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On the Risk of Leaving the Euro*

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Abstract

Following the sovereign debt crisis of 2012, some southern European countries have debated proposals to leave the Euro. We evaluate this policy change in a standard monetary model with seigniorage financing of the deficit. The main novel feature is that we depart from rational expectations while maintaining full rationality of agents in a sense made very precise. Our first contribution is to show that small departures from rational expectations imply that inflation upon exit can be orders of magnitude higher than under rational expectations. Our second contribution is to provide a framework for policy analysis in models without rational expectations.

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1 Introduction

In this paper, we study the inflationary consequences of abandoning a currency union and replacing it with a national currency. The model we use is standard, with one exception: we adopt the approach of *internal rationality,* which allows for small departures from rational expectations (RE) while maintaining full rationality of agents' behavior.[1] Agents in the model have limited information about the environment they live in, so they cannot compute the equilibrium relationship between the exogenous forces in the model and inflation, the variable they care about. Being able to compute that equilibrium relationship is a necessary condition for agents to hold rational expectations. How agents construct probability distributions over future variables in such an environment is a main theme of this paper.

In following this approach, we must make assumptions about the system of beliefs regarding expected inflation that agents hold in equilibrium. A very large share of the paper is devoted to discussing and defending those assumptions. The proposed system of beliefs implies that agents rationally update their inflation forecasts by observing inflation behavior itself. In this environment, learning is self-referential: agents' expectations influence inflation, and inflation influences expectations. As shown by Marcet and Nicolini (2003) and Sargent, Williams, and Zha (2009), this feedback process can substantially amplify the effect of seigniorage on inflation rates.[2]

Following the world recession of 2008 and the European sovereign debt crisis of 2012, the proposal to leave the Euro and reintroduce a national currency has regained support in both academic and political circles, particularly in some southern European countries. Leaving the Euro is supported in Italy by the Five Star Movement and Lega Nord, which jointly won a majority of the Parliament in the May 2018 elections. In France, Marine Le Pen, leader of the nationalist party Front National and the strongest supporter of "Frexit,"got a fifth of the vote in the first round of voting in 2017 and a third of the vote in the second. In Greece, the radical-left party Syriza won the January 2015 elections with the promise to bargain favorable bailout conditions with Europe and, if this was not possible, to leave the Euro.

Leaving the Euro would presumably bring about some benefits for these southern European economies: individual countries would be free from the fiscal chains of the European Union (EU), putting an end to austerity. In addition, relaunching national currencies would allow central banks to stimulate the economy. In contrast, remaining in the euro area would amount

[1]See Adam, Marcet, and Nicolini (2016) and Adam, Marcet, and Beutel (2017) for applications to stock market volatility, and Adam and Marcet (2011) for a discussion of some theoretical issues.

[2]Marcet and Nicolini (2003) show that such a departure from rational expectations explains key facts related to deficits, seigniorage, and inflation during the South American hyperinflations of the 1980s. Sargent et al. (2009) estimate the model using data from those same episodes.

to embracing the austerity required by some European rules and the low inflation promoted by the European Central Bank (ECB). Bernard Monot, Le Pen's economic consultant, put it this way: "Give us the Banque de France and the Finance Ministry, then France would be out of trouble in three weeks."Alessandro Di Battista from the Five Star Movement said, "We are convinced that, if we are able to take back monetary sovereignty we can raise Italy from the rubble."

An exit from the Euro could be accomplished in many different ways. We build our model around a specific exit strategy and briefly discuss other alternatives. As a summary, the exit we consider has three elements: *i)* no additional austerity will happen after exiting, and hence the deficit process will remain unchanged; *ii)* the country regains its ability to raise seigniorage; *iii)* as a consequence of exiting, the ability to access foreign markets for debt will be severely limited. We discuss these assumptions in more detail later on in the text.

Our first contribution is to show that the interaction between persistent deficits and agents that form expectations based on past observations dramatically amplifies the equilibrium inflation rates generated by the model. In particular, a small deviation from RE implies inflation rates that can be several times higher than with RE. This paper quantifies the risk of hyperinflation that may follow a departure from the euro system — a risk that has been overlooked in the recent debate. We compute the welfare implications of these events and show that they are substantial. We also show that exchange rate policies can substantially ameliorate these welfare consequences. These policies will require external funding and, therefore, an orderly exit. An example would be an exit with an agreement with the International Monetary Fund. We compute how large these funding requirements ought to be as a function of policy parameters. Our framework can therefore be used to compute the gains of a negotiated exit that includes access to limited funding in case inflation becomes too high.

Hyperinflations emerge because, due to learning, expectations have a region of instability: if expectations land in this region, inflation can only grow. Expected inflation then grows as a response to this higher inflation, leading to higher inflation and so on. The result is that hyperinflations are likely to emerge for a very persistent deficit process, as is observed in the data. Only in cases in which the deficit is by chance persistently low after exiting are hyperinflations able to be avoided.

Our second contribution is methodological, as we take a step forward in exploring the use of internal rationality (IR) for policy analysis. Departures from RE are still controversial, especially when they are used for policy analysis. Possibly the main reason that RE became the dominant paradigm in modeling expectations is that it allowed the analysis of policy reforms in a consistent manner as it addressed the Lucas critique. We claim that by explicitly modeling agents' expectations, we can gain a better understanding of policy interventions.

Under IR, agents are assumed to hold a coherent belief system about inflation, even if this distribution of inflation may not be the true one under the model considered. Therefore, a complete description of the environment requires an explicit assumption about the agents' belief system regarding inflation. Agents understand that the evolution of inflation depends on certain aggregate shocks, since they know that given their beliefs and those of other agents, the model implies a certain mapping between these aggregate shocks and inflation. As we will show in detail, however, given their available information, agents are unable to compute this mapping.[3]

We are certainly not the first to study policy analysis in models where agents have imperfect knowledge of the model. The paper innovates along two dimensions. The first one is the systematic use of IR to explore how expectations may behave after a policy change and how the feedback between inflation and inflation expectations shapes the equilibrium dynamics of the model. The second is to use the model with IR to compute the welfare effect of alternative policies using methods that are standard in the realm of rational expectations models.

First and foremost, we study the positive implications of the model for the evolution of equilibrium inflation. We show that small departures from rational expectations — in a sense made very precise in the paper — imply that the resulting equilibrium inflation rates can reach values that are orders of magnitude higher than in the model with rational expectations. This result implies that the policy implications of the rational expectations version of the model are not robust to small deviations from the expectations hypothesis. We then go on to evaluate policies that use crawling pegs on exchange rates as a way to temporarily reduce inflation rates. We parameterize those policies and compute their impact on welfare. We find that early interventions that use shock-type policies are preferred over policies that delay the intervention or that follow a gradual approach in reducing inflation.

The model we use is very similar to the one developed in **??** and estimated in Sargent et al. (2009). In terms of the positive analysis, our paper offers two innovations relative to those papers. First, we allow for a serially correlated deficit, which, as we show, is quantitatively very important. Second, and more importantly, here we adopt the approach of internal rationality: we show that a set of statistical tests are necessary and sufficient for agents not to reject their beliefs. As we show, the data in equilibrium imply that the test would not be rejected by the agents in equilibrium, even for sample sizes of 15 years. The most important innovation,

[3] A large number of papers assume that agents learn about the exogenous processes of the economy. These papers make the implicit assumption that agents are able to figure out the pricing function that maps fundamentals onto equilibrium prices; thus we refer to these models as "Bayesian/RE learning" models. This is a consistent assumption to make, but it still requires that agents know a lot from the outset about how to predict inflation. We argue in Section 2 of the paper is rational behavior does not imply knowledge of this pricing function.

however, is the policy evaluation exercises that we perform in this paper.

The use of the IR framework is certainly not standard, and it raises several methodological issues that need to be dealt with, particularly in evaluating policy. To do so, in Section 2 we discuss a monetary model with heterogeneous agents and incomplete markets. We show that, in following our approach, it is perfectly consistent to have fully rational agents that do not know the pricing function for inflation. We then go on to discuss how to choose a reasonable belief system in Section 3, where we introduce seigniorage financing and study learning equilibria. In Section 4 we assess the quantitative performance of our model and show that the presence of learning translates into recurrent hyperinflationary episodes. In Section 5 we derive testable implications of the belief system and test whether agents can reject their beliefs based on data generated by the model. In Section 6 we present the policy evaluation exercises.

2 A Model with Heterogeneous Agents

The framework of our model is represented by two equations: a government budget constraint and a money demand. In what follows, we take the simple approach of deriving money demand from an overlapping generations model. However, none of the details of this structure is of particular relevance. As will become clear, the results of the paper carry through with any structure that delivers a demand for real balances that is decreasing on the expected rate of inflation. Indeed, it is possible to derive that equation from a model with long-lived agents, and we will perform our derivations below with this extension in mind.

In this section, we consider heterogeneous agents to highlight that an individual agent would not be able to infer the pricing function from observations and her own behavior. For simplicity, however, the policy analysis, which is the core of the paper, will be done with a homogeneous agent model.

Consider a constant cohort size, overlapping generations model in which each agent lives for two periods. Agents are heterogeneous in their endowments and their preferences. The endowments of agent $j \in [0, 1]$ born at time t are normalized to 1 when young, common to all agents, and e_t^j when old, and her preferences are given by

$$\ln c_t + \alpha_t^j \ln x_{t+1}.$$

Thus, agents are heterogeneous in their endowment when old e_t^j and their discount factor α^j. The values of the pair $\{e_t^j, \alpha_t^j\}$ are drawn from some exogenously specified, possibly time-varying distribution at the time each agent is born. In solving their optimal problem, agents know their own values of $\{e_t^j, \alpha_t^j\}$.

We restrict the endowment when old to be smaller than the endowment when young ($e_t^j < 1$ for all j). We assume that agents have a relative preference for consumption when old ($\alpha_t^j \geq 1$ for all j). These assumptions are made to ensure that as long as the return on savings is not too low, young agents would save in equilibrium.

Markets are incomplete in the sense that the only asset agents can hold is fiat money. Thus, at any point in time, there is only one spot market in which agents can exchange goods for money, at a price P_t. When young, agents choose how many units of money to hold for next period, given the price level that prevails at time t. The budget constraint when young is given by[4]

$$P_t c_t^j + M_t^j \leq P_t. \tag{1}$$

In the following period, agents consume their endowment plus whatever they can buy with the money previously held, so their budget constraint when old is

$$P_{t+1} x_{t+1}^j \leq M_t^j + e_t^j P_{t+1} \tag{2}$$

for all P_{t+1}.

Agents' expectations are possibly heterogeneous as well, hence, the problem of agent of type j born in period t consists of maximizing

$$\mathbf{E}_t^j[\ln c_t + \alpha_t^j \ln x_{t+1}] \tag{3}$$

by choosing consumption and money holdings, subject to the budget constraints (1) and (2).

Agents are assumed to observe at t the values of variables dated t as well as e_t^j. However, agents do not know the value of next-period price level. Hence, the expectation is taken with respect to the price level P_{t+1}, which, owing to the presence of aggregate uncertainty, agents can not infer from their observed endowment (more on this later).

Since the budget constraints will hold with equality, once we substitute them in (3), an interior solution requires

$$\frac{1}{P_t - M_t^j} = \mathbf{E}_t^j \left[\frac{\alpha_t^j}{M_t^j + e_t^j P_{t+1}} \right],$$

which implicitly defines the individual money demand equation for agent j. Importantly, money demand must be measurable with respect to the information set available when young. Since the only source of uncertainty, namely, P_{t+1}, appears in the denominator on the right-hand

[4]As agents cannot issue money, the constraint $M_t^j \geq 0$ must be imposed. However, the assumption that the endowment in the second period is smaller than in the first period implies that this constraint will not be binding as long as the inflation rate is not too high; thus we ignore this constraint in our theoretical analysis. In the numerical section, we impose this constraint on the equilibrium.

side, we cannot solve for the money demand equation in closed form. To make progress, we study the linearized version of the eqauation, which can be written as[5]

$$\frac{M_t^j}{P_t} = \phi_t^j \left(1 - \gamma_t^j \mathbf{E}_t^j \frac{P_{t+1}}{P_t}\right),$$

which corresponds to the money demand by each agent of generation t, where

$$\phi_t^j = \frac{\alpha_t^j}{1+\alpha_t^j} \quad \text{and} \quad \gamma_t^j = \frac{e_t^j}{\alpha_t^j}.$$

By properly aggregating these money demand functions, it is possible for the theorist to compute an aggregate money demand, which is the first building block of the model.

In analyzing the policy decision in the subsequent section, we will consider a simple version of the government budget constraint that relates the evolution of the money supply to the behavior of seigniorage. Such a model will therefore jointly determine the evolution of the money supply and price level as functions of the exogenously given evolution of seigniorage. The endogenous nature of the money supply is an essential component of the policy analysis, as will become very clear in the quantitative section below.

For the discussion in this section, however, we can avoid that additional complication and take the evolution of the money supply as exogenously given. Thus, for simplicity, and only for the discussion in this section, we proceed in that fashion and let the sequence $\{M_t\}_{t=0}^{\infty}$ as exogenously given by policy.

We have been working on various applications of internal rationality for a few years now. In discussing our work, both in seminars and during the editorial process, we have found that a number of researchers in economics hold the view that a rational agent who knows the process for exogenous fundamentals of an asset cannot separately hold a view about the prices of that asset. Such "IR skeptics" sustain that the whole structure of IR is logically inconsistent: rational agents should be able to map their view of asset fundamentals onto the value of an asset price.

In the context of this model, we can formalize this view as follows. Consider the following assumption.

Assumption 1 *All agents know the evolution of* $\{M_t\}_{t=0}^{\infty}$.

An IR skeptic would claim that under Assumption 1, a rational agent should be able to infer the pricing function that maps realizations of M^s onto a price level. The rest of the subsection

[5] The linearization is standard and is offered for completeness in Appendix A.

states that this argument is flawed for a variety of reasons. Therefore, we will conclude that IR is logically consistent.

An IR skeptic would likely articulate his thoughts using a homogeneous agent version of the model, where $\alpha_t^j = \alpha$ and $e_t^j = e$. In this case, the above money demand equation is as follows

$$M_t = \phi \left(P_t - \gamma \mathbf{E}_t P_{t+1} \right). \tag{4}$$

Since knowledge of this equation is a consequence of rational behavior, it must be that IR agents know this equation. From this, it follows that the price level (in a nonbubble solution) satisfies

$$P_t = \frac{1}{\phi} \sum_{s=0}^{\infty} \gamma^s \mathbf{E}_t M_{t+s}^s. \tag{5}$$

Therefore, knowledge of the aggregate money supply M^s, plus maximizing behavior by agents, indeed determines the price level, and, according to an IR skeptic, it is then logically inconsistent to assume (as we will do later) that agents hold separate expectations about the price level.

This logic breaks down, however, once we have heterogeneous agents. In this case, the only discounted sum an agent can obtain from knowledge of optimizing behavior is

$$P_t = \frac{1}{\phi_t^j} \sum_{s=0}^{\infty} \left(\gamma_t^j \right)^s \mathbf{E}_t^j M_{t+s}^j. \tag{6}$$

The key difference is that the money demand in this expression is M^j, with a super-index corresponding to the agent j, not the exogenous supply for money as in (5). In other words, the agent does know that his own optimal decision maps *his* future demands for money onto the price level, but optimal behavior does not relate future exogenous values of M^s to price behavior. Hence, there is no contradiction in knowing the behavior of M^s and having a separate belief system for the price level; the first does not map onto the second. The optimality condition (6) that agent j knows to hold in an IR equilibrium in no way restricts what agents think about the link between M^s and P.

Since agents with different types will now face a different inference problem, the computation on the right hand side of (6) becomes a much more complicated task. In particular, it requires that each agent know the inference problem solved by all other agents in the economy so that agent i can figure out $\mathbf{E}_t^j$ for all $j \neq i$. Even if we endow each agent with knowledge of the distribution of types of all other agents in the economy, it is apparent that discovering the mapping from exogenous variables onto prices becomes a much more challenging problem.

But an IR skeptic could bring to the table the following claim: "A rational agent could use his rational behavior to infer the relationship between the aggregate money demand and the price level, and , in this way, to infer how M^s and P are related." If we add some slight knowledge about how other agents behave, individual optimization and knowledge of exogenous variables map onto a price level.

Let us see how this could work. In the above model, aggregate money demand is

$$M_t = \int_0^1 \phi^j \left(P_t - \gamma^j \mathbf{E}_t^j P_{t+1}\right) dj. \tag{7}$$

So, in addition to knowing how to solve his maximization problem (i.e., in addition to being IR), we make the following assumption.

Assumption 2 *All agents know that other agents have a utility function similar to their own, up to diversity in γ^j, ϕ^j, E^j. Furthermore, agents know that $\overline{\phi} = \int_0^1 \phi^j dj$.*

Under Assumption 2, an IR agent could obtain

$$P_t = \int_0^1 \frac{\phi^j \gamma^j}{\overline{\phi}} \mathbf{E}_t^j P_{t+1} dj + \frac{M_t^s}{\overline{\phi}}. \tag{8}$$

Is this enough to map M^s onto P?. The answer is no. All our IR agent could do is plug the optimality condition (6) into (8) to obtain

$$P_t = \int_0^1 \frac{1}{\overline{\phi}} \mathbf{E}_t^j \sum_{s=0}^{\infty} \left(\gamma^j\right)^{s+1} M_{t+1+s}^j dj + \frac{M_t^s}{\overline{\phi}}. \tag{9}$$

Now he needs to know, in addition, $\int_0^1 \mathbf{E}_t^j \left(\gamma^{j+1}\right)^s M_{t+1+s}^j dj$ for all t, s, and these quantities cannot be inferred from the knowledge given under Assumption 2.

Let us see under what assumptions the IR skeptic would be right. Consider the following.

Assumption 3 *Agents have the same system of beliefs; therefore, they have homogeneous (although possibly non-RE) expectations $E^j = E^{\mathcal{P}}$.*

Notice that under Assumptions 1 and 2, agents can figure out that

$$M_t^j = \phi^j \left(P_t - \gamma^j \mathbf{E}_t^{\mathcal{P}} P_{t+1}\right), \tag{10}$$

so that

$$\begin{aligned}
\int_0^1 \mathbf{E}_t^j \left(\gamma^j\right)^{s+1} M_{t+1+s}^j dj &= \int_0^1 \mathbf{E}_t^{\mathcal{P}} \left(\gamma^j\right)^{s+1} \phi^j \left(P_{t+s+1} - \gamma^j P_{t+s+2}\right) dj \\
&= \int_0^1 \left(\gamma^j\right)^{s+1} \phi^j \left(\mathbf{E}_t^{\mathcal{P}} P_{t+s+1} - \gamma^j \mathbf{E}_t^{\mathcal{P}} P_{t+s+2}\right) dj \\
&= \int_0^1 \left(\gamma^j\right)^{s+1} \phi^j dj \ \mathbf{E}_t^{\mathcal{P}} P_{t+s+1} - \int_0^1 \left(\gamma^j\right)^{s+2} \phi^j dj \ \mathbf{E}_t^{\mathcal{P}} P_{t+s+2},
\end{aligned}$$

for all j. But Assumptions 1-3 still do not allow for the computation of this quantity. We would additionally need to assume the following.

Assumption 4 *Agents know the whole joint distribution of* $\{\gamma, \phi\}$*.*

Assumption 4 allows agents to compute the integrals $\int_0^1 \left(\gamma^j\right)^{s+1} \phi^j dj$ and $\int_0^1 \left(\gamma^j\right)^{s+2} \phi^j dj$ in the last equation above. With this knowledge, it is indeed possible to map future values of M^s onto a price level today.[6]

In other words, it is logically consistent to assume that agents are rational and have price beliefs that do not map M^s onto P, as we do under internal rationality. All we need to assume is that agents do not know the distribution of other agents' endowments and utilities, that their beliefs are diverse, or both.

Furthermore, in this paper we consider a model where the money supply is not exogenous but is determined by the price level. Therefore, just because agents think that inflation will be different, they will have different beliefs about the money supply. This means that even Assumption 1 is not reasonable in our model: in the event of a drastic policy change, such as the one we consider in the paper, and if government deficits are going to be monetized, how could agents know from the outset the behavior of the money supply in the future given their price beliefs?

The previous discussion shows how in our model (and arguably in many models) the assumption of RE is logically unrelated to the assumption of optimal agents' behavior. Under the assumptions of incomplete markets and heterogeneous agents, it is simply impossible for consumers to compute the RE equilibrium using only their (incomplete) knowledge of the economy.[7] Therefore, agents are still making savings decisions and filtering information optimally given their beliefs about inflation. This argument, while very compelling, only justifies considering hypotheses for expectations formation that are not necessarily model consistent, as

[6]Technically, only knowledge of the covariance between e_t^j and α_t^j is required. But this is just an artifact of the linearization of the solution. In general, knowledge of the whole joint distributions of e_t^j and α_t^j is required.

[7]Adam and Marcet (2011) discuss a related issue in the context of stock markets.

RE imposes. But it does not offer any guidance on how to proceed and thereby raises several methodological issues. Here, we list three of these issues and explain how we approach them:

i) *How should expectations be modeled?*

Having accepted that the model environment does not determine agents' expectations, an explicit *assumption* in the agents' belief system about inflation is needed to fully describe the environment. Thus we treat agents' belief system the same way that utility functions, production functions, or the equilibrium concept are treated in the literature. Being explicit about this modeling choice regarding the agents' system of beliefs has some advantages. First, it highlights that RE is just one assumption about agents' beliefs from among many others. Second, it clarifies that this is the only deviation from the standard paradigm now dominant in macroeconomics; agents in our paper are completely rational given this system of beliefs.[8] Third, we can ask questions about the reasonableness of this assumption vis-à-vis the data, observations on agents' expectations, and the model itself.

ii) *What is a reasonable assumption about agents' model for inflation?*

As with any assumption, its usefulness should be judged according to its theoretical and empirical virtues. We start by assuming that the process that governs agents' beliefs is the same as the true evolution of inflation under a linearized rational expectations model. However, agents are unsure about one parameter in the formulation.[9] The specific assumption we make is that parameter, which governs inflation, is given by a mixture of a transitory and a permanent component. This has the advantage that it coincides with RE beliefs for certain parameter values, so it allows us to study the robustness of the predictions about leaving the Euro to small deviations from RE. In addition, various papers have shown that survey inflation expectations are well described by a system of beliefs similar to the one we use. More importantly, we perform a series of tests showing that for period lengths of between 10 and 15 years, agents in the model would find it hard to reject the hypothesis that their system of beliefs is the correct one under the model-generated data. In this sense, this system of beliefs is a reasonable one for agents to maintain after exiting.

[8]The literature on adaptive learning, as in, for example, Evans and Honkapohja (2001) and Marcet and Sargent (1989a), is unclear about the extent to which agents' expectations are compatible with agents' optimal behavior. That literature tends to emphasize that agents are rational in the limit if the economy converges to RE. IR clarifies this distinction: agents optimize in all periods given their system of beliefs about inflation, we make an explicit assumption about this system of beliefs, and this system is not equal to the behavior of inflation in the model.

[9]To know the value of that parameter, agents would need to be able to compute the equilibrium mapping. As discussed above, they cannot do this unless they possess all of the required information.

iii) *How do agents' beliefs about inflation change following a policy reform?*

RE ties agents' beliefs to model outcomes in a very specific way: it assumes zero distance between perceived and actual distributions. It seems unlikely that this would be the case immediately after a large change in policy, such as the one we analyze in this paper. Thus, we assume that after a policy change, the belief system is reset but in such a way that it would be difficult for agents to reject their beliefs upon observing the model equilibrium that their beliefs generate. Specifically, we allow agents to reset their prior after exiting, expressing larger uncertainty about the underlying level of inflation, and we discipline that prior so as to be consistent with the rational expectations outcome.

3 Introducing Seigniorage Financing

We now describe the complete model that we use for the policy evaluation exercises. As mentioned above, in the rest of the paper we shut down the heterogeneity considered previously and assume $\alpha^j = \alpha$, $e^j = e$. In doing so, we highlight the fact that it is the use of IR agents that matters and not heterogeneity per se, and switch the focus of the analysis to the way aggregate inflation expectations are formed. Thus, an aggregate money demand equation such as (4) will be one of the equilibrium conditions.

In contrast to the analysis of the previous section, where we took the money supply as an exogenous process, we now introduce seigniorage financing. On purpose, we innovate as little as possible on the front of model building so as to focus on the effects of a key policy decision: leaving the Euro or not. For this we adapt the model of Marcet and Nicolini (2003) and assume that increases in the money supply must be equivalent to the fraction of the deficit that the government monetizes. This choice seems reasonable since this model was shown to perform well in explaining the dynamics of chronic inflations and the burst in hyperinflations, as seen in the data. In addition, it provides policy recommendations that are in line with the standard view for the right policy in ending hyperinflations, and it is a model in which inflation expectations play a key role.

In a currency union like the Euro, that choice of seigniorage is made by the ECB, so the inflation rate in a member country is pinned down by European monetary policy.[10] Once a country leaves the euro area, however, seigniorage will be equal to the fraction of the deficit that the country cannot fund in the bond markets and must therefore monetize. The model we study is not aimed at explaining inflation while the country is a member of the euro area; rather, it is developed to simulate what equilibrium inflation rates could be if the country decides to

[10] Plus real shocks that may change relative prices in the member country.

leave the Euro. Once a country leaves the euro area, its budget constraint becomes

$$(M_t - M_{t-1}) + (B_t - B_{t-1}) = B_{t-1}r_{t-1} + (g_t - \tau_t)\, P_t,$$

where the right-hand side measures the primary deficit $(g_t - \tau_t)\, P_t$, plus the interest on the debt, $r_{t-1}B_{t-1}$, and the left-hand side shows the two ways to finance it: issuing debt or printing money. Thus, the relationship between the total deficit and money issuing depends on the ability of the government to issue bonds after leaving the euro area.

Because of large debt-to-output ratios and the difficulties that the southern European countries faced in trying to float government bonds during the debt crisis of the summer of 2011 until the ECB vigorously acted, we will consider two possible regimes. On the one hand, we will assume that after leaving the Euro, any of these countries would lose access to the ECB and would find it difficult to issue bonds. Therefore, the term

$$d_t P_t \equiv B_{t-1}r_{t-1} + (g_t - \tau_t)\, P_t - (B_t - B_{t-1}) > 0,$$

which means that there will be a need to raise seigniorage. But we will also allow the country to obtain special short-run funding — with an IMF agreement, for example — that could be used by the country in the short run to stabilize the economy in case the inflation rate accelerates. We will explain in detail why this policy may be a reasonable policy and how it could be implemented. But once this possibility is considered, the budget constraint becomes

$$(M_t - M_{t-1}) = d_t P_t - s_t F_t, \tag{11}$$

where F_t is the short-run financing provided by the IMF and $s_t \in \{0, 1\}$ indicates whether the country has an agreement with the IMF ($s_t = 1$) or not ($s_t = 0$).

3.1 Equilibrium Conditions

As our treatment of expectations is nonstandard, we now carefully describe the maximization problem faced by the internally rational representative agent. For simplicity, we describe the problem assuming that $s_t = 0$ for all t.[11] Thus, the only source of uncertainty for the economy is the realization of the seigniorage, d_t. Preferences are given by

$$\ln c_t + \alpha \mathbf{E}_t^{\mathcal{P}} \ln x_{t+1}.$$

[11] It is straightforward to extend the notation to the case in which s_t is also a state variable.

The budget constraint when young is given by

$$c_t + \frac{M_t}{P_t} \leq 1, \tag{12}$$

and the budget constraint when old is

$$x_{t+1} - e \leq \frac{M_t}{P_{t+1}}, \tag{13a}$$

for all P_{t+1}. The notation follows the one described in Section 2. Replacing the budget constraints on the preferences, the problem of the representative IR agent consists of maximizing

$$\ln(1 - \frac{M_t}{P_t}) + \alpha \mathbf{E}_t^{\mathcal{P}} \ln(e + \frac{M_t}{P_{t+1}}) \tag{14}$$

by choosing money holdings M_t, given a probability distribution over P_{t+1}. Expressed in this way, the relevant state space for the agent is given by P_t and d_t. And, from the agent's point of view, the sources of uncertainty are given by the pair $\{P_{t+1},\, d_{t+1}\}$. Notice that the agent's problem is not directly affected by the realization of d_{t+1}. Nonetheless, the agent may care about it to the extent that it can provide information about P_t.

At this level of generality, the only requirement for solving the agent's problem is a description of the joint probability distribution of $\{P_{t+1},\, d_{t+1}\}$ conditional on the pair $\{P_t,\, d_t\}$. It is this distribution that is used to compute $\mathbf{E}_t^{\mathcal{P}}$ in problem (14).

Computing a rational expectations equilibrium amounts to imposing a singularity in the joint distribution of $\{P_{t+1},\, d_{t+1}\}$, such that $P_{t+1} = P(d_{t+1})$, where $P_{t+1} = P(d_{t+1})$ is an equilibrium of the model if agents use the true distribution of d_{t+1} and the function $P(d_{t+1})$ in order to compute the distribution they use in calculating $\mathbf{E}_t^{\mathcal{P}} \ln(e + \frac{M_t}{P_{t+1}})$. In this case, the operator $\mathbf{E}_t^{\mathcal{P}}$ coincides with the expected value $\mathbf{E}_t$.

Because of the arguments laid out in Section 2, we will evaluate policies assuming that agents do not have enough information to compute the function $P_{t+1} = P(d_{t+1})$. Thus, we will need to be specific with respect to the joint distribution of $\{P_{t+1},\, d_{t+1}\}$ that agents use to compute the $\mathbf{E}_t^{\mathcal{P}}$ in problem (14).

The model is given by three equations: the money demand equation, the government budget constraint, and the law of motion for the level of seigniorage. The demand for real balances that arises from the log-linearization of the solution of (14) can be written as

$$\frac{M_t^d}{P_t} = \phi\left(1 - \gamma \pi_{t+1}^e\right), \tag{15}$$

where $\pi^e_{t+1} = \mathbb{E}^{\mathcal{P}}_t \left[\frac{P_{t+1}}{P_t}\right]$ denotes the expected gross inflation rate.

In what follows, we first analyze the behavior of the model when $s_t = 0$. The government budget constraint is then given by

$$M^s_t = M^s_{t-1} + d_t P_t, \tag{16}$$

where d_t is the level of seigniorage and evolves according to

$$d_t = (1-\rho)\delta + \rho d_{t-1} + \epsilon_t, \tag{17}$$

where ϵ_t denotes an *i.i.d.* perturbation term.

As mentioned in the introduction, this formulation is supposed to capture the feature that, upon abandoning a currency union, a country is very limited in its ability to issue new net debt, it does not default, it keeps its primary deficit as it was before exiting, and thus it must resort to inflationary financing through printing money.

An alternative would be to assume that after leaving the Euro, the country engages in serious austerity measures that radically change the trajectory of the primary deficit $(g_t - \tau_t)$. If this were the case, there would be no need to raise revenues by printing money and the analysis in the paper would not be relevant.

As the proposals to leave the Euro many times come from sectors that opposed the austerity measures imposed by Europe, we believe it is relevant to study what the consequences of leaving the Euro would be without self-imposing domestic austerity. We further discuss this issue when calibrating the model.

Equation (17) generalizes Marcet and Nicolini (2003) in that it introduces serial correlation of seigniorage, $\rho \neq 0$. This feature is very important in studying an exit from the Euro, as deficits are in fact highly serially correlated in the data, and, as we show below, a proper calibration of this process is crucial for the results.

We can combine the money demand equation (15) and the government budget constraint (16) to obtain

$$\pi_t = \frac{\phi - \phi\gamma\pi^e_t}{\phi - \phi\gamma\pi^e_{t+1} - d_t}, \tag{18}$$

where $\pi_t \equiv P_t/P_{t-1}$ denotes the realized gross inflation rate. This equation governs the evolution of inflation in any equilibrium, regardless of how expectations are formed, and we will use it repeatedly.

We start by studying the rational expectations benchmark. Under rational expectations, market prices are assumed to carry only redundant information because agents know the exact mapping from the history of seigniorage levels to prices, $P_t(d^t)$. As usual, we denote RE by dropping the superscript $\mathcal{P}$ in the expectations operator, and under RE we write

$$\pi^e_{t+1} = \mathbb{E}_t\left[\frac{P_{t+1}}{P_t}\right]. \tag{19}$$

3.2 The Rational Expectations Benchmark

We now study equilibria under RE, restricting attention first to a deterministic environment.

In the absence of uncertainty, imposing rational expectations amounts to requiring that $\pi^e_t = \pi_t$ for all t. Plugging this condition into the main equation (18) and rearranging delivers:

$$\pi_{t+1} = (1-\rho)\left(\frac{\phi+\phi\gamma-\delta}{\phi\gamma} - \frac{1}{\gamma\pi_t}\right) + \rho\left(\frac{\phi+\phi\gamma-d_{t-1}}{\phi\gamma} - \frac{1}{\gamma\pi_t}\right). \tag{20}$$

This equation will govern the dynamics of inflation in equilibrium.

The initial position of the economy is given by d_0. Notice that if the initial deficit is at the mean $d_0 = \delta$, then under no uncertainty $d_t = \delta$ for all t and the equilibrium will be stationary. In such a case, (20) admits two stationary equilibria, which are obtained as the solutions to the following quadratic equation:

$$\phi\gamma\pi^2 - (\phi+\phi\gamma-\delta)\pi + \phi = 0. \tag{21}$$

One could use this equation to trace out a stationary Laffer curve, depicting the inflation rates that allow the government to finance the level of seigniorage δ. We use $\{\boldsymbol{\pi_1}(\delta), \boldsymbol{\pi_2}(\delta)\}$ to denote the two roots of (21), where the small root $\boldsymbol{\pi_1}(\delta)$ corresponds to the "good" side of the Laffer curve.

In the case in which d_0 differs from δ, then d_t becomes a state variable of the model solution. Now we define $x_t \equiv (\pi_t, d_t)$ and write the dynamic system composed of (17) and (20) as follows:

$$x_t = \mathbf{G}\left(x_{t-1}\right) \equiv \left[\begin{array}{c} (1-\rho)\mathbf{F}(\pi_{t-1},\delta) + \rho\mathbf{F}(\pi_{t-1},d_{t-1}) \quad (1-\rho)\delta + \rho d_{t-1} \\ \\ (1-\rho)\delta + \rho d_{t-1} \end{array}\right], \tag{22}$$

where

$$\mathbf{F}(\pi, d) = \frac{\phi+\phi\gamma-d}{\phi\gamma} - \frac{1}{\gamma\pi}. \tag{23}$$

In a deterministic environment, d_t will always revert to its long-run mean δ. Hence, to characterize equilibria, it suffices to understand the behavior of π_t, conditional on the initial position d_0. To this end, it will prove convenient to ensure that stationary inflation rates are always positive and well defined, for which we assume the following.

Assumption 5 $\delta \in \mathbf{D} \equiv [0, \phi(1 + \gamma - 2\gamma^{\frac{1}{2}}))$.

One can easily check that under this assumption, stationary inflation rates are always within the interval $[1, \gamma^{-1}]$. Moreover, the upper bound of $\mathbf{D}$ can be interpreted as the maximum level of seigniorage that the government can finance, given the primitives of the economy. The following proposition summarizes the behavior of inflation under rational expectations.

Proposition 1 *Under Assumption 5, for any $d_0 \in \mathbf{D}$ there exists $\underline{\pi}(d_0)$ such that:*

1. *If $\pi_0 < \underline{\pi}(d_0)$, then $\lim_{t\to\infty} \pi_t = -\infty$.*
2. *If $\pi_0 = \underline{\pi}(d_0)$, then $\lim_{t\to\infty} \pi_t = \boldsymbol{\pi}_1(\delta)$.*
3. *If $\pi_0 > \underline{\pi}(d_0)$, then $\lim_{t\to\infty} \pi_t = \boldsymbol{\pi}_2(\delta)$.*

The proof is relegated to Appendix B. In the special case that $d_0 = \delta$, one can show that $\underline{\pi}(d_0) = \boldsymbol{\pi}_1(\delta)$ and the set of equilibria is equivalent to that corresponding to the case with no persistence, as analyzed by Sargent and Wallace (1987) or Marcet and Nicolini (2003). In general, when $d_0 \neq \delta$, there will be a stable inflation path that converges to the low inflation steady state and a continuum of paths that converge to the high inflation steady state.[12]

Thus, as in the case of i.i.d. shocks, the model exhibits equilibrium multiplicity, with one equilibrium driven by fundamentals only and a continuum of equilibria that exhibit self-fulfilling hyperinflations. The fundamentals equilibrium, which is always on the good side of the Laffer curve, is locally unique in the sense that all other equilibria converge to an inflation rate that is the high inflation steady state on the wrong side of the Laffer curve.

So far, we have not considered exchange rate policies, as the ones described above. We could apply the arguments in Obstfeld and Rogoff (1983) and use exchange rate rules to rule out all those bubble equilibria. More importantly, as shown in Nicolini (1996), those exchange rate rules will in general be imposed only off the equilibrium path, so that in equilibrium, the government never actually sets $s_t = 1$ and no special funding is required, but still the only remaining equilibrium is the one that always remains on the good side of the Laffer curve. Therefore, in the remainder of this section, we focus exclusively on that equilibrium.

To learn about the properties of the inflation process, we introduce a small amount of uncertainty in the seigniorage process and linearize (22) around the low inflation steady state.

[12] The equilibria characterized in this proposition for the case $d_0 < \delta$ is depicted in Figure 4 in Appendix F.

3.3 Inflation Persistence under Rational Expectations

To learn more about the stochastic properties of inflation in equilibrium, we linearize the main equation (18) around the low inflation steady state and introduce a small amount of uncertainty in the level of seigniorage. The linearization boils down to[13]

$$\hat{\pi}_t = \frac{\delta}{\phi - \phi\gamma\boldsymbol{\pi}_1(\delta) - \delta}\hat{d}_t \tag{24}$$

$$\hat{d}_t = \rho\hat{d}_{t-1} + \epsilon_t, \tag{25}$$

where we are using the notation $\hat{x}_t = \ln x_t - \ln \mathbf{x}$, with bold letters indicating steady state values. Thus, we can express inflation as

$$\pi_t = (1-\rho)\boldsymbol{\pi} + \rho\pi_{t-1} + \nu_t, \tag{26}$$

where $\nu_t \equiv \delta\epsilon_t/(\phi - \phi\gamma\boldsymbol{\pi}_1(\delta) - \delta)$. Hence, around the low inflation steady state, inflation behaves like an AR(1) process that inherits the persistence of seigniorage.[14]

3.4 System of Beliefs about Inflation

In order to consider expectations that perform well so that agents will be unlikely to abandon them, we proceed in the following way. We endow agents with a system of beliefs regarding the process of inflation — which agents rightly perceive as exogenous to their decisions — that is consistent with the behavior of inflation in the rational expectations equilibrium. As shown below, in the log-linearized solution, inflation follows the stochastic process described in (26), which is independent of the value of the deficit, d_t. This means that, up to a log-linear approximation, observations of the deficit do not provide additional useful information regarding the future behavior of prices, beyond the one provided by prices themselves. Based on simplicity, we assume that agents' beliefs are consistent with (26). However, we also assume that agents are not completely sure regarding the value of underlying long-run inflation in that process. This may be true, even if they know the long-run value for the deficit, since, as explained above, they are unable to compute the mapping from the deficit to inflation. We model the uncertainty of the agents by assuming they hold beliefs for the process of inflation that is the sum of a transitory and a permanent component. Given this system of beliefs, agents rationally use the data generated by the model to update their prior. In particular, given the system of beliefs, agents rationally use the Kalman filter to obtain a more precise estimate of

[13]See Appendix C for details.

[14]Appendix F displays sample paths for both inflation and seigniorage according to this linearized system.

the parameters they are uncertain about. In the next subsection, we explain why this system of beliefs has a good chance of performing well along an equilibrium path. We also explain the sense in which they imply a small departure from rational expectations.

Specifically, we assume that agents hold the following beliefs regarding the inflation process:

$$\begin{aligned} \pi_t &= (1-\rho_\pi)\,\pi_t^\star + \rho_\pi \pi_{t-1} + u_t \\ \pi_t^\star &= \pi_{t-1}^\star + \eta_t, \end{aligned} \tag{27}$$

where $u_t \sim N(0, \sigma_u^2)$ and $\eta_t \sim N(0, \sigma_\eta^2)$ are *i.i.d.* and independent of seigniorage d_t. We allow for $\rho_\pi \neq \rho$, although in practice the difference will be small. The intuition behind the proposed belief system (27) is that agents think the behavior of inflation is similar to that of seigniorage. Therefore, they think inflation is an AR(1) process, although they are unsure about the long-run average level of inflation $\pi_t^\star$, and they express their uncertainty about this long-run level by modeling $\pi_t^\star$ as a unit root process. Agents observe the realizations of inflation but not those of $\pi_t^\star$ and u_t separately. Thus, the learning problem consists of filtering long-run inflation $\pi_t^\star$ from observed inflation π_t. Since agents are rational, their filter will involve using Bayes' inference.

We denote the posterior mean of $\pi_t^\star$ entering period t given information available to agents as $\beta_t = E^{\mathcal{P}}(\pi_t^\star \mid \pi^{t-1})$. Agents are endowed with an initial prior belief about $\pi_0^\star$, which is normally distributed with mean $\beta_0 = E^{\mathcal{P}}(\pi_0^\star)$ and variance $\sigma_0^2 = E^{\mathcal{P}}(\pi_0^\star - \beta_0)^2$. In most of the paper, the prior is assumed to be centered at the low inflation steady state $\beta_0 = \boldsymbol{\pi}_1(\delta)$ with a variance guaranteeing that the gain in the Kalman filter is constant.

Notice that if we make $\sigma_\eta^2 = 0$, then we assign probability 1 to $\beta_0 = \boldsymbol{\pi}_1(\delta)$; so in this case,

$$\pi_t^e = (1-\rho_\pi)\,\boldsymbol{\pi}_1(\delta) + \rho_\pi \pi_{t-1}, \tag{28}$$

which, as long as $\rho_\pi = \rho$, is equivalent to the linearized RE equilibrium (26) for small deviations around the low inflation steady state. It is in this sense that our framework encompasses rational expectations equilibria as a special case.

Thus, we assume that the prior beliefs are normally distributed with mean $\beta_0 = \boldsymbol{\pi}_1(\delta)$ and variance σ_0^2. Under all these assumptions, optimal learning then implies that β_t evolves recursively according to

$$\beta_t = \beta_t + \frac{1}{\alpha_t}\left(\frac{\pi_t - \rho_\pi \pi_{t-1}}{1-\rho_\pi} - \beta_{t-1}\right), \tag{29}$$

where α_t denotes the optimal Kalman gain. For simplicity we set the variance of the prior to be equal to the steady state Kalman filter uncertainty about β_t, which is given by

$$\sigma_0^2 = \frac{-\sigma_\eta^2 + \sqrt{\left(\sigma_\eta^2\right)^2 + 4\sigma_\eta^2\sigma_u^2}}{2}.$$

Therefore, the value of α_t is constant and given by

$$\frac{1}{\alpha} = \frac{\sigma_\eta^2 + \sigma_0^2}{\sigma_u^2 + \sigma_\eta^2 + \sigma_0^2}.$$

If initial uncertainty is set to a value different from σ_0^2, then the optimal gain is not constant but converges monotonically to the value $\frac{1}{\alpha}$ above. Though one can imagine scenarios in which after a policy change uncertainty may initially be very high, we prefer to keep the number of free parameters to a minimum in this first analysis, so we will only consider cases where $\alpha_t = \alpha$ for all t. If there are no permanent shocks, so $\sigma_\eta^2 = 0$, and agents' priors are consistent with that, then $\sigma_0^2 = 0$ and $1/\alpha = 0$. If in addition, as we always assume in this paper, the priors are centered at $\beta_0 = \pi_1(\delta)$, the equilibrium outcome coincides with the rational expectations outcome.

Notice that the equilibrium value for α depends on the relative volatilities of the permanent and transitory shocks, so agents in the model could potentially use the law of motion for inflation (27) and past observations on inflation to estimate σ_u^2 and σ_η^2 and compute the optimal value for α. This is complicated after a policy change that ought to modify the process of inflation such as the one we study in this paper, since the behavior of the shocks would likely change. We further discuss this issue in calibrating the value for α below.

3.5 Learning Equilibria

The belief system implies that

$$\pi_{t+1}^e = (1 - \rho_\pi)\beta_t + \rho_\pi \pi_t. \tag{30}$$

Notice that if we plug this equation into (18), the solution is given by a nonlinear equation in π_t so that multiple solutions may arise. To sidestep this problem, we assume that when expectations regarding π_{t+1}^e are formed at period t, agents still do not know the realization of π_t. Therefore, in order to form their expectations regarding future inflation, they forecast inflation two periods ahead using π_{t-1} as follows:

$$\pi_{t+1}^e = \left(1 - \rho_\pi^2\right)\beta_t + \rho_\pi^2 \pi_{t-1}. \tag{31}$$

Substitutting this equation into (18) gives that equilibrium inflation follows:

$$\pi_t = \frac{\phi - \phi\gamma((1-\rho_\pi^2)\,\beta_{t-1} + \rho_\pi^2 \pi_{t-2})}{\phi - \phi\gamma((1-\rho_\pi^2)\,\beta_t + \rho_\pi^2 \pi_{t-1}) - d_t}. \tag{32}$$

To provide intuition about the behavior of inflation in this case, let us write (32) as

$$H(\beta_t, \beta_{t-1}, \pi_t, \pi_{t-1}, \pi_{t-2}, d_t) = 0$$

and define $h(\beta, \pi, d) \equiv H(\beta, \beta, \pi, \pi, \pi, d)$. The function h is useful to provide an approximation of the behavior of inflation in a situation in which $d_t = \delta$, $\beta_t \approx \beta_{t-1}$, and $\pi_t \approx \pi_{t-1} \approx \pi_{t-2} \approx \beta_{t-1}$. In such a case, (32) boils down to the quadratic equation (21), which implies that the rational expectations stationary inflation rates are also stationary inflation rates under learning. However, notice that the stationary inflation rate $\boldsymbol{\pi_i}(\delta)$ is stable under learning only if

$$\left.\frac{\partial \pi}{\partial \beta}\right|_{\beta=\boldsymbol{\pi_i}(\delta)} = -\left.\frac{\partial h/\partial \beta}{\partial h/\partial \pi}\right|_{\beta=\boldsymbol{\pi_i}(\delta)} = \frac{\phi\gamma\boldsymbol{\pi_i}(\delta) - \phi\gamma}{\phi - \phi\gamma\boldsymbol{\pi_i}(\delta) - \delta} < 1. \tag{33}$$

As long as the denominator in (33) is positive, we can verify that this is indeed the case if and only if[15]

$$\boldsymbol{\pi_i}(\delta) < \frac{\phi + \phi\gamma - \delta}{2\phi\gamma}. \tag{34}$$

Using (21), it is easy to show that this condition is satisfied by the smallest root $\boldsymbol{\pi}_1(\delta)$, but not by the largest $\boldsymbol{\pi}_2(\delta)$. Therefore, as pointed out in Marcet and Sargent (1989b) and Marcet and Nicolini (2003), the low inflation rational expectations equilibrium is stable under learning, whereas the high inflation one is unstable under learning.

These dynamics are attractive in explaining periods of relatively large but stable inflation followed by a rapid burst in inflation. The reason is that as long as expected inflation is around the stable low inflation equilibrium $\boldsymbol{\pi}_1(\delta)$, inflation itself will remain in that region. However, if a sequence of positive shocks to the deficit increase inflation to the extent that expectations eventually go beyond the value of $\boldsymbol{\pi}_2(\delta)$, the unstable dynamics on the wrong side of the Laffer curve take control and inflation can grow very quickly.

3.5.1 The role of an exchange rate regime

We assume that when inflation expectations go beyond a certain upper bound, the government switches its policy regime, establishes a crawling peg, and asks for special short-run financial

[15] Whenever there is a positive price that clears the money market, the denominator will be positive. We can always extend the model to include the case in which there are reserves that can be depleted to ensure the existence of such a price, in the spirit of Marcet and Nicolini (2003).

assistance, setting $s_t = 1$. The role of the exchange rate regime (ERR) is to stop the inflation dynamics described above and bring inflation back down to the stable region. The crawling peg implies that inflation will then be determined by the peg, so the money demand equation (15) determines the sequence of money supplies. This implies that a new source of financing is required to satisfy the government budget constraint (11). Once inflation is so stabilized, the government can switch the policy regime and stop the crawling peg, so the economy will again be governed by the money demand (15) and the government budget constraint (16) together with the evolution of the deficit.

Clearly, while $s_t = 1$, equation (11) will determine the term F_t, which measures the required financial assistance while the IMF agreement is in place. In the quantitative section, we compute the evolution of F_t over time for alternative ways of implementing the IMF agreement.

To the extent that the sequence of values for F_t while the program is active is feasible within the specific agreement, the policy of switching to an ERR successfully stabilizes the economy. In that case, the ERR opens the possibility of equilibrium cycles in which periods of relatively high but stable inflation are followed by bursts of escalating inflations that are eventually stopped by a program that involves managing the exchange rate, together with some financial assistance. These cycles can be stopped permanently only by an austerity program that generates a primary surplus large enough so that it can at least pay for the interest on the debt, so $d_t \leq 0$.[16]

The periods with stable dynamics imparts temporary shocks to the inflation rate, associated with the disturbance u_t in equation (27). On the other hand, the unstable dynamics that are eventually ended with the ERR imparts permanent shocks to the inflation rate, associated with the disturbance η_t in equation (27). Neither of these shocks is observable to the agent, who sees only the behavior of inflation in equilibrium. This equilibrium behavior may therefore reinforce the belief system of the agents in equilibrium.

3.6 Justifying the System of Beliefs

As is clear from equation (32), in the learning equilibrium π_t is a function of past seigniorage. However, agents think that inflation evolves according to the system of beliefs specified at the beginning of Section 3.4, which is obviously different from (32). This will not be surprising to the careful reader since, from the very beginning we have said that we depart from RE.

However, our aim is to consider only "reasonable" systems of beliefs. Although IR permits assuming anything you want for the system of beliefs, we do not think it is interesting to consider systems of beliefs that generate inflation processes that render the beliefs obviously wrong. For this, we follow various principles that the belief system should satisfy:

[16] This property of the model was used by Marcet and Nicolini (2003) and Sargent et al. (2009) to replicate the behavior of inflation in South America during the 1980s.

1. *Encompass RE* If the belief system encompasses RE, there is a clear sense in which the deviation from RE is small and the equilibrium does not deviate too much from the system of beliefs specified in Section 3.4.

2. *Close to the data* If the system of beliefs is close to the data behavior, and to the extent that the equilibrium outcome of our model reproduces the behavior of the data, we can expect that agents in the real world can hold this system of beliefs and that this will in fact render the system close to the model behavior.

3. *Close to the model outcome* We would like to check that if agents observe the model outcome, they cannot reject their belief system in a few periods. In this way, we can think of the considered system of beliefs as being consistent with the model of inflation that we, as economists, consider.

4. *Close to surveys* The system of beliefs should not be too different from observed surveys of expectations. Since inflation surveys are conducted continuously in many countries, it is possible to apply this criterion to inflation.

The system of beliefs specified above satisfies all of these criteria. (1) As explained in Section 3.4, the system of beliefs encompasses RE as a special case. (2) Various authors (for example, Stock and Watson (2007)) have chosen a similar model to explain actual inflation in various countries. Although they often use a more involved model including time-varying volatility, it often has the main features of our system of beliefs, namely, serial correlation and a permanent shock to average inflation. (3) In Section 5, we do a full array of tests and show that agents within the model would not reject their belief system even after two decades. This implies that, given the system of beliefs, the equilibrium is such that the agents see their belief system as a reasonable description of the inflation that they observe. The reason that this is likely to happen has been described at the end of Section 3.5. (4) Many authors (e.g., Roberts (1997), among others) have fit the above model to inflation surveys.

4 Quantitative Performance

In this section we calibrate the model and solve it numerically. We first show how likely hyperinflations are in equilibrium, as a function of the parameter $\frac{1}{\alpha}$. We also show an example of an equilibrium time series to show the difference between the rational expectations outcome and the outcome with internally rational agents. The example quantifies the amplification effect that expectations can have on the equilibrium inflation rate.

We then show the performance of the tests described in Proposition 2 below and argue that agents would not reject their beliefs in equilibrium. Finally, we compute the welfare effect of exchange rate policies that can stop the hyperinflations early on, as well as the evolution of the financial assistance required to carry out those policies, along the lines of the discussion in Section 3.5.1.

4.1 Calibration

Seigniorage process and money demand

We assume that after exiting, the total deficit, d_t, will follow a process similar to the secondary deficit before exiting. This process corresponds to a government that initially keeps austerity at a level similar to where it was before exiting, that does not default on the debt, and that can roll over the debt at the same interest rate as before. To calibrate the process, we estimate an AR(1) process for Greece's primary balance as a percentage of GDP, though very similar estimates are obtained with data from Italy, Portugal, and Spain. We calibrate the values for ρ and σ_ϵ using the results of that estimation. We also assume that austerity will eventually prevail, so we set the long-run value of the deficit, $\overline{\delta}$, to be zero. These assumptions reflect the view that the proponents of exiting the Euro see this path as an alternative to the austerity programs imposed by the monetary union.

But clearly, fiscal policy could differ after exiting. One could entertain an alternative hypothesis for the evolution of tax revenues or government spending, or allow for interactions between the real values of expenditures or government debt with the inflation that would follow exit. In that case, one could obtain the implied evolution of d_t and describe how inflation behaves according to our model. To us, a reasonable benchmark to consider is to assume that the process for the secondary deficit will stay as before and will have to be financed by monetization.

The role of preferences and the endowment in the economy boils down to the values of the money demand parameters in equation (4). These two parameters are the ones that fully determine the shape of the Laffer curve that relates the inflation rate to the amount of revenues it raises. These are key parameters, since the distance between the two solutions discussed in Section 3.5 determines the size of the stable set and therefore the likelihood of a hyperinflation to unravel. To calibrate those parameters, one would ideally observe time series in which the inflation rates are sufficiently high, events that did not occur in the countries under consideration. Thus, to calibrate those two parameters, we use data from Argentina, following Marcet and Nicolini (2003). Specifically, the money demand parameters target the inflation rate that maximizes the stationary Laffer curve and the maximum seigniorage as a percentage of GDP for the case of Argentina during the eighties.

Belief system

As explained in the introduction, the key methodological novelty in this paper is how to perform policy analysis under IR. Clearly, unlike in RE models, one can make various assumptions about the belief system. We believe that is a virtue of the model, since we do not know in fact how expectations will be set after exiting. By being explicit about our assumptions and by exploring alternatives, we are forced to express our ignorance about exactly how expectations will react to such a policy change.

We assume agents' belief system is the one specified in Section 3.4, with $\rho_\pi = \rho$, which is exactly the case with RE.[17] The point of departure from RE lies in our assumption that agents do not know for sure what the new level of long-run inflation will be following exit. That still leaves us with two free parameters, the prior β_0 and the uncertainty regarding the prior, summarized by $1/\alpha$. In all cases below, we assume the initial prior of inflation β_0 to be the low inflation equilibrium in the steady state. The first advantage of this assumption is that by setting $1/\alpha = 0$, we obtain the RE equilibrium. The second advantage is that the dynamics are not affected by asymmetric behavior in the first periods.[18] Notice that because of the seigniorage financing of the deficit, inflation in these countries would be substantially higher than during the years in which the euro was adopted. This feature recognizes that agents understand that higher inflation following exit is very likely.

The only remaining free parameter is therefore $1/\alpha$. We proceed by showing the results for values that are slightly larger than zero, which, as we mentioned before, delivers the RE equilibrium. The size of $1/\alpha$ reflects the uncertainty that agents have about their prior and can therefore be interpreted as the distance from the RE beliefs.

Exchange rate rules

We assume the government will switch to an ERR when inflation expectations are above some specified upper bound, β^U. Thus, an ERR is triggered whenever expected inflation exceeds β^U or to restore equilibrium.[19] The value of β^U will be a policy choice that we will analyze below. As explained in Section 3, an ERR can avoid too high inflations as long as the government has access to enough financing to satisfy the budget constraint (11). Switching to an ERR leaves several additional policy options. First, the ERR must specify the desired

[17]As it turns out, that is no longer true with IR, because of the feedback between inflation and inflation expectations. However, as we show in Appendix D, it is a remarkably good approximation.

[18]It is relatively easy to have hyperinflations early on just by assuming that agents start with higher initial priors.

[19]The two cases in which an ERR is required to restore equilibrium are if the money demand becomes negative or if, given the realization of seigniorage, the money demand is too low for an equilibrium to exist.

Table 1: **Parameters for Baseline Economy**

Parameter	Symbol	Value
Long-run deficit	δ	0
Persistence of deficit	ρ	.93
SD of shocks to deficit	σ_ϵ	.01
Persistence of inflation	$\rho_\pi^\star$	93
Money demand parameter	ϕ	.36
Money demand parameter	γ	.39

growth rate for the crawling peg, $\overline{\beta}$. In what follows, we always set $\overline{\beta}$ to be the inflation rate in the low inflation steady state, $\pi_1(\delta)$. Second, the ERR must specify how fast the target value for the crawling peg, $\overline{\beta}$, ought to be achieved. We let T be the number of periods after which the crawling peg will effectively be at the long-run target $\overline{\beta}$.[20] We will explore several values for T in the policy analysis below. Finally, we must specify a bound $\hat{\beta}$ such that the ERR is abandoned once equilibrium inflation falls below that bound. In what follows, we set $\hat{\beta} = (\beta^U + \overline{\beta})/2$, halfway between the belief that triggers an ERR and the target belief. This choice implies that the ERR is in place until expectations fall back down to the stable set, so the model dynamics themselves imply that — absent a new series of negative shocks — the economy converges to the low inflation equilibrium.

Note that it is possible that a sequence of good shocks brings the deficit d_t into negative territory. In this case, equation (11) implies that the money stock should go down, which may generate a deflation. In such a situation, the most natural policy choice appears to be to save those surpluses in some interest-bearing asset. Therefore, in our simulations, we assume that when $d_t < 0$, then $M_t = M_{t-1}$, and those savings are accumulated in reserves at the Treasury.

Calibration parameters

The baseline parameterization is summarized in Table 1. Below, we will show results for values of $1/\alpha = \{0, .01, .03, .05\}$. In addition, we will make our policy evaluations by solving the model for alternative values of $\{\beta^U, T\}$.

4.2 Probability of Hyperinflations

Table 2 reports the implications of the model for the probability of hyperinflations, given different values for the long-run deficit δ and initial deficit d_0. The values for $1/\alpha$ constitute the weight that agents place on past inflation to update their beliefs. Notice that the probability

[20] This variable allows the policy analysis below to consider the trade-off between "shock" and "gradualism," using the language in the literature. This choice is also motivated by the experiences of many countries that chose a crawling peg with declining rates to smoothly lower inflation, as opposed to other experiences in which the exchange rate was fixed, setting a devaluation rate of zero, at the moment of switching to the ERR.

Table 2: **Probability of n Hyperinflations**: ERR policy $(\beta^U, T) = (150\%, 1)$.

Deficit mean δ $= 0.0\%$, **Initial Deficit** d_0 $= 4.0\%$				
$1/\alpha$	0	1	2	≥ 3
0.00	100.00	0.00	0.00	0.00
0.01	57.40	26.24	11.10	5.26
0.03	22.92	28.84	23.12	25.12
0.05	15.54	24.78	24.80	34.88
Deficit mean δ $= 0.0\%$, **Initial Deficit** d_0 $= 1.0\%$				
$1/\alpha$	0	1	2	≥ 3
0.00	100.00	0.00	0.00	0.00
0.01	62.52	24.00	9.56	3.92
0.03	31.02	29.72	20.16	19.10
0.05	24.42	27.16	22.48	25.94

of experiencing a hyperinflation vanishes as the learning equilibrium approaches the rational expectations one (e.g., $1/\alpha \to 0$). This feature is also present in the case without persistence studied in Marcet and Nicolini (2003), since $1/\alpha = 0$ merely keeps expectations constant and ensures that the economy stays around the low inflation steady state. In terms of comparative statics, a higher long-run deficit δ and a higher initial deficit d_0 both increase the probability of experiencing a hyperinflation, with the largest effect coming from δ.

Figure 1 shows sample paths for inflation in a learning equilibrium. The solid blue line corresponds to the learning equilibrium with a positive constant gain, and the dotted red line to an equilibrium with fixed expectations that considers the same realizations of the shocks to seigniorage. This confirms the intuition we provided in Section 3.5: when inflation expectations are too large, it is likely that hyperinflationary paths start to appear. These are then stopped by ERR rules, but if average seigniorage is too high, these hyperinflations are activated again.

4.3 The Role of Persistence

In Table 2 we showed that hyperinflations are more likely the more we depart from rational expectations. We now investigate the role of persistence in the probability of these episodes. Table 3 displays the probability of hyperinflations for different degrees of persistence in the deficit. More precisely, we fix $1/\alpha = 0.05$ and gradually decrease ρ to assess how the frequency of hyperinflations responds. The table shows that with $\rho = 0.7$, these episodes virtually disappear.

These results emphasize that positive persistence involves an important extension relative to Marcet and Nicolini (2003), which only considered i.i.d. shocks to seigniorage. By allowing $\rho > 0$, we not only gain in realism, since shocks to the deficit are indeed very persistent, but also verify that properly calibrating its value turns out to be crucial for quantifying the risks

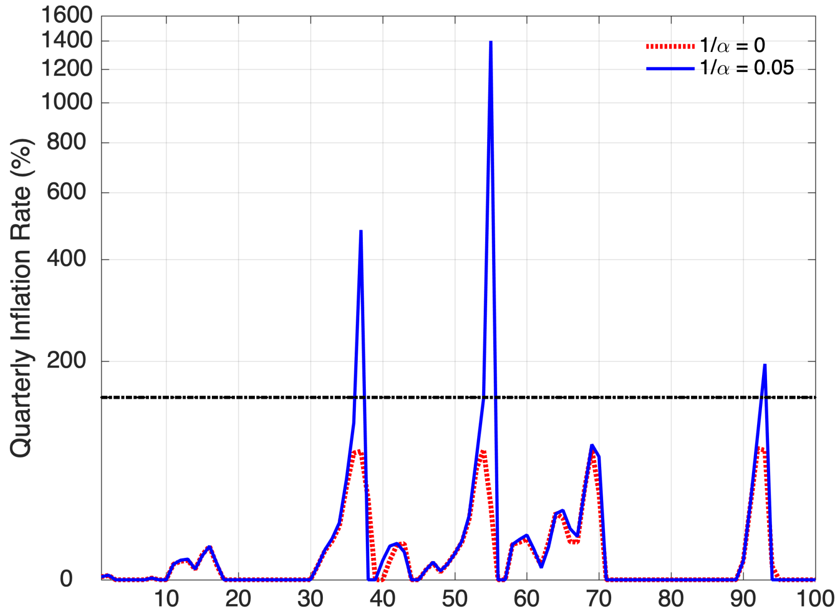

Figure 1: **Sample Path for Inflation in Learning Model.** The solid blue line corresponds to the learning equilibrium with a positive constant gain and the dotted red line to an equilibrium with fixed expectations that considers the same realizations of the shocks to seigniorage. The parameterization is discussed in the quantitative section.

of abandoning the monetary union.

Table 3: **Probability of n Hyperinflations**: ERR policy $(\beta^U, T) = (150\%, 1)$.

Deficit mean δ $= 0.0\%$, **Initial Deficit** d_0 $= 4.0\%$, **and** $1/\alpha$ $= 0.05\%$				
ρ	0	1	2	≥ 3
0.93	15.54	24.78	24.80	34.88
0.90	33.38	32.60	19.98	14.04
0.80	93.68	5.78	0.48	0.06
0.70	98.04	1.96	0.00	0.00
Deficit mean δ $= 0.0\%$, **Initial Deficit** d_0 $= 1.0\%$, **and** $1/\alpha$ $= 0.05\%$				
ρ	0	1	2	≥ 3
0.93	24.42	27.16	22.48	25.94
0.90	40.66	30.88	17.16	11.30
0.80	94.50	5.08	0.40	0.02
0.70	98.68	1.32	0.00	0.00

5 Testable Restrictions

In this section we study the conditions under which agents would question their belief system in a learning equilibrium. To do this, we consider the implications of the equilibrium conditions and the belief system for the vector $x_t = (e_t, d_t - \rho d_{t-1})$, where $e_t \equiv (\pi_t - \rho_\pi \pi_{t-1}) - (\pi_{t-1} - \rho_\pi \pi_{t-2})$, and we evaluate these implications using simulated data.

The following proposition adapts the results in Adam et al. (2016). It lists a set of necessary and sufficient second order conditions for the statement that inflation and seigniorage data are indeed generated by the model.

Proposition 2 *Let d_t be AR(1) with innovation ϵ_t as in (17). There exists a belief system as the one described in Section 3.4 consistent with the autocovariance function of $\{x_t\}$ if and only if the following restrictions hold:*

1. $\mathbb{E}[x_{t-i}e_t] = 0$ *for all* $i \geq 2$,
2. $\mathbb{E}[(\epsilon_t + \epsilon_{t-1})e_t] = 0$,
3. $\Sigma b^2 + \mathbb{E}[e_t e_{t-1}] < 0$,
4. $\mathbb{E}[e_t] = 0$,

where $\Sigma = \sigma^2_\epsilon$ and $b = \mathbb{E}[\epsilon_t e_t]$ correspond to the coefficient of a regression of e_t on ϵ_t in population.

The proof is presented in the Appendix E. We test these moment restrictions using the parameterization displayed in Table 1. If we find that these restrictions cannot be rejected in the samples we consider, we conclude that agents could be holding the system of beliefs for inflation as stated above in the model at hand. Now we provide tests for these restrictions.

5.1 Statistics

Restrictions 1, 2, and 4 represent first moment restrictions of the form $\mathbf{E}[y_t] = 0$ for $y_t = e_t q_t$, for various $q_t \in \mathbf{R}^n$. In order to test these restrictions, we estimate $\mathbf{E}[y_t]$ through its corresponding sample mean:

$$\frac{1}{T}\sum_{t=1}^{T} y_t.$$

Using standard arguments, the statistic

$$\hat{Q}_T = T\left(\frac{1}{T}\sum_{t=1}^{T} y_t\right)' \hat{S}^{-1}\left(\frac{1}{T}\sum_{t=1}^{T} y_t\right) \xrightarrow{d} \chi^2_n,$$

as $T \to \infty$ for some $\hat{S}$ that is a consistent estimator of the asymptotic variance of $\frac{1}{T}\sum_{t=1}^{T} y_t$. We use[21]

$$\hat{S} = T \cdot \mathbf{E}\left[(\bar{y}_t - \mathbf{y})(\bar{y}_t - \mathbf{y})'\right] = \sum_{\nu=-\infty}^{\infty} \Gamma_\nu = \Gamma_{-1} + \Gamma_0 + \Gamma_1.$$

In order to test restriction 3, we use a one-sided test of the form $H_0 : \alpha < 0$, where α is set to satisfy

$$\mathbf{E}\left[(\epsilon_t b + e_{t-1})e_t - \alpha\right] = 0.$$

GMM sets the estimate of b to the OLS coefficient of a regression of e_t on ϵ_t and the estimate of α precisely to $b'\Sigma b + E(e_{t-1}e_t)$.

5.2 Rejection Frequencies

The belief system can be evaluated by checking whether the rejection frequencies exceed a predetermined significance level. In what follows, we calculate rejection frequencies using the theoretical asymptotic distribution of $\hat{Q}_T$.[22] In the case of Restrictions 1, 2, and 4, asymptotic theory implies that $\hat{Q}_T \to \chi^2_n$ as the sample size increases. In testing restriction 1, we use three lags of each element of x_t, and we always include a constant term in the instrument vector q_t. Notice that by including a constant, restriction 4 is embedded in the joint hypothesis testing performed for restriction 1. In the case of restriction 3, the asymptotic properties of the GMM estimator of α imply that, under the null hypothesis, it will be normally distributed and centered at 0.

Rejection Frequencies using the Asymptotic Distribution. Table 4 displays the results of testing the restrictions of Proposition 1 using the asymptotic theoretical distribution of the statistics described in the previous paragraphs. Since restriction 1 required some discretionary choice regarding the set of instruments, we single out its results.

The results indicate that agents will find it difficult to reject their beliefs based on the observation of realized inflation in a span of 10 years (40 periods) when the signal-to-noise ratio of their beliefs is higher, since this implies a higher stationary $1/\alpha$, in this case, 0.05. On the contrary, the results show that for values closer to the RE equilibrium, the rejection frequencies are higher than 10%, particularly for restrictions 2 and 3. This emphasizes the notion that hyperinflations add a persistent component that has, because of the formation of expectations, a life on its own. Thus, for values of $1/\alpha$ that generate several hyperinflations, agents are less likely to reject the belief system, which makes hyperinflations themselves more likely.

[21] Here we exploit the MA(1) property of e_t and use the fact that beyond the first lead and lag, these autocovariances matrices must be equal to zero.

[22] The results are very similar if one uses empirical small sample distributions.

Table 4: **Rejection Frequencies at the 5% level for** $(\beta^U, T) = (150\%, 1)$
This table reports rejection frequencies obtained from testing restrictions of Proposition 2 using simulated data. The set of instruments for restriction 1 includes a constant and three lags of $d_t - \rho d_{t-1}$ (1a), three lags of e_t (1b), or three lags of x_t (1c).

	T		
	40	60	100
Deficit mean δ $= 0.0\%$, and $1/\alpha = 0.01$			
Restriction 1a	16.1 %	19.1 %	27.0 %
Restriction 1b	2.9 %	2.6 %	2.5 %
Restriction 1c	2.7 %	2.6 %	2.0 %
Restriction 2	25.8 %	31.1 %	38.9 %
Restriction 3	19.2 %	14.2 %	11.0 %
Deficit mean δ $= 0.0\%$, and $1/\alpha = 0.05$			
Restriction 1a	7.3 %	6.6 %	6.9 %
Restriction 1b	0.9 %	0.7 %	0.6 %
Restriction 1c	1.1 %	1.1 %	0.8 %
Restriction 2	10.1 %	10.9 %	11.4 %
Restriction 3	6.9 %	4.4 %	2.5 %

5.3 Alternative Belief Systems

We end this section with a brief discussion of alternative ways to model the belief system. The discussion is motivated by the observation that embracing the internal rationality approach as we do in the paper implies abandoning the hope of equilibrium uniqueness. To do so, one needs to take a strong stand on a unique specification and parameterization of the belief system. Being able to use theory and data to uniquely pin down a value for $1/\alpha$, for instance, seems to us to be beyond the ability of current macroeconomics. We see the value of this approach as finding classes of belief systems that deliver relatively similar outcomes and which cannot be rejected by the agents in the model.

For example, the probability distribution over the number of hyperinflations changes when one changes $1/\alpha$ from 0.03 to 0.05, while the test results are good for both values. How can one use the model to look into the data and distinguish the two values? Our approach does not allow one to make that distinction, and the specific welfare computations do depend on the value of that parameter. But overall, this class of belief systems implies that leaving the Euro entails substantial risks that the RE model implies do not exist. As we show below, those risks entail substantial welfare losses.

The belief system we used can be extended along several dimensions, and we have tried some of them. For instance, agents in the model never anticipate that in order to stop a hyperinflation, the government may impose an exchange rate regime. This feature can easily be incorporated into the belief system. For instance, we could alternatively assume that agents believe that inflation follows the same process as before,

$$\pi_t = \rho\pi_{t-1} + (1-\rho)\pi_t^\star + u_t,$$

but now their belief about long-run inflation is modified as follows:

$$\pi_t^\star = \begin{cases} \pi_{t-1}^\star + \eta_t & \text{if} \quad z_t = 0 \\ \overline{\beta} + v_t & \text{if} \quad z_t = 1 \end{cases},$$

where z_t indicates whether an ERR has been imposed and $\overline{\beta}$ is the ERR target. Now, the belief system must also specify the probability placed on the event that an ERR is actually imposed. We assume agents believe that

$$\Pr[z_{t+1} = 1] = \begin{cases} 0 & \text{if} \quad \pi_t \leq \overline{\pi} \\ p^{ERR} & \text{if} \quad \pi_t > \overline{\pi} \end{cases}.$$

In this way, agents understand that, once inflation goes beyond a certain threshold, the probability of an exchange rate regime becomes positive. Given a threshold, one can impose the restriction that the probability that agents use p^{ERR} is the actual probability in the model, given the policy parameters of the government. We solved the model with this alternative formulation, and the results barely change.

A natural further exploration is to allow agents to use data on the deficit in the signal extraction problem. As long as agents also use inflation data, the main results will not be affected, since the key driver of the model is the feedback between actual and perceived inflation. On the other hand, it is likely to improve the performance of the tests, since agents would now optimally use the deficit data, which in our current formulation they are ignoring.

Overall, we conjecture that these or other extensions are unlikely to change the main message of the paper: small deviations from RE imply substantial risk for countries that leave the monetary union under the assumptions we adopted in the paper.

6 Policy analysis

In this section, we first evaluate the welfare consequences of the hyperinflationary equilibria. We first present the computations when the policy parameters $\{\beta^U = 150\%, T = 1\}$. We then evaluate policies that imply an earlier intervention (lower value for β^U) or a more gradual intervention (larger values for T).

Before doing so, however, it is important to highlight that in this model, the value of the monetary system does depend on how different the endowments are. As is well known, a monetary policy that maintains a constant quantity of money so that equilibrium inflation is zero implements a Pareto-optimal allocation in which consumption is given by

$$c = \frac{1+e}{1+\alpha} \text{ and } x = \alpha\frac{1+e}{1+\alpha}.$$

Recall that we assumed that $e < \alpha$, so a monetary equilibrium exists. On the other hand, as is also well known, this economy has a nonmonetary equilibrium, which is equivalent to autarky, where consumption is given by

$$c = 1 \text{ and } x = e.$$

Therefore, the value of the monetary system, which we denote by Δ, must satisfy

$$\ln\frac{1+e}{1+\alpha} + \alpha\ln\alpha\frac{1+e}{1+\alpha} = \ln(1+\Delta) - \alpha\ln(1+\Delta)e, \tag{35}$$

and it is bounded as long as $e > 0$. Note also that Δ is decreasing on e, and it approaches zero as $e \to \alpha$.

Any equilibrium in which inflation is positive and bounded in some or all periods, will imply a utility for the different generations that is higher than in autarky. Therefore, the welfare cost of the hyperinflationary equilibria is bounded above by Δ. Thus, the choice of the parameters e and α determines the value to society of having a monetary system, which in itself puts a bound on the welfare costs of a monetary system that does not work as well. Using the values of the benchmark calibration of Table 1 in equation (35) delivers a value of Δ of roughly 0.10, or 10% of total consumption. In what follows, we present results for that benchmark calibration and for an alternative one that implies a substantially higher value for the monetary system.

6.1 Welfare costs of hyperinflations

We start by computing the compensating variation of eliminating all inflationary dynamics that arise from learning. Specifically, for any given realization of the deficit for 200 periods, we compute the utility attained in equilibrium when we set $1/\alpha = 0$, which corresponds to the RE equilibrium. Then, for the same realization, we compute the utility attained in the equilibrium when we set $1/\alpha$ equal to 0.01, 0.03, and 0.05, respectively. Then, for each value of $1/\alpha$, we compute the percentage of consumption that agents would be willing to forgo under the RE equilibrium to avoid the inflation rates that arise for positive values of $1/\alpha$. We repeat this exercise for 10,000 different simulations and compute the average.[23]

In each case, we perform two different computations. We calculate the welfare change using the full sample path and also restricting attention to the 10 periods that precede the first hyperinflation and subsequent adoption of the ERR. The results are depicted in Table 5. The first column of the table reports the value used for the parameter $1/\alpha$. The second column indicates the consumption equivalent for the 200 periods, while the third column reports the computations for the 10 periods leading to the first hyperinflation in each simulation. The first measure is the one standard in the literature, and as can be seen, the numbers are sizable. For example, when $1/\alpha = 0.05$, the cost of the hyperinflations is around 0.4% of consumption in each of the 200 periods. This corresponds to 4% of the total gain of having a monetary system. The second measure is higher by construction. Again, when $1/\alpha = 0.05$, the cost of the hyperinflations is around 1.8% of each quarter consumption, which amounts to 20% of the total value of a monetary system during those periods. We will use these computations of the welfare costs just in the 10 quarters prior to the switch to the ERR as a benchmark to discuss the financing needs of the government during the ERR.

The second and third columns just discussed correspond to the benchmark calibration presented in Table 1. The value of Δ that solves equation (35) for this calibration is around 10% of total consumption. This number appears rather low to us for modern economies. Thus, as mentioned above, we repeat the computations for the same parameter values, except that we increase the endowment in the first period and reduce it in the second period while maintaining a constant output, such that the value of Δ that solves equation (35) now becomes 30% of total output. The resulting numbers are reported in columns 4 and 5 of Table 5. The numbers are substantially larger in this case.

The numbers discussed so far correspond to the case in which the maximum tolerated

Table 5: **Consumption Equivalent Welfare Change (in %).**
ERR policy is $(\beta^U, T) = (150\%, 1)$. The two calibrations correspond to economies in which the value of the monetary system is 10% and 30%. In each case, the first column calculates the welfare change using the full sample path, and the second does it restricting attention to the 10 quarters that precede the first hyperinflation.

$1/\alpha$	Gains from Money: 10%		Gains from Money: 30%	
	Full	10q	Full	10q
0.05	0.43	1.85	0.81	2.91
0.03	0.36	1.43	0.68	2.26
0.01	0.18	1.16	0.35	1.85

[23]Notice that this procedure implies that we compute welfare using the true distribution of prices, rather than the distribution that agents believe is the true distribution.

inflation expectation is $\beta^U = 150\%$ (notice that actual inflation can in equilibrium be much higher, as depicted in Figure 1. In addition, we have so far considered only ERRs that set the crawling peg to the desired low inflation rate on impact, rather than allowing for a more gradual policy that achieves that low target after a certain number of periods.

We now compute the welfare cost for alternative values of those policy parameters. In particular, we set $\beta^U = 100\%$ and allow for values of T all the way up to 4. The results are reported in Table 6. The first row in the table considers alternative values for the policy parameter T. The first column in the table reports the two considered values for β^U and the three values for $1/\alpha$. The first three numbers in the second column correspond to numbers reported in Table 5.

As the table shows, the costs of a gradual policy that takes four periods to bring down inflation are sizable. For $1/\alpha = 0.05$, the cost increases by more than 50% when $\beta^U = 150\%$, and more than triples when $\beta^U = 100\%$. In addition, the benefits of an early intervention are very significant. For instance, when $T = 1$ and $1/\alpha = 0.05$, adopting an ERR earlier decreases the welfare cost by a factor of five, from 0.81% of consumption when $\beta^U = 150\%$, to 0.16% when $\beta^U = 100\%$.

We would like to emphasize that none of these results are qualitatively surprising: earlier interventions imply lower inflation rates, so welfare ought to be higher. Similarly, gradual policies imply higher equilibrium inflation rates, so welfare ought to be smaller. The value of these computations is that it provides magnitudes that can be compared with the costs of these ERRs: the external financing required to satisfy the government budget constraint while the ERR is in place. We turn next to that issue.

Table 6: **Consumption Equivalent Welfare Change (in %).**
Full sample comparison, welfare gain from monetary system is 30%.

$1/\alpha$	$T = 1$	$T = 2$	$T = 3$	$T = 4$
$\beta^U = 150\%$				
0.05	0.81	1.17	1.30	1.47
0.03	0.68	0.92	1.00	1.14
0.01	0.35	0.44	0.49	0.55
$\beta^U = 100\%$				
0.05	0.16	0.34	0.42	0.52
0.03	0.05	0.19	0.26	0.35
0.01	0.01	0.11	0.15	0.22

Table 7: **Cumulative Median Change in Reserves ($1/\alpha = .05$, in % of yearly GDP)**

ERR T	Periods after Intervention 1	2	3	4	5
$\beta^U = 150\%$					
1	−2.3	2.4	–	–	–
2	−1.1	0.5	2.9	–	–
3	−0.9	−0.6	0.7	1.4	–
4	−0.8	−0.9	−0.1	0.6	0.8
$\beta^U = 100\%$					
1	−2.3	1.0	–	–	–
2	−1.0	0.3	1.6	–	–
3	−0.7	0.3	1.1	1.7	–
4	−0.5	0.3	1.2	1.7	1.9

6.2 Financing requirements to stop the hyperinflations

As we mentioned in Section 3 (the model), the hyperinflations are stopped by a policy regime switch that adopts a crawling peg. To do so, the government requires external financing to satisfy the budget constraint. We now show how large and persistent these funding requirements are according to the model. In Table 7 we show the results for the case in which $\alpha = 0.05$. The first column in the table reports the values considered for β^U and T. The first row reports the accumulated balance of an account that is set to zero at the moment the ERR is implemented and which uses a real interest rate equal to zero.[24] We did 10,000 simulations and report the median value for all the simulations. For example, the number -2.3 at the top of the second column means that in the quarter in which the ERR is adopted, the median external funds required to satisfy the government budget constraint is 2.3% of yearly GDP. The number 2.4% at the top of the second column implies that by the second quarter, the government has accumulated assets. The rest of the table can be read in a similar fashion. We report the balance in the account for one period more than the policy variable T.

A remarkable feature of Table 7 is that the external financing is a very temporary phenomenon. In all cases, the government would be able to pay the debt, at the latest, one period after T and in some cases even before. In all cases, the government will end up with additional resources after paying the debt. This may seem surprising, but as the theoretical analysis in Section 3 implies, the hyperinflations are bad not only for welfare but also for tax purposes. The reason is that the hyperinflations are the result of unstable dynamics that appear on the wrong side of the Laffer curve. Along these dynamics, the inflation tax is decreasing with in-

[24] The period is a quarter, so a risk-free interest rate would be very close to zero. One could easily impute a non-zero interest rate, but given the magnitude of the numbers, the table would barely change.

flation. At the same time, real money balances are shrinking. Once the ERR is put into place, both processes revert. In particular, real money balances grow substantially, which means that nominal money is growing at a rate higher than the inflation rate.[25]

Three important messages arise from Table 7, when combined with the computations in the previous subsection. First, it shows that early interventions can be a win-win scenario. As in Table 6, an early intervention involves substantial welfare gains when β^U is set at 100% rather than 150%. In addition, the table shows that the external financing is essentially the same. Thus, all of those gains from this early intervention are net gains. Second, the table shows that shock policies can be more demanding in terms of external financing, since they require a lower negative balance. This is the case when comparing $T = 1$ with $T = 2$. This choice seems to imply a trade-off: a policy that brings down inflation quickly is better, but it may call for larger external support. However, for higher values of T, the change in the numbers is very small, suggesting that if external financing is an issue, one may have to adopt gradual policies, but ideally ones that bring inflation down in two or three quarters. Third, notice that the worst case in terms of the requirement for external financing is the case with $T = 1$ and $\beta^U = 100\%$. In that case, the financing needs amount to 2.3% of yearly GDP. We want to compare this number with the welfare cost of the hyperinflation in Table 5 when computed as a fraction of total consumption in the 10 quarters leading to the hyperinflation and the ERR. The welfare cost reported there ranges from 1.8% of GDP to 2.9% depending on what we assume is the welfare gain of the monetary system. These numbers, accumulated over 10 quarters (2.5 years) represent a range between 4.5% and 7.2% of the GDP in one year. These welfare gains represent money on the table. To grab a good share of that, a very short loan of a smaller magnitude is required — a loan that can be paid in full very quickly and that in all cases leaves the government with some extra revenue. These ERRs appear to be very close to a free lunch.

7 Conclusions

Some countries have recently been confronted with the following policy decision: is it worthwhile to leave the European Monetary Union? In this paper we analyze this policy decision when it involves no default on government debt and the government cannot increase its debt level after exiting. In this situation, leaving the Euro with the desire to regain control of the domestic central bank is just an illusion: the money supply will be driven by the need to finance the deficit.

The first contribution of this paper is to show that deficit monetization following a departure

[25]Interestingly, this process of reserve accumulation has been a common feature in the stabilization plans in Latin America in the 1980s.

from the Euro carries very high risks given the current levels of government deficit. If agents' beliefs are close to, but not precisely rational, exiting the Euro is likely to lead to hyperinflations. Furthermore, these beliefs are consistent with rational behavior since agents would not reject them using equilibrium data. As we show, and as supported by ample empirical evidence, it is possible to stop hyperinflations with appropriately designed exchange rate regimes *and* lower average seigniorage. As hyperinflations are very costly, countries exiting the Euro should find ways to reduce deficits with self-imposed austerity. We also show that if a country were to leave the Euro, an exit could be welfare improving if an agreement with an IMF-like institution that can provide short-term financing if the self-imposed austerity plan would take several years to be completed.

The second contribution of the paper is to use the framework of internal rationality to perform policy analysis. We first clearly stated the assumptions about the system of beliefs. We then justified the validity of those assumptions using various criteria that we spelled out in detail. Finally, we calibrated the parameters of the belief system, as is usually done with preference or technology parameters. As we show, the effects of the policy decision will depend on the agents' expectations in the model: if agents have RE, leaving the Euro implies higher inflation, but if agents learn about underlying inflation as a consequence of their "near-rational" belief system, rational behavior will lead to hyperinflations. That the policy outcome so critically depends on how expectations are formed is an advantage of our approach, as it highlights that this is a key element in policy decisions, since policy makers never really know how agents' expectations will react to a policy change.

This paper hardly exhausts the effects of leaving the Euro. Exiting countries could have an outright debt default, but this alternative has additional costs that should be factored in, as shown by the large literature on sovereign defaults. Exiting countries may not actually lose access to debt markets, but even if they have access to debt markets, given their very high current debt levels, it is unlikely that they can keep increasing their debt levels. Exiting countries may hope that a devaluation brings some growth. This development would be beneficial in itself and would decrease the deficit as a percentage of GDP. But past experience shows that post-devaluation growth is not always to be had: most of its effect comes through devaluating internal salaries. That is, in our model, lowering civil servant salaries is a convenient way to implement austerity and indeed lower the probability of a hyperinflation, but this is austerity in disguise anyway. Such a lowering of the deficit thanks to inflation can be modeled by making d_t depend on equilibrium inflation, a substantial complication that goes far beyond the current setup. Most importantly, we have also yet to explore different alternatives for the resetting of beliefs after leaving the Euro, to consider various paths for the deficit, to calibrate to other countries, and so on. These remaining issues can be incorporated in future research that goes beyond the scope of this paper.

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A Linearization of the Money Demand Equation

We linearize the individual money demand around a fictitious steady state with no aggregate uncertainty. In this appendix, we use the notation $m_t^j \equiv M_t^j/P_t$ and $\pi_{t+1} \equiv P_{t+1}/P_t$. The linearization boils down to

$$\left(\frac{1}{1-\tilde{m}^j}\right)^2 (m_t^j - \tilde{m}^j) = -\left(\frac{\alpha_t^j}{m_t^j + e_{t+1}^j \tilde{\pi}_{t+1}}\right)^2 \frac{1}{\alpha_t^j} \left\{\mathbf{E}_t^j[m_t^j - \tilde{m}^j] + e_{t+1}^j \mathbf{E}_t^j[\pi_t^j - \tilde{\pi}^j]\right\},$$

where the tilde variables represent steady state variables. In steady state we must have that

$$\frac{1}{1-\tilde{m}^j} = \frac{\alpha_t^j}{\tilde{m}^j + e_{t+1}^j \tilde{\pi}},$$

which also implies that

$$\frac{1+\alpha_t^j}{\alpha_t^j}\tilde{m}^j + \frac{e_{t+1}^j}{\alpha_t^j}\tilde{\pi} = 1.$$

Using these two expressions above and rearranging, we obtain

$$m_t^j = \frac{\alpha_t^j}{1+\alpha_t^j}\left\{1 - \frac{e_{t+1}^j}{\alpha_t^j}\mathbf{E}_t^j[\pi_{t+1}]\right\},$$

which corresponds to the expression in the main text.

B Proof of Proposition 1

If $d_0 = \delta$, it is straightforward that $\underline{\pi}(d_0) = \boldsymbol{\pi}_1(\delta)$. Hence, the goal is to show that $\underline{\pi}(d_0)$ exists when $d_0 \neq \delta$. The logic consists of showing that, given d_0, one can find a value for π_0 to be exactly at $\boldsymbol{\pi_1}(d_t)$ in period t, where $\boldsymbol{\pi_1}(d_t)$ denotes the solution to the quadratic equation (21) given the level of seigniorage d_t. Notice that such value is well defined for all d_t as long as $d_0 \in \mathbf{D}$.

Lemma 1 *If $d_0 \neq \delta$, there exists a monotone and bounded sequence of initial conditions $\{\beta_t\}$ such that for all t, $\pi_0 = \beta_t$ implies $\pi_t = \boldsymbol{\pi_1}(d_t)$.*

This result indicates that $\lim_{t\to\infty}\beta_t$ is well defined. We denote this limit by $\underline{\pi}(d_0)$, making explicit its dependence on the initial value of seigniorage d_0. Furthermore, it also indicates that if $\pi_0 = \underline{\pi}(d_0)$, then $\lim_{t\to\infty}\pi_t = \lim_{t\to\infty}\boldsymbol{\pi_1}(d_t) = \boldsymbol{\pi_1}(\delta)$. Hence, the second statement of Proposition 1 can be viewed as a corollary of Lemma 1. Although the two cases $d_0 < \delta$ and

$d_0 > \delta$ need to be considered separately, the proof is analogous, so we present only the one corresponding to $d_0 < \delta$.

Proof of Lemma 1. The proof is by induction. For the initial step, observe that

$$\pi_0 = \boldsymbol{\pi_1}(\delta) \implies \pi_1 = (1-\rho)\boldsymbol{\pi_1}(\delta) + \rho\mathbf{F}(\boldsymbol{\pi_1}(\delta), d_0) > \boldsymbol{\pi_1}(\delta) > \boldsymbol{\pi_1}(d_1),$$

where the first inequality follows from the following property about F:

$$\mathbf{F}(\pi, d) > \pi \iff \pi \in (\boldsymbol{\pi_1}(d), \boldsymbol{\pi_2}(d)),$$

and the second follows from the fact that $d < \delta$ implies $\boldsymbol{\pi_1}(d) < \boldsymbol{\pi_1}(\delta)$. On the other hand, we also have that

$$\pi_0 = \boldsymbol{\pi_1}(d_0) \implies \pi_1 = (1-\rho)\mathbf{F}(\boldsymbol{\pi_1}(d_0), \delta) + \rho\boldsymbol{\pi_1}(d_0) < \boldsymbol{\pi_1}(d_0) < \boldsymbol{\pi_1}(d_1).$$

Since $\mathbf{F}$ is continuous and monotone, it follows that there exists a unique $\beta_1 \in (\boldsymbol{\pi_1}(d_0), \boldsymbol{\pi_1}(\delta))$ such that if $\pi_0 = \beta_1$, then $\boldsymbol{\pi_1} = \boldsymbol{\pi_1}(d_1)$.

For the inductive step, suppose there exists β_t such that $\pi_0 = \beta_t$ implies $\pi_t = \boldsymbol{\pi_1}(d_t)$. Then it follows that

$$\pi_t = \boldsymbol{\pi_1}(d_t) \implies \pi_{t+1} = (1-\rho)\mathbf{F}(\boldsymbol{\pi_1}(d_t), d_t) + \rho\boldsymbol{\pi_1}(d_t) < \boldsymbol{\pi_1}(d_t) < \boldsymbol{\pi_1}(d_{t+1}),$$

and we again have that

$$\pi_t = \boldsymbol{\pi_1}(\delta) \implies \pi_{t+1} = (1-\rho)\boldsymbol{\pi_1}(\delta) + \rho\mathbf{F}(\boldsymbol{\pi_1}(\delta), d_t) > \boldsymbol{\pi_1}(\delta) > \boldsymbol{\pi_1}(d_{t+1}).$$

Hence, there exists $\beta_{t+1} \in (\beta_t, \boldsymbol{\pi_1}(\delta))$ such that $\pi_0 = \beta_{t+1}$ implies $\pi_{t+1} = \boldsymbol{\pi_1}(d_{t+1})$. This also implies that $\beta_{t+1} > \beta_t$ for all t, so the sequence is monotone and bounded. ■

To complete the proof of the proposition, take an arbitrary sequence $\{\pi_t\}$ that evolves according to (20) and suppose $\pi_0 > \underline{\pi}(d_0)$. Two cases need to be considered. First, if $\pi_0 \geq \max\{\boldsymbol{\pi_1}(\delta), \boldsymbol{\pi_1}(d_0)\}$, then (20) and the properties of F imply that statement 3 is satisfied. Second, if $\pi_0 \in (\underline{\pi}(d_0), \max\{\boldsymbol{\pi_1}(\delta), \boldsymbol{\pi_1}(d_0)\})$, then the proof of Lemma 1 indicates there exists a period t in which $\pi_t > \max\{\boldsymbol{\pi_1}(\delta), \boldsymbol{\pi_1}(d_t)\}$, and therefore (20) and the properties of F again tell us that statement 3 holds. The proof of the first statement, when $\pi_0 < \underline{\pi}(d_0)$, follows exactly the same steps.

C Linearization of Equation 18

To linearize equation (18), we treat π_t, π_t^e, and π_{t+1}^e as different variables. Thus, we obtain

$$\hat{\pi}_t = \frac{\delta}{\phi - \phi\gamma\boldsymbol{\pi}_1(\delta) - \delta}\hat{d}_t - \frac{\phi\gamma\boldsymbol{\pi}_1(\delta)}{\phi - \phi\gamma\boldsymbol{\pi}_1(\delta)}\hat{\pi}_t^e + \frac{\phi\gamma\boldsymbol{\pi}_1(\delta)}{\phi - \phi\gamma\boldsymbol{\pi}_1(\delta) - \delta}\hat{\pi}_{t+1}^e.$$

Rational expectations implies that, around the low inflation steady state, $\hat{\pi}_t^e = 0$ for all t. Therefore, the last two terms on the right-hand side cancel out, and we obtain the equation in the main text.

D Estimating Inflation Persistence

To calculate inflation paths, we use the following system:

$$\begin{aligned}
d_t &= (1-\rho)\delta + \rho d_{t-1} + \epsilon_t, && (36)\\
\pi_t &= \frac{\phi - \phi\gamma(\rho_\pi^2 \pi_{t-2} + (1-\rho_\pi^2)\,\beta_{t-1})}{\phi - \phi\gamma(\rho_\pi^2 \pi_{t-1} + (1-\rho_\pi^2)\,\beta_t) - d_t}, && (37)\\
\alpha_t &= 1 + \alpha_{t-1}, && (38)\\
\beta_t &= \beta_{t-1} + \frac{1}{\alpha}\left(\frac{\pi_{t-1} - \rho_\pi \pi_{t-2}}{1-\rho_\pi} - \beta_{t-1}\right), && (39)
\end{aligned}$$

with the initial condition $\{\pi_{-1}, d_0, \alpha_0, \beta_0\} = \{\boldsymbol{\pi}_1, \delta, \overline{\alpha}, \boldsymbol{\pi}_1\}$, where $\boldsymbol{\pi}_1$ is the low inflation steady state in the RE equilibrium, and $\overline{\alpha}$ is just a positive integer. We assume that the variance of the i.i.d. shock ϵ_t is small enough so that the system remains stable.

We generate N samples of length T for each variable defined in (36)-(39). Let $\hat{\rho}_\pi(N,T)$ be the bootstrap estimate of the persistence of inflation, which is a function of all primitives of the model, including ρ_π and ρ. We allow $\rho_\pi \neq \rho_\delta$ and restrict attention to positive autocorrelation coefficients. Figure 2 displays $\hat{\rho}_\pi(N,T)$ as a function of ρ_π. The figure shows that for any ρ, there is a unique ρ_π such that $\rho_\pi = \hat{\rho}_\pi(N,T)$. We are interested in those fixed points because they correspond to the case in which agents are able to estimate ρ_π using past data. Since we proved that in an RE equilibrium, $\rho = \rho_\pi$, considering $\rho_\pi \neq \rho_\delta$ implies that we are allowing the persistence of inflation to be biased in a learning equilibrium. The nature of that bias is portrayed in Figure 3, which plots the fixed point $\rho_\pi^\star = \rho_\pi = \hat{\rho}_\pi(N,T)$ as a function of ρ. The figure indicates that the bias tends to be negative for low persistence of seigniorage and positive for high persistence of seigniorage.

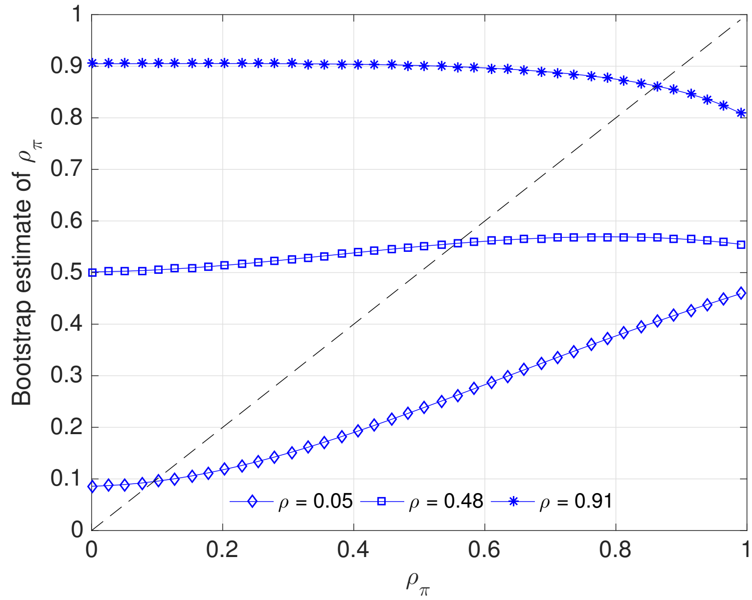

Figure 2: **Inflation persistence: bootstrap estimate vs. true parameter:** The points at which the solid lines cross the dashed line represent the fixed points $\rho_\pi = \hat{\rho}_\pi(N,T)$. For this simulation we set $N = 2,500$ and $T = 2,500$.

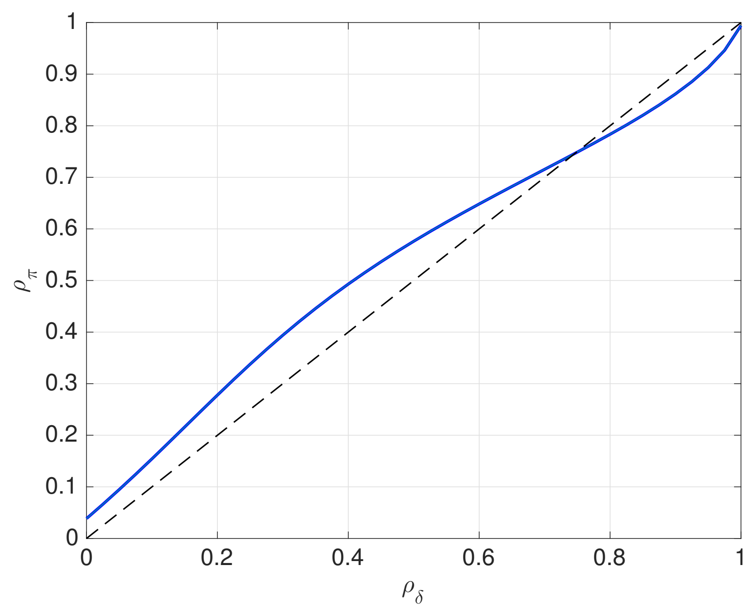

Figure 3: **Inflation persistence vs. seigniorage persistence:** The solid line represents the fixed points portrayed in Figure 2 for different values of ρ. For this simulation we set $N = 2,500$ and $T = 2,500$.

E Proof of Proposition 2

The proof considers the general formulation in which agents use information about the innovations to the deficit to adjust their expectations regarding future inflation.

Restriction 1. Note that, according to the belief system

$$\begin{aligned} e_t &= \pi_t^* + \psi\epsilon_t + u_t - \left(\pi_{t-1}^* + \psi\epsilon_{t-1} + u_{t-1}\right) \\ &= \eta_t + \psi\epsilon_t - \psi\epsilon_{t-1} + u_t - u_{t-1}, \end{aligned}$$

whereas

$$d_t - \rho d_{t-1} = (1-\rho)\delta + \epsilon_t,$$

so that Restriction 1 holds for $i \geq 2$.

Restriction 2. To prove Restriction 2, observe that

$$\mathbb{E}\left[(d_t - \rho d_{t-1})e_t\right] = \mathbb{E}\left[((1-\rho)\delta + \epsilon_t)(\eta_t + \psi\epsilon_t - \psi\epsilon_{t-1} + u_t - u_{t-1})\right] = \psi\sigma_\epsilon^2,$$

while

$$\mathbb{E}\left[(d_{t-1} - \rho d_{t-2})e_t\right] = \mathbb{E}\left[((1-\rho)\delta + \epsilon_{t-1})(\eta_t + \psi\epsilon_t - \psi\epsilon_{t-1} + u_t - u_{t-1})\right] = -\psi\sigma_\epsilon^2,$$

so that Restriction 2 also holds.

Restriction 3. Note that

$$\begin{aligned} \mathbb{E}\left[e_t e_{t-1}\right] &= \mathbb{E}\left[(\eta_t + \psi\epsilon_t - \psi\epsilon_{t-1} + u_t - u_{t-1})(\eta_{t-1} + \psi\epsilon_{t-1} - \psi\epsilon_{t-2} + u_{t-1} - u_{t-2})\right] \\ &= -\psi^2\sigma_\epsilon^2 - \sigma_u^2. \end{aligned}$$

Now consider the projection of $d_t - \rho d_{t-1}$ on e_t. The projection is given by

$$\frac{\text{Cov}\left[e_t, d_t - \rho d_{t-1}\right]}{\text{Var}\left[d_t - \rho d_{t-1}\right]}(d_t - \rho d_{t-1}),$$

where

$$\begin{aligned}
\mathrm{Cov}\left[e_t, d_t - \rho d_{t-1}\right] &= \mathrm{Cov}\left[\eta_t + \psi\epsilon_t - \psi\epsilon_{t-1} + u_t - u_{t-1}, \epsilon_t\right] = \psi\sigma_\epsilon^2, \\
\mathrm{Var}\left[d_t - \rho d_{t-1}\right] &= \sigma_\epsilon^2,
\end{aligned}$$

so that the projection is given by $\psi\epsilon_t$, and the variance of the projection is $\psi^2\sigma_\epsilon^2$. Plugging these results into the right-hand side of Restriction 3 delivers

$$\mathbb{E}\left[e_t, e_{t-1}\right] + b'^2\sigma_\epsilon^2 - \sigma_u^2 + \psi^2\sigma_\epsilon^2 = -\sigma_u^2 < 0.$$

Restriction 4. Since all the perturbation terms have zero expectation, Restriction 4 follows immediately.

F Inflation Persistence

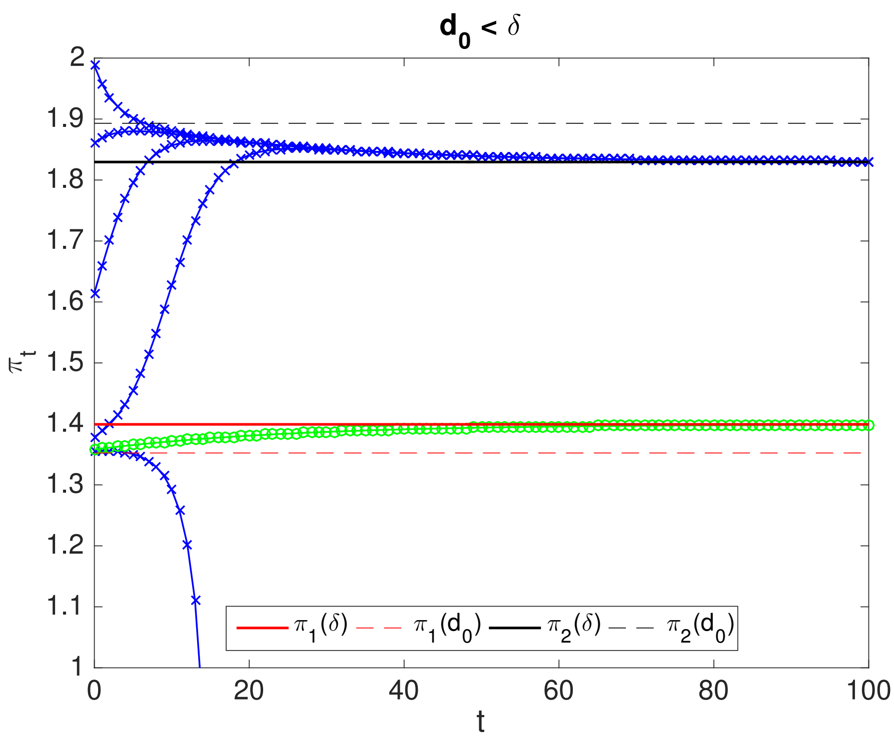

Figure 4: **Inflation paths under rational expectations that evolve according to (20).** The horizontal lines correspond to the solutions to the quadratic equation when $d = \delta$ (solid line) and $d = d_0$ (dashed line). The line with circles that converges to the low inflation steady state starts at $\pi_0 = \underline{\pi}(d_0)$, which is characterized in Proposition 1.

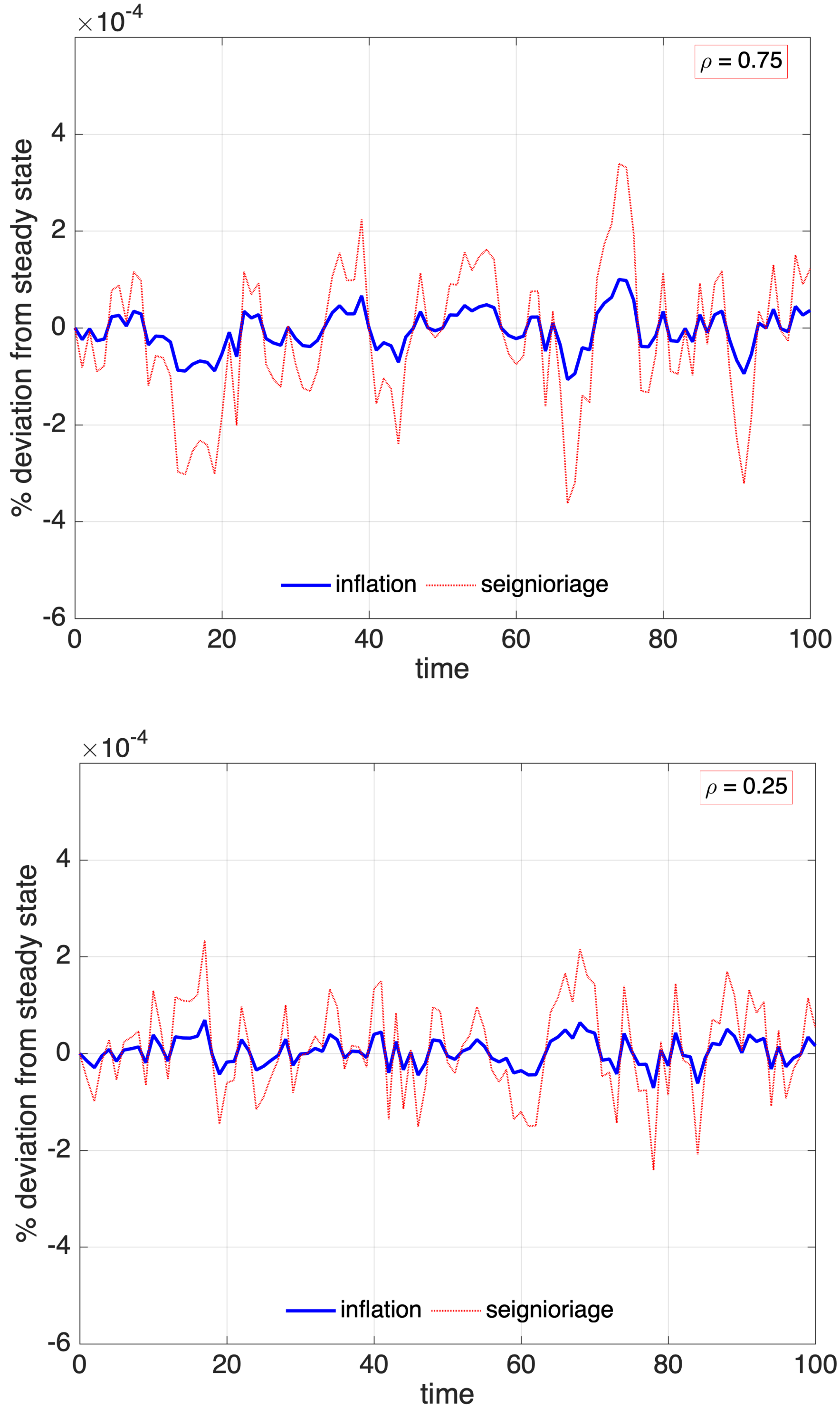

Figure 5: **Sample paths for inflation and seignioriage around the low inflation steady state in the linearized rational expectations equilibrium.**