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International Evidence on Long-Run Money Demand*

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Abstract

We explore the long-run demand for M_1 based on a data set that has comprised 32 countries since 1851. In many cases, cointegration tests identify a long-run equilibrium relationship between either velocity and the short rate or M_1, GDP, and the short rate. Evidence is especially strong for the United States and the United Kingdom over the entire period since World War I and for moderate and high-inflation countries.

With the exception of high-inflation countries—for which a "log-log" specification is preferred—the data often prefer the specification in the *levels* of velocity and the short rate originally estimated by Selden (1956) and Latané (1960). This is especially clear for the United States and other low-inflation countries.

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1 Introduction

The idea that the quantity of money in an economy can be measured and analyzed with some accuracy, and that changes in this quantity can be related in systematic ways to changes in interest rates, output, and prices, has had a long but checkered history. The postwar work of Friedman, Schwartz, Brunner, Meltzer, and others led to a common vocabulary for different definitions of money and well-documented data sets covering many countries over long time periods. Theoretical models proposed by Baumol, Tobin, and others described well how changes in the money supply affect other variables, and their predictions conformed well to evidence, at least at the low frequencies. Yet, over recent decades many economists have come to the view that monetary aggregates convey no useful information and have turned to macroeconomic models in which measures of money do not appear at all. One driver of this change was the alleged instability of the relationship between these series.

Our own conclusions in this paper are almost an exact opposite of this widespread view. We review the evidence on empirical money demand functions, using annual data on money (M_1, for us), nominal GDP, and short-term interest rates from 32 countries over periods that range in some cases to over 100 years. We find remarkable stability in long-run money demand behavior in many countries and an equally surprising sameness across different countries. In some cases of instability, anomalies have straightforward explanations. We describe these cases, and others that are less easy to dismiss, in some detail below.

In Section 2 we develop a generalized version of the Baumol-Tobin model that will guide our empirical investigation. We set up the model and then work out its main predictions. We draw the conclusions described above in two steps. The first is described in Section 3, where we simply plot the implied predictions of a particular case of the model against the data for all countries we have. We also show low-frequency evidence, using the band-pass filter. We find this informal visual evidence quite remarkable. The second is described in Sections 4 to 8, where we describe the econometric analysis of this evidence, based on cointegration methods, and we provide formal statistical tests that forcefully support the hypothesis of a stable long-run money demand.

2 A Model of Money Demand

We begin by developing a simple model that will guide our empirical investigation. We study a labor-only, representative agent economy with uncertainty in which making transactions is costly. We let s_t be the state at time t and let $s^t = \{s_0, s_1, ..., s_t\}$. The preferences of the representative agent are

$$E_0 \sum_{t=0}^{\infty} \beta^t U(x(s^t)),$$

where $x(s^t)$ is his consumption given history up through date t, and the function U is differentiable, increasing, and concave. The goods production technology is given by $y(s^t) = x(s^t) = z(s^t)l(s^t)$,where $l(s^t)$ is time devoted to the production of the consumption good and $z(s^t)$ is an exogenous stochastic process. The agent is endowed in each period with a unit of time, with $l(s^t)$ allocated to goods production and $1 - l(s^t)$ used to carry out transactions.

We assume that households choose the number n of "trips to the bank" in the manner of the classic Baumol-Tobin (BT) model. At the beginning of a period, a household begins with some nominal wealth that can be allocated to the transactional asset $M(s^t)$ or to nontransactional assets, risk-free government bonds or other state-contingent assets $Q(s^t, s_{t+1})$. During the first of the n subperiods, one member of the household uses money to buy consumption goods. During this same initial subperiod, another member of the household produces and sells goods in exchange for money. At the end of the subperiod, producers transfer to the bank the proceeds from their transactions. The situation at the beginning of the second subperiod thus replicates exactly the situation at the beginning of the first. This process is repeated n times during the period. The choice of this variable n will be the only economically relevant decision made by households. Purchases over a period are then subject to a cash-in-advance constraint $P(s^t)x(s^t) \leq M(s^t)n(s^t)$.

BT assumed that the cost of carrying out these transactions increases linearly in the number of trips. We will consider this case here and also allow for other forms for this cost function. Specifically, we describe the total cost of making transactions, measured in units of time, by a nonnegative, increasing, and smooth function $\theta(n(s^t), \nu(s^t))$, where $\nu(s^t)$ is an exogenous stochastic process. The variable $\nu(s^t)$ thus introduces some unobserved randomness into the model. This is essential to motivate the econometric analysis that is the core of the paper. It can be interpreted as changes over time in the technology to adjust portfolios available to households. We assume that $\theta(0, \nu(s^t)) = 0$, so the time involved in no trips to the banks is zero.

Equilibrium in the labor market and the equality of production and consumption imply

$$\begin{aligned} 1 &= l(s^t) + \theta(n(s^t), \nu(s^t)) \\ x(s^t) &= z(s^t)(1 - \theta(n(s^t), \nu(s^t))). \end{aligned}$$

The real wage is equal to $z(s^t)$ and the nominal wage is $z(s^t)P(s^t)$.

At the beginning of each period, an agent starts with nominal wealth $W(s^t)$, which can be allocated to $M(s^t)$, interest-bearing bonds, $B(s^t)$, or state-contingent assets $Q(s^t, s_{t+1})$. Let $P^Q(s^t, s_{t+1})$ be the price of an Arrow-Debreu security, bought at t in state s^t, which pays off one unit of money in state s_{t+1}. The agent's allocation of these assets is then restricted by

$$M(s^t) + B(s^t) + \sum_{s_{t+1}} Q(s^t, s_{t+1})P^Q(s^t, s_{t+1}) \leq W(s^t).$$

If we divide both sides by $P(s^t)$ and let $\widetilde{P}^Q(s^t, s_{t+1})$ denote the price of the state-contingent asset divided by the probability of the state, we can write this constraint as

$$m(s^t) + b(s^t) + E\left[q(s^t, s_{t+1})\pi(s^t, s_{t+1})\widetilde{P}^Q(s^t, s_{t+1})\right] \leq \omega(s^t), \tag{1}$$

where lowercase letters are real values and where $\pi(s^t, s_{t+1}) \equiv P(s^{t+1})/P(s^t)$ denotes the gross inflation rate between period t in state s^t and period $t+1$ in state (s^t, s_{t+1}).

We treat the gross nominal return on short-term bonds, $R(s^t)$, as an exogenous process determined by monetary policy.[1] This implies that the behavior of the growth rate of the money supply is restricted by other equilibrium conditions, as is well known and as we show in Online Appendix B.1.[2]

So far, we have been silent with respect to what our measure of money, $M(s^t)$, accounts for. For the theoretical analysis, we allow for money to pay a nominal return, lower than the one paid by bonds, which we call $R^m(s^t)$. As we will show, this is an important aspect of the theory. We explain our choices for both the particular monetary aggregate and its return in detail below, when discussing the empirical analysis.

We can now determine the agent's wealth next period, contingent on the actions taken in the current period and the realization of the exogenous shock s_{t+1}. In nominal units, this is

$$\begin{aligned} W(s^t, s_{t+1}) \quad \leq \quad & M(s^t)R^m(s^t) + B(s^t)R(s^t) + Q(s^t, s_{t+1}) \\ & + \left[1 - \theta(n(s^t), \nu(s^t))\right] z(s^t)P(s^t) + \tau(s^t, s_{t+1})P(s^{t+1}) - P(s^t)x(s^t), \end{aligned}$$

where $\tau(s^t, s_{t+1})$ is the real value of the monetary transfer the government makes to the representative agent. Dividing by the price level $P(s^{t+1})$, we obtain

$$\begin{aligned} \omega(s^t, s_{t+1}) \quad \leq \quad & \frac{m(s^t)R^m(s^t) + b(s^t)R(s^t)}{\pi(s^t, s_{t+1})} + q(s^t, s_{t+1}) \\ & + \frac{\left[1 - \theta(n(s^t), \nu(s^t))\right] z(s^t) - x(s^t)}{\pi(s^t, s_{t+1})} + \tau(s^t, s_{t+1}). \end{aligned} \tag{2a}$$

Finally, the cash-in-advance constraint can be written in real terms as

$$x(s^t) \leq m(s^t)n(s^t). \tag{3}$$

We now consider the decision problem of a single, atomistic agent who takes as given the prices $\widetilde{P}^Q(s^t, s_{t+1})$, the inflation rate $\pi(s^t, s_{t+1})$, the interest rate $R(s^t)$, the real wage $z(s^t)$, and the shock $\nu(s^t)$. Given the initial wealth $\omega(s^t)$, this agent chooses

[1] When policy is described as a sequence of interest rates, there may be indeterminacy of the price level. Real money balances will, however, be unique. In this paper, we ignore issues regarding the determination of the price level.

[2] The Online Appendix can be found at: https://sites.google.com/site/lucabenatiswebpage/

his consumption $x(s^t)$, the number of bank trips $n(s^t)$, and the assets $m(s^t)$, $b(s^t)$, and $q(s^t, s_{t+1})$ that he chooses to hold. These choices then determine the wealth $\omega(s^t, s_{t+1})$ that he carries into the next period conditional on s_{t+1}. These choices are restricted by equations (1), (2a), and (3).

The Bellman equation describing the decision problem is

$$\begin{aligned} V(\omega) &= \max_{x,n,m,b,q(s')} U(x) - \varepsilon\left[m + b + E\left[q(s')\pi(s')\widetilde{P}^Q(s')\right] - \omega\right] - \delta\left[x - mn\right] \\ &\quad + \beta E\left[V\left(\frac{mR^m + bR + [1-\theta(n)]\,z - x}{\pi(s')} + \tau(s') + q(s')\right)\right], \end{aligned}$$

where, for simplicity, we omitted the dependence of current variables on the state, and where s' denotes the future state.

As we show in Online Appendix B.2, the first order plus equilibrium conditions can be combined to yield a solution for the equilibrium number of portfolio adjustments, as follows:

$$r^* \equiv (R - R^m) = n^2 \frac{\theta_n(n)}{1-\theta(n)}, \tag{4}$$

which gives an extended squared root formula for the equilibrium value of n.[3]

Note first that, using just subindexes to indicate the dependency on the state, the solution for real money balances relative to output is

$$\frac{m(r_t, \nu_t)}{x(r_t^*, \nu_t)} = \frac{M(r_t^*, \nu_t)}{P(r_t^*, \nu_t)} = \frac{1}{n(r_t^*, \nu_t)},$$

which does not depend on z_t.

Now we would like to discuss the several empirical implications of this solution that do not depend on the particular functional form assumed for the function $\theta(n)$. First, the theory implies an income elasticity equal to one. This is the specification we will study for much of the paper. In Online Appendix G, we allow for a more general specification that does not restrict the income elasticity to be one, and where we are able to test this unitary income elasticity implication. Second, because $\theta(n_t, \nu_t)$ is differentiable with a strictly positive derivative, some of its properties are inherited by the function $m(r_t^*, \nu_t)$. In particular, up to a linear approximation, the stochastic properties of the money-to-output ratio, m_t/x_t, are inherited from the stochastic properties of r_t^* and ν_t. This has testable implications as long as ν_t is stationary, as we will assume throughout the paper. Specifically, if r_t^* is stationary, so should be m_t/x_t, whereas if it is the case that r_t^* has a unit root, m_t/x_t should have a unit root too. As it turns out, for the specifications of the function $\theta(n_t, \nu_t)$ that we explore in the theory and use in the empirical section, these properties hold exactly, not only in a linear approximation.

[3] The squared root formula is the by-now-classic solution of the Baumol-Tobin formulation.

2.1 Analysis of the solution

We now consider three alternative functional forms for $\theta(n_t, \nu_t)$. They deliver approximations to functional forms that have been used in empirical work and which we will explore in the following sections.

The exponential case Consider first the function $\theta(n_t, v_t) = \gamma\nu_t n_t^\sigma$. In this case, equation (4) becomes

$$n_t^{\sigma+1}\frac{\sigma\gamma\nu_t}{1-\gamma\nu_t n_t^\sigma} = r_t^*.$$

Note that $\gamma\nu_t n_t^\sigma$ is the cost of inflation in units of time, so it represents the welfare cost of inflation as a ratio of first-best output. This ratio is arbitrarily close to zero when the interest rate r_t^* is small. For moderate interest rates, the welfare cost is negligible. Even for relatively high interest rates, estimates of the welfare cost of inflation are hardly above 4%, so the denominator in the expression above would range from 1 to 0.96. We therefore use the approximation $1 - \gamma\nu_t n_t^\sigma \simeq 1$ and write the solution as $n_t^{\sigma+1}\sigma\gamma\nu_t \simeq r_t^*$. Taking logs, we then obtain

$$\ln\sigma\gamma + \ln\nu_t + (\sigma+1)\ln n_t = \ln r_t^*, \tag{5}$$

which is the log-log function typically used in the literature. The BT case is the one obtained by assuming that the function $\theta(n(r_t^*))$ is linear, or $\sigma = 1$, which implies an interest rate elasticity of 1/2.

The Selden-Latané specification A less well-known specification is obtained for the following cost function:

$$\theta(n_t, \nu_t) = b\ln(\varepsilon + n_t) + \frac{e\varepsilon + \nu_t}{n_t + \varepsilon} - \left(b\ln\varepsilon + \frac{e\varepsilon + \nu_t}{\varepsilon}\right),$$

where the term $\left(b\ln\varepsilon + \frac{e\varepsilon+\nu_t}{\varepsilon}\right)$ guarantees that $\theta(0, \upsilon_t) = 0$ and $b > e$ so the function is increasing. The function is concave, so it means that the marginal cost of making transactions is decreasing with the number of transactions (or, what is the same, decreasing with the nominal interest rate).

In this case, the solution is given by

$$n_t^2\frac{\frac{1}{(n_t+\varepsilon)^2}\left[(\varepsilon+n_t)b - \varepsilon e - \nu_t\right]}{1-\theta(n_t,\nu_t)} = r_t^*.$$

If, as before, we proceed with the approximation $1 - \theta(n, \nu_t) \simeq 1$, we obtain

$$\frac{n_t^2}{(n_t+\varepsilon)^2}\left[(\varepsilon+n_t)b - \varepsilon e - \nu_t\right] \simeq r_t^*.$$

Thus, for small values of ε, the solution can be approximated by

$$n_t b - \nu_t \simeq r_t^*, \tag{6}$$

which implies a linear relationship between velocity and the interest rate.

This empirical specification was used by Richard Selden (1956) over half a century ago, and, to the very best of our knowledge, it has been used again in the literature only once, by Henry Allen Latané (1960). The main reason for considering this long-forgotten specification is that, as we will discuss in Section 8, for several low-inflation countries—first and foremost, the United States—the data seem to quite clearly prefer it over the traditional log-log one discussed above and the semi-log specification that we discuss next.

The semi-log Finally, consider the following specification:

$$\theta(n_t, \nu_t) = -b\frac{\ln(\varepsilon + n_t)}{n_t + \varepsilon} - \frac{k + \nu_t}{n_t + \varepsilon} + \left(b\frac{\ln \varepsilon}{\varepsilon} + \frac{k + \nu_t}{\varepsilon}\right),$$

where again the term on the right-hand side implies $\theta(0, \nu_t) = 0$.

In addition, we assume $k + \nu_t > b\,(1 - \ln \varepsilon)$ for all ν_t, so that the function is always increasing in n_t. This function is also concave as the one before. The main difference between this function and the two studied above is that it asymptotes a constant (the term in parentheses on the right-hand side) as the number of trips grows arbitrarily large.

In this case, the solution is given by

$$\frac{n_t^2}{(n_t + \varepsilon)^2} \frac{[b\,(\ln(\varepsilon + n_t) - 1) + k + \nu_t]}{1 - \theta(n_t, \nu_t)} = r^*.$$

If, as before, we ignore the term $1 - \theta(n, \nu_t)$ and also consider relatively low values for ε, we obtain a linear relationship between the log of velocity and the interest rate, which corresponds to the well-known semi-log specification.

3 A First Look at the Data

The functional forms considered in the previous section deliver expressions that can be suitably taken to the data. The formal econometric analysis is presented in the following sections. As a first descriptive step, in this section we present the data and compare them to the theory. To do so, we focus on the particular case in which the function θ is linear in n, which corresponds to the BT case of the log-log specification in which the elasticity is constant and equal to 1/2.

Before doing that, we need to address the issue of how we map our theoretical construct M_t to the data. As the model makes clear, the choice of the natural aggregate

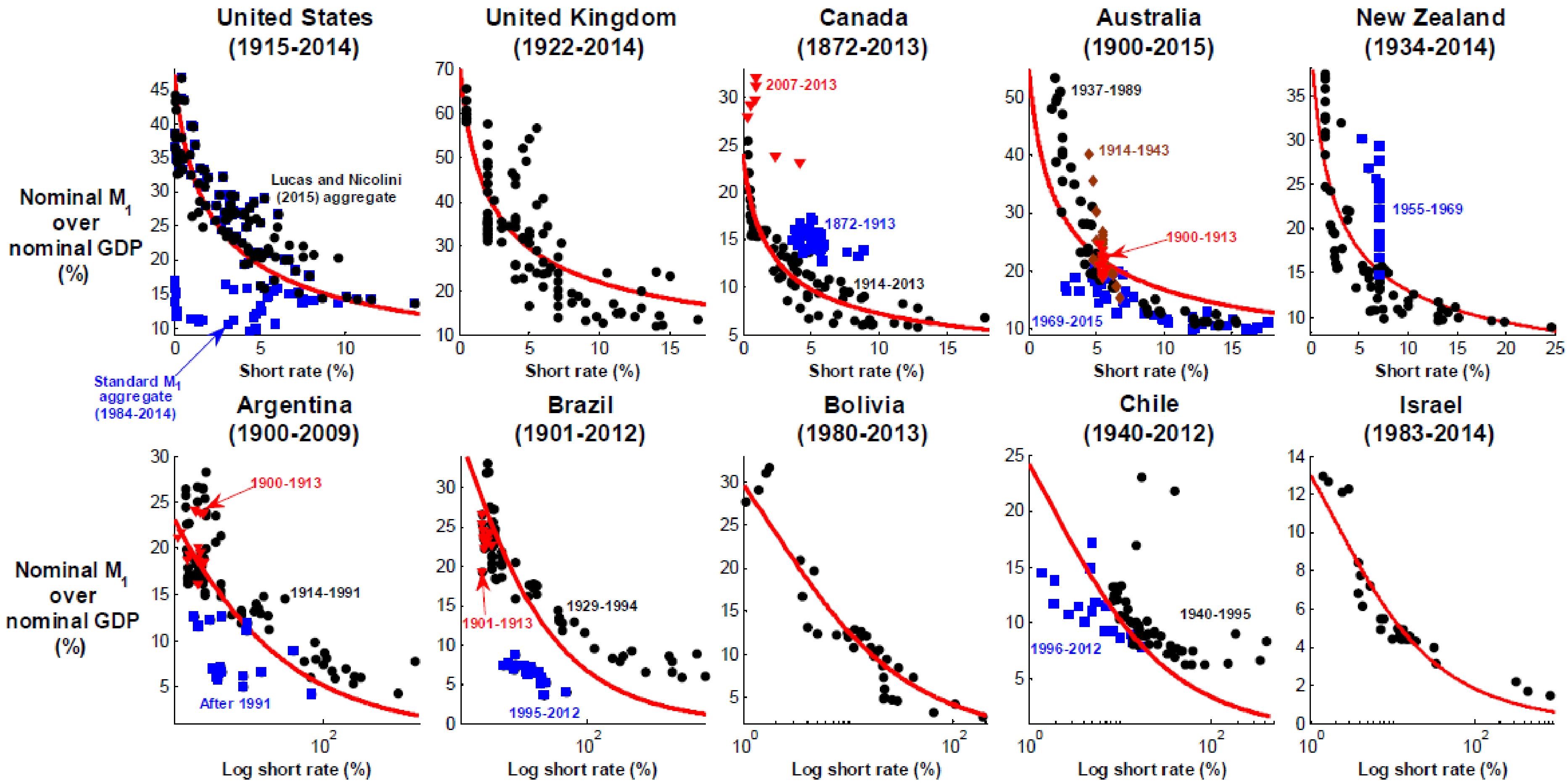

Figure 1 The raw data: short rate (plus 1%), ratio between nominal M_1 and nominal GDP, and fitted Baumol-Tobin specification

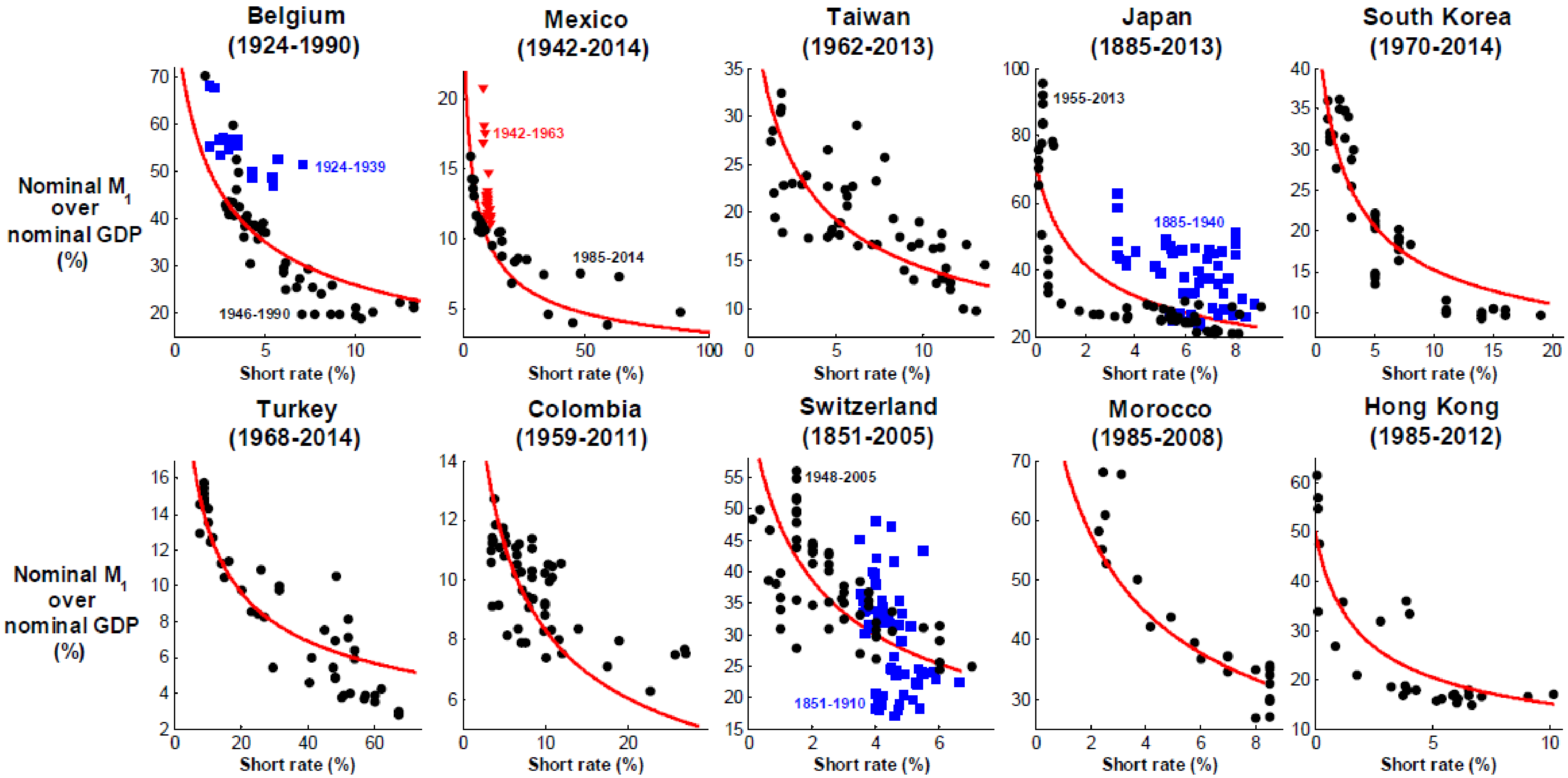

Figure 2 The raw data: short rate (plus 1%), ratio between nominal M_1 and nominal GDP, and fitted Baumol-Tobin specification

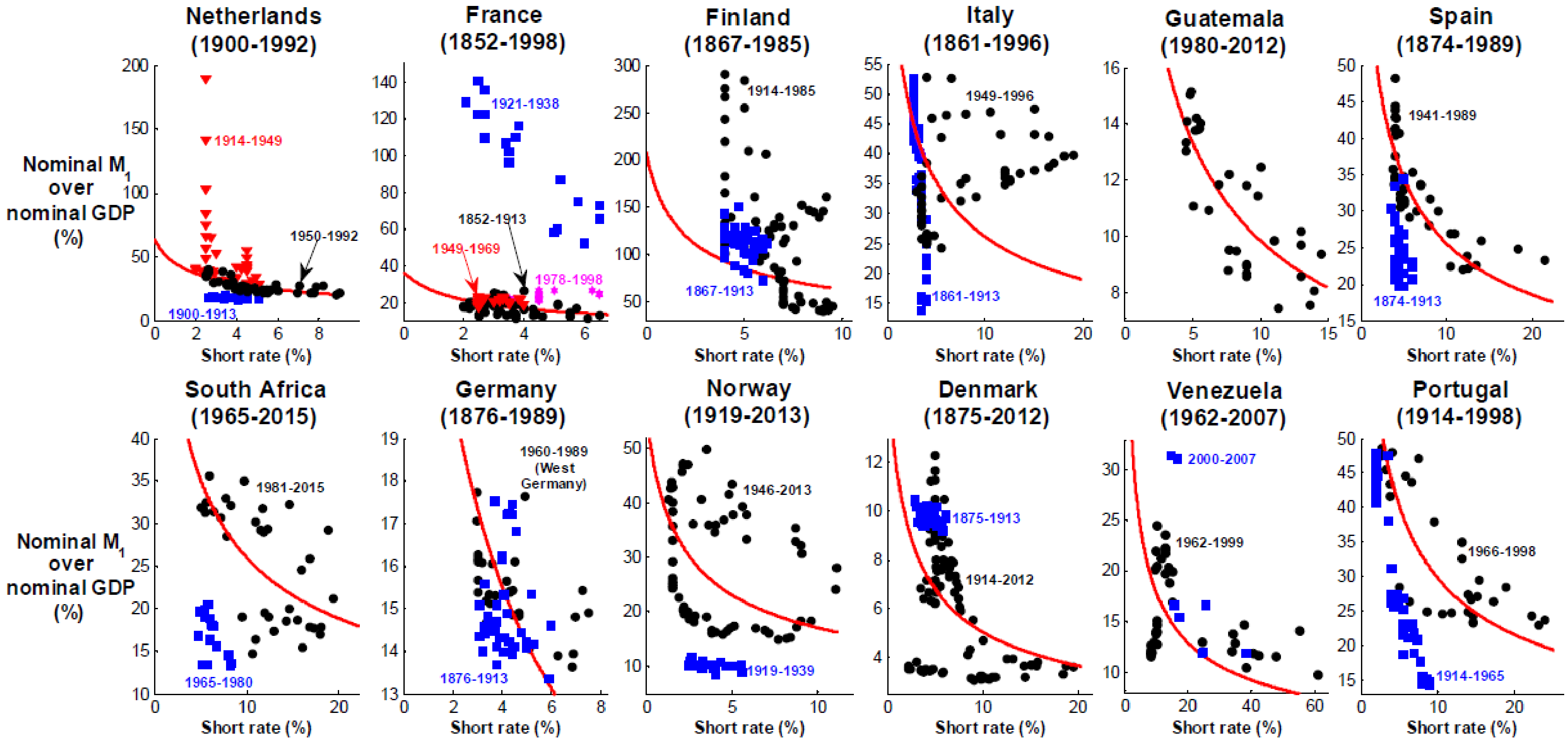

Figure 3 The raw data: short rate (plus 1%), ratio between nominal M_1 and nominal GDP, and fitted Baumol-Tobin specification

comes associated with the discussion of the nominal return of that particular aggregate R^m, since real money balances in the model depend not on the interest rate on bonds but rather on the spread between that rate and the rate paid by money. Since we do not have data on the interest rate paid by deposits, we choose to work with M_1, which in most countries includes cash and checking accounts. We will proceed under the assumption that, in the countries we study, checking accounts do not pay interest. Although this is a questionable assumption, it is certainly more appropriate for M_1 than for broader aggregates, which typically include interest-paying deposits.[4] As for cash, we follow Alvarez and Lippi (2009) and assume that it entails a *negative* return, associated with the risk of being lost or stolen. Alvarez and Lippi (2009) estimate the cost of holding cash to be close to 2% using detailed individual data from Italy. In addition, and to simplify, we assume that cash is about half of total money so that $R^m = 0.99$.

This is a very important assumption when considering the log-log specification, since it implies that real money balances have a satiation point when the interest rate on bonds is zero, as is the case for some countries in the sample. Indeed, if on the contrary we assume that $R^m = 1$, the log-log curve goes to infinity as $R \to 1$. As can be seen in the evidence we show in this section, this does not seem to be the case for countries that did experience several periods of almost zero interest rates, such as the United States and Japan. This assumption also plays an important role in the formal econometric tests because it often improves the performance of the empirical version of the log-log model.[5]

A *caveat* must be made explicit. Payments in this model are for household purchases of final goods, so they ignore other transactions where cash and deposits are used, such as paying employees and suppliers of intermediate goods and to clear asset exchanges. We are implicitly treating all these payments—which are much larger than final goods payments—as proportional to final goods payments. This will require introducing a constant of proportionality as another free parameter in the model, which will be country specific.

In other words, the theory we developed is not aimed at matching *levels* of M_1 over GDP but rather *changes* in this ratio as the interest rate changes. Therefore, one way to see our descriptive exercise is as using one free parameter per country, to allow for a country-specific intercept, while the slope will be given by the BT assumption of a linear technology, so that the elasticity is equal to 0.5.

Figures 1 to 3 show scatterplots of the short rate and of the ratio between nominal M_1 and nominal GDP (that is, the inverse of M_1 velocity), together with the

[4] It is the case, for instance, that deposits did pay interest in the United States after Regulation Q was modified in the early 1980s. It is also the case that some deposits included in M_1 did pay interest in very high-inflation countries such as Argentina and Brazil.

[5] For example, as we will see, for Switzerland for the period 1851-1906, the bootstrapped p-values for Johansen's trace and maximum eigenvalue tests of no cointegration between the logarithm of M_1 velocity and the short rate are 0.160 and 0.113 without the Alvarez-Lippi 1% correction to the short rate, but they fall to 0.094 and 0.057 with the correction.

theoretical curve that corresponds to an approximation of equation (5), namely, the BT case, so

$$\frac{M_t^j}{Y_t^j} = \frac{A^j}{(r_t^j + 1)^{1/2}}, \tag{7}$$

where Y_t^j is nominal income at time t in country j and A^j is a country-specific constant. As mentioned above, we let $r_{t,j}^* = R_t^j - 0.99$, where R_t^j is the gross short term interest rate at time t in country j. In three cases in which we could not find a (sufficiently long) interest rate series,[6] we use inflation as a *proxy* for the opportunity cost of money. For a detailed description of the data and the sources for each country, see Online Appendix B.

The grouping of countries has been largely arbitrary. The first row of Figure 1 contains countries that belonged, at some point, to the Commonwealth. The second row contains countries that experienced very high inflation rates, and the interest rate (i.e., the horizontal axis) is in a logarithmic scale because of the magnitudes reached by inflation and interest rates in these countries. In the second row of Figure 1, there are two countries, Argentina and Brazil, for which we highlight the most recent period (since 1991 and since 1995, respectively). These are the two countries in our sample that experienced recurrent periods of very high inflation that lasted over a decade. The blue squares correspond to the periods that followed the successful stabilization years: 1991 for Argentina and 1995 for Brazil. These points are highlighted because in both cases, the points following a successful stabilization lie below the theoretical curve that matches the previous period.

Figure 2 reports countries for which the theoretical curve is visually a still decent approximation to the data. The first row of Figure 3 shows countries for which the fit gets worse,[7] but still there seems to be some relationship between the theory and the data, whereas the second row of Figure 3 shows countries for which there seems to be no connection between theory and data.

In all of these figures, the data are shown with different colors and markers (dot, square, triangle, and star) under four circumstances: (i) data for the gold standard, up until 1913,[8] are always shown with a color different from that used for subsequent years; (ii) when we have data for nonconsecutive subperiods (e.g., as in the case for France); (iii) when we have different series for the short rate that cannot be linked (e.g., as is the case for Venezuela); and (iv) when we want to highlight drastic changes

[6] Specifically, Mexico, Chile (for the period 1941-2012), and Brazil (for the period 1934-2012).

[7] For the Netherlands, the two world wars and their aftermath had been characterized by an anomalous behavior of velocity, which in some cases reached values ranging between 50 and almost 200. Because of this, in our econometric analysis we will uniquely focus on the period 1950-1992.

[8] Although we take the gold standard to have ended in August 1914 with the outbreak of WWI, in fact, marking the *exact* date of its end is all but impossible because Richard Nixon's closing of the "gold window" in August 1971 was the culmination of a decades-long unraveling process that had started with WWI. (For a fascinating discussion of such progressive unraveling, see, e.g., Barro (1982).) We take August 1914 as the date marking the end of the gold standard mostly because we regard WWI as the single most important shock to the system.

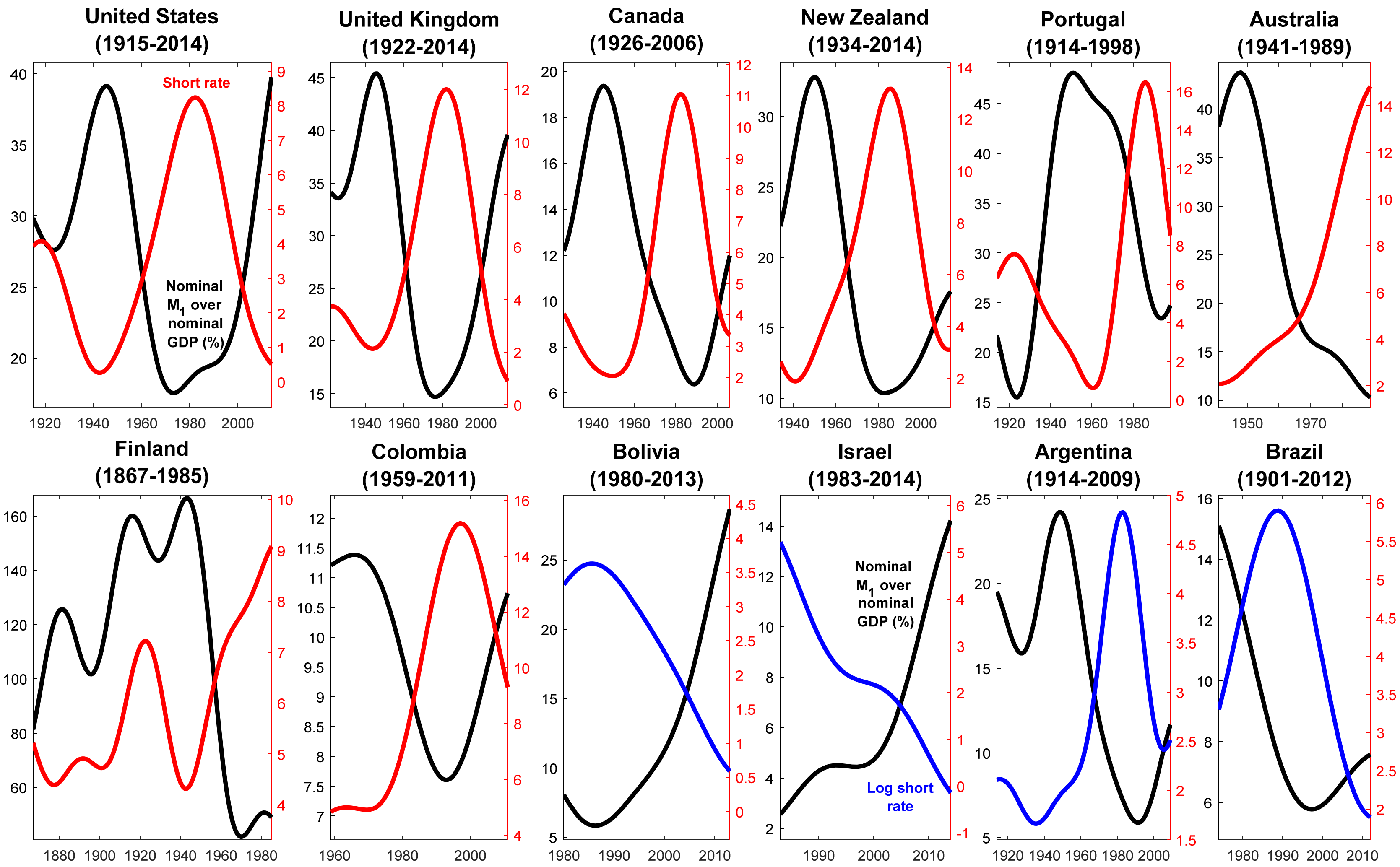

Figure 4 Low-frequency components of short rate and ratio between nominal M_1 and nominal GDP for selected countries

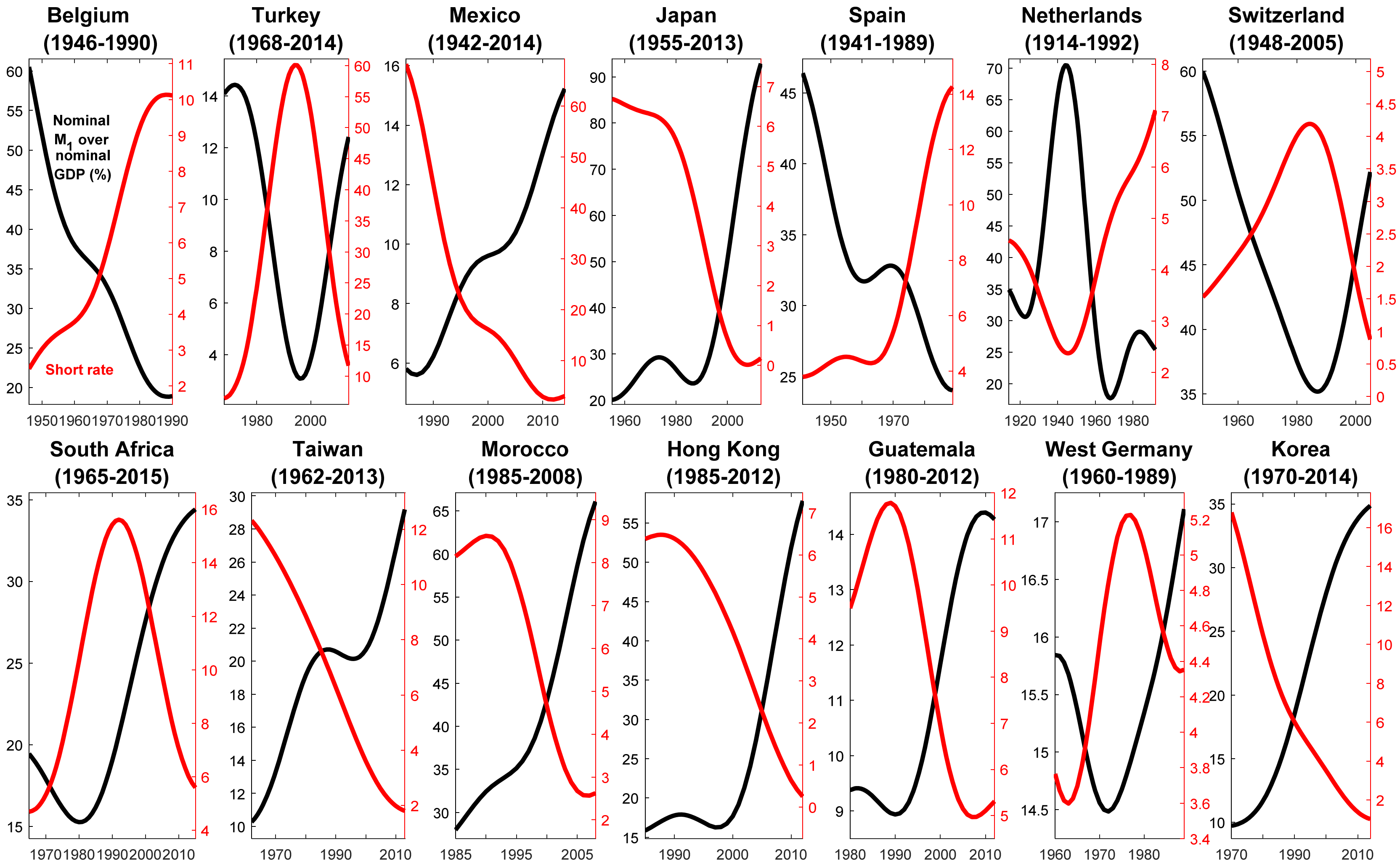

Figure 5 Low-frequency components of short rate and ratio between nominal M_1 and nominal GDP for selected countries

in the relationship between velocity and the short rate (e.g., as is the case for the Netherlands and Portugal). Finally, for the United States we show with a different color the "standard" M_1 aggregate for the period since 1984, in order to highlight how the failure to correct M_1, as in Lucas and Nicolini (2015), leads to the apparent breakdown of the relationship between velocity and the short rate documented by several authors.[9] In our view, it is remarkable how well this simple theory performs in this first inspection for a large set of countries, in spite of a few apparent failures.

Figures 4 and 5 present evidence in the spirit of Lucas (1980) by plotting the low-frequency components of the same series shown in the scatterplots in Figures 1-3.[10] The components have been extracted *via* the filter proposed by Christiano and Fitzgerald (2003).[11] We find this evidence, consistently pointing toward a negative correlation between M_1 velocity and the short rate at very low frequencies, quite simply remarkable. Although the main empirical body of the paper will be based on cointegration tests, the evidence in Figures 4-5 is, possibly, even *more convincing* because it is based on a simple technique such as linear filtering, which uniquely hinges on defining a specific frequency band of interest.

Despite the attractiveness of looking at simple plots, however, the previous analysis has several limitations. One would like to formally test whether, as some of our simple technologies imply, the ratio between real money balances and output inherits a unit root when the short-term interest rate exhibits a unit root. We also want to formally test whether the estimated elasticities are indeed equal to 1/2, as the simple BT specification suggests, when using the log-log specification. In addition, we would also like to let the data indicate which of the three specifications appear to provide a better fit, and therefore learn something regarding the shape of the function $\theta(n_t, \nu_t)$. To the extent that the interest rate and velocity exhibit a unit root—which, as we discuss in Section 5, appears to be overwhelmingly the case—we can use cointegration techniques to test whether there is a statistical long-run relationship between the two series, and therefore between the interest rate and the ratio of money balances to GDP. This is a particularly important question, since the distinction between short run and long run has always been at the center of the discussion in empirical studies of the money demand. Cointegration techniques are particularly suited to address this distinction formally, with the advantage that the cointegration residual provides a measure of the short-run deviations. The remainder of the paper contains the formal econometric analysis of our data set.

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[9] See, first and foremost, Friedman and Kuttner (1992).

[10] To be precise: we left out six countries for which evidence was weaker.

[11] Specifically, if the sample length, T, is greater than 50 years, we extract the components of the series with cycles slower than 30 years. If $40<T \leq 50$, $30<T \leq 40$, $20<T \leq 30$, we extract the components with cycles slower than 25, 20, and 15 years, respectively.

4 Main Features of Our Approach

In this paper we explore the long-run demand for M_1 *via* cointegration methods. The main reason is that, as we will show, the hypothesis that the short-term interest rate (in levels or in logs) exhibits a unit root is clearly supported by the data for most countries. At the same time, the debate over the stability of the money demand has long made the distinction between the short and the long run. This distinction is totally absent in our model, but a large theoretical literature has developed to try to understand the large and sustained deviations of observed real money balances from their theoretical counterparts: the "short-run" deviations of money demand.[12]

The entire notion of cointegration boils down to the existence of a *long-run relationship* between series driven by *permanent shocks*. Those permanent shocks are the main source of identification of the relationship between the short-term interest rate and real money balances over GDP, which we will discuss in what follows. The existence of the cointegration relationship implies that, in the long run, any permanent increase in the interest rate maps into a corresponding permanent decrease in real money balances: the exact amount will be described by the estimated cointegrating vector. In addition, any deviation of the two series from their long-run relationship—that is, what is technically known as the *cointegration residual*—is transitory, and it is bound to disappear in the long run. Accordingly, the persistence of this cointegration residual became an explicit measure of the "short-run" deviations. This is why, since the early 1990s, cointegration has become the standard approach for searching for a long-run money demand.[13]

It is important to highlight two aspects of our empirical strategy. First, we will perform tests that take either *cointegration* or *no cointegration* as the *null hypothesis* (specifically, Shin's, and Johansen's). Although the overwhelming majority of the papers in the literature have been based on Johansen's procedure, there is no reason—especially within the present context—to regard no cointegration as the "natural null hypothesis." Rather, it might reasonably be argued that, since we are here searching for the presence of a long-run money demand for transaction purposes, cointegration should be regarded as the natural null,[14] so that tests should just be based on Shin's (1994) procedure. A key reason for not eschewing Johansen's approach altogether, however, is that, as we document *via* Monte Carlo simulations, Johansen's procedure exhibits an overall better performance, and it produces more informative results. This is why, in the end, we decided to use both approaches.

Second, we perform our analysis separately for the gold standard and for the

[12] See Grossman and Weiss (1983) or Rotemberg (1984) for early contributions or Alvarez and Lippi (2014) for a recent one.

[13] See, in particular, Friedman and Kuttner (1992), Stock and Watson (1993), and Ball (2001).

[14] When dealing with a money demand *for transaction purposes*, cointegration should be regarded as the "natural null hypothesis" because basic economic logic suggests that—up to fluctuations in the opportunity cost of money—the nominal quantity of money demanded should be proportional to the nominal volume of transactions (i.e., to nominal GDP).

subsequent period. As has been extensively documented,[15] the stochastic properties of inflation in the former period had been radically different from the latter, with inflation being statistically indistinguishable from white noise most of the time. By the Fisher equation, this implies that, unless the natural rate of interest had contained a sizeable permanent component (due, e.g., to permanent shifts in trend productivity growth), nominal interest rates should be expected to have been stationary, too, which would preclude them from being entered in any cointegrated system or cointegrating regression.[16] Therefore, the integration properties of nominal interest rates during the gold standard period ought to be separately checked, or otherwise we would run the risk of performing cointegration analysis based on a series that had been stationary for a significant portion of the sample period.

In the next section, we study the integration properties of the data, where we show that the presence of unit roots seems very robust. The following section discusses the bootstrapping procedures that are involved in the cointegration tests and in the estimations procedures. We then discuss the Monte Carlo evidence that provides measures of the performance of the tests. Finally, we discuss how to interpret the outcome of the tests, in view of the Monte Carlo evidence, before analyzing the results.

5 Integration Properties of the Data

A necessary condition for using cointegration methods is that all series feature a unit root. In this section, we briefly summarize the main results from the extensive investigation of the integration properties of the data. For a full discussion, see Online Appendix C. Although in the main text we will only study bivariate systems with M_1 velocity and the short-term interest rate, we now discuss the integration properties of all variables, since in Online Appendix G we report estimates of systems including money balances and GDP separately, in order to test the hypothesis of unitary income elasticity.

Tables C.1a-C.1b report, for the series in our data set, bootstrapped p-values for Elliot, Rothenberg, and Stock (1996) tests.[17] For nominal GDP and nominal M_1, which exhibit obvious trends, the tests are based on models including an intercept

[15] See Barsky (1987) and Benati (2008).

[16] A key assumption underlying both Johansen's and Shin's cointegration tests is that all of the variables entering either the multivariate system (in the former case) or the single-equation cointegrating regression (in the latter case) are integrated of order one. See Hamilton (1994, very first sentence of p. 636) and Shin (1994, p. 92).

[17] For either series, p-values have been computed by bootstrapping 10,000 times estimated ARIMA(p,1,0) processes. In all cases, the bootstrapped processes are of a length equal to the series under investigation. As for the lag order, p, since, as it is well known, results from unit root tests may be sensitive to the specific lag order that is being used, for reasons of robustness we consider two alternative lag orders, either 1 or 2 years.

and a time trend.[18] For the short rate and velocity, on the other hand, they are based on models including an intercept but no time trend. For the short rate, the rationale for not including a time trend is obvious: the notion that nominal interest rates may follow an upward path,[19] in which they grow over time, is manifestly absurd.[20] For M_1 velocity, on the other hand, things are, at first sight, less obvious. The reason for not including a time trend originates from the fact that here we are focusing on a demand for money *for transaction purposes* (so this argument holds for M_1, but it would not hold for broader aggregates). The resulting natural assumption of unitary income elasticity logically implies that, if the demand for M_1 is stable, M_1 velocity should inherit the stochastic properties of the opportunity cost of money. In turn, this implies that the type of unit root tests we run for M_1 velocity should be *the same* as those we run for the nominal rate.

For both velocity and the short rate, we report results for either the levels or the logarithms of the series. For GDP and M_1, on the other hand, we only consider tests based on the logarithms of the series. The reason is that the level of either series is manifestly characterized by exponential-type growth, which is why these series are never considered in levels but rather are always considered in logarithms. For our purposes, this would not be a problem if Elliot *et al.*'s unit root tests allowed for the alternative of stationarity around an *exponential* time trend rather than a linear one. Since this is not the case, for both GDP and M_1 we are necessarily compelled to only consider tests based on their logarithms.

Finally, for the short-term rate we report results based on both the simple series (either in levels or in logarithms) and the simple series corrected along the lines of Alvarez and Lippi (2009), by adding to it a 1% cost of either losing cash or having it stolen.

Evidence of a unit root in M_1 velocity and the short rate is typically strong, with the bootstrapped p-values being almost uniformly greater than the 10% significance level we take as our benchmark throughout the entire paper, and often significantly so. The following exceptions ought to be briefly discussed:

(i) In a few cases, results based on the two alternative lag orders we consider produce contrasting evidence.[21] In these cases, we regard the null of a unit root as not having been convincingly rejected, and in what follows we therefore proceed under the assumption that these series are I(1).

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[18] We include a time trend because, as discussed by Hamilton (1994, p. 501), for example, the model used for unit root tests should also be a meaningful one under the alternative.

[19] The possibility of a downward path is ruled out by the zero lower bound.

[20] This does *not* rule out the possibility that, over specific sample periods in which inflation exhibits permanent variation (such as post-WWII samples dominated by the Great Inflation episode), nominal interest rates are I(1), too. Rather, by the Fisher effect, we *should* expect this to be the case. Historically, however, a unit root in inflation has been the exception rather than the rule—see Benati (2008).

[21] This is the case, for example, for the logarithms of velocity and the short rate for Israel, for log velocity for Chile for the period 1940-1995, and for the short rate for West Germany.

(*ii*) Under the gold standard, a unit root is rejected for both the level and the logarithm of the short rate (either with or without the 1% correction) for Canada, Finland, France, and Spain, and it is rejected for Switzerland based on the logarithm of the short rate with the 1% correction. In all of these cases, stationarity of the short rate precludes it from being entered in any cointegrated system or cointegrating regression.[22] By the same token, a unit root in the level of the short rate is rejected for Argentina, Brazil for the period 1934-2012, and Chile for the period 1941-2012, whereas in neither case is it rejected based on the logarithms. For all of these three cases, we will therefore eschew the Selden-Latané specification. Under the gold standard a unit root in either the level or the logarithm of velocity is rejected for Italy: in this case, we will therefore uniquely consider unrestricted specifications for GDP, M_1, and the short rate.

(*iii*) For Taiwan, a unit root in velocity is rejected based on the level but not based on the logarithm. In this case, we will eschew the Selden-Latané specification.

Evidence of a unit root in the logarithms of nominal GDP and nominal M_1 is, likewise, typically strong.[23] For GDP, a unit root is rejected only for Bolivia and for France under the gold standard (the latter rejection is ultimately irrelevant, since, as previously mentioned, for France the interest rate is stationary, so that it is not possible to analyze cointegrated systems). As for M_1, it is rejected only for Israel, Canada (1967-2013), and Finland (1914-1985). For Bolivia, Israel, Canada (1967-2013), and Finland (1914-1985), we will therefore eschew unrestricted specifications for GDP, M_1, and the short rate, and we will uniquely focus on bivariate systems with velocity and the short rate.[24]

We now turn to a brief discussion of methodological issues pertaining to bootstrapping cointegrated processes.

[22] See footnote 16.

[23] Again, in those few cases in which results based on the two alternative lag orders produce contrasting evidence, we regard the null of a unit root as not having been convincingly rejected, and we proceed under the assumption that the series is I(1).

[24] A necessary condition for performing either Johansen's or Shin's cointegration tests is that the series under investigation contain a unit root, and that their order of integration is not greater than one. Tables C.2*a*-C.2*b* in the Online Appendix C report bootstrapped *p*-values for Elliot *et al.*'s (1996) unit root tests with an intercept but no time trend, for either the log- or the first-differences of M_1 velocity and the short rate, and for the log-differences of nominal M_1 and nominal GDP. In brief (for details, see the discussion in Online Appendix C.3), in a few cases it is not possible to reject a unit root in, for example, the log-differences or the first-differences of M_1 velocity and the short rate (this is the case for Morocco and for Portugal under the gold standard), or the log-differences of nominal GDP and/or nominal M_1. In all of these cases, we will therefore eschew the relevant specifications, and in what follows we will therefore uniquely focus on specifications for which all series are I(1).

6 Issues Pertaining to Bootstrapping

Everything in this paper is bootstrapped[25]—specifically, both the p-values for the cointegration tests and, more generally, all of the objects of interest, such as the coefficients on the short rate in the estimated long-run money demand functions. In this section, we therefore briefly discuss (i) details of the bootstrapping procedures we use and (ii) how such procedures perform, in particular in terms of *comparative* performance. In our discussion, we will extensively refer to Online Appendices D and E, which contain the Monte Carlo evidence motivating both some of our choices and the way in which we will interpret the empirical evidence.

6.1 Details of the bootstrapping procedures

We bootstrap Johansen's tests *via* the procedure proposed by Cavaliere *et al.* (2012; henceforth, CRT). In a nutshell, CRT's procedure is based on the notion of computing critical and p-values by bootstrapping the model that is *relevant under the null hypothesis*.[26] All of the technical details can be found in CRT, to which the reader is referred. We select the VAR lag order as the maximum[27] between the lag orders chosen by the Schwartz and the Hannan-Quinn criteria[28] for the VAR in levels.

As for Shin's tests, to the very best of our knowledge, no one has yet provided anything comparable to what CRT did for Johansen's procedure (in fact, we were not able to find a single paper discussing how to bootstrap Shin's test statistic). The bootstrap procedure we propose in Online Appendix E is based on exactly the same idea underlying CRT, that is, computing critical and p-values by bootstrapping the process that is relevant under the null hypothesis. Within the present context, this implies that the process to be bootstrapped is the vector error-correction model

[25] As for Johansen's tests, the rationale for bootstrapping critical and p-values was provided by Johansen (2002) himself, who showed how, in small samples, trace and maximum eigenvalue tests based on asymptotic critical values typically tend to perform poorly. Since this is a small-sample issue, as a matter of logic we should expect Shin's (1994) tests to suffer from an analogous poor performance, thus justifiying the use of a bootstrapping procedure. Appendix B provides an additional rationale for bootstrapping Shin's tests: as we show there, even in very large samples, the distributions of Shin's test statistics coincide with the asymptotic distribution reported in Shin's (1994) Table 1 *only* if the cointegration residual has no persistence.

[26] This means that for tests of the null of no cointegration against the alternative of one or more cointegrating vectors, the model that is being bootstrapped is a simple, noncointegrated VAR in differences. For the maximum eigenvalue tests of h versus $h+1$ cointegrating vectors, on the other hand, the model that ought to be bootstrapped is the VECM estimated under the null of h cointegrating vectors.

[27] We consider the maximum between the lag orders chosen by the SIC and HQ criteria because the risk associated with selecting a lag order smaller than the true one (model misspecification) is more serious than the one resulting from choosing a lag order greater than the true one (overfitting).

[28] On the other hand, we do not consider the Akaike Information Criterion since, as discussed by Luetkepohl (1991), for example, for systems featuring I(1) series, the AIC is an inconsistent lag selection criterion, in the sense of not choosing the correct lag order asymptotically.

(VECM) estimated under the null of one cointegration vector. Apart from this, and with the exception of two comparatively less important technical issues we discuss in Section E.2.1 of Online Appendix E, the procedure we are proposing for Shin's tests is very similar to the one proposed by CRT for Johansen's ones.

6.2 Monte Carlo evidence

Table 1 below and Table E.1 in the Online Appendix report Monte Carlo evidence on the performance of the two bootstrapping procedures for Johansen's and Shin's tests, respectively, which is discussed in detail in Sections E.3.1 and E.3.2 of Appendix E. In either case, we perform the Monte Carlo simulations based on two types of data generation processes (DGPs), featuring *no cointegration* and *cointegration*, respectively. For either DGP, we consider five alternative sample lengths: $T = 50$, 100, 200, 500, and 1,000.

The main findings emerging from Table 1 can be summarized as follows.

If the true DGP features *no cointegration*, CRT's procedure performs very well irrespective of the sample size, with empirical rejection frequencies (henceforth, ERFs) very close to the 10% significance level. This is in line with the Monte Carlo evidence reported in CRT's Table I, p. 1731, and with the analogous evidence reported in Benati (2015).

Table 1 Monte Carlo evidence on the performance of Johansen's tests of the null of no cointegration, bootstrapped as in Cavaliere *et al.*'s (2012):[a] fractions of replications for which no cointegration is rejected[b] at the 10% level

	Sample length:				
	$T = 50$	$T = 100$	$T = 200$	$T = 500$	$T = 1000$
	True data generation process: no cointegration				
	0.116	0.098	0.105	0.107	0.119
Persistence of the cointegration residual	*True data generation process: cointegration*				
$\rho = 0$	0.774	1.000	1.000	1.000	1.000
$\rho = 0.25$	0.584	0.993	1.000	1.000	1.000
$\rho = 0.5$	0.350	0.882	1.000	1.000	1.000
$\rho = 0.75$	0.184	0.433	0.937	1.000	1.000
$\rho = 0.9$	0.117	0.167	0.328	0.958	1.000
$\rho = 0.95$	0.114	0.120	0.164	0.533	0.966

[a] Based on the trace test of the null of no cointegration against the alternative of 1 or more cointegrating vectors. [b] Based on 1,000 Monte Carlo replications and, for each of them, on 5,000 bootstrap replications.

If, however, the true DGP does feature *cointegration*, Johansen's tests[29] perform well *only* if the persistence of the cointegration residual is sufficiently low or the sample size is sufficiently large (or both). If, however, the cointegration residual is persistent and the sample size is small, the test fails to detect cointegration a nonnegligible fraction of the time.[30] This is conceptually in line with some of the evidence reported by Engle and Granger (1987), and it has a straightforward explanation: as the cointegration residual becomes more and more persistent, it gets closer and closer to a random walk (in which case there would be no cointegration), and the procedure therefore needs larger and larger samples to detect the truth (i.e., that the residual is highly persistent but ultimately stationary).

Turning to Shin's tests, the main findings emerging from Table E.1 in Online Appendix E can be summarized as follows.

If the true DGP features *cointegration*, then the greater the persistence of the cointegration residual, the more the proposed bootstrap procedure improves upon Shin's asymptotic critical values.[31]

If the DGP features *no cointegration*, however, even in large samples the proposed bootstrapped procedure produces ERFs far from the ideal of 100%. For $T = 1000$, for example, cointegration is rejected only about 38% of the time and, based on smaller sample lengths, much less than that.

6.2.1 Summing up

The preceding discussion can be summarized as follows.

If Johansen's tests do detect cointegration, we should have a reasonable presumption that cointegration is indeed there. If, on the other hand, they do not detect it, a possible explanation is that the sample period is too short or the cointegration residual is highly persistent (or both).

Lack of rejection of the null of cointegration based on Shin's tests and our bootstrapping procedure does not represent strong evidence that cointegration truly is there. Further, rejection of the null of cointegration does not appear to be especially informative about the true nature of the DGP because the ERFs are not significantly different conditional on the two possible states of the world. Another way to put all this is that results from Shin's tests appear, overall, as less informative than the corresponding results produced by Johansen's tests bootstrapped as in CRT.

We now turn to the issue of what we should expect to obtain from cointegration tests *before running them*, based on (i) the persistence of the cointegration residuals

[29] Table 1 only reports results based on the trace test, but evidence based on the maximum eigenvalue test is near-identical.

[30] For example, with $T = 100$, cointegration will be detected, at the 10% level, 43.3% of the time if $\rho = 0.75$ and just 12.0% of the time if $\rho = 0.95$.

[31] For example, for $T = 100$, if $\rho = 0.95$, tests based on asymptotic critical values would lead a researcher to reject the null of cointegration at the 10% level 72.1% of the time, whereas the bootstrap-based procedure only rejects 25.1% of the time.

and (*ii*) the just-discussed Monte Carlo evidence on how such persistence affects the performance of the tests for a given sample length.

7 What Should We Expect from Cointegration Tests?

In Section 8 we will perform cointegration tests based on about *three dozen samples*. Performing such a large number of tests implies that, even if cointegration truly is there in *all* samples and even under ideal conditions (e.g., Shin's tests incorrectly rejecting the null of cointegration $x\%$ of the times at the $x\%$ level), a certain number of fluke results is to be expected. Further, the Monte Carlo evidence we discussed in the previous section suggests that—in line with Engle and Granger (1987)—detecting a cointegration relationship can be extremely difficult when the sample period is comparatively short or the cointegration residual is highly persistent (or both). In this section, we therefore start by exploring how persistent "candidate cointegration residuals" (defined below) actually are. Based on this and on the series' actual sample lengths, we then discuss what we should reasonably expect to obtain from cointegration tests before running them.

7.1 How persistent are "candidate cointegration residuals"?

Tables SELA.1, SL.1, LL.1, and LLCO.1 in the online appendix report Hansen (1999) "grid bootstrap" median-unbiased (henceforth, MUB) estimates of the sum of the AR coefficients in AR(2) representations for the "candidate cointegration residuals" in our data set.[32] By "candidate cointegration residual" (henceforth, CCR), we mean the linear combination of the I(1) variables in the system that will indeed be regarded as a cointegration residual *if* cointegration is detected. We label it as "candidate" because, as the Monte Carlo evidence in the previous section has shown, if a cointegration residual is highly persistent, cointegration might well *not* be detected *even if it is there*, which would prevent the candidate from being identified as a true cointegration residual. For reasons of robustness, we report results based on two alternative estimators of the cointegration vector: Johansen's and Stock and Watson's (1993).

Results based on either estimator are qualitatively similar and point toward a nonnegligible extent of persistence of the CCRs. At the same time, our data set exhibits a wide extent of heterogeneity in terms of the estimated persistence. Focusing on the results based on the log-log specification for high-inflation countries, and on the Selden-Latané specification for all other countries, the MUB estimate based on Johansen's estimator of the cointegration vector—let us label it as $\hat{\rho}^J_{MUB}$—ranges from a minimum of 0.30 for Australia to a maximum of 1.00 for Portugal (1966-1998). By classifying the $\hat{\rho}^J_{MUB}$'s, in an admittedly quite arbitrary fashion, as "highly persistent"

[32] Results are based on 2,000 bootstrap replications for each possible value of the sum of the AR coefficients in the grid. Bootstrapping has been performed as in Diebold and Chen (1996).

($\hat{\rho}^J_{MUB}$ ≥0.8), "moderately persistent" (0.4< $\hat{\rho}^J_{MUB}$ <0.8), and "not very persistent" ($\hat{\rho}^J_{MUB}$ ≤0.4), we end up with 16 $\hat{\rho}^J_{MUB}$'s in the first group, 15 in the second, and 3 in the third. Results based on Stock and Watson's estimator point toward an even greater extent of persistence, with the number of MUB estimates in the three groups being equal to 21, 10, and 3, respectively.

7.2 Implications

Under these circumstances, statistical tests will often have a hard time detecting cointegration even if it truly is there. This will be especially so in those cases in which $\hat{\rho}_{MUB}$ is high and the sample period is comparatively short, such as Turkey. As we discuss in the next section, based on none of the three specifications (Selden-Latané, semi-log, and log-log) do Johansen's tests detect any evidence of cointegration between velocity and the short rate for this country. Such a failure of Johansen's tests to detect evidence of cointegration is especially startling in the light of the evidence reported in panel (1,2) of Figure 5, highlighting very strongly the correlated fluctuations in velocity and the short rate at low frequencies. This finding has a straightforward explanation: given the comparatively short sample period (46 years) and the high persistence of the CCR (with, e.g., $\hat{\rho}_{MUB}$ in Table SELA.1 equal to either 0.92 or 0.94), it is not surprising that Johansen's procedure does not reject the null of no cointegration. Rather, based on the evidence reported in Table 1, this is *to be expected*: when the true DGP features cointegration, with $T = 50$ and $\rho = 0.9$ Johansen's tests correctly reject the null of no cointegration *only 11.7% of the time.* This is qualitatively and even quantitatively in line with the Monte Carlo evidence reported by Engle and Granger (1987) for cointegration tests based on the Dickey-Fuller test statistic. This means that if cointegration were truly there in Turkish data, given the sample length we are working with and the specific characteristics of the DGP, we would have a *nearly 90% chance of not detecting it.* The same logic applies to several other countries for which Johansen's tests will not detect cointegration, in spite of the strong visual evidence in Figures 4-5.

The takeaway for the reader is that the results from cointegration tests we discuss in the next section should *not* be taken strictly at face value. Rather, they ought to be interpreted in light of the Monte Carlo evidence on the performance of cointegration tests reported in Tables 1 and E.1, and of the evidence on the persistence of the CCRs reported in Tables SELA.1, SL.1, LL.1, and LLCO.1.[33]

Let us now turn to the results from cointegration tests.

[33] This is very much in the spirit of Lucas' (1988) interpretation of econometric results that, taken at face value, appeared to contradict the findings of Meltzer (1963).

8 Searching for a Long-Run Money Demand

Table 2 reports results of tests from either Johansen or Shin for cointegration between log velocity (the inverse of the ratio of nominal M_1 to nominal GDP) and the log of the short rate, which corresponds to the basic Baumol-Tobin constant elasticity specification. As mentioned above, following Alvarez and Lippi (2009), the short rate has been corrected by adding the expected cost of either losing cash or having it stolen, which we calibrate to 1%. In Table 3 we report the results for the Selden-Latané specification, which corresponds to an elasticity that is increasing over time.

The corresponding set of results based on the semi-log specification are reported in Table SL2 in the online appendix and are discussed in Online Appendix G. We do not discuss them here because they are systematically weaker than those based on either the log-log or the Selden-Latané specifications. In a nutshell, as we will see, the data seem to prefer the Selden-Latané specification at comparatively low interest rates (i.e., those associated with countries such as the United States or the United Kingdom) and the log-log specification at high or very high interest rates (e.g., for countries such as Argentina or Israel), whereas evidence based on the semi-log specification is never strong.

Figures 6 to 10 report the estimation results for the log-log specification.[34] In the top rows, we report the candidate cointegration residuals produced by either Johansen's or Stock and Watson's (1993) estimators, and in the bottom rows the bootstrapped distributions[35] of the corresponding estimates of the coefficient on the log of the short rate (i.e., the interest rate elasticity of money demand). For each bootstrapped distribution we also report the mean, the median, and the 5th and 95th percentiles. Figures SELA.1 to SELA.6 in the online appendix report the corresponding set of results based on the Selden-Latané specification, whereas Figures SL.1 to SL.6 report results for the semi-log specification. For the reasons discussed above, in all cases we report both candidate cointegration residuals and estimates of the coefficients on the short rate for *all* countries rather than only for those for which statistical tests detect evidence of cointegration.

8.1 Evidence from cointegration tests

8.1.1 Unrestricted tests of the null of cointegration

Although this paper mostly focuses on the results produced by bivariate systems, we want to briefly discuss those produced by Shin's tests of the null of cointegration applied to unrestricted specifications featuring (the logarithm of) the short rate and the logarithms of nominal GDP and M_1. The reason for doing so is that they represent

[34] Again, these results are based on the "corrected" short rate, incorporating the expected cost of either losing cash or having it stolen.

[35] Bootstrapping has been implemented as in Cavaliere *et al.* (2012) based on the estimated VECM conditional on one cointegration vector.

Table 2 Cointegration tests between the logarithms of M_1 velocity and a short-term rate[a]

		I: Johansen's tests		
Country	*Period*	*Trace*[b]	*Maximum eigenvalue*[c]	II: Shin's tests
Argentina	1914-2009	21.303 (0.032)	18.866 (0.023)	0.567 (0.288)
Australia	1941-1989	6.111 (0.800)	6.102 (0.709)	0.369 (0.126)
	1969-2015	10.268 (0.506)	9.373 (0.405)	0.245 (0.395)
Belgium	1946-1990	23.319 (0.011)	21.225 (0.010)	0.106 (0.736)
Bolivia	1980-2013	15.480 (0.255)	15.134 (0.154)	0.156 (0.249)
Brazil	1974-2012	20.904 (0.049)	15.221 (0.093)	0.325 (0.104)
	1934-2012	20.270 (0.034)	16.842 (0.037)	2.043 (0.011)
Canada	1926-2006	12.533 (0.306)	11.202 (0.229)	0.722 (0.182)
	1967-2012	27.310 (0.010)	27.262 (0.003)	0.079 (0.705)
Chile	1940-1995	26.453 (0.013)	18.953 (0.033)	0.178 (0.244)
	1941-2012	18.541 (0.059)	13.224 (0.119)	0.127 (0.725)
Colombia	1959-2011	6.603 (0.830)	4.896 (0.872)	0.225 (0.502)
Finland	1914-1985	7.225 (0.736)	5.019 (0.839)	1.447 (0.023)
Germany	1876-1913	9.947 (0.559)	8.689 (0.532)	0.522 (0.177)
Guatemala	1980-2012	18.939 (0.077)	17.261 (0.052)	0.072 (0.737)
Japan	1885-1913	11.938 (0.408)	10.737 (0.331)	0.435 (0.099)
	1955-2013	13.502 (0.199)	13.502 (0.120)	0.098 (0.975)
Korea	1970-2014	6.698 (0.746)	6.075 (0.715)	0.282 (0.269)
Israel	1983-2014	41.66 (0.001)	40.773 (0.000)	0.135 (0.350)
Mexico	1985-2014	15.569 (0.230)	14.027 (0.205)	0.132 (0.285)
Netherlands	1950-1992	15.054 (0.166)	9.309 (0.401)	0.216 (0.413)
New Zealand	1934-2014	17.917 (0.075)	16.454 (0.044)	0.604 (0.354)
Norway	1946-2013	24.004 (0.016)	20.698 (0.015)	0.736 (0.157)
Portugal	*1914-1965*	20.699 (0.061)	19.887 (0.032)	0.120 (0.360)
	1966-1998	19.392 (0.086)	14.975 (0.125)	0.074 (0.546)
South Africa	1967-2014	16.776 (0.131)	15.686 (0.080)	0.336 (0.160)
Spain	1941-1989	7.850 (0.642)	7.632 (0.537)	0.261 (0.256)
Switzerland	1851-1906	15.520 (0.094)	15.377 (0.057)	0.780 (0.192)
	1948-2005	31.284 (0.001)	27.586 (0.001)	0.975 (0.064)
Taiwan	1962-2013	6.108 (0.816)	5.508 (0.794)	0.387 (0.131)
United Kingdom	1922-2014	15.684 (0.159)	15.361 (0.077)	0.951 (0.058)
United States				
standard M_1	1915-2014	11.224 (0.342)	9.563 (0.320)	3.021 (0.015)
Lucas-Nicolini M_1	1915-2014	14.623 (0.187)	13.107 (0.137)	0.369 (0.290)
Venezuela	1962-1999	6.616 (0.771)	4.389 (0.888)	0.364 (0.094)
West Germany	1960-1989	12.243 (0.419)	12.194 (0.261)	0.442 (0.076)

[a] Bootstrapped p-values (in parentheses) are based on 10,000 bootstrap replications.
[b] Null of no cointegration against alternative of 1 or more cointegration vectors.
[b] Null of 0 *versus* 1 cointegration vectors.

Table 3 Cointegration tests between M_1 velocity and a short rate[a]				
		I: Johansen's tests		
Country	*Period*	*Trace*[b]	*Maximum eigenvalue*[c]	II: Shin's tests
Australia	1941-1989	6.699 (0.735)	6.613 (0.642)	0.434 (0.103)
	1969-2015	16.903 (0.116)	15.890 (0.063)	0.278 (0.227)
Belgium	1946-1990	12.892 (0.339)	10.528 (0.361)	0.099 (0.906)
Bolivia	1980-2013	19.339 (0.089)	18.519 (0.053)	0.090 (0.976)
Brazil	1974-2012	30.987 (0.005)	25.024 (0.008)	0.640 (0.018)
Canada	1926-2006	23.244 (0.017)	21.714 (0.007)	0.800 (0.197)
	1967-2013	26.139 (0.016)	25.195 (0.007)	0.090 (0.558)
Chile	1940-1995	24.191 (0.024)	14.026 (0.133)	0.696 (0.024)
	1941-2012	23.304 (0.020)	18.084 (0.035)	0.411 (0.307)
Colombia	1959-2011	8.435 (0.673)	6.439 (0.717)	0.251 (0.433)
Finland	1914-1985	6.825 (0.742)	6.765 (0.622)	1.391 (0.071)
Germany	1876-1913	9.882 (0.571)	8.996 (0.503)	0.490 (0.197)
Guatemala	1980-2012	20.282 (0.058)	18.014 (0.049)	0.053 (0.872)
Japan	1885-1913	11.870 (0.408)	10.834 (0.333)	0.455 (0.094)
	1955-2013	9.846 (0.511)	9.240 (0.427)	0.141 (0.888)
Korea	1970-2014	18.407 (0.074)	16.909 (0.060)	0.175 (0.351)
Israel	1983-2014	154.166 (0.000)	154.098 (0.000)	0.137 (0.282)
Italy	1949-1996	15.767 (0.145)	12.474 (0.171)	0.457 (0.230)
Mexico	1985-2014	47.085 (0.000)	29.609 (0.007)	0.110 (0.312)
Netherlands	1950-1992	14.491 (0.211)	10.052 (0.349)	0.253 (0.381)
New Zealand	1934-2014	15.384 (0.155)	14.282 (0.093)	0.965 (0.175)
Norway	1946-2013	22.770 (0.021)	17.992 (0.031)	0.932 (0.084)
Portugal	1914-1965	26.827 (0.012)	25.749 (0.004)	0.086 (0.495)
	1966-1998	11.733 (0.422)	8.818 (0.511)	0.278 (0.004)
South Africa	1967-2014	17.877 (0.117)	16.635 (0.068)	0.489 (0.109)
Spain	1941-1989	14.260 (0.183)	13.569 (0.120)	0.272 (0.272)
Switzerland	1851-1906	15.883 (0.109)	12.625 (0.158)	0.635 (0.225)
	1948-2005	38.892 (0.000)	35.289 (0.000)	0.985 (0.033)
Turkey	1968-2014	6.817 (0.814)	4.614 (0.896)	0.164 (0.523)
United Kingdom	1922-2014	23.261 (0.019)	21.680 (0.011)	0.900 (0.046)
United States				
standard M_1	1915-2014	7.152 (0.767)	4.822 (0.870)	3.507 (0.007)
Lucas-Nicolini M_1	1915-2014	20.769 (0.038)	16.557 (0.048)	0.554 (0.121)
Venezuela	1962-1999	7.635 (0.724)	5.836 (0.776)	0.412 (0.112)

[a] Bootstrapped p-values (in parentheses) are based on 10,000 bootstrap replications.
[b] Null of no cointegration against alternative of 1 or more cointegration vectors.
[c] Null of 0 *versus* 1 cointegration vectors.

one extreme end of the spectrum within the full set of results. As we discuss in Online Appendix G.1, based on unrestricted three-variables systems, *it is almost impossible to reject the null of cointegration.*[36] For the reasons discussed in Section 6.2,[37] however, these results should be downplayed. As we stressed there, lack of rejection of the null of cointegration based on Shin's tests and our bootstrapping procedure does not represent strong evidence that cointegration is truly there[38]

Let us now turn to bivariate systems for velocity and the short rate.

8.1.2 Evidence from bivariate systems for velocity and the short rate

The log-log specification Based on the log-log specification, evidence of cointegration is uniformly very strong for all of the high-inflation countries, with the single exception of Bolivia, as well as for Belgium, Canada (1967-2013), Guatemala, New Zealand, Norway, Portugal, and Switzerland. In all of these cases, p-values for Johansen's tests are below 10%, and p-values for Shin's tests are above 10%.[39]

On the other hand, in four cases—Finland, Venezuela, West Germany, and Japan under the gold standard—the opposite is true, with Johansen's tests not rejecting the null of no cointegration and Shin's tests instead rejecting cointegration.

Finally, in almost all of the other cases, neither Johansen's nor Shin's tests reject the null, thus producing contradictory evidence (e.g., this is the case for both the United States and the United Kingdom). In this respect and based on our previous discussion in Sections 6 and 7, two things ought to be stressed: on the one hand, as we showed *via* Monte Carlo simulations, Johansen's procedure exhibits an overall better performance and produces more informative results. On the other hand, despite even bootstrapping critical and p-values as in CRT (2012), Johansen's tests still suffer, in small samples, from the problem highlighted by Engle and Granger (1987). The former issue suggests giving more weight to the results from Johansen's tests—pointing toward no cointegration—whereas the latter suggests that this result might well be the figment of a comparatively short sample period or a highly persistent cointegration residual (or both).[40] So, in the end, evidence for this group of countries

[36] Specifically, at the 10% level, we obtain *just four* rejections of the null out of 33 tests based on the semi-log specification, whereas based on the log-log specification with the 1% correction to the short rate, we obtain *only one* rejection. (For the Selden-Latané specification, it is not possible to consider unrestricted specifications.)

[37] See also Section E.3.2 in Appendix E.

[38] There is no need to remind the reader that using the asymptotic critical values reported in Shin's (1994) Table 1 is a nonstarter. On this, see the extended discussion in Appendix D.

[39] To be precise, for Portugal (1966-1998) the p-value for the maximum eigenvalue tests is 0.125. The overall picture for this country, however, clearly points toward cointegration. For Switzerland, the p-value for Shin's test for the period 1948-2005 is 0.064, but once again, overall evidence clearly points toward cointegration.

[40] For example, for the United States (based on the Lucas-Nicolini aggregate) $\hat{\rho}_{MUB}$ in Table LLCO.1 in the online appendix is equal to either 0.77 or 0.79, whereas the corresponding figures for the United Kingdom are 0.81 and 0.83.

is not clear-cut. Based on the discussion in Sections 6 and 7, on the respective sample lengths and on the estimated persistence of the CCRs reported in Table LLCO.1 in the online appendix, and based on the CCRs themselves—which in several cases appear quite clearly stationary (particularly for the United States and the United Kingdom)—our own reading of the overall evidence is that in many cases, it is *at the very least compatible* with the existence of a cointegration relationship between velocity and the short rate.

The Selden-Latané specification Turning to the Selden-Latané specification, evidence of cointegration is, once again, almost uniformly strong for high-inflation countries. As for other countries, it is strong for Canada, Guatemala, Korea, Norway, Portugal (1914-1965), and the United States based on Lucas and Nicolini's (2015) M_1 aggregate.

On the other hand, in three cases—Finland, Japan under the Gold Standard, and Portugal (1966-1998)—evidence clearly points toward no cointegration, with Johansen's tests not rejecting the null of no cointegration and Shin's tests instead rejecting cointegration.

In several other cases, neither Johansen's nor Shin's tests reject the null, thus producing contradictory evidence. This is the case, for example, for Colombia, Japan (1955-2013), the Netherlands, Spain, and Switzerland under the gold standard. For all of these countries, the same considerations we made in the previous subsection still apply, so that in these cases, the overall evidence is typically compatible with the presence of cointegration between velocity and the short rate. Symmetrically, for Norway, Switzerland (1948-2005), and the United Kingdom, both Johansen's and Shin's tests reject the null. For Switzerland and the United Kingdom, our own reading of the overall evidence (moreover, as discussed in Section 6, Johansen's procedure is more reliable than Shin's) suggests that it is compatible with cointegration between velocity and the short rate. As for Norway, things are less clear-cut: in particular, the CCR shown in Figure SELA.4 does not appear as manifestly stationary.

8.1.3 Unrestricted tests of the null of no cointegration

Turning to specifications in which we do not impose unitary income elasticity, Tables SL.4, LL.4, and LLCO.4 in the online appendix report results from Johansen's tests of no cointegration based on unrestricted specifications for (the logarithm of) the short rate and the logarithms of nominal GDP and M_1. As we discuss more extensively in Online Appendix G.3, based on the log-log specification with the 1% correction to the short rate, cointegration is detected based on both the trace and the maximum eigenvalue tests for Argentina, Brazil (1974-2012), Canada (1967-2013), Japan (1955-2013), Korea, Israel, the Netherlands, Norway, Portugal (1914-1965), and Switzerland (1948-2005), whereas the two tests produce opposite results for Bolivia, Germany (1876-2013), New Zealand, and South Africa.

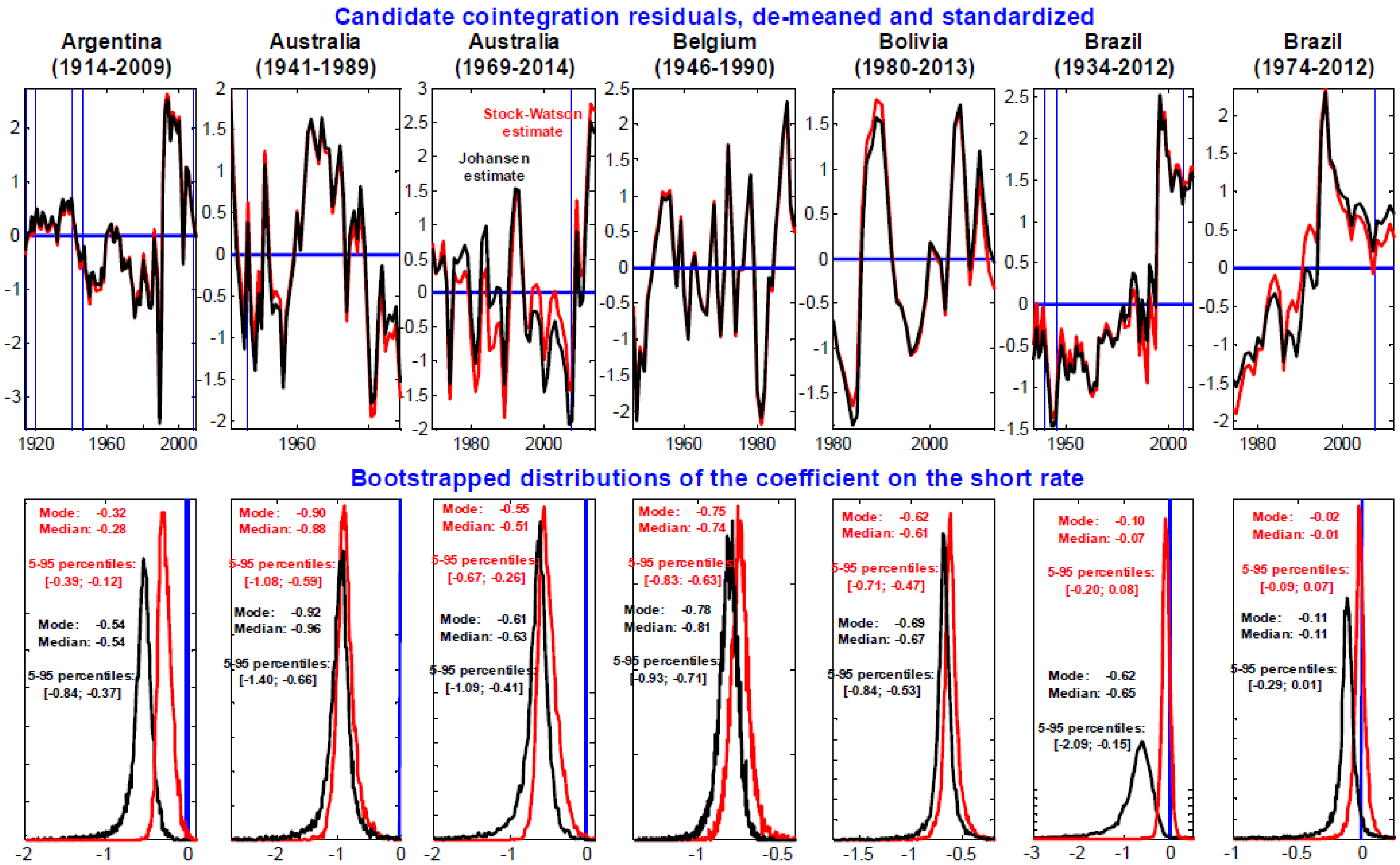

Figure 6 Log-log specification with the 1% correction to the short rate, imposing unitary income elasticity: cointegration residuals and bootstrapped distributions of the coefficients on the log of the short rate

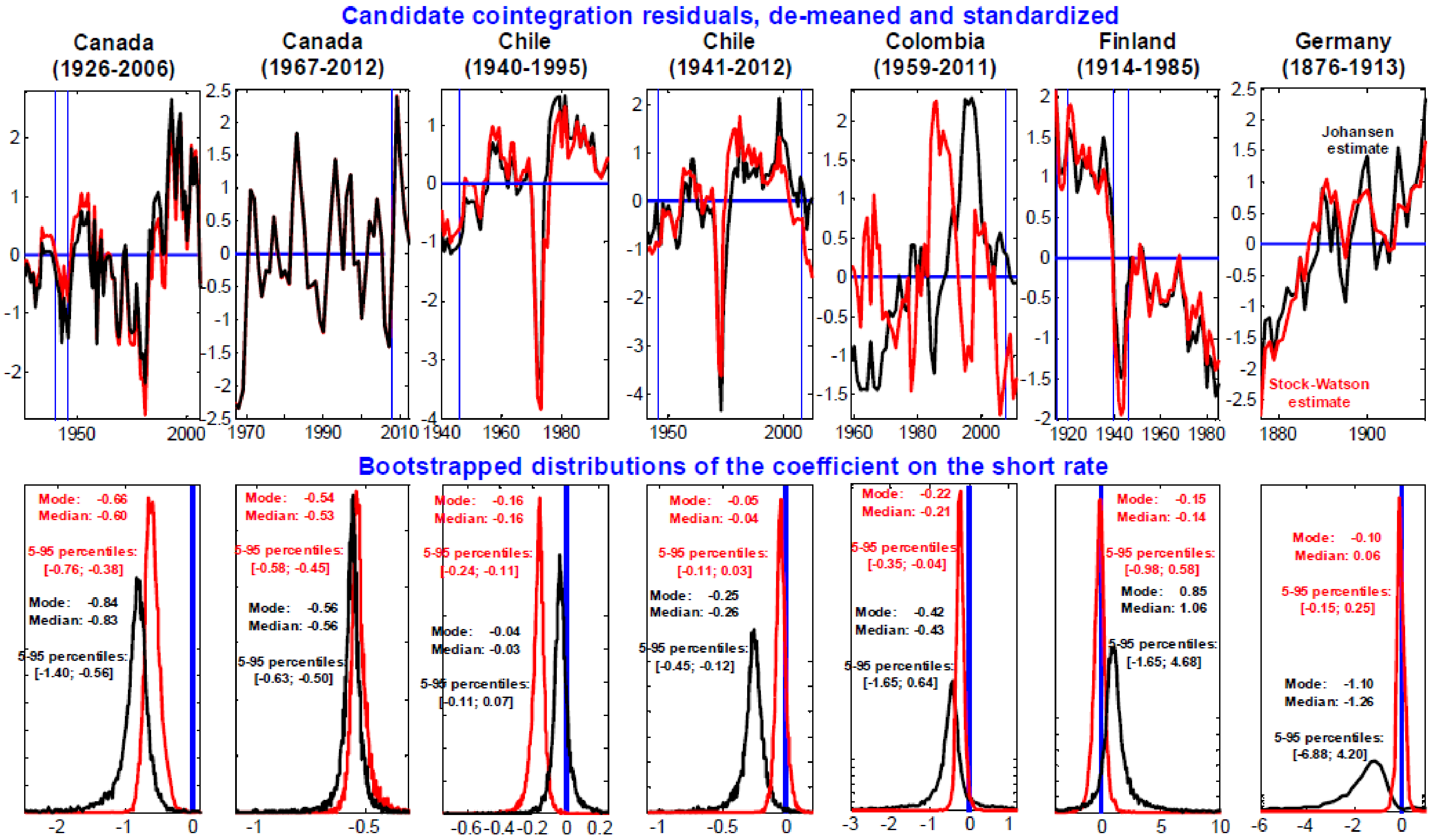

Figure 7 Log-log specification with the 1% correction to the short rate, imposing unitary income elasticity: cointegration residuals and bootstrapped distributions of the coefficients on the log of the short rate

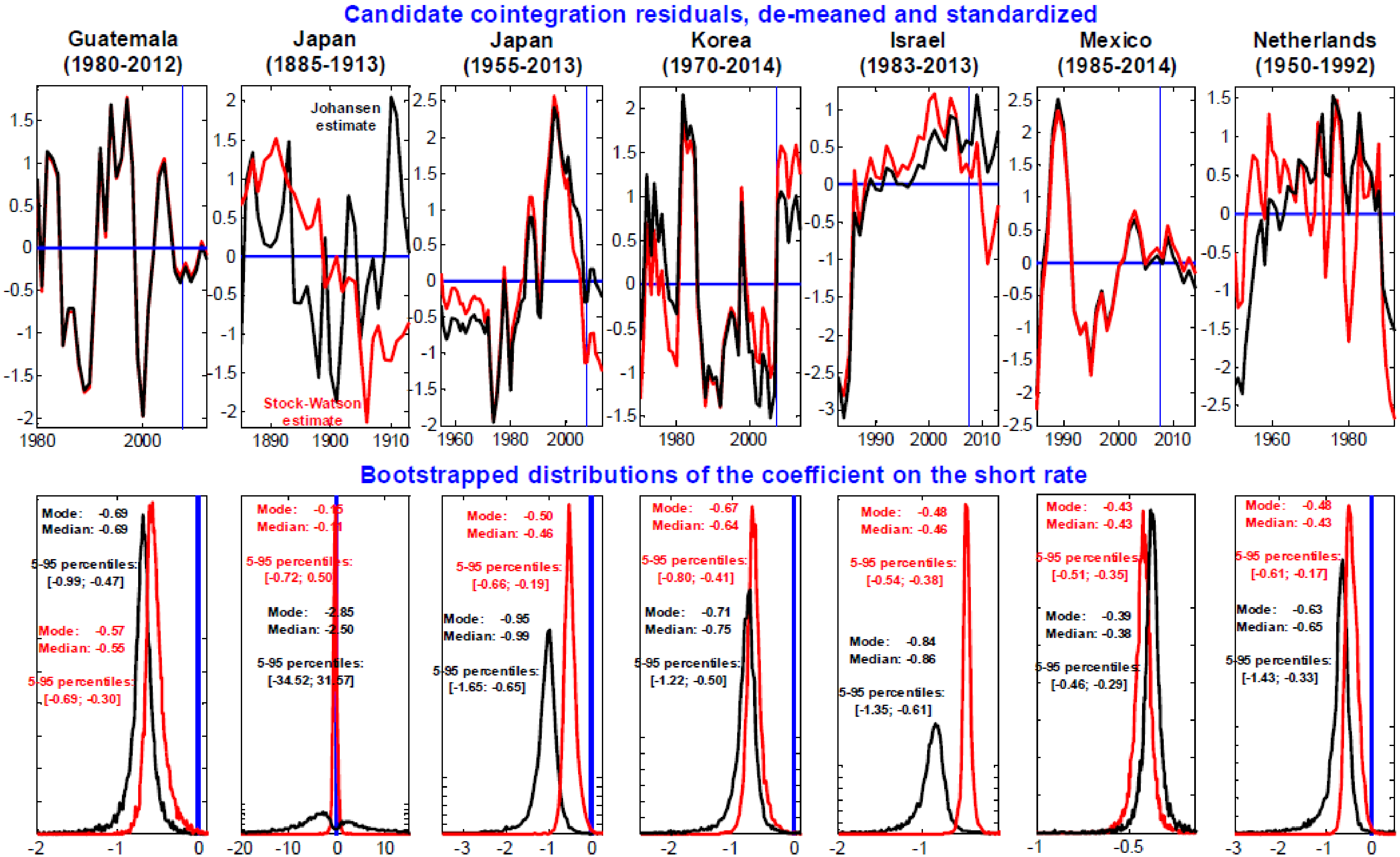

Figure 8 Log-log specification with the 1% correction to the short rate, imposing unitary income elasticity: cointegration residuals and bootstrapped distributions of the coefficients on the log of the short rate

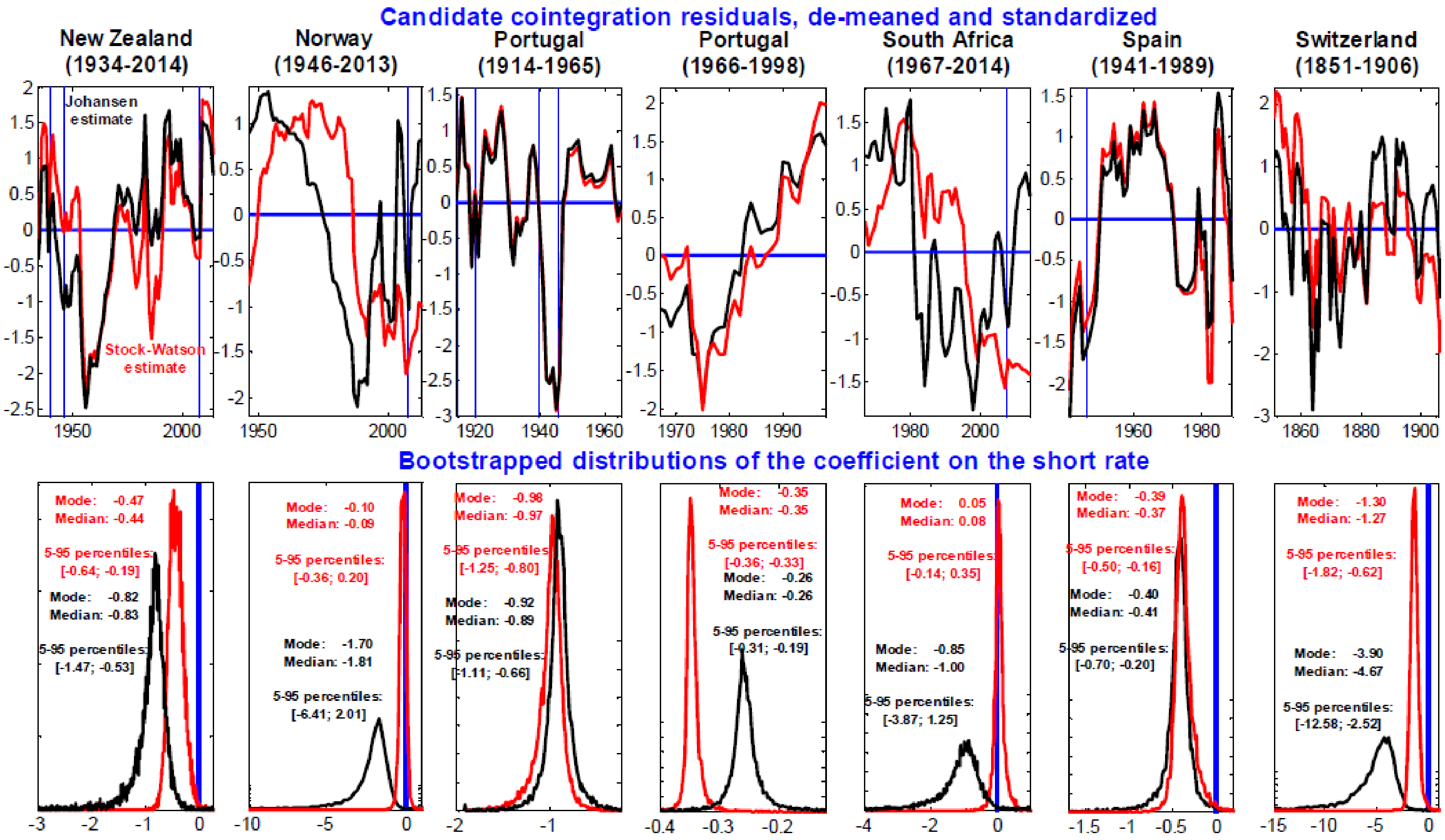

Figure 9 Log-log specification with the 1% correction to the short rate, imposing unitary income elasticity: cointegration residuals and bootstrapped distributions of the coefficients on the log of the short rate

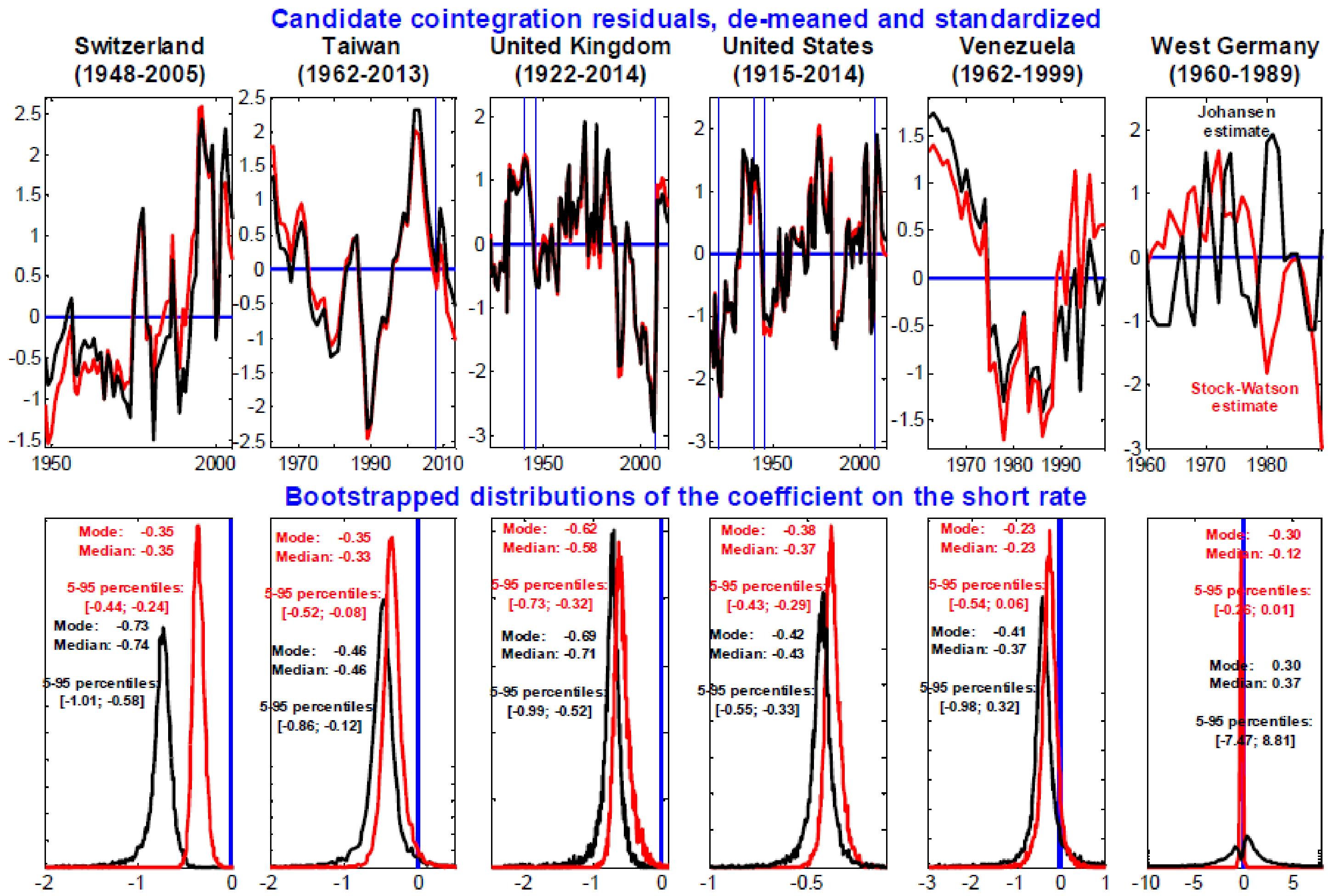

Figure 10 Log-log specification with the 1% correction to the short rate, imposing unitary income elasticity: cointegration residuals and bootstrapped distributions of the coefficients on the log of the short rate

8.2 The estimated coefficients on the short rate

We now turn our discussion to the bottom rows of Figures 6 to 10—showing the estimated interest rate elasticities, which according to the Baumol-Tobin specification ought to be equal to -1/2—and to Table LLCO.3 in the online appendix, reporting bootstrapped p-values for testing the null hypothesis that the elasticity should be equal to -1/2. Overall results are mixed, with the null being rejected in 17 cases out of 32 based on Johansen's estimator of the cointegration vector, and in 21 cases based on Stock and Watson's estimator.

The bottom rows of Figures SELA.1 to SELA.6 in the online appendix show *minus* the estimated coefficients on the short rate[41] based on the Selden-Latané specification, whereas Table SELA.3 reports bootstrapped p-values for testing the null hypothesis that the coefficients should be equal to -0.4. Since, different from Baumol-Tobin, theory does not provide us with a numerical benchmark that can be used in order to perform statistical tests, we have set such a benchmark to -0.4, which is roughly equal to the median or modal estimates we obtain for the United States based on the Lucas-Nicolini aggregate (see Figure SELA.6). (This is why Table SELA.3 does not report results for the United States based on the Lucas-Nicolini aggregate.) The null of -0.4 is rejected in 19 cases out of 33 based on Johansen's estimator of the cointegration vector, and in 25 cases based on Stock and Watson's.

8.3 Toward a unified framework?

Is there any way to learn from this exercise which specification—log-log or Selden-Latané—better fits the data? The econometric approach used herein does not nest the two specifications, and it therefore does not allow us to formally test which of them is better. Visual inspection, however, might favor the log-log specification. To see this, we compare the point estimates of the parameters of both specifications for two sets of countries. Both sets provide very good visual evidence, as reported in Section 3. For all countries in both sets, there is strong evidence of cointegration in at least one of the specifications. The first set comprises the United States, the United Kingdom, Australia, Canada, and New Zealand. All of these countries experienced important variations on their nominal interest rates, but they are low-inflation countries. The second group is composed of Brazil, Bolivia, Chile, and Israel, all high-inflation countries.[42] In the case of the log-log specification (see Figures 6 to 10) the estimates are very similar and are around 1/2, as the BT linear technology implies, for most countries. Sometimes they are a bit smaller (as for the United States, Chile, or Brazil) and sometimes a bit higher (as for the United Kingdom, New Zealand, or Israel),

[41] We report minus the coefficient on the short rate in order to make these results as comparable as possible to those based on the log-log and semi-log specifications.

[42] For Argentina, the Selden-Latané specification could not be estimated because, as explained in the text, the necessary conditions were not fulfilled.

depending on the specific details of the statistical procedures used, but overall, the sense that "one size fits all" dominates the estimates.

In contrast, when considering the Selden-Latané specification—see Figures SELA.1-SELA.6 in Online Appendix II—the first set of countries consistently delivers estimates between -0.5 and -0.4, whereas for some countries belonging to the second group, the coefficient on the short rate can get to values such as -0.009 for Brazil or -0.06 for Chile and is very precisely estimated.

Thus, if we are in search of a unified framework, the Baumol-Tobin specification should be the preferred one. On the other hand, when focusing on the experience of low-inflation countries, such as the United States, the United Kingdom, Canada, and Australia, the Selden-Latané specification appears as the preferred one.

Therefore, one possible interpretation is that the technology that relates the number of transactions to total costs exhibits a decreasing marginal cost for low values of interest rates, as implied by the Selden-Latane functional form, but that it eventually becomes constant, as implied by the Baumol-Tobin specification.

9 Conclusions

We use a simple model of a transaction demand for money to guide a thorough investigation of the stability of the long-run relationship between the ratio of money to output and a short-term nominal interest rate. Our data set comprises 32 countries for periods that range from 35 to 100 years. The log-log specification, which roughly corresponds to the linear cost function assumed by Baumol and Tobin, with an income elasticity of 1 and an interest rate elasticity close to 0.5, performs remarkably well for almost all the countries. It is also the case, however, that a specification in which the ratio of money to output is inversely related to a linear function of the short-term nominal interest rate is a better description of the data if we focus on countries in which the inflation rate has been in the low range, such as the United States or the United Kingdom. Overall, while there are a few countries for which the relationship cannot be detected, we find very strong evidence of a stable long-run money demand.

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