy. Then the equilibrium allocations and prices in the sticky wage economy coincide with those in the prototype economy.

The proof is immediate from comparing the first order conditions, the budget constraints and the resource constraints for the two economies.

Suppose next that the utility function of consumers in the sticky wage economy is additively separable in money, so that $U(c, l, m) = u(c, l) + v(m)$. Consider a real prototype economy which is a one sector real growth model with labor income taxes. Let the utility function be

$$\sum_{t=0}^{\infty} \sum_{s^t} \beta^t \pi(s^t) \, u\left(c(s^t), l(s^t)\right) \tag{28}$$

and the technology be the same as in the monetary prototype economy. Define the economy and the equilibrium in the standard way. The following is immediate.

Corollary 1. Consider a real prototype economy with a given stochastic process for labor wedges

$$1 - \tau_l(s^t) = -u_l^*(s^t)/u_c^*(s^t)F_l^*(s^t)$$

where $u_l^*(s^t), u_c^*(s^t)$ and $F_l^*(s^t)$ are evaluated at the equilibrium at the sticky wage economy with preferences of the form (28). Then the equilibrium allocations in the sticky wage economy coincide with those in the real prototype economy.

B. Unions

In this section, we describe an economy with unions and then map it into the prototype economy with labor wedges.

The detailed economy with unions

Consider the following economy in which fluctuations in policies towards unions show up as fluctuations in labor market distortions in the prototype economy. (See Cole and Ohanian 2001 for a discussion of such policies in the Great Depression.) The economy is a non-monetary version of the sticky wage economy. The technology for producing final goods is given by (12) and (13). Capital is accumulated according to (14). The problem faced by the final goods producer is

$$\max \sum_{t=0}^{\infty} q(s^t)) \left[y(s^t) - x(s^t) - w(s^t)l(s^t)\right] \tag{29}$$

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where $q(s^t)$ is the price of a unit of consumption goods at s^t in an abstract unit of account and $w(s^t)$ is the aggregate real wage at s^t. The producer's problem can be stated in two parts. First, the producer chooses sequences for capital, $k(s^{t-1})$, investment, $x(s^t)$, and aggregate labor, $l(s^t)$, subject to (12) and (14). The demand for labor of type j by the final goods producer is

$$l^d(j, s^t) = \left(\frac{w(s^t)}{w(j, s^t)} \right)^{\frac{1}{1-v}} l(s^t), \tag{30}$$

where $w(s^t) \equiv \left[\int w(j, s^t)^{\frac{v}{v-1}} \, dj \right]^{\frac{v-1}{v}}$ is the aggregate wage.

Analogously to the sticky wage economy, the representative union faces a downward-sloping demand for its type of labor given by (30) in setting its wage. The problem of the jth union is to maximize

$$\sum_{t=0}^{\infty} \sum_{s^t} \beta^t \pi(s^t) u \left(c(j, s^t), l(j, s^t) \right). \tag{31}$$

subject to the budget constraints

$$c(j, s^t) + \sum_{s_{t+1}} q(s^{t+1}|s^t) b(j, s^{t+1}) \leq w(s^t) l^d(j, s^t) + b(j, s^t) + d(s^t).$$

and the borrowing constraint $b(s^{t+1}) \geq -\bar{b}$ where $l^d(j, s^t)$ is given by (30). Here $b(j, s^t, s_{t+1})$ denotes the consumers' holdings of one period state contingent bond purchased in period t and state s^t with payoffs contingent on some particular state s_{t+1} at $t+1$ and $q(s^{t+1}|s^t)$ is its corresponding price. Clearly, $q(s^{t+1}|s^t) = q(s^{t+1})/q(s^t)$. Also, $d(s^t) = y(s^t) - x(s^t) - w(s^t)l(s^t)$ are the dividends paid by the firms. The initial conditions $b(j, s^0)$ are given and assumed to be the same for all j.

The only distorted first-order condition for this problem is

$$w(j, s^t) = -\frac{1}{v} \frac{U_l(j, s^t)}{U_c(j, s^t)}. \tag{32}$$

Notice that real wages are set as a markup over the marginal rate of substitution between labor and consumption. Given the symmetry among the unions it is clear that all of them

choose the same consumption, labor, bond-holdings and wages which we denote by simply $c(s^t)$, $l(s^t)$, $b(s^{t+1})$ and $w(s^t)$ and the resource constraint is as in (22).

We think of government pro-competitive policy as limiting the monopoly power of unions putting pressure on them to limit their anti-competitive behavior. We model the government policy as enforcing provisions the make the unions price competitively if the markups exceed, say $1/\bar{v}(s^t)$ where $\bar{v}(s^t) \leq v$. Under such a policy it is immediate that the markup charged by unions is $1/\bar{v}(s^t)$.

The associated prototype economy with labor wedges

Consider next a prototype economy in which the firm maximizes the present discounted value of profits

$$\max \sum_{t=0}^{\infty} q(s^t)) \left[F(k(s^{t-1}), l(s^t)) - x(s^t) - w(s^t)l(s^t) \right] \tag{33}$$

subject to $k(s^t) = (1 - \delta)k(s^{t-1}) + x(s^t)$. Consumers maximize

$$\sum_{t=0}^{\infty} \sum_{s^t} \beta^t \pi(s^t) \, u\left(c(s^t), l(s^t) \right) \tag{34}$$

subject to

$$c(s^t) + \sum_{s_{t+1}} q(s^{t+1}|s^t)b(s^{t+1}) \leq (1 - \tau(s^t))w(s^t)l(s^t) + b(s^t) + d(s^t) + T(s^t).$$

where the dividends $d(s^t) = F(k(s^{t-1}), l(s^t)) - x(s^t) - w(s^t)l(s^t)$ and the lump sum transfers $T(s^t) = \tau(s^t)w(s^t)l(s^t)$. The resource constraint is as in (22). The only distorted first order condition is

$$(1 - \tau(s^t))w(s^t) = -\frac{U_l(j, s^t)}{U_c(j, s^t)}.$$

The following proposition is immediate.

Proposition 3. Consider this prototype economy with a given stochastic process for labor income taxes

$$1 - \tau(s^t) = \bar{v}(s^t).$$

Then the allocations and prices of the prototype economy coincide with those of the unionized economy.

4. Investment Wedges

A variety of investment frictions affect the economy by raising the cost of investment. These frictions show up in the prototype economy as a tax on investment. Some investment frictions also show up in the prototype economy as wasted resources in both the resource constraint and the capital accumulation equation. One such example is due to Carlstrom and Fuerst (1997) who exposit a quantitative version with infinitely-lived households of the Bernanke-Gertler (1989) model. In this section we show the equivalence between the Carlstrom and Fuerst model and a particular prototype growth model.

A. A detailed economy with investment frictions

In the Carlstrom-Fuerst model, there is a continuum of risk neutral entrepreneurs of mass η and a continuum of consumers of mass 1. The timing is as follows. At the beginning of each period each consumer supplies l_t units of labor, each entrepreneur supplies l_{et} units of labor and each consumer and each entrepreneur rents capital denoted k_{ct} and k_{et} to firms that produce output according to $F(k_{ct} + \eta k_{et}, l_t, \eta l_{et})$. These firms solve

$$\max \ F(k_{ct} + \eta k_{et}, l_t, \eta l_{et}) - r_t(k_{ct} + \eta k_{et}) - w_t l_t - w_{et} l_{et}$$

where r_t is the rental rate on capital and w_t and w_{et} are the wage rates of consumers and entrepreneurs.

Consumers solve the problem

$$\max \ \sum_{t=0}^{\infty} \beta^t U(c_t, l_t)$$

subject to

$$c_t + q_t[k_{ct+1} - (1 - \delta)k_{ct}] = w_t l_t + r_t k_{ct} + T_t$$

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where q_t is the price of the investment good in units of the consumption good and T_t is a lump-sum transfer. Combining the first-order conditions for the firms and consumers gives

$$-\frac{U_{lt}}{U_{ct}} = F_{lt} \tag{35}$$

$$q_t U_{ct} = \beta U_{ct+1}[q_{t+1}(1-\delta) + F_{kt+1}] \tag{36}$$

Consumption goods can be transformed into capital goods only by entrepreneurs. Each entrepreneur owns a technology that transforms i_t units of consumption goods at the beginning of any period t into $\omega_t i_t$ units of capital goods at the end of the period where ω_t is i.i.d. across entrepreneurs and time and has density ϕ and c.d.f. Φ. The realization of ω_t is private information to the entrepreneur. At the beginning of each period each entrepreneur supplies one unit of labor inelastically, receives labor income w_{et}, receives rental income $r_t k_{et}$ and pays taxes T_{et}. In addition, the value of the entrepreneur's capital is $q_t k_{et}(1-\delta)$. Thus, the entrepreneur's net worth in period t

$$a_t = w_{et} + k_e[r_t + q_t(1-\delta)] - T_{et}. \tag{37}$$

Entrepreneurs can use their net worth together with funds borrowed from financial intermediaries to purchase consumption goods and transform them into capital goods. The financial intermediaries can monitor the realized output $\omega_t i_t$ by paying μi_t units of the capital good. The key restriction on trades is that entrepreneurs are allowed only to trade in within period deterministic contracts that are made before the realization of ω_t and payoff after its realization. (In particular, the risk neutral entrepreneurs are prohibited from entering into contracts that share aggregate risk with the consumers.) With such a restriction, following Townsend (1979), it is straightforward to show that the optimal contract is a type of risky debt in which the entrepreneur pays a fixed amount $R_t(i_t - n_t)$ if ω_t is greater than some cutoff level $\bar{\omega}_t$ and $\omega_t i_t$ otherwise where $R_t(i_t - n_t) = \bar{\omega}_t i_t$. The intermediaries monitor the entrepreneur if and only if $\omega_t < \bar{\omega}_t$. Under such a contract the expected income of the entrepreneur is

$$q_t i_t \left[\int_{\bar{\omega}_t}^{\infty} (\omega_t - \bar{\omega}_t)\phi(\omega)d\omega\right] \equiv q_t i_t f(\bar{\omega}_t)$$

and the expected income of the financial intermediary is

$$q_t i_t \left[\int_0^{\bar{\omega}_t} (\omega_t - \mu)\phi(\omega)d\omega + (1 - \Phi(\bar{\omega}_t))\bar{\omega}_t \right] \equiv q_t i_t g(\bar{\omega}_t)$$

The funds the intermediary lends out are from the consumers. The consumers can either store their consumption goods from the beginning until the end of the period at a zero rate of return or they can lend them to the entrepreneur through the financial intermediaries. The mass of entrepreneurs is sufficiently small such that the optimal contract maximizes their expected income subject to the constraint that the intermediary's gross return on the investment of $i_t - n_t$ is at least one. The contract then solves

$$\max_{i_t, \bar{\omega}_t} \; q_t i_t f(\bar{\omega}_t)$$

subject to

$$q_t i_t g(\bar{\omega}_t) \geq i_t - a_t \tag{38}$$

The first order conditions imply

$$\frac{f'(\bar{\omega}_t)}{f(\bar{\omega}_t)} + \frac{q_t g'(\bar{\omega}_t)}{1 - q_t g(\bar{\omega}_t)} = 0 \tag{39}$$

and, since (38) holds with equality, the optimal investment level is given by

$$i_t = \frac{a_t}{1 - q_t g(\bar{\omega}_t)}. \tag{40}$$

The expected income of each entrepreneur is thus

$$q_t i_t f(\bar{\omega}_t) = \frac{a_t q_t f(\bar{\omega}_t)}{1 - q_t g(\bar{\omega}_t)}. \tag{41}$$

which, by the law of large numbers, is the aggregate income of entrepreneurs. From (40) it is clear that investment by each entrepreneur is linear in that entrepreneur's net worth so that aggregate investment is linear in aggregate net worth. This aggregation result plus the law of law numbers implies that the aggregate capital held by entrepreneurs has the following law of motion

$$c_{et} + q_t k_{et+1} = [w_{et} + k_e(r_t + q_t(1 - \delta)) - T_{et}] \frac{q_t f(\bar{\omega}_t)}{1 - q_t g(\bar{\omega}_t)} \tag{42}$$

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where the right side is simply $q_t i_t f(\bar{\omega}_t)$ after substituting from (37) and (41). The entrepreneur's utility function is

$$\sum_{t=0}^{\infty} (\beta\gamma)^t c_{et} \qquad (43)$$

where γ is less than one. We assume that entrepreneurs discount the future at a higher rate than consumers. This assumption is needed because the within period rate of return earned by entrepreneurs is (weakly) greater than the rate of return earned by consumers. If entrepreneurs discounted the future as the same rate as consumers the entrepreneurs would postpone consumption indefinitely and no equilibrium would exist. Given the risk neutrality of the entrepreneurs and the aggregation result, it should be clear that the optimal decisions of the entrepreneurs can be obtained as a solution to maximize (43) subject to (42). The lump sum tax levied on entrepreneurs is redistributed to the consumers and hence $T_t = \eta T_{et}$.

B. The associated prototype economy with investment wedges

In the prototype economy the resource constraint is given by

$$c_t + x_t + g_t = F(k_t, l_t, \eta).$$

The firm solves

$$\pi_t = \max_{k_t, l_t} \ F(k_t, l_t, \eta) - w_t l_t - r_t k_t.$$

with first order conditions $F_{kt} = r_t$ and $F_{lt} = w_t$. Consumers maximize

$$\max \sum_{t=0}^{\infty} \beta^t U(c_t, l_t)$$

subject to

$$c_t + (1 + \tau_{xt})x_t = w_t l_t + r_t k_t + T_t + \pi_t$$

$$k_{t+1} = (1 - \delta)k_t + x_t(1 - \theta_t).$$

where the lump-sum transfer T_t in equilibrium is given by $\tau_{xt}x_t$. The first order conditions are summarized by

$$-\frac{U_{lt}}{U_{ct}} = w_t \tag{44}$$

$$\frac{1 + \tau_{xt}}{1 - \theta_t}U_{ct} = \beta U_{ct+1}[r_{t+1} + \frac{1 + \tau_{xt+1}}{1 - \theta_{t+1}}(1 - \delta)] \tag{45}$$

Denoting the equilibrium allocations in the Carlstrom-Fuerst economy with asterisks we have the following.

Proposition 4. Consider this prototype economy with a given stochastic processes for adjustment costs $\theta_t = \Phi(\bar{\omega}_t^*)\mu$, capital income taxes $1 + \tau_{xt} = q_t^*(1 - \theta_t)$, and government consumption $g_t = \eta c_{et}$. Then the aggregate allocations for the prototype economy coincide with those of the Carlstrom-Fuerst economy, where aggregate consumption in the prototype economy is $c_t + g_t$.

In this proposition we are imagining that aggregate consumption in the Carlstrom-Fuerst economy is measured by $c_t + \eta_t c_{et}$ and that in the associated prototype economy it is measured by $c_t + g_t$.

5. Business Cycle Accounting

Now we try to measure our three wedges and determine how much of actual output fluctuations they can account for in the great depressions of the United States, Germany, and Canada. (For recent attempts to assess the neoclassical growth model's performance in accounting for depressions in these countries see Cole and Ohanian (1999), Fisher and Hornstein (2002), and Amaral and MacGee (2002).)

Given data on y_t, k_t, l_t, and c_t, we manipulate the equations (1) and (2) to obtain efficiency and labor wedges:

$$A_t = \frac{F(k_t, l_t)}{y_t}, \tag{46}$$

$$1 - \tau_{\ell t} = \frac{A_t F_{lt},}{-U_{lt}/U_{ct}}. \tag{47}$$

Notice that equations (46) and (47) are static so obtaining a time series for these wedges only requires that we specify functional forms. Given such a time series we estimate a stochastic process for the efficiency and labor wedges. We then ask, What fraction of output fluctuations can be accounted for by the efficiency and labor wedges? We answer this question as follows. We solve our prototype economy with our estimated stochastic processes for the wedges. We then simulate our prototype economy with the realizations of the wedges measured from (46) and (47). The contribution of these wedges is measured by comparing the realizations of variables like output, labor and investment to the actual data on these variables.

To assess the contribution of the investment wedge, we utilize the result that the sum of the contribution of all three wedges is simply the data. To see this note that (1)-(3) give 3 equations for each period and state. Using the data on y_t, k_t, l_t, and c_t, together with an initial condition, these equations can be solved for realizations of each of the wedges. Thus, simulating the economy with these realized wedges simply reproduces the data. This result implies that the contribution of the investment wedge can be measured residually as the difference between the contribution of the efficiency and labor wedges and the data.

In our experiments we assume that the production function has the form $F(k, l) = k^\alpha l^{1-\alpha}$, and the utility function has the form $U(c, l) = \log c + \psi \log(\bar{l} - l)$. In our experiments we added government spending. We assumed that the technology grows at a constant rate so that on the balanced growth path the economy grows at 1.6 per cent per year, the rate of growth of technology for the United States in the early part of the twentieth century. We assume that $\beta = .97$, $\psi = 2.25$ and $\bar{l}$ is 5,000 hours per year. For the capital share, α, the depreciation rate, δ, and the share of government spending in output, g/y we used the national income accounts from the individual countries. The values of these parameters are reported in Table 1. We chose initial conditions for the wedges so that in the starting year the economies would be on a balanced growth path, given their observed consumption, investment, capital stock and labor input.

Parameters:		U.S.	Germany	Canada
Capital share	α	.340	.250	.330
Depreciation rate	δ	.061	.014	.025
Government spending share	g/y	.076	.121	.136

For now we have assumed the following simple stochastic process for the wedges. For each of the first 3 years following the start of the depression we assumed that each of the following three events is equally likely: the wedges revert to their values at the beginning of the depression, the wedges stay at their current value and the wedges take on the values in the subsequent year. Following these three years we assume perfect foresight until the end of the depression, in the sense that agents foresee the actual values the wedges take on. At the end of the depression we assume that the labor wedge is constant at its last period value and the technology parameter grows from its last period value at the balanced growth rate.

For our three countries we measure output by real GNP per capita after removing a 1.6% trend and the labor input by total civilian manhours per working age person. In each country we measure consumption by personal consumption expenditures and investment as the sum of fixed investment and inventory investment by businesses. We use the capital stocks that correspond to our measures of investment. (Details are available upon request.)

A. The U.S. Great Depression

We begin with the U.S. Great Depression. Both output and the labor input are normalized to equal 100 in 1929, the base year. Investment in each period is divided by the the 1929 level of real GNP per capita.

In Figure 1 we see that output is 35% below trend in 1933 and is still 20% below trend by 1939. We also see that the labor input declines by 28% from 1929 to 1933 and stays relatively low throughout the rest of the decade. Investment declines sharply from 1929 to

1933 and partially recovers by the end of the decade. Interestingly, about half of the 35% fall in output from 1929 to 1933 is due to the decline in investment.

In Figure 2 we display the efficiency wedge and the labor wedge. The efficiency wedge is measured as in (46) after removing a 1.6% trend. From 1929 to 1933 the efficiency wedge falls by 17% but by 1939 it has essentially recovered to trend. The labor wedge falls by 28% from 1929 to 1933 and in 1939 it is still about that low. Thus, the underlying distortions that manifested themselves as efficiency and labor wedges became substantially worse from 1929 to 1933. By 1939, the efficiency wedge had disappeared, but the labor wedge remained as large as it had been in 1933.

We start by assessing the separate contributions of the efficiency wedge and the labor wedge. To do so we include these wedges one at a time in the prototype model and set the other wedges to their 1929 levels. With the efficiency wedge alone, the prototype economy generates much of the observed downturn in output, but much too rapid a recovery. As can be seen in Figure 2, for example, by 1933 output falls about 26% in the model and about 35% in the data. By 1939, the efficiency wedge model generates an output decline of only 6% rather than the observed 20%. As can also be seen in Figure 2, the reason for this rapid recovery is that the efficiency wedge model completely misses the continued sluggishness in labor from 1933 onward. For investment, this model shows a similar fall as in the data from 1929–33 but a faster recovery.

In our model with only labor wedges, output falls only about half as much by 1933 as output actually fell: 17% vs. 35%. By 1939, output in both this model and the data have fallen about 20%. The labor wedge model misses the sharp decline in investment from 1929 to 1933, but it does generate the sluggishness in labor input after 1933.

We assess the combined contribution of the efficiency and labor wedges by simulating the economy with our constructed series for the efficiency and labor wedges holding the investment wedge at its 1929 level. Figure 4 shows that the resulting model captures the

downturn in output from 1929 to 1933 remarkably well. From 1929 to 1933 the model produces a slightly greater reduction in output than is seen in the data. The model also captures the slow recovery from 1933 to 1939. In 1939, in the data, output is about 20% below trend while in the model it is about 20% below trend. The model also captures the dynamics of labor and investment reasonably well.

Given that the efficiency and labor wedges account for essentially all of the movements in output, we conclude that essentially none of the output fluctuations can be accounted for by the investment wedge. If anything, the model suggests that investment is less distorted in 1933 than it is in 1929.

B. The German Great Depression

The German Great Depression is similar to the U.S. Great Depression. We normalize the data in a similar fashion that for the U.S. Great Depression, except that 1928 is the base year.

In Figure 5 we see that output is 30% below trend in 1932 and is 5% above trend in 1938. We also see that the labor input declines by 28% from 1928 to 1932 and in 1938 is 9% below its 1929 level. Investment declines sharply from 1928 to 1932 and recovers fully by the end of the decade. Interestingly, about half of the fall in output from 1928 to 1932 is due to the decline in investment.

In Figure 6 we display the efficiency wedge and the labor wedge. From 1928 to 1932 the efficiency wedge falls by 10% but by 1938 it is 15% above trend. The labor wedge falls by 28% from 1928 to 1932 and by 1938 it is 34% below its 1928 level.

In Figure 7, we assess the separate contributions of the efficiency wedge and the labor wedge in a similar way to that for the U.S. Great Depression. With the efficiency wedge alone, in the prototype economy, output declines but is above the level in the data in both 1932 and 1938. With only labor wedges, output falls by about two-thirds as much as in the data by 1932 and is far below its level in the data in 1938.

In Figure 8, we see that together the efficiency and labor wedges account for essentially all of the decline and all of the recovery in output. The model also captures the dynamics of labor and investment reasonably well. As with the U.S. Great Depression we conclude that essentially none of the output fluctuations can be accounted for by the investment wedge.

C. The Canadian Great Depression

The Canadian Great Depression is broadly similar to the U.S. Great Depression. We normalize the data in a similar fashion that for the U.S. Great Depression.

In Figure 9 we see that output is 39% below trend in 1933 and is 28% below trend in 1939. We also see that the labor input declines by 28% from 1929 to 1933 and in 1939 is 18% below its 1929 level. Investment declines sharply from 1929 to 1933 and is still relatively depressed by the end of the decade.

In Figure 10 we display the efficiency wedge and the labor wedge. From 1929 to 1933 the efficiency wedge falls by 18% and is still 12% below trend by 1939. The labor wedge falls by 28% from 1929 to 1933 and by 1939 it is still 18% below its 1929 level.

In Figure 11, we assess the separate contributions of the efficiency wedge and the labor wedge in a similar way to that for the U.S. Great Depression. Taken separately either the efficiency wedge or the labor wedge can account for about half of the observed decline in output from 1929 to 1933. The efficiency alone accounts for little of the decline in labor while the labor wedge alone can account for most of this decline.

In Figure 12, we see that together the efficiency and labor wedges account for both the decline and the of the recovery in output. Relative to the data, the model predicts a larger fall and a sharper recovery.

6. Alternative Measures of Capital Utilization

One possible source of the efficiency wedge is mismeasurement of capital services. We assume that the flow of capital services is proportional to the capital stock. Numerous

authors have argued that capital utilization rates fluctuate systematically over the business cycle so that the flow of capital services fluctuates more than the capital stock. It is difficult to measure the flow of capital services directly. We follow Kydland and Prescott (1988) and Hornstein and Prescott (1993) in considering an alternative specification of the technology which allows for variable capital utilization.

In this specification, the production function is

$$y = A(kh)^\alpha (nh)^{1-\alpha}$$

where n is the number of workers and h is the length of the workweek. Total labor input is given by $l = nh$.

One interpretation of the *benchmark* specification for the production function used in previous sections is that the workweek h is constant and all of the variation in labor input is in the number of workers n. Our procedure for measuring the efficiency wedge uses data on the capital stock k and total labor input l. Under this interpretation our procedure correctly measures the efficiency wedge (up to the constant h).

Here we pursue the opposite extreme: we assume that the number of workers n is constant and all the variation in the labor input is in the workweek h. Under this *variable capital utilization* specification, the services of capital kh are proportional to the product of the stock k and the labor input l, so that variations in the labor input induce variations in the flow of capital services. Thus, the capital utilization rate is proportional to the labor input l and the efficiency wedge is proportional to y/k^a.

In Figure 13 we compare the two wedges for benchmark and variable capital utilization specifications. With variable capital utilization the efficiency wedge falls less and recovers more quickly than under the benchmark specification.

The labor wedge is identical, up to a scale factor, in the two specifications. To see this note that with our Cobb-Douglas production function in the benchmark specification $F_{lt} = (1-\alpha)y_t/l_t$ while with variable capital utilization $F_{lt} = y_{lt}/l_t$. The result follows from

(47).

In Figure 14 we compare equilibrium output, labor, and investment for the two specifi-
cations. The figure shows that there is essentially no difference in outcomes. Variable capital
utilization reduces the volatility in the measured efficiency wedge, and it makes the labor
input more sensitive to a given change in this wedge. These two effects offset each other and
the equilibrium is essentially unaffected.

7. Related Literature

Our paper is related to the existing literature in terms of methodology, the interpre-
tation of the wedges and analysis of the Great Depression in the U.S. and other countries.
We discuss each in turn.

A. Related Methodology

The basic methodology used in this paper is the following. We use restrictions from
economic theory to back out the wedges from the data. We then formulate stochastic processes
for these wedges which we put back into a into a quantitative general equilibrium model for
our accounting exercise. This basic idea is at the heart of an enormous number of papers in
the real business cycle theory literature. For example, Prescott (1986) explicitly asks what
fraction of the variance of output can plausibly attributed to productivity shocks, which we
refer to as the efficiency wedge.

Papers in the subsequent literature have expanded this general equilibrium accounting
exercise to include a wide variety of other shocks. For example, for shocks to the marginal
efficiency of investment see Greenwood, Hercowitz, and Huffman (1988), for money shocks see
Cooley and Hansen (1989), for broadly interpreted preference shocks see Bencivenga (1992)
and Stockman and Tesar (1995), for terms of trade shocks see Mendoza (1991), for foreign
technology shocks see Backus, Kydland and Kehoe (1992) and Baxter and Crucini (1995),
for shocks to the home production technology see Benhabib, Rogerson, and Wright (1991)

and Greenwood and Hercowitz (1991), for government spending shocks see Christiano and Eichenbaum (1992), for shocks to markups see Rotemberg and Woodford (1992), for shocks to taxes see Braun (1994) and McGrattan (1994)

An important difference between our method and many of those in the later real business cycle literature is that in our method we back out the labor wedge and the investment wedge from the combined household and firm first order conditions while most of this later literature uses direct measures of these shocks. One of the most closely related precursors to our method is that of McGrattan (1991) who, for the post-war U.S. data, decomposes the movements in output into the fraction that comes from the efficiency wedge, the labor wedge and the investment wedge, which she refers to as productivity shocks, taxes on labor income and taxes on capital income. She uses no data on taxes but instead simply uses the equilibrium to infer the implicit wedges. Ingram, Kocherlakota and Savin (1994) advocate a similar approach.

B. Interpreting and Assessing the Three Wedges

We show that the three wedges in our model can arise from a variety of detailed economies. In terms of the theory, there are large number of papers that have shown how distortions in economies manifest themselves as at least one of our three wedges. In terms of applications, a large number of papers have used one or more of the wedges to assess aspects of the model.

Theory

The idea that taxes of various kinds distort the relation between various marginal rates is the cornerstone of public finance. The specific ideas that taxes on intermediate goods lead to aggregate production inefficiency and thus give rise to an efficiency wedge, taxes on labor income distort the within period marginal rates of substitution from the within period marginal rates of transformation and thus give rise to a labor wedge and finally, that taxes on capital income or investment distort the intertemporal marginal rates of substitution from

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the intertemporal marginal rates of transformation and thus give rise to a investment wedge are all well-known. (See, for example, Atkinson and Stiglitz 1980). So is the specific idea that monopoly power by unions or firms give rise to a labor wedge.

The idea that sticky wages or sticky prices give rise to a labor wedge is the cornerstone of the New Keynesian approach to business cycles. (See, for example, the recent survey by Rotemberg and Woodford 1999.)

One contribution of this paper is to show the precise map between general equilibrium models with frictions and these various wedges. As we have seen, it is not true that each distortion in the underlying economy maps into one and only one wedge. For example, input financing frictions, in general, distort all three wedges simultaneously. For another, while it is true that models with one period of either wage or price stickiness affect give rise to only labor wedges, models with staggered wage or price setting give rise to efficiency wedges as well. (See Chari, Kehoe, McGrattan 2002). Finally, as noted by Carlstrom and Fuerst (1997) the investment frictions from costly state verification give rise to wedges in the capital accumulation equation as well as to investment wedges.

Applications to Postwar Data

Many papers have plotted and interpreted one or more of the three wedges. As the theoretical discussion makes obvious, having a very clear understanding of the map between the underlying detailed economy and the prototype growth model is critical for interpreting any patterns in the wedges. In particular, is the correlation between the wedges driven by a common underlying distortion or is it driven by multiple distortions that happen to be correlated? Most of the applied papers never comes to terms with this subtlety but nonetheless conduct some interesting analysis.

Many papers in the real business cycle literature plot the efficiency wedge and hint at other interpretations for the postwar data. Numerous papers try to sort out whether this wedge is coming from misspecified production functions (increasing returns instead of

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constant returns) , mismeasured factor inputs (unobserved utilization of capital or labor) or procyclical productivity. See, among many papers, Burnside, Eichenbaum and Rebelo (1993) and the survey by Basu and Fernald (1999).

A number of papers have plotted the labor wedge for the postwar data and discussed various interpretations it. For example, Parkin (1988), Hall (1997) and Gali, Gertler and Lopez-Salido (2002) all graph and interpret the labor wedge for the postwar data. Parkin (1988) discusses how monetary shocks might drive the this wedge. Hall (1997) mostly interprets the wedge as a preference shock but also discusses a search interpretation. Gali, Gertler and Lopez-Salido (2002) discuss a variety of interpretations. Rotemberg and Woodford discuss a wide variety of interpretations of this wedge and its behavior in the postwar data in a number of papers, including Rotemberg and Woodford (1991, 1995, and 1999).

The investment wedge has also been investigated by a number of authors. In addition to the work of Braun (1994) and McGrattan (1994) see Carlstrom and Fuerst (1999) and Cooper and DeJarque (2000).

The Neoclassical Approach to the Great Depression

There has been a surge of interest in reinterpreting the Great Depression in the United States and other countries through lens of neoclassical theory. Some of the early proponents of this view are Cole and Ohanian (1999 and 2000) and Prescott (1999). Cole and Ohanian (1999) find that for the United States the efficiency wedge can account for only 15% of the observed 38% decline in detrended output from 1929 to 1933. They argue that some force other than the efficiency wedge is needed, especially to account for the slow recovery. They consider and dismiss fiscal policy shocks and trade shocks and leave open the possibility of monetary shocks, financial intermediation shocks and sticky wages. Bordo, Erceg, and Evans (1999) use a quantitative equilibrium model to argue that monetary shocks interacting with sticky wages can account for much of the decline and some of the slow recovery in output in the U.S. Great Depression. Crucini and Kahn (1996) find that tariff shocks can account

for only about 2% of the decline in U.S. output. Mulligan (2002a and 2002b) plots the labor wedge for the United States for much of the 20th century, including the Great Depression period. He interprets movements in this wedge as arising from changes in labor market instititutions and regulation, including features discussed in this paper.

8. Conclusion

This paper is aimed at applied theorists who are building detailed models of economic fluctuations. Once such theorists have chosen the primitive sources of shocks they need to choose the mechanisms through which such shocks lead to fluctuations. We have shown that these mechanisms can be summarized by their effects on three wedges. Our accounting procedure, by focusing on the wedges, can be used to suggest promising mechanisms and rule out others a priori.

In this paper we have applied our procedure to three great depressions. We found that efficiency and labor wedges, in combination, account for essentially all of the fall and recovery in these depressions and that investment wedges play, at best, a minor role. Future theoretical work should focus on developing models which lead to fluctuations in efficiency and labor wedges. There are many models that give rise to fluctuations in labor wedges. The challenging task is to develop detailed models in which primitive shocks lead to fluctuations in efficiency wedges.

It would also be interesting to ask whether our substantive findings on the role of various wedges holds up for other business cycle episodes.

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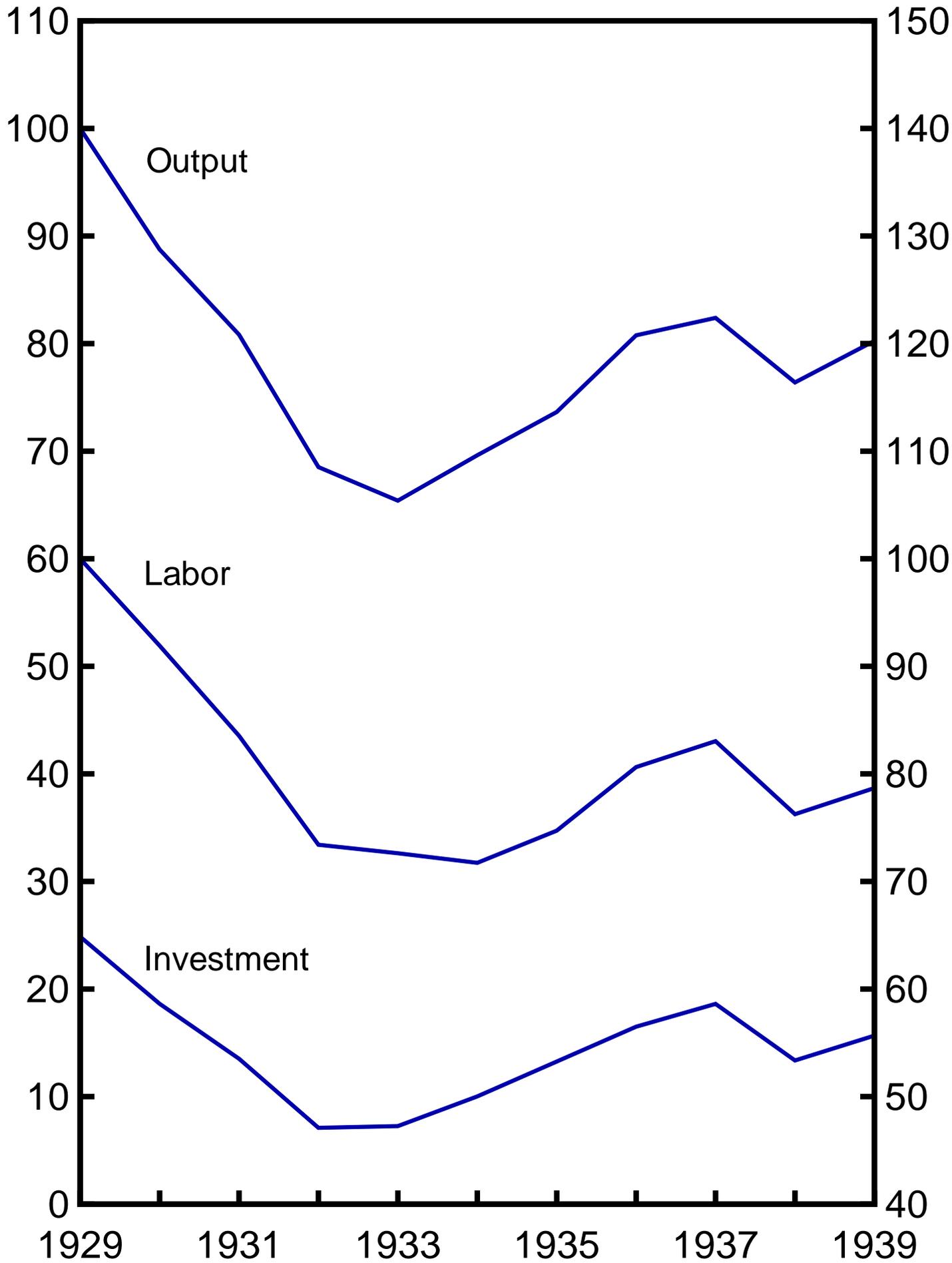Figure 1.   The U.S. Great Depression

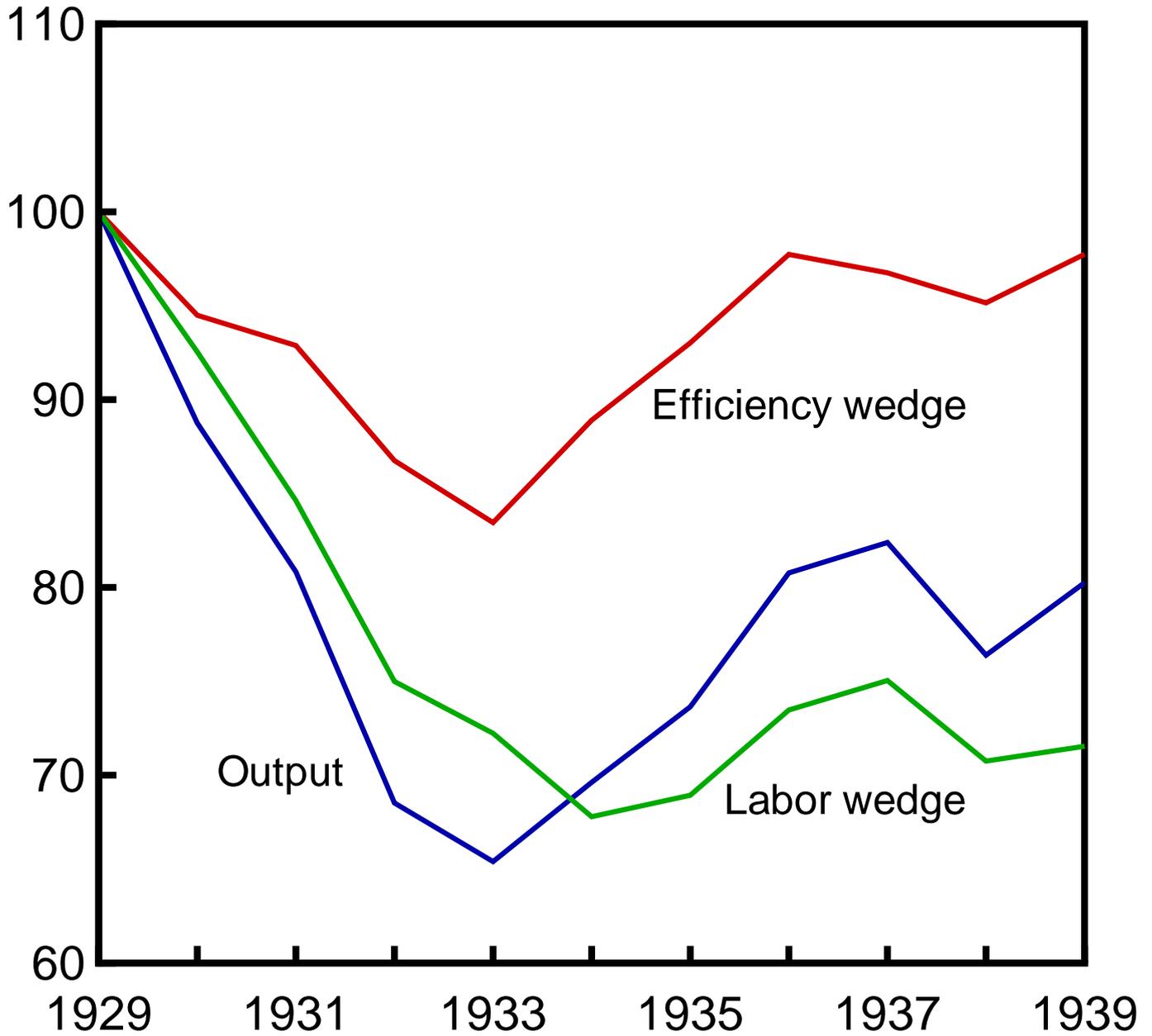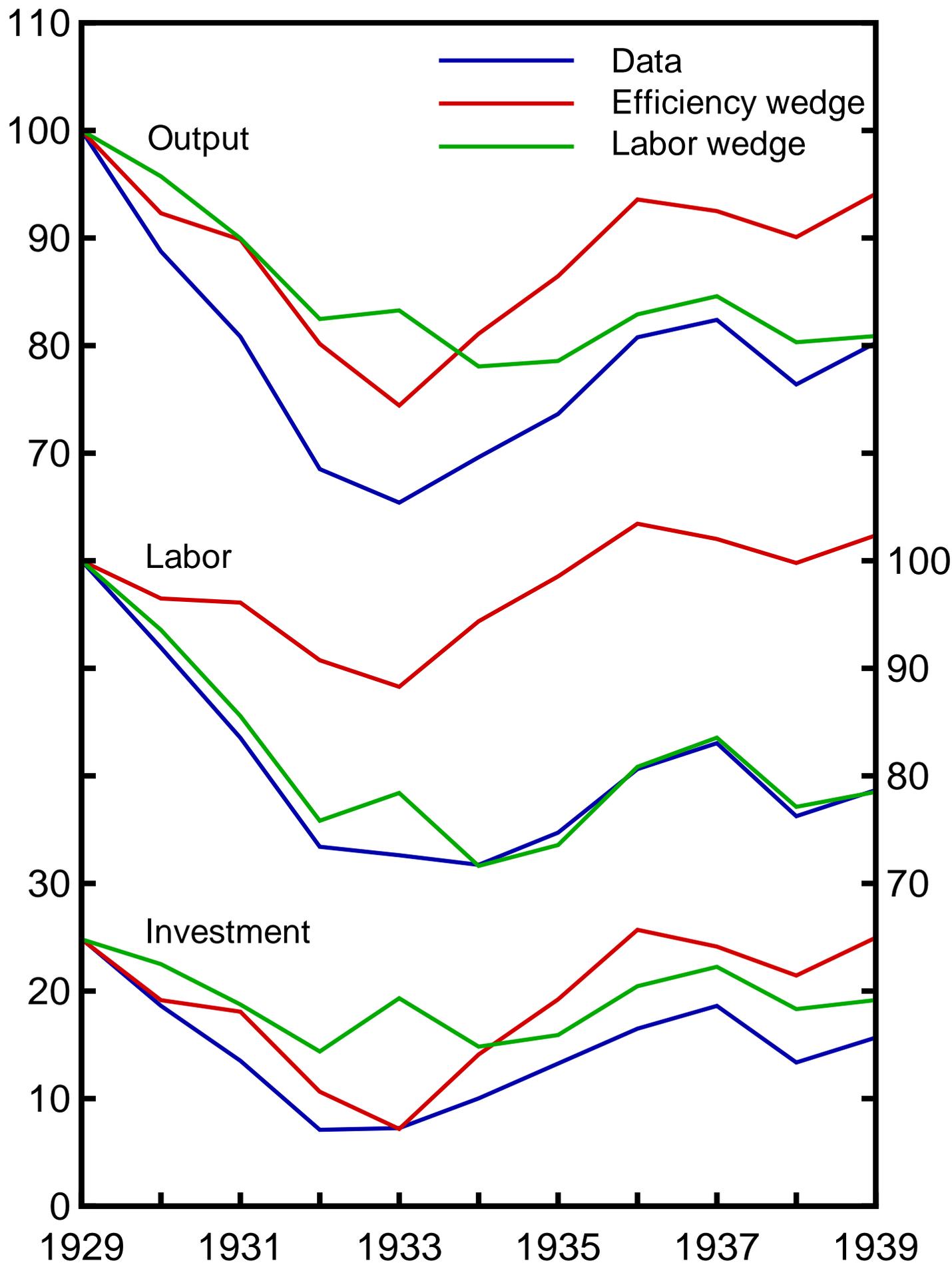Figure 2. U.S. Output and Measured Efficiency and Labor Wedges

Figure 3. U.S. Data and Models with One Wedge

Figure 4. U.S. Data and Model with Efficiency and Labor Wedges
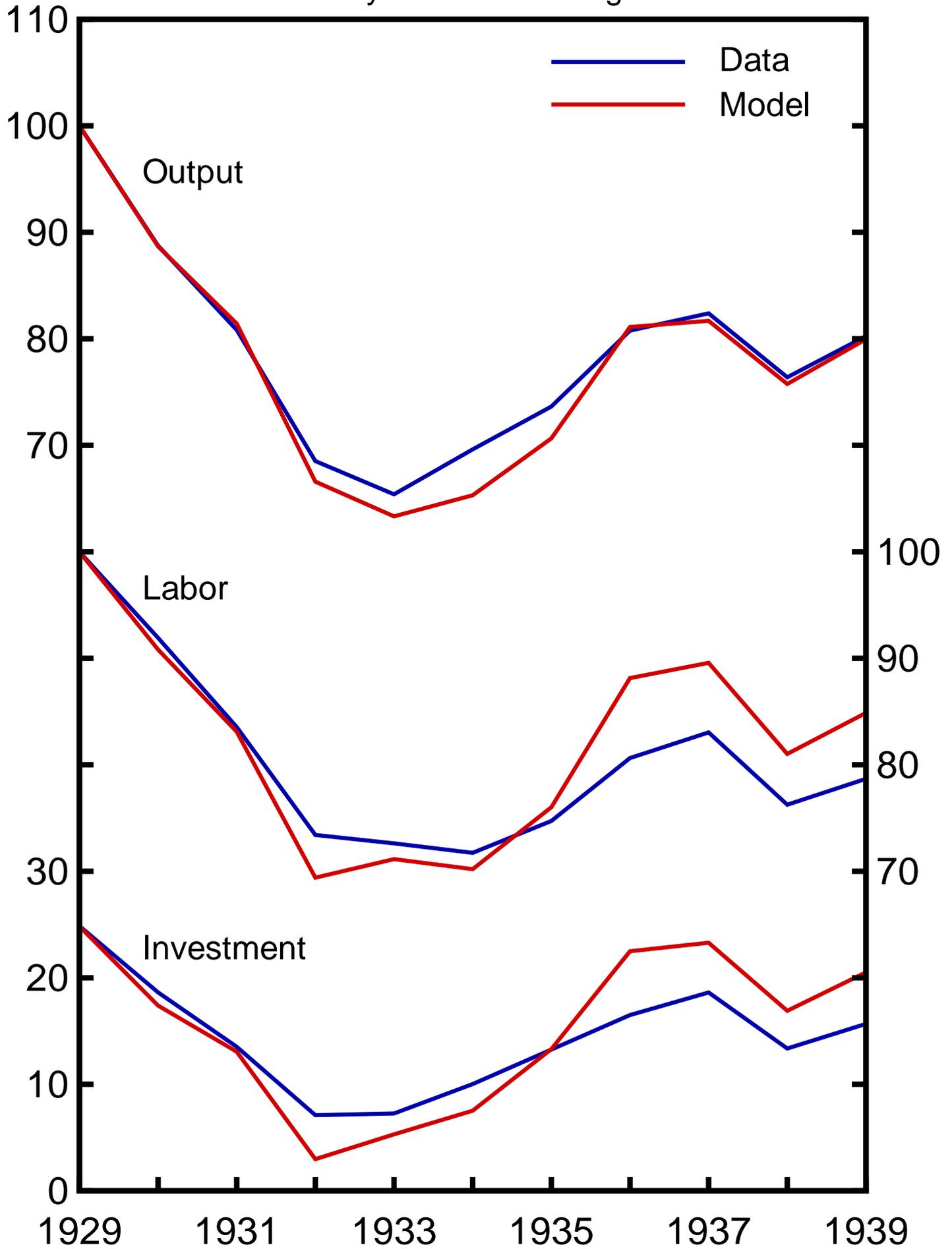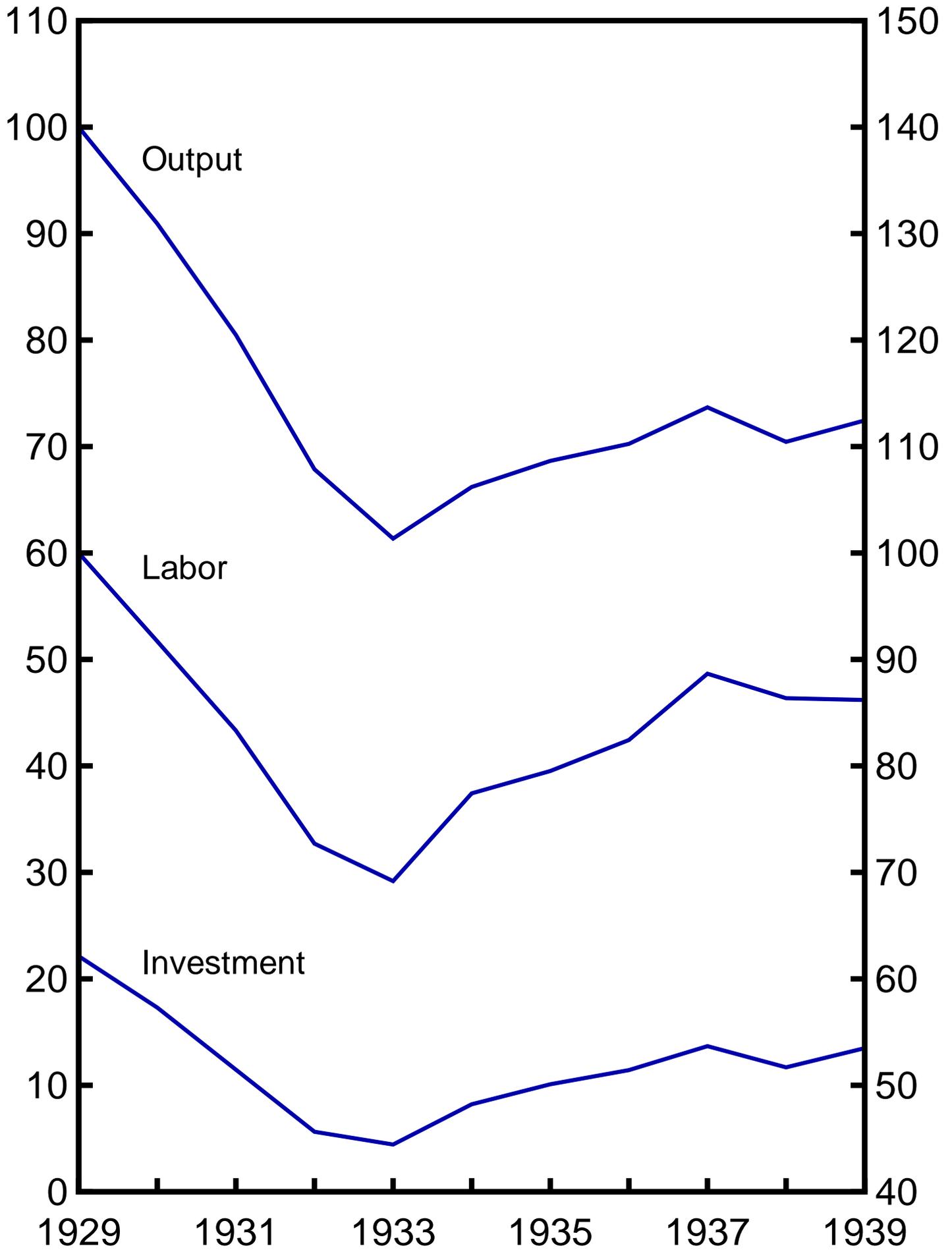
Figure 5. The German Great Depression

Output

Labor

Investment

1929 1931 1933 1935 1937
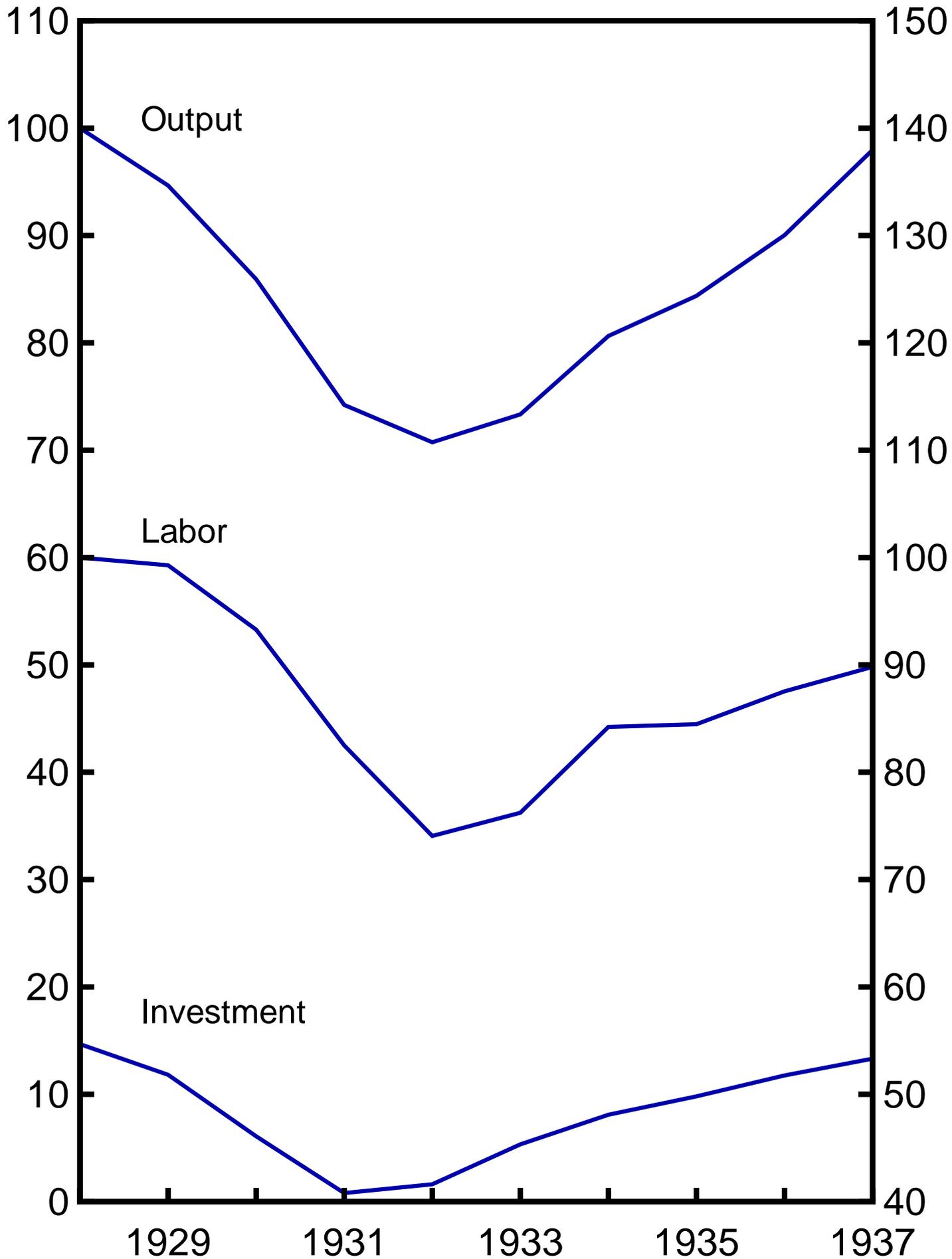
Figure 6. German Output and Measured Efficiency and Labor Wedges
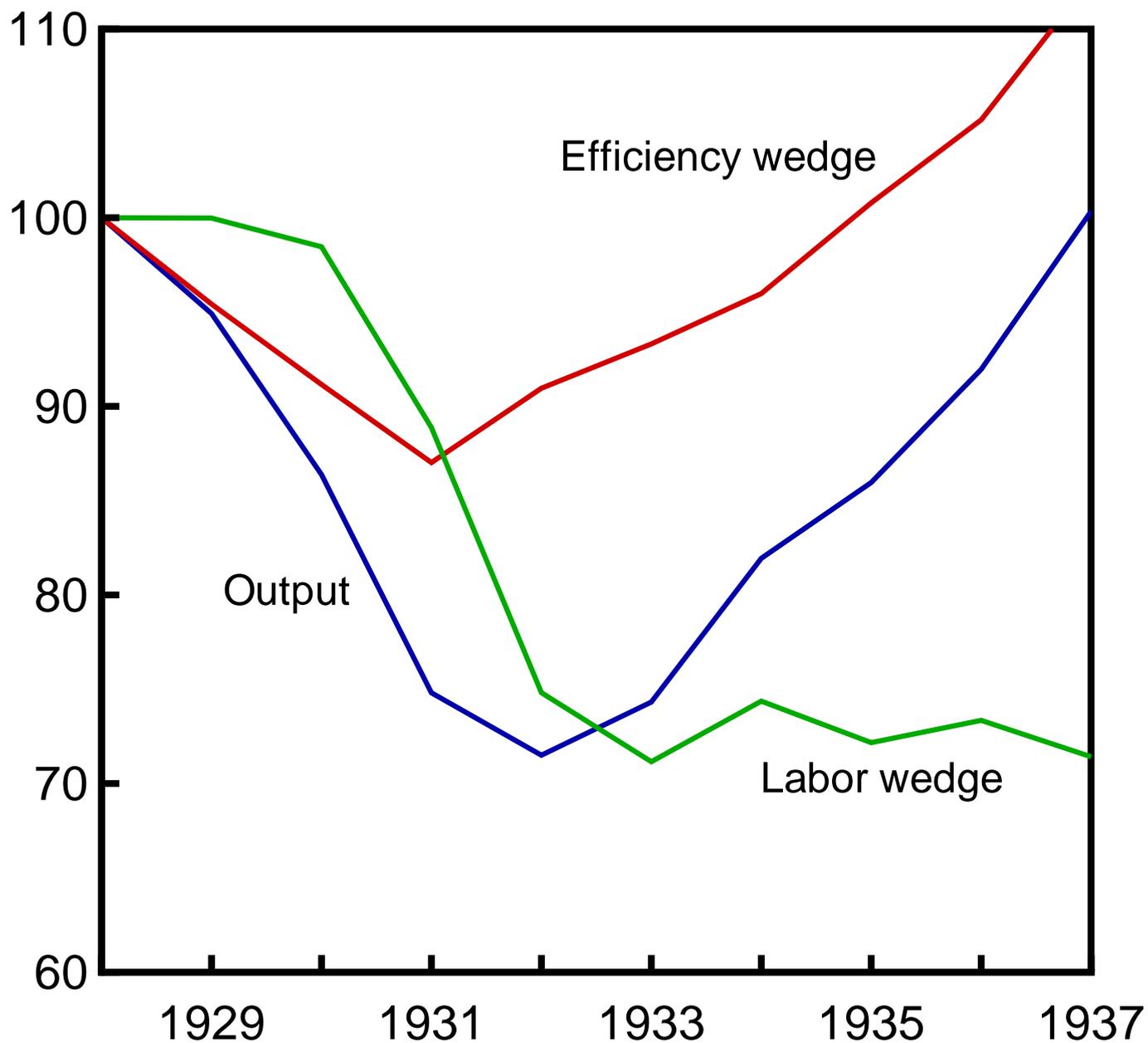
Figure 7. German Data and Models with One Wedge
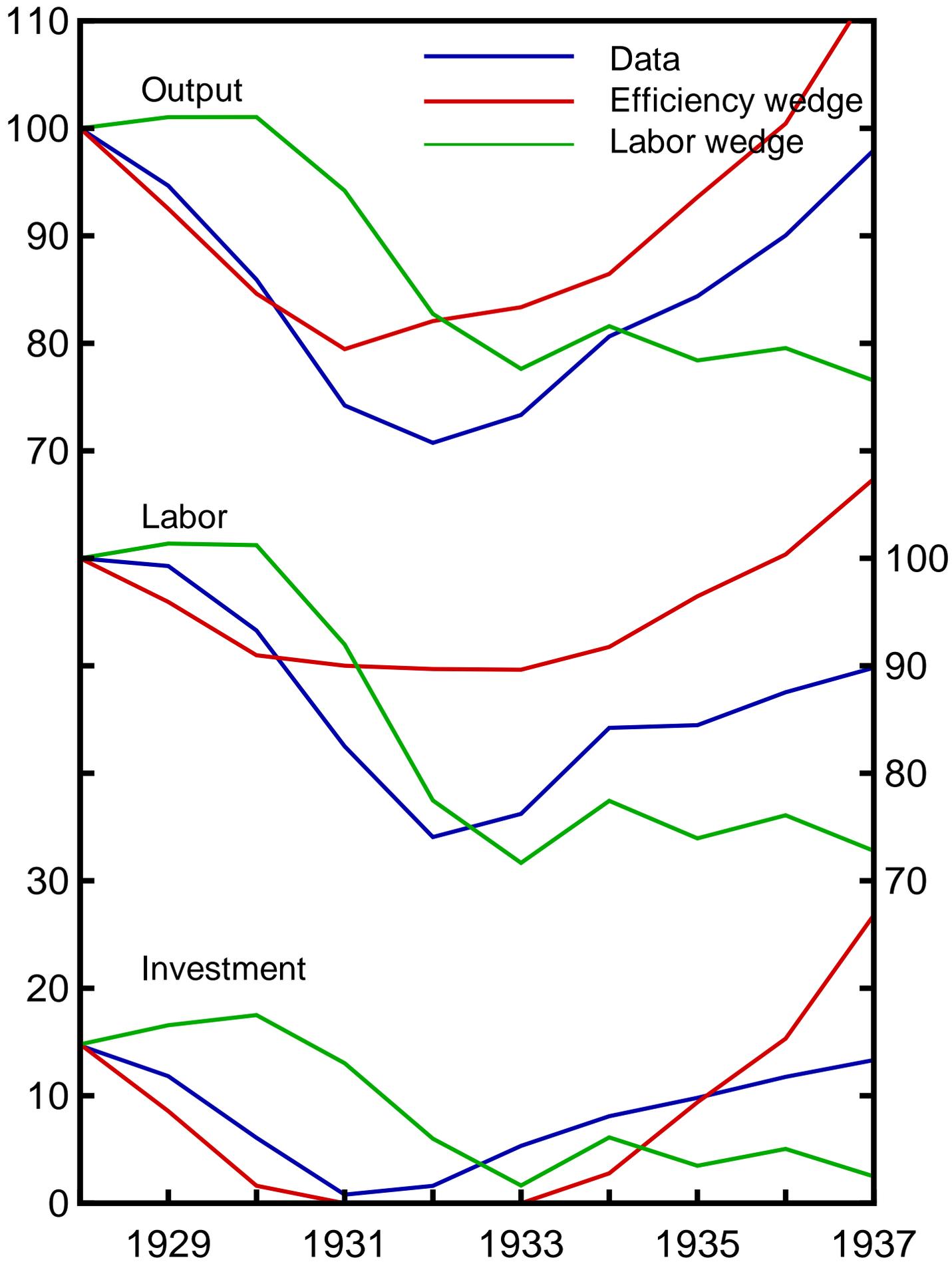
Figure 8. German Data and Model with Efficiency and Labor Wedges
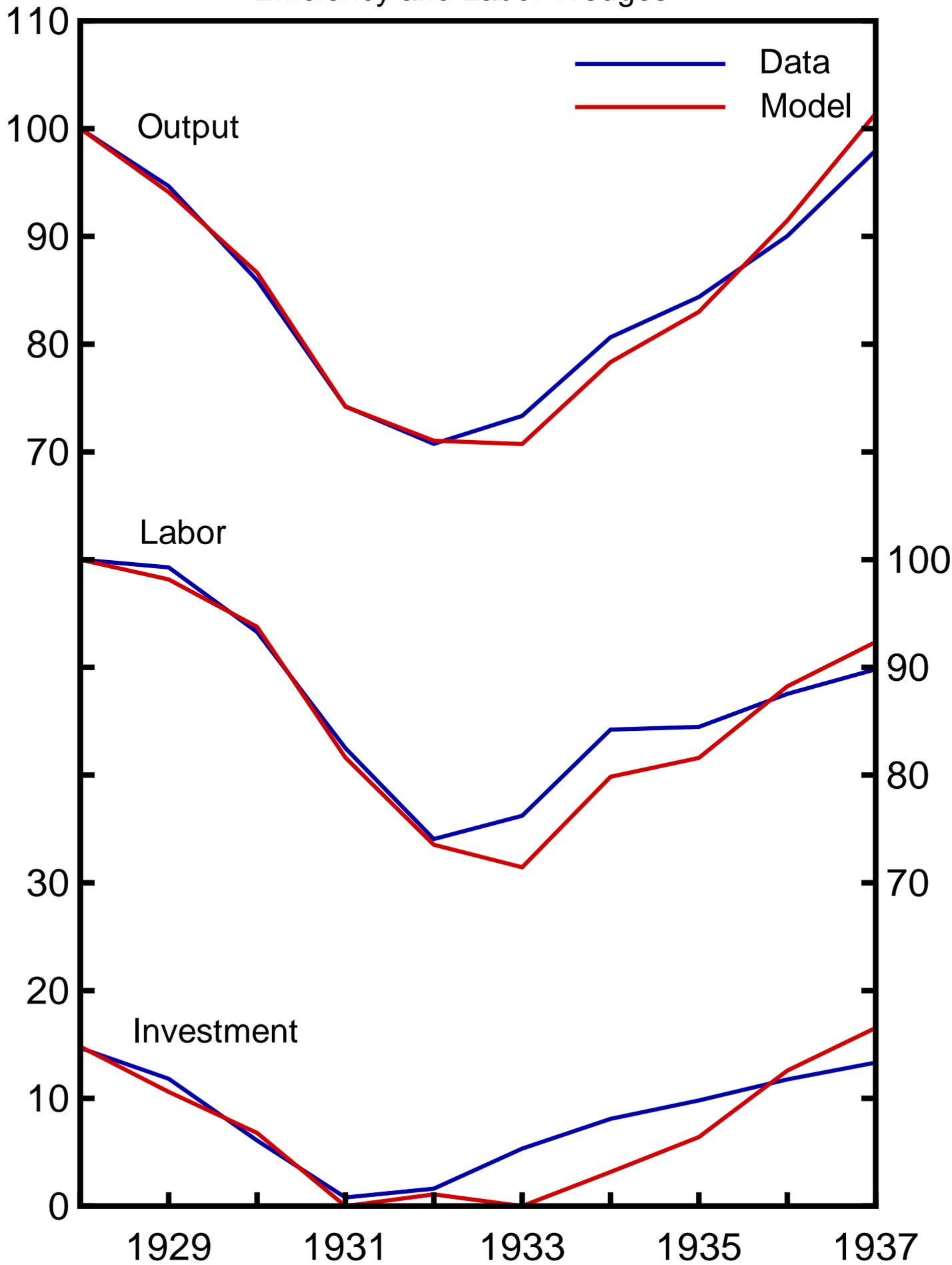
Figure 9. The Canadian Great Depression

Figure 10. Canadian Output and Measured Efficiency and Labor Wedges
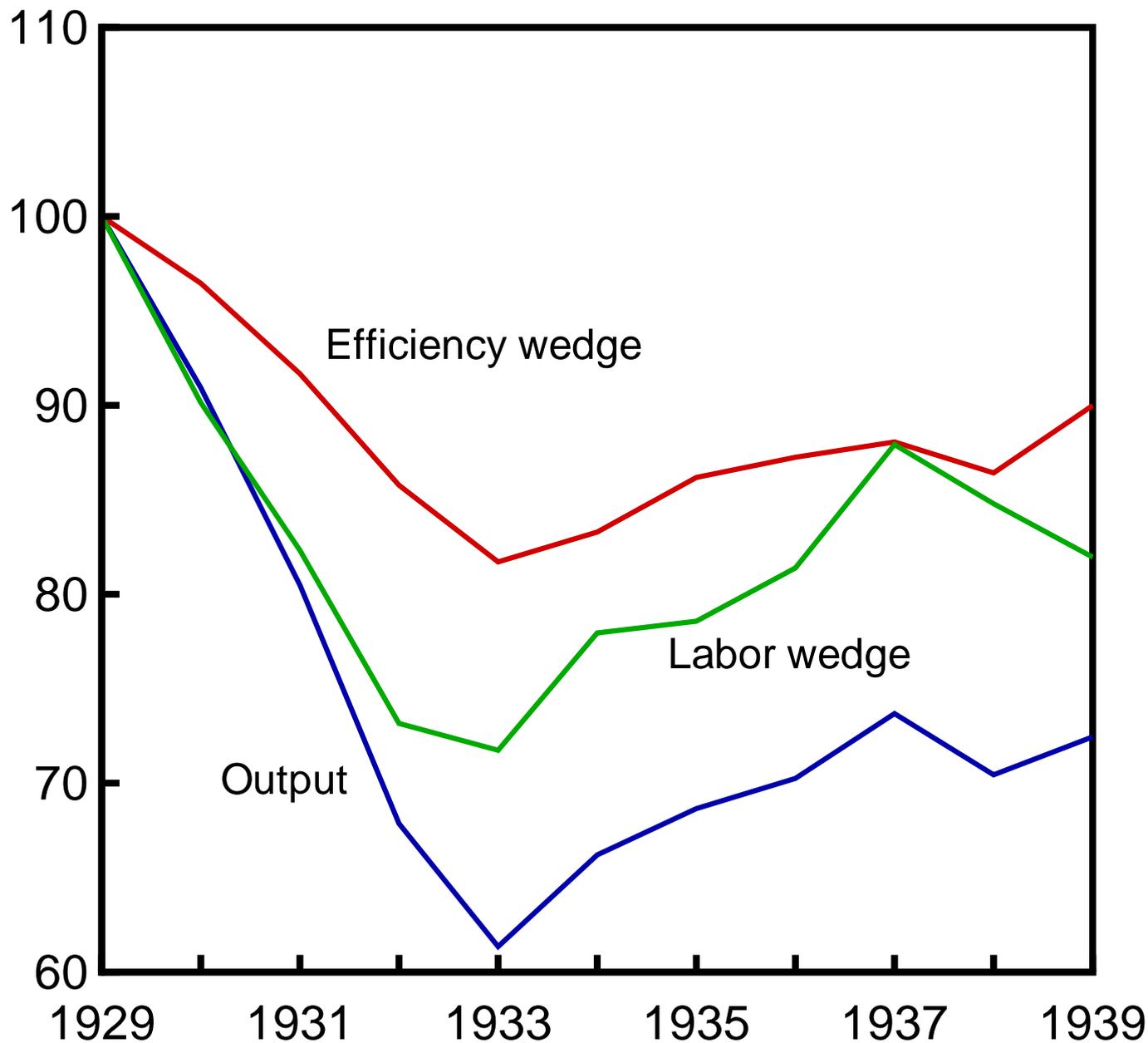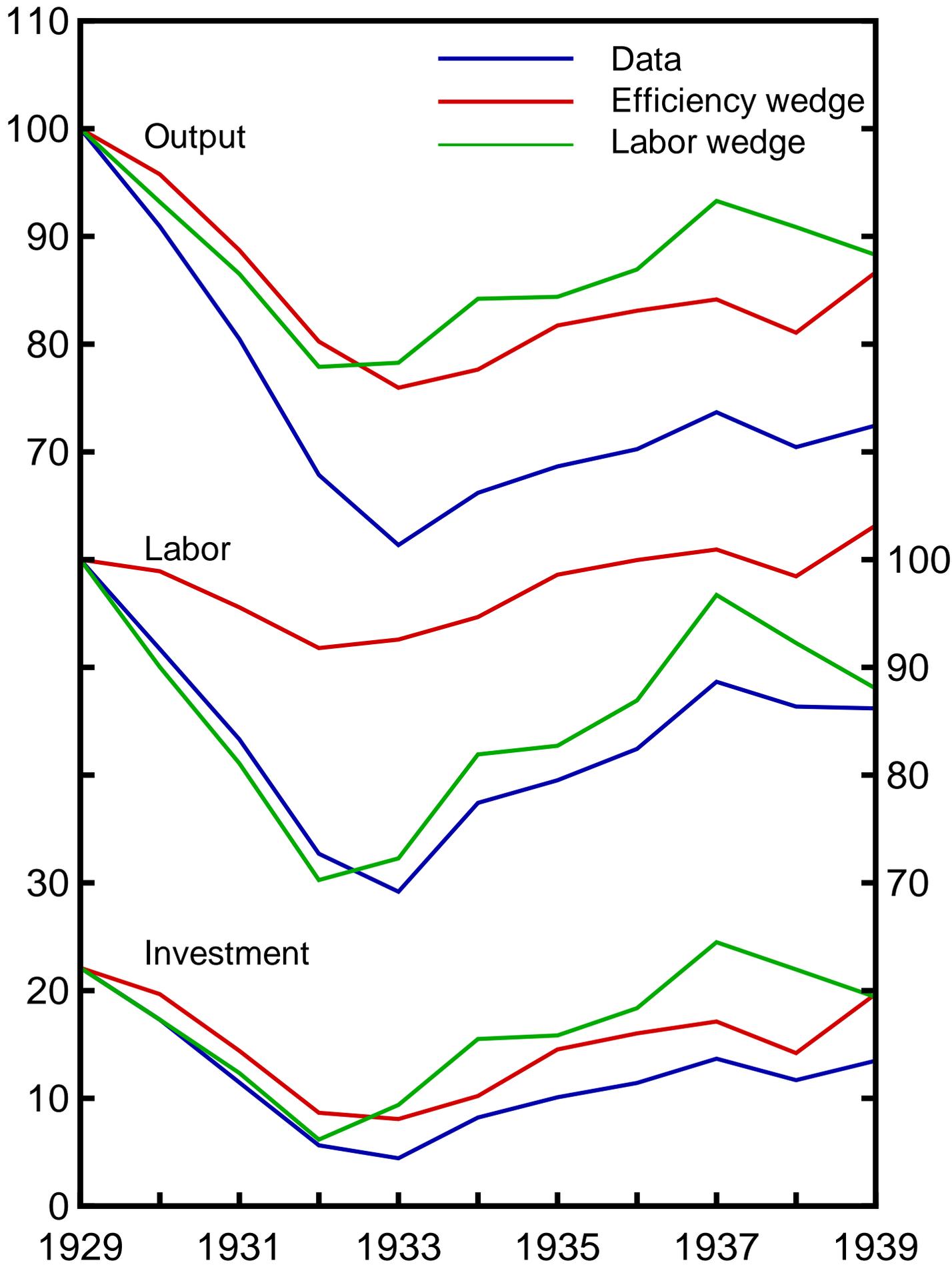
Figure 11. Canadian Data and Models with One Wedge

Figure 12. Canadian Data and Model with Efficiency and Labor Wedges
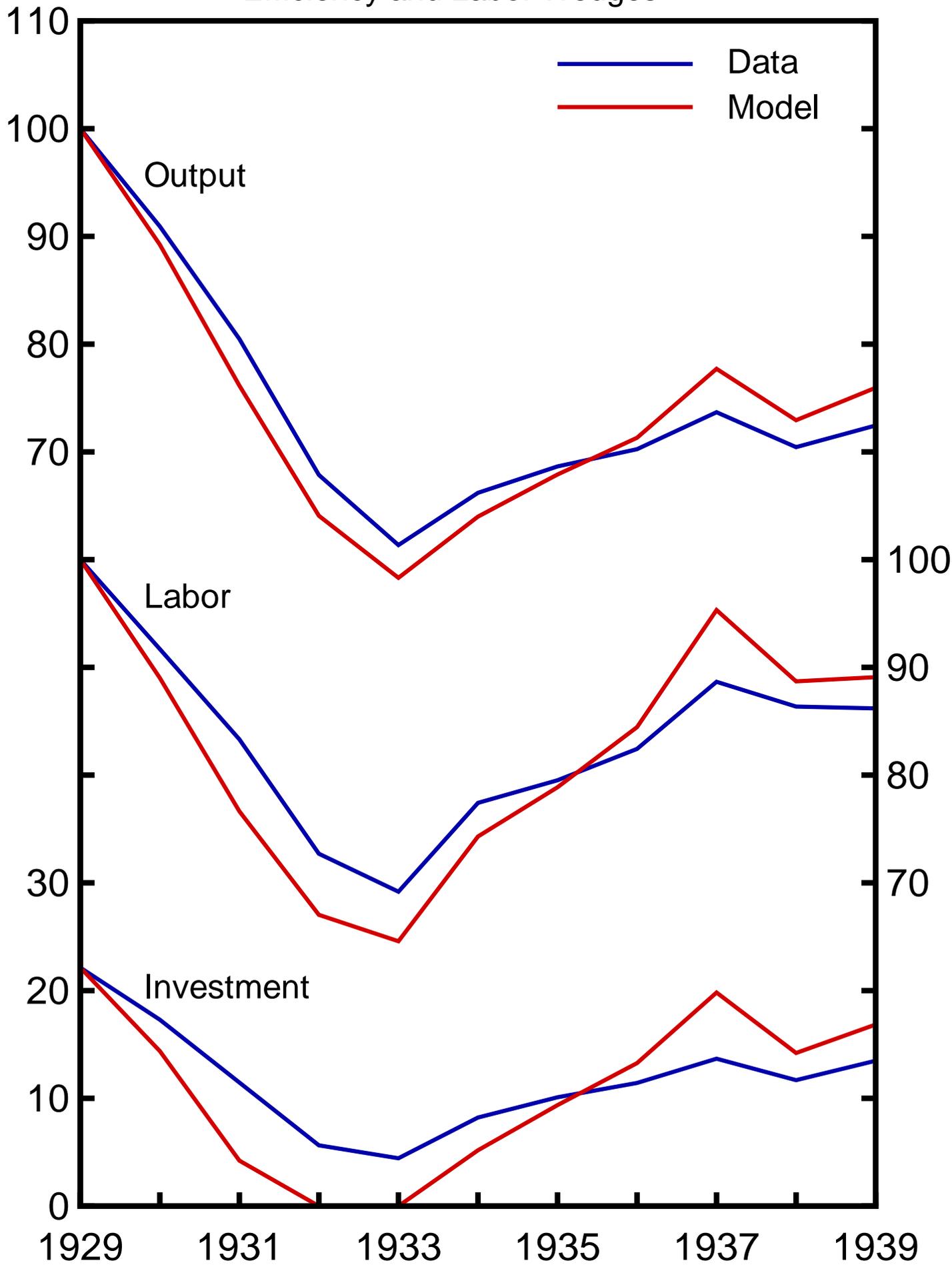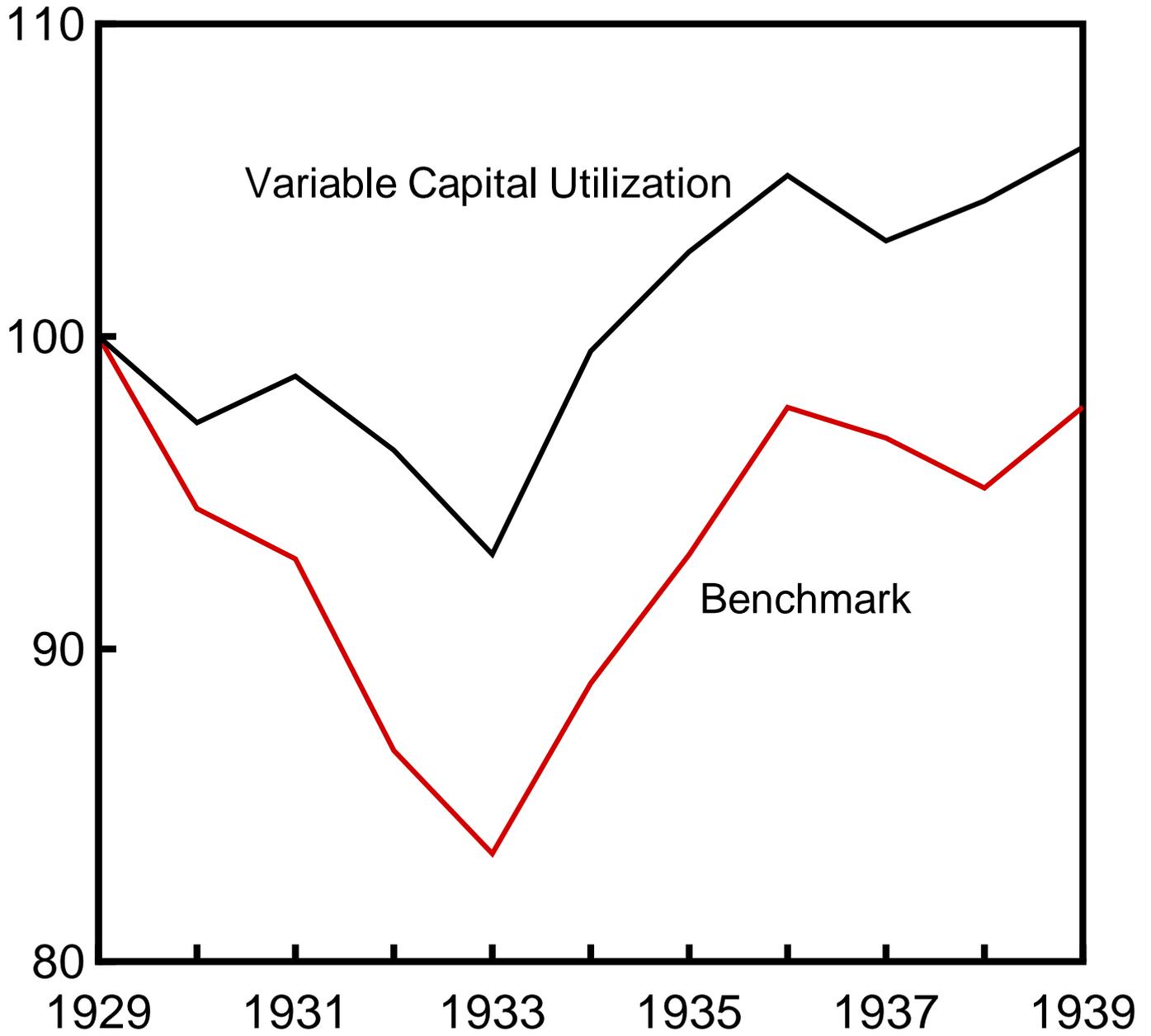
Figure 13. U.S. Measured Efficiency Wedges

Figure 14. Results of Two Models with Efficiency and Labor Wedges