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Liquidity Traps, Prudential Policies, and International Spillovers *

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Abstract

We present a simple open economy framework to study the transmission channels of monetary and macroprudential policies and evaluate the implications for international spillovers and global welfare. Using an analytical decomposition, we first identify three transmission channels: intertemporal substitution, expenditure switching, and aggregate income. Quantitatively, expenditure switching plays a prominent role for monetary policy, while macroprudential policy operates almost entirely through intertemporal substitution. Turning to the normative analysis, we show that the risk of a liquidity trap generates a monetary policy tradeoff between stabilizing output today and reducing capital flows to lower the likelihood of a future recession. However, leaning against the wind is not necessarily optimal, even in the absence of capital controls. Finally, we argue that contrary to emerging policy concerns, capital controls are not beggar-thy-neighbor and can enhance global macroeconomic stability.

Keywords: Capital flows, monetary and macroprudential policies, liquidity traps, international spillovers

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1 Introduction

Low interest rates have become a major feature of the international monetary system. With little room to lower policy rates, central banks face increasing challenges to stabilizing exchange rate and macroeconomic fluctuations.[1] Amid this context, macroprudential policy has emerged as a new pillar of the traditional macroeconomic toolkit. The premise is that prudentially regulating capital flows will reduce the vulnerability to deep economic contractions. Despite much progress, however, our understanding of how macroprudential policy should be integrated within the traditional macroeconomic toolkit remains limited.

This paper aims to fill this gap by tackling three key questions. First, how interrelated are the transmission channels of monetary and macroprudential policy? Second, how should monetary policy be used in conjunction with macroprudential policy? Third, if macroprudential policy can improve stability to an individual country, what happens with global welfare when it is implemented simultaneously by many countries?

To answer these questions, we provide an integrated analysis of monetary and macroprudential policies in a dynamic open economy with aggregate demand externalities. Taking first a positive approach, we analytically and quantitatively trace out how monetary and macroprudential policies affect, through three distinct channels, output, capital flows, and exchange rates. Then, we take a normative approach and characterize how a central bank should optimally use these policies jointly and how much is gained from integrating macroprudential policy into the policy toolkit. Finally, we evaluate the extent to which adopting prudential policies can trigger adverse international spillovers and generate scope for coordination.

The environment we consider is an infinite horizon small open economy model with nominal rigidities and an occasionally binding zero lower bound (ZLB) constraint on nominal interest rates. The model features a single final tradable good and a non-tradable good with sticky prices. We consider two government policies: a nominal interest rate and a tax on capital flows. In this context, we analyze the transmission channels of these policies and their interactions, the optimal policy design, and the policies' global implications.

Our first set of results is an analytical decomposition of the transmission channels of monetary policy that extends the work of Kaplan, Moll and Violante (2018) and Auclert (2019) to an open economy framework. Two channels are common with their closed economy analysis, intertemporal substitution and aggregate income. When there is a

[1] The recent COVID-19 crisis has again brought interest rates close to zero in several economies, both advanced and emerging. Kiley and Roberts (2017) estimate that the zero lower bound will be binding 30-40 percent of the time going forward.

reduction in the nominal interest rate, given prices and income, current consumption becomes cheaper, leading to an increase in aggregate demand. Moreover, increases in demand generate higher output, which gives rise to higher income and further increases consumption. In open economies, there is also an *expenditure switching* channel. Through the depreciation of the exchange rate, the price of non-tradables becomes relatively cheaper, leading households to substitute consumption toward domestically produced goods. In general equilibrium, this feeds into further increases in income and consumption. Notably, the effects of monetary policy in an open economy are potentially dynamic as the effects on capital flows shift demand over time.

Building on this decomposition, we show that capital flows may amplify or attenuate the effects of monetary policy. Depending on the elasticities of substitution over time and across sectors, a monetary policy expansion can lead to larger or smaller effects on output relative an economy with a closed capital account. If the elasticity of substitution over time is larger than the elasticity across sectors, an open capital account amplifies the effects of monetary policy. The logic of this result when household demand for consumption is more sensitive to changes in the intertemporal price than to the intratemporal price, the demand for tradable consumption goes up in response to a monetary expansion—or equivalently, the demand for total consumption rises more than total output, which implies that households increases borrowing from abroad. In this situation, closing the capital account leads households to postpone consumption to the future, and therefore generates a smaller output expansion than with an open capital account. These results revert when the elasticity of substitution over time is lower than the elasticity across sectors. In this case, households reduce borrowing in response to a monetary expansion, and a closed capital account generates a larger output expansion.

We then provide an analogous decomposition of the transmission channels of capital controls. Like an increase in the nominal interest rate, a tax on capital inflows generates a contraction in consumption through an intertemporal substitution effect. Moreover, the resulting decrease in capital inflows imply that the exchange rate appreciates in the future. Given a nominal interest rate, the exchange rate appreciates today, generating a contraction in demand for non-tradables through expenditure switching effects. The aggregate income channel implies that as income contracts, aggregate demand and output fall further.

A calibration of the model underscores that monetary policy and capital controls operate through very different channels. While the intertemporal substitution channel accounts for about 95% of the effects of capital controls, expenditure switching accounts for about one-third of the effects of monetary policy.

Our second set of results concerns the normative properties of the joint use of monetary and macroprudential policy. We first show that in the absence of capital controls, monetary policy faces an intertemporal tradeoff. Contrary to a widespread policy view, however, a policy of leaning against the wind is not necessarily optimal. In fact, if the elasticity of substitution across sectors is higher than the elasticity across time, a rise in the interest rate may be counterproductive. The logic of this result can be traced back to the effects discussed above. When tradable and non-tradable goods are highly substitutable, a rise in the nominal interest rates generates a large negative expenditure switching effect and a substantial drop in income. In addition, when consumption is not very substitutable over time, a rise in the interest rate generates only a modest decline in consumption. These two forces together imply that in general equilibrium, a rise in the interest rate leads to capital inflows and a larger accumulation of external debt. In turn, the increase in external debt exacerbates aggregate demand externalities, as in Schmitt-Grohé and Uribe (2016) and Farhi and Werning (2016), and makes the economy more vulnerable to a severe output contraction in the future. As a result, a prudential monetary policy requires lowering the interest rate ahead of a liquidity trap.

When capital controls are available, the central bank uses monetary policy to stabilize output and taxes inflows away from the zero lower bound. While capital controls can help stimulate or cool down the economy, output stability is best achieved using monetary policy. We also show that the macroprudential tax on debt is positive only if the zero lower bound is likely to bind in the following period, whereas monetary policy is used prudentially—in the absence of capital controls—as long as the zero lower bound binds in some distant future. The lesson is that because monetary policy is a blunter instrument, it has to be used even more preemptively than capital controls. Moreover, we show that the central bank may restrict outflows during a liquidity trap. This occurs when the liquidity trap is either temporary or very severe.

Our quantitative evaluation shows that capital controls can substantially improve macroeconomic stabilization. In the absence of capital controls, the average unemployment conditional on a liquidity trap is about 6% and the unconditional welfare cost of liquidity traps is 0.4% of permanent consumption. With capital controls, unemployment becomes 1.5% and the welfare costs fall to 0.1%. In terms of policies, the ex-ante prudential tax on inflows and the ex-post tax on outflows are respectively 0.2% and -0.05% on average. We also find that while liquidity traps are less frequent and severe with capital controls, perhaps surprisingly, they tend to last longer.

Our final set of results is concerned with international spillovers and is motivated by the policy discussions on currency wars and capital control wars.[2] Expanding our analysis to a multi-country model, we show that when there is a change in the stance of monetary policy abroad, an individual country can remain insulated from negative spillovers through the use of capital controls. In this sense, capital controls can help prevent the outbreak of a currency war. Moreover, we revisit the possibility of a global paradox of thrift and provide general conditions under which a regime of uncoordinated capital controls can dominate a laissez-faire one. Furthermore, we show that while there may be a role for coordination, it is only desirable during a liquidity trap, and it stimulates flows rather than preventing them.

Related literature. Our paper relates to several strands of the literature. First, our paper belongs to the literature on aggregate demand externalities that emerge from nominal rigidities and constraints on monetary policy (Schmitt-Grohé and Uribe, 2016; Farhi and Werning, 2016). Specifically, we share the focus on the zero lower bound with Korinek and Simsek (2016), who consider a closed economy analysis. We contribute to this literature by analyzing the interrelation between monetary and macroprudential policy and characterizing their transmission channels and global implications.[3]

Our work also relates to the literature in liquidity traps in open economies.[4] Our set of results on international spillovers may be surprising in light of the more negative view that emerges from important recent contributions. Caballero, Farhi and Gourinchas (2021) present a model of global liquidity traps and show how a recession in one block is exported abroad through goods and asset markets. Eggertsson, Mehrotra, Singh and Summers (2016) argue that neo-mercantilist policies in some countries can bring the whole world economy into a state of secular stagnation with a permanently depressed level of output. In these two studies, each country produces a tradable good, which is equally demanded by domestic and foreign households. We consider instead the polar opposite case, in which the goods produced subject to nominal rigidities are consumed exclusively by domestic households. Our analysis uncovers how this feature implies that foreign policies that

[2] For an overview of the policy discussions, see Rey (2013), Rajan (2014), Blanchard (2021), and Kalemli-Ozcan (2019).

[3] A review of the literature on prudential policies due to aggregate demand and pecuniary externalities is presented in Bianchi and Lorenzoni (2021).

[4] Examples include Cook and Devereux (2013); Devereux and Yetman (2014); Eggertsson, Mehrotra, Singh and Summers (2016); Caballero, Farhi and Gourinchas (2021); Acharya and Bengui (2018); Fornaro and Romei (2019); Jeanne (2009); Benigno and Romei (2014); Fornaro (2018); Corsetti, Mueller and Kuester (2019b); Corsetti, Mavroeidi, Thwaites and Wolf (2019a); Kollmann (2021) and Amador, Bianchi, Bocola and Perri (2020). Notable closed economy studies include Krugman (1998), Eggertsson and Woodford (2003), and Werning (2011).

favor savings actually increase the demand for domestic goods via asset markets and can become stabilizing at the zero lower bound.

Fornaro and Romei (2019) is another notable study that examines international spillovers in a model with a similar production and monetary structure. Unlike us, they find that capital account policies may lead to a global paradox of thrift, in which uncoordinated capital control policies at the global level lead to worse output and welfare outcomes compared with those of a laissez-faire economy without capital controls. Two key features of our model explain the stark differences in our results: positive liquidity and the presence of ex-post capital controls. The former implies that a decline in the world interest rate due to prudential capital controls abroad generates a rise in the demand for consumption, counteracting the tighter zero lower bound constraint resulting from the lower world interest rate. The latter implies that the central bank can actively manage capital flows ex post during a liquidity trap, which can help offset potential adverse spillover effects.

Our paper is also related to a growing literature on the interaction between monetary and macroprudential policies. Coulibaly (2020) and Basu, Boz, Gopinath, Roch and Unsal (2020) consider these interactions and study optimal policies in models with pecuniary externalities, but they abstract from any constraints on monetary policy. Farhi and Werning (2020) examine, as we do, a model with aggregate demand externalities that emerge in the presence of a zero lower bound. Their analysis is complementary to ours in that, they study a closed economy and focus on behavioral aspects, whereas we focus on international dimensions and take a more quantitative approach.[5]

Finally, Kaplan, Moll and Violante (2018) and Auclert (2019) provide a decomposition of the effects of monetary policy in a closed economy model and evaluate the differences in transmission channels between representative agent and heterogeneous agent models. One contribution of our paper is to develop an open economy decomposition and uncover the central role of expenditure switching and exchange rates. In parallel and independent work, Auclert, Rognlie, Souchier and Straub (2021) also provide a related open economy decomposition of the effects of monetary policy. Their focus is on the interaction between terms of trade and the real income channel and the role of heterogeneity in shaping the monetary transmission.[6] Our focus is instead on understanding the interplay between

[5] In a setting with dollar currency pricing, Egorov and Mukhin (2020) also study optimal monetary policy and capital controls, but find no welfare role for the latter despite the lack of insularity. Several other contributions consider monetary and macroprudential interactions but do not characterize optimal policies (e.g., Aoki, Benigno and Kiyotaki, 2016; Van der Ghote, 2021) .

[6] A burgeoning literature is incorporating heterogeneity within open economy New Keynesian models (Bianchi, Ottonello and Presno, 2019; De Ferra, Mitman and Romei, 2020; Guo, Ottonello and Perez, 2020; Auclert et al., 2021)

monetary policy and capital controls from a positive and normative perspective.

Outline. Section 2 presents the model. Section 3 provides a decomposition of the channels of monetary policy and capital controls. Section 4 studies optimal monetary and macroprudential policy. Section 5 analyzes international spillovers. Section 6 concludes.

2 Model

We consider a small open economy with nominal rigidities and an occasionally binding zero lower bound constraint. There is an infinite horizon and two types of goods: tradables and non-tradables.

2.1 Households

There is a continuum of identical households of measure one. Households' preferences

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t z_t \left[U(c_t) - v(h_t) \right], \tag{1}$$

where $\mathbb{E}_t$ denotes the time t expectation operator, $\beta \in (0,1)$ is the discount factor and z_t represents a discount factor shock. The utility function over consumption $u(\cdot)$ is strictly increasing and concave, and $v(\cdot)$ denotes an increasing and convex disutility function of labor. We assume that these functions are isoelastic of the form

$$U(c_t) = \frac{c_t^{1-\frac{1}{\sigma}}}{1-\frac{1}{\sigma}}, \quad v(h_t) = \frac{h_t^{1+\phi}}{1+\phi},$$

where σ is the elasticity of intertemporal substiution and ϕ is the inverse of the Frisch elasticity. The consumption good c_t is a composite of tradable consumption c_t^T and non-tradable consumption c_t^N, according to a constant elasticity of substitution aggregator:

$$c_t = \left[\omega (c_t^T)^{1-\frac{1}{\gamma}} + (1-\omega)(c_t^N)^{1-\frac{1}{\gamma}} \right]^{\frac{1-\gamma}{\gamma}}, \quad \text{where } \omega \in (0,1).$$

The elasticity of substitution between tradable and non-tradable consumption is γ. For convenience, we use $u(c^T, c^N)$ to denote the utility as a function of the two consumption goods.

In each period t, households supply h_t units of labor and are endowed with y_t^T units of tradable goods. We assume that y_t^T is stochastic and follows a first-order Markov process. Households receive a wage rate, W_t, collect profits, ϕ_t^N, all expressed in terms of domestic currency, which serves as the numeraire, and receive government transfers T_t. Households trade two types of one-period non-state-contingent bonds in credit markets: a real bond b_{t+1}^*, which pays a gross return R_t^* units of tradables, and a nominal bond b_{t+1}, which pays R_t in units of domestic currency. The domestic government controls the nominal rate R_t. Both bonds are potentially subject to a tax/subsidy τ_t. When $\tau_t > 0$, households face a tax on debt issuance and a subsidy on savings. Conversely, when $\tau_t < 0$, households face a subsidy on debt issuance and taxes on savings.

The budget constraint of the representative household is therefore given by

$$P_t^N c_t^N + P_t^T c_t^T + \frac{1}{1+\tau_t}\left[\frac{b_{t+1}}{R_t} + P_t^T \frac{b_{t+1}^*}{R_t^*}\right] = \phi_t^N + W_t h_t + P_t^T (y_t^T + T_t) + b_t + P_t^T b_t^\star, \quad (2)$$

where P_t^N and P_t^T denote respectively the price of non-tradables and tradables (in terms of the domestic currency). The left-hand side represents total expenditures in tradable and non-tradable goods and purchases of bonds while the right-hand side represents total income, including the returns from bond holdings.

Optimality conditions. The households' problem consists of choosing sequences of $\{c_t^N, c_t^T, h_t, b_{t+1}, b_{t+1}^*\}$ to maximize the expected present discounted value of utility (1), subject to (2) and taking as given the sequence of tradable endowments $\{y_t^T\}$, profits $\{\phi_t^N\}$, transfers $\{T_t\}$, and prices $\{W_t, P_t^N, P_t^T, R_t, R_t^*\}$.

The first-order conditions for consumption and labor yield

$$\frac{W_t}{P_t^N} = \frac{v'(h_t)}{u_N(t)} \quad (3)$$

$$\frac{P_t^N}{P_t^T} = \frac{1-\omega}{\omega}\left(\frac{c_t^T}{c_t^N}\right)^{\frac{1}{\gamma}}, \quad (4)$$

where u_N denotes the marginal utility of non-tradable consumption in period t. Condition (3) is the labor supply optimality condition equating the marginal rate of substitution between leisure and non-tradable consumption with the wage rate in terms of non-tradables. Condition (4) equates the marginal rate of substitution between tradables and non-tradables to the relative price.

The first-order conditions for the nominal and real bond holdings yield

$$u_T(c_t^T, c_t^N) = \beta R_t^*(1+\tau_t)\mathbb{E}_t\left[\frac{z_{t+1}}{z_t}u_T(c_{t+1}^T, c_{t+1}^N)\right] \tag{5}$$

$$\frac{u_T(c_t^T, c_t^N)}{P_t^T} = \beta R_t(1+\tau_t)\mathbb{E}_t\left[\frac{z_{t+1}}{z_t}\frac{u_T(c_{t+1}^T, c_{t+1}^N)}{P_{t+1}^T}\right]. \tag{6}$$

where u_T denoting the marginal utility of tradable consumption. Households equate the marginal benefit from saving in nominal or real bonds to the marginal costs of cutting tradable consumption today to buy the bonds.

2.2 Firms

The non-tradable good is produced by a continuum of firms in a perfectly competitive market. Each firm produces a non-tradable good according to a production technology given by $y_t^N = n_t^\alpha$ and perceive profits given by

$$\phi_t^N = P_t^N n_t^\alpha - W_t n_t. \tag{7}$$

We assume that prices are perfectly rigid, $P_t^N = \bar{P}^N$, and that firms produce goods to satisfy demand (i.e., labor demand in equilibrium is given by $n = (c^N)^{1/\alpha}$). We note that results would be similar with optimal price setting, as long as there are costs from changing prices.[7] In a sense, this modelling of price rigidities is closer to an earlier vintage of disequilibrium models (e.g., Barro and Grossman, 1971). In terms of the welfare analysis that will follow, an advantage of this formulation is that the results on optimal policy are guided only by aggregate demand management considerations.

2.3 Government

The government sets a nominal interest rate $R_t \geq 1$ and a tax on all forms of bond issuances τ_t. As is common in the literature, this tax can be interpreted as a capital control (see e.g., Bianchi, 2011 and Schmitt-Grohé and Uribe, 2016). The tax is assumed to be rebated lump-sum to households, an assumption that is without loss of generality given that

[7] We have conducted simulations with one-period-in-advance price setting with similar results.

Ricardian equivalence holds.[8] That is, the government budget constraint is

$$T_t = -\frac{\tau_t}{1+\tau_t}\left[\frac{b_{t+1}}{P_t^T R_t} + \frac{b_{t+1}^*}{R_t^*}\right]. \tag{8}$$

2.4 Prices, UIP and Exchange Rates

We assume that the law of one price holds for the tradable good, that is, $P_t^T = e_t P_t^{T*}$, where e is the nominal exchange rate defined as the price of the foreign currency in terms of the domestic currency, and P^{T*} is the price of the tradable good denominated in foreign currency.

Using the Euler equations for international bond (5) and domestic bond (6), we can equate the marginal benefits from buying the real and nominal bond. Together with the law of one price, this implies that the nominal exchange rate must satisfy the risk-adjusted uncovered interest parity condition:

$$R_t^* = R_t\,\mathbb{E}_t\left[\Lambda_{t+1}\frac{e_t}{e_{t+1}}\frac{P_t^{T,*}}{P_{t+1}^{T,*}}\right], \tag{9}$$

where $\Lambda_{t+1} \equiv z_{t+1}u_T(c_{t+1}^T, c_{t+1}^N)/\mathbb{E}_t\left[z_{t+1}u_T(c_{t+1}^T, c_{t+1}^N)\right]$ represents a stochastic discount factor. Condition (9) is a standard condition that relates the the foreign real interest rate and the domestic nominal interest rate to the expected depreciation of the domestic currency.

2.5 Competitive Equilibrium

Market clearing for labor requires that the units of labor supplied by households equal the aggregate labor demand by firms:

$$h_t = n_t. \tag{10}$$

Market clearing for the non-tradable good requires that output be equal to non-tradable consumption:

$$y_t^N = c_t^N. \tag{11}$$

[8] We abstract from other the so-called unconventional fiscal policies that can relax the zero lower bound (see e.g. Correia, Farhi, Nicolini and Teles, 2013). As examined in Acharya and Bengui (2018), differential taxes on bonds across currencies can also help relax the zero lower bound. As long as there are some limitations on the use of these policies (either political or economic), the first best cannot be implemented and the scope of our analysis would be similar.

We assume that the bond denominated in domestic currency is traded only domestically. We make this assumption to abstract from portfolio problems and from the possibility of inflating away external debt. Market clearing therefore implies

$$b_{t+1} = 0. \tag{12}$$

Combining the budget constraints of households, firms, and the government, as well as market clearing conditions, we arrive at the resource constraint for tradables, or the balance of payment condition:

$$c_t^T - y_t^T = b_t^* - \frac{b_{t+1}^*}{R_t^*}, \tag{13}$$

which says that the trade balance must be financed with net bond issuances.

An equilibrium, given government policies, is defined as follows.

Definition 1. Given an initial condition b_0^*, exogenous process $\{R_t^*, y_t^T, z_t\}_{t=0}^{\infty}$, a rigid price $\bar{P}^N$, and government policies $\{R_t, \tau_t\}_{t=0}^{\infty}$, an equilibrium is a stochastic sequence of prices $\{e_t, P_t^{T*}, W_t\}$ and allocations $\{c_t^T, c_t^N, b_{t+1}^*, n_t, h_t\}_{t=0}^{\infty}$ such that

(i) households optimize, and hence the following conditions hold: (3), (4), (5), (6);

(ii) firms choose hours to meet demand, $h^\alpha = c^N$;

(iii) labor market clears (10) and the domestic currency bond is in zero net supply (12);

(iv) the government budget constraint (8) is satisfied;

(v) the law of one price holds: $P_t^T = e_t P_t^{T*}$.

2.6 First-Best Allocation

We conclude the description of the model by presenting the first-best allocation.

We consider a benevolent social planner of the small open economy who chooses allocations subject to resource constraint. The planner's problem can be written as

$$\max_{\{b_{t+1}^*, c_t^N, c_t^T\}} \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t z_t \left[u\left(c_t^T, c_t^N\right) - v\left(\left(c_t^N\right)^{1/\alpha}\right)\right], \tag{14}$$

subject to

$$c_t^T = y_t^T + b_t^* - \frac{b_{t+1}^*}{R_t^*}.$$

The first-best allocation equates the value of one additional employed unit of labor to the marginal cost of leisure

$$\alpha h_t^{\alpha-1} u_N(c_t^T, c_t^N) = v'(h_t) \tag{15}$$

It also equates the marginal utility of current consumption to the marginal utility of saving one extra unit and consuming in the next period:

$$u_T(c_t^T, c_t^N) = \beta R_t^* \mathbb{E}_t \left[\frac{z_{t+1}}{z_t} u_T(c_{t+1}^T, c_{t+1}^N) \right]. \tag{16}$$

It should be clear that the allocations in a competitive equilibrium with flexible prices would coincide with the first best. This can be seen by noting that if firms could adjust prices, we would have $\alpha h_t^{\alpha-1} = W_t / P_t^N$, which, combined with households' labor supply decision (3), would yield (15).[9] Moreover, as we will see, with sticky prices, a government that can choose monetary policy without any constraints would choose to replicate the flexible price allocation and hence implement the first-best allocations.[10]

Before we analyze monetary policy and macroprudential policy from a normative point of view, it is useful to first analyze the transmission channels of both monetary and macroprudential policies.

3 Transmission of Monetary and Macroprudential Policy

In this section, we construct an analytical decomposition of the effects of monetary policy and macroprudential policy. This decomposition is instrumental to understand the transmission channels of these policies, their interactions, and the normative analysis that we will conduct in Sections 4 and 5.

Our approach consists of first studying how an individual household responds in partial equilibrium to these policies, then analyzing how these polices play out in general equilibrium by disentangling the direct effects of policies on households decisions from the indirect effects emerging from general equilibrium. For analytical convenience, we consider a version of the model starting with zero net foreign asset position and without uncertainty. We then show how the decomposition can be applied to a stochastic version

[9]In addition, notice that (16) coincides with households' optimality (5) when $\tau_t = 0$.

[10]Often New Keynesian models feature monopolistic competition which create an additional wedge between competitive equilibrium with nominal rigidities and flexible price equilibria. In addition, by considering a single tradable good with perfect pass through, we are abstracting from issues related to terms of trade externalities that create additional welfare considerations. Our framework allows us to focus squarely on aggregate demand management considerations.

of the model, which we solve numerically.

3.1 Partial Equilibrium Response

Our approach requires us to first characterize the households policies as a function of an arbitrary sequence of prices and government policies. We place no restrictions on prices and policies other than the condition that the return of the domestic bond in real terms equals the foreign bond.[11]

Households' policies. We can express consumption of tradables and non-tradables and foreign bond holdings as a function of the sequence of income levels, relative prices, interest rates and taxes. That is,

$$c_0^N = \mathcal{C}^N \left(\left\{ \bar{P}^N / (P^{T*} e_t), R_t, \tau_t, Y_t \right\}_{t \geq 0} \right), \tag{17a}$$

$$c_0^T = \mathcal{C}^T \left(\left\{ \bar{P}^N / (P^{T*} e_t), R_t, \tau_t, Y_t \right\}_{t \geq 0} \right), \tag{17b}$$

$$b_1^* = \mathcal{B}^* \left(\left\{ \bar{P}^N / (P^{T*} e_t), R_t, \tau_t, Y_t \right\}_{t \geq 0} \right), \tag{17c}$$

where Y_t is defined value of the income in period t in units of the composite consumption, or the "real income". That is,

$$\mathcal{P}_t Y_t \equiv y_t^T + T_t + \frac{W_t h_t + \phi_t^N}{P_t^T}, \tag{18}$$

and $\mathcal{P}_t$ denotes the ideal price index (i.e., the minimum expenditure, denominated in units of tradables, required to buy one unit of the composite good c_t):

$$\mathcal{P}_t = \left[\omega^\gamma + (1-\omega)^\gamma \left(\frac{\bar{P}^N}{e_t P_t^{T*}} \right)^{1-\gamma} \right]^{\frac{1}{1-\gamma}}. \tag{19}$$

In addition, for future reference, the share of expenditures in tradables is denoted by

$$\tilde{\omega}_t \equiv P_t^T c_t^T / (P_t^T c_t^T + P^N c_t^N). \tag{20}$$

We have the following lemma characterizing the solutions of the household problem in

[11] Otherwise, the deviation from arbitrage would imply that households' policies would be unbounded.

closed form.

Lemma 1. *For any sequence of nominal exchange rates $\{e_t\}$, taxes $\{\tau_t\}$, and households' income $\{Y_t\}$, we have that savings decision, tradable consumption, and non-tradable consumption at date 0 are given by*

$$c_0^T = \omega^\gamma \left(\mathcal{P}_0\right)^{\gamma-1} \ \mu_0 \sum_{t=0}^{\infty} Q_{t|0}\mathcal{P}_t Y_t. \tag{21a}$$

$$c_0^N = \frac{P^{T*}e_0}{\bar{P}^N}\left[1-\omega^\gamma \left(\mathcal{P}_0\right)^{\gamma-1}\right] \ \mu_0 \sum_{t=0}^{\infty} Q_{t|0}\mathcal{P}_t Y_t, \tag{21b}$$

$$b_1^* = R_0^*(1+\tau_0)\left[\mathcal{P}_0 Y_0 - \mu_0 \sum_{t=0}^{\infty} Q_{t|0}\mathcal{P}_t Y_t\right], \tag{21c}$$

where $Q_{t|0} \equiv \prod_{s=0}^{t-1}[R_s^*(1+\tau_s)]^{-1}$ and $\mu_0 = 1/\sum_{t=0}^{\infty}\beta^{t\sigma}\left[Q_{t|0}\frac{\mathcal{P}_0}{\mathcal{P}_t}\right]^{\sigma-1}$ is the marginal propensity to consume.

Proof. In Appendix A.1 □

Because preferences are additively separable over time, the household problem can be split into choosing the optimal expenditures on the composite consumption over time, then choosing how expenditures at each point in time are allocated between tradables and non-tradables.[12] In particular, households spend at $t=0$ a fraction μ_0 of the annuity value of lifetime income and split it between tradables and non-tradables, with weights given by $\omega^\gamma \left(\mathcal{P}_0\right)^{\gamma-1}$ and $1-\omega^\gamma \left(\mathcal{P}_0\right)^{\gamma-1}$, respectively. In turn, the marginal propensity to consume, μ_0, depends on the price of the period 0 composite consumption relative to all future prices and the intertemporal elasticity of substitution. Likewise, the share of expenditures between tradables and non-tradables in period 0 depends on the relative price of tradables and non-tradables in period 0 and the elasticity of substitution across the two goods.

3.2 Transmission Channels of Monetary Policy

In this section, we propose an analytical decomposition of the effects of monetary policy. The analysis extends the analysis of Auclert (2019) and Kaplan, Moll and Violante (2018) to

[12] Notice that by taking as given the household income Y as exogenously given for the household, we are, in effect, taking as given a number of hours worked. Thanks to the separability of consumption and leisure in the utility function, the solutions from Lemma 1 apply to any choice of hours. When we consider the general equilibrium response, we consider the optimally supplied amount of hours.

an open economy setting. As we will see, there are two critical elements of the transmission channel not present in the closed economy. First, there is an exchange rate between the domestic and foreign currencies. In fact, through the risk-adjusted UIP condition (9), a temporary decline in the nominal interest rate leads to a depreciation of the nominal exchange rate and shifts the relative price between tradables and non-tradables. Second, monetary policy can potentially affect capital flows and shift demand over time.

We consider a one-period decline in the nominal interest rate at $t = 0$, which reverts for simplicity to the steady state value at $t = 1$. We restrict attention to the case in which $\beta R^* = 1$, so that tradable consumption remains constant in the steady state. Moreover, we assume that for $t \geq 1$, the economy features a level for the exchange rate that is consistent with the first-best allocation. Given these assumptions, we have a stationary equilibrium for $t > 1$ with a constant exchange rate, employment, and non-tradable consumption. Notice, however, that the stationary level for the exchange rate and allocations do not revert to the initial steady state values. Indeed, the new steady state values depend on how the economy reacts to the temporary shock at $t = 0$.

In Lemma 1, we derived the individually optimal policies for consumption and savings for arbitrary prices and government policies. We can now use the fact that the non-tradable goods market clears in general equilibrium to obtain a decomposition of the channels by which monetary policy affects the economy. The results are displayed in the proposition below.

Proposition 1 (Monetary policy decomposition)**.** *Consider a change in the nominal interest rate R_0. To a first-order, the effects on output are given by*

$$\frac{dy_0^N}{y_0^N} = \underbrace{-\beta\sigma(1-\tilde{\omega})\frac{dR_0}{R_0}}_{\text{Intertemporal Subst.}} + \underbrace{\tilde{\omega}\gamma\frac{de_0}{e_0}}_{\text{Expend. Switching}} - \underbrace{(1-\beta)(1-\tilde{\omega})\left[(1-\kappa)\sigma+\kappa\gamma\right]\frac{dR_0}{R_0}}_{\text{Aggregate Income}} \tag{22}$$

and the effects on capital flows are given by

$$\frac{db_1^*}{\mathcal{P}c} = \underbrace{\sigma(1-\tilde{\omega})\frac{dR_0}{R_0}}_{\text{Intertemporal Subst.}} - \underbrace{\left[(1-\tilde{\omega})\sigma+\tilde{\omega}\gamma\right](1-\tilde{\omega})\frac{dR_0}{R_0}}_{\text{Aggregate Income}} \tag{23}$$

where $de_0/e_0 = -\left[1-(1-\beta)(1-\kappa)\left(1-\sigma\gamma^{-1}\right)\right] dR_0/R_0$, $\kappa \equiv \tilde{\omega}\left[1+\frac{\alpha(1-\tilde{\omega})(\gamma-\sigma)}{\alpha\sigma+(1-\alpha+\phi)\gamma\sigma}\right]^{-1}$*. We also have that* $\kappa \in (0,1)$*, and the effects on* y_t^N *for* $t > 0$ *and* c_t^T *are presented in Appendix A.2*

Proof. In Appendix A.2. □

Equation (22) decomposes the effects of a change in the interest rate on non-tradable output in three channels: intertemporal substitution, expenditure switching and aggregate income.

Consider first the intertemporal substitution channel, a channel that is common with closed economy studies. To fix ideas, consider a reduction in the nominal interest rate, which reduces the relative price of consuming in period 0 relative to future periods. The intertemporal substitution channel denotes the shift in consumption from the future to the present that originates from the change in the relative intertemporal price of consumption. To a first order, the optimal households' response in terms of non-tradable consumption is given by the discounted change in the real interest rate, times the IES. Notice that a 1% reduction in the nominal interest rate gives rise to a reduction in the interest rate in terms of the composite consumption (i.e., the real interest rate) of $(1-\tilde{\omega})\%$ where $\tilde{\omega}$ is defined in (20).[13]

Given a sticky price for non-tradables in domestic currency and the price for tradables in foreign currency, the depreciation reduces today's relative price of non-tradables and leads households to switch consumption toward non-tradables. The expenditure switching channel represents the increase in non-tradable consumption that results from the reduction in the relative price for non-tradables, keeping lifetime constant in units of the compositive consumption. For a 1% depreciation of the nominal exchange rate, the magnitude of this effect is given by $\tilde{\omega}$ times the elasticity of substitution between tradables and non-tradables. Notice that the more open the economy is, as measured by $\tilde{\omega}$, the larger is the expenditure switching channel (and the smaller is the intertemporal substitution channel).

These two channels, expenditure switching and intertemporal substitution, concern the direct partial equilibrium response of households.[14] This partial equilibrium response assumes that production of non-tradable goods is not affected by the monetary expansion. However, in general equilibrium, as demand increases, firms respond by producing more. The increased production raises households' labor income and profits and generates second-round effects on demand. By a classic Keynesian cross logic, higher demand feeds into higher output. Notice that the marginal propensity to consume that matters is the one associated with non-tradables. In particular, given relative prices, a one unit increase in lifetime income leads to a $(1-\beta)(1-\tilde{\omega})$ increase in c^N. The total effect is given by

[13]See Lemma A.1 in the appendix for this result.

[14]We note that an old tradition in international macro describe the expenditure switching and expenditure tilting effect (also called expenditure changing or in our case intertemporal substitution), but without a formal decomposition.

the marginal propensity to consume non-tradables times an average of the elasticities of substitution across sectors and time weighted by κ.

Capital flows and dynamic effects. It is important to highlight that the effects of a monetary policy shock are dynamic to the extent that monetary policy affects capital flows. In particular, if monetary policy generates capital outflows, this lowers the level of the exchange rate in the future e_{t+1} since a larger bond position implies that implementing the first-best allocation requires a more appreciated exchange rate. In turn, through (9), a more appreciated exchange rate translates into a more appreciated exchange rate today and thus a smaller expenditure switching channel. This feature of our open economy model contrasts with the canonical closed economies where a purely transitory monetary policy shock does not alter future output and prices.

The second item in Proposition 1 characterizes the effects of monetary policy on capital flows. A monetary expansion has two opposite effects on capital flows. For a given level of aggregate income, a decline in the domestic nominal interest rate lowers the price of consuming today versus tomorrow, leading households to substitute toward current consumption. Given aggregate income, this reduces aggregate savings and leads to an increase in capital inflows. On the other hand, the temporary increase in income, resulting from expenditure switching and intertemporal substitution, leads to higher savings, and therefore capital outflows.

The net effects of a decrease in the nominal interest rate on aggregate savings depend crucially on the relative importance of the two elasticities. If the elasticity across time σ is higher than the elasticity across sectors γ, the intertemporal substitution channel dominates the aggregate income one and the increase in consumption exceeds the increase in income. As a result, we have an increase in capital inflows, and savings decrease. On the other hand, if the elasticity across time is lower than the elasticity across sectors, we have that the aggregate income channel dominates the intertemporal substitution one and the increase in income exceeds the increase in consumption, resulting in capital outflows. In the knife-edge case in which the elasticities are equal ($\sigma = \gamma$), a monetary expansion does not induce movement in the country's net foreign asset position, as aggregate income and current consumption increase by the same amount.[15,16]

These results are summarized in the following corollary.

Corollary 1. *Consider a reduction in R. If $\sigma < \gamma$, aggregate savings increase, and if $\sigma > \gamma$, aggregate savings decrease. Finally, if $\sigma = \gamma$, savings remain constant.*

Proof. In Appendix A.3. □

Capital flows: Amplification or attenuation? An important question is whether capital flows amplify or attenuate the effects of monetary policy. To examine this, we analyze the effects of monetary policy when we shut down the capital account and compare the response with our baseline model with an open capital account. As it turns out, the result depends on the relative elasticities of substitution across time and sectors. When $\sigma > \gamma$, households decrease savings in response to a monetary policy expansion, as characterized in Corollary 1. This implies that households shift aggregate demand from the future to the present. In this context, closing the capital account contracts aggregate demand and raises output. Thus, capital flows amplify the effects of monetary policy (i.e., output increases by more than if the capital account were closed). On the other hand, when $\sigma < \gamma$, households increase savings, and shutting down the capital account boosts aggregate demand. Thus, capital flows attenuate the effects of monetary policy. Finally, when $\sigma = \gamma$, the response of monetary policy is independent of whether the capital account is open or closed.

The following proposition summarizes these results.

Proposition 2. *Consider a monetary expansion under a closed capital account and an open capital account. We have that*

i) If $\sigma > \gamma$, output increases by more with an open capital account

ii) If $\sigma < \gamma$, output increases by more with a closed capital account

iii) If $\sigma = \gamma$, the output response is the same under open and closed capital account.

Proof. In Appendix A.4. □

[15] The role of these two elasticities in driving the current account response to a monetary policy shock was first highlighted by Lane (2001). A related result by Werning (2015) establishes conditions under which idiosyncratic risk does not affect how aggregate consumption responds to monetary policy (see also Auclert et al., 2021 for an open economy analysis).

[16] An alternative intuition for how the relative elasticities matter for the response of capital flows to monetary policy can be obtained by noting that given the preference structure, $\sigma > \gamma$ if and only if $u_{TN} > 0$. That is, given that a monetary policy expansion increases c^N, a positive cross partial derivative implies that households value more tradables and therefore borrow more.

3.3 Transmission Channel of Macroprudential Policy

We now turn to describing the transmission channels of capital controls. We keep the nominal interest rate constant and consider a one-period change in the tax on debt at $t = 0$ (which reverts to the steady state value at $t = 1$). [17]

Analogously to Proposition 1, the next proposition presents the channels by which capital controls affect the economy.

Proposition 3 (Capital controls decomposition). *Consider a change in the tax on debt τ_0. To a first order, the effects on output are given by*

$$\frac{dy_0^N}{y_0^N} = \underbrace{-\beta\sigma d\tau_0}_{\text{Intertemporal subst.}} + \underbrace{\tilde{\omega}\gamma\frac{de_0}{e_0}}_{\text{Expend. Switching}} \underbrace{-(1-\beta)(1-\kappa)\sigma d\tau_0}_{\text{Aggregate Income}}$$

and the effects on capital flows are given by

$$\frac{db_1^*}{\mathcal{P}c} = \underbrace{\sigma d\tau_0}_{\text{Intertemporal Subst.}} - \underbrace{(1-\tilde{\omega})\sigma d\tau_0}_{\text{Aggregate Income}}$$

where $\frac{de_0}{e_0} = -(1-\beta)\frac{1-\kappa}{1-\tilde{\omega}}\sigma\gamma^{-1}d\tau_0$.

Proof. In Appendix A.5. □

A tax on debt increases the return on savings and leads households to substitute consumption intertemporally toward the future. The intertemporal substitution channel is again proportional to $\beta\sigma$, but now the change in the real interest rate is given by $d\tau_0$. So the total effect is given by $\beta\sigma d\tau_0$, implying that for changes of the same magnitude in the nominal interest rate and the tax rate, we have a larger intertemporal substutitution effect with the latter.

As foreign savings increase, the future exchange rate appreciates to close the output gap. Through the UIP condition, this implies that the current exchange rate also appreciates, generating an expenditure switching effect away from non-tradable consumption. The form of the expenditure switching channel is isomorphic to the one in (22). However, as we will see, it is smaller than the one for monetary policy because the exchange rate responds only indirectly through the future changes in the exchange rate.

[17] As we proceeded above, we assume that for $t \geq 1$, the central bank implements a monetary policy consistent with the first-best allocation.

Finally, we have the aggregate income channel. As output contracts in response to the reduction in the demand for consumption, income falls, and this further reduces consumption.

3.4 Quantitative Inspection of Transmission Channels

We now solve the stochastic version of our model economy and simulate the impulse response to a monetary policy shock and a capital control shock. The calibration is standard and described in Section 4.4.[18]

The overall response in output and decomposition are displayed in Table 1. We illustrate the results for different values of the intra and inter-temporal elasticities of substitution, keeping the rest of the parameters constant. We highlight that the results are very close to those that one would obtain from directly applying Proposition 1, suggesting that the approximation remains accurate in the presence of uncertainty and higher-order terms.

Table 1 shows that monetary policy and capital controls operate quantitatively through entirely different channels. While capital controls work almost entirely through intertemporal substitution, expenditure switching plays a key role for monetary policy. In the case of unitary elasticities, expenditure switching accounts for about one-third of the overall effects of monetary policy. As the elasticity of substitution across sectors γ increases, demand becomes more sensitive to changes in the nominal exchange rate, leading to a larger increase in output following a monetary expansion. When $\gamma = 2$, expenditure switching accounts for about one-half of the overall effects. Meanwhile, the aggregate income accounts for at most 5% of the overall effects of monetary policy, a finding that is common with the closed economy representative agent model. Moreover, the modest contribution of the aggregate income channel also extends to the effects of capital controls.

The table also shows that for values of $\gamma = 1$, the effects on output of a 1 percentage point cut in the nominal interest rate and a 1 percentage point decrease in the tax on debt are about the same. While this suggests that these policies may be substitutes in terms of the output response, they work through distinct channels, as we showed, and they also have different implications for capital flows. In the next section, we leverage this analysis and take a normative perspective on how policy makers should jointly use these policies.

[18]The only exception is that the world interest rate is set to a deterministic value of 1.4%.

Table 1: Transmission channels of monetary policy and capital controls

	$\sigma = 0.5$			$\sigma = 1$		
	$\gamma = 0.5$	$\gamma = 1$	$\gamma = 2$	$\gamma = 0.5$	$\gamma = 1$	$\gamma = 2$
Monetary Policy						
Change in y^N (pp)	0.50	0.62	0.91	0.89	1.00	1.27
Intertemporal subst.	0.37	0.35	0.34	0.74	0.71	0.69
Expend. switching	0.11	0.22	0.53	0.12	0.25	0.53
Aggregate income	0.02	0.04	0.04	0.03	0.04	0.05
Capital controls						
Change in y^N (pp)	0.50	0.50	0.50	1.01	1.01	1.01
Intertemporal subst.	0.48	0.48	0.47	0.97	0.96	0.96
Expend. switching	0.00	0.00	0.01	0.01	0.01	0.01
Aggregate income	0.02	0.02	0.02	0.03	0.04	0.04

Note: The monetary policy shock we consider is a 1 percentage point decrease in the nominal interest rate for one quarter (annualized). The capital controls shock is a 1 percentage point decrease in the tax on debt for one quarter (annualized). All responses are reported in annualized terms.

4 Optimal Monetary and Macroprudential Policies

In this section, we take a normative approach and consider optimal monetary and macroprudential policies. To shed light on the policy interactions, we first study optimal macroprudential policy given monetary policy, we then study joint optimal monetary and macroprudential policy, and finally, we study optimal monetary policy given a macroprudential policy.

4.1 Macroprudential Policy

The instrument for macroprudential policy is a tax on debt. As is common to the literature on aggregate demand externalities, the role of the tax on debt is to tilt aggregate demand intertemporally and help improve macroeconomic stabilization (see, e.g., Schmitt-Grohé and Uribe, 2016 and Farhi and Werning, 2016).

The approach we follow here considers a generic monetary policy that depends on the history of all shocks. We use $\{e_t\}$ to denote the nominal exchange rate policy sequence chosen by the government. An advantage of this formulation is that we are able to provide a general characterization of macroprudential policy encompassing multiple monetary policy regimes.

Under an arbitrary monetary policy, the production of non-tradable goods is, in general, inefficient. For example, given the sticky price $\bar{P}^N$, a low exchange rate implies a high relative price for non-tradables, in turn generating lower household demand for non-tradable goods and leading firms to reduce production. The departure of the equilibrium allocations from the first best can be conveniently summarized in the labor wedge, defined below:

$$\psi_t \equiv 1 - \frac{1}{\alpha h_t^{\alpha-1}} \frac{v'(h_t)}{u_N(c_t^T, c_t^N)}. \tag{24}$$

At a first-best allocation $\psi_t = 0$. A positive labor wedge, $\psi_t > 0$, reflects a marginal value of employment that exceeds the marginal cost from providing labor. In this sense, the economy experiences a recession. Conversely, a negative labor wedge, $\psi_t < 0$, reflects a marginal value of employment that is too low relative to the marginal cost from providing labor. In this case, the economy experiences overheating.

Given a sequence of $\{e_t\}$, the government chooses the state-contingent tax on debt $\{\tau_t\}$ that maximizes private agents' welfare among the set of competitive equilibria. The problem can be written as

$$\max_{\{b_{t+1}^*, c_t^N, c_t^T\}} \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t z_t \left[u\left(c_t^T, c_t^N\right) - v\left(\left(c_t^N\right)^{1/\alpha}\right)\right], \tag{25}$$

subject to

$$c_t^T = y_t^T + b_t^* - \frac{b_{t+1}^*}{R_t^*},$$

$$c_t^N = \left[\frac{1-\omega}{\omega} \frac{P_t^{T*}}{\bar{P}^N} e_t\right]^{\gamma} c_t^T.$$

The last constraint in problem (25) relates non-tradable consumption to tradable consumption and the relative price of non-tradables. More tradable resources increase aggregate demand for both goods. Given a fixed price for non-tradables, higher resources translate into more demand for non-tradable goods, to which firms respond by raising employment. This general equilibrium feedback is key for the characterization of the optimal macroprudential tax presented below.

Proposition 4 (Optimal macroprudential policy given $\{e_t\}$)**.** *Consider an exogenous exchange* *rate policy* $\{e_t\}$. *The optimal tax on borrowing that implements* (25) *satisfies*

$$\tau_t = \frac{1}{\beta R_t^* \mathbb{E}_t \frac{z_{t+1}}{z_t} u_T(t+1)} \left\{ -\frac{1-\tilde{\omega}_t}{\tilde{\omega}_t} u_T(t)\psi_t + \beta R_t^* \mathbb{E}_t \frac{z_{t+1}}{z_t} \left[\frac{1-\tilde{\omega}_{t+1}}{\tilde{\omega}_{t+1}} u_T(t+1)\psi_{t+1} \right] \right\}, \tag{26}$$

where ψ *is defined as in* (24).

Proof. In Appendix B.1. □

Equation (26) provides an analytical characterization of the optimal tax that emerges to correct the aggregate demand externality at work in the model. When households take savings decisions, they do not internalize that redirecting consumption over time affects firms' demand for non-tradable goods and can move production closer or further away from the first best.

A key result is that the sign of the tax is ambiguous and depends on the sign of the current and future labor wedges. When the current labor wedge is zero, the tax on debt takes the sign of the expected risk-adjusted labor wedge. The intuition for the analytical expression is that the government internalizes that an increase in one of savings today is associated with an increase in aggregate demand tomorrow, which stimulates employment and reduces the labor wedge. When the labor wedge today and tomorrow are both positive, the government trades-off the marginal benefits from stimulating future demand and easing the recession tomorrow with the marginal costs from reducing current demand and deepening the recession today. On the other hand, if the labor wedge is negative, taxing borrowing and postponing consumption helps to reduce overheating.

4.2 Joint Monetary and Macroprudential Policies

In the previous section, we considered optimal capital controls when monetary policy follows an exogenous process. We now consider a government that conducts jointly macroprudential and monetary policy. Importantly, the government is subject to a zero lower bound that restricts its ability to achieve the first-best allocations.

In contrast to the previous section, here the optimal policy for the government is subject to a time inconsistency problem, common in environments with a zero lower bound (e.g., Eggertsson and Woodford, 2003). We examine the optimal policy without commitment, which we see as the one that is practically most relevant. In particular, we study Markov perfect equilibrium in which the policies of the government at each point in time depend on the relevant payoff states. We use $s_t \equiv \{R_t^*, P_t^{T*}, y_t^T, z_t\}$ to denote the date-t realizations of exogenous shocks, $\mathcal{E}(b^{*\prime},s^{\prime}),\mathcal{C}^{\mathcal{T}}(b^{*\prime},s^{\prime}),\mathcal{C}^{\mathcal{N}}(b^{*\prime},s^{\prime})$ to denote the stationary policy functions for the exchange rate and tradable and non-tradable consumption followed by future governments, and $V(b^{*},s)$ to denote the value function for the government.

The optimal time-consistent problem of the government can be expressed recursively as follows:

$$V\left(b^{*},s\right)=\max_{R,e,b^{\prime},c^{N},c^{T}}u\left(c^{T},c^{N}\right)-v\left((c^{N})^{1/\alpha}\right)+\beta\mathbb{E}_{s^{\prime}|s}\frac{z^{\prime}}{z}V\left(b^{*\prime},s^{\prime}\right), \tag{27}$$

subject to

$$\begin{aligned}
c^{T}&=y^{T}+b^{*}-\frac{b^{*\prime}}{R^{*}}\\
c^{N}&=\left[\frac{1-\omega}{\omega}\frac{P^{T*}}{\bar{P}^{N}}e\right]^{\gamma}c^{T}\\
R^{*}&=R\,\mathbb{E}_{s^{\prime}|s}\left[\Lambda\left(\mathcal{C}^{T}(b^{*\prime},s^{\prime}),\mathcal{C}^{N}(b^{*\prime},s^{\prime})\right)\frac{P^{T*}}{P^{T*\prime}}\frac{e}{\mathcal{E}(b^{*\prime},s^{\prime})}\right]\\
R&\geq 1.
\end{aligned}$$

The key difference compared with problem (25), is that now the exchange rate and the nominal interest rate are choices for the government.

In a Markov perfect equilibrium as defined below, the conjectured policies for future governments have to be consistent with the actual policies chosen.

Definition 2. A Markov perfect equilibrium is defined by policies $\mathcal{R}(b^{*},s),\tau(b^{*},s),\mathcal{E}(b^{*},s)$, $\mathcal{B}^{*}(b^{*},s),\mathcal{C}^{T}(b^{*},s),\mathcal{C}^{N}(b^{*},s)$ and a value function $V(b^{*},s)$ that solve Problem 27

The following proposition characterizes the optimal policy of the government.

Proposition 5 (Optimal monetary and macroprudential policy)**.** *Consider the optimal monetary and capital controls policy. We have that the labor wedge satisfies $\psi_t\geq 0$, with equality if the ZLB does not bind. Moreover, e_t is given by*

$$e_{t}=\frac{\omega}{1-\omega}\frac{\bar{P}^{N}}{P_{t}^{T*}}\left[\alpha^{\frac{\sigma}{\gamma}}(1-\omega)\left(\frac{e_{t}P_{t}^{T*}}{\bar{P}^{N}}\mathcal{P}(e_{t})\right)^{\frac{\gamma-\sigma}{\gamma}}\right]^{\frac{\alpha}{(1-\alpha+\phi)\sigma+\alpha}}(c_{t}^{T})^{-\frac{1}{\gamma}}. \tag{28}$$

In addition, the optimal tax on debt is given by

$$\tau=\frac{1}{\beta R^{*}\mathbb{E}_{s^{\prime}|s}\frac{z^{\prime}}{z}\left[u_{T}\left(c^{T\prime},c^{N\prime}\right)\right]}\left\{-(1+\Theta)\frac{\xi}{\gamma c^{T}}+\beta R^{*}\mathbb{E}_{s^{\prime}|s}\frac{z^{\prime}}{z}\left[\frac{\xi^{\prime}}{\gamma c^{T\prime}}\right]\right\}, \tag{29}$$

where $\Theta \equiv \gamma c^T \frac{\partial}{\partial b^{*\prime}} \mathbb{E}_{s'|s}\left[\frac{\Lambda(b^{*\prime},s')}{\mathcal{E}(b^{*\prime},s')}\frac{eP^{T*}}{P^{T*\prime}}\right]$ *and* ξ *is the non-negative Lagrange multiplier on the ZLB constraint.*

Proof. In Appendix B.2. □

An important lesson from Proposition 5 that when the government can choose optimal monetary policy and capital controls, the government fully stabilizes the labor wedge whenever the zero lower bound is not binding.[19] Moreover, the exchange rate policy can be implemented by setting a nominal interest rate such that

$$R_t = \frac{R_t^*}{e_t}\left\{\mathbb{E}_t\left[\frac{\Lambda_{t+1}}{\mathcal{E}_{t+1}}\frac{P_t^{T*}}{P_{t+1}^{T*}}\right]\right\}^{-1}. \tag{30}$$

In addition, the economy never experiences overheating. Intuitively, the zero lower bound imposes a constraint on the ability to depreciate the exchange rate, but the government can always appreciate the exchange rate and reduce demand of non-tradables by raising the nominal interest rate. As it turns out, these two lessons hinge on the ability of the government to use capital controls. We next examine the role of the tax on debt, and in the following section, we study the optimal monetary policy in the absence of macroprudential policy.

On the other hand, if the zero lower bound binds, the government is unable to depreciate the exchange rate by lowering the nominal interest rate. Notice, however, that through the choice of borrowing, the government can still affect the current exchange rate by altering expectations about the future exchange rate. To the extent that altering the borrowing level distorts the intertemporal allocation of consumption, the zero lower bound remains costly for the small open economy.

Equation (29) presents the optimal tax on debt. The tax depends crucially on the current and future Lagrange multipliers on the zero lower bound constraints, denoted by ξ. Because ξ is non-negative, it follows that in a state in which the zero lower bound is not currently binding, the tax on debt is always positive. On the other hand, equation (29) also reveals that if the zero lower bound is currently binding but is not in the future, the tax is negative.

Using the first-order conditions for c^N and e in (27), we obtain the following relationship

[19] To see this, notice that if the ZLB constraint is slack, we can drop all constraints but the resource constraint. Thus, we obtain a static condition that delivers a zero labor wedge and back out R and e that implement those allocations.

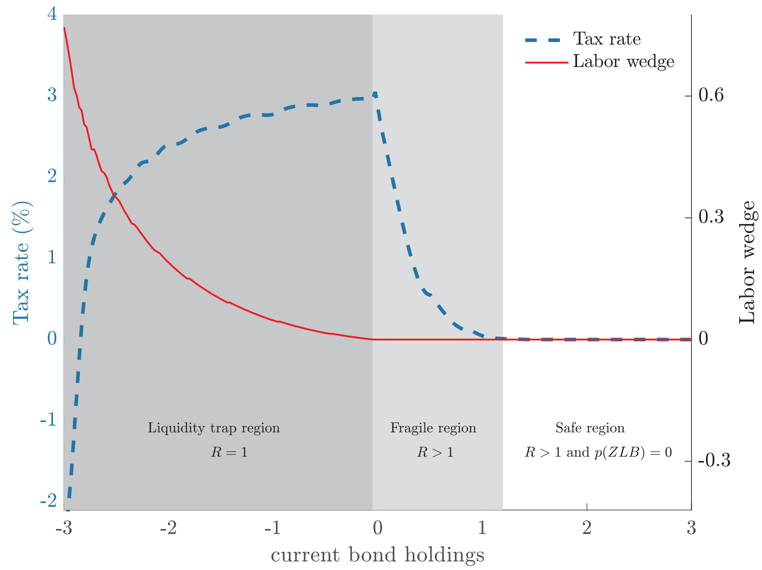

Figure 1: Capital control policy

between the labor wedge and the Lagrange multiplier on the zero lower bound:

$$\frac{\xi_t}{\gamma c_t^T} = \frac{1 - \tilde{\omega}_t}{\tilde{\omega}_t} u_T(c_t^T, c_t^N)\psi_t. \tag{31}$$

Moreover, replacing (31) into (29), we arrive at an equation analogous to (26), the optimal tax under an arbitrary exchange rate policy. That is, it is possible to write the tax as a function of how savings affect the next-period labor wedge or as a function of how savings affect the tightness of the zero lower bound. The two expressions are linked by the government optimization and are, in fact, equivalent.[20]

Figure 1 shows that the tax on debt is non-monotonic on the current level of bond holdings.[21] (In a different axis, the figure also shows the current labor wedge.) There are three distinct regions. For low bond holdings, the economy is in a liquidity trap region in which $R = 1$ and $\psi > 0$. In this region, the tax is increasing in bond holdings. In particular, notice that when bond holdings are sufficiently low, the tax becomes negative. Consistent with (29), the current labor wedge exceeds the expected future ones, and the planner finds optimal to subsidize inflows. For intermediate levels of bond holdings, the economy is in a fragile region in which $R > 1$ but the ZLB may become binding in the next period.

[20] A difference between the two tax expressions (26) and (29) is that the latter carries an additional term, Θ, related to the restriction that the policy is consistent with an optimal time-consistent equilibrium. The additional term captures that an increase in savings alters both next-period consumption and the exchange rate followed by the next government.

[21] The figure considers values of the shocks equal to the mean values. The calibration will be described below. The overall pattern, however, is general and does not hinge on specific parameters.

In this region, the tax is increasing in bond holdings. Intuitively, the higher the level of bond holdings, the lower the likelihood of a binding ZLB. Finally, for high bond holdings, the economy is in a safe region in which the tax becomes zero because there is a zero probability of a binding ZLB in the next period.

4.3 Optimal Monetary Policy without Capital Controls

We now study optimal monetary policy when the government does not have access to capital controls. As we will see, there is a sharp distinction between the optimal conduct of monetary policy in this case and in the previous section.

We consider again the optimal problem under lack of commitment. In this case, the optimal monetary policy faces an additional binding implementability constraint: the intertemporal Euler equation for the foreign asset. This constraint was slack in Section 4.2 because the government could use a tax on borrowing to control the borrowing decision. The government problem is

$$V\left(b^{*}, s\right)=\max_{R, b^{*\prime}, c^{N}, c^{T}} u\left(c^{T}, c^{N}\right)-v\left(\left(c^{N}\right)^{1 / \alpha}\right)+\beta \mathbb{E}_{s^{\prime} \mid s} \frac{z^{\prime}}{z} V\left(b^{*\prime}, s^{\prime}\right), \tag{32}$$

subject to

$$\begin{aligned}
c^{T} &= y^{T}+b^{*}-\frac{b^{*\prime}}{R^{*}} \\
c^{N} &= \left[\frac{1-\omega}{\omega} \frac{P^{T *}}{\bar{P}^{N}} e\right]^{\gamma} c^{T} \\
u_{T}(c^{T}, c^{N}) &= \beta R^{*}\, \mathbb{E}_{s^{\prime} \mid s}\left[\frac{z^{\prime}}{z} u_{T}\left(\mathcal{C}^{T}(b^{*\prime}, s^{\prime}), \mathcal{C}^{N}(b^{*\prime}, s^{\prime})\right)\right] \\
R^{*} &= R\, \mathbb{E}_{s^{\prime} \mid s}\left[\Lambda\left(\mathcal{C}^{T}(b^{*\prime}, s^{\prime}), \mathcal{C}^{N}(b^{*\prime}, s^{\prime})\right) \frac{P^{T *}}{P^{T * \prime}} \frac{e}{\mathcal{E}(b^{*\prime}, s^{\prime})}\right] \\
R &\geq 1 .
\end{aligned}$$

We saw in Section 4.2 that when the government has access to capital controls, it chooses to implement a zero labor wedge allocation whenever the zero lower bound is not binding. In the absence of capital controls, however, a key question is whether the government should use monetary policy prudentially as a substitute for capital controls and, if so, the government should raise the interest rate. Based on the analysis on Section 3, a partial equilibrium perspective suggests that the nominal interest rate should be raised but in general equilibrium, this is not necessarily the case.

The next proposition characterizes the optimal monetary policy in target form.

Proposition 6 (Optimal monetary policy without capital controls)**.** *When the government does not have access to capital controls, the optimal monetary policy satisfies*

$$u_T(t)\psi_t = \frac{\tilde{\omega}_t(\sigma-\gamma)}{(1-\tilde{\omega}_t)\sigma+\tilde{\omega}_t\gamma}\mathbb{E}_t\sum_{k=1}^{\infty}\beta^k\frac{z_{t+k}}{z_t}\left(\prod_{\ell=0}^{k-1}\frac{R^*_{t+\ell}}{1+\bar{\Theta}_\ell}\right)\frac{\xi_{t+k}}{\gamma c^T_{t+k}}, \tag{33}$$

where $\bar{\Theta}_t \equiv \beta R^*_t \frac{1}{u_{TT}(t)}\mathbb{E}_t\frac{z_{t+1}}{z_t}\frac{\partial u_T(\mathcal{C}^T(b^*_{t+1},s_{t+1})),\mathcal{C}^N(b^*_{t+1},s_{t+1})}{\partial b^*_{t+1}}$.

Proof. In Appendix B.3. □

Whether monetary policy is used prudentially and whether it leans *with* or *against* the wind turns out to depend on the elasticities of substitution. The logic stems from the aggregate response of saving to a change in the nominal interest rate, characterized in Section 3. In the absence of capital controls, monetary policy can potentially be used as a prudential tool to stimulate precautionary savings and reduce the likelihood of future liquidity traps. However, when $\sigma = \gamma$, saving does not respond to a change in the nominal interest rate. In that case, monetary policy focuses solely on stabilizing output and is not used prudentially. When $\sigma > \gamma$, the government optimally raises the nominal interest rate to stimulate savings and reduce the likelihood of future liquidity traps at the expense of a recession today. The optimal monetary policy leans against wind. Conversely, when $\sigma < \gamma$, the government optimally cuts down the nominal interest rate to reduce the likelihood of future liquidity traps at the expense of an overheating economy. The optimal monetary policy leans with wind.

Notice also that the optimal monetary policy—except in the knife-edge case of equal intra and inter-temporal elasticities—is used prudentially as long as the zero lower bound binds in some distant future state. This contrasts with the optimal macroprudential policy, in which a tax is imposed only if the zero lower bound binds in the next period. To put it differently, monetary policy needs to act even more preemptively than macroprudential policy. The reason for this result is that monetary policy is a blunter instrument than macroprudential policy. A binding zero lower bound in some future state k implies that the government needs to reduce overborrowing at $k-1$. With macroprudential policy, the government introduces a tax on borrowing at $k-1$ while preserving a zero labor wedge. On the other hand, without macroprudential policy, the government must introduce a labor wedge at $k-1$. Doing so implies that from the perspective of $k-2$, the government also needs to deviate from a zero labor wedge. Proceeding backwards, it implies a strong history dependent result: as long as there is a binding ZLB in some future state, the government will deviate from full-employment at any period before.

4.4 Quantitative Results

Calibration. The time period is one-quarter, and data are calibrated using United Kingdom data between 1980 and 2019.[22] The labor supply elasticity is set to one-third, as in Gali and Monacelli (2005) and α is set one.

The stochastic processes for $\{y_t^T\}$, $\{R_t^*\}$ and $\{z_t\}$ are assumed to be independent and specified as follows. The tradable output y_t^T is measured with the cyclical component of value added in agriculture, mining, fishing and manufacturing from the World Development Indicators. The world interest rate R_t^* is measured by the United States real interest rate, which corresponds to the U.S. federal funds rate deflated with the expected US. CPI inflation. Each process is assumed to be a first-order univariate autoregressive process. The estimated processes are, $\ln y_t^T = 0.6771 \ln y_{t-1}^T + \varepsilon_t^y$ with $\varepsilon_t^y \sim N(0, 0.0377^2)$ and $\ln(R_t^*/R^*) = 0.9173 \ln(R_{t-1}^*/R^*) + \varepsilon_t^{R*}$ with $R^* = 1.0036$ and $\varepsilon_t^{R*} \sim N(0, 0.0026^2)$.

Table 2: Calibration

Description	Parameter Value	Source/Target
Intertemporal elasticity	$\sigma = 1$	Standard value
Technology	$\alpha = 1$	Standard value
Frisch elasticity parameter	$\phi = 3$	Gali and Monacelli (2005)
Weight on tradables in CES	$\omega = 0.25$	Share of tradable output = 24%
Discount factor	$\beta = 0.99$	Standard value
Discount factor shocks	$g^H = 0.006$	Average NFA-GDP ratio = −17.4%
Transition prob. g^L to g^H	$P(g^L \vert g^H) = 0.20$	3 liquidity traps every century
Transition prob. g^H to g^H	$P(g^L \vert g^L) = 0.39$	2 years duration of liquidity traps
Intratemporal elasticity	$\gamma \in \{0.5, 1, 2\}$	

The discount factor shock $g_t \equiv \frac{z_{t+1}}{z_t}$ follows a two state regime-switching Markov process, where $g_t \in \{g^L, g^H\}$ with $g^L < g^H$. We set $g^L = 1$, which represents the normal regime in which households discount the future at rate β. The discount factor heightens with probability $P(g^H|g^L)$ and returns to its normal value with probability $P(g^L|g^H)$. The

[22] We focus on the United Kingdom because an as an example of an advanced small open economy. We note that the problem of the zero lower bound has indeed been more pervasive for advanced economies although a side effect of the recent increase in central bank credibility in emerging markets appears to be the increase in vulnerability to liquidity traps, as can be seen from the recent experiences of countries such as Chile and Peru (see Matthew Bristow "Paul Krugman Says the Liquidity Trap Has Spread to Emerging Markets" Bloomberg May 12, 2020).

transition probability matrix P is set to target the frequency and duration of a liquidity trap episode. The discount factor is set to $\beta = 0.99$. To set g^H, we target the historical average net foreign asset position (NFA) as a share of GDP of -17.4 % in the UK. The resulting values are presented in Table 2.

Finally, we use $\gamma = 1$ as a baseline value but consider also $\gamma = 0.5$ and $\gamma = 2$. The weight on tradable consumption in the CES function ω is calibrated to match a 24% share of tradable output in the total value of production observed in the data over the period 1980-2019, implying that $\omega = 0.252$.

Long-run moments. Table 3 displays the likelihood and duration of liquidity trap episodes in an economy in which monetary policy is set optimally both with and without capital controls. A first lesson is that capital controls are effective at reducing the likelihood of a liquidity trap. By taxing borrowing when the economy is in the fragile region, the government is successful at making the economy less vulnerable to a liquidity trap. The ex-ante prudential capital control imply an average tax rate on inflows of 0.2% percent.

A second lesson is that, perhaps surprisingly, liquidity traps last longer when capital controls are used jointly with monetary policy. This occurs because in a liquidity trap, the government may tax outflows, which implies that the deleveraging process is slowed down. In fact, the average tax during a liquidity trap is -0.05%. Notice that because taxes on inflows are more frequent, capital controls generate a reduction in external debt of about 7 percentage points of GDP.

Table 3: Frequency and duration of liquidity traps

	Monetary Policy Only		Monetary & Macroprudential			
	Frequency	Duration	Frequency	Duration	mean(τ)	$corr(R,\tau)$
$\gamma = 0.5$	3.4%	7.7	3.2%	11.0	0.2%	-0.1
$\gamma = 1$	3.8%	7.8	3.4%	9.6	0.2%	-0.3
$\gamma = 2$	4.3%	8.1	3.8%	9.0	0.2%	-0.5

Note: Duration expressed in quarters.

Table 4 examines the average welfare cost of the ZLB and the unemployment rate during a liquidity trap under optimal monetary policy with and without capital controls. [23] For a given state (b^*, s), the welfare cost of the ZLB under a policy regime is calculated as

[23] The unemployment rate is defined as the gap between the current level of employment and the efficient employment level (that is, the level that would equate the marginal value of employment to the marginal cost from providing an extra unit of labor).

the compensating consumption variations that equalize the expected utility of a household living in an economy under that policy regime and the expected utility in the efficient allocation (without ZLB).[24]

Table 4 reports an average unemployment rate of about 1.5% with capital controls versus 6.0% when the government refrains from using capital controls. The significant reduction in both the frequency and the severity of liquidity trap episodes points toward substantial quantitative gains from capital controls. Capital controls cut the welfare cost of the ZLB by more than fourfold. The average welfare cost of the ZLB when monetary policy is supplemented with capital controls is about 0.1 percentage points of permanent consumption versus 0.4 percentage points of permanent consumption in the case without capital controls.

Table 4: Welfare costs and unemployment rate

	Monetary Policy Only		Monetary & Macroprudential	
	Welfare costs	Unemployment*	Welfare costs	Unemployment*
$\gamma = 0.5$	0.47%	7.80%	0.08%	1.44%
$\gamma = 1$	0.40%	5.99%	0.09%	1.46%
$\gamma = 2$	0.45%	5.10%	0.12%	1.65%

Note: Unemployment is the average unemployment rate conditional on a liquidity trap.

5 International Spillovers and Policy Interdependence

So far, we have considered a small open economy. Motivated by policy discussions regarding global imbalances and currency wars, we now extend our framework to tackle the interdependence of monetary and macroprudential policies at the global level. In particular, we are interested in analyzing how policies abroad affect welfare at home and how they alter the optimal policy response.

[24]Formally, the welfare cost associated with a policy regime G, for a given state (b^*, s), corresponds to the value of $q(b^*, s)$ that satisfies

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \frac{z_t}{z_0} \left[\log((1+q)c_t^G) - v(h_t^G) \right] = \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \frac{z_t}{z_0} \left[\log(c_t^E) - v(h_t^E) \right],$$

where c^E and h^E denote consumption and hours worked in the efficient allocation.

World economy extension. We consider a world economy that is composed of a continuum of small open economies of the type described in Section 2. We assume that there are two blocs of countries, Home and Foreign, with measure n and $1-n$, respectively. All countries within each block are identical, and we assume that all countries start with zero net foreign asset position. We use subscript H to denote the countries that belong to Home and F to denote countries that belong to the Foreign block. For simplicity, we assume there is no uncertainty.

We now use the arbitrage condition between domestic currency bonds and the real global bond in the two blocks to obtain an implied arbitrage condition between bonds in different currencies. Using $e^F_{H,t}$ to denote the bilateral exchange rate between a country in block H and a country in block F, we obtain

$$R_{F,t} = R_{H,t}(e^F_{H,t}/e^F_{H,t+1}),$$

where by definition $e^F_{H,t} = 1/e^H_{F,t}$.

The definition of competitive equilibrium extends the definition of Section 2. We now have a continuum of prices and allocations, one for each country. In addition, the real interest rate R^*_t is endogenous, and we have a market clearing condition for the bond market:

$$\int_0^n B_{H,t+1}\, dH + \int_n^1 B_{F,t+1}\, dF = 0. \tag{34}$$

5.1 Monetary Policy Spillovers

We start by examining the spillover effects of monetary policy. As a preview, our main finding is that the ability to use capital controls to insulate from the foreign shock is key for the welfare effects.

A monetary policy shock abroad entails potential spillovers on the home country through the effects on the equilibrium world interest rate, the law of one price, and the risk-adjusted UIP condition (9). The following proposition characterizes the effects on welfare.

Proposition 7. *Consider a change in the foreign nominal interest rate. To a first order, the effects on the home country's welfare are given by*

$$\frac{\partial V_{H,0}}{\partial R_{F,0}} = \frac{u_T(c^T_{H,0}, c^N_{H,0})}{R^*_0}\left[\frac{1-\tilde{\omega}_{H,0}}{\tilde{\omega}_{H,0}}\gamma c^T_{H,0}\psi_{H,0} + \left(\tilde{\omega}_{H,0} + (1-\tilde{\omega}_{H,0})\frac{\gamma}{\sigma}\right) v_{H,0}\right]\frac{\partial R^*_0}{\partial R_{F,0}}, \tag{35}$$

where $v_{H,0}$ is the Lagrange multiplier on the households' Euler equation (16).

Proof. In Appendix C.1 □

Proposition 7 underscores how a change in the monetary policy stance abroad has effects on the SOE only to the extent that it may change the foreign real rate. Because tradable goods have flexible prices, a reduction in the foreign nominal interest rate leads simultaneously to an appreciation of the domestic currency and to an increase in the foreign price level. Through the law of one price, this implies that the domestic price remains the same.

When $\sigma = \gamma$, as discussed above, it follows that the real interest rate is not affected, and therefore an economy is not affected by changes in monetary policy abroad. We highlight that this form of insulation is related to but distinct from the one in Corsetti and Pesenti (2001), which operates through a terms of trade channel.

When $\sigma \neq \gamma$, there potentially are spillover effects. Let us examine the case in which the home country is away from a liquidity trap. Using the results from Proposition 6 and eq. (35), we have that the welfare effects of a monetary expansion abroad reduce to [25]

$$\frac{\partial V_{H,0}}{\partial R_{F,0}} = \left[\frac{u_T(c^T_{H,0}, c^N_{H,0})}{R^*_0} v_{H,0} \right] \frac{\partial R^*_0}{\partial R_{F,0}}. \tag{36}$$

Notice that now the sign of the welfare effect depends on the interaction of $v_{H,0}$, the Lagrange multiplier on the households' Euler equation (16), and the direction of the world interest rate in response to the monetary policy abroad. There are two cases to consider depending on whether capital controls are available.

Without capital controls. From the analysis in Section 4.3, we have that $v_{H,t} > 0$ when the ZLB is not currently binding but is expected to bind in the future.[26] A strictly positive Lagrange multiplier reflects that agents tend to overborrow relative to the constrained-efficient benchmark (see Section 4). If the monetary policy abroad generates a reduction in the world real interest rate, this causes a reduction in welfare in the home country. Intuitively, the reduction in the real interest rate generates incentives for households to borrow more—from an already inefficiently high level—and increases the vulnerability to a liquidity trap.

[25] Away from the ZLB, as shown in (B.28) in Appendix B.3, the optimal monetary policy in its target form (33) can be rewritten as $\psi_{H,0} = \tilde{\omega}(\gamma^{-1} - \sigma^{-1}) v_{H,0} / c^T_{H,0}$. Combining this with (35), we obtain (36).

[26] Formally, this can be seen in (B.32) in Appendix B.3.

As shown in Proposition 6, a small open economy indeed has incentives to increase the net foreign asset position so as to become less vulnerable to a binding zero lower bound, lowering the interest rate when $\gamma > \sigma$ and increasing it otherwise. As a result, this generates a form of currency war, in which every country has incentives to push savings up and this ends up distorting output. However, countries have incentives to pursue a depreciation of their currency only if $\gamma > \sigma$. Otherwise, the world equilibrium is such that all countries raise the interest rate relative to the level that would be collectively optimal. We summarize this result in the corollary below.

Corollary 2 (Currency wars)**.** *A prudential monetary intervention abroad lowers home welfare, strictly so if the zero lower bound binds in the future.*

Proof. In Appendix C.2. □

With capital controls. On the other hand, when the government has access to capital controls, there is no inefficiency stemming from domestic households' saving decisions ($v_{H,0} = 0$). The effects of a foreign monetary expansion from Proposition 7 reduce to

$$\frac{\partial V_{H,0}}{\partial R_{F,0}} = \frac{u_T(c^T_{H,0}, c^N_{H,0})}{R^*_0} \left[\frac{1 - \tilde{\omega}_{H,0}}{\tilde{\omega}_{H,0}} \gamma c^T_{H,0} \psi_{H,0} \right] \frac{\partial R^*_0}{\partial R_{F,0}}. \tag{37}$$

Hence, it turns out that a foreign monetary policy intervention has no effect on domestic households' welfare, as monetary policy optimally closes the labor wedge ($\psi_{H,0} = 0$) away from the liquidity trap. Capital controls thus render the domestic country insulated from foreign policy shocks outside of a liquidity trap. Our results suggest that in a low natural interest rates environment away from the ZLB, monetary policy shocks originating abroad are not welfare-reducing in the domestic economy as long as macroprudential policies are set optimally.

Corollary 3 (No currency wars with capital controls)**.** *Consider a government that uses capital controls. A prudential monetary intervention abroad does not affect welfare away from the zero lower bound.*

Proof. In Appendix C.3 □

5.2 Capital Control Spillovers

We now turn to analyze the spillovers of capital controls. As in Fornaro and Romei (2019), we compare the welfare of domestic households under laissez-faire (no capital controls at home or abroad) versus that of a capital control regime in which all countries use capital controls.

We use V_{LF} and V_τ to denote the welfare of households in the home country, under the two regimes, laissez-faire and capital controls. Notice that the key general equilibrium effect with capital controls is through the risk-free rate. As established in the following lemma, by making borrowing more expensive, a positive tax abroad ($\tau_{F,0} > 0$) leads in equilibrium to a decline in the world real rate.

Lemma 2 (Spillovers of capital controls)**.** *A positive tax on inflows by the foreign block at time 0 leads to a decline in the world real interest rate at time 0.*

Proof. In Appendix C.4 □

Using the results of this lemma, we establish the following welfare results.

Proposition 8 (Welfare dominance of capital control regime)**.** *Consider the welfare of the home country under laissez-faire versus the welfare under a capital control regime, starting from a symmetric equilibrium with zero net positions.*

i) Away from a liquidity trap, home welfare is higher in a capital control regime, strictly so if the ZLB binds at some future date.

ii) In a liquidity trap, home welfare is higher in a capital control regime if $\tau_{F,t} < 0$ or if $\tau_{F,t} > 0$ and $\sigma \geq \gamma$.

Proof. In Appendix C.5 □

Item (i) of Proposition 8 demonstrates that when all countries are away from a liquidity trap, welfare is unambiguously higher under capital controls. When starting from a symmetric equilibrium, countries are neither net borrowers nor net creditors. As a result, changes in the world interest rate from capital controls abroad do not carry first-order effects. In addition, as shown in Proposition 5, countries can achieve a zero output gap away from the zero lower bound and use taxes on outflows in a future state with a binding zero lower bound. Thus, the use of capital controls allows the home country to strictly improve welfare relative to a laissez-faire regime.

Item (ii) of Proposition 8 examines a situation in which the home economy is in a liquidity trap. At the zero lower bound, the decline in the world interest rate tightens the zero lower bound and can potentially reduce welfare, as highlighted by Fornaro and Romei (2019). They consider an economy with only prudential capital controls ($\tau \geq 0$) and zero liquidity (i.e., no cross-border borrowing and lending) and prove that welfare indeed falls, assuming that $\sigma = \gamma = 1$.[27] Our results show that once we allow for equilibrium borrowing and ex-post capital controls, the laissez-faire regime becomes dominated by a capital control regime if $\sigma \geq \gamma$.

This result reflects two opposing forces. First, we have Fornaro and Romei's contractionary effect. When countries in the foreign block introduce positive taxes on borrowing, this causes a reduction in the world interest rate. From the risk-adjusted UIP condition (9), this immediately implies that the zero lower bound is tighter. For a given future exchange rate, this leads to an appreciation of the exchange rate today. The higher is γ, the higher is the expenditure switching channel, and the larger is the contraction in output. Second, once we allow taxes for capital flows and ex-post capital controls, the government can, in effect, control the amount of capital inflows and tradable consumption. In particular, the government can impose a large tax on outflows in such a way that it offsets the exchange rate appreciation and therefore closes the output gap. Doing so is optimal only to the extent that the wedge introduced in the intertemporal Euler equation is not too large. When σ is high, consumption is highly substitutable over time and induces a lower intertemporal wedge. Critically, the decline in the world interest makes it less expensive for the planner to reallocate consumption toward the present. A value of $\sigma \geq \gamma$ turns out to be sufficient to ensure that welfare goes up in the capital control regime relative to the laissez-faire. Table 5 presents all the cases. [28]

Table 5: Welfare at home: Capital control regime vs. laissez-faire

		Away from a liquidity trap	In a liquidity trap	
			$\sigma \geq \gamma$	$\sigma < \gamma$
Policy abroad	ex-ante	$V_\tau > V_{LF}$	$V_\tau > V_{LF}$	ambiguous
	ex-post	$V_\tau > V_{LF}$	$V_\tau > V_{LF}$	$V_\tau > V_{LF}$

Note: Ex-ante (ex-post) policy abroad refers to $\tau_{F.t} > 0$ ($\tau_{F.t} < 0$).

[27] Technically, they consider a government than can choose savings on behalf of agents but implicitly precludes the use of subsidies on capital inflows or restrictions on capital outflows.

[28] We can also show that under zero liquidity, or a binding borrowing constraint, a capital control regime becomes unambiguously welfare improving once we allow for ex-post capital controls. The logic in this case is that a subsidy to borrowing does not distort consumption relative to the first-best, while it does stimulate the demand for non-tradable consumption and help reduce the output gap.

5.3 Coordination

So far, we have focused on global spillovers when government policies at the country level are set in an uncoordinated fashion. We have argued that in the absence of coordination, capital controls still deliver significant gains. One question, however, is how the possibility of coordination affects the analysis. In particular, are there any gains from coordination? If so, does the coordinated solution imply a more or less active use of capital controls?[29]

We consider the problem of a global regulator that coordinates on capital controls. We look for Pareto improvements. In particular, we analyze an optimal policy problem without commitment, in which the global regulator maximizes the welfare of one of the two blocks, subject to leaving the other block as well-off.[30]

The main results are summarized in the following proposition.

Proposition 9 (Gains from macroprudential coordination)**.** *Consider the welfare of countries under an uncoordinated capital control regime vs. that under a coordinated capital control regime. Starting from a symmetric equilibrium with zero net positions, we have that*

i) the ZLB never binds in a coordinated capital control regime;

ii) in a state in which the ZLB does not bind in the uncoordinated solution for any country, there are no gains from coordination.

Proof. In Appendix C.6 □

The proposition establishes two powerful results on the desirability of coordination. First, when the ZLB binds in some countries, there are strong benefits from coordination. In fact, a coordinated intervention is able to fully relax the ZLB (while weakly improving welfare in all countries). As we show in the Appendix, the policy consists of coordinated subsidies on capital flows. To the extent that all countries apply the subsidies, there is a rise in the risk-free rate, without necessarily any effects on capital flows. However, the increase in the real rate provides more monetary space to close the output gap.

Second, if the ZLB is not currently binding for any country, then there are no benefits from coordination *regardless of whether the ZLB binds in the future*. In other words, if a

[29]The notion of spillovers is often used in policy circles to justify the need for coordination. However,the presence of spillovers does not mean that it is possible to find Pareto improving policies (see e.g. Obstfeld and Rogoff (2002), Blanchard (2021), and Korinek (2019).

[30]See Appendix C.6 for details about the global government problem. Because Home and Foreign are not symmetric – for example, one country can be in a liquidity trap and the other one away from a liquidity trap – this approach is not equivalent to considering a utilitarian global government that seeks to maximize a weighted average of home and foreign households' welfare weighted by the relative size of each country.

country seeks to reduce the likelihood of a future liquidity trap, this may or may not generate adverse spillovers abroad, but crucially, it is not possible to find a coordinated solution that improves welfare for all countries.

6 Conclusion

In this paper, we provide an integrated analysis of monetary and macroprudential policies in an open economy. An analytical decomposition reveals that the expenditure switching channel accounts for about one-quarter of the overall effects of monetary policy. Macroprudential policy instead operates primarily through intertemporal substitution. On the normative side, we show that although monetary policy can be used prudentially, leaning with the wind may not be optimal. Capital controls on both inflows and outflows ameliorate monetary policy tradeoffs and provide substantial welfare gains. Our analysis also provides a more benign perspective on international spillovers in contrast to widespread policy concerns. We show that a country's ability to deploy capital controls can provide insulation from adverse effects and help prevent the outbreak of a currency war. Finally, there are gains from coordinating macroprudential policies, but they apply only during liquidity traps and require stimulating rather than contracting the volume of capital flows.

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APPENDIX TO "LIQUIDITY TRAPS, PRUDENTIAL POLICIES AND INTERNATIONAL SPILLOVERS"

A Proofs for Section 3

A.1 Proof of Lemma 1 (Household problem)

Proof. We split the optimization problem of the households into an intra and an inter-temporal problem. The inter-temporal problem determines the consumption-savings and labor supply decision of the households, while the intra-temporal problem determines the allocation of consumption expenditures among tradable goods and non-tradable goods.

The intra-temporal problem of the households consists of choosing (c_t^N, c_t^T) to maximize the consumption bundle $c = \left[\omega (c^T)^{\frac{\gamma-1}{\gamma}} + (1-\omega)(c^N)^{\frac{\gamma-1}{\gamma}}\right]^{\frac{\gamma}{\gamma-1}}$ for any given expenditure level $\mathcal{Z}_t$

$$c_t^T + \frac{\bar{P}^N}{P_t^{T*} e_t} c_t^N = \mathcal{Z}_t \tag{A.1}$$

Maximizing c subject to (A.1) yields the following optimality conditions

$$c_t^T = \omega^\gamma (\mathcal{P}_t)^{\gamma-1} \mathcal{Z}_t \tag{A.2}$$

$$\frac{\bar{P}^N}{P_t^{T*} e_t} c_t^N = \left[1 - \omega^\gamma (\mathcal{P}_t)^{\gamma-1}\right] \mathcal{Z}_t \tag{A.3}$$

where $\mathcal{P}$ is defined as the price index that equates total consumption expenditures $\mathcal{Z}_t$ to $\mathcal{P}_t c_t$, which is given by

$$\mathcal{P}_t = \left[\omega^\gamma + (1-\omega)^\gamma \left(\frac{P^N}{P_t^{T*} e_t}\right)^{1-\gamma}\right]^{\frac{1}{1-\gamma}}. \tag{A.4}$$

Let us consider now the inter-temporal problem of the households. Recall that Y_t is defined as:

$$\mathcal{P}_t Y_t \equiv y_t^T + T_t + (W_t h_t + \phi_t^N)/P_t^T$$

Using this definition and the budget constraint (2) then becomes

$$\mathcal{P}_t c_t + \frac{1}{1+\tau_t}\left[\frac{b_{t+1}}{R_t} + P_t^T \frac{b_{t+1}^*}{R_t^*}\right] = \mathcal{P}_t Y_t \tag{A.5}$$

Let's define the discount rate as $Q_{t|0} \equiv \prod_{s=0}^{t-1} \left(R_s^*(1+\tau_s)\right)^{-1}$. Integrating forward (A.5) and the standard terminal conditions, the inter-temporal problem of the households can be formulated as follow

$$\max_{\{c_t,h_t\}} \sum_{t=0}^{\infty} \beta^t \left[\frac{(c_t)^{1-\frac{1}{\sigma}}}{1-\sigma^{-1}} - \frac{(h_t)^{1+\phi}}{1+\phi} \right]$$

subject to

$$\sum_{t=0}^{\infty} Q_{t|0} \mathcal{P}_t c_t = \sum_{t=0}^{\infty} Q_{t|0} \mathcal{P}_t Y_t \tag{A.6}$$

The first-order condition of that problem yields the following Euler equation:

$$c_t = \left[\beta R_t^* (1+\tau_t) \frac{\mathcal{P}_t}{\mathcal{P}_{t+1}} \right]^{-\sigma} c_{t+1} \tag{A.7}$$

Plugging (A.7) into the inter-temporal budget constraint (A.6), we obtain that consumption of the composite good in period 0 is given by

$$c_0 = \mu_0 \sum_{t=0}^{\infty} \left(Q_{t|0} \frac{\mathcal{P}_t}{\mathcal{P}_0} \right) Y_t, \tag{A.8}$$

where

$$\mu_0 \equiv \left[\sum_{t=0}^{\infty} (\beta^{\sigma})^t q_{t|0}^{1-\sigma} \right]^{-1}$$

is the marginal propensity to consume and

$$q_{t|0} \equiv Q_{t|0} \mathcal{P}_t / \mathcal{P}_0 \tag{A.9}$$

is the effective real interest rate on consumption between period 0 and period t.

We derive the optimal savings decision of the household as the difference between the net income and consumption expenditures. The optimal savings in period 0 is given by

$$b_1^* = R_0^* (1+\tau_0) \left| \mathcal{P}_0 Y_0 - \mu_0 \sum_{t=0}^{\infty} Q_{t|0} \mathcal{P}_t Y_t \right| . \tag{A.10}$$

Finally, we substitute (A.8) into (A.1) and the demand equations (A.2) and (A.3) to obtain households policies for tradable consumption and non-tradable expenditures at date 0

$$c_0^T = \omega^{\gamma} (\mathcal{P}_0)^{\gamma-1} \mu_0 \sum_{t=0}^{\infty} Q_{t|0} \mathcal{P}_t Y_t, \tag{A.11a}$$

$$\frac{\bar{P}^N}{P^{T*} e_0} c_0^N = \left[1 - \omega^{\gamma} (\mathcal{P}_0)^{\gamma-1} \right] \mu_0 \sum_{t=0}^{\infty} Q_{t|0} \mathcal{P}_t Y_t. \tag{A.11b}$$

□

A.2 Proof of Proposition 1 (Decomposition of monetary policy)

Preliminaries. We decompose the overall effect of a temporary decline in R at time $t = 0$ which reverts to the steady state value at $t = 1$. Moreover, we assume that for $t \geq 1$, the economy features a level for the exchange rate that is consistent with the first-best allocation. Because we restrict attention to the case in which $\beta R^* = 1$ so that tradable consumption remains constant in the steady state, it is straightforward to see that $R = R^*$ in the steady state. Note that $\tau_t = 0$ which implies that $T_t = 0$.

The proof covers both the decomposition of the overall effect of a monetary expansion on the demand for nontradables c_0^N and the decomposition of the effect a monetary expansion on the demand for tradables c_0^T.

Recall that by definition, the consumption functions and the function for foreign bond holdings are given by:

$$
\begin{aligned}
c_0^N &= \mathcal{C}^N\left(\left\{\bar{P}^N/(P^{T*}e_t), R_t, \tau_t, Y_t\right\}_{t\geq 0}\right),\\
c_0^T &= \mathcal{C}^T\left(\left\{\bar{P}^N/(P^{T*}e_t), R_t, \tau_t, Y_t\right\}_{t\geq 0}\right),\\
b_1^* &= \mathcal{B}^*\left(\left\{\bar{P}^N/(P^{T*}e_t), R_t, \tau_t, Y_t\right\}_{t\geq 0}\right),
\end{aligned}
$$

where recall that Y_t is defined as in (18).

Using the results from Proposition 1 and (A.9), we can express the households' policies (A.10), (A.11a) and (A.11b) as:

$$c_0^N = \frac{P^{T*}e_0}{\bar{P}^N}\left[1-\omega^\gamma(\mathcal{P}_0)^{\gamma-1}\right]\left[\sum_{t=0}^{\infty}\beta^{t\sigma}q_{t|0}^{1-\sigma}\right]^{-1}\sum_{t=0}^{\infty}q_{t|0}\mathcal{P}_0 Y_t \tag{A.13a}$$

$$c_0^T = \omega^\gamma(\mathcal{P}_0)^{\gamma-1}\left[\sum_{t=0}^{\infty}\beta^{t\sigma}q_{t|0}^{1-\sigma}\right]^{-1}\sum_{t=0}^{\infty}q_{t|0}\mathcal{P}_0 Y_t \tag{A.13b}$$

$$\frac{b_1^*}{R_0^*} = \mathcal{P}_0 Y_0 - \left[\sum_{t=0}^{\infty}\beta^{t\sigma}q_{t|0}^{1-\sigma}\right]^{-1}\sum_{t=0}^{\infty}q_{t|0}\mathcal{P}_0 Y_t \tag{A.13c}$$

The overall effects of a decline in the nominal interest rate on initial consumption of non-tradables c_0^N can thus be decomposed as follow:

$$dc_0^N = \underbrace{\sum_{t=0}^{\infty} \frac{\partial \mathcal{C}^N}{\partial e_t} de_t}_{\text{Expend. switching}} + \underbrace{\sum_{t=0}^{\infty} \frac{\partial \mathcal{C}^N}{\partial q_{t|0}} dq_{t|0}}_{\text{Intertemporal subst.}} + \underbrace{\sum_{t=0}^{\infty} \frac{\partial \mathcal{C}^N}{\partial Y_t} dY_t}_{\text{GE effects}} \tag{A.14}$$

This decomposition also applies to the overall effects of a decline in the nominal interest rate on tradable consumption c_0^T and for foreign bond holdings b_1^*.

It will be useful to derive the the effects of an exchange depreciation on the consumer price index. As established in Lemma A.1, when the exchange rate depreciates by 1%, the price index falls by $(1 - \tilde{\omega})\%$.

Lemma A.1. *The impact of an exchange rate depreciation on the consumer price index is given by*

$$\frac{d\mathcal{P}_t}{\mathcal{P}_t} = -(1 - \tilde{\omega}_t) \frac{de_t}{e_t} \tag{A.15}$$

Proof. Differentiating the consumer price index (A.4) with respect to e_t yields

$$\begin{aligned} \frac{d\mathcal{P}_t}{de_t} &= (1-\omega)^\gamma \left(\frac{\bar{P}^N}{P_t^{T*} e_t} \right)^{1-\gamma} \frac{1}{e_t} \left[\omega^\gamma + (1-\omega)^\gamma \left(\frac{\bar{P}^N}{P_t^{T*} e_t} \right)^{1-\gamma} \right]^{\frac{\gamma}{1-\gamma}} \\ &= -(1-\omega)^\gamma \left[\frac{P_t^{T*} e_t}{\bar{P}^N} \mathcal{P}_t \right]^{\gamma - 1} \frac{\mathcal{P}_t}{e_t} \end{aligned} \tag{A.16}$$

Next, use the optimality that describes the household's demand for nontradable goods and substitute (A.2) to obtain

$$\begin{aligned} c_t^N &= \left[\frac{1-\tilde{\omega}}{\omega} \frac{P^{T*} e_t}{\bar{P}^N} \right]^\gamma c_t^T \\ &= \frac{P_t^{T*} e_t}{\bar{P}^N} (1-\omega)^\gamma \left[\frac{P_t^{T*} e_t}{\bar{P}^N} \mathcal{P}_t \right]^{\gamma-1} \mathcal{P}_t c_t \end{aligned} \tag{A.17}$$

where we substitute $\mathcal{Z}_t = \mathcal{P}_t c_t$ in (A.2). Using the definition of the expenditure share in tradables (20), we have that $\frac{\bar{P}^N}{P_t^{T*} e_t} c_t^N = (1 - \tilde{\omega}_t) \mathcal{P}_t c_t$. Thus, (A.17) implies that

$$1 - \tilde{\omega}_t = (1-\omega)^\gamma \left[\frac{P_t^{T*} e_t}{\bar{P}^N} \mathcal{P}_t \right]^{\gamma-1} \tag{A.18}$$

Finally, substitute (A.18) into (A.16) to get

$$\frac{d\mathcal{P}_t}{\mathcal{P}_t} = -(1 - \tilde{\omega}_t) \frac{de_t}{e_t}$$

□

We now proceed by calculating the different terms in the decomposition.

Expenditure switching. We start with the expenditure switching channel. Using equation (A.15) in Lemma A.1, the derivatives of the policy functions, (A.13b), (A.13a) and (A.13c), with respect to e_t (starting from the stationary equilibrium, with $b_1^* = 0$) are

$$\sum_{t=0}^{\infty} \frac{\partial \log \mathcal{C}^N}{\partial e_t} de_t = \tilde{\omega}\gamma \, d\log e_0 \tag{A.19a}$$

$$\sum_{t=0}^{\infty} \frac{\partial \log \mathcal{C}^T}{\partial e_t} de_t = -(1-\tilde{\omega})\gamma \, d\log e_0 \tag{A.19b}$$

$$\sum_{t=0}^{\infty} \frac{\partial \mathcal{B}^*}{\partial e_t} de_t = 0 \tag{A.19c}$$

Intertemporal substitution. Consider next the derivative of the policy functions with respect to $q_{t|0}$. Letting $\iota_0 \equiv \frac{P^{T*}e_0}{\bar{P}^N}(1-\tilde{\omega}_0)$, we have

$$\frac{\partial \mathcal{C}^N}{\partial q_{t|0}} = -(1-\sigma)\beta^{\sigma t} q_{t|0}^{-\sigma} \left[\sum_{t=0}^{\infty} \beta^{t\sigma} q_{t|0}^{1-\sigma}\right]^{-2} \iota_0 \sum_{t=0}^{\infty} q_{t|0}\mathcal{P}_0 Y_t + \iota_0 \left[\sum_{t=0}^{\infty} \beta^{t\sigma} q_{t|0}^{1-\sigma}\right]^{-1} \mathcal{P}_0 Y_t$$

Using that $q_{0|0} = 1$ and that $q_{t|0}$ is constant for all $t > 0$, given that the economy reaches the stationary equilibrium at $t = 1$.

$$\begin{aligned}
\sum_{t=0}^{\infty} \frac{\partial \mathcal{C}^N}{\partial q_{t|0}} dq_{t|0} &= (\sigma - 1) \sum_{t=1}^{\infty} \beta^{\sigma t} q_{t|0}^{1-\sigma} \frac{dq_{t|0}}{q_{t|0}} \cdot \mu_0^2 \iota_0 \sum_{t=0}^{\infty} q_{t|0}\mathcal{P}_0 Y_t + \mu_0 \iota_0 \sum_{t=1}^{\infty} \mathcal{P}_0 Y_t \, dq_{t|0} \\
&= \left[(\sigma-1)(1-\mu_0)\mu_0\iota_0 \sum_{t=0}^{\infty} q_{t|0}\mathcal{P}_0 Y_t + \mu_0\iota_0 \sum_{t=1}^{\infty} q_{t|0}\mathcal{P}_0 Y_t\right] \frac{dq_{t|0}}{q_{t|0}}
\end{aligned} \tag{A.20}$$

Moreover, note that

$$\begin{aligned}
\sum_{t=1}^{\infty} q_{t|0}\mathcal{P}_0 Y_t &= \sum_{t=0}^{\infty} q_{t|0}\mathcal{P}_0 Y_t - \mathcal{P}_0 c_0 + \mathcal{P}_0(c_0 - Y_0) \\
&= \sum_{t=0}^{\infty} q_{t|0}\mathcal{P}_0 Y_t - \mathcal{P}_0 \, \mu_0 \sum_{t=0}^{\infty} q_{t|0} Y_t + \mathcal{P}_0(c_0 - Y_0) \\
&= (1-\mu_0) \sum_{t=0}^{\infty} q_{t|0}\mathcal{P}_0 Y_t - \frac{b_1^*}{R_0^*}
\end{aligned} \tag{A.21}$$

Plugging (A.21) into (A.20), we get

$$
\begin{aligned}
\sum_{t=0}^{\infty} \frac{\partial \mathcal{C}^N}{\partial q_{t|0}} dq_{t|0} &= \left[\sigma(1-\mu_0)\mu_0\iota_0 \sum_{t=0}^{\infty} q_{t|0}\mathcal{P}_0 Y_t - \mu_0\iota_0 \frac{b_1^*}{R_0^*}\right] \frac{dq_{t|0}}{q_{t|0}} \\
&= \left[\sigma(1-\mu_0)c_0^T - \mu_0\iota_0 \frac{b_1^*}{R_0^*}\right] \frac{dq_{t|0}}{q_{t|0}}
\end{aligned} \tag{A.22}
$$

Differentiating $q_{t|0} = Q_{t|0}\mathcal{P}_t/\mathcal{P}_0$, we obtain that the change in the relative price induced by the temporary decline in the nominal interest rate yields $d\log q_{0|0} = 0$ and

$$
d\log q_{t|0} = -(1-\tilde{\omega})d\log R_0 \quad \text{for } t \geq 1 \tag{A.23}
$$

Finally substituting (A.23) into (A.22) and starting from the stationary equilibrium, with $b_1^* = 0$, we arrive to the expression in the proposition.

$$
\sum_{t=0}^{\infty} \frac{\partial \log \mathcal{C}^N}{\partial q_{t|0}} dq_{t|0} = -\beta\sigma(1-\tilde{\omega})d\log R_0 \tag{A.24}
$$

Proceeding similar for tradable consumption (with $\iota_0 = \tilde{\omega}_0$), we have

$$
\sum_{t=0}^{\infty} \frac{\partial \log \mathcal{C}^T}{\partial q_{t|0}} dq_{t|0} = -\beta\sigma(1-\tilde{\omega})d\log R_0 \tag{A.25}
$$

Next, we turn to deriving the effects on foreign bond holdings. We have

$$
\begin{aligned}
\sum_{t=0}^{\infty} \frac{\partial \mathcal{B}^*}{\partial q_{t|0}} dq_{t|0} &= -R_0^* \left[(\sigma-1)(1-\mu_0)\mu_0 \sum_{t=0}^{\infty} q_{t|0}\mathcal{P}_0 Y_t + \mu_0 \sum_{t=1}^{\infty} q_{t|0}\mathcal{P}_0 Y_t\right] \frac{dq_{t|0}}{q_{t|0}} \\
&= -R_0^* \left[\sigma(1-\mu_0)\mathcal{P}_0 c_0 - \mu_0 \frac{b_1^*}{R_0^*}\right] \frac{dq_{t|0}}{q_{t|0}}
\end{aligned} \tag{A.26}
$$

where the second equality uses (A.21) and (A.7). We then substitute (A.23) into (A.26), and starting from the stationary equilibrium, with $b_1^* = 0$, we arrive to the expression in the proposition

$$
\sum_{t=0}^{\infty} \frac{\partial \mathcal{B}^*}{\partial q_{t|0}} dq_{t|0} = \mathcal{P}c \cdot \sigma(1-\tilde{\omega}) \frac{dR_0}{R_0} \tag{A.27}
$$

Effects through income. Using the definition of Y_t and totally differentiating with respect to e_t and y_t^N, we obtain

$$
dY_t = \frac{\partial Y_t}{\partial e_t} de_t + \frac{\partial Y_t}{\partial y_t^N} dy_t^N \tag{A.28}
$$

The total effect of the change in income on non-tradable consumption is thus given by

$$\sum_{t=0}^{\infty}\frac{\partial\mathcal{C}^N}{\partial Y_t}dY_t=\underbrace{\sum_{t=0}^{\infty}\frac{\partial\mathcal{C}^N}{\partial Y_t}\left[\frac{\partial Y_t}{\partial e_t}de_t\right]}_{\text{valuation effects}}+\underbrace{\sum_{t=0}^{\infty}\frac{\partial\mathcal{C}^N}{\partial Y_t}\left[\frac{\partial Y_t}{\partial y_t^N}dy_t^N\right]}_{\text{aggregate income channel}} \tag{A.29}$$

where

$$\frac{\partial\mathcal{C}^N}{\partial Y_t}=\frac{P^{T*}e_0}{\bar{P}^N}\left[1-\omega^\gamma(\mathcal{P}_0)^{\gamma-1}\right]\mu_0 q_{t|0}\mathcal{P}_0 \tag{A.30}$$

We also use (A.28) to split the general equilibrium through income on tradable consumption and foreign bond holdings into the valuation effects and the aggregate income channel where

$$\frac{\partial\mathcal{C}^T}{\partial Y_t}=\omega^\gamma(\mathcal{P}_0)^{\gamma-1}\mu_0 q_{t|0}\mathcal{P}_0,$$

$$\frac{\partial\mathcal{B}^*}{\partial Y_0}=R_0^*(1-\mu_0)\cdot\mathcal{P}_0 \quad \text{and} \quad \frac{\partial\mathcal{B}^*}{\partial Y_t}=-R_0^*\mu_0\cdot q_{t|0}\mathcal{P}_0 \quad \text{for } t\geq 1 \tag{A.31}$$

Valuation effects. From (18), we can see that an increase in the nominal exchange rate reduces the value of income measured in units of the CPI. In particular,

$$\begin{aligned}\frac{\partial Y_t}{\partial e_t}&=-\frac{1}{\mathcal{P}_t}\frac{W_t h_t+\phi^N}{P^{T*}e_t}\frac{1}{e_t}-\frac{1}{\mathcal{P}_t^2}\frac{\partial\mathcal{P}_t}{\partial e_t}Y_t\\&=-\frac{1}{\mathcal{P}_t}\frac{\bar{P}^N y_t^N}{P_t^T}\frac{1}{e_t}+(1-\tilde{\omega}_t)\frac{1}{e_t}Y_t\\&=-(1-\tilde{\omega}_t)(c_t-Y_t)\frac{1}{e_t}\end{aligned} \tag{A.32}$$

where the last equality combine (A.3), (A.18) and the market clearing condition for the non-tradable good $c_t^N=y_t^N$ to get $\bar{P}^N c_t^N/P_t^T=(1-\tilde{\omega}_t)\mathcal{P}_t c_t$. From (A.30) and (A.32), we have

$$\begin{aligned}\sum_{t=0}^{\infty}\frac{\partial\mathcal{C}^N}{\partial Y_t}\left[\frac{\partial Y_t}{\partial e_t}de_t\right]&=\iota_0\mu_0\sum_{t=0}^{\infty}Q_{t|0}\mathcal{P}_t(Y_t-c_t)(1-\tilde{\omega}_t)\frac{de_t}{e_t}\\&=\iota_0\mu_0\mathcal{P}_0(Y_0-c_0)\left[(1-\tilde{\omega}_0)\frac{de_0}{e_0}-(1-\tilde{\omega}_1)\frac{de_1}{e_1}\right]\\&=\iota_0\mu_0\frac{b_1^*}{R_0^*}\left[(1-\tilde{\omega}_0)\frac{de_0}{e_0}-(1-\tilde{\omega}_1)\frac{de_1}{e_1}\right]\end{aligned} \tag{A.33}$$

where the second equality uses $\sum_{t=1}^{\infty}Q_{t|0}\mathcal{P}_t(Y_t-c_t)=\mathcal{P}_0(c_0-Y_0)$ from (A.6) and $e_t=e_1$ for $t\geq 1$. The last equality uses the flow budget constraint, that is $P_0(c_0-Y_0)=b_1^*/R_0^*$.

Hence, starting from the stationary equilibrium, with $b_1^* = 0$, we arrive to

$$\sum_{t=0}^{\infty} \frac{\partial \log \mathcal{C}^N}{\partial Y_t} \left[\frac{\partial Y_t}{\partial e_t} de_t\right] = 0 \tag{A.34}$$

Proceeding similar for tradable consumption $\mathcal{C}^T$ (with $\iota_0 = \tilde{\omega}_0$), we have that starting from the stationary equilibrium, with $b_1^* = 0$

$$\sum_{t=0}^{\infty} \frac{\partial \log \mathcal{C}^T}{\partial Y_t} \left[\frac{\partial Y_t}{\partial e_t} de_t\right] = 0 \tag{A.35}$$

For foreign bond holdings, we use again $\sum_{t=1}^{\infty} Q_{t|0}\mathcal{P}_t(Y_t - c_t) = \mathcal{P}_0(c_0 - Y_0)$ from (A.6) and $e_t = e_1$ for $t \geq 1$, to obtain

$$\begin{aligned}
\sum_{t=0}^{\infty} \frac{\partial \mathcal{B}^*}{\partial Y_t} \left[\frac{\partial Y_t}{\partial e_t} de_t\right] &= R_0^*(1-\mu_0)\mathcal{P}_0 \frac{\partial Y_0}{\partial e_0} de_0 - R_0^* \mu_0 \sum_{t=1}^{\infty} Q_{t|0}\mathcal{P}_t \frac{\partial Y_t}{\partial e_t} de_t \\
&= R_0^*(1-\mu_0)\mathcal{P}_0(Y_0 - c_0)(1-\tilde{\omega}_0)\frac{de_0}{e_0} - R_0^*\mu_0 \sum_{t=1}^{\infty} Q_{t|0}\mathcal{P}_t(Y_t - c_t)(1-\tilde{\omega}_t)\frac{de_t}{e_t} \\
&= b_1^* \left[(1-\mu_0)(1-\tilde{\omega}_0)\frac{de_0}{e_0} + \mu_0(1-\tilde{\omega}_1)\frac{de_1}{e_1}\right]
\end{aligned} \tag{A.36}$$

Thus, starting from the stationary equilibrium, with $b_1^* = 0$, we arrive to

$$\sum_{t=0}^{\infty} \frac{\partial \mathcal{B}^*}{\partial Y_t} \left[\frac{\partial Y_t}{\partial e_t} de_t\right] = 0 \tag{A.37}$$

Aggregate income. We now deriving the change in real labor income, dy_0^N induced by the temporary decline in nominal interest rate. In equilibrium $y^N = c^N$, and from (A.2) and using Lemma A.1 we have

$$y_t^N = \frac{P_t^{T*}}{\bar{P}^N} e_t \left[1 - \omega^{\gamma}(\mathcal{P}_t)^{\gamma-1}\right] \mathcal{P}_t c_t \quad \Rightarrow \quad \frac{dy_t^N}{y_t^N} = \gamma\tilde{\omega}\frac{de_t}{e_t} + \frac{dc_t}{c_t} \tag{A.38}$$

Given that we reach a stationary equilibrium at $t = 1$, we have $e_{t+1} = e_t$ and $c_{t+1}^T = c_1^T$ for all $t \geq 1$. In addition the stationary level of tradable consumption is $c_1^T = y^T + (1-\beta)b_1^*$. Thus, the next period level of consumption can be expressed as $c_{t+1} = \omega^{-\gamma}\mathcal{P}_{t+1}^{-\gamma}[y^T + (1 - \beta)b_1^*]$ and replacing this in (A.7) yields

$$c_t = \left[\beta R^*(1+\tau_t)\frac{\mathcal{P}_t}{\mathcal{P}_{t+1}}\right]^{-\sigma} \omega^{-\gamma}\mathcal{P}_{t+1}^{-\gamma}\left[y^T + (1-\beta)b_1^*\right] \tag{A.39}$$

Differentiating this equation and using (A.39) we obtain

$$\frac{dc_t}{c_t} = -\sigma d\tau_t - \sigma(1-\tilde{\omega})\frac{dR_t}{R_t} + \gamma(1-\tilde{\omega})\frac{de_{t+1}}{e_{t+1}} + (1-\beta)\frac{db_1^*}{c^T} \tag{A.40}$$

Recall that absent capital controls $d\tau_t = 0$ for all t. We then plug (A.40) into (A.38) to get

$$\frac{dy_t^N}{y_t^N} = -\left[(1-\tilde{\omega})\sigma + \tilde{\omega}\gamma\right]\frac{dR_t}{R_t} + \gamma\frac{de_1}{e_1} + (1-\beta)\frac{db_1^*}{c^T} \tag{A.41}$$

Use (A.41) to determine the effects of changes in y^N on foreign bond holdings

$$\begin{aligned}\sum_{t=0}^{\infty}\frac{\partial\mathcal{B}^*}{\partial Y_t}\left[\frac{\partial Y_t}{\partial y_t^N}dy_t^N\right] &= (1-\mu_0)R_0^*(1-\tilde{\omega}_0)\mathcal{P}_0c_0\frac{dy_0^N}{y_0^N} - \mu_0R_0^*\sum_{t=1}^{\infty}Q_{t|0}\mathcal{P}_tc_t(1-\tilde{\omega}_t)\frac{dy_t^N}{y_t^N}\\ &= \beta R^*(1-\tilde{\omega})\mathcal{P}c\left[\frac{dy_0^N}{y^N} - \frac{dy_1^N}{y^N}\right]\\ &= -(1-\tilde{\omega})\mathcal{P}c\ \left[(1-\tilde{\omega})\sigma + \tilde{\omega}\gamma\right]d\log R_0 \end{aligned} \tag{A.42}$$

The second equality evaluates the first one at the stationary equilibrium. We then combine (A.27), (A.37) and (A.42) and arrive to

$$db_1^* = (1-\tilde{\omega})(\sigma-\gamma)c^T d\log R_0 \tag{A.43}$$

Next, we determine de_1. Because e_1 is consistent with the first-best allocation e_1 satisfies (28). We define $\kappa \equiv \tilde{\omega}\left[1 + \frac{\alpha(1-\tilde{\omega})(\gamma-\sigma)}{\alpha\sigma+(1-\alpha+\phi)\gamma\sigma}\right]^{-1}$. It is straightforward to see that $\kappa \in [0,1]$. The derivative of e_1 evaluated at the stationary equilibrium is given by

$$d\log e_1 = -\frac{1-\kappa}{1-\tilde{\omega}}\frac{1}{\gamma}(1-\beta)\frac{db_1^*}{c^T} \tag{A.44}$$

$$= (1-\kappa)(1-\beta)(\gamma-\sigma)\frac{1}{\gamma}d\log R_0 \tag{A.45}$$

where the second equality uses (A.43) to substitute for db_1^*. We substitute (A.43) and (A.45) into to get the change in output induced by a change in R_0,

$$\frac{dy_t^N}{y_t^N} = -\left[(1-\tilde{\omega})\sigma + \tilde{\omega}\gamma\right]\frac{dR_t}{R_t} + (1-\beta)(\kappa-\tilde{\omega})(\sigma-\gamma)\frac{dR_0}{R_0} \tag{A.46}$$

The aggregate income effect on nontradable consumption is given by

$$\sum_{t=0}^{\infty}\frac{\partial\mathcal{C}^N}{\partial Y_t}\left[\frac{\partial Y_t}{\partial y_t^N}dy_t^N\right] = \mu_0\iota_0\sum_{t=0}^{\infty}Q_{t|0}\mathcal{P}_tc_t(1-\tilde{\omega}_t)\frac{dy_t^N}{y^N}$$

Starting from the stationary equilibrium and plugging equation (A.46) into this equation, we have that

$$\begin{aligned}\sum_{t=0}^{\infty} \frac{\partial \log \mathcal{C}^N}{\partial Y_t}\left[\frac{\partial Y_t}{\partial y_t^N} dy_t^N\right] &= (1-\beta)(1-\tilde{\omega})\frac{dy_0^N}{y^N} + (1-\beta)(1-\tilde{\omega})\sum_{t=1}^{\infty}\beta^t \frac{dy_t^N}{y^N} \\ &= -(1-\beta)(1-\tilde{\omega})\left[(1-\tilde{\omega})\sigma + \tilde{\omega}\gamma + (\kappa-\tilde{\omega})(\gamma-\sigma)\right] d\log R_0 \\ &= -(1-\beta)(1-\tilde{\omega})\left[(1-\kappa)\sigma + \kappa\gamma\right] d\log R_0 \qquad \text{(A.47)}\end{aligned}$$

Proceeding similar for $\mathcal{C}^T$, we use (A.43) and (A.45) to get

$$\sum_{t=0}^{\infty} \frac{\partial \log \mathcal{C}^T}{\partial Y_t}\left[\frac{\partial Y_t}{\partial y_t^N} dy_t^N\right] = -(1-\beta)(1-\tilde{\omega})\left[(1-\kappa)\sigma + \kappa\gamma\right] d\log R_0 \qquad \text{(A.48)}$$

Conclusion of Proof of Proposition 1

We combine (A.19a), (A.24), (A.34) and (A.47) to obtain the decomposition of the overall effect of a temporary change in the nominal interest rate R_0 on non-tradable consumption into three channels

$$\frac{dc_0^N}{c_0^N} = \underbrace{\tilde{\omega}\gamma\frac{de_0}{e_0}}_{\text{Expend. switching}} \underbrace{-\beta\sigma\,(1-\tilde{\omega})\frac{dR_0}{R_0}}_{\text{Intertemporal subst.}} \underbrace{-(1-\beta)\left[(1-\kappa)\sigma+\kappa\gamma\right](1-\tilde{\omega})\frac{dR_0}{R_0}}_{\text{aggregate income channel}} \qquad \text{(A.49)}$$

Because of market clearing, this also corresponds to the decomposition of the effect of a temporary decrease in the nominal interest rate on output as in proposition 1. We use the UIP condition and (A.45) to obtain the change in the nominal exchange rate

$$\frac{de_0}{e_0} = -\frac{dR_0}{dR_0} + (1-\kappa)(1-\beta)(\gamma-\sigma)\frac{1}{\gamma}\frac{dR_0}{dR_0} \qquad \text{(A.50)}$$

and the overall effects on y_t^N for $t > 0$ follows from (A.46) with $dR_t = 0$

$$\frac{dy_t^N}{y_t^N} = (1-\beta)(\kappa-\tilde{\omega})(\sigma-\gamma)\frac{dR_0}{R_0}, \quad \text{for } t > 0$$

Similarly, we combine (A.19b), (A.25), (A.35) and (A.48) to obtain the decomposition of the overall effect of a temporary change in the nominal interest rate R_0 on tradable consumption

$$\frac{dc_0^T}{c_0^T} = -\underbrace{(1-\tilde{\omega})\gamma\frac{de_0}{e_0}}_{\text{Expend. switching}} \underbrace{-\beta\sigma\,(1-\tilde{\omega})\frac{dR_0}{R_0}}_{\text{Intertemporal subst.}} \underbrace{-(1-\beta)\left[(1-\kappa)\sigma+\kappa\gamma\right](1-\tilde{\omega})\frac{dR_0}{R_0}}_{\text{aggregate income channel}}$$

Finally, from (A.27), (A.37) and (A.42) it follows that the overall effect of a temporary change in the nominal interest rate on savings can be decomposed as

$$\frac{db_1^*}{\mathcal{P}c} = \underbrace{\sigma\,(1-\tilde{\omega})\frac{dR_0}{R_0}}_{\text{Intertemporal subst.}} - \underbrace{[(1-\tilde{\omega})\sigma + \tilde{\omega}\gamma]\,(1-\tilde{\omega})\frac{dR_0}{R_0}}_{\text{aggregate income channel}}$$

A.3 Proof of Corollary 1

Proof. The proof of the corollary follows directly from equation (23) in Proposition 1. From equation (23),

$$\begin{aligned}\frac{db_1^*}{\mathcal{P}c} &= \sigma\,(1-\tilde{\omega})\frac{dR_0}{R_0} - [\sigma + \tilde{\omega}(\gamma-\sigma)]\,(1-\tilde{\omega})\frac{dR_0}{R_0} \\ &= \tilde{\omega}(1-\tilde{\omega})(\sigma-\gamma)\frac{dR_0}{R_0} \qquad \text{(A.51)}\end{aligned}$$

It then follows that

$$\text{sign}\left(\frac{db_1^*}{dR_0}\right) = \text{sign}\,(\sigma-\gamma)$$

The response of aggregate savings to a monetary expansion thus follows. □

A.4 Proof of Proposition 2

Proof. Because next period foreign bond holdings is the only state variable for future income and exchange rates, under closed capital capital $db_1^* = 0$, a temporary monetary expansion has no effect on future income and exchange rate, $dy_t^N = 0$ and $de_t = 0$ for $t \geq 1$. We then turn to calculating each component of the decomposition of the effects of a change in R_0 on nontradable consumption (A.14) under closed capital account. Differentiating (A.13b) with respect to e_0 and using (A.15), for given $q_{t|0}$ and Y_t, the expenditure switching effect is given by

$$\begin{aligned}\frac{\partial \log C^N}{\partial e_0}de_0 &= \tilde{\omega}\gamma\; d\log e_0 \\ &= -\tilde{\omega}\gamma\; d\log R_0 \qquad \text{(A.52)}\end{aligned}$$

noting that from the UIP condition $d\log e_0 = -d\log R_0 + d\log e_1$. It can easily be shown that the expression for intertemporal substitution effect, the direct effect from changes in $q_{t|0}$, is not affected by the assumption that the capital account is closed. We again arrive to

(A.24), that is

$$\sum_{t=0}^{\infty} \frac{\partial \log \mathcal{C}^N}{\partial q_{t|0}} dq_{t|0} = -\beta\sigma(1-\tilde{\omega})d\log R_0 \tag{A.53}$$

It is straightforward to see from (A.32) that the valuation effects are once again nil,

$$\frac{\partial \log \mathcal{C}^N}{\partial Y_0}\left[\frac{\partial Y_0}{\partial e_0} de_0\right] = 0 \tag{A.54}$$

Next, we turn to the aggregate income effects. First, we have

$$\frac{\partial \mathcal{C}^N}{\partial Y_0} = \frac{P^{T*}e_0}{\bar{P}^N}\left[1-\omega^\gamma(\mathcal{P}_0)^{\gamma-1}\right]\mu_0 \tag{A.55}$$

Then, use (A.41) with $db_1^* = 0$ and $de_1 = 0$ to obtain

$$\frac{dy_0^N}{y_0^N} = -\left[(1-\tilde{\omega})\sigma + \tilde{\omega}\gamma\right]\frac{dR_0}{R_0} \tag{A.56}$$

Equations (A.55) and (A.56) in turn imply, starting from the stationary equilibrium, that

$$\frac{\partial \log \mathcal{C}^N}{\partial Y_0}\left[\frac{\partial Y_0}{\partial y_0^N} dy_0^N\right] = -(1-\beta)\left[(1-\tilde{\omega})\sigma + \tilde{\omega}\gamma\right]\frac{dR_0}{R_0} \tag{A.57}$$

Finally, combine (A.52), (A.53), (A.54) and (A.57) to get the decomposition of the overall effect of a temporary change in R_0 on c^N under closed capital account

$$\frac{dc_0^N}{c_0^N} = \underbrace{\tilde{\omega}\gamma\frac{dR_0}{R_0}}_{\text{Expend. switching}} \underbrace{-\beta\sigma(1-\tilde{\omega})\frac{dR_0}{R_0}}_{\text{Intertemporal subst.}} \underbrace{-(1-\beta)\left[(1-\tilde{\omega})\sigma + \tilde{\omega}\gamma\right](1-\tilde{\omega})\frac{dR_0}{R_0}}_{\text{aggregate income channel}} \tag{A.58}$$

Because of market clearing, this also corresponds to the decomposition of the effect of a temporary decrease in the nominal interest rate on output under closed capital account.

Next, subtract (A.58) from (A.49) and use (A.50) to obtain the difference between the effect of a change in R_0 on output under open capital account, $d\log y_0^{N,o}$ and its effect on output under closed capital account $d\log y_0^{N,c}$. We have

$$\begin{aligned} d\log y_0^{N,o} - d\log y_0^{N,c} &= \tilde{\omega}(1-\beta)(1-\kappa)(\gamma-\sigma)\frac{dR_0}{R_0} - (1-\beta)(1-\tilde{\omega})(\kappa-\tilde{\omega})(\gamma-\sigma)\frac{dR_0}{R_0} \\ &= \tilde{\omega}(2-\tilde{\omega}-\kappa)(1-\beta)(\gamma-\sigma)\frac{dR_0}{R_0} \end{aligned}$$

Note that $2-\tilde{\omega}-\kappa = (1-\tilde{\omega})+(1-\kappa) > 0$. Thus, if $\gamma > \sigma$, the effect of a monetary expandion $dR_0 < 0$ on output is larger with a closed capital account. Conversely, if $\gamma < \sigma$, the effect of a monetary expandion is larger with a open capital account. The effect is the same with open and closed capital account if $\gamma = \sigma$. □

A.5 Proof of Proposition 3 (Decomposition of capital controls)

We decompose the overall effect of a temporary increase in a capital control τ at time $t = 0$ which reverts to the steady state value at $t = 1$ under passive monetary policy response (i.e., R remains constant). We assume that for $t \geq 1$ the economy features a level for the exchange rate that is consistent with the first-best allocation.

Proof. We follow the same procedure as in section A.2. The expression of the households' policy for non-tradable consumption and foreign bond holdings are given by (A.13b) and (A.13c) where $q_{t|0} = (\mathcal{P}_t/\mathcal{P}_0) \prod_{s=0}^{t-1} \left(R_s^*(1+\tau_s)\right)^{-1}$. Totally differentiate these function, we obtain the following decomposition of the overall effects of an increase in τ_0 on non-tradable consumption

$$dc_0^N = \underbrace{\sum_{t=0}^{\infty} \frac{\partial \mathcal{C}^N}{\partial e_t} de_t}_{\text{Expend. switching}} + \underbrace{\sum_{t=0}^{\infty} \frac{\partial \mathcal{C}^N}{\partial q_{t|0}} dq_{t|0}}_{\text{Intertemporal subst.}} + \underbrace{\sum_{t=0}^{\infty} \frac{\partial \mathcal{C}^N}{\partial Y_t} dY_t}_{\text{GE effects}}$$

$$db_1^* = \underbrace{\sum_{t=0}^{\infty} \frac{\partial \mathcal{B}^*}{\partial e_t} de_t}_{\text{Expend. switching}} + \underbrace{\sum_{t=0}^{\infty} \frac{\partial \mathcal{B}^*}{\partial q_{t|0}} dq_{t|0}}_{\text{Intertemporal subst.}} + \underbrace{\sum_{t=0}^{\infty} \frac{\partial \mathcal{B}^*}{\partial Y_t} dY_t}_{\text{GE effects}}$$

Expenditure switching. start by characterizing the expenditure switching channel. Using equation (A.15) in Lemma A.1, the derivative of (A.13b) and (A.13c) with respect to e_t, starting from the stationary equilibrium, are

$$\sum_{t=0}^{\infty} \frac{\partial \log \mathcal{C}^N}{\partial e_t} de_t = [1 - \tilde{\omega}(1-\gamma)]\, d\log e_0 \tag{A.59}$$

$$\sum_{t=0}^{\infty} \frac{\partial \log \mathcal{B}^*}{\partial e_t} de_t = 0 \tag{A.60}$$

Intertemporal substitution. First, notice that the change in $q_{t|0}$ induced by the decline in τ_0 is given by $d\log q_{0|0} = 0$ and for $t \geq 1$

$$\begin{aligned} d\log q_{t|0} &= -d\tau_0 - (1-\tilde{\omega})d\log R_0, \\ &= -d\tau_0, \end{aligned} \tag{A.61}$$

Substituting (A.61) into (A.22) starting from the stationary equilibrium, with $b_1^* = 0$, we arrive to the expression in the proposition

$$\sum_{t=0}^{\infty} \frac{\partial \log \mathcal{C}^N}{\partial q_{t|0}} dq_{t|0} = -\sigma\beta d\tau_0 \tag{A.62}$$

Substituting (A.61) into (A.26) starting from the stationary equilibrium, with $b_1^* = 0$, we arrive to the expression in the proposition

$$\sum_{t=0}^{\infty} \frac{\partial \log \mathcal{B}^*}{\partial q_{t|0}} dq_{t|0} = \sigma\beta d\tau_0 \tag{A.63}$$

Effects through income. Using the definition of Y_t, with $T_t \neq 0$, and totally differentiating with respect to e_t, T_t and y_t^N, we have that

$$dY_t = \frac{\partial Y_t}{\partial e_t} de_t + \frac{\partial Y_t}{\partial T_t} dT_t + \frac{\partial Y_t}{\partial y_t^N} dy_t^N$$

The total effect of the change in Y_t on non-tradable consumption is thus given by

$$\sum_{t=0}^{\infty} \frac{\partial \mathcal{C}^N}{\partial Y_t} dY_t = \underbrace{\sum_{t=0}^{\infty} \frac{\partial \mathcal{C}^N}{\partial Y_t} \left[\frac{\partial Y_t}{\partial e_t} de_t\right]}_{\text{valuation effects}} + \underbrace{\sum_{t=0}^{\infty} \frac{\partial \mathcal{C}^N}{\partial Y_t} \left[\frac{\partial Y_t}{\partial T_t} dT_t\right]}_{\text{fiscal channel}} + \underbrace{\sum_{t=0}^{\infty} \frac{\partial \mathcal{C}^N}{\partial Y_t} \left[\frac{\partial Y_t}{\partial y_t^N} dy_t^N\right]}_{\text{aggregate income channel}}$$

where $\partial \mathcal{C}^N / \partial Y_t$ is given by (A.30). Similarly, we decompose the total effect of a change in Y_t on foreign bond holdings where $\partial \mathcal{B}^* / \partial Y_t$ is given by (A.31). Consider first the valuation effects. Using directly (A.33) and (A.36), and starting at a stationary equilibrium, we have

$$\sum_{t=0}^{\infty} \frac{\partial \log \mathcal{C}^N}{\partial Y_t} \left[\frac{\partial Y_t}{\partial e_t} de_t\right] = 0 \quad \text{and} \quad \sum_{t=0}^{\infty} \frac{\partial \log \mathcal{B}^*}{\partial Y_t} \left[\frac{\partial Y_t}{\partial e_t} de_t\right] = 0 \tag{A.64}$$

Differentiating the government budget, $T_t = -\frac{\tau_t}{1+\tau_t} \frac{b_{t+1}^*}{R_t^*}$, leads to $dT_0 = -\frac{b_1^*}{R^*} d\tau_0$ and $dT_t = 0$ for $t \geq 1$. Thus, starting at the stationary equilibrium with $b_1^* = 0$, $dT_0 = 0$ and we have

$$\sum_{t=0}^{\infty} \frac{\partial \log \mathcal{C}^N}{\partial T_t} \left[\frac{\partial Y_t}{\partial T_t} dT_t\right] = 0 \quad \text{and} \quad \sum_{t=0}^{\infty} \frac{\partial \log \mathcal{B}^*}{\partial T_t} \left[\frac{\partial Y_t}{\partial T_t} dT_t\right] = 0 \tag{A.65}$$

Finally, for the aggregate income channel, substitute (A.40) into into (A.38), along with $dR_t = 0$ for all t as the nominal interest R remain constant, to get

$$dy_t^N = \left[-\sigma d\tau_t + \gamma \frac{de_1}{e_1} + (1-\beta) \frac{db_1^*}{c^T}\right] y_t^N \tag{A.66}$$

We use (A.66) to evaluate the effects of changes in y^N on foreign bond holdings starting from the stationary equilibrium. We have

$$\sum_{t=0}^{\infty} \frac{\partial \mathcal{B}^*}{\partial Y_t} \left[\frac{\partial Y_t}{\partial y_t^N} dy_t^N \right] = \beta R^* (1-\tilde{\omega}) \mathcal{P} c \left[\frac{dy_0^N}{y^N} - \frac{dy_1^N}{y^N} \right]$$
$$= -(1-\tilde{\omega}) \mathcal{P} c \, \sigma d\tau_0 \qquad \text{(A.67)}$$

Combining (A.60), (A.63)-(A.65) and (A.67), we arrive to the decomposition of a change in the tax on debt τ_0 on capital flows as in Proposition 3

$$\frac{db_1^*}{\mathcal{P}c} = \underbrace{\sigma d\tau_0}_{\text{Intertemporal Subst.}} - \underbrace{(1-\tilde{\omega})\sigma d\tau_0}_{\text{Aggregate Income}}$$

This in turn implies that the overall effect of capital controls on foreign bond holdings is given by

$$db_1^* = \sigma c^T d\tau_0 \qquad \text{(A.68)}$$

We then substitute (A.68) into (A.44) to obtain

$$d \log e_1 = -\frac{1-\kappa}{1-\tilde{\omega}} (1-\beta) \sigma \frac{1}{\gamma} d\tau_0 \qquad \text{(A.69)}$$

Next, we plug (A.68) and (A.69) into (A.66) and use (A.30), starting from the stationary, to obtain the effects of changes in y^N on non-tradable consumption. Formally, we have

$$\sum_{t=0}^{\infty} \frac{\partial \log \mathcal{C}^N}{\partial Y_t} \left[\frac{\partial Y_{t|0}}{\partial y_t^N} dy_t^N \right] = (1-\beta)(1-\tilde{\omega}) \frac{dy_0^N}{y^N} + (1-\beta)(1-\tilde{\omega}) \sum_{t=1}^{\infty} \beta^t \frac{dy_t^N}{y^N}$$
$$= (1-\beta) \left[-\sigma(1-\tilde{\omega}) - (1-\kappa)\sigma + \sigma(1-\tilde{\omega}) \right] d\tau_0$$
$$= -(1-\beta)(1-\kappa)\sigma d\tau_0 \qquad \text{(A.70)}$$

We finally combine (A.59), (A.62)-(A.65) and (A.70) to get the decomposition of the overall effect of capital controls on non-tradable consumption

$$d \log c_0^N = \underbrace{\tilde{\omega}\gamma \, d \log e_0}_{\text{Expend. switching}} - \underbrace{\beta \sigma d\tau_0}_{\text{Intertemporal subst.}} - \underbrace{(1-\beta)(1-\kappa)\sigma d\tau_0}_{\text{aggregate income channel}}$$

Because of market clearing, this also corresponds to the decomposition of the effect of capital controls on output. □

B Proofs for Section 4

B.1 Proof of Proposition 4

Proof. The optimization problem of the government consists in choosing the optimal tax on debt to maximize private agents' welfare subject to the equilibrium conditions (3), (4), (5), (6) and (13).

For a given exogenous path of the nominal exchange rate $\{e_t\}$, we solve a relaxed government problem of the government as follows:

$$\max_{\{b^*_{t+1}, c^N_t, c^T_t\}} \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t z_t \left[u\left(c^T_t, c^N_t\right) - v\left((c^N_t)^{1/\alpha}\right)\right] \tag{B.1}$$

subject to

$$c^T_t = y^T_t + b^*_t - \frac{b^*_{t+1}}{R^*_t} \tag{B.2}$$

$$c^N_t = \left[\frac{1-\omega}{\omega}\frac{P^{T*}_t}{\bar{P}^N} e_t\right]^{\gamma} c^T_t \tag{B.3}$$

After solving the relaxed problem, (B.1), we can back out the optimal tax on debt τ by using (5). Taking the first-order conditions, we arrive to

$$c^T_t: \quad \lambda_t = u_T(t) + \vartheta_t \frac{c^N_t}{c^T_t} \tag{B.4}$$

$$c^N_t: \quad \vartheta_t = u_N(t) - \frac{1}{\alpha}(h_t)^{1-\alpha} v'\left((c^N_t)^{1/\alpha}\right) \tag{B.5}$$

$$b^*_{t+1}: \quad \frac{z_t \lambda_t}{R^*_t} = \beta \mathbb{E}_t z_{t+1} \lambda_{t+1} \tag{B.6}$$

where $\lambda_t \geq 0$ and ϑ_t are the Lagrange multipliers on constraints (B.2) and (B.3) respectively. Combining (B.4) and (B.5), we have

$$\begin{aligned}\lambda_t &= u_T(t) + u_N(t)\frac{c^N_t}{c^T_t}\left[1 - \frac{1}{\alpha h_t^{\alpha-1}}\frac{v'(h_t)}{u_N(t)}\right] \\ &= u_T(t) + u_T(t)\frac{\bar{P}^N c^N_t}{P^T_t c^T_t}\psi_t \end{aligned} \tag{B.7}$$

where the second equality uses $\frac{\bar{P}^N}{P^T_t} = \frac{u_N(t)}{u_T(t)}$. We then substitute (B.7) into (B.6) to get

$$u_T(t)\left[1 + \frac{\bar{P}^N c^N_t}{P^T_t c^T_t}\psi_t\right] = \beta R^*_t \mathbb{E}_t \left\{\frac{z_{t+1}}{z_t} u_T(t+1)\left[1 + \frac{\bar{P}^N c^N_{t+1}}{P^T_{t+1} c^T_{t+1}}\psi_{t+1}\right]\right\} \tag{B.8}$$

We now turn to deriving the optimal tax rate on debt. We plug the implementability constraint (5) given by

$$u_T(t) = \beta R_t^*(1+\tau_t)\mathbb{E}_t\left[\frac{z_{t+1}}{z_t}u_T(t+1)\right]$$

into the optimality condition (B.8) and obtain

$$\tau_t = \frac{1}{\beta R_t^* \mathbb{E}_t \frac{z_{t+1}}{z_t} u_T(t+1)}\left\{-\frac{1-\tilde{\omega}_t}{\tilde{\omega}_t}u_T(t)\psi_t + \beta R_t^* \mathbb{E}_t \frac{z_{t+1}}{z_t}\left[\frac{1-\tilde{\omega}_{t+1}}{\tilde{\omega}_{t+1}}u_T(t+1)\psi_{t+1}\right]\right\}.$$

□

B.2 Proof of Proposition 5 (Monetary Policy and Capital Controls)

Proof. The government sets its policy $\{R, \tau\}$ to maximize households' welfare subject to the resource and the implementability constraints, and a zero lower bound constraint on nominal interest rate. The government problem in recursive form is:

$$V(b^*, s) = \max_{R,\tau,e,b^{*\prime},c^N,c^T} u\left(c^T, c^N\right) - v\left((c^N)^{1/\alpha}\right) + \beta \mathbb{E}_{s'|s}\frac{z'}{z}V\left(b^{*\prime}, s'\right)$$

subject to

$$c^T = y^T + b^* - \frac{b^{*\prime}}{R^*} \tag{B.9}$$

$$c^N = \left[\frac{1-\omega}{\omega}\frac{P^{T*}}{\bar{P}^N}e\right]^\gamma c^T \tag{B.10}$$

$$u_T(c^T, c^N) = \beta R^*(1+\tau)\,\mathbb{E}_{s'|s}\left[\frac{z'}{z}u_T\left(\mathcal{C}^T(b^{*\prime}, s'), \mathcal{C}^N(b^{*\prime}, s')\right)\right] \tag{B.11}$$

$$R^* = R\mathbb{E}_{s'|s}\left[\Lambda\left(\mathcal{C}^T(b^{*\prime}, s'), \mathcal{C}^N(b^{*\prime}, s')\right)\frac{P^{T*}}{P^{T*\prime}}\frac{e}{\mathcal{E}(b^{*\prime}, s')}\right] \tag{B.12}$$

$$R \geq 1 \tag{B.13}$$

Let $\lambda \geq 0$ be the multiplier on the resource constraint (B.9), ϑ, v and χ be the multiplier on implementability constraints (B.10), (B.11) and (B.12) respectively, $\xi \geq 0$ be the multiplier zero lower bound constraint (B.13).

Notice how τ only appears in (B.11). The first-order condition with respect to τ thus equates the lagrange multiplier on (B.11) to zero, $v = 0$. In other words, the tax rate τ can be dropped from the government's problem along with the implementability constraint (B.11) (i.e., the relaxed problem of the government presented in the text). The remaining optimality conditions, after eliminating χ, are

$$\xi = \gamma c^N \vartheta \tag{B.14}$$

$$\vartheta = u_N(c^T, c^N) - \frac{v'(h)}{\alpha h^{\alpha-1}} \tag{B.15}$$

$$\lambda = u_T(c^T, c^N) + \frac{c^N}{c^T}\vartheta \tag{B.16}$$

$$\lambda = -\xi\, \mathbb{E}_{s'|s} \frac{\partial}{\partial b^{*\prime}} \left[\frac{\Lambda(b^{*\prime}, s')}{\mathcal{E}(b^{*\prime}, s')} \frac{eP^{T*}}{P^{T*\prime}} \right] + \beta R^* \mathbb{E}_{s'|s} \frac{z'}{z} \lambda' \tag{B.17}$$

We combine (B.14) and (B.15) to obtain

$$\begin{aligned} \xi &= \gamma c^N u_N(c^T, c^N)\psi \\ &= \gamma c^T \, \frac{1-\tilde{\omega}}{\tilde{\omega}} u_T(c^T, c^N)\psi \end{aligned} \tag{B.18}$$

This corresponds to (31) in the text. Next we determine the level of the nominal exchange rate when monetary policy is not constraint by the ZLB. Notice from (B.18) that a non binding ZLB constraint on nominal interest rate $\xi = 0$ implies that $\psi = 0$, and using the definition of the labor wedge we have

$$\frac{1}{\alpha (c^N)^{\frac{\alpha-1}{\alpha}}} \frac{(c^N)^{\frac{\phi}{\alpha}}}{u_N(c^T, c^N)} = 1 \quad \Rightarrow \quad (c^N)^{\frac{1-\alpha+\phi}{\alpha} + \frac{1}{\sigma}} = \alpha(1-\omega) \left(\frac{c}{c^N} \right)^{\frac{1}{\gamma} - \frac{1}{\sigma}}$$

We then plug in the expression for the composite consumption $c_t(c^T, c^N)$ and use equation (B.10) to eliminate the consumption ratio c^T/c^N to get

$$(c^N)^{\frac{(1-\alpha+\phi)\sigma+\alpha}{\alpha\sigma}} = \alpha(1-\omega)^{\frac{\gamma}{\sigma}} \left[\omega^\gamma \left(\frac{eP^{T*}}{\bar{P}^N} \right)^{1-\gamma} + (1-\omega)^\gamma \right]^{\frac{1}{\gamma-1} \frac{\sigma-\gamma}{\sigma}}$$

Using (B.10) to substitute for c^N and using the ideal price index $\mathcal{P}$ defined in (19) to simplify the right hand side of this equation, we arrive to

$$\left[\left(\frac{1-\omega}{\omega} \frac{P^{T*}}{\bar{P}^N} e \right)^\gamma c^T \right]^{\frac{(1-\alpha+\phi)\sigma+\alpha}{\alpha}} = \alpha^\sigma (1-\omega)^\gamma \left[\frac{eP^{T*}}{\bar{P}^N} \mathcal{P} \right]^{\sigma-\gamma}$$

which in turn implies that

$$e = \frac{\omega}{1-\omega} \frac{\bar{P}^N}{P^{T*}} \left[\alpha^{\frac{\sigma}{\gamma}} (1-\omega) \left(\frac{eP^{T*}}{\bar{P}^N} \mathcal{P} \right)^{\frac{\gamma-\sigma}{\gamma}} \right]^{\frac{\alpha}{(1-\alpha+\phi)\sigma+\alpha}} (c^T)^{-\frac{1}{\gamma}}$$

Finally, we turn to deriving the optimal tax on debt. Defining $\Theta \equiv \gamma c^T \frac{\partial}{\partial b^{*\prime}} \mathbb{E}_{s'|s}\left[\frac{\Lambda(b^{*\prime},s')}{\mathcal{E}(b^{*\prime},s')}\frac{eP^{T*}}{P^{T*\prime}}\right]$ and plugging (B.16) into (B.17), we get the following Euler equation for foreign bonds

$$u_T(c^T,c^N) + (1+\Theta)\frac{\xi}{\gamma c^T} = \beta R^* \mathbb{E}_{s'|s}\frac{z'}{z}\left[u_T\left(c^{T\prime},c^{N\prime}\right) + \frac{\xi'}{\gamma c^{T\prime}}\right] \tag{B.19}$$

We then use the implementability constraint (B.11) and substitute it into (B.19) to back out the optimal tax on debt

$$\tau = \frac{1}{\beta R^* \mathbb{E}_{s'|s}\frac{z'}{z}\left[u_T\left(c^{T\prime},c^{N\prime}\right)\right]}\left\{-(1+\Theta)\frac{\xi}{\gamma c^T} + \beta R^* \mathbb{E}_{s'|s}\frac{z'}{z}\left[\frac{\xi'}{\gamma c^{T\prime}}\right]\right\}$$

□

B.3 Proof of Proposition 6 (Monetary Policy without Capital Controls)

Preliminaries. Absent capital controls, the government sets its policy $\{R\}$ to maximize households' welfare subject to resource and implementability constraints, and a zero lower bound constraint on nominal interest rate. The government problem is given by:

$$V(b^*,s) = \max_{R,e,b^{*\prime},c^N,c^T} u\left(c^T,c^N\right) - v\left(\left(c^N\right)^{1/\alpha}\right) + \beta \mathbb{E}_{s'|s}\frac{z'}{z}V\left(b^{*\prime},s'\right) \tag{B.20}$$

subject to

$$c^T = y^T + b^* - \frac{b^{*\prime}}{R^*} \tag{B.21}$$

$$c^N = \left[\frac{1-\omega}{\omega}\frac{P^{T*}}{\bar{P}^N}e\right]^{\gamma} c^T \tag{B.22}$$

$$u_T(c^T,c^N) = \beta R^* \, \mathbb{E}_{s'|s}\frac{z'}{z}\left[u_T\left(\mathcal{C}^T(b^{*\prime},s'),\mathcal{C}^N(b^{*\prime},s')\right)\right] \tag{B.23}$$

$$R^* = R\mathbb{E}_{s'|s}\left[\Lambda\left(\mathcal{C}^T(b^{*\prime},s'),\mathcal{C}^N(b^{*\prime},s')\right)\frac{P^{T*}}{P^{T*\prime}}\frac{e}{\mathcal{E}(b^{*\prime},s')}\right] \tag{B.24}$$

$$R \geq 1 \tag{B.25}$$

We let $\lambda \geq 0$ be the multiplier on the resource constraint (B.21), ϑ, v and χ be the multiplier on implementability constraints (B.22), (B.23) and (B.24) respectively, $\xi \geq 0$ be the multiplier zero lower bound constraint (B.25).

We proceed by first defining a Markov perfect equilibrium in the absence of capital controls and then characterizing the the optimal monetary policy.

Definition B.1 (Markov perfect equilibrium absent capital controls)**.** A Markov perfect equilibrium is defined by the current government policy functions $R(b^*,s)$, $\mathcal{E}(b^*,s)$ with associated decision rules $c^T(b^*,s)$, $b'(b^*,s)$ $c^N(b^*,s)$, and value function $V(b^*,s)$, and the conjectured function characterizing the decision rule of future governments $\mathcal{R}(b^*,s)$, $\mathcal{B}(b^*,s)$ and the associated decision rules $\mathcal{C}^T(b^*,s)$, $\mathcal{C}^N(b^*,s)$, such that: (i) given the conjecture of future policies, the value function and the policy functions solve the government problem (B.20); and (ii) The conjectured policy rules that represent optimal choices of future governments coincide with the solutions to (B.20).

Optimal monetary policy. Next, we characterize the optimal monetary policy.

Proof. Using the first-order condition with respect R, given by $\chi R^*/R = \xi$, to eliminate χ, the first-order conditions with respect to e and c^N are

$$\xi = \gamma c^N \vartheta \tag{B.26}$$

$$\vartheta = u_N(c^T,c^N) - \frac{v'(h)}{\alpha h^{\alpha-1}} - u_{TN}(c^T,c^N)v \tag{B.27}$$

To derive the optimal monetary policy when the nominal interest rate is not constraint by the ZLB, we start by substituting (B.26) into (B.27) to get

$$\begin{aligned}\xi &= \gamma c^N \left[u_N(c^T,c^N)\psi - u_{TN}(c^T,c^N)v\right] \\ &= \gamma c^N u_N(c^T,c^N)\left[\psi - \frac{\tilde{\omega}(\sigma-\gamma)}{\sigma\gamma}\frac{v}{c^T}\right]\end{aligned}$$

Thus, when the ZLB does not bind

$$\psi = \frac{\tilde{\omega}(\sigma-\gamma)}{\sigma\gamma}\frac{v}{c^T} \tag{B.28}$$

We now need to determine v. Using the first order conditions with respect to tradable consumption c^T and foreign bonds b'^*

$$\lambda = u_T(c^T,c^N) - u_{TT}(c^T,c^N)v + \frac{c^N}{c^T}\vartheta \tag{B.29}$$

$$\begin{aligned}\lambda = \beta R^* \mathbb{E}_{s'|s}\frac{z'}{z}\lambda' - \xi\, \mathbb{E}_{s'|s}\frac{eP^{T*}}{P^{T*\prime}}\frac{\partial}{\partial b^{*\prime}}\left[\frac{\Lambda(b^{*\prime},s')}{\mathcal{E}(b^{*\prime},s')}\right] \\ + v\,\beta R^* E_{s'|s}\frac{z'}{z}\frac{\partial}{\partial b^{*\prime}}\left[u_T\left(\mathcal{C}^T(b^{*\prime},s'),\mathcal{C}^N(b^{*\prime},s')\right)\right]\end{aligned} \tag{B.30}$$

and plugging (B.29) into (B.30), we get

$$u_T(c^T,c^N) - (1+\bar{\Theta})u_{TT}(c^T,c^N)v = \beta R^* \mathbb{E}_{s'|s}\frac{z'}{z}\left[u_T(c^{T\prime},c^{N\prime}) - u_{TT}(c^{T\prime},c^{N\prime})v' + \frac{\xi'}{\gamma c^{T\prime}}\right]$$

where $\bar{\Theta} \equiv \frac{1}{u_{TT}(c^T, c^N)} \beta R^* \mathbb{E}_{s'|s} \frac{z'}{z} \frac{\partial}{\partial b^{*\prime}} u_T \left(c^{T\prime}, c^{N\prime}\right) > 0$. Then, substituting the implementability constraint (B.23) into this equation leads to

$$- u_{TT}(c^T, c^N) v = \beta R^* (1 + \bar{\Theta})^{-1} \mathbb{E}_{s'|s} \frac{z'}{z} \left[-u_{TT}(c^{T\prime}, c^{N\prime}) v' + \frac{\xi'}{\gamma c^{T\prime}} \right] \tag{B.31}$$

Iterating forward (B.31) and using the transversality condition, we obtain (for convenience, the equations are written in their sequential form)

$$\begin{aligned} v_t &= \frac{1}{-u_{TT}(t)} \mathbb{E}_t \sum_{k=1}^{\infty} \bar{Q}_{k|0} \frac{\xi_{t+k}}{\gamma c^T_{t+k}} \\ \frac{v_t}{c^T_t} &= \frac{1}{u_T(t)} \frac{\sigma \gamma}{(1 - \tilde{\omega}_t) \sigma + \tilde{\omega}_t \gamma} \mathbb{E}_t \sum_{k=1}^{\infty} \bar{Q}_{t+k|t} \frac{\xi_{t+k}}{\gamma c^T_{t+k}} \end{aligned} \tag{B.32}$$

where $\bar{Q}_{t+k|t} = \beta^k \frac{z_{t+k}}{z_t} \prod_{s=0}^{k-1} \frac{R^*_{t+s}}{1+\bar{\Theta}_s}$. Finally, we substitute (B.32) into (B.28) to get the optimal monetary policy in its target form

$$u_T(t) \psi_t = \frac{\tilde{\omega}_t (\sigma - \gamma)}{(1 - \tilde{\omega}_t) \sigma + \tilde{\omega}_t \gamma} \mathbb{E}_t \sum_{k=1}^{\infty} \bar{Q}_{t+k|t} \frac{\xi_{t+k}}{\gamma c^T_{t+k}}.$$

□

C Proofs for Section 5

C.1 Proof of Proposition 7

Proof. From the perspective of the SOE, we have to infer the effects of the foreign monetary policy shock on P^{T*}, R^*, which are taken as given by the SOE. Let $V_{H,0}(b^*_{H,0}, \{P^{T*}, R^*\})$ denote the welfare of households in the SOE at the initial period and $c^N_{H,0}(b^*_{H,0}, \{P^{T*}, R^*\})$, $c^T_{H,0}(b^*_{H,0}, \{P^{T*}, R^*\})$, $b^*_{H,1}(b^*_{H,0}, \{P^{T*}, R^*\})$, $e_{H,0}(b^*_{H,0}, \{P^{T*}, R^*\})$ the associated policy functions. The effect on welfare is then given by

$$\frac{dV_{H,0}}{dR_{F,0}} = \sum_{t=0}^{\infty} \frac{\partial V_{H,0}}{\partial P^{T*}_t} \frac{dP^{T*}_t}{dR_{F,0}} + \sum_{t=0}^{\infty} \frac{\partial V_{H,0}}{\partial R^*_t} \frac{dR^*_t}{dR_{F,0}} \tag{C.1}$$

We determine $\partial V_{H,0}/\partial P^{T*}$ and $\partial V_{H,0}/\partial R^*$ by applying the envelope theorem to the SOE problem that follows

$$V_{H,0} = \max_{c^N_{H,0}, c^T_{H,0}, b^*_{H,1}, e_{H,0}} u\left[y^T_{H,0} + b^*_{H,0} - \frac{b^*_{H,1}}{R^*_0}, c^N_{H,0}\right] - v\left[(c^N_{H,0})^{1/\alpha}\right] + \beta \frac{z_{H,1}}{z_{H,0}} V_{H,1}(b^*_{H,1})$$

subject to

$$c^T_{H,0} = y^T_{H,0} + b^*_{H,0} - \frac{b^*_{H,1}}{R^*_0} \qquad (\times \lambda_{H,0})$$

$$c^N_{H,0} = \left[\frac{1-\omega}{\omega} \frac{P^{T*}_0}{\bar{P}^N} e_{H,0}\right]^{\gamma} c^T_{H,0} \qquad (\times \vartheta_{H,0})$$

$$u_T(c^T_{H,0}, c^N_{H,0}) = \beta R^*_0 (1+\tau_{H,0}) \frac{z_{H,1}}{z_{H,0}} \left[u_T\left(\mathcal{C}^T(b^*_{H,1}), \mathcal{C}^N(b^*_i)\right)\right] \qquad (\times v_{H,0})$$

$$1 \geq \frac{1}{R^*_0} \frac{e_{H,0}}{\mathcal{E}_H(b^*_{H,1})} \frac{P^{T*}_0}{P^{T*}_1} \qquad (\times \xi_{H,0})$$

where we omitted the arguments for the value function $V_{H,0}$ and policy functions $c^N_{H,0}$, $c^T_{H,0}$, $b^*_{H,1}$, $e_{H,0}$ to simplify the expressions. We therefore have using the envelope condition that the partial derivative of the home households' welfare with respect to P^{T*}_0 is given by

$$\frac{\partial V_{H,0}}{\partial P^{T*}_0} = \gamma c^N_{H,0} \vartheta_{H,0} - \xi_{H,0} = 0 \tag{C.2}$$

where the second equality uses the government's first order condition with respect to the nominal exchange rate. For the derivative with respect to P^{T*}_1,

$$\frac{\partial V_{H,0}}{\partial P^{T*}_1} = -\gamma c^N_{H,0} \vartheta_{H,0} + \xi_{H,0} = 0 \tag{C.3}$$

Next, applying the envelope condition to $\partial V_{H,0}/\partial P_t^{T*}$ for $t > 1$, and $\partial V_{H,0}/\partial R_t^*$ for $t \geq 1$, it is straightforward to see that

$$\frac{\partial V_{H,0}}{\partial P_t^{T*}} = 0 \;\text{ for } t > 1, \quad \text{and} \quad \frac{\partial V_{H,0}}{\partial R_t^*} = 0 \;\text{ for } t \geq 1. \tag{C.4}$$

It remains to determine $\partial V_{H,0} \partial R_0^*$. Use once again the envelope condition to arrive to

$$\frac{\partial V_{H,0}}{\partial R_0^*} = \lambda_{H,0} \frac{b_{H,1}^*}{(R_0^*)^2} + \frac{1}{R_0^*} \left[u_T(c_{H,0}^T, c_{H,0}^N) v_{H,0} + \xi_{H,0} \right] \tag{C.5}$$

Then, combine the government's first order condition with respect to e and c^N to get

$$\begin{aligned} \xi_{H,0} &= \gamma c_{H,0}^N \left[u_N(c_{H,0}^T, c_{H,0}^N) \psi_{H,0} - u_{TN}(c_{H,0}^T, c_{H,0}^N) v_{H,0} \right] \\ &= \gamma c_{H,0}^N u_N(c_{H,0}^T, c_{H,0}^N) \psi_{H,0} - \gamma c_{H,0}^N \frac{\tilde{\omega}}{c_{H,0}^T} u_N(c_{H,0}^T, c_{H,0}^N) \frac{\sigma - \gamma}{\sigma \gamma} v_{H,0} \\ &= u_T(c_{H,0}^T, c_{H,0}^N) \left[\frac{1 - \tilde{\omega}_{H,0}}{\tilde{\omega}_{H,0}} \psi_{H,0} - (1 - \tilde{\omega}_{H,0}) \frac{\sigma - \gamma}{\sigma \gamma} v_{H,0} \right] \end{aligned} \tag{C.6}$$

Plugging (C.6) into (C.5), and given that we start from $b_{H,1}^* = 0$, we get

$$\frac{\partial V_{H,0}}{\partial R_0^*} = \frac{u_T(c_{H,0}^T, c_{H,0}^N)}{R_0^*} \left[v_{H,0} + \frac{1 - \tilde{\omega}_{H,0}}{\tilde{\omega}_{H,0}} \gamma c_{H,0}^T \psi_{H,0} - (1 - \tilde{\omega}_{H,0}) \frac{\sigma - \gamma}{\sigma} v_{H,0} \right] \tag{C.7}$$

Finally, we substitute (C.2), (C.3), (C.4) and (C.7) into (C.1) to obtain the equation (35) in the text, that is

$$\frac{dV_{H,0}}{dR_{F,0}} = \frac{u_T(c_{H,0}^T, c_{H,0}^N)}{R_0^*} \left[\frac{1 - \tilde{\omega}_{H,0}}{\tilde{\omega}_{H,0}} \gamma c_{H,0}^T \psi_{H,0} + \left(\tilde{\omega}_{H,0} + (1 - \tilde{\omega}_{H,0}) \frac{\gamma}{\sigma} \right) v_{H,0} \right] \frac{dR_0^*}{dR_{F,0}}.$$

□

C.2 Proof of Corollary 2

Proof. The proof of the corollary follows from equation (36). Consider a prudential monetary policy intervention that aims at increasing aggregate savings in order to mitigate overborrowing. From (C.20) in Appendix C.4, the overall effect of a change in the foreign nominal interest rate on aggregate savings is given by

$$db_{F,1}^* = R^*(1 - \mu_F) c_F^T \, (1 - \tilde{\omega}_F)(\sigma - \gamma) dR_{F,0} \tag{C.8}$$

Thus, an increase in aggregate savings $db^*_{F,1} > 0$ requires

$$(\sigma - \gamma) dR_{F,0} > 0 \tag{C.9}$$

In line with Proposition 6, equation (C.9) states that when $\sigma > \gamma$ the prudential monetary intervention is contractionary $dR_{F,0} > 0$ and it turns out be expansionary $dR_{F,0} < 0$ when $\sigma < \gamma$. Next, we turn to deriving $dR^*_0 / dR_{F,0}$. In addition to its direct effect on aggregate savings, a prudential monetary intervention has an indirect effect on aggregate savings at home and foreign through the potential change in the world interest rate. From (C.20) in Appendix C.4,, we have that,

$$db^*_{F,1} = R^*(1-\mu_F)c^T_F \left[(\sigma\tilde{\omega}_F + \gamma(1-\tilde{\omega}_F)) \frac{\partial \log R^*_0}{\partial R_{F,0}} \right] dR_{F,0} \tag{C.10}$$

$$db^*_{H,1} = R^*(1-\mu_H)c^T_H \left[(\sigma\tilde{\omega}_H + \gamma(1-\tilde{\omega}_H)) \frac{\partial \log R^*_0}{\partial R_{F,0}} \right] dR_{F,0} \tag{C.11}$$

Then, combine (C.8), (C.10) and (C.11) and use the market clearing condition for bond, $n\, db^*_{H,1} + (1-n) db^*_{F,1} = 0$, to obtain

$$d \log R^*_0 = -\frac{(1-n)(1-\mu_F)c^T_F\,(1-\tilde{\omega}_F)}{\sum_{i \in \{H,F\}}(1-\mu_i)c^T_i(\sigma\tilde{\omega}_i + \gamma(1-\tilde{\omega}_i))} (\sigma-\gamma) dR_{F,0} < 0$$

where the inequality uses (C.9). Therefore, from equation (36), it follows that

$$\frac{\partial V_{H,0}}{\partial R_{F,0}} = \left[\frac{u_T(c^T_{H,0}, c^N_{H,0})}{R^*_0} v_{H,0} \right] \frac{\partial R^*_0}{\partial R_{F,0}} \leq 0 \tag{C.12}$$

where $v_{H,0} > 0$ if the ZLB binds in the future as shown in (B.28). Thus, prudential monetary intervention abroad lowers home welfare, strictly so if the ZLB binds in the future. □

C.3 Proof of Corollary 3

Proof. The proof of the corollary follows from equation (37)

$$\frac{\partial V_{H,0}}{\partial R_{F,0}} = \frac{u_T(c^T_{H,0}, c^N_{H,0})}{R^*_0} \left[\frac{1-\tilde{\omega}_{H,0}}{\tilde{\omega}_{H,0}} \gamma c^T_{H,0} \psi_{H,0} \right] \frac{\partial R^*_0}{\partial R_{F,0}}$$

As shown in Proposition 5, away from the ZLB the labor wedge under optimal monetary and capital controls policy satisfies $\psi_{H,0} = 0$. Therefore,

$$\frac{\partial V_{H,0}}{\partial R_{F,0}} = 0.$$

A prudential monetary intervention abroad does not affect welfare away from the zero lower bound when the government uses capital controls. □

C.4 Proof of Lemma 2

Proof. To determine the effects of a positive tax on capital flows in foreign on the equilibrium real interest rate, we start by showing the effect of a temporary change in $\tau_{i,0}$ and R_0^* on bond holdings. For convenience, we also consider here the effect a temporary change in $R_{i,0}$ which we use in appendix C.2. For any sequence of the nominal exchange rate, the policy function for bond holdings is given by

$$\frac{b_{i,1}^*}{R_0^*} = \mathcal{P}_{i,0}Y_{i,0} - \left[\sum_{t=0}^{\infty} \beta^{t\sigma} q_{i,t|0}^{1-\sigma}\right]^{-1} \sum_{t=0}^{\infty} q_{i,t|0}\mathcal{P}_{i,0}Y_{i,t} \quad \text{for } i \in \{H,F\} \tag{C.13}$$

where $q_{i,t|0} = (\mathcal{P}_{i,t}/\mathcal{P}_{i,0}) \prod_{s=0}^{t-1} \left(R_s^*(1+\tau_{i,s})\right)^{-1}$. Totally differentiating (C.13) yields

$$db_{i,1}^* = \frac{b_{i,1}^*}{R_0^*} dR_0^* + \sum_{t=0}^{\infty} \frac{\partial \mathcal{B}_i^*}{\partial e_{i,t}} de_{i,t} + \sum_{t=0}^{\infty} \frac{\partial \mathcal{B}_i^*}{\partial q_{i,t|0}} dq_{i,t|0} + \sum_{t=0}^{\infty} \frac{\partial \mathcal{B}_i^*}{\partial Y_{i,t}} \left[\frac{\partial Y_{i,t}}{de_{i,t}} de_{i,t} + \frac{\partial Y_{i,t}}{dy_{i,t}^N} dy_{i,t}^N\right] \tag{C.14}$$

Using (A.36) and starting from $b_{i,1}^* = 0$, we have

$$\begin{aligned}\sum_{t=0}^{\infty} \frac{\partial \mathcal{B}_i^*}{\partial e_{i,t}} de_{i,t} + \sum_{t=0}^{\infty} \frac{\partial \mathcal{B}_i^*}{\partial Y_{i,t}} \left[\frac{\partial Y_{i,t}}{de_{i,t}} de_{i,t}\right] &= b_1^* \mu_{i,0} \left[(1-\tilde{\omega}_i)\frac{de_{i,1}}{e_{i,1}} - \mu_{i,0}(1-\tilde{\omega}_{i,0})\frac{de_{i,0}}{e_{i,0}}\right] \\ &= 0\end{aligned} \tag{C.15}$$

Next, use (A.26) and the change in $q_{t|0}$ (evaluated at the initial equilibrium) $d\log q_{0|0} = 0$, and for $t \geq 1$ we have

$$\begin{aligned} d\log q_{i,t|0} &= -\sum_{k=0}^{t-1} \left[d\tau_{i,k} + \tilde{\omega}_i d\log R_k^* + (1-\tilde{\omega}_i) d\log R_{i,k}\right], \\ &= -d\tau_{i,0} - \tilde{\omega}_i d\log R_0^* - (1-\tilde{\omega}_i) d\log R_{i,0}, \end{aligned} \tag{C.16}$$

where the second equality follows from the fact that the changes considered are temporary, to arrive to

$$\begin{aligned}\sum_{t=0}^{\infty} \frac{\partial \mathcal{B}_i^*}{\partial q_{i,t|0}} dq_{i,t|0} &= R_0^* \left[\sigma(1-\mu_{i,0})\mathcal{P}_{i,0}c_{i,0} - \mu_{i,0}\frac{b_{i,1}^*}{R_0^*}\right] \left[d\tau_{i,0} + \tilde{\omega}\frac{dR_0^*}{R_0^*} + (1-\tilde{\omega})\frac{dR_{i,0}}{R_{i,0}}\right] \\ &= R^*(1-\mu_i)\mathcal{P}_i c_i \left[\sigma d\tau_{i,0} + \sigma\tilde{\omega}_i \frac{dR_0^*}{R_0^*} + \sigma(1-\tilde{\omega}_i)\frac{dR_{i,0}}{R_{i,0}}\right]\end{aligned} \tag{C.17}$$

starting from the initial equilibrium with $b^*_{i,1} = 0$. Next, combine the Euler equation (A.7) and (A.38) to get

$$y^N_{i,t} = \left[\beta R^*_t (1+\tau_{i,t}) \frac{\mathcal{P}_{i,t}}{\mathcal{P}_{i,t+1}}\right]^{-\sigma} \left[\frac{P^{T*}_t e_{i,t}}{P^{T*}_{t+1} e_{i,t+1}} \frac{\mathcal{P}_{i,t}}{\mathcal{P}_{i,t+1}}\right]^{-\gamma} y^N_{i,t+1}$$

This is the small open economy version of the New Keynesian dynamic IS curve. Differentiating this equation yields

$$d\log(y^N_{i,t}/y^N_{i,t+1}) = -\sigma\left[d\tau_{i,t} + \tilde{\omega}_i d\log R^*_t + (1-\tilde{\omega}_i) d\log R_{i,t}\right] + \gamma\tilde{\omega}_i(d\log R^*_t - d\log R_{i,t})$$

since we consider only temporary changes, it follows from (C.18) that

$$d\log y^N_{0,t} - d\log y^N_{i,t} = -\sigma d\tau_{i,0} - (\sigma-\gamma)\tilde{\omega}_i d\log R^*_0 - [\sigma(1-\tilde{\omega}_i) + \gamma\tilde{\omega}_i] d\log R_{i,0} \quad \text{(C.18)}$$

for any $t \geq 1$. Use now (A.31) to get

$$\begin{aligned}
\Delta_y &\equiv \sum_{t=0}^{\infty} \frac{\partial \mathcal{B}^*}{\partial Y_{i,t}} \left[\frac{\partial Y_{i,t}}{\partial y^N_{i,t}} dy^N_{i,t}\right] \\
&= R^*_0(1-\mu_{i,0})(1-\tilde{\omega}_{i,0})\mathcal{P}_{i,0} c_{i,0} \frac{dy^N_{i,0}}{y^N_{i,0}} - R^*_0 \mu_{i,0} \sum_{t=1}^{\infty} q_{i,t|0} \mathcal{P}_{i,0} c_{i,t} (1-\tilde{\omega}_{i,t}) \frac{dy^N_{i,t}}{y^N_{i,t}} \\
&= R^*_0(1-\mu_{i,0})(1-\tilde{\omega}_{i,0})\mathcal{P}_{i,0} c_{i,0} \frac{dy^N_{i,0}}{y^N_{i,0}} - R^*_0 \mu_{i,0} \sum_{t=1}^{\infty} \beta^{t\sigma} q^{1-\sigma}_{i,t|0} \mathcal{P}_{i,0} c_{i,0} (1-\tilde{\omega}_{i,t}) \frac{dy^N_t}{y^N_t}
\end{aligned}$$

where the second equality uses (A.7). Note that $1-\mu_{i,0} = \mu_{i,0}\sum_{t=1}^{\infty} \beta^{t\sigma} q^{1-\sigma}_{i,t|0}$. Thus, starting from the initial equilibrium and substituting (C.18) into the previous equation yields

$$\Delta_y = -R^*(1-\mu_i)\mathcal{P}_i c_i (1-\tilde{\omega}_i)\left[\sigma d\tau_{i,0} + \tilde{\omega}_i(\sigma-\gamma)\frac{dR^*_0}{R^*_0} + ((1-\tilde{\omega}_i)\sigma + \tilde{\omega}_i\gamma)\frac{dR_{i,0}}{R_{i,0}}\right] \quad \text{(C.19)}$$

Finally, substituting (C.15), (C.17), (C.19) into (C.14) and using $c^T_i = \tilde{\omega}_i \mathcal{P}_i c_i$, we arrive to

$$db^*_{i,1} = R^*(1-\mu_i)c^T_i \left[\sigma d\tau_{i,0} + (\sigma\tilde{\omega}_i + \gamma(1-\tilde{\omega}_i))\frac{dR^*_0}{R^*_0} + (\sigma-\gamma)(1-\tilde{\omega}_i)\frac{dR_{i,0}}{R_{i,0}}\right] \quad \text{(C.20)}$$

We now turn to deriving the effects of a positive tax on capital flows in foreign $\tau_{F,0} > 0$ on the equilibrium real interest rate R^*_0. In addition to its direct effect on aggregate savings, a positive tax on inflows by the foreign block at time 0 has an indirect effect on aggregate savings in both the home and the foreign blocks through the change in the equilibrium world interest rate. From (C.20), we have

$$db^*_{F,1} = R^*(1-\mu_F)c^T_F \left[\sigma + (\sigma\tilde{\omega}_F + \gamma(1-\tilde{\omega}_F))\frac{\partial \log R^*_0}{\partial \tau_{F,0}}\right] d\tau_{F,0}$$
$$db^*_{H,1} = R^*(1-\mu_H)c^T_H \left[(\sigma\tilde{\omega}_H + \gamma(1-\tilde{\omega}_H))\frac{\partial \log R^*_0}{\partial \tau_{F,0}}\right] d\tau_{F,0}$$

and using the market clearing condition for bond, $n\, db^*_{H,1} + (1-n)db^*_{F,1} = 0$, we obtain

$$d\log R^*_0 = -\frac{(1-n)(1-\mu_F)c^T_F\,\sigma}{\sum_{i\in\{H,F\}}(1-\mu_i)c^T_i(\sigma\tilde{\omega}_i + \gamma(1-\tilde{\omega}_i))} d\tau_{F,0} < 0$$

Thus, a positive tax on inflows by the foreign block at time 0 leads to a decline in the world real interest rate at time 0. It is useful to note that in the special case where $n = 0$, that is the domestic block is a small open economy (SOE), and preferences are separable $\sigma = \gamma$, we have $d\log R^*_0 = -d\tau_{F,0}$. That is, the world interest rate varies one-to-one with the tax on inflows by the foreign block. □

C.5 Proof of Proposition 8

In this section we compare the welfare of a generic small open economy (SOE) under a laissez-faire regime without policy intervention (i.e., no capital controls at home or abroad) versus the welfare under an uncoordinated capital controls (i.e., the SOE faces a real interest rate different from the one under laissez-faire because other countries are using capital controls, and the SOE can impose a tax τ on capital flows).

$$V(b^*) = \max_{b^{*\prime},e,c^N,c^T} u\left[y^T + b^* - \frac{b^{*\prime}}{R^*}, c^N\right] - v\left[(c^N)^{1/\alpha}\right] + \beta\frac{z'}{z}V(b^{*\prime}) \tag{C.21}$$

subject to

$$c^N = \left[\frac{1-\omega}{\omega}\frac{P^{T*}}{\bar{P}^N}e\right]^{\gamma}\left(y^T + b^* - \frac{b^{*\prime}}{R^*}\right) \tag{C.22}$$

$$u_T(c^T, c^N) = \beta R^*\frac{z'}{z}\left[u_T\left(\mathcal{C}^T(b^{*\prime}), \mathcal{C}^N(b^{*\prime})\right)\right] \tag{C.23}$$

$$R^* \geq \frac{e}{\mathcal{E}(b^{*\prime})}\frac{P^{T*}}{P^{T*\prime}} \tag{C.24}$$

We start from a symmetric equilibrium with zero net positions $b^* = b'^*_{LF} = 0$. The welfare of in the SOE under laissez-faire is given by

$$V_{LF}\left(b^*,\{R^*_{LF}\}\right) = u\left[y^T + b^* - \frac{b'^*_{LF}}{R^*_{LF}}, c^N_{LF}\right] - v\left[(c^N_{LF})^{1/\alpha}\right] + \beta\frac{z'}{z}V_{LF}\left(b'^*_{LF},\{R^*\}\right)$$
$$= u\left(y^T, c^N_{LF}\right) - v\left[(c^N_{LF})^{1/\alpha}\right] + \beta\frac{z'}{z}V_{LF}\left(b'^*_{LF},\{R^*\}\right) \tag{C.25}$$

Consider now that the SOE faces a real interest rate R_τ because other countries are using capital controls. We denote by $V_\tau(b^*,\{R^*_\tau\})$ the welfare of households in the SOE under capital controls.

Away from a liquidity trap. When the ZLB does not bind in the SOE, the effects of a change in R on households' welfare is given by

$$\frac{\partial V}{\partial R^*} = \frac{u_T(c^T,c^N)}{R^*}\frac{\sigma c^T}{(1-\tilde{\omega})\sigma + \tilde{\omega}\gamma}\mathbb{E}_t\sum_{k=1}^{\infty}\beta^k\frac{z_k}{z}\left(\prod_{\ell=0}^{k-1}\frac{R^*_\ell}{1+\bar{\Theta}_\ell}\right)\frac{\xi_k}{\gamma c^T_k} \geq 0 \tag{C.26}$$

where $\xi \geq 0$ is the non-negative lagrange multiplier on the ZLB constraint (C.24). Thus, when capital inflows are subsidized abroad $\tau^* \leq 0$ leading to an increase in the real rate $R^*_\tau \geq R^*_{LF}$, it is straightforward to see that the SOE is better-off in the uncoordinated capital control regime as $V_\tau\left(b^*,\{R^*_\tau\}\right) \geq V_{\tau=0}\left(b^*,\{R^*_\tau\}\right) \geq V_{LF}\left(b^*,\{R^*_{LF}\}\right)$. Let's now turn to analyzing the case where $\tau^* > 0$ leading to $R^*_\tau < R^*_{LF}$. Start by setting capital controls in the SOE at $\tau_o > 0$ to close the capital account (that is, $b^*_{\tau_o} = b^*_{LF}$). Because with capital control the government closes the labor wedge, it follows that

$$u\left(y^T, c^N_{\tau_o}\right) - v\left[(c^N_{\tau_o})^{1/\alpha}\right] \geq u\left(y^T, c^N_{LF}\right) - v\left[(c^N_{LF})^{1/\alpha}\right]$$

which implies that

$$V_{\tau_o}\left(b^*,\{R^*_\tau\}\right) = u\left(y^T, c^N_{\tau_o}\right) - v\left[(c^N_{\tau_o})^{1/\alpha}\right] + \beta\frac{z'}{z}V_{LF}\left(b'^*_{LF},\{R^*\}\right)$$
$$\geq u\left(y^T, c^N_{LF}\right) - v\left[(c^N_{LF})^{1/\alpha}\right] + \beta\frac{z'}{z}V_{LF}\left(b'^*_{LF},\{R^*\}\right) = V_{LF}\left(b^*,\{R^*_{LF}\}\right)$$

there exists a tax rate $\tau_o > 0$ under capital controls such that $V_{\tau_o}\left(b^*,\{R^*_\tau\}\right) \geq V_{LF}\left(b^*,\{R^*_{LF}\}\right)$. Hence, with optimal capital control, the SOE is better-off in a capital control regime.

In a liquidity trap. We now compare the welfare of households under laissez faire versus capital controls when SOE is in a temporary liquidity trap. We first consider the case where $\tau^* \leq 0$ leading to $R^*_\tau \geq R^*_{LF}$. Start by setting capital controls in the SOE at $\tau_o > 0$ to close the capital account (that is, $b^*_{\tau_o} = b^*_{LF}$). Using (C.24) to substitute for the nominal exchange rate in (C.22), we get

$$c^N_{\tau_o} = \left[\frac{1-\omega}{\omega}\frac{P^{T*\prime}}{\bar{P}^N}\mathcal{E}(b^{*\prime}_{LF})\, R^*_\tau\right]^\gamma y^T$$
$$\geq \left[\frac{1-\omega}{\omega}\frac{P^{T*\prime}}{\bar{P}^N}\mathcal{E}(b^{*\prime}_{LF})\, R^*_{LF}\right]^\gamma y^T = c^N_{LF} \tag{C.27}$$

Since the SOE is in a recession at the ZLB (i.e., the labor wedge is positive $\psi > 0$), it follows that

$$u\left(y^T, c^N_{\tau_o}\right) - v\left[(c^N_{\tau_o})^{1/\alpha}\right] \geq u\left(y^T, c^N_{LF}\right) - v\left[(c^N_{LF})^{1/\alpha}\right]$$

Using this inequality, we obtain

$$V_{\tau_o}\left(b^*, \{R^*_\tau\}\right) = u\left(y^T, c^N_{\tau_o}\right) - v\left[(c^N_{\tau_o})^{1/\alpha}\right] + \beta\frac{z'}{z}V_{LF}\left(b'^*_{LF}, \{R^*\}\right)$$
$$\geq u\left(y^T, c^N_{LF}\right) - v\left[(c^N_{LF})^{1/\alpha}\right] + \beta\frac{z'}{z}V_{LF}\left(b'^*_{LF}, \{R^*\}\right) = V_{LF}\left(b^*, \{R^*_{LF}\}\right)$$

Therefore, with optimal capital control, the SOE is better-off in a capital control regime.

Second, we consider the case where $\tau^* > 0$ leading to $R^*_\tau < R^*_{LF}$. The effects of the decline in R^* on households' welfare is given by

$$\frac{\partial V}{\partial R^*} = \frac{\tilde{\omega}\left(\gamma - \sigma\right)}{\left(1-\tilde{\omega}\right)\sigma + \tilde{\omega}\gamma}\frac{\xi}{R^*} \tag{C.28}$$

Hence, for $\sigma \geq \gamma$ we have $\partial V / \partial R^* < 0$. This implies that $V_\tau\left(b^*, \{R^*_\tau\}\right) > V_{LF}\left(b^*, \{R^*_\tau\}\right) \geq V_{LF}\left(b^*, \{R^*_{LF}\}\right)$. The welfare of households in the SOE is higher in the uncoordinated capital controls regime compared to the laissez-faire regime for $\sigma \geq \gamma$.

When $\sigma < \gamma$, however, the result is ambiguous. On one hand $V_{LF}\left(b^*, \{R^*_\tau\}\right) < V_{LF}\left(b^*, \{R^*_{LF}\}\right)$ since $\partial V / \partial R^* > 0$ and one the other hand $V_\tau\left(b^*, \{R^*_\tau\}\right) > V_{LF}\left(b^*, \{R^*_\tau\}\right)$. Note here that setting a positive tax rate on debt issuance $\tau_o > 0$ to close the capital account (that is, $b^*_{\tau_o} = b^*_{LF}$) would not make the SOE weakly better-off under uncoordinated capital control since the lower real rate would imply that $c^N_{\tau_o} < c^N_{LF}$ by (C.24).

C.6 Proof of Proposition 9

The global planner maximizes the welfare of households in the home country subject to the implementability and participation constraints. It solves

$$V_H\left(b^*_{H,0}\right) = \max_{\{c^N_{k,0},c^T_{k,0},P^T_{k,0},b^*_{k,1},R^*_0\}} \left[u\left(c^T_{H,0},c^N_{H,0}\right) - v\left(\left(c^N_{H,0}\right)^{1/\alpha}\right)\right] + \beta\frac{z_{H,1}}{z_{H,0}}V_H\left(b^*_{H,1}\right) \quad \text{(C.29)}$$

subject to

$$c^T_{k,0} = y^T_{k,0} + b^*_{k,0} - \frac{b^*_{k,1}}{R^*_0}, \qquad \forall k \in \{H,F\} \quad \text{(C.30)}$$

$$c^N_{k,0} = \left[\frac{1-\omega}{\omega}\frac{P^T_{k,0}}{\bar{P}^N}\right]^{\gamma} c^T_{k,0}, \qquad \forall k \in \{H,F\} \quad \text{(C.31)}$$

$$R^*_0 \geq \frac{P^T_{k,0}}{\mathcal{P}^T_{k,1}(b^*_{k,1})}, \qquad \forall k \in \{H,F\} \quad \text{(C.32)}$$

$$\bar{V}_F\left(b^*_{F,0}\right) \leq u\left(c^T_{F,0},c^N_{F,0}\right) - v\left(\left(c^N_{F,0}\right)^{1/\alpha}\right) + \beta V_F\left(b^*_{F,1}\right) \quad \text{(C.33)}$$

$$0 = \int_0^n b^*_H dH + \int_n^1 b^*_F dF \quad \text{(C.34)}$$

(C.33) ensures that Foreign does want to cooperate. We denote by $q_{F,0} \geq 0$ the non-negative lagrange multiplier on the participation constraint (C.33). $\lambda_{k,0}, \vartheta_{k,0}, \xi_{k,0} \geq 0$ represents the multipliers on (C.30), (C.31) and (C.32). Notice that this formulation of the global planner problem abstract from the dynamic implementability constraints

$$u_T\left(c^T_{k,0},c^N_{k,0}\right) = \beta R^*_0(1+\tau_{k,0})\frac{z_{k,1}}{z_{k,0}}\left[u_T\left(c^T_{k,1},c^N_{k,1}\right)\right] \quad \text{(C.35)}$$

which is used to back out the optimal tax rates on debt, $\tau_{k,0}$, for a given allocation.

We first proceed by showing that, when the ZLB does not bind, the uncoordinated solution is a solution of the global planner problem. We use the optimality conditions for $P^T_{k,0}$, given by $\vartheta_{k,0} = \xi_{k,0}/\gamma c^N_{k,0}$, to eliminate the multipliers $\vartheta_{k,0}$. The first order conditions with respect to $c^N_{k,0}$ and $c^T_{k,0}$ are

$$\left[c^N_{k,0}\right]: \quad q_k u_N\left(c^T_{k,0},c^N_{k,0}\right)\psi_{k,0} = \frac{\xi_{k,0}}{\gamma c^N_{k,0}} \quad \text{(C.36)}$$

$$\left[c^T_{k,0}\right]: \quad \lambda_{k,0} = q_k u_T\left(c^T_{k,0},c^N_{k,0}\right) + \frac{\xi_{k,0}}{\gamma c^T_{k,0}} \quad \text{(C.37)}$$

for $k \in \{i, j\}$, with $q_i = 1$. The first order conditions with respect to R_0^* is given by

$$\left[\lambda_{H,0} - \frac{n}{1-n}\lambda_{F,0}\right]\frac{b_{H,1}^*}{R_0^*} + \xi_{H,0} + \xi_{F,0} = 0 \tag{C.38}$$

The last optimality condition combines the first order conditions with respect to $b_{H,1}^*$ and $b_{F,1}^*$ and uses the envelope condition that is $V'(b_{k,0}) = \lambda_{k,0}$. We have

$$\lambda_{H,0} - \frac{n}{1-n}\lambda_{F,0} = \beta R_0^*\left[\frac{z_{H,1}}{z_{H,0}}\lambda_{H,1} - \frac{n}{1-n}\frac{z_{F,1}}{z_{F,0}}\lambda_{F,1}\right] + \frac{P_{H,1}^{T\prime}(b_{H,1}^*)}{P_{H,1}^T}\xi_{H,0} - \frac{n}{1-n}\frac{P_{F,1}^{T\prime}(b_{F,1}^*)}{P_{F,1}^T}\xi_{F,0} \tag{C.39}$$

Recall that, when the ZLB does not bind, the uncoordinated solution features

$$\psi_{k,0} = 0 \tag{C.40}$$

$$\tau_{k,0} = \frac{1}{u_T\left(c_{k,1}^T, c_{k,1}^N\right)}\frac{\xi_{k,1}}{\gamma c_{k,1}^T} \tag{C.41}$$

where (C.41) is (29) absent uncertainty. Let us show that (C.40) and (C.41) solves the first order conditions of the global planner. Using (C.40), it follows that (C.36) is satisfied. Condition (C.36) basically states that, as in the uncoordinated regime, monetary policy in the coordinated regime always closes the labor wedge away from the ZLB. Combining (C.37) and (C.39) we obtain

$$\begin{aligned} u_T(c_{H,0}^T, c_{H,0}^N) &- \frac{n}{1-n}u_T(c_{F,0}^T, c_{F,0}^N) \\ &= \beta R_0^*\left[\frac{z_{H,1}}{z_{H,0}}u_T(c_{H,1}^T, c_{H,1}^N) + \frac{\xi_{H,1}}{\gamma c_{k,1}^T} - \frac{n}{1-n}\frac{z_{F,1}}{z_{F,0}}\left(u_T(c_{F,1}^T, c_{F,1}^N) + \frac{\xi_{F,1}}{\gamma c_{F,1}^T}\right)\right] \end{aligned}$$

we then use (C.41) to substitute for $\xi_{k,1}$ and rearrange the expression to get

$$\begin{aligned} u_T\left(c_{H,0}^T, c_{H,0}^N\right) &- \beta R_0^*(1+\tau_{H,0})\frac{z_{H,1}}{z_{H,0}}u_T\left(c_{H,1}^T, c_{H,1}^N\right) \\ &= \frac{n}{1-n}\left[u_T(c_{F,0}^T, c_{F,0}^N) - \beta R_0^*(1+\tau_{F,0})\frac{z_{F,1}}{z_{F,0}}u_T\left(c_{F,1}^T, c_{F,1}^N\right)\right] \end{aligned}$$

From (C.35) this equality is satisfied as both sides of the equation are equal to zero. Thus, the uncoordinated solution is also a solution of the global planner problem. Moreover, (C.38) pins down the relative Pareto weight which is given by the ratio of the inverse of the marginal utilities of tradables,

$$q_{F,0} = \frac{(1-n)/u_T(c_{F,0}^T, c_{F,0}^N)}{n/u_T(c_{H,0}^T, c_{H,0}^N)}.$$

We now turn showing that the coordinated solution and the uncoordinated solution do not coincide when at least one block of countries is at the ZLB. In particular, starting from zero net asset positions the global planner always sets capital controls to relax the ZLB. To clearly see it and why this allocation makes all countries better off (and strictly better off for the country is at the ZLB) compared to their outcome in the uncoordinated regime, consider the problem of the global planner (C.29). For convenience, substitute (C.32) and (C.34) to rewrite the problem as follow

$$V_i\left(-\frac{1-n}{n}b^*_{F,0}\right) = \max_{R^*_0, c^N_{H,0}, c^N_{F,0}, c^T_{F,0}} u\left[y^T_{H,0} + \frac{1-n}{n}\left(y^T_{F,0} - c^T_{F,0}\right), c^N_{H,0}\right] - v\left((c^N_{H,0})^{1/\alpha}\right)$$
$$+ \beta\frac{z_{H,1}}{z_{H,0}} V_i\left(\frac{1-n}{n}R^*_0\left[c^T_{F,0} - y^T_{F,0} - b^*_{F,0}\right]\right)$$

subject to

$$c^N_{H,0} \leq \left[\frac{1-\omega}{\omega}\frac{\mathcal{P}^T_{H,1}}{\bar{P}^N}R^*_0\right]^\gamma \left(y^T_{H,0} + \frac{1-n}{n}\left(y^T_{F,0} - c^T_{F,0}\right)\right) \tag{C.42}$$

$$c^N_{F,0} \leq \left[\frac{1-\omega}{\omega}\frac{\mathcal{P}^T_{F,1}}{\bar{P}^N}R^*_0\right]^\gamma c^T_{F,0} \tag{C.43}$$

$$\bar{V}_j\left(b^*_{F,0}\right) \leq u\left(c^T_{F,0}, c^N_{F,0}\right) - v\left((c^N_{F,0})^{1/\alpha}\right) + \beta\frac{z_{F,1}}{z_{F,0}}V_F\left(R^*_0\left[y^T_{F,0} + b^*_{F,0} - c^T_{F,0}\right]\right)$$

Consider an initial uncoordinated equilibrium with zero net positions, that is $b^*_{H,1} = b^*_{F,1} = 0$, and at least one block of countries is at the ZLB. Without loss of generality we assume that the ZLB binds at home, that is (C.42) is satisfied with equality. This equilibrium is represented by countries' net asset position $\{\hat{b}^*_{H,1}, \hat{b}^*_{F,1}\}$, the consumption allocation $\{\hat{c}^N_{H,0}, \hat{c}^N_{F,0}\}$, the equilibrium world interest rate $\{\hat{R}^*_0\}$ and the implied macroprudential policies $\{\hat{\tau}_{H,0}, \hat{\tau}_{F,0}\}$.

We argue that in the coordinated solution, the global planner fully relaxes the ZLB constraint.[31] To understand why, consider an decline in $\tau_{H,0}$ and $\tau_{F,0}$ that keeps the capital account of each country closed:

$$\tau_{H,0} < \hat{\tau}_{H,0} \quad \text{and} \quad \tau_{F,0} < \hat{\tau}_{F,0} \tag{C.44}$$
$$c^T_{H,0} = \hat{c}^T_{H,0} \quad \text{and} \quad c^T_{F,0} = \hat{c}^T_{F,0} \tag{C.45}$$

The increased subsidy on borrowing in each country lowers the demand for international bonds which in turn pushes an upward pressure on the equilibrium world interest rate

$$R^*_0 > \hat{R}^*_0 \tag{C.46}$$

[31] This can also be seen from the first order condition with respect to R^*_0 which yields $\gamma\xi_{H,0} + \gamma\xi_{F,0} = 0$ where $\xi_{H,0} \geq 0$ and $\xi_{F,0} \geq 0$ are the multipliers on (C.42) and (C.43).

Since $b_{k,0} = \hat{b}_{k,0}$ for $k \in \{i, j\}$, the continuation values and future prices remain unchanged $V_k(b^*_{k,1}) = \hat{V}_k(\hat{b}^*_{k,1})$ and $\mathcal{P}^T_k(b^*_{k,1}) = \hat{\mathcal{P}}^T_k(\hat{b}^*_{k,1})$. Thus, from (C.45) and (C.46) we have

$$\begin{aligned} c^N_{H,0} &= \left[\frac{1-\omega}{\omega}\frac{\hat{\mathcal{P}}^T_{H,1}}{\bar{P}^N}R^*_0\right]^\gamma \left[y^T_{H,0} + \frac{1-n}{n}\left(y^T_{F,0} - \hat{c}^T_{F,0}\right)\right] \\ &> \left[\frac{1-\omega}{\omega}\frac{\hat{\mathcal{P}}^T_{H,1}}{\bar{P}^N}\hat{R}^*_0\right]^\gamma \left[y^T_{H,0} + \frac{1-n}{n}\left(y^T_{F,0} - \hat{c}^T_{F,0}\right)\right] = \hat{c}^N_{H,0} \end{aligned} \tag{C.47}$$

This in turn implies that

$$\begin{aligned} V_H\left(b^*_{H,0}\right) &= u\left(\hat{c}^T_{H,0}, c^N_{H,0}\right) - v\left[(c^N_{H,0})^{1/\alpha}\right] + \beta\frac{z_{H,1}}{z_{H,0}}\hat{V}_i\left(\hat{b}^*_{H,1}\right) \\ &> u\left(\hat{c}^T_{H,0}, \hat{c}^N_{H,0}\right) - v\left[(\hat{c}^N_{H,0})^{1/\alpha}\right] + \beta\frac{z_{H,1}}{z_{H,0}}\hat{V}_i\left(b'^*_{H,1}\right) = \hat{V}_H\left(b^*_{H,0}\right) \end{aligned}$$

where the second equality uses (C.47) and $\psi_{H,0} > 0$. Hence, Home is strictly better-off. If Foreign is also at the ZLB, the same analysis applies and we have $V_F(b^*_{F,0}) > \hat{V}_F(\hat{b}^*_{F,0})$. Because the ZLB does not bind in Foreign, ffrom the first order condition with respect to $c^N_{F,0}$, the labor wedge is closed $\psi_{F,0} = 0$ and we have

$$\begin{aligned} \psi_{F,0} = 0 \quad &\Rightarrow \quad \frac{1}{\alpha}(c^N_{H,0})^{1/\alpha-1}v'\left((c^N_{H,0})^{1/\alpha}\right) = u_N\left(c^T_{F,0}, c^N_{F,0}\right) \\ &\Rightarrow \quad \frac{1}{\alpha}(c^N_{H,0})^{\frac{1}{\alpha}-1}v'\left((c^N_{H,0})^{1/\alpha}\right) = u_N\left(\hat{c}^T_{F,0}, c^N_{F,0}\right) \\ &\Rightarrow \quad c^N_{F,0} = \hat{c}^N_{F,0} \end{aligned}$$

hence, $V_F(b^*_{F,0}) = \hat{V}_F(\hat{b}^*_{F,0})$. When the ZLB binds in at least one block of countries, by reducing the tax rate on debt in all countries in a way that keeps capital accounts closed, the global planner raises the world interest rate, makes countries at the ZLB stricly better off without making countries outside of a liquidity trap worse off. Therefore, the ZLB never binds in the coordinated solution.