Historical

A Comparison of Gasoline Sales Taxes and Automobile Efficiency Taxes as Methods for Reducing Gasoline Consumption

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Members of the present administration have recently resurrected the notion of an automobile efficiency tax, i.e., an excise tax which is negatively related to mileage ratings, as a means for reducing gasoline consumption. This proposal has been offered either as an alternative to, or in conjunction with, gasoline sales taxes. It is clear to most economists that a sales tax is a more direct method of affecting gasoline consumption than an efficiency tax and, hence, likely to be more effective in achieving that end. There is evidently some disagreement with this position within the economics profession though, since many of those policy makers advocating efficiency taxes are economists.

In this paper I argue for the widely held view that more direct methods are more efficient. I compare sales tax and automobile efficiency tax schemes within a very simplified general equilibrium model of an economy. Firms are assumed to operate with one input constant coefficient production functions, and all individuals are assumed to have identical utility functions and initial endowments. I find that within this simple model, gasoline sales taxes will definitely lead to a reduction in gasoline consumption, while automobile efficiency taxes may actually increase gasoline consumption. Furthermore, I show that for any efficiency tax program which does reduce fuel consumption, the sales tax program which accomplishes the same reduction in fuel consumption yields a Pareto superior allocation of other resources.

What alternative specification of the economy would favor efficiency taxes over sales taxes? It is my impression that the appeal of efficiency taxes is based on the observation that individual endowments differ, contrary to the assumptions of the present model, and this tax redistributes wealth from rich Cadillac owners to poor Vega owners. If this impression is accurate, then one should ask if an automobile efficiency tax is an efficient way to redistribute wealth, and I believe that question is even more easily answered than the one considered in this paper.

The Model

We assume that there are I individuals with identical strictly quasi-concave and twice continuously differentiable utility functions, V. V has as its arguments $\alpha_A Q_A$, $\alpha_B Q_B$, H-L, and Q_C; where α_i is the number of miles driven per automobile of type i, Q_i is the number of automobiles of type i purchased, H is the total hours available for work or leisure, L is the hours worked, and Q_C is the quantity of a composite good consumed. The partial derivatives of V, with respect to each of its arguments, are assumed to be strictly positive.

Individual budget constraints prior to any tax-cum-subsidy programs are of the form

1) $$wL \geq P_A Q_A^i + P_B Q_B^i + P_C Q_C^i + P_G Q_G^i$$

2) $$Q_G^i \geq \alpha_A^i Q_A^i b_A + \alpha_B^i Q_B^i b_B$$

$$Q_A^i = 0 \text{ or } 1,\ Q_B^i = 0 \text{ or } 1,\ Q_C^i \geq 0, \text{ and } Q_G^i \geq 0$$

where w is the wage rate and P_A, Q_A^i, P_B, Q_B^i, P_C, Q_C^i, P_G, and Q_G^i are the price and quantity of automobiles of type A, automobiles of type B, the consumption good, and gasoline, respectively. b_A and b_B are the different gallons of gasoline used per mile of travel in automobiles of type A and of type B.

Production of each commodity in the economy is accomplished by means of a constant returns to scale technology. In particular,

$$Q_A = aL_A,\ Q_B = bL_B,\ Q_C = cL_C, \text{ and, } Q_G = gL_G$$

where L_j, j=A, B, C, and G is the quantity of labor devoted to the production of Q_j. Production of Q_j is carried out by N_j firms. Each firm, n_j, seeks to maximize $P_j Q_j^{n_j} - wL_j^{n_j}$. The total labor demand,

$$\sum_{n_A=1}^{N_A} L_A^{n_A} + \sum_{n_B=1}^{N_B} L_B^{n_B} + \sum_{n_C=1}^{N_C} L_C^{n_C} + \sum_{n_G=1}^{N_G} L_G^{n_G},$$

will equal the total labor supply, IL^i, in equilibrium.

The absolute price level in a nonmonetary economy such as this one is indeterminate. As a consequence it will prove convenient to designate one commodity as numeraire and to express all prices in units of that commodity. We have chosen labor services as the numeraire for the present analysis, and prices hereafter are interpretable as the rate at which any particular market good can be acquired in exchange for labor services.

We assume that the economy is at an initial competitive equilibrium with positive Q_A, Q_B, Q_C, and Q_G being produced. Recall that a competitive equilibrium is a market clearing allocation of goods and services resulting from consumer utility maximization and firm profit maximization at prices which are viewed as parameters by all market participants. Prices and quantities associated with this initial competitive equilibrium are denoted with a single asterisk. (Thus, the aggregate outputs and prices in the no tax equilibrium are: Q_A^*, Q_B^*, Q_C^*, Q_G^*, L^*, P_A^*, P_B^*, P_C^*, P_G^*, 1.)

Gasoline Sales Tax

Instituting a sales tax on gasoline and rebating the proceeds to consumers leads to a new competitive equilibrium. We denote the prices and quantities associated with this equilibrium with two asterisks. Aggregate outputs and prices with the sales tax are thus Q_A^{**}, Q_B^{**}, Q_C^{**}, Q_G^{**}, L^{**}, P_A^{**}, P_B^{**}, P_C^{**}, P_G^{**}, 1. Our first theorem asserts that aggregate gasoline consumption will decline in response to the gasoline sales tax levy.

Theorem 1: $Q_G^{*} > Q_G^{**}$.

Proof:

There are four possibilities for equilibrium automobile production, since all individuals are identical and individual automobile demand is a zero-one variable:

1) Q_A^{**} and $Q_B^{**} = 0$

2) $Q_A^{**} = I$ and $Q_B^{**} = 0$

3) $Q_A^{**} = 0$ and $Q_B^{**} = I$

4) $Q_A^{**} = I$ and $Q_B^{**} = I$.

The theorem's validity is obvious for Case 1. There are only minor differences in the proof of the theorem for Cases 2, 3, and 4, so only Case 2, $Q_A^{**} = I$ and $Q_B^{**} = 0$, is dealt with here.

Notice first that profit maximizing firms with the production functions given on page 2 will produce positive but finite quantities of Q_A, Q_B, Q_C, and Q_G only if $P_A = 1/a$, $P_B = 1/b$, $P_C = 1/c$, and $P_G = 1/g$. Therefore, these must be the pre-tax competitive equilibrium prices.

But this implies that the sales tax equilibrium individual consumption bundles must have been feasible at the pre-tax equilibrium prices. That is,

(I) $L^{**} = P_A^* Q_A^{**} + P_C^* Q_C^{**} + P_G^* Q_G^{**}$,

since the right-hand side is merely the sum of the labor demands in the three active industries and all individuals are identical so the $(Q_A^{i}{**}, Q_C^{i}{**}, Q_G^{i}{**}, L^{i}{**})$ must be feasible and equal for all i. Thus, by revealed preference

(II) $V(\alpha_A^{i}{**}Q_A^{i}{**}, 0, H-L^{i}{**}, Q_C^{i}{**}) \leq V(\alpha_A^{i}{*}Q_A^{i}{*}, \alpha_B^{i}{*}Q_B^{i}{*}, H-L^{i}{*}, Q_C^{i}{*})$.

Since Q_A^{**} and Q_G^{**} are positive and finite, we must again have $P_A^{**} = 1/a$ and $P_G^{**} = 1/g$. Also, Q_C^{**} and Q_B^{**} are finite, so $P_C^{**} \leq 1/c$ and $P_B^{**} \leq 1/b$. P_B^{**} is net of taxes though, and hence each individual's budget constraint in the sales tax equilibrium has a different slope than in the initial equilibrium. This fact, together with (II) and the continuous differentiability of V, imply

(III) $P_G^{**}tQ_G^{**} + L^* < P_A^{**}Q_A^* + P_B^{**}Q_B^* + P_C^{**}Q_C^* + (1+t)P_G^{**}Q_G^*$.

Substracting (I) from (III) we obtain

(IV) $(1+t)P_G^{**}(Q_G^{**}-Q_G^*) + (L^*-L^{**}) < P_B^{**}Q_B^* + P_C^{**}(Q_C^*-Q_C^{**})$,

since $P_G^* = P_G^{**}$ and $P_C^{**} \leq P_C^*$ with inequality holding only if $Q_C^{**} = 0$.

We know from (I) that

$$(L^*-L^{**}) + P_G^*(Q_G^{**}-Q_G^*) = P_B^*Q_B^* + P_C^*(Q_C^*-Q_C^{**}),$$

but this implies

(V) $(L^*-L^{**}) + (1+t)P_G^*(Q_G^{**}-Q_G^*) \geq P_B^*Q_B^* + P_C^*(Q_C^*-Q_C^{**})$, if $(Q_G^{**}-Q_G^{*}) \geq 0$, since $t > 0$. However,

$$P_B^{*} \geq P_B^{**} \text{ and } P_C^{*}(Q_C^{*}-Q_C^{**}) \geq P_C^{**}(Q_C^{*}-Q_C^{**})$$

because $P_C^{*} = P_C^{**}$ whenever $Q_C^{**} > 0$.

Thus, replacing the right-hand side of (V) with the smaller quantity

$$P_B^{**}Q_B^{**} + P_C^{**}(Q_C^{*}-Q_C^{**})$$

contradicts (IV), hence (V) cannot be true, and we must have

$$Q_G^{**} < Q_G^{*}.$$

Automobile Efficiency Tax

We may next analyze the effects of the gallons per mile based excise tax or "efficiency tax" scheme. As in the preceding case, a new equilibrium may be associated with this scheme. We shall denote the prices and quantities existing in such an equilibrium by the symbol $\hat{}$. Thus, the vector of equilibrium aggregate quantities and prices is

$$(\hat{Q}_A,\hat{Q}_B,\hat{Q}_C,\hat{Q}_G,\hat{L},\hat{P}_A,\hat{P}_B,\hat{P}_C,\hat{P}_G,1).$$

Each consumer will be maximizing V subject to

$$\frac{b_A\tau\hat{Q}_A + b_B\tau\hat{Q}_B}{I} + L^i = (\hat{P}_A+b_A\tau)Q_A^i + (\hat{P}_B+b_B\tau)Q_B^i + \hat{P}_CQ_C^i + \hat{P}_GQ_G^i$$

and

$$Q_G^i = \alpha_A^iQ_A^ib_A + \alpha_B^iQ_B^ib_B$$

where τ is the efficiency tax factor. Each producer will be maximizing profits at the prices, $\hat{P}_A$, $\hat{P}_B$, $\hat{P}_C$, $\hat{P}_G$, and $\hat{w} = 1$.

Theorem 2:

The institution of an automobile efficiency tax and offsetting rebate may increase, decrease, or leave unchanged equilibrium gasoline consumption.

Proof (by example):

In order to simplify the following examples we shall assume labor supply is fixed for each individual.[1/]

Example 1--Increase in Gas Consumption

$$V(\alpha_A^i Q_A^i, \alpha_B^i Q_B^i, H-L^i, Q_C^i) = (\alpha_A^i Q_A^i)^{\frac{1}{2}} + \alpha_B^i Q_B^i/40 + 10\ell n Q_C^i +$$

$$[H-L^i+1]^{\frac{1}{2}}/100 - 1/100$$

$$b_A = .2,\ b_B = .1,\ a = (19)^{-1},\ b = (10)^{-1},$$

$$c = 1,\ g = 1,\ H = 129,\ I = 100.$$

Equilibrium prior to the institution of the tax is:

$$Q_A^* = 100,\ Q_B^* = 100,\ \alpha_A^* = 100,\ \alpha_B^* = 400,\ Q_C^* = 4000,\ Q_G^* = 6000,$$

$$P_A^* = 19,\ P_B^* = 10,\ P_C^* = 1,\ P_G^* = 1, \text{ and } w^* = 1.$$

Let $\tau = 50$. Since all consumers are alike and Q_A^i and Q_B^i are zero-one decision variables, there are only four possibilities for zero-net-revenue equilibrium individual rebates, 0, $b_A\tau = 10$, $b_B\tau = 5$, and $b_B\tau + b_B\tau = 15$.

Also note that because of our simple production functions, we need only consider $P_A = 19$, $P_B = 10$, $P_C = 1$, $P_G = 1$, and $w = 1$.

Table I indicates maximum consumer utility associated with auto purchases indicated in the column headings and rebate magnitudes indicated in the left-hand column.

Only when the largest row element appears on the main diagonal will the economy be in a zero-net-government revenue competitive equilibrium. This occurs when the rebate magnitude is 5 per household. The equilibrium values for the various outputs are:

$$\hat{Q}_A = 0,\ \hat{Q}_B = 100,\ \hat{Q}_C = 4000,\ \hat{Q}_G = 7900,\ \hat{\alpha}_B = 790.$$

Notice that gasoline consumption has increased by 1,900 gallons.

Example 2--Decrease in Gasoline Consumption

$$V(\alpha_A^i Q_A^i, \alpha_B^i Q_B^i, H-L, Q_C^i) = (\alpha_A Q_A)^{\frac{1}{2}} + 2.5 \log \alpha_B Q_B + 10 \log Q_C +$$

$$[H-L^i+1]^{\frac{1}{2}}/100 - 1/100$$

$$b_A = .2,\ b_B = .1,\ a = (40)^{-1},\ b = (25)^{-1},$$

$$c = 1,\ g = 1,\ H = 135,\ I = 100.$$

Equilibrium prior to the institution of the tax is:

$$Q_A^* = 100,\ Q_B^* = 100,\ \alpha_A^* = 100,\ \alpha_B^* = 100,\ Q_C^* = 4000,\ Q_G^* = 3000,$$

$$P_A^* = 40,\ P_B^* = 25,\ P_C^* = 1,\ P_G^* = 1, \text{ and } w^* = 1.$$

Let $\tau = 50$. Again, we may construct a table indicating the maximum consumer utility associated with each possible auto purchase scheme and government rebate magnitude (see Table II). Once again a zero-net-government revenue equilibrium is attained with a rebate of 5. Equilibrium values for the various outputs are:

$$\hat{Q}_A = 0,\ \hat{Q}_B = 100,\ \hat{Q}_C = 8800,\ \hat{\alpha}_B = 220,\ \hat{Q}_G = 2200,\ \hat{P}_A = 40,$$

$$\hat{P}_B = 25,\ \hat{P}_C = 1,\ \hat{P}_G = 1, \text{ and } \hat{w} = 1.$$

Gasoline consumption has fallen by 800 gallons as a result of the efficiency tax-cum-rebate.

Example 3--No Change in Gasoline Consumption

$$V(\alpha_A^i Q_A^i, \alpha_B^i Q_B^i, H-L^i, Q_C^i) = \ell n \alpha Q_A + 2.5 \ell n \alpha_B Q_B + 10 \ell n Q_C +$$

$$[H-L^i+1]^{\frac{1}{2}}/100 - 1/100$$

$$b_A = .2,\ b_B = .1,\ a = (40)^{-1},\ b = (25)^{-1},$$

$$C = 1,\ G = 1,\ H = 135,\ I = 100.$$

The competitive equilibrium for this economy has:

$$Q_A^* = 100,\ Q_B^* = 100,\ \alpha_A = 25.9,\ \alpha_B = 129.7,\ Q_C^* = 5188,$$

$$Q_G^* = 1815,\ P_A^* = 40,\ P_B^* = 25,\ P_C^* = 1,\ P_G^* = 1, \text{ and } w^* = 1.$$

The form of the utility function insures that for any level of income greater than the cost of one type A and one type B car, both will be purchased. Thus, letting $\tau = 50$ once again, it is clear that a zero-net-government revenue competitive equilibrium will obtain only when the rebate is 15. The equilibrium in this situation is precisely that which existed in the absence of the efficiency tax scheme.

Welfare Comparisons of Sales and Efficiency Taxes

It was demonstrated in the preceding section that an automobile efficiency tax may or may not succeed in reducing the consumption of gasoline in our simple economy. In this section we show that even if an efficiency tax will reduce gasoline consumption, a gasoline sales tax having the same impact on gasoline consumption would be strictly preferred by the consumers in our economy.

Again, we use the symbols *, **, and ^ to designate equilibrium quantities and prices in the economy when there are no taxes, when there is a gasoline sales tax rate $\bar{t}$ and offsetting subsidy, and when there is an automobile efficiency tax of rate $\bar{\tau}$ and offsetting subsidy, respectively.

Theorem 3:

If $0 < Q_G^{**} = \hat{Q}_G < Q_G^{*}$

then

$$V(\alpha_A^{i}{}^{**}Q_A^{i}{}^{**},\alpha_B^{i}{}^{**}Q_B^{i}{}^{**},H-L^{i}{}^{**},Q_C^{i}{}^{**}) > V(\hat{\alpha}_A^{i}\hat{Q}_A^{i},\hat{\alpha}_B^{i}\hat{Q}_B^{i},H-\hat{L}^{i},\hat{Q}_C^{i}).$$

Proof:

Note first that either $\hat{Q}_A$ or $\hat{Q}_B$ equals zero, since if both are positive, the pre- and post-tax equilibria will be identical,[2/] violating the conditions of the theorem. We assume, without loss of generality, that $\hat{Q}_A = I$ and $\hat{Q}_B = 0$.

Finally, since net taxes are zero for all individuals under both schemes and $\hat{Q}_G^{i} = Q_G^{i}{}^{**}$, one can manipulate the post-tax equilibrium budget constaints to obtain

$$-L^{**} + P_A^{**}Q_A^{i}{}^{**} + P_B^{**}Q_B^{i}{}^{**} + P_C^{**}Q_C^{i}{}^{**} = P_A^{**}\hat{Q}_A^{i} + P_C^{**}\hat{Q}_C^{i} - \hat{L}.$$

Consumers, however, will be facing budget constraints with differing slopes under the two tax schemes, since $P_A^{**} = \hat{P}_A$, $P_C^{**} = \hat{P}_C$ but $(1+t)P_G^{**} \neq \hat{P}_G$ and $P_A^{**} \neq \hat{P}_A + b_A\tau$. Our assumption on the continuous differentiability of V thus rules out identical utility maximizing consumption bundles under the two tax schemes. Therefore, $(\hat{Q}_A^{i},\hat{Q}_C^{i},\hat{Q}_G^{i})$ is feasible but not chosen at prices $(P_A^{**},P_B^{**},P_C^{**},P_G^{**},1)$, and the theorem follows by revealed preference//.

Conclusions

In this note we have compared the efficiency of automobile efficiency taxes and gasoline sales taxes as means for reducing domestic consumption of gasoline. We found that within a very simple general equilibrium model the sales tax is to be preferred to the efficiency tax for at least two reasons. First, an automobile efficiency tax with offsetting income subsidy can not be counted on to reduce gasoline consumption (Theorem 2), whereas a gasoline sales tax and rebate will definitely achieve a reduction in fuel use (Theorem 1). Second, if gasoline consumption can be reduced by the same amount with either an automobile efficiency tax-cum-rebate or a gasoline sales tax-cum-rebate, the latter program will yield an allocation of resources, which is Pareto superior to the allocation which is attained with the former program (Theorem 3).

Footnotes

1/ This is not inconsistent with our assumptions regarding individual preferences, since the marginal utility of leisure could be made positive but small so that the maximum possible labor supply always would be forthcoming.

2/ Each individual would be maximizing utility along the same budget hyperplane he faced in the no tax equilibrium if $\hat{Q}_A$ and $\hat{Q}_B$ were both positive.

Table I

Per Person Rebate	Auto Purchases			
	$Q_A^i = 0$ $Q_B^i = 0$	$Q_A^i = 0$ $Q_B^i = 1$	$Q_A^i = 1$ $Q_B^i = 0$	$Q_A^i = 1$ $Q_B^i = 1$
0	48.60	55.39	55.10	53.14
5	48.99	56.64	55.95	55.39
10	49.35	57.89	56.77	55.64
15	49.71	59.14	57.56	56.89

Table II

Per Person Rebate	Auto Purchases			
	$Q_A^i = 0$ $Q_B^i = 0$	$Q_A^i = 0$ $Q_B^i = 1$	$Q_A^i = 1$ $Q_B^i = 0$	$Q_A^i = 1$ $Q_B^i = 1$
0	$-\infty$	57.67	$-\infty$	54.35
5	$-\infty$	58.25	$-\infty$	55.71
10	$-\infty$	58.80	$-\infty$	57.02
15	$-\infty$	59.34	$-\infty$	58.40