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A Ramsey Theory of Financial Distortions*

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Abstract

The interest rate on government debt is significantly lower than the rates of return on other assets. From the perspective of standard models of optimal taxation, this empirical fact is puzzling: typically, the government should finance expenditures either through contingent taxes, or by previously-issued state-contingent debt, or by labor taxes, with only minor effects arising from intertemporal distortions on interest rates. We study how this answer changes in an economy with financial frictions, where the government cannot directly redistribute towards the agents in need of liquidity, but has otherwise access to a complete set of linear tax instruments. We establish a stark result. Provided this is feasible, optimal policy calls for the government to increase its debt, up to the point at which it provides sufficient liquidity to avoid financial constraints. In this case, capital-income taxes are zero in the long run, and the returns on government debt and capital are equalized. However, if the fiscal space is insufficient, a wedge opens between the rate of return on government debt and capital. In this case, optimal long-run tax policy is driven by a trade-off between the desire to mitigate financial frictions by subsidizing capital and the incentive to exploit the quasi-rents accruing to producers of capital by *taxing* capital instead. This latter incentive magnifies the wedge between rates of return on publicly and privately-issued assets.

JEL classification: E22, E44, E62;

Key words: Financing Constraints; Asset Directed Search; Capital Tax; Low Interest Rates; Optimal Level of Government Debt

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1 Introduction

How should governments finance their expenditures in the least costly way when capital is present? This question has attracted much interest in the past. Judd (1985), Chamley (1986), Chari, Christiano, and Kehoe (1994), and Siu (2004), and a large literature that follows, argues that taxing capital in the long run is a bad idea and that the interest rate on government debt, which is a perfect substitute for capital, should not be distorted as well.[1]

More recently, a lot of attention has been devoted to the study of financial frictions that generate imperfect substitution between assets. In this paper, we revisit the issue of capital taxation and intertemporal distortions in this context. In doing so, we uncover a tight connection between the two, that is at work both in the short run and even more so in the eventual long-run limit.

We adopt an otherwise standard neoclassical growth model, in which the government aims at an exogenous stream of expenditures, that is financed with taxes on income from labor and capital and by issuing debt. Our key point of departure is that investment is undertaken by entrepreneurs whose net worth affects their ability to access external sources of finance. In the model, private agents face idiosyncratic investment opportunities, as in Kiyotaki and Moore (2012). Some of them have investment projects, while others do not. When private agents have investment projects, they seek outside financing. But, because of asset liquidity frictions, only part of their claims to future investment or existing capital can be pledged. In contrast, government bonds are fully liquid instead and therefore can better finance any potential investment opportunity that arises. For this reason, households have a precautionary motive to buy them.

We first illustrate the optimal policy in a simple 2-period deterministic model in which entrepreneurs finance their investment by selling up to a fixed fraction of their investment, as well as their entire endowment of liquid government debt.[2] When entrepreneurs start with scarce liquidity, financial constraints drive a wedge between the rate of return accruing to buyers of capital, and that perceived by the constrained entrepreneurs, and the constraints reduce the elasticity of the supply of capital to its after-tax rate of return.

In the special case of a perfectly inelastic supply of capital, increasing capital-income taxes has no effect on investment and is simply a way of extracting what is a rent that entrepreneurs receive on their inframarginal units of investment. However, when financial frictions are such that investment can react to Tobin's q, a countervailing force emerges: by *subsidizing* capital, the government can push up Tobin's q and alleviate underinvestment. Which one of these forces dominates is a quantitative question, except when the government starts with enough assets: when the need to raise

[1] More recently, Lansing (1999), Bassetto and Benhabib (2006) and Straub and Werning (2020) show examples of economies where the Chamley-Judd result does not apply, and taxes on capital remain high in the limit. The economy that we study does not fall in this category; in the absence of financial frictions, the Chamley-Judd result would apply.

[2] An alternative equivalent interpretation is that entrepreneurs borrow and pledge as collateral up to a fraction of their investment and all of their government bonds.

distortionary taxes is zero (or close to it), optimal policy calls for undoing the financial distortions by subsidizing capital. Conversely, when the government is desperate for funds, its labor-income tax policy may depress the labor supply so much that investment drops to the point where financial constraints cease to bind, in which case the Chamley-Judd result reemerges and the optimal capital-income tax is zero. Positive capital taxation can emerge in an intermediate range, in which the government finds it optimal to exploit the low elasticity of the supply of capital to raise funds, as we show in a numerical example.

We then extend the analysis to an infinite-horizon economy and one in which the fraction of capital that can be sold can itself be endogenously determined from primitive assumptions about the intermediation technology, and we study the long-run properties of an optimal allocation. A stark result emerges. If the government is able to issue enough debt to completely eliminate financial frictions, it will choose to do so, and capital-income taxes will be zero in the limit. However, if this level of debt cannot be sustained by raising enough labor-income tax revenues, then the economy converges to a steady state with binding financing constraints, a positive capital tax, and a lower interest rate on government debt than the rate of time preference. In this case, government debt commands a liquidity premium because of the better liquidity service.

For the above result, it is crucial that financial frictions are specified in such a way that investment reacts to Tobin's q. As in the two-period economy, when investment is inelastically supplied as long as constraints bind, the planner always has an incentive to equalize the returns on government debt and capital by taxing the latter to the point at which constraints stop binding: this tax raises revenue without introducing any new distortions. The interplay between Tobin's q and rate of return differentials connects our theory to the corporate/banking finance view of public finance, in which other policies related to financial distortions are introduced, such as capital requirements, capital controls, liquidity coverage ratios, and other instrument that drive a wedge between rates of return of assets in different classes, thereby lowering the interest rate on government debt.

The paper starts by discussing its relationship with existing literature in Section 1.1. The two-period economy in which entrepreneurs can sell a fixed fraction of capital is our subject of interest in Section 2; in Section 3 we posit a more primitive intermediation technology and endogenize the fraction of capital sold, proving robustness of our conclusions to this slightly more tractable environment; Section 4 extends the analysis to an infinite-horizon economy and studies the properties of the limiting allocation, and Section 5 concludes.

1.1 Related Literature

Our paper builds on a large literature that introduced financial frictions in the form of imperfect asset liquidity. In addition to Kiyotaki and Moore (2012), similar economic environments appear in Shi (2015), Nezafat and Slavik (2010), Del Negro, Eggertsson, Ferrero, and Kiyotaki (2017), Ajello

(2016), and Bigio (2012), among many others. In particular, Cui and Radde (2016a,b) and Cui (2016) propose a framework where asset liquidity is determined by search frictions and the supply of government debt can affect the participation in asset markets.[3] Search frictions exist in many markets, such as those for corporate bonds, IPO, and acquisitions. They can also capture many aspects of frictional financial markets with endogenous market participation (see e.g., Vayanos and Wang, 2013; Rocheteau and Weill, 2011), while still keeping the simple structure of the neoclassical growth model. This tractability is crucial since one can use all the insights from a standard Ramsey plan. In particular, we use the "primal approach" (see e.g., Lucas and Stokey, 1983; Chari and Kehoe, 1999) to show the allocations chosen by a Ramsey planner. While not essential for our results, this asset-search specification carries the benefit of smoothing some of the kinks inherent in the financing constraints, thereby improving tractability and intuition.

The presence of liquidity constraints opens the possibility for government bonds or even fiat money to circulate, as in Holmström and Tirole (1998). If private liquidity is not enough, public liquidity can improve efficiency.[4] In this paper, government debt provides liquidity services and has a "crowding-in" effect, similar to Woodford (1990).[5] At the same time, the need to raise distortionary taxes limits the government's ability to flood the market with liquidity so that an optimal supply of public liquidity emerges. Our work is complementary to Collard, Dellas, and Angeletos (2020), who study a model where non-state-contingent government bonds also may crowd in private investment.[6] An important difference between their setup and ours is that we allow for capital-income taxes, so that the tax system is complete at the macroeconomic level. This separates the role of interest-rate distortions as a way of indirectly taxing capital (whose production is facilitated by debt due to the financing frictions) from their germane role as a manipulation of relative intertemporal prices.[7] The completeness of the tax system implies that our results would extend to implementations that use other tax instruments, such as a consumption tax, or an investment credit.

While taxes impinge on all of the intratemporal and intertemporal margins of the choices faced by households, the timing we assume rules out the possibility for the government to directly send differential payments to agents in need of liquidity at the moment in which they experience the need. In this, our paper is different from Itskhoki and Moll (2019), who study the mix of labor

[3] Recent work by Lagos and Rocheteau (2008), Rocheteau (2011), and Cao and Shi (2014) also use search models to endogenize liquidity and asset prices, but they do not study the individual trade-off that agents face between asset liquidity and prices. This channel gives rise to different degrees of liquidity constraints and risk sharing.

[4] Changing the portfolio compositions of the two assets can potentially affect the real economy. More recent papers enriched the basic structure by explicitly introducing financial intermediaries that are subject to independent frictions. See, for example, Gertler and Karadi (2011) and Gertler and Kiyotaki (2010).

[5] This aspect is in contrast with Aiyagari and McGrattan (1998), where government debt is a perfect substitute to private assets (or capital stock). There, government debt relaxes agents' borrowing constraints but also crowds out capital accumulation.

[6] A similar setup is used in Cao (2014) to analyze inflation as a shock absorber in the government budget constraint.

[7] Capital only appears in the appendix of Collard, Dellas, and Angeletos (2020). In the main text version, the untaxed good is the "morning" good, and government debt serves a liquidity role in its consumption, rather than in investment.

and capital-income taxes as a way of redistributing across different actors along the development path of an economy with two classes of agents and financial constraints. Redistribution across different agents also plays the dominant role in Azzimonti and Yared (2017, 2019), who consider the optimal supply of public liquidity with lump-sum taxes when agents differ in their income. Their framework also generates an incentive for the government to manipulate debt prices, keeping interest rate low and some agents liquidity constrained. Finally, redistribution takes center stage also in Chien and Wen (2018, 2020), that revisit capital-income taxation and debt in incomplete-markets models à la Bewley. Our paper is complementary to theirs: while the frictions in their papers and ours are substantially different, so that capital tends to be *over*provided in Bewley models while it is *under*provided in models of financial constraints on capital, a common theme emerges that the government is pushed to move away from tax smoothing towards path of increasing debt to relax the household constraints to the extent possible, and resorts to distorting capital accumulation through taxes only when this avenue is exhausted. In contrast, the specific nature of optimal tax distortions is different in the two settings and has to be tailored to the friction that impinges on capital accumulation.

Finally, a different motive for manipulating interest rates is analyzed in Farhi, Golosov, and Tsyvinski (2009), where this distortion is introduced to alleviate the impossibility of signing exclusive contracts with financial intermediaries in the presence of private information.

2 A Simple Two-period Framework

We start our analysis with a two-period model. Both the provision of public liquidity and the degree of illiquidity of private assets are exogenous in this section. We analyze how liquidity frictions affect the choice of distorting return on capital and interest rates, and how this choice in turn depends on the fiscal constraints faced by the government. Throughout the paper, we use lowercase variables for individual choices, and uppercase for aggregate allocations, except for prices and taxes.

2.1 The Environment

In period $t = 1$, a continuum of firms can produce output by using labor using a constant returns technology, with one unit of labor normalized to produce 1 unit of output. In period 2, the firms have a technology $F(K_1, L_2)$, where K_1 and L_2 are capital and labor utilized in period 2. We assume that F satisfies Inada conditions, so we can neglect corner solutions. Firms hire labor and rent capital in competitive markets at the wage rates w_1 and w_2, and the rental rate $\tilde{r}_2$.

The economy is populated by a continuum of families, each of which is composed by a continuum of agents. In period 1, they start with some (exogenous) government debt B_0. A fraction χ of agents from each household are revealed to be entrepreneurs and the remainder $1-\chi$ are workers. Entrepreneurs and workers are separated at the beginning of the period. The entrepreneurs have in total B_0^e units of government bonds, whereas workers have B_0^w units,[8] and we define total per-capita bonds to be[9]

$$B_0 \equiv B_0^e + B_0^w.$$

Period 1

Workers supply labor to the firms. Entrepreneurs do not supply labor. Rather, in period 1, they can turn one unit of the firms' output into one unit of new capital to be used in the subsequent period. This ability will only be used in the first period, since the economy ends after period 2. The amount that each entrepreneur invests is k_1^e, the amount of capital available at the beginning of period 2.

Entrepreneurs cannot sell the capital directly, but they can sell claims to the capital k_1^e in a frictional competitive market, in the amount s_1^e:

$$s_1^e \leq \phi_1 k_1^e, \tag{1}$$

where ϕ_1 is asset liquidity. An entrepreneur has internal funds arising from her holdings of government debt, equal to $R_1 B_0^e/\chi$, where R_1 is an exogenous initial return on government debt which we only include for symmetry of notation with the second period. The entrepreneur's budget constraint is

$$k_1^e \leq R_1 B_0^e/\chi + q_1 s_1^e : \tag{2}$$

entrepreneurs can only "borrow" by selling claims to capital at the price q_1, and any left-over funds after investment has taken place are brought back to the family at the end of the period. We will typically be interested in equilibria where constraint (2) is binding and entrepreneurs use all of their available funds to undertake new investment.

Workers use some of their income to purchase new claims to capital from entrepreneurs and new government debt b_1^w, and return the remaining funds to the family. Their period-1 budget constraint is

$$q_1^w s_1^w + b_1^w \leq R_1 B_0^w/(1-\chi) + w_1 \ell_1, \tag{3}$$

where $s_1^w \geq 0$ is the end-of-period private claims on capital that they purchase, ℓ_t is their labor supply, q_1^w is the price at which claims to capital can be bought.

[8] The per-entrepreneur level of initial bonds that entrepreneurs have is therefore B_0^e/χ, and the per-worker amount owned by workers is $B_0^w/(1-\chi)$.

[9] In multi-period versions, the identity of entrepreneurs will not be known ex ante and $B_0^e/\chi = B_0^w/(1-\chi)$. We keep the two initial conditions separate because this allows us to study how the problem changes as a function of the entrepreneurs' initial net worth.

At the end of the first period, entrepreneurs and workers rejoin their family, pool their capital and their left-over funds, pay taxes, and consume. Their constraint is

$$c_1 = (1-\tau_1^\ell)w_1\ell_1(1-\chi) + R_1B_0 - (1-\chi)b_1^w - (1-\chi)q_1^w s_1^w - \chi(k_1^e - q_1 s_1^e), \tag{4}$$

where c_t is the family's consumption in period t, and τ_t^ℓ is the tax rate on labor income.

Claims to capital are subject to an intermediation cost. Intermediaries are competitive and their cost is η per unit of capital intermediated; therefore we have

$$q_1^w = \eta + q_1. \tag{5}$$

In period 1, the government budget constraint ensures that its revenues from labor-income taxation and new borrowing cover debt repayments that become due as well as any government spending G_1:[10]

$$G_1 + R_1B_0 = B_1 + \tau_1^\ell w_1 L_1. \tag{6}$$

Period 2

The second and final period is similar to the first, except that no new investment takes place, so that entrepreneurs no longer have any role. We can then collapse the two subperiods, and simply write the joint family budget constraint as

$$c_2 = (1-\tau_2^\ell)w_2(1-\chi)\ell_2 + \left[(1-\tau_2^k)\tilde{r}_2\right]\left[\chi(k_1^e - s_1^e) + (1-\chi)s_1^w\right] + R_2(1-\chi)b_1^w, \tag{7}$$

where τ_2^ℓ is the labor income tax in period 2, and τ_2^k is the capital income tax in period 2. R_2 is the return of government bonds between period 1 and period 2.

The government budget constraint is:

$$G_2 + R_2B_1 = \tau_2^k\tilde{r}_2K_1 + \tau_2^\ell w_2 L_2, \tag{8}$$

with G_2 being government spending in the second period.

Contrary to period 1, the government is allowed to tax (or subsidize) capital in the second period at a rate τ_2^k, and our goal is to study how this power is used in the presence of financial frictions, together with interest rate R_2.

[10]Note that the individual labor supply is normalized in per-worker terms, while the aggregate labor supply is in per-capita terms. So, an aggregate labor supply L_1 corresponds to $L_1/(1-\chi)$ for each worker. Similar normalizations occur for aggregate capital K_1, bonds B_1, and intermediated capital S_1.

The household preferences are represented by:

$$\sum_{t=1}^{2} \beta^{t-1} \left[u(c_t) - v\left((1-\chi)\ell_t\right) \right], \tag{9}$$

where u and v are strictly increasing and continuously differentiable functions, u is weakly concave, and v is strictly convex.[11]

2.2 Competitive Equilibrium

We next characterize a competitive equilibrium.

The household maximizes (9), subject to (1), (2), (4), and (7), taking prices and taxes as given. We note that, in any equilibrium in which $q_1 < 1$, there would be no sales of capital.[12] With this observation, we can limit our analysis to $q_1 \geq 1$ without loss of generality.[13]

From the intermediaries' and firms' optimality conditions, we obtain (5),

$$w_1 = 1, \quad w_2 = F_L(K_1, L_2), \tag{10}$$

$$\text{and } \tilde{r}_2 = F_K(K_1, L_2). \tag{11}$$

From the families' necessary and sufficient first-order conditions we obtain:

- Labor supply in period $t = 1, 2$:

$$(1 - \tau_t^{\ell}) w_t u'(C_t) = v'(L_t); \tag{12}$$

- Demand for government bonds:

$$1 = \frac{\beta u'(C_2)}{u'(C_1)} R_2; \tag{13}$$

- Demand for claims on capital:

$$q_1^w \geq \frac{\beta u'(C_2)}{u'(C_1)} (1 - \tau_2^k) \tilde{r}_2, \tag{14}$$

  with equality if $S_1 > 0$;

[11] The particular choice of scale for the function v is a pure normalization that is convenient to obtain simpler expressions when studying the aggregate allocation.

[12] To see this, consider a family whose entrepreneurs are selling capital. By reducing investment and capital sales one for one, the family can simultaneously relax the constraints (1), (2), and (4). The last budget constraint is necessarily binding, since families would otherwise increase their consumption, hence the original plan cannot be optimal.

[13] For any competitive equilibrium in which $q_1 < 1$, there exists a competitive equilibrium with the same allocation and the same prices, except for $q_1 = 1$ and $q_1^w = 1 + \eta$.

- Investment and supply of claims:

$$u'(C_1) \leq \beta u'(C_2)(1 - \tau_2^k)\tilde{r}_2, \tag{15}$$

with equality if $S_1 = 0$, and the financing constraint of an entrepreneur implies

$$q_1 = \max\left\{1, \frac{K_1 - R_1 B_0^e}{\phi_1 K_1}\right\}. \tag{16}$$

In addition, a competitive-equilibrium allocation must satisfy the government budget constraints (6) and (8) and the resource constraints:

$$L_1 = C_1 + K_1 + (q_1^w - q_1)S_1 + G_1, \tag{17}$$

$$\text{and } F(K_1, L_2) = C_2 + G_2. \tag{18}$$

By Walras' law, the household budget constraints (4) and (7) must be satisfied as an equality by the aggregate allocation (chosen by the representative family). Furthermore, consumption is always strictly positive and it is always weakly preferable for a household to first use the workers' resources to fund consumption and only after these are exhausted to potentially cut back on the entrepreneurs' investment. Because of this, equation (3) does not bind in equilibrium: this equation constrains workers not to invest more than all of their earnings and assets and is slack if something is left for consumption.

Definition. A competitive equilibrium is an allocation $\{C_t\}_{t=1}^2$, $\{L_t\}_{t=1}^2$, and K_1, asset prices q_1^w and q_1, wage rate $\{w_t\}_{t=1}^2$, capital income rate $\tilde{r}_2$, and an interest rate R_2, such that (5), (6), (8), and (10)-(18) are satisfied, given a labor income tax rate $\{\tau_t^\ell\}_{t=1}^2$, capital tax rate τ_2^k, and bond supply B_1,

In any competitive equilibrium, market clearing implies that $S_1 \equiv S_1^w = S_1^e$, where S_1 is the per-capita level of intermediated capital. If $q_1 > 1$, then both the financing constraint and the entrepreneurs' budget constraint (1) and (2) bind; if $q_1 = 1$, (1) is certainly slack and (2) may or may not bind.

2.3 Optimal Policy with Zero Intermediation Costs

We start our analysis from the case in which intermediation costs are absent ($\eta = 0$), so that the financing constraint is the only departure from a standard neoclassical growth model. We also set $R_1 = 1$. We return to the role of these two parameters in the next section, where we endogenize intermediation costs and allow the government to determine the supply of public liquidity by setting

R_1.

Forming the Policy Problem We study the Ramsey outcome, that is, the best competitive equilibrium that maximizes (9) . To do so, we follow the primal approach, deriving a set of necessary and sufficient conditions for an allocation to be part of a competitive equilibrium, without reference to prices and tax rates. These conditions include a restriction that allows us to derive intermediated capital S_1 given the other variables (equation (19) below), and it is thus convenient to also substitute out this variable from the policy problem.

Given any allocation, we can ensure that (10) and (11) hold by setting factor prices w_t and r_t to the appropriate marginal product. Similarly, we can ensure that (12) hold with a suitable choice of τ_t^ℓ, for $t = 1, 2$; (13) holds for the appropriate choice of R_2.

Next, in order for (1) and (2) to hold and for S_1 to be optimally chosen, we must have

$$S_1 = \begin{cases} 0 & \text{if } K_1 \leq (1-\phi_1)K^*, \\ K_1 - (1-\phi_1)K^* & \text{if } K_1 \in ((1-\phi_1)K^*, K^*], \\ \phi_1 K_1 & \text{if } K_1 > K^*, \end{cases} \tag{19}$$

where

$$K^* := \frac{B_0^e}{1-\phi_1}.$$

K^* is the maximum level of investment that entrepreneurs can finance when $q_1 = 1$, and $(1-\phi_1)K^*$ is the maximum that they can finance using internal funds only.

τ_2^k can then be chosen so that either (14) or (15) hold as an equality, depending on whether S_1 is 0 or positive, with the remaining of the two equations holding as the appropriate inequality. Finally, equation (16) can be used to determine q_1 and (5) to determine q_1^w.

The remaining conditions that characterize a competitive equilibrium are the following:

- The resource constraints (17) and (18); and
- The household budget constraints evaluated at the aggregate allocation, (2), (4), and (7).[14]

Substituting prices and tax rates from the first-order conditions, we can aggregate the household budget constraints into the following implementability constraint for period 1 and period 2:

$$\sum_{t=1}^{2} \beta^{t-1}[u'(C_t)C_t - v'(L_t)L_t] = u'(C_1)B_0 + \begin{cases} 0 & \text{if } K_1 \leq K^* \\ \left(\frac{1}{\phi_1} - 1\right) u'(C_1)(K_1 - K^*) & \text{if } K_1 > K^* \end{cases}. \tag{20}$$

[14]The financing constraint (1) holds by construction when (19) holds.

As usual in Ramsey problems, the implementability constraint represents the cost for the government not to have access to lump-sum taxation.[15]

The implementability constraint has two branches, corresponding to the two possible types of equilibria in our economy. In the first case, the financing constraint is slack and the price of capital q_1 is 1. This happens when either the entrepreneurs are sufficiently wealthy to finance all of the investment internally, or when they issue claim to capital that fall short of the constraint (1). In this case, our economy behaves as a standard neoclassical growth model. In the second case, when the financing constraint is binding, a new term appears in Equation (20); this term captures the fact that, when financing constraints are binding, entrepreneurs face a different intertemporal trade-off than workers. When the present-value budget constraint is evaluated at the trade-off faced by workers, who are the unconstrained agents in the family, capital appears as an extra source of revenues. This happens because the entrepreneurs require only one unit of period-1 good to produce 1 unit of capital, but the price of capital is $q_1 > 1$, and the last term in (20) captures the family's profits from investment. These profits emerge because entrepreneurial net worth plays the same role as a factor of production: it expands the economy's ability to produce capital.[16]

Taxation of capital in the Ramsey outcome is shaped by two countervailing effects:

- The presence of entrepreneurial net worth as a fixed factor implies that the government has an incentive to tax the associated rents, which can be done through capital-income taxes. To see this transparently, consider a modification of the environment in which the price of capital does not enter in the entrepreneurs' constraints, but rather (1) and (2) are replaced by a single collateral constraint that only involves the entrepreneurs' initial net worth:

$$\theta k_1^e \leq R_1 B_0^e / \chi. \tag{21}$$

In this case, as long as the financing constraint is binding, investment is fixed by initial conditions and exogenous parameters, and is completely unresponsive to taxes. With this modified constraint, the implementability constraint would change to

$$\sum_{t=1}^{2} \beta^{t-1}[u'(C_t)C_t - v'(L_t)L_t] = u'(C_1)B_0 + \begin{cases} 0 & \text{if } \theta K_1 < R_1 B_0^e \\ [\beta u'(C_2)\tilde{r}_2(1-\tau_2^k) - u'(C_1)]K_1 & \text{if } \theta K_1 = R_1 B_0^e. \end{cases} . \tag{22}$$

In the presence of a binding financing constraint, taxation of capital is equivalent to taxing a

[15] We assume here that any lump-sum transfers would be paid to the households *after* investment has taken place, so that they do not relax the financing constraint (2). In this case, the binding side of the constraint is that the left-hand side must be no smaller than the right-hand side, as is the case in standard Ramsey problems.

[16] Note that K_1^* is proportional to entrepreneurial net worth.

pure rent, or to have access to a lump-sum tax: it relaxes the implementability constraint and has no further direct effect on the allocation.[17] The government would thus optimally choose τ_2^k sufficiently high that $\beta u'(C_2)\tilde{r}_2(1-\tau_2^k) - u'(C_1) = 0$ and so that (21) is not binding.[18] Compared to a standard neoclassical growth model, financing constraints thus introduce an extra motive to tax capital.

- In our more realistic case in which investment can respond to changes in Tobin's q, capital-income taxes retain their ability to capture some of the pure profits arising from entrepreneurial net worth, but at the same time they may depress investment, which is already inefficiently low. In this case, it is possible that the government may want to *subsidize* capital to increase its price and relax the entrepreneurs' financing constraint.

We illustrate these countervailing forces in a special case.

2.4 A Special Case

To clarify the role of different distortions, we consider the special case of a Cobb-Douglas production function, $F(K_{t-1}, L_t) = AK_{t-1}^{\alpha}L_t^{1-\alpha} + (1-\delta)K_{t-1}$, where $\alpha \in (0,1)$, and preferences given by

$$u(c_t) - v(\ell_t) = c_t - \frac{\mu \ell_t^{1+\nu}}{1+\nu},$$

where $\mu > 0$ and $\nu > 0$. These preferences are convenient because they abstract from the usual incentive to distort intertemporal prices and devalue the families' initial claims, as emphasized by Armenter (2008). In the absence of financing constraints, they imply that the optimal tax on capital income is zero not just in the long run, but in every period (except period 1, in which our model has no capital). We can thus focus on the new channels of intertemporal distortions that arise from financial frictions.

We now derive the first-order conditions that must hold if the Ramsey plan is interior. However, it is possible that the plan will be at the kink, which needs to be checked separately. We are particularly interested in studying comparative statics when the financing constraint is binding and Tobin's q responds to investment, which will be the case when the entrepreneurs' wealth is sufficiently low relative to the return on capital and the government's resources are sufficiently scarce relative to its spending.

[17] Of course, there would be an indirect beneficial effect on the allocation from the government's ability to use the extra revenues to reduce distortionary labor-income taxes.

[18] In a competitive equilibrium, household optimality implies that the government cannot drive $\beta u'(C_2)\tilde{r}_2(1-\tau_2^k) - u'(C_1) < 0$: once the financing constraint becomes slack, further increases in capital-income taxation would depress incentives to invest in the same way they do in an economy that is not subject to financing constraints.

Let $\beta^{t-1}\lambda_t$ be the Lagrange multiplier on the resource constraint, for $t = 1$ and $t = 2$, and let Ψ_1 be the Lagrange multiplier on the implementability constraint. The planner's first-order conditions for consumption C_1 and C_2 are

$$\begin{aligned} & u'(C_1)(1+\Psi_1) + \Psi_1 u''(C_1)C_1 - \lambda_1 - \Psi_1 u''(C_1)B_0 \\ &= \begin{cases} 0 & \text{if } K_1 \leq K^* \\ \Psi_1(1/\phi_1 - 1)u''(C_1)(K_1 - K^*) & \text{if } K_1 > K^* \end{cases} \end{aligned} \tag{23}$$

$$u'(C_2)(1+\Psi_1) + \Psi_1 u''(C_2)C_2 - \lambda_2 = 0 \tag{24}$$

The planner's first-order conditions for labor supply L_1 and L_2 are

$$v'(L_1)(1+\Psi_1) + \Psi_1 v''(L_1)L_1 = \lambda_1 A \tag{25}$$

$$v'(L_2)(1+\Psi_1) + \Psi_1 v''(L_2)L_2 = \lambda_2 F_L(K_1, L_2) \tag{26}$$

The first-order conditions for capital K_1 is

$$-\lambda_1 + \beta\lambda_2 A F_K(K_1, L_2) \begin{cases} = 0 & \text{if } K_1 < K^* \\ \in [0, \Psi_1 u'(C_1)(1/\phi_1 - 1)] & \text{if } K_1 = K^* \\ = \Psi_1 u'(C_1)(1/\phi_1 - 1) & \text{if } K_1 > K^* \end{cases} \tag{27}$$

In our special case, the marginal utility of consumption is one. From the planner's first-order condition for consumption, we have $\lambda_1 = \lambda_2 = 1 + \Psi_1$. The labor supply in period 1 is simply a function of Ψ_1, and we can express the other two first-order conditions for labor supply and capital used in period 2 as follows:

$$\mu L_2^{\nu}\frac{1+\Psi_1(1+\nu)}{1+\Psi_1} = A(1-\alpha)\left(\frac{K_1}{L_2}\right)^{\alpha} \Longrightarrow L_2 = \left[\frac{A(1-\alpha)}{\mu}\frac{1+\Psi_1}{1+(1+\nu)\Psi_1}\right]^{\frac{1}{\alpha+\nu}} K_1^{\frac{\alpha}{\alpha+\nu}} \tag{28}$$

and

$$\beta A\alpha(K_1/L_2)^{\alpha-1} = 1 - \beta(1-\delta) + \begin{cases} 0 & \text{if } K_1 < K^* \\ \in [0, \frac{\Psi_1(\phi_1^{-1}-1)}{1+\Psi_1}] & \text{if } K_1 = K^* \\ \frac{\Psi_1(\phi_1^{-1}-1)}{1+\Psi_1} & \text{if } K_1 > K^* \end{cases} \tag{29}$$

Consumption C_1 and C_2 can be derived from the resource constraints.

Comparing the planner's optimality condition for capital (29) with the household optimality conditions (14) and (15) (also using the fact that $\tilde{r}_2 = A\alpha(K_1/L_2)^{\alpha-1} + 1 - \delta$), we can establish the following:

- If the allocation is such that the financing constraint of the entrepreneurs is not binding, then capital-income taxes are optimally set to zero, independently of the tightness of the government budget constraint (captured by the multiplier Ψ_1). In this case, we recover the standard result that it is not optimal to tax capital, which is an intermediate input.[19] This case can arise either when entrepreneurs have enough wealth to finance investment internally, in which case the private cost of investment is 1 and the social cost is $1+\Psi_1$, or when they need to sell part of their capital, but not to the point at which q need to exceed 1. In both cases, the private reward in the second period is $\beta\tilde{r}_2$ and the social reward is $\beta\tilde{r}_2(1+\Psi_1)$. Thus, private and social costs are proportional to each other and capital-income taxes are zero; moreover, in both cases the trade-off coincides with the marginal rate of transformation coming from technology alone, taking into account the costs of intermediation.[20]

- When entrepreneurs are sufficiently poor that the financing constraint binds, we obtain a very different result. In this case, in the absence of capital taxes or subsidies, the private rate of return does not coincide with the marginal rate of transformation. Furthermore, changes in the level of investment have an effect on the price of capital, and a higher price of capital tightens in turn the implementability constraint, forcing the government to raise more funds through distortionary taxes.[21] If the government has abundant resources and $\Psi_1 \approx 0$, comparing (29) and (14) (taking into account $K_1 > K^*$) we can see that the optimal policy calls for a capital subsidy. By subsidizing capital income in the second period, the government can raise the price of period-1 claims to capital, which in turn relaxes the entrepreneurs' constraints and allows the economy to attain a higher (and more efficient) level of investment. However, as the cost of public funds Ψ_1 increases, the rents that we isolated in the case of a fixed collateral constraint become more important: financing constraints may weaken the link between investment and future capital income, so that capital-income taxes may be less distortionary than they would be in a world of perfect capital markets. For this reason, it eventually becomes ambiguous whether a government strapped for cash would subsidize or tax capital.

By assuming linear preferences, we automatically imposed from equation (13) that $R_2 = 1/\beta$, that is, the government choice of taxes or subsidies has no effect on the rate of return on government debt.

[19] Of course, this result also relies on the fact that the preferences that we assumed rule out distorting intertemporal prices to devalue initial claims or to enhance the present value of taxes on labor. For more general preferences, both of these forces would be in play, as they are in a standard neoclassical growth model, and our effect would appear in addition to those.

[20] Positive intermediation costs $\eta > 0$ would not change this result, because both private/social rewards and private/social costs would be multiplied by the same factor $1 + \eta$.

[21] The Lagrange multiplier Ψ can be viewed as the cost to the planner of starting with an extra unit of government debt in period 0.

A further channel at work when preferences are not linear is that a capital-income tax reduces the after-tax return on capital, and hence further favors government debt, which is a further beneficial force in the case of a constrained government. This effect appears on the right-hand side of equation (23) and we will analyze it later in the infinite-horizon economy.

The above simplification implies that we can express all equilibrium outcomes in closed form. There are three cases, with the last case having K_1 at the kink K^*. Rather than varying B_0 and finding the implied value of Ψ_1, we find it more intuitive to graph directly the optimal solution treating Ψ_1 as a parameter, and then backing out the corresponding B_0 from the resulting allocation and prices and the government budget constraint.[22]

Case 1: when $K_1 \leq K^*$. This case occurs when the financing constraint is slack. The planner's first-order condition for capital becomes $\beta\tilde{r}_2 = 1$ and the price $q_1^w = q_1 = 1$. This condition and the household's first-order condition for capital (Equation (14)) imply no taxes on capital: $\tau_2^k = 0$. From the first-order conditions, we know that $K_1 = K_1^u(\Psi_1)$, where

$$K_1^u(\Psi_1) := \left[\frac{1+\Psi_1}{1+(1+\nu)\Psi_1}\frac{A(1-\alpha)}{\mu}\right]^{\frac{1}{\nu}}\left[\frac{1}{A\alpha}\left[\frac{1}{\beta}-(1-\delta)\right]\right]^{\frac{\alpha+1}{(\alpha-1)\nu}}. \tag{30}$$

which is a decreasing function of Ψ_1. A higher Ψ_1 implies higher labor-income taxes, which (given our preferences) reduce the labor supply and discourage investment.

Case 2: when $K_1 > K^*$. We can express labor supply L_2 and capital stock K_1 as functions of Ψ_1, we know that $K_1 = K_1^c(\Psi_1)$ where

$$K_1^c(\Psi_1) := \left[\frac{1+\Psi_1}{1+(1+\nu)\Psi_1}\frac{A(1-\alpha)}{\mu}\right]^{\frac{1}{\nu}}\left[\frac{1}{A\alpha}\left[\frac{\phi_1+\Psi_1}{\phi_1(1+\Psi_1)\beta}-(1-\delta)\right]\right]^{\frac{\alpha+1}{(\alpha-1)\nu}}, \tag{31}$$

which is also a decreasing functions of Ψ_1 after noticing that $\phi_1 \in (0,1)$ and $\alpha \in (0,1)$.

Since $(\phi_1+\Psi_1)/\phi_1(1+\Psi_1) > 1$, $K_1^u(\Psi_1) \geq K_1^c(\Psi_1)$, with the inequality being strict for any $\Psi_1 > 0$. It follows that case 1 will occur when $K_1^u(\Psi_1) < K^*$, so that the financing constraint is slack and case 2 will occur when $K_1^c(\Psi_1) > K^*$, in which case the financing constraint binds and the level $K_1^c(\Psi_1) > K^*$ can only be financed because $q_1 > 1$. When $K^* \in [K_1^c(\Psi_1), K_1^u(\Psi_1)]$, we obtain the following:

Case 3: $K_1 = K^*$. Labor in period 2 can be still expressed as in (28) by setting $K_1 = K^*$. At this kink, the incentive for the government to tax or subsidize capital undergoes a jump represented by the two branches of Equation (29). As Ψ_1 increases and the government budget constraint becomes tighter, the tax on labor must increase, discouraging labor supply in the second period. However, because of the kink, the optimal level of capital stays constant for a range of values of Ψ_1, with

[22] In this experiment, B_0^e is kept fixed, so that B_0 affects the shadow cost of public funds, but not the entrepreneurs' financing constraints; all the residual bonds are allocated to the workers.

capital taxation adjusting to ensure that this is the case.

To further characterize the solution, we note that both $K_1^u(\Psi_1)$ and $K_1^c(\Psi_1)$ are strictly decreasing in Ψ_1. When $\Psi_1 = 0$, the government has sufficient wealth at the beginning that the shadow cost of resources in the government budget constraint is zero. In this case, the government can undo the effect of financial constraints by subsidizing the return on capital in the second period, thereby raising the price of capital q_1 to a level which replicates the efficient level of investment in the absence of constraints, which is why $K_1^c(0) = K_1^u(0)$. In contrast, for any $\Psi_1 > 0$, that is, when the government is forced to raise revenues through distortionary means, if the solution to the Ramsey problem ignoring the financing constraint would violate the constraint itself, we have $K_1^u(\Psi_1) > K^*$ and it is never optimal for the planner to subsidize capital to the point that the constraint is slack from the perspective of the households and that households thus invest $K_1^u(\Psi_1)$.

The properties of the functions $K_1^u(\Psi_1)$ and $K_1^c(\Psi_1)$ that we established above allow us to describe how the Ramsey allocation changes with Ψ_1, fixing other parameters. This is summarized in the following proposition:

Proposition 1. *The Ramsey allocation can be characterized as follows:*

- *The economy under the planner's allocation is going to be financially unconstrained, regardless of Ψ_1, if*
$$K^* \geq K_1^c(0) = K_1^u(0),$$

- *If*
$$K_1^u(0) > K^* > K_1^u(\infty),$$
then the economy is financially constrained for small levels of Ψ_1 and capital is given by $K^1 = K^c(\Psi_1)$; the economy is financially unconstrained for large values of Ψ_1 and capital is given by $K^1 = K^u(\Psi_1)$, and there is an intermediate range of values of Ψ_1 for which the Ramsey allocation has capital exactly at the kink.

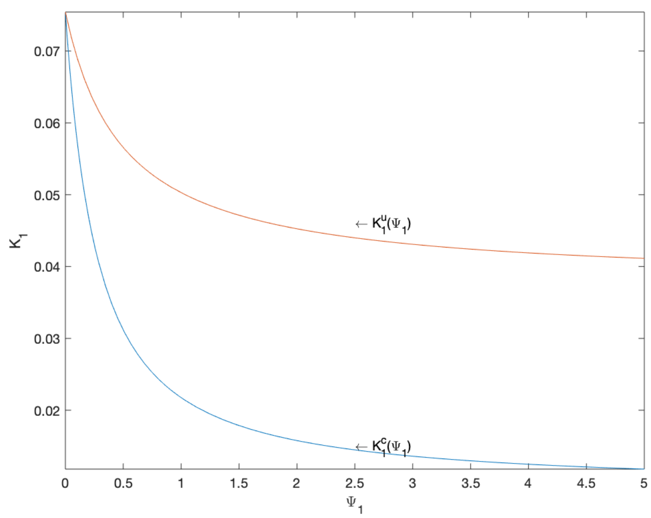
- *If*
$$K_1^u(\infty) \geq K^* > K_1^c(\infty),$$
then the economy is financially constrained for small levels of Ψ_1 and capital is given by $K_1 = K_1^c(\Psi_1)$, and it is at the kink for higher values of Ψ_1.

- *Finally, the economy is always financially constrained for any $\Psi_1 \geq 0$ and capital is $K_1 = K_1^c(\Psi_1)$ if*
$$K_1^c(\infty) \geq K^*. \quad \square$$

A Numerical Example We use a numerical example to illustrate the previous results. We consider parameter combinations that lead to a binding financing constraint and also others that make it slack.

First, consider the following parameters: $\beta = 0.96$ (discount factor), $\alpha = 0.33$ (capital share), $\delta = 0.95$ (depreciation rate), $\mu = 1$ (disutility parameter of labor supply), $\nu = 1$ (labor supply elasticity), $A = 1$ (productivity), and $\phi = 0.5$ (asset liquidity). With linear utility in consumption, the Ramsey allocation depends on government spending only through the multiplier Ψ_1, and we thus do not need to specify it explicitly as explained above.[23] We plot K_1, τ_2, L_2, and q_1 against Ψ_1

Figure 1: The functions $K_1^u(\Psi_1)$ and $K_1^c(\Psi_1)$



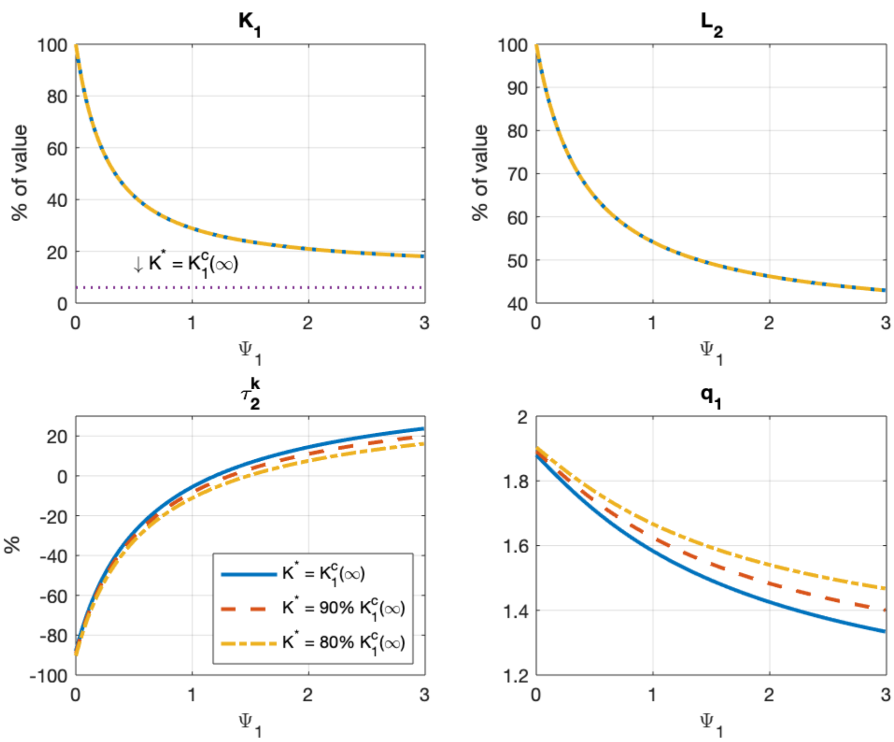
We first plot the functions of $K_1^u(\Psi_1)$ and $K_1^c(\Psi_1)$. As shown before, both are downward sloping, and $K_1^u(0) = K_1^c(0)$. However, the two curves converge to different levels with $K_1^u(\infty) > K_1^c(\infty)$. The critical value $K^* = B_0^e/(1 - \phi_1)$ is a horizontal line whose position depends on the parameter values. As shown by the proposition, four possibilities emerge. The first case is trivial: when $K_1^* > K_1^u(0) = K_1^c(0)$, the economy is always unconstrained. We now show the remaining three cases.

(1). When the liquidity is insufficient, the economy is always constrained. This happens when $K^* \leq K_1^c(\infty)$. Figure 2 illustrates this case, for three possible values of K^*, equal to 100%, 90% and 80% of the critical threshold $K_1^c(\infty)$. As Ψ_1 goes from zero to infinity, the planner implements initially a capital subsidy (between 80% and 90%), but as the budget becomes tighter this turns into a capital tax. The capital tax converges to a level in the range of 20%-25% as $\Psi_1 \to \infty$. For this special example, the allocation in terms of capital and labor used in period 2 is independent of the initial level of internal funds in the hands of the entrepreneurs, as long as they remain such that $K^* \leq K_1^c(\infty)$. Taxes and asset prices adjust to exactly offset the effect of tighter financing

[23] The level of initial debt B_0 that corresponds to a given Ψ_1 is of course different based on the spending process.

constraints. The less liquidity is given to entrepreneurs, the more capital is subsidized when Ψ_1 is small and the less it is taxed for large values of Ψ_1.

Figure 2: Economies with always binding financing constraints.

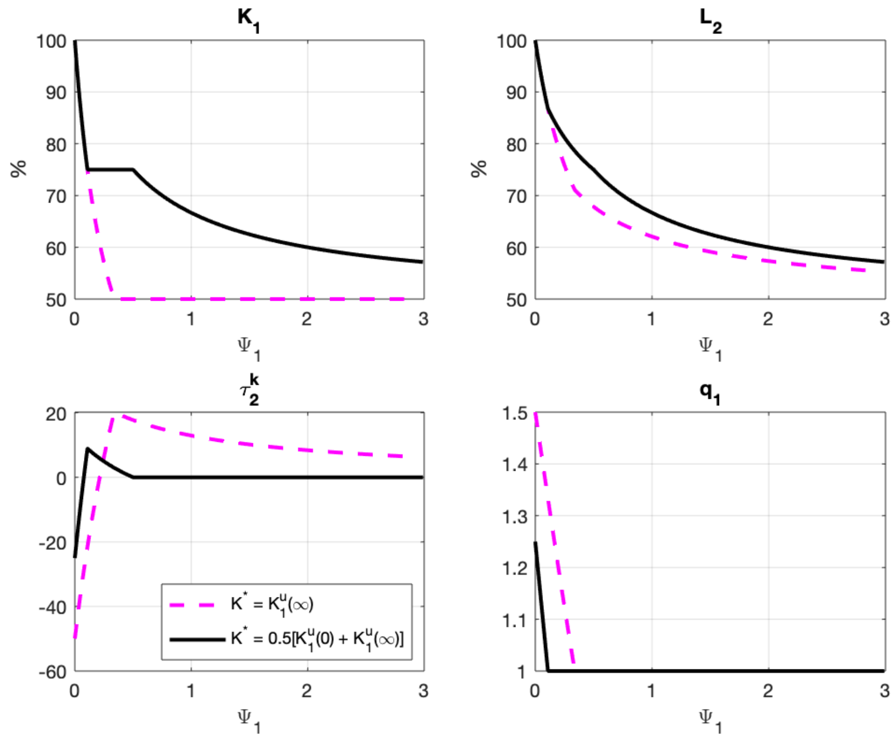


(2). For higher values of liquidity held by entrepreneurs, we enter the range illustrated in Figure 3. The pink dashed line shows what happens when the Ramsey solution hits the kink and remains there as $\Psi_1 \to \infty$.[24] As in the previous case, it is optimal to implement subsidies for low values of Ψ_1 and taxes for higher values. The difference is that the capital income tax becomes decreasing once the government hits K^*. When Ψ_1 goes up, the government needs to tax more. As the labor-income tax increases, in order to keep investment at K^*, it is necessary to reduce the capital-income taxes.

(3). For even higher levels of liquidity for the entrepreneurs, (e.g., when $K^* = [K_1^u(0) + K_1^u(\infty)]/2$ represented by the blue line in Figure 3), we get $K_1 = K_1^*$ for intermediate ranges of Ψ_1. When public resources are abundant and Ψ_1 is low, the planner always finds it better to choose the allocation in the constrained region, which calls for a capital subsidy. As the government budget becomes tighter, the optimal capital level drops, and the subsidy eventually turns into a tax, until the kink K^* is attained. As Ψ_1 grows further, the government keeps increasing labor taxes, but it maintains capital at the kink K^* by lowering capital-income taxes. Finally, for even higher values of Ψ_1, the

[24]The specific value that we choose to illustrate this case is $K^* = K_1^u(\infty)$.

Figure 3: Economies with possible kinks and slack financing constraints.



labor-income tax is so large that the optimal level of capital is below the one at which financing constraints bind. From here on, there are no further quasi-rents to be extracted, and the optimal capital-income tax is zero. As the economy becomes unconstrained, q_1 falls to unity.

3 Endogenous Asset Liquidity

To study the long-run properties of the Ramsey policy and explore some quantitative implications we need to extend the model to an infinite horizon. However, before we do so in Section 4, we provide microfoundations that allow for asset liquidity ϕ and the cost of trading to be endogenous. We make this choice for two reasons. First and foremost, this gives us tractability in computing numerical examples. In an infinite-horizon economy, the kink in each period at the infinite-horizon equivalent of K^* would lead to a proliferation of kinks in optimal policy in previous periods. The microfoundations upon which we build smooth that kink;[25] without affecting the economic intuition that we developed in the previous section, as we will show in this section. Moreover, there is a further economic reason to move in this direction. Cui and Radde (2016b) have shown that endogenizing asset liquidity is crucial to generate the positive co-movement of ϕ and q, which is empirically

[25]While a kink remains at the point at which entrepreneurs start accessing external funds, the price of capital smoothly moves from one region to the other, so that no kinks are present in the implementability constraint.

supported and crucial for amplifying financial shocks.[26]

3.1 Competitive Equilibrium with Search-and-Matching

As in the original paper by Moen (1997) with directed search in labor markets, we assume that intermediaries set up markets where trade occurs and compete by offering a given cost of trading (measured by the bid/ask spread) and market tightness. The competition among market makers implies that they will offer contracts that are Pareto optimal for buyers and sellers. Beyond that, the price will reflect market clearing.

ϕ_1, the fraction of claims to capital that entrepreneurs are able to sell, is the relevant measure of market tightness. We assume that financial intermediaries need to pay a cost $\eta = \eta(\phi_1)$ to intermediate one unit of capital in a market in a market with tightness ϕ_1. We assume that $\eta(0) = \eta'(0) = 0$, $\eta(.)$ is convex and twice continuously differentiable. Cui and Radde (2016b) and Cui (2016) provide complete microfoundations for these assumptions in a world in which intermediation is subject to search frictions which prevent a full match between buyers and sellers of capital.

Under these assumptions, the competitive financial intermediation sector will set the bid/ask spread according the following revised version of (5), for each value of ϕ_1:

$$q_1^w - q_1 = \eta(\phi_1). \tag{32}$$

The left-hand side of (32) is the revenue for intermediating capital and the right-hand side is the cost. Hence, given a price q_1^w paid by workers to acquire one unit of capital, entrepreneurs face a trade-off between asset liquidity ϕ_1 and the price q_1 that they fetch for their sale. The assumption that $\eta(0) = \eta'(0) = 0$ implies that there is no kink at the point in which entrepreneurs stop selling capital. At this point, there are no intermediation costs and both q_1 and q_1^w converge smoothly to 1.

Consider an entrepreneur who participates in a market of tightness ϕ_1 where the price is q_1. Combining the entrepreneurs' budget and financing constraints, we obtain that the claims to capital that an entrepreneur brings back to the household $k_1^e - s_1^e$ satisfy

$$q_1^r \left(k_1^e - s_1^e\right) \leq R_1 B_0^e / \chi,$$

where

$$q_1^r \equiv \frac{1 - \phi_1 q_1}{1 - \phi_1}. \tag{33}$$

[26]In a general equilibrium framework like this one, an exogenous negative shock to asset liquidity pushes up asset price q to reflect the scarcity of assets. The key is that entrepreneurs' financing constraint is also tied to asset liquidity so part of q reflects the tightness of the financing constraint. See Shi (2015) for a critique of models relying on exogenous financial shocks. A negative co-movement can also stabilize financial shocks, because ϕq together matters for investment financing.

q_1^r can be interpreted as the replacement cost of capital. To bring back a claim to one unit of capital to the household, an entrepreneur produces $1/(1-\phi_1)$ units; this investment is financed by selling claims to $\phi_1/(1-\phi_1)$ units at a price q_1 and by the initial assets.[27]

In equilibrium, since all sellers and all buyers are identical, only one market will be open. Specifically, in a directed search environment, an entrepreneur chooses the market which will offer her the lowest value of q_1^r, which maximizes the amounts of claims to capital that she can bring to the household at the end of the period. Market makers will only offer contracts on the Pareto frontier. We index the Pareto frontier by the price q_1^w at which workers can buy claims to capital. We can then trace it by solving

$$\min_{(\phi_1,q_1)} q_1^r = \frac{1-\phi_1 q_1}{1-\phi_1}, \quad \text{subj. to (32).}$$

Substituting the constraint and taking first-order conditions, we obtain

$$q_1^w = 1 + \eta(\phi_1) + (1-\phi_1)\phi_1\eta'(\phi_1). \tag{34}$$

Equation (34) defines an implicit positive relationship between the price workers are willing to pay for a claim to one unit of capital and the entrepreneurs' search intensity, which maps into the fraction of capital that they sell. When $q_1^w = 1$, $\phi_1 = 0$ and entrepreneurs retain all of the capital that they produce. As q_1^w increases above 1, $\phi_1 > 0$, so that entrepreneurs sell some of their capital. The specific point on the Pareto frontier between buyers and sellers that corresponds to the open market is determined by market clearing. Under endogenous liquidity constraints, the competitive equilibrium is thus characterized by conditions (10)-(18) plus (32) and (34). Equations (32) and (34) imply that $q_1 \geq 1$, so that the relevant term in the maximum in equation (16) is always the second one.

Proposition 1. *In a competitive equilibrium, either the financing constraint is slack, in which case* $\phi_1 = 0$ *and* $q_1 = q_1^w = 0$*, or* $\phi_1 < 1$.

Proof. If $\eta'(1) > 1$, equations (32), (33), and (34) imply that there exists a unique value $\hat{\phi}$ such that $q_1\hat{\phi} \equiv \hat{\phi}[1 + (1-\hat{\phi})\hat{\phi}\eta'(\hat{\phi})] = 1$. For $\phi_1 \geq \hat{\phi}$, entrepreneurs would have access to an arbitrage: by producing an extra unit of capital at a unit cost in terms of the period-1 consumption good, they would be able to sell a fraction ϕ_1 and receive a payment $q_1\phi_1 \geq 1$, while retaining the extra $1-\phi_1$ units of capital. In this case, a competitive equilibrium will necessarily have $\phi_1 < \hat{\phi}$.

If $\eta'(1) \leq 1$, the same equations imply $q_1\phi_1$ remains below 1 even as $\phi_1 \to 1$; by continuity, we can define $\hat{\phi} = 1$, since $\lim_{\phi_1 \to 1} q_1\phi_1 = 1$. In this case, note that $\phi_1 > 0$ implies that the financial constraint is binding, so that $K_1 = R_1 B_0^e/(1-q_1\phi_1)$. As $\phi_1 \to 1$, the amount of capital that

[27]This equation remains valid even when entrepreneurs do not sell any claims to their new investment. In this case, $\phi_1 = 0$ and $q_1 = 1$.

entrepreneurs optimally produce diverges to infinity. This would violate the feasibility constraint (and workers would not find it optimal to buy claims to such a large amount of capital), proving that in this case too any competitive equilibrium will feature $\phi_1 < \hat{\phi} = 1$. □

3.2 The Ramsey Outcome with Endogenous Financial Constraints

Combining equations (16) and (34) we obtain

$$(1-\phi_1)\left[1-\phi_1^2\eta'(\phi_1)\right]K_1 \equiv x(\phi_1)K_1 \leq R_1 B_0^e. \tag{35}$$

We have $x(0)=1$, $x'(\phi)<0$ for $\phi \in [0,\hat{\phi}]$, and $x(\hat{\phi})=0$.

This equation links K_1 and ϕ_1, and replaces equation (19) in the previous section, along with $S_1 = \phi_1 K_1$. When $K_1 < R_1 B_0^e$, the constraint is slack, entrepreneurs finance investment only through internal funds, and $\phi_1 = 0$.

Substituting prices and taxes from the first-order conditions, we can aggregate the household budget constraints into the following implementability constraint:

$$\begin{aligned}\sum_{t=1}^{2}\beta^{t-1}[u'(C_t)C_t - v'(L_t)L_t] &= u'(C_1)R_1B_0 + u'(C_1)\left[(q_1^w-1)K_1 - (q_1^w - q_1)\,\phi_1 K_1\right] \\ &= u'(C_1)R_1B_0 + u'(C_1)K_1(1-\phi_1)\left[\eta(\phi_1)+\phi_1\eta'(\phi_1)\right]\end{aligned} \tag{36}$$

Let us define $z(\phi_1) \equiv (1-\phi_1)\left[\eta(\phi_1)+\phi_1\eta'(\phi_1)\right]$. Notice that $z(0)=z'(0)=0$.

Equations (35) and (36) generate two regions in which competitive equilibria can be found, depending on whether $K_1 \leq R_1 B_0^e$ or $K_1 > R_1 B_0^e$. These regions have the same interpretation that applied in the case of exogenous constraints: when investment is small or bond holdings are large, the economy behaves as in the standard neoclassical growth model, whereas an extra term appears when the financing constraint is binding and a wedge appears between the after-tax rate of return on capital and the intertemporal marginal rate of substitution of the households.[28] The only difference is that the implementability constraint (20) features a kink at K^*, while in the case of endogenous liquidity constraints equations (35) and (36) imply a smooth transition of ϕ_1 and K_1 at $K_1 = R_1 B_0^e$: the unit cost of accessing external funds converges to zero when intermediated funds become zero; this greatly simplifies the numerical analysis that will follow.

The planner maximizes the household utility (9), subject to the resources constraints (17) and (18) (with $S_1 = \phi_1 K_1$ as the amount of transaction of claims), the implementability constraint (36), and the equilibrium relationship between ϕ_1 and K_1, equation (35). Let $\beta^{t-1}\lambda_t$ be the Lagrange multiplier on the resource constraint for period $t = 1, 2$ and Ψ_1 be the Lagrange multiplier on the imple-

[28] Mathematically, note that, when $K_1 \leq R_1 B_0^e$, $\eta(\phi_1) = \phi_1 = 0$.

mentability constraint (as before), and let $\gamma_1 u'(C_1)$ be the Lagrange multiplier on equation (35).

Appendix A derives the necessary first-order conditions. As in the case of exogenous trading costs, optimal capital taxation in the presence of financing constraints is driven by a trade-off between the desire to subsidize investment, to alleviate its underprovision, and the benefit of taxing the rents accruing to entrepreneurs in the face of binding financing constraints. To show this most transparently, we return to the special case of preferences and technology of Section 2.4, while introducing endogenous intermediation costs.

3.3 The Special Case Again

As before, we vary the multiplier of the implementability constraint Ψ_1 as an indicator of the tightness of the government budget; the value of B_0 that corresponds to different degrees of tightness can then be backed out by the implementability constraint itself.

First, if we know K_1, we can express labor in period 2 as in (28). Second, from (55) and using the planner's FOC for capital (56),

$$\beta A\alpha \left(\frac{K_1}{L_2}\right)^{\alpha-1} = 1-\beta(1-\delta)+\phi_1\eta(\phi_1)+\frac{\Psi_1}{1+\Psi_1}h(\phi_1)-g(\phi_1),$$

where $h(\phi_1)\equiv z(\phi_1)-\frac{x(\phi_1)z'(\phi_1)}{x'(\phi_1)}$ and $g(\phi_1)=\frac{\eta(\phi_1)+\phi_1\eta'(\phi_1)}{x'(\phi_1)}x(\phi_1)$. Finally, using the relationship between L_2 and K_1, we have

$$\beta A\alpha\left[\frac{\mu}{(1-\alpha)A}\frac{1+(1+\nu)\Psi_1}{1+\Psi_1}K_1^{\nu}\right]^{\frac{\alpha-1}{\alpha+\nu}} = 1-\beta(1-\delta)+\phi_1\eta(\phi_1)+\frac{\Psi_1}{1+\Psi_1}h(\phi_1)-g(\phi_1). \quad (37)$$

Given Ψ_1, the Ramsey outcome can be found by jointly solving (35) and (37) for K_1 and ϕ_1, taking into account that $\phi_1=0$ whenever (35) holds as an inequality. Now, η is a function of ϕ_1.

We first establish conditions under which $\phi_1=0$ and internal financing is sufficient for entrepreneurs to achieve the Ramsey level of K_1. In this case, the capital level is given by $K^u(\Psi_1)$, as defined in equation (30). Note that K^u is strictly decreasing in Ψ, for the same reasons previously identified. We thus have three possibilities:

1. If $K^u(0)\leq B_0^e$, the financial constraint does not bind, independently of the state of government finances. In this case, the solution of the standard neoclassical growth model applies.

2. If $K^u(0)>B_0^e\geq K^u(\infty)$, then there exists a threshold Ψ_1^* such that the Ramsey outcome is not affected by financial constraints if $\Psi_1>\Psi^*$, and is otherwise constrained.

3. Finally, if $K^u(\infty)>B_0^e$, financing constraints are binding, no matter how tight government finances are.

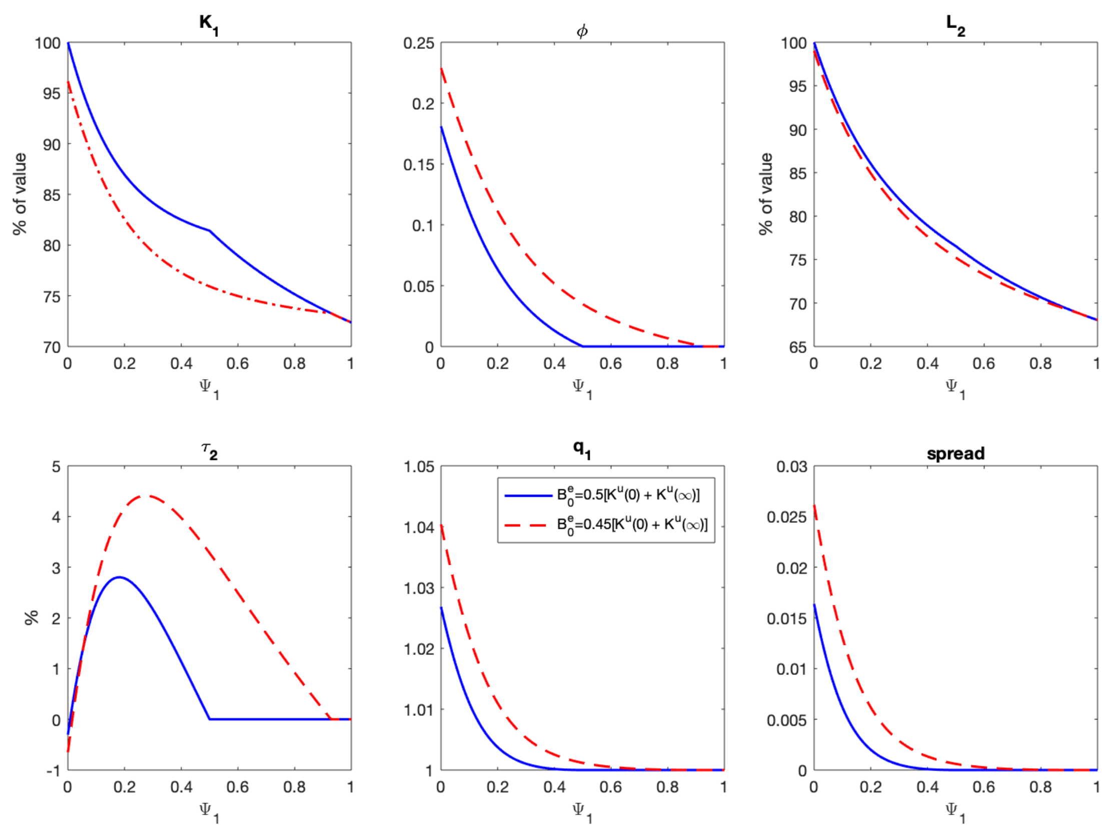
When $K^u(\Psi_1) > R_1 B_0^e$, we can use (35) and substitute $K_1 = B_0^e / x(\phi_1)$ into equation (37), thereby obtaining a single equation to be solved for ϕ_1. It is straightforward to prove that this equation has at least one solution in $(0, \hat{\phi})$, since the expression implies opposite inequalities at the extrema, but in principle it may have multiple solutions, in which case each one should be checked to obtain the global maximum. In our numerical examples, we find a unique solution.

Another Numerical Example Assume the same parameter values as in Section 2, except that now

$$\eta(\phi) = \omega_0 \phi^{\omega_1}$$

with $\omega_0 = 0.2$ and $\omega_1 = 2$.[29]

Figure 4: Ramsey outcome with Endogenous Asset Liquidity



The solid lines in Figure 4 correspond to the economy when $B_0^e = 0.5[K^u(0) + K^u(\infty)]$, so that

[29]We have experimented with different sets of values for ω_0 and ω_1 and the qualitative results shown below are robust. An increase of ω_0 leads to higher capital taxes, while an increase of ω_1 does the opposite. Intuitively, an increase of ω_1 makes the intermediation cost more elastic to the quantity to be sold, which calls for smaller taxes and the planner should reduce intervention. ω_0 increases financial frictions and the quasi-rents accruing to entrepreneurs.

the amount of liquidity held by entrepreneurs satisfies $K^u(\infty) < B_0^e < K^u(0)$. When $\Psi_1 = 0$, as in the case of the previous section, it is optimal to subsidize capital and overcome its underprovision. As Ψ_1 increases from zero and government finances become tighter, the subsidy turns into a tax, but the capital tax eventually vanishes when Ψ_1 becomes large enough, as q_1 approaches 1 and the rents accruing to the entrepreneurs vanish. Therefore, we have a hump-shaped pattern, a smoothed outcome of the economy with exogenous asset liquidity.

Before reaching the cut-off level Ψ_1^* , asset liquidity ϕ falls as the search activity drops and eventually the economy is going to rely purely on public liquidity to finance capital investment (i.e., when the demand for capital is low). This numerical example illustrates how the key variables react in the constrained and unconstrained regions. We can also confirm these constrained and unconstrained regions from q_1 and the spread $\eta(\phi_1)$ schedules.

To further understand the importance of liquidity held by entrepreneurs, when B_0^e falls about 10% (the red dashed lines), the region of Ψ_1 indicating financing-constrained economy is widened, as the cut-off Ψ_1^* is almost doubled. The capital tax implemented is higher (and so is the capital subsidy). Because of the greater shortage in public liquidity, more intermediation takes place and the private liquidity measured by the tightness ϕ_1 is higher for any given Ψ_1. q_1 and the spread $\eta = \eta(\phi_1)$ are then higher for any given Ψ_1.

In summary, when $K^u(\infty) < B_0^e < K^u(0)$, the planner may implement binding financing constraints when Ψ_1 is small. As Ψ_1 becomes larger, the planner eventually finds it optimal to raise labor taxes to the point that optimal entrepreneurial investment can be achieved using internal funds alone.

4 The Infinite Horizon Economy

To discuss the long-run properties of the Ramsey policy, we now extend the model to an infinite horizon.

4.1 The Setup

We adopt the same notation of the previous sections. The household's utility in (9) is now:

$$\sum_{t=0}^{\infty} \beta^t \left[u(c_t) - v((1-\chi)\ell_t)\right] \tag{38}$$

Production in each period occurs according to a constant-returns-to-scale technology $A_t F(K_{t-1}, L_t)$ employing capital and labor, and capital depreciates at the rate δ. The government again has an exogenous stream of spending G_t for any $t \geq 0$.

All households start with some initially given capital K_{-1} and bonds B_{-1}. In our two-period economy, we distinguished between the bonds issued by the government and those held by entrepreneurs, so that we could better explain the economic forces at work by discussing independently the consequences of tightening government finances (by increasing B_0) and loosening financing constraints (by increasing B_0^e). We now assume that each member of a household has an i.i.d. chance χ of being an entrepreneur and a $1-\chi$ chance of being a worker in each period, and this opportunity is realized after the household has allocated bonds, so that $b_t^w = b_t^e$ at the individual family level.[30] Similarly, each member of a household will start period t with k_{t-1} units of capital. An entrepreneur can finance new investment by selling her government bonds as well as claims to capital; we treat existing and new capital symmetrically, with both subject to intermediation costs. We thus have

$$k_t^e \leq R_t b_{t-1} + q_t s_t^e$$

and

$$s_t^e = \phi_t \left[k_t^e + (1-\delta)k_{t-1}\right].$$

These two constraints can be combined as

$$(1-\phi_t q_t)k_t^e \leq R_t b_{t-1} + \phi_t q_t (1-\delta)k_{t-1}. \tag{39}$$

The household budget constraint is

$$c_t + (1-\chi)b_t^w + (1-\chi)q_t^w s_t^w + \chi\left(k_t^e - q_t s_t^e\right) = (1-\tau_t^\ell)w_t(1-\chi)\ell_t + R_t b_{t-1} + (1-\tau_t^k)r_t k_{t-1}.$$

The asset positions evolve according to

$$b_t = (1-\chi)b_t^w \text{ and } k_t = (1-\delta)k_{t-1} + (1-\chi)s_t^w - \chi s_t^e + \chi k_t^e.$$

As before, only workers accumulate government bonds. A household's claims to capital at the beginning of period $t+1$ (which are k_t) include claims to undepreciated capital from the previous period, which are$(1-\delta)k_{t-1}$, new purchases from workers $(1-\chi)s_t^w$, and physical investment by entrepreneurs χk_t^e net of claims sold χs_t^e.[31]

For convenience, we will work with the following budget constraint, which uses the budget

[30] In per-capita aggregate terms, entrepreneurs will thus have χB_t units of government debt.

[31] In a symmetric equilibrium, $(1-\chi)s_t^w = S_t^w$, $\chi s_t^e = S_t^e$, and market clearing requires $S_t^w = S_t^e$. Capital evolves according to $K_t = (1-\delta)K_{t-1} + K_t^e$, where K_t^e is the aggregate investment undertaken by entrepreneurs.

constraint above and the evolution of assets, so that b_t and k_t show up on the left-hand side:

$$c_t + b_t + q_t^w k_t = (1-\tau_t^\ell) w_t (1-\chi)\ell_t + R_t b_{t-1} + (1-\tau_t^k) r_t k_{t-1} \\ + \left[q_t^w - \chi\phi_t \left(q_t^w - q_t\right)\right](1-\delta)k_{t-1} + \left[q_t^w - 1 - \phi_t\left(q_t^w - q_t\right)\right]\chi k_t^e. \quad (40)$$

The intermediation of assets follow directed search implemented by financial intermediaries with free entry:

$$q_t^w - q_t = \eta(\phi_t). \quad (41)$$

4.2 Competitive Equilibrium

A typical household maximizes (38), subject to the financing constraint (39) and the budget constraint (40). The wage rate and the rental rate of capital are the marginal products of labor and capital

$$w_t = A_t F_L(K_{t-1}, L_t); \quad (42)$$

$$r_t = A_t F_K(K_{t-1}, L_t). \quad (43)$$

For asset intermediation, we can immediately extend the result from (34) in the two-period model to any arbitrary period t:[32]

$$q_t = 1 + (1-\phi_t)\phi_t \eta'(\phi_t). \quad (44)$$

In equilibrium, the aggregate quantities are the same as individual quantities, because all households are identical: that is, $K_t = k_t$, $B_t = b_t$, $L_t = (1-\chi)\ell_t$, and $C_t = c_t$. Additionally, the total assets being intermediated are

$$S_t = (1-\chi)s_t^w = \chi s_t^e = \phi_t\left[K_t - (1-\delta)K_{t-1}\right] + \phi_t\chi(1-\delta)K_{t-1} : \quad (45)$$

entrepreneurs sell a fraction ϕ_t of new investment and of their holdings of previous undepreciated capital. The goods market clearing condition is thus

$$C_t + G_t + K_t + \eta(\phi_t)\phi_t[K_t - (1-\chi)(1-\delta)K_{t-1}] = A_t F(K_{t-1}, L_t) + (1-\delta)K_{t-1},$$

[32] To be more specific, an entrepreneur maximizes the amount of claims to capital brought to the household, which is $(1-\phi_t)\left[k_t^e + (1-\delta)k_{t-1}\right]$ because a fraction ϕ_t of $k_t^e + (1-\delta)k_{t-1}$ is issued. The financing constraint (53) can be rewritten as

$$\frac{1-\phi_t q_t}{1-\phi_t}(1-\phi_t)\left[k_t^e + (1-\delta)k_{t-1}\right] \le R_t b_{t-1} + (1-\delta)k_{t-1}$$

so that the entrepreneurs will again minimize $q_t^r = (1-\phi_t q_t)/(1-\phi_t)$ to achieve her goal.

where G_t is the exogenous stream of government expenditures. Substituting (45), this becomes

$$C_t + G_t + [1 + \phi_t \eta(\phi_t)] K_t = A_t F(K_{t-1}, L_t) + [1 + (1-\chi)\phi_t \eta(\phi_t)] (1-\delta) K_{t-1}. \tag{46}$$

Given our assumption of a representative household, the aggregate allocation must satisfy the individual households' optimality conditions. The first-order condition for labor is

$$(1-\tau_t^\ell) w_t u'(C_t) = v'(L_t), \tag{47}$$

for any $t \geq 0$. Let $\beta^t u'(C_t)\chi\rho_t$ be the Lagrange multiplier attached to the financing constraint (39), where the scaling $u'(c_t)\chi$ simplifies the derivation in the following. ρ_t is determined from the first-order condition for k_t^e

$$q_t^w - 1 - \phi_t(q_t^w - q_t) = \rho_t(1 - \phi_t q_t) \to \rho_t = \frac{q_t - 1 + (1-\phi_t)\eta(\phi_t)}{1 - \phi_t q_t} = \frac{\phi_t \eta'(\phi_t) + \eta(\phi_t)}{1 - \phi_t^2 \eta'(\phi_t)}, \tag{48}$$

for any $t \geq 0$. ρ_t reflects the liquidity service provided by government debt. It is only positive when entrepreneurs' financing constraints are binding.

The household first-order condition for government bonds b_t implies

$$1 = \frac{\beta u'(C_{t+1})}{u'(C_t)} R_{t+1} \left(1 + \chi \rho_{t+1}\right). \tag{49}$$

The term $\chi\rho_{t+1}$ in equation (49) represents the liquidity services that government bonds offer to the entrepreneurs, arising from the fact that bonds can be liquidated with no intermediation costs by the fraction χ of household members that turn out to be entrepreneurs in any given period. This liquidity service pushes down the interest rate R_{t+1}.

The first-order condition for capital k_t implies

$$q_t^w = \frac{\beta u'(C_{t+1})}{u'(C_t)} \Big\{ (1-\tau_{t+1}^k) r_{t+1} + (1-\delta) q_{t+1}^w + \chi(1-\delta)\phi_{t+1} \left[q_{t+1}(1+\rho_{t+1}) - q_{t+1}^w \right] \Big\}. \tag{50}$$

q_t^w represents the cost for a worker to acquire one unit of capital.[33] In the next period, the household receives a payoff $(1-\tau_{t+1}^k) r_{t+1} + (1-\delta) q_{t+1}^w$ from the investment. In addition, the fraction $\chi\phi_{t+1}$ of undepreciated capital that entrepreneurs will sell to finance further investment has an extra liquidity value captured by $q_{t+1}(1+\rho_{t+1}) - q_{t+1}^w$, the difference between the price at which entrepreneurs sell their capital, adjusted for the shadow value of liquidity, and the price at which workers can buy the

[33]When the financing constraint is slack, $q_t^w = 1$ and an individual household is indifferent whether to purchase an extra unit in the market or to increase its own entrepreneurs' investment. Hence, q_t^w remains the correct shadow cost of acquiring an extra unit of capital. This is true even though in the aggregate we must have $\phi_t = 0$ and hence no trade in capital claims takes place.

capital back.

Definition. A competitive equilibrium is an allocation $\{C_t, L_t, K_t, K_t^e, \phi_t\}_{t=0}^{\infty}$, a sequence of asset market prices $\{q_t^w, q_t, r_t, R_t\}_{t=0}^{\infty}$, wage rates $\{w_t\}_{t=0}^{\infty}$, government policies $\{G_t, B_t, \tau_t^{\ell}, \tau_t^k\}_{t=0}^{\infty}$, shadow values of liquidity $\{\rho_t\}_{t=0}^{\infty}$, and an exogenous sequence of productivity $\{A_t\}_{t=0}^{\infty}$ such that (39) – (44) and (46) – (50) are satisfied, and capital evolves according to $K_t = (1-\delta)K_{t-1} + K_t^e$.

4.3 The Ramsey Outcome

To find the best equilibrium, in a frictionless economy it is possible to write a planner problem that collapses all the constraints into feasibility (equation (46)) and a single present-value implementability condition. The presence of financing constraints implies that we cannot collapse the implementability constraints into a single present-value condition, but rather we have a sequence of them. To simplify notation, from here on we will write η_t, q_t^w, q_t, and ρ_t to denote the functions of ϕ_t that are defined by $\eta(\phi_t)$, and equations (41), (44), and (48).[34] We also define

$$d_t := 1 - q_t^w + \phi_t \eta_t - \chi \rho_t \phi_t q_t,$$

which is also a function of ϕ_t alone.

Using the household budget constraint and the first-order conditions, the implementability constraint at $t \geq 1$ can be written as:

$$\begin{aligned} & u'(C_t)C_t - v'(L_t)L_t + u'(C_t)B_t + u'(C_t)\left(1 + \phi_t \eta_t\right) K_t \\ =& u'(C_{t-1}) \frac{B_{t-1}}{\beta(1+\chi\rho_t)} + u'(C_{t-1}) \frac{q_{t-1}^w K_{t-1}}{\beta} + u'(C_t) d_t (1-\delta) K_{t-1}. \end{aligned} \tag{51}$$

The implementability constraint at $t = 0$ is

$$\begin{aligned} & u'(C_0)C_0 - v'(L_0)L_0 + u'(C_0)B_0 + u'(C_0)\left(1 + \phi_0 \eta_0\right) K_0 \\ =& u'(C_0)R_0 B_{-1} + u'(C_0)(1-\tau_0^k)A_0 F_K(K_{-1}, L_0)K_{-1} + u'(C_0)\left[1 + (1-\chi)\phi_0\eta_0\right](1-\delta)K_{-1}, \end{aligned} \tag{52}$$

with B_{-1}, K_{-1}, R_0, and τ_0^k exogenously given. We follow the tradition of exogenously limiting capital-income taxation in period 0, since this would be otherwise a lump-sum tax. Using the individual entrepreneur's financing constraint (39) and the first-order condition for bonds, we obtain

[34] In computing an optimum, we will take into account that in a competitive equilibrium these variables are functions of ϕ_t and of no other variable that enters into the planner's maximization problem.

that a competitive equilibrium satisfies the following condition in the aggregate for any period $t > 0$

$$(1 - \phi_t q_t)\left[K_t - (1-\delta)K_{t-1}\right] \leq \chi \left[\frac{u'(C_{t-1})}{\beta u'(C_t)(1+\chi\rho_t)} B_{t-1} + \phi_t q_t (1-\delta) K_{t-1}\right]. \tag{53}$$

In period 0, the financing constraint is

$$(1 - \phi_0 q_0)\left[K_0 - (1-\delta)K_{-1}\right] \leq \chi \left[R_0 B_{-1} + \phi_0 q_0 (1-\delta) K_{-1}\right]. \tag{54}$$

Therefore, the planner maximizes the household utility (38) subject to the sequence of resource constraints represented by (46), the implementability constraints (51) and (52), and the financing constraints (53) and (54). The planner chooses the allocation $\{C_t, L_t, K_t, B_t, \phi_t\}_{t=0}^{\infty}$, which are consumption, labor hours, capital stock, government bonds, and asset liquidity. We can back out the taxes and prices from the allocation and the other necessary conditions for a competitive equilibrium.

4.4 Long-run Public Liquidity Provision, Capital Tax, and Interest Rates

Let $\beta^t \Psi_t$ and $\beta^t \gamma_t u'(C_t)$ be the Lagrange multipliers attached to implementability constraints and the financing constraints. AppendixB contains the derivation of the planner's first-order conditions. In particular, the planner's first-order condition for bonds is

$$\Psi_t - \frac{\Psi_{t+1}}{1+\chi\rho_{t+1}} + \frac{\chi\gamma_{t+1}}{1+\chi\rho_{t+1}} = 0 \Longrightarrow \Psi_{t+1} = (1+\chi\rho_{t+1})\Psi_t + \chi\gamma_{t+1}.$$

An additional unit of debt issuance relaxes the current government budget (or implementability constraint) measured by Ψ_t. Without frictions, this would be exactly offset by a tighter budget constraint in period $t+1$, leading to $\Psi_t = \Psi_{t+1}$. This is what happens if the financing constraint is slack in period $t+1$. If instead the financing constraint is binding, two forces lead to $\Psi_t < \Psi_{t+1}$. First, since bonds can be liquidated without incurring intermediation costs, households are willing to hold them at a lower interest rate, which accounts for the term $1+\chi\rho_{t+1}$ as in equation (49). Second, the additional liquidity provided by the increased supply of bonds directly relaxes the financing constraint of the entrepreneurs in period $t+1$, which justifies the term $\chi\gamma_{t+1}$.

When the financing constraint is slack, $\Psi_t = \Psi_{t+1}$ corresponds to the standard tax-smoothing principle. In contrast, with $\Psi_{t+1} < \Psi_t$, the tightness of government budget is increasing over time. We thus obtain the following result:

Proposition 2. *Assume that the economy converges to a steady state with finite allocations (finite C, K, L, and B, given finite G and A).*

- *If the government finds it feasible to flood the economy with public liquidity, it is optimal to do so. More precisely, the government issues enough debt to fully relax the financing constraints* *in the limit. In this case, Ψ_t converges to a constant, capital-income taxes are zero in the limit, and the interest rate on government debt is $1/\beta$ in the limit.*

- *If the amount of debt that fully relaxes financing constraints exceeds the fiscal capacity of the government, Ψ_t grows without bounds and the economy converges to a dynamic equivalent of the top of the Laffer curve. In this case, the interest rate on government debt is lower than $1/\beta$ in the limit. In addition, if either utility is quasilinear or the shadow cost of relaxing the financing constraint is sufficiently low in the limit, then the limiting tax rate on capital is strictly positive: $\lim_{t\to\infty} \tau_t^k > 0$, and the interest rate on government debt is lower than $1/\beta$ in the limit.*

□

For the infinite horizon economy we cannot obtain analytical expressions even with quasi-linear utility. Moreover, for general preferences we are limited in our ability to use comparative statics by the possibility that the solution "jumps" in the presence of nonconvexities. Nonetheless, when such jumps do not occur, Proposition 2 provides a generalization of what we observe in Figure 2: as we gradually move from the region in which the financing constraint is slack to that in which it is binding, taxes on capital become unambiguously positive.[35]

4.5 Numerical Examples: Comparative Statics in the Long Run

We illustrate our results with numerical exercises. We assume that preferences are given by

$$\sum_{t=0}^{\infty} \beta^t \left[\frac{c^{1-\sigma} - 1}{1-\sigma} - \mu \ell^{\nu} \right]$$

and that technology in period t is Cobb-Douglas, $AK_{t-1}^{\alpha} L_t^{1-\alpha}$, with a share of capital $\alpha = 1/3$, and A normalized to 1. Capital depreciates at $\delta = 0.1$. The baseline parameters for preferences are $\beta = 0.96$, $\alpha = 1/3$, and $\nu = 1/1.5$, which are all standard parameters for a yearly calibration for a macroeconomic model. μ is set to 1.35 so that labor supply is 1/3 (which does not matter for later results). We set $\sigma = 0.2$. A high degree of intertemporal elasticity of substitution (low σ) is needed for the solution to feature a dynamic Laffer curve. For high values of σ, households are so desperate to consume in each period that the government is able to extract even the entire GDP in taxes; in this case, the steady state will necessarily occur at a point in which financing constraints are not binding.

We choose government spending and the fraction of entrepreneurs χ so that the steady state occurs at the point at which the financing constraint is slack while holding exactly as an equality,

[35] There are several parameters that can be adjusted for this comparative statics exercise. The most natural one is a proportional shift in the cost of the intermediation technology.

and debt-to-GDP is 200%.[36] and $\chi = 0.115$. By construction, we thus have $q = q^w = 1$, $\phi = 0$, and $R = 1/\beta$ in the limit.

Finally, we set $\eta(\phi) = \omega_0 \phi^{\omega_1}$. We choose $\omega_1 = 2$, which results from a matching function where the elasticity of matches to buy orders and saleable assets is the same and it is costly to process the buy orders, as shown in Cui and Radde (2016b) and Cui (2016). We pick ω_1 so that the debt-to-output ratio drops from 200% to 100% in the steady state in which $\Psi_t/\Psi_{t-1} = 1.01$ in the experiment of Table 1 below; this gives us $\omega_0 = 0.58$, but we then experiment with alternative values.[37]

Our first comparative-statics exercise analyzes the effect of changing government spending, and is illustrated in Table 1. When the economy converges to a steady state with a binding implementability constraint, Appendix C proves that the Lagrange multiplier on the implementability constraint grows at a constant rate in the limit. We pick values of G such that it grows at 1% a year, 2% a year, or 3% a year (remember that it is constant in the limit for the baseline economy).

Table 1: Steady state of the Ramsey allocation for different government expenditures

G/Y	26.00%	31.20%	32.76%	33.26%
Capital: K	100%	88.19%	82.13%	79.38 %
Capital tax: τ^k	0%	2.88%	4.37%	4.99%
Labor tax: τ^ℓ	52%	50.64%	49.45%	48.36%
Interest rate:	4.17%	3.19%	2.24%	1.20%
Debt-to-output: B/Y	200%	100%	60.51%	29.30%
Asset Liquidity ϕ	0	0.2157	0.3017	0.3713

As G increases, the maximum sustainable level of debt in the steady state *decreases*. The government is forced to cut back on public liquidity. With smaller amounts of public liquidity, entrepreneurs increasingly rely on financial intermediaries to sell some of their capital and fund their investment: the fraction ϕ of capital that is intermediated increases. From our theoretical results we know that it is ambiguous whether capital-income taxes become positive or negative. In this numerical example, the incentive to tax quasi-rents dominates and capital income is taxed, while the tax on labor income drops somewhat. Government debt commands a liquidity premium, and its interest rate drops as it becomes scarcer the higher G is.

[36] This means that, for any higher level of G, the financing constraint is strictly binding and the interest rate on government debt is below $1/\beta$, while for any lower value it is slack and the interest rate is $1/\beta$. This gives us $G = 0.1363$, $G/GDP \approx 26\%$. This ratio is higher than the value in the United States, but our model does not feature transfers. If some of the government expenditure takes the form of transfers, the wealth effect is lessened, which lowers the maximum sustainable level of debt.

[37] Notice that the cost of intermediation does not matter for the steady state of the baseline economy, since no intermediation takes place in that steady state.

Next, we explore the role of financial intermediation costs. Specifically, we increase ω_0 in 3 steps of 20% each. At the baseline steady state, this would be irrelevant, since no intermediation takes place. We thus use government spending from the second column of Table 1.[38]

Perhaps surprisingly, when intermediation is more costly it is used *more* in the limit. The reason is that the fiscal capacity of the economy contracts, so the government is less able to issue debt. As a substitute for the inability to relax financing constraints by providing public debt, the government increases the capital-income tax instead.

Table 2: The long-run economies with different financial intermediation

	$\omega_0 = 0.58$	$\omega_0 = 0.696$	$\omega_0 = 0.812$	$\omega_0 = 0.928$
Capital: K	100%	96.38%	92.37%	87.53%
Capital tax: τ^k	2.88%	3.93%	5.21%	6.87%
Labor tax: τ^ℓ	50.64%	50.21%	49.56%	48.79%
Interest rate:	3.19%	2.97%	2.70%	2.30%
Debt-to-output: B/Y	100%	96.47%	91.23%	82.56%
Asset Liquidity ϕ	0.2157	0.2187	0.2238	0.2358

5 Conclusion

Within the context of a Ramsey model of capital taxation, we identified a force that operates as in Sargent and Wallace (1982) and pushes the government to increase its indebtedness to mitigate frictions in private asset markets. We also showed that, when it is impossible to completely undo those frictions in the long run, it is optimal to tax capital even though its provision is already inefficiently low: this happens because the frictions that prevent efficient investment also alter the elasticity of the supply of capital. We considered here an economy with no aggregate risk, where no force countervails the upward drift in government debt. In a stochastic economy with non-contingent debt, Aiyagari, Marcet, Sargent, and Seppälä (2002) identify an opposite force, that induces the government to accumulate assets for self insurance. In our next step, we plan to study how capital-income taxes and government debt are optimally chosen when both of these forces are present.

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A The Two-period Planner's Problem (with Endogenous Asset Liquidity)

Here, we write down the planner's problem in detail and derive the first-order necessary conditions. First, the planner's objective can be stated as the following Lagrangian:

$$\mathbb{L} = \sum_{t=1}^{2} \beta^{t-1} \Big\{ u(C_t) - v(L_t) + \Psi_1 \left[u'(C_t)C_t - v'(L_t)L_t \right] \Big\} - \Psi_1 u'(C_1) z(\phi_1) K_1 + \gamma_1 u'(C_1) \left[R_1 B_0^e - x(\phi_1) K_1 \right]$$
$$+ \lambda_1 \left[L_1 - C_1 - K_1 - \eta(\phi_1)\phi_1 K_1 - G_1 \right] + \beta \lambda_2 \left[F(K_{1,} L_2) - C_2 - G_2 \right]$$

where we use the household's utility, the implementability constraint, the financing constraint, and the resource constraints. Thanks to the smoothness of function $\eta(.)$, we do not need to impose the constraint $\phi_1 \geq 0$. Second, we derive all the planner's first-order necessary conditions here. The first-order conditions for consumption C_1 and C_2 are

$$u'(C_1)(1 + \Psi_1) + \Psi_1 u''(C_1)C_1 - \lambda_1 = \Psi_1 u''(C_1) K_1 z(\phi_1);$$

$$u'(C_2)(1 + \Psi_1) + \Psi_1 u''(C_2)C_2 - \lambda_2 = 0.$$

The first-order conditions for labor supply L_1 and L_2 are

$$v'(L_1)(1 + \Psi_1) + \Psi_1 v''(L_1) L_1 = \lambda_1;$$

$$v'(L_2)(1 + \Psi_1) + \Psi_1 v''(L_2) L_2 = \lambda_2 F_L(K_1, L_2).$$

The first-order condition for asset liquidity ϕ_1 is

$$-\lambda_1 \left[1 + \phi_1 \eta(\phi_1) \right] + \beta \lambda_2 F_K(K_1, L_2) = u'(C_1) \left[\Psi_1 z(\phi_1) + \gamma_1 x(\phi_1) \right]. \tag{55}$$

The first-order condition for capital K_1 is

$$\lambda_1 \left[\eta(\phi_1) + \phi_1 \eta'(\phi_1) \right] + u'(C_1) [\Psi_1 z'(\phi_1) + \gamma_1 x'(\phi_1)] = 0. \tag{56}$$

Notice that $\phi_1 = 0$ if and only if $\gamma_1 = 0$: this happens when the planner optimally chooses an allocation such that the financing constraint is slack.

B The Infinite-horizon Planner's Problem

Let $\beta^t \lambda_t$ be the Lagrange multiplier on constraint (46), Ψ_0 be the Lagrange multiplier on constraint (52), $\beta^t \Psi_t$ the Lagrange multiplier on (51), $u'(C_0)\gamma_0$ on (54), and $\beta_t u'(C_t)\gamma_t$ on (53). For brevity, we denote by η'_t, $(q^w_t)'$, q'_t, ρ'_t, and d'_t the derivatives of each (previously defined) function with respect to ϕ_t.

The necessary first-order conditions for a Ramsey outcome are the following:

- Consumption in period 0:

$$\begin{aligned}&(1+\Psi_0)u'(C_0)+\Psi_0 u''(C_0)[C_0+B_0+(1+\phi_0\eta_0)K_0]\\&-\Psi_0 u''(C_0)[R_0B_{-1}+[(1-\tau_0^k)A_0F_K(K_{-1},L_0)+(1+(1-\chi)\phi_0\eta_0)(1-\delta)]K_{-1}]\\&+\gamma_0 u''(C_0)[\chi(R_0B_{-1}+\phi_0q_0(1-\delta)K_{-1})-(1-\phi_0q_0)(K_0-(1-\delta)K_{-1})]-\lambda_0\\&=-\gamma_1 u''(C_0)\frac{\chi B_0}{\beta(1+\chi\rho_1)};\end{aligned}$$

- Consumption in period $t\geq 1$

$$\begin{aligned}&(1+\Psi_t)u'(C_t)+\Psi_t u''(C_t)C_t+\Psi_t u''(C_t)\left[B_t+(1+\phi_t\eta_t)K_t\right]-\Psi_t u''(C_t)d_t(1-\delta)K_{t-1}\\&+\gamma_t u''(C_t)\left[\left[1-(1-\chi)\phi_tq_t\right](1-\delta)K_{t-1}-(1-\phi_tq_t)K_t\right]-\lambda_t\\&=-\gamma_{t+1}u''(C_t)\frac{\chi B_t}{1+\chi\rho_{t+1}}+\Psi_{t+1}u''(C_t)\left(q_t^wK_t+\frac{B_t}{1+\chi\rho_{t+1}}\right);\end{aligned}\tag{57}$$

- Leisure in period 0:

$$v'(L_0)(1+\Psi_0)+\Psi_0 v''(L_0)L_0=\lambda_0A_0F_L(K_{-1},L_0)-\Psi_0u'(C_0)(1-\tau_0^k)A_0F_{KL}(K_{-1},L_0)K_{-1};$$

- Leisure in period $t\geq 1$:

$$v'(L_t)(1+\Psi_t)+\Psi_t v''(L_t)L_t=\lambda_tA_tF_L(K_{t-1},L_t);\tag{58}$$

- Liquidity in period 0:

$$[\Psi_0u'(C_0)-\lambda_0](\eta_0+\phi_0\eta_0')+\gamma_0(q_0+\phi_0q_0')=0;$$

- Liquidity in period $t\geq 1$:

$$\begin{aligned}&\Psi_tu'(C_t)K_t(\eta_t+\phi_t\eta_t')-\gamma_tu'(C_{t-1})\frac{\chi B_{t-1}}{\beta}\frac{\chi\rho_t'}{(1+\chi\rho_t)^2}+\gamma_tu'(C_t)\left[K_t-(1-\chi)(1-\delta)K_{t-1}\right](q_t+\phi_tq_t')\\&+\lambda_t\left[(1-\chi)(1-\delta)K_{t-1}-K_t\right](\eta_t+\phi_t\eta_t')+\Psi_tu'(C_{t-1})\frac{\chi B_{t-1}\rho_t'}{\beta(1+\chi\rho_t)^2}-\Psi_tu'(C_t)(1-\delta)K_{t-1}d_t'\\&=\Psi_{t+1}u'(C_t)K_t(q_t^w)';\end{aligned}\tag{59}$$

- Capital in period $t\geq 0$:

$$\begin{aligned}&\lambda_t\left(1+\phi_t\eta_t\right)-\Psi_tu'(C_t)\left(1+\phi_t\eta_t\right)+\gamma_tu'(C_t)(1-\phi_tq_t)\\&=\beta\lambda_{t+1}\left[A_{t+1}F_K(K_t,L_{t+1})+\left[1+(1-\chi)\phi_{t+1}\eta_{t+1}\right](1-\delta)\right]-\Psi_{t+1}u'(C_t)q_t^w\\&-\beta\Psi_{t+1}u'(C_{t+1})d_{t+1}(1-\delta)+\beta\gamma_{t+1}u'(C_{t+1})\left[1-(1-\chi)\phi_{t+1}q_{t+1}\right](1-\delta);\end{aligned}\tag{60}$$

- Bond choice for $t\geq 0$:

$$\Psi_t=\frac{\Psi_{t+1}}{1+\chi\rho_{t+1}}-\frac{\chi\gamma_{t+1}}{1+\chi\rho_{t+1}}\to\Psi_{t+1}=(1+\chi\rho_{t+1})\Psi_t+\chi\gamma_{t+1}.\tag{61}$$

C Proof to Proposition 2

We denote steady-state allocations by a bar over each variable. From the first-order conditions for bonds, equation (61), we know that Ψ_t is weakly increasing. Moreover, it is constant if and only if $\rho_{t+1} = 0$ and $\gamma_{t+1} = 0$, which happens if and only if the financing constraint is slack. If the Ramsey allocation converges to a constant, we then have two possibilities as follows.

Case 1: Ψ_t converges to a finite constant $\bar{\Psi} > 0$.[39] In this case, the Lagrange multiplier of the financing constraint converges to zero in the limit and so does the financial-market trading in (claims to) capital, that is $\phi_t \to 0$. The limiting first-order conditions look like those of a standard neoclassical growth model. In particular, the limit of the planner's first-order condition with respect to capital becomes

$$\beta[\bar{A}F_K(\bar{K}, \bar{L}) + 1 - \delta] = 1,$$

which coincides with the first-order condition for capital of the households with $\tau_t^k = \bar{\tau}^k = 0$.[40] With $\bar{\rho} = 0$, the households' first-order condition for bonds evaluated at steady state implies that $\bar{R} = 1/\beta$.

Case 2: Ψ_t diverges to infinity. In this case, we use equations (58) and (61) to substitute for λ_t and γ_t in equations (57), (59), and (60). If the Ramsey allocation converges to a steady state, these three equations in the limit turn into linear second-order difference equations in Ψ_t. These equations are generically distinct. In order for the system to have a solution, it must be that the 5 variables $(\bar{C}, \bar{L}, \bar{K}, \bar{B}, \bar{\phi})$ are such that equations (46), (51), and (53) (the resources, implementability, and financing constraints respectively) are satisfied in the steady state, and such that the three difference equations share at least one root. This gives us 5 (nonlinear) conditions to solve for the 5 variables. In addition, Ψ_{t+1}/Ψ_t must converge to a constant ζ.[41]

Also, for the first-order conditions to be optimal, Ψ_t cannot grow at rate larger than $1/\beta$ (the transversality condition), i.e. $\zeta < \beta^{-1}$. The economy can be captured by finite levels of K, B, C, L, ϕ, ζ, $\tilde{\gamma} := \lim_{t\to\infty} \gamma_t/\Psi_t$, and $\tilde{\lambda} := \lim_{t\to\infty} \lambda_t/\Psi_t$. We can thus write the limiting conditions that hold in steady state as follows:

The financing constraint

$$\frac{\chi B}{\beta(1+\chi\rho)} + \left[\left[1 - (1-\chi)\phi q\right](1-\delta) - (1-\phi q)\right] K = 0. \tag{62}$$

The implementability condition:

$$C - \frac{v'(L)}{u'(C)} L + B + (1+\phi\eta) K = \frac{B}{\beta(1+\chi\rho)} + \frac{q^w}{\beta} K + d(1-\delta)K. \tag{63}$$

Tbe FOC for consumption:

[39] If $\Psi_t = 0$ at any time t, it is straightforward to show that it must be the case that $\Psi_t = 0$ in all periods and that the Ramsey solution attains the first best. In this case, capital is subsidized if the financing constraint is binding, as we discussed in the context of the two-period example.

[40] That λ_t converges to a constant follows from the first-order conditions with respect to consumption or labor.

[41] Expressing the second-order difference equations as two-equation systems of first-order difference equations for the vector (Ψ_{t+1}, Ψ_t), the constant ζ corresponds to the ratio Ψ_{t+1}/Ψ_t in the eigenvector associated with the common eigenvalue across the three systems. Note that this eigenvalue must be real; if the systems had complex eigenvalues, matching eigenvalues would imply 2 additional constraints, giving us 7 conditions for 5 variables and implying that generically there would be no solution.

$$
\begin{aligned}
&\frac{u'(C)}{u''(C)} + C + B + (1+\phi\eta)K + \tilde{\gamma}\left[\left[1-(1-\chi)\phi q\right](1-\delta) - (1-\phi q)\right]K \\
=&d(1-\delta)K + \tilde{\lambda}\frac{1}{u''(C)} - \tilde{\gamma}\zeta\frac{\chi B}{1+\chi\rho} + \zeta\left(q^w K + \frac{B}{1+\chi\rho}\right)
\end{aligned}
$$

and after we use the financing constraint

$$
\frac{u'(C)}{u''(C)} + C + B + (1+\phi\eta)K - \tilde{\gamma}\frac{\chi B}{1+\chi\rho}\frac{1-\beta\zeta}{\beta} = d(1-\delta)K + \tilde{\lambda}\frac{1}{u''(C)} + \zeta\left(q^w K + \frac{B}{1+\chi\rho}\right). \tag{64}
$$

The FOC for capital:

$$
\begin{aligned}
&\tilde{\lambda}(1+\phi\eta) - u'(C)(1+\phi\eta-\zeta q^w) + \tilde{\gamma}u'(C)(1-\phi q) \\
=&\beta\tilde{\lambda}\zeta\left[AF_K(K,L) + \left[1+(1-\chi)\phi\eta\right](1-\delta)\right] - \beta u'(C)d(1-\delta)\zeta + \beta\tilde{\gamma}\zeta u'(C)\left[1-(1-\chi)\phi q\right](1-\delta).
\end{aligned} \tag{65}
$$

The FOC for bonds:

$$
(1-\chi\tilde{\gamma})\zeta = 1+\chi\rho.
$$

In such a steady state, the entrepreneurs' financing constraint binds, so that $\phi > 0$ and $\rho > 0$. This means that the interest rate $R = \frac{1}{\beta(1+\chi\rho)} < \frac{1}{\beta}$. We are also ready to show that capital tax $\tau^k > 0$, which can be seen from comparing the planner's first-order condition for capital in (65) and the household's first-order condition for capital (50):

$$
\begin{aligned}
\beta\left[F_K(K,L) + \left[1+(1-\chi)\phi\eta\right](1-\delta)\right] = &\frac{u'(C)}{\tilde{\lambda}}q^w + \frac{\tilde{\gamma}}{\tilde{\lambda}\zeta}u'(C)(1-\phi q) + \left[1-\frac{u'(C)}{\tilde{\lambda}}\right]\frac{1+\phi\eta}{\zeta} \\
&+ \frac{\beta u'(C)}{\tilde{\lambda}}d(1-\delta) - \beta\frac{\tilde{\gamma}}{\tilde{\lambda}}u'(C)\left[1-(1-\chi)\phi q\right](1-\delta);
\end{aligned}
$$

$$
\beta\left[(1-\tau^k)F_K(K,L) + (q^w - \chi\phi\eta + \chi\rho\phi q)(1-\delta)\right] = q^w.
$$

Taking the difference of the two and using the relationship $d_t = d(\phi_t) = 1 + \phi_t\eta_t - q_t^w - \chi\rho_t\phi_t q_t$, we obtain

$$
\tau^k\beta F_K(K,L) = \left[\frac{u'(C)}{\tilde{\lambda}} - 1\right]\left[q^w - \frac{1+\phi\eta}{\zeta} + \beta(1-\delta)d\right] + \frac{\tilde{\gamma}}{\tilde{\lambda}}u'(C)\left[\frac{1-\phi q}{\zeta} - \beta(1-\delta)\left[1-(1-\chi)\phi q\right]\right] \tag{66}
$$

The transversality condition requires $\zeta < 1/\beta$. Morever, (62) implies that

$$
\left[1-\phi q-(1-\delta)\left[1-(1-\chi)\phi q\right]\right] = \chi RB/K > 0.
$$

Using these facts, we have

$$
\frac{\tilde{\gamma}}{\tilde{\lambda}}u'(C)\left[\frac{1-\phi q}{\zeta} - \beta(1-\delta)\left[1-(1-\chi)\phi q\right]\right] > \frac{\tilde{\gamma}}{\tilde{\lambda}}u'(C)\left[(1-\phi q)-(1-\delta)\left[1-(1-\chi)\phi q\right]\right] > 0
$$

Consider next the first term in equation (66). The planner's first-order condition for consumption (64) can be rearranged as:

$$
\frac{u'(C)}{\tilde{\lambda}} - 1 = \frac{u''(C)}{\tilde{\lambda}}\left[d(1-\delta)K + \zeta\left(q^w K + \frac{B}{1+\chi\rho}\right) - C - B - (1+\phi\eta)K + \tilde{\gamma}\frac{\chi B}{1+\chi\rho}\frac{1-\beta\zeta}{\beta}\right]. \tag{67}
$$

If the utility function is quasi-linear, $u'(C) = 1$, $u''(C) = 0$, and in the limit $\tilde{\lambda} = 1$ according to (67). Equation (66) then implies that $\tau^k > 0$. Alternatively, the implementability condition (63), along with $\zeta < 1/\beta$, implies

$$d(1-\delta)K + \zeta\left(q^w K + \frac{B}{1+\chi\rho}\right) - C - B - (1+\phi\eta)\,K < -\frac{v'(L)}{u'(C)}L.$$

Substituting this equation into (67) we obtain

$$\frac{u'(C)}{\tilde{\lambda}} - 1 \geq \frac{u''(C)}{\tilde{\lambda}}\left[-\frac{v'(L)}{u'(C)}L + \tilde{\gamma}\frac{\chi B}{1+\chi\rho}\frac{1-\beta\zeta}{\beta}\right]. \tag{68}$$

In a neighborhood of the point at which the financing constraint just starts to bind when debt is at the top of the dynamic Laffer curve (that is, as $\Psi_t \to \infty$), we have that $\tilde{\gamma}$ is arbitrarily close to zero. Hence, in such a neighborhood we have

$$\frac{u'(C)}{\tilde{\lambda}} - 1 > 0. \tag{69}$$

Using the fact that $\zeta \in (1, 1/\beta)$ and $\rho = (q^w - 1 - \phi\eta)/(1-\phi q)$, we get

$$q^w - \frac{1+\phi\eta}{\zeta} + \beta(1-\delta)d > \frac{q^w}{\zeta} - \frac{1+\phi\eta}{\zeta} + \beta(1-\delta)d > \beta\rho\,[1 - \phi q - (1-\delta)\,[1-(1-\chi)\phi q]] > 0, \tag{70}$$

where the last inequality was proven earlier. Substituting (69) and (70) into (66), we complete the proof that $\tau^k > 0$. □