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Business Start-Ups and
Productive Efficiency[*]

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ABSTRACT

This paper studies efficient allocation of resources in an economy in which agents are initially heterogeneous with regard to their wealth levels and whether they have ideas or not. An agent with an idea can start a business that generates random returns. Agents have private information about (1) their initial types, (2) how they allocate their resources, and (3) the realized returns. The unobservability of returns creates a novel motive for subsidizing agents who have ideas but lack resources to invest in them. To analyze this motive in isolation, the paper assumes that agents are risk-neutral and abstracts away from equality and insurance considerations. The unobservability of initial types and actions implies that the subsidy that poor agents with ideas receive is limited by incentive compatibility: the society should provide other agents with enough incentives so that they do not claim to be poor and have ideas. The paper then provides an implementation of the constrained-efficient allocation in an incomplete markets setup that is similar to the U.S. Small Business Administration's Business Loan Program. Finally, the paper extends the model in several dimensions to show that the results are robust to these generalizations of the model.

1   Introduction

Starting a business requires two main ingredients: a productive idea and resources to invest in that idea. Unfortunately, it is not necessarily the case that whoever has one of these ingredients also has the other one. Consequently, there is a potential mismatch among individuals in a society in terms of who holds productive resources and who can use them most efficiently. In a frictionless world, a solution to this mismatch is private markets: those with ideas (potential start-ups) can borrow from those with resources, invest, and then pay back. This paper explores how a society should cope with this mismatch in an environment in which individuals possess asymmetric information at all stages of the process of starting a business. Ex ante, individuals privately know their wealth levels and whether they have ideas or not; interim, their actions are unobservable to others; and ex post, they privately observe investment returns. The paper shows that under these informational frictions, the society has to subsidize poor agents who start businesses so as to cope with this mismatch. Therefore, the paper provides a novel rationale for governments to subsidize business start-ups.

Individuals in the model economy live for two periods and are risk-neutral. In period one, agents are heterogeneous with respect to wealth levels and whether they have ideas or not. Agents with ideas can create businesses that generate risky returns in the next period and feature diminishing marginal returns to capital. In the absence of informational frictions, efficient resource allocation involves two separate steps: (1) productive efficiency requires transferring resources to poor and productive agents initially to make sure that all productive agents can invest at the socially efficient level; (2) distributive efficiency then requires making transfers between agents so as to achieve the desired consumption distribution, which depends on the welfare criterion of the society.

Unfortunately, it is hardly the case that all relevant information about business start-ups are known publicly.[1] The paper assumes that agents' ex ante types (wealth-idea), how they allocate their resources, and ex post returns to business start-ups are private information. The result that poor agents with ideas should be subsidized depends solely on the assumption of unobservability of returns. The assumption that ex ante types and actions of agents are private information only limits the scope of this subsidy. The role of each informational assumption on the results will be discussed in detail in section 3.

In order to understand the intuition for the subsidy result, one first needs to know what

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[1]See Hubbard (1998) for a survey of the literature on informational problems in capital markets.

society cares about in this economy. I assume the social welfare function to be utilitarian with equal weights on every agent. This assumption, together with risk neutrality of agents, implies that society has a preference only for the amount of total consumption, not for how it is distributed across agents. The society is only concerned about agents making right amounts of investment. Therefore, the problem that the society is facing is maximizing production subject to incentive compatibility and feasibility.

The intuition for the subsidy result is simple. Since there are diminishing marginal returns to capital, it is socially optimal to have all agents with ideas invest at the same socially efficient level. However, since returns to start-ups are unobservable, agents cannot write contracts with state-contingent repayment schedules. This market incompleteness then implies that agents can, at most, borrow an amount that they can pay back the next period in the lowest return state.[2] This borrowing constraint binds for poor agents with ideas when they want to invest at the efficient level. If the society can transfer some of its resources to these individuals, it would relax their budget constraints, enabling them to produce at a level closer to the social optimal, which is the social objective.

Consequently, this paper focuses solely on productive efficiency, leaving aside distributional concerns. The motivation for subsidy in this model is the need to finance the investment of poor agents with ideas. In fact, due to the choice of social welfare function and risk-neutrality assumption, this is the only reason why subsidy is socially desirable.

If the society knew who were the poor agents with ideas, then it would be very easy to implement the subsidy. However, when there are benefits at stake, such as a subsidy, people can pretend to be poor and to have productive ideas, get the subsidy, and consume it. As a result, the amount of subsidy going to poor agents with ideas is constrained by incentive compatibility: agents should not find it optimal to lie about their wealth and ideas, and use the subsidy for reasons other than investment.

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[2]Observe that I do not allow for default in the model. Following Diamond (1984), one can add default to this model by assuming that if a start-up continues to operate after period two, this brings a continuation value to the owner; if not, then at least some strictly positive fraction of this value gets destroyed. Then, agents can write state-contingent contracts by conditioning the continuation of a start-up business on the level of repayment. The fear of losing a fraction of the continuation value can make the agent make the payment associated with her true return level. In such a world, θ_l state can be interpreted as a default state. Even though in such a model poor agents with ideas would be able to borrow more than they can in the original model, one can show that this level would still be strictly less than the amount they need to finance socially efficient investment level. Therefore, the subsidy result would still be true under this alternative model. The reason why such an extension can be interesting is because it can make the details of the efficient social contract more realistic, giving rise to a more realistic implementation. This is further discussed in section 4.

Of course, it is possible that the society can try to understand whether people's ideas are productive, and monitor their wealth and how they use the subsidy. However, these activities are all costly. The assumption that it is impossible to pursue such monitoring activities corresponds to assuming that monitoring costs are prohibitively high.[3] I accept that this is an extreme assumption; however, assuming that agents' wealth, ideas, and actions are perfectly, costlessly observable is also extreme. I focus on the less studied of the extremes. I conjecture that the subsidy result would still be true if I allowed for monitoring technologies as long as the cost of monitoring is not zero.

It is important to note that the subsidy result is not an artifact of risk neutrality; it survives even if agents have strictly concave utility functions. However, in that case, society would also have a taste for equality that would force a redistribution from the rich to the poor. Furthermore, since agents would be risk-averse, society would like to smooth their consumption across states and periods. The risk-neutrality assumption makes it possible to abstract away from these additional distributive forces and focus solely on what productive efficiency dictates.

A corollary that follows from the subsidy result is that how productive activity (distribution of investment in the current context) *should be* organized in the economy depends on the distribution of wealth. This result depends crucially on the existence of informational frictions. The result and the assumptions behind it are further explained in section 3.

The paper provides a decentralization of the constrained efficient allocation in an incomplete markets setup where people trade risk-free bonds in a competitive market. Given that markets cannot provide subsidies on their own, an incomplete markets equilibrium under laissez-faire cannot attain constrained efficiency.[4] In order to implement the efficient allocation, the paper introduces two separate institutions to the market environment: a government and a government agency that deals with start-up firms. The government taxes all agents in a lump-sum manner and subsidizes its agency from its budget. The agency then subsidizes some individuals from a pool of applicants based only on their level of bond

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[3]Formally introducing monitoring technologies to the model that allow the user to observe another agent's wealth, idea, or actions and setting the cost of this technology to infinity restores the current model.

[4]Note that I do not allow for markets to open ex ante, meaning before agents know whether they are rich or poor and whether they have ideas or not. If that is allowed, then the interpretation of the optimal contract would be completely different. Instead of calling the transfers in the optimal social contract subsidies, we would call them state-contingent payment schedules of the optimal financial contract written between agents behind the veil of ignorance. Thus, constrained efficiency requires either markets to open ex ante or government to execute subsidies.

holdings. The tax-subsidy system is chosen such that only agents with ideas get subsidized. A comparison of the implementation with the U.S. Small Business Administration's (SBA) Business Loan Program is provided in section 4.

This is not the first paper to put forth the idea that, under informational frictions, productive efficiency may require subsidizing a certain group of individuals in a society. Aghion and Bolton (1997) also focuses on output maximization and show when there are moral hazard problems due to unobservable effort, redistributing resources from the rich to the poor may increase total output, boosting economic growth. However, the underlying mechanism behind the subsidy result in the current paper and that paper are completely different. Another difference of the current paper from Aghion and Bolton (1997) is that these authors focus on a particular notion of equilibrium, whereas the current paper analyzes constrained-efficient allocations.

Loury (1981), Banerjee and Newman (1991), and Galor and Zeira (1993) are also related to the current paper. These papers share a common result: in the presence of capital market imperfections, the distribution of wealth affects the distribution of investment, and hence aggregate output.[5] This is akin to the following result I derive in this paper: the distribution of wealth affects the distribution of productive activity in the constrained-efficient allocation. However, there is an important distinction between the two results. Each paper mentioned above assumes that a particular incomplete market structure exists and show that the wealth distribution affects equilibrium the distribution of investment under this assumption. The contribution of the current paper is that, instead of making arbitrary assumptions on the space of contracts available to agents, it takes as given the informational frictions and shows that the distribution of wealth affects productive activity in an economy *even* in the constrained-efficient allocation. Consequently, this paper directly establishes that it is due to informational frictions that the distribution of wealth affects the distribution of productive activity.[6]

Another strand of literature that is related to this paper is on optimal venture capital contracts since both this literature and the current paper consider the question of how to

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[5]Aghion et al. (1999), section 2, not only proves a similar result but also provides a discussion of related papers.

[6]Banerjee and Newman (1993) is closer to the current paper in the sense that it explicitly models an informational friction that causes the market imperfection. However, it restricts the contract space available to the agents arbitrarily. Therefore, essentially, it also focuses on some exogenously specified equilibrium notion, not on constrained efficiency.

finance business start-ups.[7] In general, venture capital literature focuses on characterizing the structure of optimal contracts in principal-agent relationships in which venture capitalists monitor everything but entrepreneurs' effort. The current paper assumes less transparency between agents by assuming that people cannot monitor each other's investment levels or output realizations. This rules out the existence of venture capital in the current model.[8] Consequently, this paper deals with the complementary problem of how a society should allocate productive resources in an environment in which agents are more opaque and hence less capable of allocating resources themselves.

The rest of the paper is organized as follows. Section 2 introduces the baseline model formally and analyzes the full information benchmark. Section 3 defines and solves for the constrained efficient allocation. Section 4 provides an implementation of constrained efficient allocation similar to the U.S. SBA's loan program. Section 5 studies some extensions and generalizations of the model and shows how robust results are. Finally, section 6 concludes.

2   Model

2.1   Environment

The economy is populated by a continuum of unit measure of agents who live for two periods. Agents are risk-neutral with the instantaneous utility function $u : \Re \to \Re$ defined as $u(c) = c$, for $c \geq 0$ and $u(c) = -\infty$, for $c < 0$.[9] They are expected utility maximizers with

$$E_1\{u(c_1) + \beta u(c_2)\},$$

where c_t is period t consumption and $\beta \in (0, 1)$ is the discount factor.

At the beginning of period one, some agents are born with ideas and some without. Let i denote whether an agent has an idea or not. Those who have ideas are called $i = 1$

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[7]See Admati and Pfleiderer (1994), Gompers (1995), and Jovanovic and Szentes (2007) for important contributions to this literature.

[8]The paper does not claim that venture capital does not exist in real life or it is not important. However, given that it requires some resources that are limited in supply (like time of experts) and, hence, serves a relatively small portion of business start-ups, an alternative less transparent relationship is also present.

[9]Allowing for negative consumption but setting utility derived from it to negative infinity is a convenient way of securing non-negativity of consumption in the solution. The reason for requiring non-negativity of consumption is as follows. The only reason why there is any exchange between agents in this model is to finance investment. If utility function were $u(c) = c$ for all c, then any agent can finance her own investment by consuming a negative amount. Then, even under autarky full information efficiency would be achieved.

types, and those who do not are called $i = 0$ types. Let $I = \{0, 1\}$. The fraction of agents born with (without) an idea is η_1 (η_0). In order to produce, an agent must have an idea. Agents are also born with different levels of initial endowment of the only consumption good, $w \in W = \{p, r\}$, $p < r$. Fraction ζ_w are born with initial wealth level w. There is no endowment in period two. So, there are four types of agents initially, at the beginning of period one: $\{(p, 0), (p, 1), (r, 0), (r, 1)\}$.

Agent of type (w, i) operates the following production technology:

$$ y = i\theta k^{\alpha}, \alpha \in (0, 1), {}^{10} $$

where k is the amount invested in period one, θ is the random return on capital, and y is the random output produced in period two. θ is drawn from the set $\Theta = \{\theta_l, \theta_h\}$, where $\theta_l < \theta_h$, according to the probability distribution μ, independently across agents.[11] The probability of drawing θ_l is μ_l and θ_h is μ_h. An agent gets to learn the realization of return after the investment is made. Hence, agents face idiosyncratic investment risk. The term i is in the production function to denote that only agents with ideas can start businesses.

There is also a risk-free, linear storage technology that is available to all agents. An agent who stores s_1 units in period one wakes up with As_1 units in period two. The assumption below says that the storage technology is wasteful.

Assumption 1. $A < 1/\beta$.

The information structure and timing of events are as follows: An agent's initial type, actions, and period two realized returns are private information. The rest of the data of the economy is public information. Given her initial type, an agent chooses how much to consume, invest, and store in period one. Then, in period two, θ is realized and hence output is produced, and the agent consumes.[12]

One way to think about resource allocation is to consider a benevolent social planner who chooses allocations for agents. Since consumption-investment choice is unobservable,

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[10]This specific form of the production function is not needed for any of the results; all the results go through if $y = \theta f(k)$, where $f(0) = 0$ and $f', -f'' > 0$.

[11]The assumption that the cardinality of the set of returns is two is immaterial for any result. All the results go through if Θ has any finite number or a continuum of elements.

[12]Whether θ is realized in period one or two is immaterial for the results; the important thing is that it is realized after the investment decision is made.

the planner cannot choose allocations directly. Instead, each period the planner makes transfers between agents based on their reports of their private histories. This way the planner manipulates agents' actions. In addition, there is no outside party, which means the planner cannot save or borrow resources through time.[13]

An allocation in this economy is a vector $(c, k, s, \delta) \equiv (c_1, c_2, k_1, s_1, \delta_1, \delta_2)$, where

$$c_1 : W \times I \to \Re$$
$$k_1 : W \times I \to \Re_+$$
$$s_1 : W \times I \to \Re_+$$
$$c_2 : W \times I \times \Theta \to \Re$$
$$\delta_1 : W \times I \to \Re$$
$$\delta_2 : W \times I \times \Theta \to \Re.$$

In the above, $c_1(w, i)$, $k_1(w, i)$, and $s_1(w, i)$ refer to period one levels of consumption, investment, and storage of the agent who has initial wealth w and idea i. Similarly, $c_2(w, i, \theta)$ is the consumption level of the agent of type (w, i) who has a realized return θ in period two.[14] $\delta_1(w, i)$ and $\delta_2(w, i, \theta)$ are the levels of transfers received by corresponding types.

Feasibility. An allocation (c, k, s, δ) is *feasible* if

$$\sum_{w,i} \zeta_w \eta_i \delta_1(w, i) \leq 0,$$
$$\sum_{w,i} \sum_{\theta} \zeta_w \eta_i \mu_\theta \delta_2(w, i, \theta) \leq 0, \tag{1}$$

and for every $(w, i) \in W \times I$

$$c_1(w, i) + k_1(w, i) + s_1(w, i) \leq w + \delta_1(w, i),$$
$$c_2(w, i, \theta) \leq i\theta k_1(w, i)^\alpha + As_1(w, i) + \delta_2(w, i, \theta), \tag{2}$$

$$k_1(w, i), s_1(w, i) \geq 0.^{[15]} \tag{3}$$

Here, (1) is the *aggregate feasibility* condition, which says that the planner should balance

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[13] All results go through if the planner can borrow and save at a risk-free rate of $1/\beta$.

[14] Since an agent with no idea cannot produce, her period two consumption is independent of θ. So $c_2(w, 0, \theta_l) = c_2(w, 0, \theta_h)$.

[15] As it can be understood from (2), we assume that there is full depreciation of capital. This assumption is only made for notational simplicity and is not needed for any of the results.

its budget every period. (2) is *individual feasibility* and stands for the fact that allocation assigned to each agent should be affordable by him. (3) is just the non-negativity constraint on investment and storage.

Incentive compatibility. Using the terminology of mechanism design literature, there are two sources of private information in the model. First, there is hidden information: an agent's initial type and period two investment returns are observed privately by the agent. Second, agents are involved in hidden action: their consumption and investment levels are hidden. Hence, they can deviate from an allocation recommended by the planner in two ways: they can lie about their private information and/or they can choose an investment level that is different from what the planner recommended. Due to these informational frictions, only incentive-compatible allocations are achievable. I invoke a powerful revelation principle introduced by Myerson (1982) and characterize the set of incentive-compatible allocations as follows.

Let $(\tilde{w}, \tilde{i}) \in W \times I$ and $\tilde{\theta} : \Theta \rightarrow \Theta$ be agent's period one and period two reporting strategies, respectively. Also, define $(\tilde{k}_1, \tilde{s}_1) \in \Re_+^2$ as agent's investment strategy. Then, $\tilde{\gamma} \equiv (\tilde{w}, \tilde{i}, \tilde{\theta}, \tilde{k}_1, \tilde{s}_1)$ is a complete strategy of agent (w, i).[16] Let Γ be the set of all complete strategy profiles.

Given the allocation (c, k, s, δ), for any (w, i), the utility of following a strategy $\tilde{\gamma}$ is:

$$V_{w,i}(\tilde{\gamma}; c, k, s, \delta) \equiv u[w + \delta_1(\tilde{w}, \tilde{i}) - \tilde{k}_1 - \tilde{s}_1] + \beta \sum_\theta \mu_\theta u[i\theta\tilde{k}_1^{\ \alpha} + A\tilde{s}_1 + \delta_2(\tilde{w}, \tilde{i}, \tilde{\theta}(\theta))]$$

Define $\gamma \equiv (w, i, \theta, k_1, s_1)$ to be the strategy consisting of truthful reporting and obeying recommendations, where $\theta(\theta) = \theta$ denotes the truth-telling reporting strategy.

An allocation (c, k, s, δ) is *incentive-compatible* if for each $(w, i) \in W \times I$,

$$V_{w,i}(\gamma; c, k, s, \delta) \geq V_{w,i}(\tilde{\gamma}; c, k, s, \delta), \text{for all } \tilde{\gamma} \in \Gamma. \tag{4}$$

An allocation that is feasible and incentive-compatible is called *incentive-feasible*.

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[16]Myerson (1982) calls this *participation strategy*. Also, note that consumption is not a part of the strategy since it is implied by the choice of other actions.

2.2 Benchmark: Full Information Efficiency

The aim of this subsection is to analyze what society can achieve when everything in the economy is publicly observable. Full information efficient allocation turns out to be a useful benchmark for the constrained-efficient allocation. Under the utilitarian objective, the efficient allocation with full information is the solution to the following problem:

Planner's full information problem.

$$\max_{c,k} \sum_{w,i} \zeta_w \eta_i \Big\{ u(c_1(w,i)) + \beta \sum_{\theta} \mu_\theta u(c_2(w,i,\theta)) \Big\}$$

s.t.

$$\sum_{w,i} \zeta_w \eta_i \Big\{ c_1(w,i) + k_1(w,i) + s_1(w,i) \Big\} \leq \sum_w \zeta_w w,$$

$$\sum_{w,i} \zeta_w \eta_i \sum_{\theta} \mu_\theta c_2(w,i,\theta) \leq \sum_{w,i} \zeta_w \eta_i \sum_{\theta} \mu_\theta \Big\{ i\theta k_1(w,i)^\alpha + A s_1(w,i) \Big\},$$

$$k_1(w,i), s_1(w,i) \geq 0, \text{ for all } (w,i) \in W \times I.$$

Since s_1 is wasteful, it is obvious that in the full information efficient allocation $s_1(w,i) = 0$, for all $(w,i) \in W \times I$.

Assuming that total initial wealth in period one is large enough, the first-order optimality condition for investment of agents with an idea reads:

$$1/\beta = \alpha k_1(w,1)^{\alpha-1} \sum_{\theta} \mu_\theta \theta.$$

The left-hand side of the equation is the marginal social cost of investing an additional unit in terms of period two utility. The right-hand side is the marginal social benefit of investment in the same units. This condition defines

$$k^{fi} = \Big\{ \beta\alpha \sum_i \mu_i \theta_i \Big\}^{\frac{1}{1-\alpha}}$$

as the full information efficient level of investment provided that the following holds:

Assumption 2. *Total resources in the economy in period one are sufficient to finance*

k^{fi} investment for each $(w, 1)$ agent, or

$$\eta_1 k^{fi} \leq \sum_w \zeta_w w.$$

Assumption 2 formally states that cumulative initial wealth is sufficiently large.[17]

Lemma 1. *Suppose Assumption 2 holds. Then,*

1. *The full information level of investment for agents with ideas is equal to k^{fi}, irrespective of their wealth. The full information level of investment for agents without ideas is zero.*

2. *The full information level of storage is zero for all agents.*

3. *As long as it provides non-negative consumption to all agents and uses all output, distribution of individual consumption does not matter.*

Looking at the objective function of the full information problem, one can see that utilitarian welfare with equal weights and risk neutrality together imply that society has no preference for how total consumption should be distributed, as long as no one gets negative consumption. The society is only concerned about the right agents making the right amounts of investment. Therefore, there is a set of full information efficient allocations that are unique up to the distribution of consumption.

The next section analyzes a problem with exactly the same objective function, but this time with a different constraint set due to private information. As a result, that problem will be one of maximizing production subject to feasibility and incentive compatibility. Thanks to Lemma 1, it is clear now that the challenge that awaits the society under private information is to make agents with ideas invest as close to the full information efficient level as possible.[18]

3 Constrained-Efficient Allocation

In analyzing the benchmark case, the only assumption made was about total initial wealth. However, with private information, the comparison of p, r and k^{fi} becomes important. The

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[17]If Assumption 2 does not hold, then the full information level of investment will be a corner solution: $\frac{\sum_w \zeta_w w}{\eta_1}$, and all the results of the paper go unchanged.

[18]Of course, without making any agent consume a negative amount.

first assumption about this comparison is the following:

Assumption 3. $p < k^{fi} < r$.

The first part of this assumption, $p < k^{fi}$, says that the initial wealth of the poor is not large enough to cover the full information level of investment. Thus, a poor agent with an idea cannot operate her idea at the most efficient level on her own. If, to the contrary, $p \geq k^{fi}$ were the case, the economy would reach full information without agents interacting at all. Obviously, this case is neither interesting nor realistic. The second part of the assumption, that $k^{fi} < r$, simply says that a rich agent who has an idea can invest at the efficient level even under autarky. Therefore, Assumption 3 ensures that there is a reason for the planner to intervene in this economy: to ensure that poor agents with ideas invest at the efficient level.

The remainder of this section first defines and then characterizes constrained-efficient allocation.[19]

Definition 1. *An allocation $(c^*, k^*, s^*, \delta^*)$ is called constrained-efficient if it solves the following social planner's problem:*

$$\max_{c,k,\delta} \sum_{w,i} \zeta_w \eta_i \Big\{ u(c_1(w,i)) + \beta \sum_{\theta} \mu_\theta u(c_2(w,i,\theta)) \Big\}$$

subject to (1), (2), (3), and (4).

As in the benchmark case, the objective function clearly shows that society does not care about how consumption is going to be distributed among individuals. Consequently, the above problem is one of constrained productive efficiency. This implies there can be many constrained efficient allocations, all of which have the same investment allocation and hence the same total production and welfare, but different consumption allocations. Nonetheless, it should also be noted that incentive compatibility arising from private information does put some discipline on the distribution of consumption across agents compared to the full information benchmark.

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[19]Throughout the paper, I refer to efficient allocation under the informational problems as constrained-efficient allocation so as to distinguish it from the full information efficient allocation.

3.1 Characterizing the Constrained-Efficient Allocation

First make the following observation, which simplifies the analysis. If transfer levels depend on period two announcements of agents, then any agent will report the type that brings the highest level of transfers in period two. Therefore, any transfer mechanism in which a transfer level depends on a period two shock cannot be incentive-compatible. Consequently, without loss of generality, the rest of the paper restricts attention to allocations in which transfers are functions of period one announcements only, $\delta_1, \delta_2 : W \times I \to \Re$.

Now I make the second assumption comparing p and k^{fi}.

Assumption 4. $\frac{k^{fi}-p}{\beta} > \theta_l k^{fi\alpha}$.

To understand this assumption, suppose it does not hold. Observe that in order to invest at the full information efficient level, the poor agent with an idea needs at least $k^{fi} - p$ additional resources in period one. Also observe that the most this agent can pay back in period two in low-return state is $\theta_l k^{fi\alpha}$. When Assumption 4 does not hold, even in the worst contingency in period two, the poor agent with an idea would be able to pay back the amount she borrowed in period one to finance the full information level of investment. Obviously, in this case, the fact that entrepreneurial returns are private information would not have an effect. Therefore, no subsidy would be necessary for constrained efficiency. The society can implement the full information outcome by just making sure that simple, not state-contingent debt contracts are perfectly enforced (by punishing very harshly anyone who does not pay back the amount she borrowed in period one). Agents, then, sign these contracts that an set interest rate of $1/\beta$ and optimal investment would be attained. However, that even in the worst case an entrepreneur can pay back her debt is highly unrealistic, especially for businesses that are newly forming.[20]

The proposition below formally shows that when Assumption 4 does not hold, the full information allocation is trivially reached without any net present value (NPV) of transfers between agents.

Before getting to the proposition, define $\Delta(w,i) = \delta_1(w,i) + \beta\delta_2(w,i)$ as the NPV of transfers an agent gets under a given allocation. An agent (w, i) is said to be *subsidized* by

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[20]That the lowest return is sufficiently dire is a standard assumption in financial contracting literature. Among others, see Diamond (1984), Gale and Hellwig (1985), Bolton and Scharfstein (1990), and DeMarzo and Fishman (2007).

the society under allocation (c, k, s, δ) if $\Delta(w, i) > 0$.

Proposition 1. *Suppose that $\frac{k^{fi} - p}{\beta} \leq \theta_l k^{fi\alpha}$. Then, in the constrained-efficient allocation:*

1. *$k_1^*(w, 1) = k^{fi}$ and $k_1(w, 0) = 0$, for all $w \in W$;*

2. *$s_1^*(w, i) = 0$, for all $(w, i) \in W \times I$;*

3. *$\delta_1^*(p, 1) = k^{fi} - p$ and $\delta_2^*(p, 1) = \frac{-(k^{fi} - p)}{\beta}$;*

4. *$(\delta_1^*(w, i), \delta_2^*(w, i))_{(w,i) \neq (p,1)}$ satisfy $\Delta^*(w, i) = 0$, aggregate feasibility, and individual feasibility with non-negative consumption for all.*

In words, no agent gets subsidized in the constrained-efficient allocation.

Proof. Choose $\delta_1^*(w, i)$ for $(w, i) \neq (p, 1)$ such that:

$$\sum_{(w,i) \neq (p,1)} \zeta_w \eta_i \delta_1^*(w, i) = -(k^{fi} - p)\zeta_p \eta_1, \tag{5}$$

$$\delta_1^*(r, 1) \geq k^{fi} - r, \tag{6}$$

$$0 \leq \delta_1^*(w, 0) \leq w, \tag{7}$$

and

$$\delta_2^*(w, i) = -\frac{\delta_1^*(w, i)}{\beta}. \tag{8}$$

By Assumption 2, such a δ^* exists. Observe that conditions (5) and (8) guarantee that transfers sum to zero in periods one and two, respectively. Thus, aggregate feasibility is satisfied.

Next, one has to show that non-negative consumption is feasible for each agent under the proposed allocation. Observe that the NPV of transfers of any agent is equal to zero in this allocation. In period one, a poor agent with an idea faces the budget $c_1 + k_1 + s_1 \leq k^{fi}$ and chooses $k_1^*(p, 1) = k^{fi}$. In period two in the low-return state, her consumption is $c_2^*(p, 1, \theta_l) = \theta_l k^{fi\alpha} - \frac{k^{fi} - p}{\beta} \geq 0$ by assumption. This clearly implies that $c_2(p, 1, \theta_h) \geq 0$, too. Condition (6) guarantees that $(r, 1)$ agent can choose investment equal to k^{fi} and still consume a non-negative amount in period one. The consumption levels of agents without ideas are non-negative in both periods by (7).

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The only thing left is to check that given δ^* agents will tell the truth about their types, but this is straightforward given that the NPV of transfers of any type is equal to zero. $\quad\square$

From now on, the paper analyzes the more interesting case in which Assumption 4 holds: the lowest return to an idea, θ_l, is sufficiently low.

Remember that, under the ex ante welfare criterion, the only reason why the planner intervenes in this economy ($\delta \neq 0$) is because a poor agent with an idea invests at a very low level, p, on her own. In order to make her invest at the full information level, the planner has to set $\delta_1(p, 1) \geq k^{fi} - p$. Since returns to a business start-up, θ, are private information, period two transfers cannot depend on the returns. Therefore, an agent who is poor and has an idea can pay back to the society an amount that is at most equal to output she produces in the low-return state, $\delta_2(p, 1) \geq -\theta_l k^{fi\alpha}$. This implies that in order to attain full information efficiency, the NPV of transfers going to the poor agent with an idea should at least be $\Delta^{fi} \equiv k^{fi} - p - \beta\theta_l k^{fi\alpha}$, which is strictly positive by Assumption 4.

In what follows, without loss of generality, I restrict attention to constrained-efficient allocations in which $\Delta^*(p, 1) \leq \Delta^{fi}$. This is without loss of generality for the following reason. The discussion above shows that if $\Delta^*(p, 1) = \Delta^{fi}$, then $k_1^*(p, 1) = k^{fi}$, meaning full information efficiency is attained. Thus, in any allocation in which NPV of transfers going to $(p, 1)$ is higher than Δ^{fi}, the value of the objective function is equal full information level. Thus, increasing the NPV of transfers going to $(p, 1)$ above Δ^{fi} does not change social objective but only changes the distribution of consumption.

Lemma 2 below shows that the constrained-efficient level of investment for the poor agent with an idea is strictly increasing in the NPV of transfers she gets under the constrained-efficient allocation. Therefore, if the society wants to increase the investment level for poor agents with ideas, it has to increase the NPV of transfers going to these agents.

Lemma 2. *In the constrained-efficient allocation, $\delta_1^*(p, 1) = k_1^*(p, 1) - p$ and $\delta_2^*(p, 1) = -\theta_l k_1^*(p, 1)^{\alpha}$.*

Proof. Observe that $\Delta^*(p, 1) \leq \Delta^{fi}$ implies that $\delta_1^*(p, 1) \leq k^{fi} - p$. If not, then $c_2^*(p, 1, \theta_l) < 0$, a contradiction. This implies automatically that $c_1^*(p, 1) = 0$ and hence $\delta_1^*(p, 1) = k_1^*(p, 1) - p$. For if not, the agent can decrease her consumption by a small amount and increase her

investment by the same amount. This would increase her welfare strictly since her investment level is strictly below k^{fi} when $c_1^*(p, 1) > 0$, a contradiction.

Now, suppose that $\delta_2^*(p, 1) > -\theta_l k_1^*(p, 1)^\alpha$. Define a new allocation with transfers $\tilde{\delta}_2(p, 1) = \delta_2^*(p, 1) - \epsilon$ and $\tilde{\delta}_1(p, 1) = \delta_1^*(p, 1) + \beta\epsilon$. The resulting allocation is incentive-feasible since the NPV of transfers is unchanged for all agents. For agents $(w, i) \neq (p, 1)$, welfare is unchanged. The change in $(p, 1)$'s welfare will be

$$\underbrace{\beta\epsilon\alpha k_1(p, 1)^{\alpha-1} \sum_\theta \mu_\theta \theta}_{\text{Gain in pd. 2}} - \underbrace{\epsilon}_{\text{Loss in pd. 2}} > 0, \tag{9}$$

which means the new allocation improves over the constrained-efficient one, a contradiction. □

Lemma 2 implies the following has to hold in the constrained-efficient allocation:

$$\Delta^*(p, 1) = k_1^*(p, 1) - p - \beta\theta_l k_1^*(p, 1)^\alpha. \tag{10}$$

Taking the derivative of both sides with respect to $k_1^*(p, 1)$ gives

$$\frac{d\Delta^*(p, 1)}{dk_1^*(p, 1)} = 1 - \beta\theta_l k_1^*(p, 1)^{\alpha-1}\alpha > 0, \tag{11}$$

for $k_1^*(p, 1) \leq k^{fi}$, where strict inequality follows from k^{fi} being the unconstrained optimal investment level. Therefore, $\Delta^*(p, 1)$ is strictly increasing in $k_1^*(p, 1)$. Since $k_1^*(p, 1) - p - \beta\theta_l k_1^*(p, 1)^\alpha$ is one-to-one, the converse is also true: in order to increase $k_1^*(p, 1)$, the planner needs to increase $\Delta^*(p, 1)$. Therefore, society has to increase the NPV of transfers going to poor agents with ideas so as to bring these agents' investment levels closer to the full information level and thus so as to bring social welfare closer to the full information level.

Proposition 2 below shows that as long as storage technology is wasteful (Assumption 1), it is incentive-feasible to make transfers to poor agents with ideas with strictly positive NPV. As a result, Proposition 2 shows that constrained efficiency requires transferring a positive NPV of resources from agents without ideas to ones with ideas when the lowest return to start-ups is sufficiently dire. It also proves that there is a threshold level of the return to storage technology, $\bar{A}$, such that it is incentive-feasible to make transfers to $(p, 1)$ with NPV equal to Δ^{fi} if and only if the return to storage is less than or equal to $\bar{A}$. Furthermore,

Proposition 2 provides an exact calculation of the constrained-efficient allocation in either case.[21]

I have made Assumption 5 below. This assumption is not substantial in the sense that it is not necessary for the subsidy result. It is assumed merely for the sake of simplicity.

Assumption 5.

a. $\eta_1[k^{fi} - p - \beta\theta_l k^{fi\alpha}] \leq \eta_0 p$.

b. $\eta_1[k^{fi} - p] \leq \eta_0 \sum_w \zeta_w w$.

Proposition 2. *Suppose Assumptions 4 and 5 hold. Then there exists a unique $\bar{A} \equiv \frac{\eta_0\theta_l k^{fi\alpha}}{k^{fi}-p-\eta_1\beta\theta_l k^{fi\alpha}} \in (0, \beta^{-1})$ such that in the constrained-efficient allocation:*

1. *$k_1^*(r,1) = k^{fi}$ and $k_1^*(w,0) = 0$, for all $w \in W$;*

2. *$k_1^*(p,1) = k^{fi}$, if $A \leq \bar{A}$,*
   $k_1^*(p,1) < k^{fi}$ is given by the unique solution to $A = \frac{\eta_0\theta_l k_1^*(p,1)^\alpha}{k_1^*(p,1)-p-\eta_1\beta\theta_l k_1^*(p,1)^\alpha}$, if $A > \bar{A}$;

3. *$s_1^*(w,i) = 0$ for all $(w,i) \in W \times I$;*

4. *$\delta_1^*(w,1) = k_1^*(p,1) - p$ and $\delta_2^*(w,1) = -\theta_l k_1^*(p,1)^\alpha$ for all $w \in W$;*

5. *$(\delta_1^*(w,0), \delta_2^*(w,0))_{(w \in W)}$ satisfy: $\Delta^*(w,0) = -\frac{\eta_1}{\eta_0}\Delta^*(w,1)$ for all $w \in W$, and individual feasibility with non-negative consumption for all.*

More importantly, $\Delta^*(w,1) > 0$, i.e., in the constrained-efficient allocation society transfers a strictly positive NPV of resources from agents without ideas to the ones with ideas.

Proof. First, I show that $\Delta^*(p,1) = \Delta^*(r,1)$. If $\Delta_{p,1}^* > \Delta_{r,1}^*$, then $(r,1)$ lies to be $(p,1)$ and gets the transfers with higher NPV; therefore, this cannot be true. On the other hand, if $\Delta_{p,1}^* < \Delta_{r,1}^*$, then one can propose a new allocation with a transfer system $\tilde{\delta}$ that is the same as δ^*, except for $\tilde{\delta}_1(p,1) = \delta_1^*(p,1) + \epsilon$ and $\tilde{\delta}_1(r,1) = \delta_1^*(r,1) - \frac{\zeta_p}{\zeta_r}\epsilon$. Clearly, this transfer mechanism is a part of a feasible allocation. This new allocation is also incentive-compatible: $(r,1)$ does not lie to be $(p,1)$ since $\epsilon > 0$ is small and agents without ideas do not lie to be $(p,1)$ since with original transfers they were not lying to be $(r,1)$ and the NPV of transfers

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[21]Under the restriction of $\Delta^*(p,1) \leq k^{fi} - p - \beta\theta_l k^{fi\alpha}$, the constrained-efficient allocation is unique.

of $(p, 1)$ is still lower than that of $(r, 1)$ under the original transfer mechanism. But the allocation that is attained by this transfer mechanism has strictly greater aggregate utility. The reason is $(p, 1)$ agent's utility increases strictly more than ϵ with the new allocation since she was investment-constrained under δ^*. Then, Assumption 5.b allows the planner to set $\delta_1^*(r, 1) = k_1^*(p, 1) - p$ and $\delta_2^*(r, 1) = -\theta_l k_1^*(p, 1)^\alpha$, without loss of generality.

One can similarly show that in the constrained-efficient allocation $\Delta^*(p, 0) = \Delta^*(r, 0)$. Aggregate feasibility then implies

$$\Delta^*(w, 0) = -\frac{\eta_1}{\eta_0} \Delta^*(p, 1). \tag{12}$$

So far it has been shown that in order to increase ex ante welfare, the planner has to increase $\Delta^*(w, 1)$, and this has to be financed by taking resources from agents without ideas.

The only incentive-compatibility condition that one needs to check is the one regarding deviations in which an agent without an idea lies to have an idea. The allocation has to satisfy

$$w + \Delta^*(w, 0) \geq \begin{cases} w + \Delta^*(w, 1) + \frac{\delta_1^*(w', 1) - \Delta^*(w, 1)}{\beta A}[-1 + \beta A], & \text{if } A(\delta_1^*(w', 1) + w) \geq -\delta_2^*(w', 1); \\ -\infty, & \text{if else.} \end{cases} \tag{13}$$

Here, the left-hand side of the equation is the utility of truth-telling, whereas right-hand side is the utility of lying to be $(w, 1)$. The left-hand side already takes into account the fact that transfers are such that when agents without ideas tell the truth, they do not have to use risk-free technology. Hence, they do not use it. The right-hand side already has the fact that $(w, 0)$ has to set $s_1' \geq \frac{-\delta_2^*(w', 1)}{A} = \frac{\delta_1^*(w', 1) - \Delta^*(w, 1)}{\beta A}$ to have non-negative consumption in period two. When $A(\delta_1^*(w', 1) + w) \geq -\delta_2^*(w', 1)$ this is possible. Otherwise, lying to be $(w, 1)$ implies they have to consume a negative amount in either period one or period two, meaning they get negative infinity utility.

Plugging in $\delta_1^*(w, 1) = k_1^*(p, 1) - p$ and $\delta_2^*(w, 1) = -\beta \theta_l k_1^*(p, 1)^\alpha$, (13) implies that $k_1^*(p, 1)$ units of investment for the poor agent with an idea is attained in the constrained-efficient allocation if and only if it satisfies, for any $w \in W$:

$$A < \frac{\theta_l k_1^*(p, 1)^\alpha}{k_1^*(p, 1) - p + w}, \text{ or } A \leq \frac{\theta_l k_1^*(p, 1)^\alpha}{k_1^*(p, 1) - p - \Delta^*(w, 0)}. \tag{14}$$

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It follows from Assumption 5.a and aggregate feasibility that $\frac{\theta_l k_1^*(p,1)^\alpha}{k_1^*(p,1)-p+w} < \frac{\theta_l k_1^*(p,1)^\alpha}{k_1^*(p,1)-p-\Delta^*(w,0)}$. Hence, $k_1^*(p,1)$ is attained at the constrained-efficient allocation if and only if $A \leq \frac{\theta_l k_1^*(p,1)^\alpha}{k_1^*(p,1)-p-\Delta^*(w,0)}$. Plugging in the value $\delta^*(w,0)$, we get $k_1^*(p,1)$ is constrained-efficient if and only if

$$A \leq \frac{\eta_0 \theta_l k_1^*(p,1)^\alpha}{k_1^*(p,1)-p-\eta_1\beta\theta_l k_1^*(p,1)^\alpha}. \tag{15}$$

Hence, for $A \leq \bar{A}$, as it is defined in Proposition 2, $k_1^*(p,1) = k^{fi}$ is incentive-compatible. To see that for $A > \bar{A}$, $k_1^*(p,1)$ is given by $A \leq \frac{\eta_0 \theta_l k_1^*(p,1)^\alpha}{k_1^*(p,1)-p-\eta_1\beta\theta_l k_1^*(p,1)^\alpha}$, suppose for a contradiction that $A < \frac{\eta_0 \theta_l k_1^*(p,1)^\alpha}{k_1^*(p,1)-p-\eta_1\beta\theta_l k_1^*(p,1)^\alpha}$. Then, define a new transfer system $\tilde{\delta}$ which is identical to the constrained-efficient one, δ^*, except for $\tilde{\delta}_1(w,1) = \delta_1^*(w,1) + \epsilon$ and $\tilde{\delta}_1(w,0) = \delta_1^*(w,0) - \frac{\eta_1}{\eta_0}\epsilon$, $\epsilon > 0$. This increases $k_1^*(p,1)$ to $\tilde{k}_1 = k_1^*(p,1) + \epsilon$ and hence decreases $\frac{\theta_l k_1^*(p,1)^\alpha}{k-p+\frac{\eta_1}{\eta_0}\Delta^*(w,1)}$. However, for ϵ small, $A < \frac{\theta_l \tilde{k}_1^\alpha}{\tilde{k}_1-p+\frac{\eta_1}{\eta_0}\tilde{\Delta}(w,1)}$ still holds, where $\tilde{\Delta}(w,1)$ is the NPV of transfers going to $(w,1)$ agents under the new allocation. Thus, this new allocation is incentive-compatible. It is clearly feasible. Finally, it strictly increases total welfare since the increase in $(p,1)$ agents' welfare is strictly greater than ϵ. Then, δ^* cannot be constrained-efficient, a contradiction. $\square$

The intuition for why the NPV of transfers going to poor agents with ideas is related to the returns to storage is simple. Since an agent who is rich and has an idea can imitate being poor, she has to get transfers with the same NPV. This is why all agents with ideas, potential start-ups, should be subsidized by an equal amount in the constrained-efficient allocation. Someone has to finance the subsidy going to agents with ideas. Consequently, individuals without ideas end up getting strictly negative NPV of resources. But is it incentive-compatible to transfer resources from agents without ideas to ones with them? Or do agents with no ideas lie to have an idea and get the subsidy? The answer depends on the returns to the storage technology, A. The reason is that period two transfers of agents with ideas is strictly negative, and hence if agents without ideas want to pretend to have ideas, they have to pay back to the society in period two. For agents without ideas, the only way to carry resources into period two is via the storage technology.

If $A = 0$, then it is impossible for agents without ideas to carry resources to period two. In that case, they cannot pretend to have ideas; therefore, planner can transfer Δ^{fi} to agents with ideas and attain full information efficiency. As A increases, storing resources instead of consuming in period one becomes less wasteful. There is a threshold level of the return to storage technology, $\bar{A}$, such that above this level, the benefit of lying to have an idea (not

financing but enjoying the subsidy) exceeds the cost of doing so. As a result, when $A > \bar{A}$, agents with ideas cannot be subsidized Δ^{fi}, and hence poor agents with ideas cannot invest at the full information level, k^{fi}.

Nonetheless, as long as $A < \beta^{-1}$, some subsidy is still incentive-compatible since $A < \beta^{-1}$ implies that it is costly to store resources and hence lie for agents without ideas to have ideas. In this case, the amount of subsidy going to agents with ideas is determined by equating the benefit and cost to the agents without ideas of lying to have ideas.

Proposition 2 shows that under some parameters, the society attains the full information efficient allocation, even under the informational constraints. This result is an artifact of risk neutrality and hence will vanish if more general utility functions are assumed. On the other hand, the main result of Proposition 2, that due to informational problems productive efficiency requires transferring resources from agents without ideas to ones with them, holds with risk-averse preferences as well.

Proposition 2 also points to an interesting property of the model economy: the distribution of wealth affects the constrained-efficient distribution of productive activity in the economy. To see this, remember that Proposition 2 tells that when $A > \bar{A}$, the constrained-efficient level of investment for a poor agent with an idea depends on her wealth level, p. Now, consider another wealth distribution with ζ_p fraction of agents having initial wealth $p + \epsilon$ and ζ_r fraction having $r - \epsilon \frac{\zeta_p}{\zeta_r}$, where $\epsilon > 0$ and small. This new wealth distribution is a perturbation of the old one in a way that preserves the mean. By Proposition 2, in the economy with the perturbed wealth distribution, $k_1^*(r, 1) = k^{fi}$ and investment level for a poor agent with an idea is given by $A = \frac{\eta_0 \theta_l k_1^*(p,1)^\alpha}{k_1^*(p,1) - (p+\epsilon) - \eta_1 \beta \theta_l k_1^*(p,1)^\alpha}$.[22] This means that in the current model, when Assumptions 4 and 5 hold and $A > \bar{A}$, the constrained-efficient distribution of productive activity depends on how initial wealth is distributed across agents. This result is summarized in the following corollary.

Corollary 1. *Suppose Assumptions 4 and 5 hold, and $A > \bar{A}$. Distribution of productive activity in the constrained-efficient allocation depends on the distribution of wealth.*

It is important to note that this result crucially depends on private information assump-

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[22]Here, I abuse the notation, hoping this does not cause any confusion. In the original economy, p denotes two things: poor agents and their wealth level. In the perturbed economy, p denotes poor agents, whereas $p + \epsilon$ denotes their wealth level. The same is true for r.

tions. In the full information efficient allocation, investment levels for both agents with ideas is k^{fi}, independent of how a total of $\sum_w \zeta_w w$ units is distributed across agents. The intuition for why the constrained-efficient level of productive activity depends on the distribution of wealth is as follows. Under private information, the marginal social cost of investment is not only equal to its resource cost. For a given distribution of initial wealth, increasing the investment level of poor agents with ideas tightens some incentive-compatibility conditions. Thus, there is an incentive cost of increasing investment in addition to the resource cost. Changing the wealth distribution changes this incentive cost of investment while leaving the resource cost untouched. Consequently, between two otherwise identical economies with different distributions of wealth, the resource cost of investment is the same, which implies that the full information efficient allocation is the same. However, the incentive costs in these two economies are potentially different, making the social marginal costs of investment different, which results in different distributions of constrained-efficient productive activity.

3.2 Discussion of Assumptions

This section discusses the role of informational assumptions on the subsidy result. The assumption that θ, the returns to a start-up, is unobservable is the sole cause of the subsidy result. To see this, consider a version of the model in which, for each agent, θ is realized publicly. Assume that initial type, (w, i), and actions are still private information. In that case, the planner can attain full information efficiency without subsidizing any agent, even under Assumption 4. This result is shown in Proposition 3 below.

Proposition 3. *Suppose that θ is observable for each agent. Then, in the constrained-efficient allocation:*

1. *$k_1^*(w, 1) = k^{fi}$ and $k_1(w, 0) = 0$, for all $w \in W$;*

2. *$s_1^*(w, i) = 0$, for all $(w, i) \in W \times I$;*

3. *$\delta_1^*(p, 1) = k^{fi} - p$ and $\delta_2(p, 1, \theta_l) = -\theta_l k_1^{fi\alpha}$, and $\delta_2(p, 1, \theta_h)$ such that $\Delta^*(p, 1) = 0$;*

4. *$(\delta_1^*(w, i), \delta_2^*(w, i))_{(w,i) \neq (p,1)}$ satisfy $\Delta^*(w, i) = 0$, aggregate feasibility, and individual feasibility with non-negative consumption for all.*

In words, no agent gets subsidized in the constrained-efficient allocation.

Proof. Since the allocation described attains productive efficiency, we only need to check that it is incentive-compatible and satisfies aggregate and individual feasibility conditions with no agent consuming a negative amount. Incentive compatibility directly follows from the fact that the NPV of transfers is zero for each agent. Aggregate and individual feasibility is by construction. That each agent consumes a non-negative amount in any period is obvious except for $(p, 1)$ agent in θ_h state. So, we need to show that $\delta_2(p, 1, \theta_h) \leq \theta_h k^{fi\alpha}$.

For a contradiction, suppose that $\delta_2(p, 1, \theta_h) > \theta_h k^{fi\alpha}$. By construction, $\delta_2(p, 1, \theta_h)$ satisfies

$$k^{fi} - p - \beta[\mu_l \theta_l k^{fi\alpha} + \mu_h \delta_2(p, 1, \theta_h)] = 0.$$

Therefore, we get:

$$k^{fi} - p > \beta \sum_\theta \mu_\theta \theta k^{fi\alpha}. \tag{16}$$

Remember that the first-order condition that gives k^{fi} is:

$$1 = \beta \alpha k^{fi\alpha-1} \sum_\theta \mu_\theta \theta.$$

Multiplying both sides by k^{fi} and then subtracting p from both sides gives

$$k^{fi} - p = \beta \alpha k^{fi\alpha} \sum_\theta \mu_\theta \theta - p,$$

which contradicts with (16). $\square$

The intuition is simple. When θ is observable, the planner can make period two transfers depend on the realization of θ. Therefore, even if the low state return, θ_l, is very low (Assumption 4), the agent can still pay back to the society the future value of resources transferred to her in period one, $k^{fi} - p$, by paying a sufficiently high amount in the high state. This proposition precisely establishes that the only reason in the model why the society has to subsidize agents with ideas is because start-up returns are private information.

The assumptions that initial type and actions are observable imply that the planner has to respect incentive-compatibility conditions when subsidizing poor agents with ideas.

Consider, for instance, a model that is identical to the baseline model introduced in section 2, except that initial type, (w, i), is publicly known at no cost. As long as θ is unobservable, the society still has to make Δ^{fi} units of transfers to poor agents with ideas. However, now it is trivial to make this transfer since the planner knows exactly the agents who have ideas but lack resources to invest in them.

Similarly, if investment is assumed to be observable, keeping the rest of the model the same as the baseline model, subsidizing agents with ideas would be trivial. It is not beneficial for an agent without an idea to lie to have one and get the subsidy since she has to invest it, and hence cannot consume it. The exercise in which everything else is kept the same but storage is assumed to be observable is the same as assuming there is no storage technology, or $A = 0$. From Proposition 2, it follows that in this case, the planner can make transfers with NPV that is sufficient to attain full information efficiency.

4 Implementation

This purpose of this section is to provide an implementation of the constrained-efficient allocation via a program like the U.S. SBA's Business Loan Program.[23] I first introduce the paper's implementation and then compare it to the SBA's Business Loan Program.

The physical and informational environment is the same as described in section 2. The main difference is that there is an incomplete markets structure that allows agents to competitively trade risk-free bonds in period one.[24] Bonds pay back a gross return R in period two that is determined in equilibrium. Individual trades in the bonds market are public information and there is full enforcement, meaning that no one can die without paying back their debt.

There are two institutions: a government and an institution of the government that aids start-up businesses. The government taxes all individuals in the society lump-sum, by an amount T, and transfers these funds to its institution. Any individual can apply to this institution for a subsidy. The institution asks the agent to report her wealth, business idea,

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[23]The SBA is the major government institution in the United States assisting business start-ups in particular and small businesses in general. The total amount of outstanding business loans that small businesses acquired through the SBA's loan program was \$75.5 billion as of fiscal year 2007. This amounts to 30% of all small business borrowing.

[24]That I restrict attention to incomplete markets from the outset is without loss of generality. It is easy to show that under the given informational assumptions, incomplete markets trade is the best agents can attain on their own.

and investment plan, w', i', and k'_1, respectively. Then, after observing the amount borrowed (or lent) and the reports, the institution decides whether or not to provide the subsidy, $\tau(b_1, w', i', k'_1)$.

Taking the tax-subsidy system (T, τ) and the interest rate R as given, an agent (w, i) who decided to apply to the institution for a subsidy solves the following problem:

Agent's problem.

$$\max_{c_1, c_2, k_1, s_1, b_1, w', i', k'_1} u(c_1) + \beta \sum_\theta \mu_\theta u(c_{2\theta}) \tag{17}$$

s.t.

$$c_1 + k_1 + s_1 + b_1 \leq w - T(b_1) + \tau(b_1, w', i', k'_1),$$

$$c_{2\theta} \leq i\theta k_1^\alpha + Rb_1,$$

$$k_1, s_1 \geq 0.$$

An agent who does not apply for a subsidy ($a = 0$ agent) would solve a very similar problem. The only difference is there would be no $\tau(b_1, w', i', k'_1)$ in that agent's problem, and hence there would not be any w' and k'_1 choice. However, since in the current setup there is no cost of applying for a subsidy, without loss of generality, assume that all agents apply.

Below is the definition of incomplete markets equilibrium with a tax-subsidy system.

Definition 2. *Given (T, τ), an incomplete markets (IM) equilibrium is individual choices $(c_1(w, i), c_2(w, i, \theta), k_1(w, i), s_1(w, i), b_1(w, i), w'(w, i), i'(w, i), k'_1(w, i))_{w \in W, i \in I, \theta \in \Theta}$ and interest rate R s.t.*

1. *Given R, for each agent, $(c_1(w, i), c_2(w, i, \theta), k_1(w, i), s_1(w, i), b_1(w, i), w'(w, i), i'(w, i), k'_1(w, i))$ solves (17),*

2. *Bond market clears: $\sum_w \sum_i \zeta_w \eta_i b_1(w, i) = 0$,*

3. *The institution's budget balances: $\sum_w \sum_i \zeta_w \eta_i \tau(b_1(w, i), w'(w, i), i'(w, i), k'_1(w, i)) = T$.*

An allocation (c, k, s, δ) is *implementable in the market with a tax-subsidy system* (T, τ) if, given (T, τ), (c, k, s, b) with some interest rate R constitute an IM equilibrium.

4.1 Incomplete Markets under Laissez-Faire

Before providing an actual tax-subsidy system that implements constrained-efficient allocation, this subsection first analyzes what happens under no government intervention, i.e., $(T, \tau) = 0$.

Proposition 4. *Suppose Assumptions 4 and 5 hold. Then, constrained-efficient allocation cannot be attained in the equilibrium of IM under laissez-faire.*

Proof. Suppose for contradiction that the constrained-efficient allocation can be achieved. Then, $b_1(p, 1) \leq -(-k^{fi} - p)$. Due to linearity of preferences, $R = 1/\beta$. Thus, $c_2(p, 1, \theta_l) \leq \theta_l k^{fi\alpha} - \frac{k^{fi} - p}{\beta} < 0$, by Assumption 3. But this cannot be an optimal choice for the agent since the agent could do better just by setting $b_1(p, 1) = 0$. Thus, we have a contradiction. $\square$

Proposition 2 already proved that the constrained-efficient allocation involves transferring strictly positive NPV of resources from agents without ideas to those with ideas. Proposition 4 then follows since markets cannot make such transfers on their own. A separate entity, like a government, should intervene and make the necessary transfers between agents.[25]

4.2 Optimal Tax-Subsidy System

This subsection provides the actual tax-subsidy system that implements the constrained-efficient allocation.

In that regard, define

$$T = \frac{\eta_1}{\eta_0} \Delta^*(p, 1),$$

$$\tau(b_1, w', i', k_1') = \begin{cases} \frac{\Delta^*(w, 1)}{\eta_0}, & \text{if } b_1 \leq -\beta \theta_l k_1^*(p, 1)^{\alpha}; \\ 0, & \text{otherwise.} \end{cases}$$

(18)

Proposition 5. *Suppose Assumptions 4 and 5 hold. Then, the incomplete markets equilibrium with the tax-subsidy system defined in* (18) *implements the constrained-efficient*

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[25]It is important to note that laissez-faire IM equilibrium is ex post Pareto efficient. However, it is not output maximizing (ex ante Pareto efficient), which is the focus of this paper.

allocation.[26]

Proof. Now I construct an IM equilibrium where $R = 1/\beta$ and agents with ideas invest at their corresponding constrained-efficient investment level.

Under the specified taxes, an agent faces the following problem:

Agent's problem with taxes.

$$\max_{c,k,b,s} u(c_1) + \beta \sum_{\theta} \mu_\theta u(c_{2\theta}) \tag{19}$$

s.t.

$$c_1 + k_1 + b_1 + s_1 \leq \begin{cases} w + \Delta^*(w, 1), & \text{if } b_1 \leq -\beta\theta_l k_1^*(p, 1)^\alpha; \\ w - \frac{\eta_1}{\eta_0}\Delta^*(w, 1), & \text{if else,} \end{cases}$$

$$c_{2\theta} \leq i\theta k_1^\alpha + Rb_1 + As_1,$$

$$s_1, k_1 \geq 0.$$

First, consider an agent who has an idea in period one. If a poor agent with an idea chooses $b_1 \leq -\beta\theta_l k_1^*(p, 1)^\alpha$, she chooses $k_1 = k_1^*(p, 1)$ and $b_1 = p - k_1^*(p, 1) + \Delta^*(w, 1) = -\beta\theta_l k_1^*(p, 1)^\alpha$. Suppose for contradiction that this is not true. Then, there is (k_1', b_1', s_1'), where $(k_1', s_1') \neq (k_1^*(p, 1), 0)$, which gives strictly greater utility to the agent. $s_1' = 0$ follows immediately from the fact that the return to bonds is strictly greater than the risk-free return; hence, it has to be that $k_1' \neq k_1^*(p, 1)$. Then, define a new allocation with transfers $\delta_1'(w, 1) = k_1' - p$, $\delta_2'(w, 1) = -\frac{k_1' - p - \Delta^*(w, 1)}{\beta}$, and $(\delta_1'(w, 0), \delta_2'(w, 0))_{w \in W}$ such that $\Delta'(w, 0) = -\eta_1/\eta_0\Delta'(w, 1)$ and individual consumption levels are non-negative. $(p, 1)$ chooses (k_1', b_1') in the market, implying that she chooses k_1' in the planner's problem when she faces δ'. The only thing left to check is incentive compatibility. That holds because of the way in which the new allocation is constructed, $\Delta'(w, 1) = \Delta^*(w, 1)$. Therefore, this new allocation is incentive-feasible, keeps the welfare of $(w, i) \neq (p, 1)$ unchanged compared to the constrained-efficient allocation, and provides strictly greater welfare than the constrained-efficient level for poor agents with ideas. This means that the new allocation is an improvement over the constrained-efficient

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[26]Observe that the subsidy function does not depend on any of the agent's reports. This is due to the fact that the current model abstracts away from any form of monitoring. Hence, these reports are just cheap talk.

allocation, a contradiction.

Similarly, one can show that when $b_1(r,1) \leq -\beta\theta_l k_1^*(p,1)^\alpha$, $(r,1)$ agent chooses to invest at the constrained-efficient level, k^{fi}.

Now we need to show that agents with ideas choose $b_1 \leq -\beta\theta_l k_1^*(p,1)^\alpha$. The utility of $(w,1)$ type when she chooses $b_1 \leq -\beta\theta_l k_1^*(p,1)^\alpha$, is:

$$w + \Delta^*(w,1) - k_1^*(w,1) - b_1(w,1) + \beta \sum_\theta \mu_\theta [\theta k_1^*(w,1)^\alpha + b_1(w,1)/\beta] \tag{20}$$

$$= w + \Delta^*(w,1) - k_1^*(w,1) + \beta \sum_\theta \mu_\theta \theta k_1^*(w,1)^\alpha. \tag{21}$$

On the other hand, if an agent with an idea chooses $b_1 > -\beta\theta_l k_1^*(p,1)^\alpha$, then, letting her optimal choices be $\tilde{k}_1(w,1), \tilde{b}_1(w,1)$, her utility would be:

$$w - \frac{\eta_1}{\eta_0}\Delta^*(w,1) - \tilde{k}_1(w,1) - \tilde{b}_1(w,1) + \beta \sum_\theta \mu_\theta [\theta \tilde{k}_1(w,1)^\alpha + \tilde{b}_1(w,1)/\beta]$$

$$= w - \frac{\eta_1}{\eta_0}\Delta^*(w,1) - \tilde{k}_1(w,1) + \beta \sum_\theta \mu_\theta \theta \tilde{k}_1(w,1)^\alpha. \tag{22}$$

The difference between the maximized values of $(w,1)$ agents' problem in the market under $b_1 \leq -\beta\theta_l k_1^*(p,1)^\alpha$ and under $b_1 > -\beta\theta_l k_1^*(p,1)^\alpha$ then is given by subtracting (22) from (20) :

$$\frac{\Delta^*(w,1)}{\eta_0} + [-k_1^*(w,1) + \beta \sum_\theta \mu_\theta \theta k_1^*(w,1)^\alpha] - [-\tilde{k}_1(w,1) + \beta \sum_\theta \mu_\theta \theta \tilde{k}_1(w,1)^\alpha]. \tag{23}$$

Given that k^{fi} maximizes the function $-k + \beta \sum_\theta \mu_\theta \theta k^\alpha$, it is obvious that this difference is strictly positive for $(r,1)$. For $(p,1)$, under $b_1 > -\beta\theta_l k_1^*(p,1)^\alpha$, $\tilde{k}_1(w,1) < k_1^*(p,1) \leq k^{fi}$. This, combined with the fact that the function $-k + \beta \sum_\theta \mu_\theta \theta k^\alpha$ is strictly increasing in k, for $k \leq k^{fi}$, implies that the expression in (23) is also strictly positive. Hence, we showed that agents with ideas act according to the constrained-efficient allocation in the market.

Now consider agents who do not have an idea in period one. If they choose $b_1 \leq -\beta\theta_l k_1^*(p,1)^\alpha$, then $c_2(w,0) \leq -\theta_l k_1^*(p,1)^\alpha + As_1(w,0)$. To keep consumption non-negative, $s_1(w,0) \geq \frac{\theta_l k_1^*(p,1)^\alpha}{A}$. Since $A < \beta^{-1}$, these agents will invest as little as possible in risk-free technology. This implies they choose $b_1 = -\beta\theta_l k_1^*(p,1)^\alpha$ and $s_1(w,0) = \frac{\theta_l k_1^*(p,1)^\alpha}{A}$. The utility then is $w + \Delta^*(w,1) + \beta\theta_l k_1^*(p,1)^\alpha - \frac{\theta_l k_1^*(p,1)^\alpha}{A}$.

When an agent with no ideas chooses $b_1 > -\beta\theta_l k_1^*(p,1)^\alpha$, she sets $s_1 = 0$ and chooses the constrained-efficient allocation. The utility she gets is $w - \frac{\eta_1}{\eta_0}\Delta^*(w,1)$.

We need to show that, for agents without ideas, utility under $b_1 > -\beta\theta_l k_1^*(p,1)^\alpha$ is greater than utility under $b_1 \leq -\beta\theta_l k_1^*(p,1)^\alpha$. The difference is equal to

$$w - \frac{\eta_1}{\eta_0}\Delta^*(w,1) - [w + \Delta^*(w,1) + \beta\theta_l k_1^*(p,1)^\alpha - \frac{\theta_l k_1^*(p,1)^\alpha}{A}] \tag{24}$$

$$= \frac{-\Delta^*(w,1)}{\eta_0} - \theta_l k_1^*(p,1)^\alpha(\beta - 1/A) \tag{25}$$

$$= 0, \tag{26}$$

where the last inequality follows from $A = \frac{\eta_0\theta_l k_1^*(p,1)^\alpha}{k_1^*(p,1)-p-\eta_1\beta\theta_l k_1^*(p,1)^\alpha}$.

Market clearing and government budget balance conditions are immediate from the fact that the constrained-efficient allocation satisfies aggregate feasibility and has non-negative consumption for all agents.

□

The way the implementation works is as follows. An agent who borrows above the threshold gets a net subsidy of $-\frac{\eta_1}{\eta_0}\Delta^*(p,1) + \Delta^*(p,1)/\eta_0 = \Delta^*(p,1)$. Remember that this is exactly the amount of the NPV of transfers agents with ideas get in the planner's problem. Therefore, agents with ideas borrow at the threshold level, get the subsidy, and invest at the constrained-efficient level. Agents without ideas would like to do the same; however, for them, the only way to pay back in period two is to save through the storage technology, s_1, which is costly since s_1 is wasteful. The threshold amount of borrowing required to get the subsidy is chosen such that this cost is weakly higher than the benefit of getting the subsidy. Therefore, only agents with ideas get the subsidy, and hence the budget of the agency balances.

A comparison between the implementation provided above and the actual Business Loan Program is in order. The above implementation is similar to the actual system in the United States in the sense that in both, the government taxes all citizens and transfers some of its tax revenue to the agency that deals with start-ups (more generally, small businesses), with the intention of subsidizing potential start-ups (or, more generally, small businesses in the actual program) that are financially constrained.[27] Also, in both the model and the Business

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[27]The fact that in the paper's implementation all the tax revenue goes to the agency dealing with start-ups

Loan Program, only borrowers (start-ups) get subsidized.

However, since the model is very simple, there are significant differences between the paper's implementation and the actual system in the United States. Here, I stress two of those discrepancies and what causes them.

First, there is no default in the model economy; agents only sign non-state-contingent that they have to honor by assumption. This creates a discrepancy between the model and the actual program because the actual loan program does not give out direct subsidies but rather provides loan guarantees to some borrowers. These guarantees ensure the lenders that in case of default the SBA will pay back a certain percentage of the loan. This, in turn, causes the interest rate on SBA backed loan to be lower relative to other loans, thereby effectively subsidizing borrowers.

Second, in real life it is possible to monitor some features of start-ups at some cost, while the model abstracts away from any sort of monitoring. As a result, the actual Business Loan Program takes people's reports about, say, their ideas more seriously and spends some resources (labor) to determine whether or not the ideas are worth subsidizing. In the model, once an agent sends a report to the agency saying she has an idea, there is no way to check whether she is lying or not. Therefore, in the model's implementation, the function τ, which determines who gets subsidized and potentially depends on agents' reports, does not actually depend on agents' reports.

5   Generalizations

The purpose of this section is to convince the reader that the results of the paper are in fact quite general. I try to do this by changing the model in various dimensions and explaining how the results still hold in the resulting new environments. For each alteration of the model, I keep everything else the same so as to focus on the specific feature being altered. Also, for simplicity I am going to set $A = 0$, so there is no storage technology.[28]

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is immaterial. One can add exogenous government to the model, G, and then only $T - G$ units would be transferred to the agency. As long as this G is also subtracted from the right-hand side of the aggregate feasibility condition in the planner's problem, all the analysis goes unchanged.

[28]Formal analysis of the first extension is omitted for the sake of brevity; however, it is available from the author upon request.

5.1 All Agents Have Ideas

Instead of having some agents have ideas and some not, suppose that all agents have ideas, but fraction η_g have good ideas ($i = g$) and fraction η_b have bad ideas ($i = b$).

An agent of type (w, i) operates the following production technology:

$$y = \theta k^{\alpha}, \alpha \in (0, 1),$$

where, as before, k is the amount invested in period one, $\theta \in \{\theta_l, \theta_h\}$ is the idiosyncratic random return, and y is the output. The probability that an agent (w, i) receives return θ is $\mu_i(\theta)$, where

$$\mu_g(\theta_h) > \mu_b(\theta_h).$$

So, both agents with good and bad ideas produce, but the former is more likely to get a high return.

Define $k^{i,fi}$ as the full information efficient level of investment for agents with idea type i. Obviously, $k^{g,fi} > k^{b,fi}$. An analog of Assumption 4 here is that $\frac{k^{g,fi}-p}{\beta} > \theta_l k^{g,fi\alpha}$. Also, suppose for simplicity that $k^{b,fi} < p$. So, even the poor agents with bad ideas can invest at the full information level on their own. It is possible to show that θ being unobservable, together with this version of Assumption 4, implies that constrained efficiency requires that agents with good ideas receive a subsidy. That agents with bad ideas can pretend to have good ideas limits the amount of the subsidy going to agents who have good ideas. Nevertheless, this does not completely eliminate the subsidy since it is costly for agents with bad ideas to lie to have good ideas. The same implementation also works in this environment with some simple adjustments to the tax-subsidy system.

5.2 Allowing for Negative Consumption

Suppose utility function is of the form

$$u(c) = \begin{cases} c, & \text{if } c \geq 0; \\ \lambda c, & c < 0, \end{cases} \tag{27}$$

where $\lambda > 1$ is a constant.

The purpose of this extension is to show that our main subsidy result does not crucially

hinge upon non-negativity restriction on consumption. However, we need $\lambda > 1$, meaning marginal disutility of decreasing consumption when $c \leq 0$ is strictly greater than marginal disutility of decreasing consumption when $c > 0$.[29] As long as this holds, the constrained-efficient allocation involves NPV of transfers from unproductive to productive agents.

Full information efficiency is the same as it is in the benchmark case: both agents with ideas invest at the socially efficient level, k^{fi}, and consumption distribution is such that aggregate feasibility holds with equality and no agent consumes a negative amount in any period and any state.

Now consider constrained efficiency. Since the only change in the physical environment compared to the baseline model is the utility function, the definition of constrained efficiency remains the same as in section 3. Exact calculation of the constrained-efficient allocation for any $\lambda > 1$ is very lengthy and tedious due to numerous cases and hence is omitted.[30] Instead, the paper first provides a proposition that shows if λ is greater than or equal to a threshold level $\underline{\lambda}$, then the amount of NPV of transfers that poor agents with ideas have to receive in order to attain the full information solution is incentive-compatible. Hence, the constrained-efficient allocation coincides with a full information efficient allocation.[31] Then, the paper goes on to prove that any constrained-efficient allocation features subsidizing agents who have ideas, as long as $\lambda > 1$.

Proposition 6. *Suppose Assumptions 4 and 5 hold. Then, there exists a unique $\underline{\lambda} = 1 + \frac{k^{fi}-p-\beta\theta_l k^{fi\alpha}}{\eta_0 \beta \theta_l k^{fi\alpha}} > 1$ such that if $\lambda > \underline{\lambda}$, then in the constrained-efficient allocation:*

1. *$k_1^*(w,1) = k^{fi}$ and $k_1^*(w,0) = 0$, for all $w \in W$;*

2. *$\delta_1^*(w,1) = k^{fi} - p$ and $\delta_2^*(w,1) = -\theta_l k^{fi\alpha}$, for all $w \in W$;*

3. *$(\delta_1^*(w,0), \delta_2^*(w,0))_{(w \in W)}$ satisfy: $\Delta^*(w,0) = -\frac{\eta_1}{\eta_0}\Delta^*(w,1)$ and individual feasibility with non-negative consumption for all.*

More importantly, in the constrained-efficient allocation society transfers strictly positive NPV of resources from agents without ideas to those with ideas.

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[29]The reason why $\lambda > 1$ is crucial is as follows. The only reason why there is any exchange between agents in this model is to finance investment. If $\lambda = 1$, then any agent can finance her own investment by consuming a negative amount, and hence even under autarky full information efficiency would be achieved.

[30]This calculation is available from the author upon request.

[31]Remember that the full information efficient allocation is indeterminate in terms of individual consumption as long as no one consumes a negative amount.

Proof. The allocation described in Proposition 6 attains full information efficiency, provided that it is in the constraint set of the planner. Therefore, all one needs to do is to prove that this allocation is incentive-feasible. Individual and aggregate feasibility conditions and non-negativity of consumption for all agents at all times and states hold by construction.

Given that $\Delta^*(w, 1)$ is independent of w, an agent with an idea does not lie to be a different agent with an idea. The same is true for agents without ideas. Agents with ideas do not lie to be agents without ideas, since that brings transfers with strictly smaller NPV. Therefore, one only needs to check that agents without ideas do not lie to have ideas under the proposed allocation.

Given that no one gets to consume a negative amount in the constrained-efficient allocation, the payoff to those without ideas from telling the truth is $w + \delta^*(w, 0)$. The value from lying to be $(w', 1)$ is $w + \delta_1^*(w', 1) + \beta\lambda\delta_2^*(w', 1)$. Thus, this allocation is incentive-compatible if

$$-\tfrac{\eta_1}{\eta_0}\Delta^*(w, 1) \geq \Delta^*(w, 1) + \beta(\lambda - 1)\delta_2^*(w', 1)$$
$$\Rightarrow \lambda \geq 1 - \tfrac{\Delta^*(w,1)}{\eta_0\beta\delta_2^*(w,1)}.$$

Rearranging (28) and plugging in $\delta_1^*(w, 1)$ and $\Delta^*(w, 1)$, we get that the proposed allocation is incentive-compatible if $\lambda \geq 1 + \frac{k^{fi} - p - \beta\theta_l k^{fi\alpha}}{\eta_0\beta\theta_l k^{fi\alpha}}$, which is the assumption made in the proposition.

$\square$

Consequently, if λ is large enough, the amount of subsidy needed to achieve the full information investment level, Δ^{fi}, can be reached even in the case with private information. The intuition is as follows. From the perspective of agents without ideas, the benefit of lying to be an agent with an idea is getting transfers with NPV equal to $(1 + \tfrac{\eta_1}{\eta_0})\Delta^{fi}$, whereas the cost comes from consuming a negative amount in period two. When λ is sufficiently high, the cost outweighs the benefit and hence deters those without an idea from reporting to have one.

The rest of this subsection considers the constrained-efficient allocation when $\lambda < \underline{\lambda}$. The social planner still transfers strictly positive NPV of resources to agents with an idea; however, now the amount is smaller than Δ^{fi} due to incentive compatibility.

The social planner's goal is still to make poor agents with an idea invest as close to the full information level as possible and do this without making her consume a huge negative amount. This pushes for a subsidy from other agents to poor agents with ideas, and

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incentive-compatibility constraints push in the reverse direction. The constrained-efficient allocation arises from this trade-off. The following corollary, which follows directly from Proposition 6 and the discussion above, states formally that in the constrained-efficient allocation there is a transfer of resources from agents with no ideas to those with ideas, even when $\lambda \in (1, \underline{\lambda})$.

Corollary 2. *Suppose Assumptions 4 and 5 hold, and $\lambda \in (1, \underline{\lambda})$. Then, in the constrained efficient allocation, $\Delta^*(w, 1) > 0$, for all $w \in W$.*

6 Conclusion

This paper provides a novel rationale for governments to subsidize agents who have ideas (potential start-ups) but do not have enough resources to invest in them. If we accept that returns to start-up firms are privately observed by the owners of the firms, then constrained efficiency calls for subsidizing poor agents with ideas. If society knew who has ideas but lacks resources to invest in them, then it is simple to implement the subsidy. However, I assume here that people's wealth levels, whether they have ideas or not, and how they use their resources are unobservable to others. These additional private information assumptions imply that the subsidy going to poor agents with ideas is limited by incentive compatibility.

The paper also provides an implementation of the constrained-efficient allocation similar to the U.S. SBA's Business Loan Program. Even though the main idea behind both the implementation in the model and the actual Business Loan Program are the same, to subsidize financially constrained individuals with productive ideas, there are still significant discrepancies between the model's implementation and the actual program. This is due mainly to the fact that the model economy is very simple. Introducing default and/or monitoring technologies to the model can help to make the implementation look much more like the SBA's actual loan program. This may be interesting direction for future work.

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