Federal Reserve Bank of Minneapolis
Research Department

Intermediated Quantities
and Returns

Rajnish Mehra, Edward C. Prescott,
and Facundo Piguillem

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ABSTRACT _____

There is a large amount of intermediated borrowing and lending between households. Some of it is intergenerational, but most is between older households. The average difference in borrowing and lending rates is over 2 percent. In this paper, we develop a model economy that displays these facts and matches not only the returns on assets but also their *quantities*. The heterogeneity giving rise to borrowing and lending and differences in equity holdings depends on differences in the strength of the bequest motive. In equilibrium, the lenders are annuity holders and the borrowers are those who have equity holdings, who live off its income when retired, and who leave a bequest. The borrowing rate and return on equity are the same in the absence of aggregate uncertainty. The divergence between borrowing and lending rates can thus give rise to an equity premium, even in a world without aggregate uncertainty.

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1. Introduction:

A limitation of the *homogenous household* construct is that it precludes the modelling of borrowing and lending amongst agents. In equilibrium, the shadow price of consumption at date t+1 in terms of consumption at date t is such that the amount of borrowing and lending is zero. Homogenous household models are thus incapable of matching the *quantities* of assets held and *intermediated*.

To address this issue, we construct a model economy that incorporates agent heterogeneity in the form of differences in the strength of their bequest motive. In light of our earlier finding (1985) that the premium for bearing non-diversifiable aggregate risk is small, our analysis abstracts from *aggregate risk.* The only uncertainty that agents face is idiosyncratic risk about the duration of their lifetime after retirement. All agents have identical preferences for consumption; however, they differ with respect to their intensity for bequests. In equilibrium, those with a strong bequest motive accumulate equity assets, borrow, and upon retirement, live off the income of these assets. Households with no bequest motive buy annuities during their working years and consume the annuity benefits over their retirement years.

The incorporation of agent heterogeneity allows us to capture a key empirical fact: there is a large amount of borrowing and lending between households, in particular, between older households. This borrowing is done either directly, by issuing mortgages to finance owner occupied housing or indirectly, by owning partially debt financed rental properties through direct or limited partnerships or REITS. We abstract from the small amount of borrowing and lending that occurs directly between households and assume that all of it is intermediated through financial institutions such as banks and pension

funds. For the United States, in 2005 the amount intermediated was approximately 1.3 times the GDP[1]

The intermediation technology is constant returns to scale with intermediation costs being proportional to the amount intermediated. To calibrate the constant of proportionality, we use Flow of Funds statistics and data from National Income and Product Accounts. The calibrated value of this parameter equals the net interest income of financial intermediaries, divided by the quantity of intermediated debt and is a little over 2 percent[2].

In the absence of aggregate uncertainty, the return on equity and the borrowing rate are identical, since the agents who borrow are also marginal in equity markets. In our framework, government debt is not intermediated and thus its return is equal to the lending rate. The equity premium relative to government debt is the intermediation spread. The divergence between borrowing and lending rates gives rise to an equity premium even in a world *without aggregate uncertainty.*

The paper is organized as follows: the economy is defined in Section 2. In Section 3, we discuss the decision problem of the agents. Sections 4 and 5 deal with aggregation. Section 6 presents the balance sheets, while section 7 characterizes the equilibrium. We calibrate the economy in Section 8. In Section 9, we present and discuss our results. Section 10 concludes the paper.

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[1] See section 8 (calibration) for details
[2] See section 8 (calibration) for details

2. **The Economy**

To build a parsimonious model that captures the large amount of borrowing and lending and financial intermediation, we postulate households with identical preferences with respect to consumption over their lifetime but differentiated along *one dimension –* the strength of their bequest motive parameter, α.

What motivates bequests? While a casual consideration of bequests naturally assumes that they exist because of parents' altruistic concern for the economic well being of their offspring, results in Hurd (1989) and Kopczuk and Lupton (2004), among others (see also Wilhelm (1996), Laitner and Juster (1996), Altonji et al. (1997), and Laitner and Ohlsson (2001)), suggest otherwise: households with children do not, in general, exhibit behavior in greated accord with a bequest motive than do childless households. As a result, the existing literature is largely agnostic as to bequest motivation, attributing bequests to general idiosyncratic, egoistic reasons. These empirical results lead us to eschew the perspective of Becker and Barro (1988), who postulate that each generation receives utility from the consumption of the generations to follow, and bequests as being motivated by a well defined "joy of giving"[3] as in Abel and Warshawsky (1998)) and Constantinides et al. (2007)

Any systematic consideration of bequests mandates that the analysis be undertaken in an overlapping generations model context. Consequently, we analyze an overlapping generations economy in balanced growth. Each period, a set of individuals, of measure one, enter the economy. The measure of types, $\mu(d\alpha)$, is defined on the Borel sigma algebra of $\mathbb{R}_+$.

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[3] See also De Nardi, Imrohoroglu and Sargent (1999) and Hansen and Imrohoroglu (2007)

All individuals have finite expected lives. They enter the labor force at age 22, work for T years and then retire[4]. Individuals receive a wage income during their working years but not during their retirement years. At retirement, individuals face idiosyncratic uncertainty about the length of their remaining lifetime. Their lifetimes are exponentially distributed. Once retired, the probability of surviving to the next period is $(1-\delta)$, where δ is the probability of death. Expected life is $T + 1/\delta$. There is no aggregate uncertainty

An individual who is born at time t and dies at age j, bequeaths $b_{t,t+j}$ units of the period t consumption good and consumes nothing at that or latter ages. For an individual of type α (born at time t) the expected utility over age contingent bequests and consumption, conditional on being alive at age j, $c_{t+j,j}$, is

(2.1) $$\sum_{j=0}^{T} \beta^j \log c_{t+j,j} + \sum_{j=T+1}^{\infty} \beta^j (1-\delta)^{j-T-1}[(1-\delta)\log c_{t+j,j} + \delta \alpha \log b_{t+j,j}]$$

Here $\beta < 1$ is the discount factor and α is a parameter that governs the strength of the bequest motive. Each generation supplies one unit of labor inelastically for $j = 0,1,...,T-1$. Thus, aggregate labor supply is $L = T$ given the measure of each generation is 1.

We only need to analyze the decision problems of an individual of a type α individual born at time $t = 0$. The solution to the problem for groups born at any other t can be found using the fact that

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[4] We implicitly assume that parents finance the consumption of their children under the age of 22 – in other words, children's consumption is a part of their parents consumption. For simplicity (and without loss in generality) we assume that individuals are '22 year old' when born and model their consumption and investment decisions only after they enter the workforce at age 22.

(2.2) $$c_{t,j} = (1+\gamma)^t c_{0,j}$$

Further, to simplify the notation, we use c_j to denote the consumption of a j year old at time j rather than $c_{j,j}$. An analogous change of notation applies to the other variables.

Production Technology

The aggregate production function is

(2.3) $$Y_t = F(K_t, A_t L_t) = K_t^\theta (z_t L_t)^{1-\theta}$$

(2.4) $$z_{t+1} = (1+\gamma)z_t.$$

K_t is the capital, L_t is labor, and z_t is the labor augmenting technological change parameter, which grows at a rate γ. The parameter z_0 is chosen so that $Y_0 = 1$.

Output is produced competitively so that

(2.5) $$\delta_k + r_e = F_K(K_t, z_t L_t)$$

(2.6) $$e_t = F_L(K_t, z_t L_t)$$

where δ_k is the depreciation rate, r_e is the borrowing rate and the return on equity, and e_t is the wage rate.

Income is received either as wage income E_t or gross capital income R_t. Thus

(2.7) $$Y_t = E_t + R_t.$$

The wage rate is e_t; $E_t = L_t e_t = (1-\theta)Y_t$; and $R_t = (\delta_k + r_e)K_t = \theta Y_t$. Components of output are consumption C_t, investment X_t and intermediation I_t; thus

(2.8) $$Y_t = C_t + X_t + I_t$$

In balanced growth investment, $X_t = (\delta_k + \gamma)K_t$ and $K_{t+1} = (1+\gamma)K_t$.

Financial Intermediary Technology

There are three assets: private debt, government debt, and capital. A unit of debt is a promise to deliver one unit of the consumption good. The intermediary can hold private debt, government debt and capital, and can issues annuities. An annuity contract specifies a stream of premiums and payments contingent on being alive at each possible age and a payment at death. The intermediary's liabilities are the expected present value of annuity benefits less the present value of future premiums summed over all its current set of annuity contracts. The lending rate is used in this present value calculation.

The intermediary's liabilities are denoted by L^I, its holding of private debt by $D^{P,I}$, its holding of government debt $D^{G,I}$, and its holding of capital by K^I. Financial intermediation services are

(2.9)
$$I = \phi(D^{P,I} + K^I)$$

Note there are no costs of intermediating government debt and the net worth of the intermediary is zero. The intermediary balance sheet identity is

(2.10)
$$L^I = (1+r)D^{G,I} + (1+r_e)D^{P,I} + (1+r_e)K^I$$

In the above, $(1+r_e)$ is the price of a unit of capital in terms of the consumption good.

This being a constant returns to scale technology, an equilibrium condition is

(2.11)
$$r_e - r = \phi,$$

where r_e is the interest rate the intermediary receives on its lending and the return it earns on its equity holdings.

We refer to r as the household lending rate because households are lending to the intermediaries when they pay premiums and once they start receiving benefits, the benefit stream is de facto a debt contract. In equilibrium, an intermediary will offer an annuity

contract with the property that the expected present value of benefits is equal to the present value of premiums using r in the present values calculations.

During their working years, individuals can accumulate equity and borrow. If an individual enters into an annuity contract at age $j = 0$, the pension fund reserves for that contract is an asset of that individual[5]. Thus, an agent's asset holdings at point in time are pension fund reserves and equities. The agent's liabilities are the agent's private debt.

Government Policy

The government finances interest payments by issuing new debt and by a tax τ on labor income. The government's period t budget constraint is

(2.12) $$(1+r)D_t = \tau E_t + D_{t+1}$$

$D_{t+1} = (1+\gamma)D_t$ in balanced growth . Therefore,

(2.13) $$(r - \gamma)D = \tau(1 - \theta)Y$$

In addition, the government pursues a tax-rate policy that pegs[6] r_e. This being a balanced growth analysis, government debt grows at rate $\gamma > 0$, which means the government deficits are positive and grows at rate γ.

3. Optimal Individual Decisions

We consider the optimal individual decision problem, taking as given the size of the bequest that he will receive at age 30 and the labor income tax rate, τ . At time t,

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[5] The Flow-of-Funds household sector net worth sector lists pension fund reserves as part of household net worth.
[6] In this paper, we fix this to be 5%. This is discussed further in the section on calibration.

people age 30 receive inheritance $b_t = \bar{b}(1+\gamma)^t$. At time $t = 0$, people aged 30 receive bequests $\bar{b}$.

We use the term 'annuity' to denote a contractual arrangement between an individual and a financial intermediary where the individual makes premium payments during their working years and receives payments when retired. These payments are used to finance consumption when retired. The annuity strategy thus allows the individual to hedge the idiosyncratic risk associated with the uncertainty of his time of death.

In this model, for a reasonable set of parameters, depending on their intensity for bequests, some individuals *at birth* find it optimal to save for retirement in the form of annuities and other individuals find it optimal to save in the form of equity. Some of those holding equity will also be debtors. Given borrowing rates and returns on equity are equal, changing equity and debt position has no consequence for an individual provided the individual doesn't become a creditor. In the aggregate the total amount of borrowing is determined, but not that of individuals.

We will show there exists an $\alpha^* > 0$ that partitions individuals into two groups: individuals with $0 \le \alpha < \alpha^*$ choose to annuitize while those with $\alpha > \alpha^*$ hold equity and possibly borrow. The function $\Delta U(\alpha)$ for a given economy specifies the differences in utilities for the two strategies. Plotted in Figure 1 is this difference for a particular economy. We see for the economy considered that agents with bequest intensity $\alpha < 5$ choose to annuitize for the illustrative set of parameters considered. Our finding that agents with a low "intensity" for bequest will annuitize is consistent with the result in Yaari (1965).

Figure 1

Utility Difference between the Best No Annuity and Best Annuity Strategy:

$$U^B(\alpha) - U^A(\alpha)$$
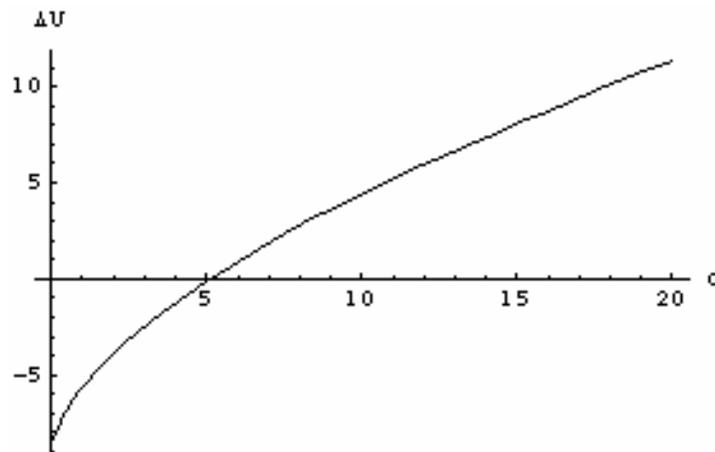
In light of this, we restrict our further analysis to an economy with two types of agents: those who have a zero intensity for bequests and follow the annuity strategy (Type A) and those with a high intensity for bequests, who follow the no annuity strategy (Type B). The measures of these two types are denoted μ^A and μ^B. The motivation for the names is that type A will choose the annuity strategy and type B will follow the no annuity strategy and leave a bequest.

A convention followed is that a bar over a variable denotes a constant. In the case where the constant depends upon a person type, that is on α, this functional dependence will be indicated. This is necessary, as the best strategy will differ across agent types.

3.1 The Best No Annuity Strategy

We first consider the problem of an individual given that individual's wealth, which for this strategy is the present value of inheritance, $(1+\gamma)^{30}\bar{b}$ by age 30 plus the present value of wages discounted at the rate r_e.

This problem becomes stationary and recursive at retirement age T with net worth w being the state variable. The value function $f(w)$ is the maximal obtainable expected current and future utility flows if a retiree is alive and has net worth w. The optimality equation is

$$f(w) = \max_{c,w'} \{\log c + (1-\delta)\beta f(w') + \delta\beta\alpha\log w'\}$$

(3.1)
$$s.t. \quad c + \frac{w'}{(1+r_e)} \le w$$

The solution to this optimality equation has the form:

(3.2) $\qquad f(w) = \bar{f_1}(\alpha) + \bar{f_2}(\alpha)\log w,$

where

(3.3) $\qquad \bar{f_2}(\alpha) = \dfrac{1+\alpha\ \beta\delta}{1-(1-\delta)\beta}.$

The optimal consumption and implicit bequest strategies are:

(3.4) $\qquad c = w / \bar{f_2}(\alpha)$

$\qquad\qquad w' = (1+r_e)(w-c)$

The assets of a person born at time 0 at age j are w_j in units of the period j consumption good. Bequests are

(3.5) $\qquad b_j = w_j \qquad\qquad j \ge T$

The problem facing an individual at birth that follows the no annuity strategy, (which we call strategy B because it is the one that those with a strong bequest motive) is,

$$U^B(\alpha) = \max_{\{c_j\}_{j=0}^{T-1}, w_T} \{\log c + (1-\delta)\beta f(w') + \delta\beta\alpha\log w'\}$$

(3.6)

subject to

$$\sum_{j=0}^{T-1} \frac{c_j}{(1+r_e)^j} + \frac{w_T}{(1+r_e)^T} \le v_0^B = \sum_{j=0}^{T-1} \frac{(1-\tau)e_0(1+\gamma)^j}{(1+r_e)^j} + \frac{\overline{b}}{(1+r_e)^{30}}$$

Here v_0^B is the present value of wages and bequest at birth of someone born at $t=0$. The solution (see appendix for the expression for $\overline{c}^A(\alpha)$) is

$$c_j = \overline{c}^A(\alpha)\beta^j(1+r_e)^j v_0^B \qquad j < T$$

(3.7)

$$w_T^B = (1 - \sum_{j=0}^{T-1}\overline{c}^A(\alpha)\beta^j)(1+r_e)^T\, v_0^B$$

The age j net worth of an individual following this strategy satisfies

$$w_0 = 0$$

(3.8)

$$w_{j+1} = (1+r_e)(w_j + e_j(1-\tau) - c_j) \qquad\qquad j \ne 30$$

$$w_{j+1} = (1+r_e)(w_j + e_j(1-\tau) - c_j + \overline{b}) \qquad j = 30$$

3.2. The Best Annuity Strategy

The best annuity strategy for a type-α is the solution to the following :

$$U^A(\alpha) = \max_{\{b_j,c_j\}}\{ \sum_{j=0}^T \beta^j \log c_j + \sum_{j=T+1}^\infty \beta^j(1-\delta)^{j-T-1}[(1-\delta)\log c_j + \delta\alpha\log b_j]\}$$

(3.9)

s.t

$$\sum_{j=0}^T \frac{c_j}{(1+r)^j} + \sum_{j=T+1}^\infty \frac{(1-\delta)^{j-T-1}[(1-\delta)c_j + \delta b_j]}{(1+r)^j} \le v_0^A$$

where r is the lending rate and

$$(3.10) \qquad v_0^A = \sum_{t=0}^{T-1} \frac{(1-\tau)e_0(1+\gamma)^t}{(1+r)^t} + \frac{\overline{b}}{(1+r)^{30}}$$

The constant v_0^A is the present value of future wage income and inheritances using the lending rate r of a person born at $t = 0$. The superscript A denotes the annuity strategy and not an individual type. It will be the case that in equilibrium type A will choose strategy A.

There are other constraints, specifically, that the worker choosing this strategy does not borrow, that is $e_j - c_j \geq 0$. For the economies considered in this study, these constraints are not binding and can therefore be ignored. If, however, the economy were specified such that the no-borrowing constraint were binding for some j, then the solution below would not be the solution to the problem formulated above.

The nature of the annuity contract is that the payment to someone alive at age $j \geq T$ is c_j. If the individual dies at age j, payment b_j is made to that person's estate. The solution to this program is

$$(3.11) \qquad c_j^A = \overline{c}(\alpha)(1+r)^j \beta^j\ v_0^A \qquad j \geq 0$$

$$(3.12) \qquad b_j^A = \alpha\overline{c}(\alpha)(1+r)^j \beta^j v_0^A \qquad j \geq T+1$$

The net worth of an individual choosing this strategy is his pension fund reserves. Pension fund reserves (from the point of view of the company) for a given annuity contract for an individual born at $t = 0$ at age j is the expected present value at time

$t = j$ of payments that would be made less the present value (at time $t = j$ as well) of premiums that would be received.

For workers they can be determined as the present value of past premiums. Thus, pension fund reserves for individuals born at $t = 0$ at age j satisfy

$$w_0^A = 0$$

(3.13)
$$w_j^A = \sum_{k=0}^{j-1} ((1-\tau)e_k - c_k^A)(1+r)^{j-k} \qquad j = 1,\ldots,30$$

$$w_j^A = \sum_{k=0}^{j-1} ((1-\tau)e_k - c_k^A)(1+r)^{j-k} + \bar{b}(1+r)^{(j-30)} \qquad j = 31,\ldots,T-1$$

For retirees, conditional on being alive, pension fund reserves for individuals born at $t = 0$ at age j must be equal to the expected present value of the future payments

$$w_T^A = c_T^A + \sum_{t=1}^{\infty} (1-\delta)^t \frac{c_{T+t}^A}{(1+r)^t} + \sum_{t=1}^{\infty} \delta(1-\delta)^{t-1} \frac{b_{T+t}^A}{(1+r)^t}$$

(3.14)
$$w_j^A = \sum_{t=0}^{\infty} (1-\delta)^{t+1} \frac{c_{j+t}^A}{(1+r)^t} + \sum_{t=0}^{\infty} \delta(1-\delta)^t \frac{b_{j+t}^A}{(1+r)^t} \qquad j > T$$

3.3 Best Strategy

The best strategy is the no annuity strategy if $U^B(\alpha) > U^A(\alpha)$. The best strategy is the annuity strategy if $U^B(\alpha) < U^A(\alpha)$.

Section 4: Aggregate Behavior of the Household Sector

Aggregate Consumption

This aggregate consumption demand depends upon the labor tax rate τ and inheritance $\bar{b}$ as well as the prices $\{e, r, r_e\}$, which are determined by policy and by firms. Having formulated the optimal consumption strategies for the two types of individuals, we can characterize the aggregate consumption, asset holdings and bequest at time $t = 0$ by individual type given $\bar{b}_0$ and e_0. Given this is a balanced growth analysis period zero is as good as any.

There are two types of agents $k \in \{A, B\}$. The A-type has $\alpha = 0$ and will in equilibrium choose the annuity strategy given the model economy. The measure of type k of age j at $t = 0$ is

(4.1)
$$\mu_j^k = \begin{cases} \mu_0^k & j \leq T \\ (1 - \delta)^{j-T} \mu_0^k & j > T \end{cases}$$

The aggregate consumption of both types of agents at time 0 is C^k

(4.2)
$$C^k(\bar{b}, \tau) = \mu^k \sum_{j=0}^{T-1} c_j^k (1 + \gamma)^{-j} + \mu^k \sum_{j=T}^{\infty} (1 - \delta)^{j-T} c_j^k (1 + \gamma)^{-j}$$

Where we have used the fact that each subsequent generation has a consumption-age profile that is higher by a factor of $(1 + \gamma)^j$ in balanced growth.

Aggregate consumption is

(4.3)
$$C(\bar{b}, \tau) = C^A(\bar{b}, \tau) + C^B(\bar{b}, \tau)$$

Aggregate Asset Holdings

The aggregate net worth of types $k \in \{A, B\}$ are

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$$(4.4) \qquad W(\bar{b}, \tau) = \mu_0^k \sum_{j=0}^{T} w_j^k (1+\gamma)^{-j} + \mu_0^k \sum_{j=T+1}^{\infty} (1-\delta)^{j-T} w_j^k (1+\gamma)^{-j}$$

Net worths are as of the beginning of a period and are in units of the consumption good.

Aggregate Inheritance

At time zero the measure of the people aged $j > T$ who *die* and leave a bequest is $\mu_0^B \delta (1-\delta)^{j-T-1}$, thus the total bequests given by these agents is:

$$B_j = \mu_0^B \delta (1-\delta)^{j-T-1} w_j^B \qquad\qquad j > T$$

Hence the *aggregate bequests* at time 0 are:

$$(4.5) \qquad\qquad B_0 = \sum_{j=T+1}^{\infty} B_j (1+\gamma)^{-j}$$

Since we assume that bequests are equally distributed and received at age 30, the *inheritance* of someone who is 30 years old at time 0 is:

$$\bar{b}_0 = \frac{B_0}{\mu_0^A + \mu_0^B}$$

Thus, the inheritance received at time 30 by an individual who is born at time zero is

$$(4.6) \qquad\qquad \bar{b} = \bar{b}_0 (1+\gamma)^{30}$$

This is the quantity used in computing the present value wealth of individuals in equations (3.6) and (3.10).

Aggregate Private Debt

The aggregate indebtedness of a type B is

(4.7) $\qquad D^B(\bar{b},\tau) = K - W^B(\bar{b},\tau)/(1+r_e)$

as the price of existing capital in terms of the consumption good is $(1+r_e)$ and the

household is obligated to make payment $(1+r_e)D^B(\bar{b},\tau)$.

Section 5: Equilibrium Relations

From the Production Side

We determine the value of a set of balanced growth state variables at $t = 0$. All

variables grow at rate γ except aggregate labor supply, which is constant and equal to

40. Given Y has been normalized to 1 at time zero, the cost share relations determine

time zero K and wage e:

(5.1) $\qquad (r_e + \delta_k)K = \theta Y$

(5.2) $\qquad e L = (1-\theta)Y$

From the intermediary's problem, the lending rate is determined by

(5.3) $\qquad r = r_e - \phi$

Two Equilibrium Conditions

Prices $\{e, r, r_e\}$ are determined from policy and therefore only $\bar{b}$ and τ are

needed to completely specify the household budget constraints. Aggregate

consumption $C(\bar{b},\tau)$ and aggregate intermediation $I(\bar{b},\tau)$ will be determined by

aggregating household variables. An aggregate equilibrium condition is the aggregate

resource constraint,

$$(5.4) \qquad C(\overline{b},\tau) + X + \phi\, I(\overline{b},\tau) = K^\alpha L^{1-\alpha}.$$

where $X = (\delta_k + \gamma)K$ is investment. Intermediation services satisfy

$$(5.5) \qquad I(\overline{b},\tau) = K - \frac{W^B(\overline{b},\tau)}{(1 + r_e)}$$

We assume that type B hold all the capital and the intermediary none. This is done to resolve the unimportant indeterminacy. Increasing the amount of capital held by a type B and type B indebtedness by the same value amount does not affect a type B wealth, and that is all that matters. This portfolio shift of the B type individual is offset by a portfolio shift of the intermediary. The aggregate indebtedness of a type B is denoted by $D^B(\overline{b},\tau)$. An expression for aggregate intermediation is $I(\overline{b},\tau) = \phi\, D^B(\overline{b},\tau)$

The second equilibrium condition is that inheritance of people at a point in time equals aggregate bequests at that point in time. We consider $t = 0$ and let $B(\overline{b},\tau)$ be the aggregate bequest at that time. Given $\overline{b}$ is the inheritance that someone born at $t = 0$ receives at age 30 and the economy grows at rate γ, the second equilibrium condition is

$$(5.6) \qquad \overline{b} = \frac{(1+\gamma)^{30} B(\overline{b},\tau)}{\mu_A + \mu_B} = (1+\gamma)^{30} B(\overline{b},\tau) \quad \text{as} \quad \mu_A + \mu_B = 1$$

There is a third equilibrium condition, namely the government's budget constraint. Equating payments to receipts, $(1 + r)D_t = \tau E_t + D_{t+1}.$ Given

$D_{t+1} = (1+\gamma)D_t$, $E_0 = (1-\theta)Y_0$, and Y_0 has been normalized to 1.0, the time zero government budget constraint is

(5.7) $\qquad (r-\gamma)D(\bar{b},\tau) = \tau(1-\theta)\,\text{Y}$

Section 6: Balance Sheets

Assets and liabilities are beginning of period numbers and are in units of the consumption good.   We consider only economies for which there is intermediated borrowing and lending in equilibrium. Given there is a large amount of intermediated borrowing and lending, these economies are the empirically interesting ones. The balance sheet relations, (assets equal liabilities plus net worth) are:

$\qquad$ *Type B:* $\qquad\qquad (1+r_e)K = (1+r_e)D^B(\bar{b},\tau) + W^B(\bar{b},\tau)$

$\qquad$ *Type A:* $\qquad\qquad W^A(\bar{b},\tau) = W^A(\bar{b},\tau)$

$\qquad$ *Intermediary:* $\qquad (1+r)(D^B(\bar{b},\tau) + D^G(\bar{b},\tau)) = W^A(\bar{b},\tau) + W^I(\bar{b},\tau)$

$\qquad$ *Government:* $\qquad \dfrac{\tau(1-\theta)Y}{(r-\gamma)} = D^G(\bar{b},\tau)) + W^G(\bar{b},\tau)$

The net worth of the government $W^G(\bar{b},\tau)$ and of the intermediary $W^I(\bar{b},\tau)$ are both zero. Further, the value of private debt held by the intermediary is slightly different than the value of the associated liability because of intermediation costs. In our model, (see section 9 on results) the present value of the tax on wages using a discount rate of 3% is precisely equal to the value government debt. Since labor is supplied inelastically and taxed at a rate τ, the government effectively owns a fraction τ of an individual's time endowment (now and in all future periods). Hence, the net worth of the government is zero and government debt is an asset for debt holders in our model.

Section 7: Equilibrium

The two equilibrium conditions are linear in $(\bar{b}, \tau)$, so solving for a candidate solution is straightforward. To be an equilibrium it must be the case that (i) the best strategy for type B is the no annuity strategy; (ii) the best strategy for type A is the annuity strategy; (iii) $D^B > 0$; and (iv) $c_{0,j}^A < (1-\tau)e_o$. The reason for the last constraint is that these equilibrium conditions hold provided that the no borrowing constraint of annuity holders is not binding and it will not be binding if (iv) holds.

Section 8: Calibration

The parameters that needs to be "calibrated" are the parameters related to the model people $\{\alpha^A, \alpha^B, \beta, \mu^A, \mu^B, T_W, T_I, T, \delta\}$; the intermediation technology parameter ϕ; the goods technology parameters $\{\theta, \delta_k, \gamma\}$; the policy parameter r_e. The other policy parameters $\{\tau, D^G\}$ are endogenous. Many of these parameters are well documented in the literature; others are not.

We proceed by listing them with selected values and a brief motivation

Parameters associated with individuals

$\beta = 0.99$

$\delta = 0.05$ (Implies a post retirement life expectancy of 20 years)

$\alpha^A = 0$ (Assumption. Type A individuals have low bequest intensity)

$\alpha^B = 10$ (Assumption. Type B individuals have high bequest intensity)

$T_W = 22$ (Age at which an agent enters the workforce)

$T_I = 52$ (Age at which inheritance is received)

$T = 62$ (Age at retirement)

$\mu^A = 0.88$ (Specified such that the amount intermediated matched U.S. data)

$\mu^B = 0.12$ (equal to $(1 - \mu^A)$)

Intermediation parameters

$\phi = .02$ (Consistent with the average difference in borrowing and lending rates)

Goods production parameters

$\theta = 0.3$ (Share of capital in output)

$\gamma = 0.02$ (Consistent with observations on labor productivity)

$\delta_k = 0.05$ (Consistent with capital output ratio = 3, given r_e)

Policy parameters

$r_e = 0.05$ (Assumption about government fiscal policy)

The motivation for this policy is that this has been the approximate after tax return of capital in the corporate sector (See McGrattan and Prescott, 2005).

In calibrating ϕ we proceed by estimating the value added by the financial intermediation sector. The major source of revenue for this sector is the difference in interest payments received from borrowers and interest payments paid to lenders. Using data from NIPA[7] for 2000 the former amounted to $1,480 billion (0.148 times GNI) and the latter to $940 billion (0.094 times GNI. To estimate the services associated with intermediating borrowing and lending, we first subtracted services furnished without payment by the financial intermediaries, because we see these services as corresponding mostly to transaction services. These amounted to $187 billion. Thus, the value added by

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[7] The data used is from NIPA (2000) tables 7.11 and 2.4.5.

the financial intermediation sector is $ 353 billion or about 3.5% of GNI. A significant

amount of intermediation services is purchased by non financial business. We do not yet

have a good measure of this number. We guess that it is about 0.8 times GNI which leads

to a number of 0.026 times GNI being household borrowing/lending intermediation

services.

Using data from the flow of funds[8], we estimate the total amount of intermediated

borrowing and lending between households to be 1.3 times GNI (See Table 1 below). The

implied intermediation spread is thus 2.0 percent. Some intermediate borrowing is by

young Type A people in the form of consumer debt. This led us to estimate the difference

in average household borrowing and lending rates to be 2 percent and in turn the

calibrated $\phi = 0.02$.

We estimated borrowing and lending between households by determining total

household holdings of debt assets in year 2000. Not all of this corresponds to the

household debt in our model. Some is intermediated borrowing and lending between

young people of the same type. Some is lending for precautionary reasons and for

transaction purposes (including currency held). Considerations such as these led us to

calibrate the measure of Type B so the amount of intermediated borrowing and lending

was 1.5 times GNI.

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[8] The data is from the Flow of Funds (2000) table B.100.e.

Table 1		
Interest received by financial intermediaries	0.14480 GNI	Table 7.11 NIPA Line 28
Interest paid by financial intermediaries	0.0940 GNI	Table 7.11 NIPA Line 4
Services furnished by financial intermediaries without payment	0.0167 GNI	Table 2.4.5 NIPA Line 89
Intermediation services associated with household borrowing and lending[9]	0.0353 GNI	
Total amount intermediated[10]	1.3076 GNI	

Section 9: Results

We considered three values for α^B, a parameter for which we have little information. For each value of α^B we search for the μ^B for which the intermediate borrowing and lending between households is approximately 1.5 times GNI. The results are summarized in Table 2. The results are not sensitive to the strength of the bequest parameter α^B.

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[9] Net interest less transaction services, which are assumed equal to Services furnished without payment by FI.

[10] From FoF year 2000 Table B.100b.e. This number is Assets (line 1) minus Tangible Assets (line 2) minus Equity Shares at Market Value (line 6) minus equity of unincorporated business. The last number was obtained from Table B.100 (line 28).

Table 2

Summary of Results

Economy	$\alpha^B = 6$	$\alpha^B = 10$	$\alpha^B = 15$
μ^A	0.863	0.880	0.895
μ^B	0.137	0.120	0.105
Nation Accounts			
C_A	0.658	0.672	0.685
C_B	0.106	0.092	0.079
X	0.210	0.210	0.210
I	0.026	0.026	0.026
Y	1.000	1.000	1.000
Depreciation	0.15	0.15	0.15
Compensation	0.70	0.70	0.70
Profits	0.15	0.15	0.15
Net Worth			
Type A	6.63	6.75	6.85
Type B	1.78	1.78	1.78
Government Debt/Y	5.13	5.24	5.25
Bequest/Y	0.0700	0.0746	0.0799
Tax rate	0.0732	0.0748	0.0764

Table 3

Inheritance as Fraction of Wealth at Enter into Workforce

	$\alpha^B = 6$	$\alpha^B = 10$	$\alpha^B = 15$
Type A	**0.051**	**0.054**	**0.058**
Type B	**0.040**	**0.043**	**0.045**

Total bequests in our model are large, larger than for the U.S. economy. Total bequests reported on U.S. estate tax forms plus total charitable contribution reported on individual tax forms were only 0.4 percent of GNI in year 2000. This number is far smaller than the 7.5 percent number for our model economy. We do not view this as problematic given the nature of our abstract, and more importantly, because much of what we think of as bequests is not reported to the tax authorities. This number would be significantly smaller if there were population growth at say 1 percent a year, which has been the approximate U.S. population growth rate. The fraction of people leaving bequests in a given year would be far smaller and the fraction working higher. Introducing population growth would reduce the bequest to GNI ratio by at least a factor of 2. The big adjustment is for bequests not reported on tax records.

Some bequests are given prior to death for estate tax reasons and for the joy of seeing others benefiting from them. There are hidden bequests when family businesses are transferred to a younger generation. Further, most estates in year 2000 were less that $600,000 and therefore not reported on estate tax forms. Converting the inheritance to the annual wage, in our model economy an individual receives 3.4 times their annual wage when 52 years old. With 1 percent population growth this would be reduced by over a factor of 2. These consideration suggest that bequest are not excessive in our model world.

One variable of interest is the fraction of wealth that is inherited. A significant component of wealth is human capital, which is the present value of wages. It is about 94.5 percent and would be higher if there is population growth. These results are for a

type A, who discount using a 3 percent rate. The share is a little lower for type B who use a 5 percent discount rate.

Government debt may appear large relative to explicit U.S. government debt, which is only 0.3 times GNI. In fact, the estimates of implicit Social Security Retirement and Medicare promises are over 3 times GNI by most estimates. Further, with population growth this number would be significantly smaller. Perhaps the consumption value of Medicare payments is less than the cost to the government. Thus government debt is not large. If the government prohibited bequests, the steady state capital stock would be the same, namely 3 times GNI with the given government policy.

Lifetime and cross sectional consumption patterns

Figure 2 plots the lifetime consumption patterns of the two types. Type A's consumption grows at a constant rate $\beta(1+r)-1 \approx 0.02$ conditional on being alive. Type B's have a higher saving rate during their working life. Once retired their consumption grows at a lower rate, which can be negative if the bequest motive is sufficiently strong.

Figure 2

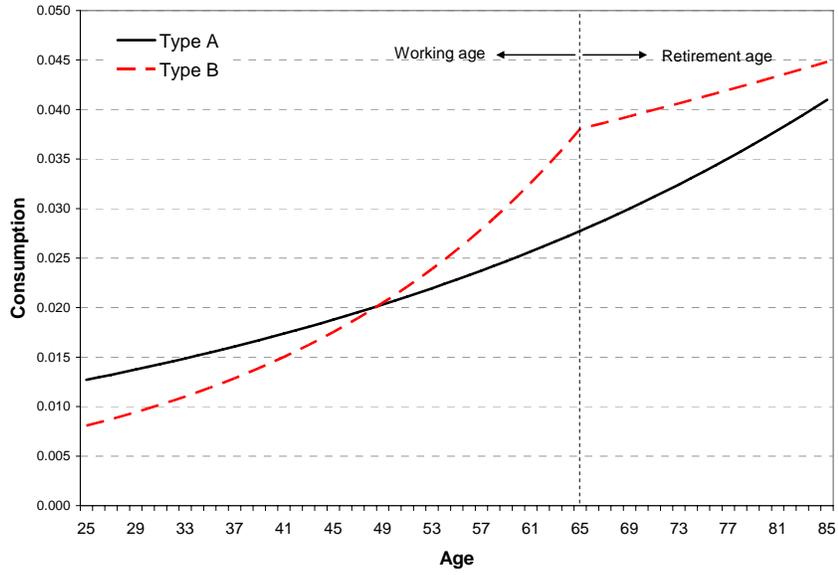## Life time pattern of consumption

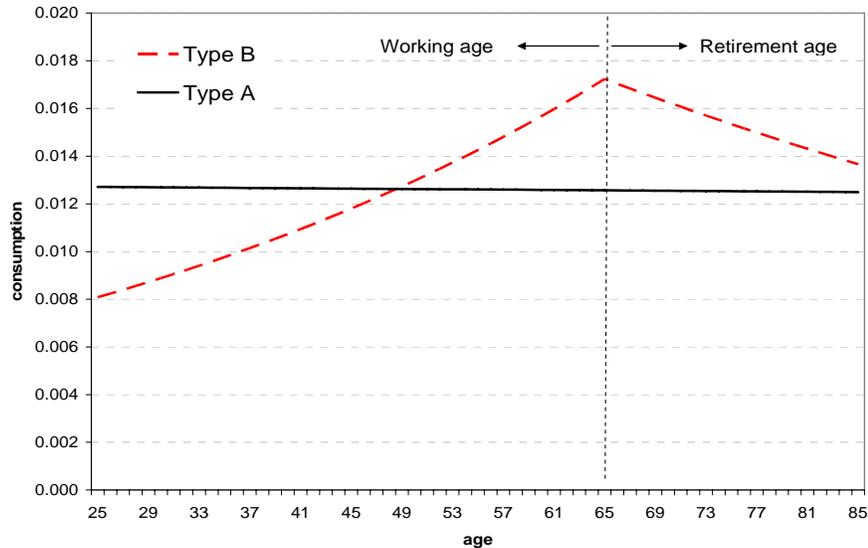**Figure 3**

Consumption by age at period zero

Figure 3 plots consumption by age at a point in time. Young and old of Type A consume the same. For type B, however, consumption starts low and increases through out the working life and declines throughout the retirement period. However, it is worth noting that there little dispersion of consumption between the two types of agents.

*Equity holdings by age and concentration of equity holdings*

Only type B hold the capital. Figure 4 plots their holding by age at a point in time. There is a high concentration of capital ownership with 33 percent of type B holding half the capital stock. But Type B is only 12 percent of the population so less than 4. percent of the households own half the capital stock in this economy even though all have the same inheritance and the same lifetime pattern of wages if they are born at the same time. While there is considerable dispersion in net worth over the life cycle, it is considerably muted when adjusted for age.

Figure 4
Net worth by age at time zero
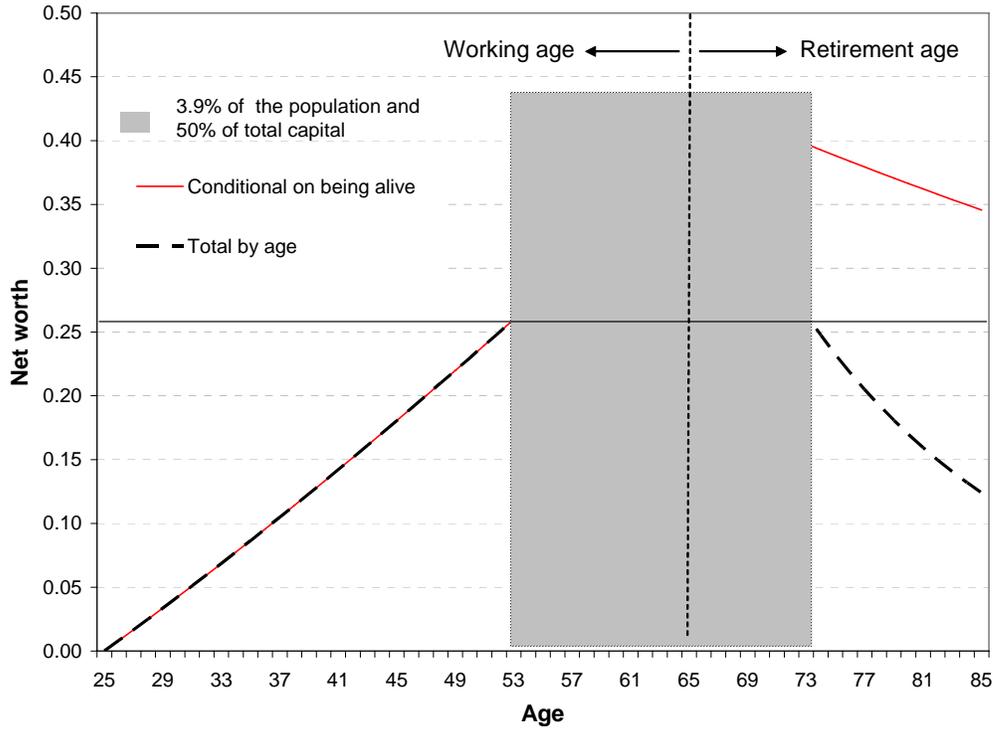
The red line is the Type B net worth conditional on being alive, that is $w_j^B(1+\gamma)^{-j}$, while the black dotted line is the total net worth of an agent type B at age j at period zero. That is, $w_j^B(1+\gamma)^{-j}$ if $j<T$ and $w_j^B(1+\gamma)^{-j}(1-\delta)^{j-T}$ if $j \geq T$

Lorenz curve for Consumption, total Net Worth and Capital
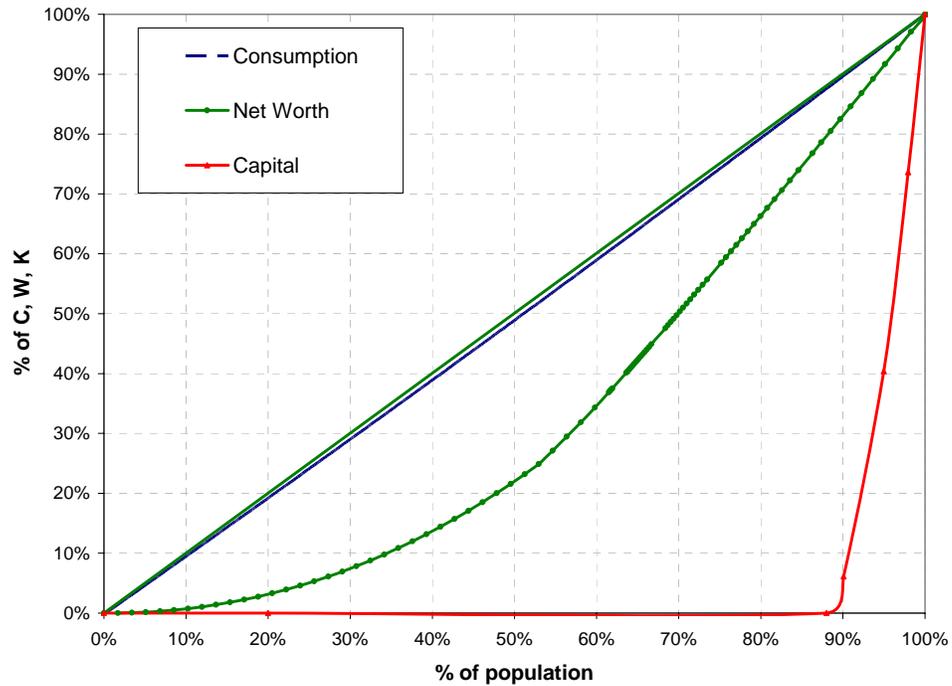
This picture shows the usual Lorenz curve for consumption, total wealth and capital.

Cost of financial market constraints

What are the gains to a household of having access to the equity market at no intermediation cost? Table 4 reports the cost of not having this access, (which was the case for most Americans prior to the development of low cost indexed mutual funds) as being about 25 percent of wealth at entry into workforce. This wealth is the present value of labor income and inheritance.

Table 4

Cost to an *A* of not Having Access to the

Annuity Market

α^B	Change in v_0^A
6	0.99%
10	1.07%
15	1.15%

Table 5

Cost to a *B* of not Being Permitted to hold Equity Directly

in Units of Wealth at Entry into Workforce

α^B	Change in v_0^B
6	15.68%
10	21.65%
15	27.43%

These tables shows the percentage increase in either e_0 or v_0^k necessary to compensate agent k in utility if he is forced to switch to a system other than his most preferred choice. Since both, consumption and bequest are linear functions of initial wealth; the percentage changes in both consumption and bequest are the same as the percentage change in initial wealth.

What are the costs to a type *A* if for some reason such as adverse selection problems or legal constraints, they do not have access to annuity markets, and must use the equity option for saving?

Table 6

Cost to an A of not Having Access to Annuity Markets

in Units of Wealth at time of Entry into Workforce

α^B	Cost
6	0.9%
10	1.0%
15	1.0%

Section 10: Concluding Comments

In this paper, we develop a heterogeneous agent economy where agents differ as to the strength of their bequest motive. In equilibrium, households with a low motivation to bequeath lend and hold annuities, while those with a well-articulated preference for bequests borrow and hold equity. This is important, for the amount borrowed by households must equal the amount lent by households. In our framework, we are able to account for *both* the amount of intermediated borrowing and lending between households *and* the average spread in borrowing and lending rates resulting from intermediation costs.

We find that incorporating the divergence between borrowing and lending rates can account for a third of the historically observed equity premium of 6%, *even in a world without aggregate uncertainty*. This supports the conclusion of our 1985 paper that the premium for bearing systematic risk is small.

Our analysis in this paper is admittedly stylized. However, we believe the abstraction is well suited to address the impact of the costs associated with financial intermediation on the equity premium. We view this as a first step in what we conceive of as an important research agenda. Possible extensions include building in differential survival rates and addressing the issues of adverse selection and moral hazard when pricing annuities. We expect these extensions to yield theories that, in addition to matching the quantity intermediated and the intermediation spreads, also match the stocks of assets held. We will, of course need detailed statistics on individual asset holdings to investigate these issues.

This research program, if successful, will interface with the literature on household lifetime consumption behavior. Such an interface will require an extension as the bequest motive is not the only factor that differentiates people. There surely are differences in preferences with respect to consumption today versus consumption in the future and differences in preferences that give rise to differences in lifetime labor supply. Our analysis suggests that asset holdings and consumption over the lifetime should be jointly considered.

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