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Online Appendix for: Mussa Meets Backus-Smith: The Role of Primary Commodities

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Supplement to "Mussa meets Backus-Smith: The Role of Primary Commodities"

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A Appendix: Data

A.1 Data description

Input-output table Data for the input-output tables of the United States and Japan come from the 2005 Japan-US Input-Output Table published by the Ministry of Economy, Trade, and Industry (METI) of Japan. We map each of the 174 sectors into five sectors: final good, intermediate good, primary commodity 1 (energy), primary commodity 2 (agriculture), and primary commodity 3 (metals and minerals). When constructing the mapping, we took into account the fraction of the production in each sector that is used for final consumption and also the production structure in our model. That is, final goods use intermediate goods as inputs and intermediate goods use primary commodities as inputs. The exact mapping with sector codes is the following:

Final goods: 022, 027, 030, 038, 059, 065, 091–092, 107, 109, 113, 117–118, 132–137, 147, 149–150, 152–154, 160–161, 167–171.

Intermediate goods: 018–021, 023–026, 028–029, 033–037, 042, 044–058, 060–064, 066–074, 078–090, 093–106, 108, 110–112, 114–116, 119–131, 138–146, 148, 151, 155–159, 162–166, 172–174.

Primary commodity 1 (energy): 016–017.

Primary commodity 2 (agriculture): 001–012, 015, 031–032, 039–041, 043.

Primary commodity 3 (metals and minerals): 013–014, 075–077.

Trade data Trade data were obtained from the United Nations Comtrade Database. We use total exports by primary commodity group in 2005 to compute their respective shares in world trade. Using the SITC Revision 3, the exact mapping from primary commodities into the three primary commodity groups is the following:

Primary commodity 1 (energy): 3.

Primary commodity 2 (agriculture): 0, 2, 4, 12.

Primary commodity 3 (metals and minerals): 27–28, 67–68.

Sectoral data for the rest of the world Data are from the 10-Sector Database from the Groningen Growth and Development Centre. We group the sectors into three groups: final good, intermediate good, and primary commodities. The exact mapping is the following:

Primary commodities: agriculture and mining.

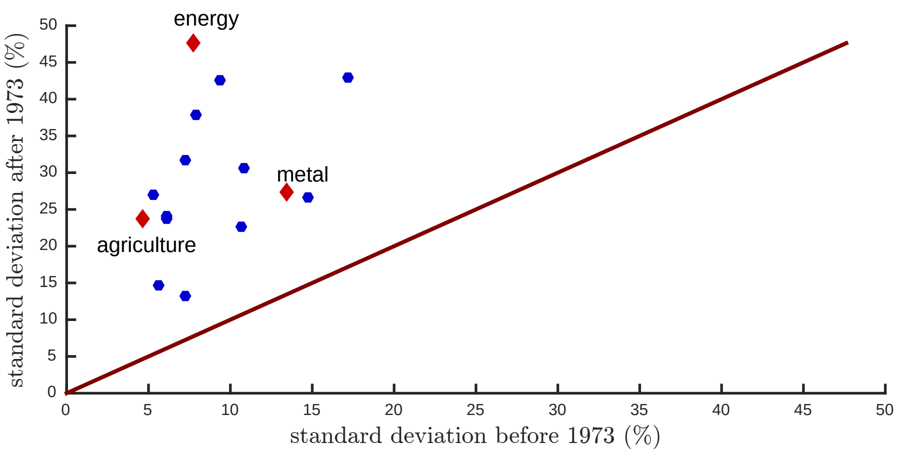
Intermediate good: manufacturing and construction

Final good: utilities, trade services, transport services, business services, government services, and personal services.

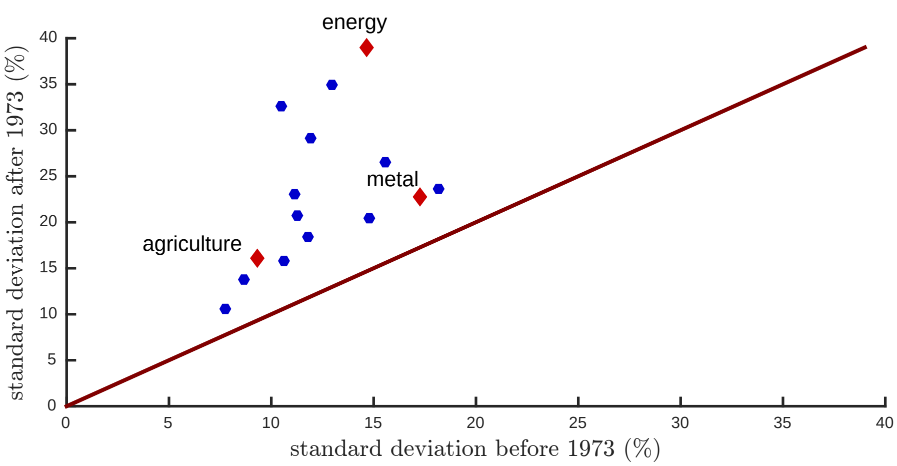
A.2 Volatility of commodity prices before and after 1973

Figure 1: Standard deviation of primary commodity prices before and after 1973

(a) Prices expressed in Deutsche marks (euros after 2000) and normalized by German CPI



(b) Prices normalized by the price of wheat



B Appendix: Model

B.1 Log-linearized equilibrium equations

This appendix presents the (log-linearized) equilibrium equations of the model described in Section 3 in the main text. We use the log-linearized version of the model to compute the model simulations based on the method of undetermined coefficients discussed in **?**. Variables without time subscript denote steady-state values, and $\widetilde{X}$ denote the log-deviation of variable X from its steady-state level.

We begin by describing the equilibrium equations related to the household problem in Country $i \in \{1,2,3\}$. The household inelastically supplies the endowment of labor and natural resources, and the optimal choices of consumption and bond holdings are characterized by the following equations:

$$P^{c_i}C^i\widetilde{P^{c_i}_t} + P^{c_i}C^i\widetilde{C^i_t} + P^b\widetilde{B^i_{t+1}} = W^i\overline{N}^i\widetilde{W^i_t} + P^{e^i_1}e^i_1\widetilde{P^{e^i_1}_t} + P^{e^i_2}e^i_2\widetilde{P^{e^i_2}_t} + P^{e^i_3}e^i_3\widetilde{P^{e^i_3}_t} + P^{x_3}\widetilde{B^i_t}, \quad (1)$$

$$\widetilde{P^b_t} + \frac{P^{c_i}}{P^{x_3}}\frac{\kappa}{\beta}\widetilde{B^i_{t+1}} = \gamma\widetilde{C^i_t} + \widetilde{P^{c_i}_t} - \gamma\mathbb{E}_t\widetilde{C^i_{t+1}} - \mathbb{E}_t\widetilde{P^{c_i}_{t+1}}. \quad (2)$$

Equation (1) is simply the budget constraint, in which P^b denotes the price of the uncontingent bond B that pays in units of commodity X_3. Equation (2) is the standard Euler equation, in which $\mathbb{E}[\cdot]$ is the expectation operator and the parameter $\kappa > 0$ determines the cost of moving bond holdings away from their steady-state level (assumed to be zero).[1] The latter is expressed in units of the final good in each country.

Next, we move to the final-good sector in Country $i \in \{1,2,3\}$. The equilibrium conditions are represented by the feasibility constraint and the optimality conditions for

[1] We set $\kappa = 1.0e^{-5}$. It is a device to make bond holdings stationary.

the choice of inputs:

$$
\begin{aligned}
\widetilde{C_t^i} &= \widetilde{Z_t^i} + \alpha_1^i \widetilde{q_{1,t}^i} + \alpha_2^i \widetilde{q_{2,t}^i} + \alpha_3^i \widetilde{q_{3,t}^i} + \alpha_4^i \widetilde{n_{c,t}^i}, && (3) \\
\widetilde{P_t^{q_1}} + \widetilde{q_{1,t}^i} &= \widetilde{P_t^{c_i}} + \widetilde{C_t^i}, && (4) \\
\widetilde{P_t^{q_2}} + \widetilde{q_{2,t}^i} &= \widetilde{P_t^{c_i}} + \widetilde{C_t^i}, && (5) \\
\widetilde{P_t^{q_3}} + \widetilde{q_{3,t}^i} &= \widetilde{P_t^{c_i}} + \widetilde{C_t^i}, && (6) \\
\widetilde{W_t^i} + \widetilde{n_{c,t}^i} &= \widetilde{P_t^{c_i}} + \widetilde{C_t^i}. && (7)
\end{aligned}
$$

The assumption of a Cobb-Douglas production function implies that input costs are a fixed proportion of total revenues. This means that their log-deviations from steady-state must be the same, as equations (4)–(7) show.

The same applies to the intermediate-good sector:

$$
\begin{aligned}
\widetilde{Q_t^i} &= \widetilde{Z_t^i} + \beta_1^i \widetilde{x_{1,t}^i} + \beta_2^i \widetilde{x_{2,t}^i} + \beta_3^i \widetilde{x_{3,t}^i} + \beta_4^i \widetilde{n_{q,t}^i}, && (8) \\
\widetilde{P_t^{x_1}} + \widetilde{x_{1,t}^i} &= \widetilde{P_t^{q_i}} + \widetilde{Q_t^i}, && (9) \\
\widetilde{P_t^{x_2}} + \widetilde{x_{2,t}^i} &= \widetilde{P_t^{q_i}} + \widetilde{Q_t^i}, && (10) \\
\widetilde{P_t^{x_3}} + \widetilde{x_{3,t}^i} &= \widetilde{P_t^{q_i}} + \widetilde{Q_t^i}, && (11) \\
\widetilde{W_t^i} + \widetilde{n_{q,t}^i} &= \widetilde{P_t^{q_i}} + \widetilde{Q_t^i}. && (12)
\end{aligned}
$$

We assume a CES production function in the primary commodity sectors $j = 1, 2, 3$, so the proportions of input costs are allowed to vary. The equilibrium equations in the primary-commodity sector X^i_j in Country $i \in \{1,2\}$ are:

$$
\begin{aligned}
\widetilde{X^i_{j,t}} &= \widetilde{Z^i_t} + \left(\phi^i_j\right)^{\frac{1}{\sigma^i_{x_j}}} \left(\frac{X^i_j}{Z^i n^i_{x_j}}\right)^{\frac{1-\sigma^i_{x_j}}{\sigma^i_{x_j}}} \widetilde{n^i_{x_{j,t}}}, && (13)\\
\widetilde{P^{e^i_j}_t} &= \widetilde{P^{x_j}_t} + \frac{\sigma^i_{x_j}-1}{\sigma^i_{x_j}} \widetilde{Z^i_t} + \frac{1}{\sigma^i_{x_j}} \widetilde{X^i_{j,t}}, && (14)\\
\widetilde{W^i_t} + \frac{1}{\sigma^i_{x_j}} \widetilde{n^i_{x_{j,t}}} &= \widetilde{P^{x_j}_t} + \frac{\sigma^i_{x_j}-1}{\sigma^i_{x_j}} \widetilde{Z^i_t} + \frac{1}{\sigma^i_{x_j}} \widetilde{X^i_{j,t}}. && (15)
\end{aligned}
$$

We assume that labor cannot move across countries, only across sectors within each country. That implies the following market-clearing condition for labor in Country $i \in \{1,2\}$:

$$
0 = n^i_c \widetilde{n^i_{c,t}} + n^i_q \widetilde{n^i_{q,t}} + n^i_{x_1} \widetilde{n^i_{x_1,t}} + n^i_{x_2} \widetilde{n^i_{x_2,t}} + n^i_{x_3} \widetilde{n^i_{x_3,t}}. \qquad (16)
$$

The market clearing condition for labor in Country 3 is similar, with the exception that labor is not used in the production of primary commodities:

$$
0 = n^3_c \widetilde{n^3_{c,t}} + n^3_q \widetilde{n^3_{q,t}}. \qquad (17)
$$

Finally, the following market-clearing conditions must hold in equilibrium for the tradable goods and bond holdings:

$$
\begin{aligned}
q^1_1 \widetilde{q^1_{1,t}} + q^2_1 \widetilde{q^2_{1,t}} + q^3_{1,t} \widetilde{q^3_{1,t}} &= Q^1 \widetilde{Q^1_t}, && (18)\\
q^1_2 \widetilde{q^1_{2,t}} + q^2_2 \widetilde{q^2_{2,t}} + q^3_{2,t} \widetilde{q^3_{2,t}} &= Q^2 \widetilde{Q^2_t}, && (19)\\
q^1_3 \widetilde{q^1_{3,t}} + q^2_3 \widetilde{q^2_{3,t}} + q^3_{3,t} \widetilde{q^3_{3,t}} &= Q^3 \widetilde{Q^3_t}, && (20)\\
x^1_1 \widetilde{x^1_{1,t}} + x^2_1 \widetilde{x^2_{1,t}} + x^3_{1,t} \widetilde{x^3_{1,t}} &= X^1_1 \widetilde{X^1_{1,t}} + X^2_1 \widetilde{X^2_{1,t}} + X^3_1 \widetilde{X^3_{1,t}}, && (21)\\
x^1_2 \widetilde{x^1_{2,t}} + x^2_2 \widetilde{x^2_{2,t}} + x^3_{2,t} \widetilde{x^3_{2,t}} &= X^1_2 \widetilde{X^1_{2,t}} + X^2_2 \widetilde{X^2_{2,t}} + X^3_2 \widetilde{X^3_{2,t}}, && (22)\\
x^1_3 \widetilde{x^1_{3,t}} + x^2_3 \widetilde{x^2_{3,t}} + x^3_{3,t} \widetilde{x^3_{3,t}} &= X^1_3 \widetilde{X^1_{3,t}} + X^2_3 \widetilde{X^2_{3,t}} + X^3_3 \widetilde{X^3_{3,t}}, && (23)\\
\widetilde{B^1_t} + \widetilde{B^2_t} + \widetilde{B^3_t} = 0. & && (24)
\end{aligned}
$$

The equations above represent a system of 64 equations with 63 variables. Walras's Law implies that one equation is redundant, so we drop the budget constraint in Country 3 to compute the simulations. Note that we have three state variables, B^1, B^2, and B^3, and three expectation equations represented by the Euler equation (2). Finally, we need to describe the stochastic processes of the productivities in Country 1 and 2, $\widetilde{Z_t^1}$ and $\widetilde{Z_t^2}$, and endowments of primary commodities in Country 3, $\widetilde{X_{1,t}^3}$, $\widetilde{X_{2,t}^3}$, and $\widetilde{X_{1,t}^3}$. We assume the following (stationary) autoregressive processes:

$$\ln(Z_t^1) = (1-\rho^{z_1})\ln(Z^1) + \rho^{z_1}\ln(Z_{t-1}^1) + \varepsilon_t^{z_1},$$
$$\ln(Z_t^2) = (1-\rho^{z_2})\ln(Z^2) + \rho^{z_2}\ln(Z_{t-1}^2) + \varepsilon_t^{z_2},$$
$$\ln(X_{1,t}^3) = (1-\rho^{x_1^3})\ln(X_1^3) + \rho^{x_1^3}\ln(X_{1,t-1}^3) + \varepsilon_t^{x_1},$$
$$\ln(X_{2,t}^3) = (1-\rho^{x_2^3})\ln(X_2^3) + \rho^{x_2^3}\ln(X_{2,t-1}^3) + \varepsilon_t^{x_2},$$
$$\ln(X_{3,t}^3) = (1-\rho^{x_3^3})\ln(X_3^3) + \rho^{x_3^3}\ln(X_{3,t-1}^3) + \varepsilon_t^{x_3},$$

where the vector of innovations $\left[\varepsilon_t^{z_1}, \varepsilon_t^{z_2}, \varepsilon_t^{x_1}, \varepsilon_t^{x_2}, \varepsilon_t^{x_3}\right]$ is normally distributed with zero mean and arbitrary covariance matrix. Variables without time subscripts represent long-run means.

Complete markets The linear system characterizing the equilibrium in the economy under complete markets is similar to the one described above. The difference is that we can drop the budget constraints, and we replace the Euler equations by the following (perfect) risk-sharing conditions:

$$\gamma\widetilde{C_t^2} - \gamma\widetilde{C_t^1} = \widetilde{P_t^{c_1}} - \widetilde{P_t^{c_2}}, \tag{25}$$
$$\gamma\widetilde{C_t^3} - \gamma\widetilde{C_t^1} = \widetilde{P_t^{c_1}} - \widetilde{P_t^{c_3}}. \tag{26}$$

In this case, we have a system of 59 equations and 58 variables, without any endogenous state variables or expectation equations.

Financial autarky The linear system characterizing the equilibrium of the economy under financial autarky is also similar to the cases above. The difference from the complete markets economy is that we replace the (perfect) risk-sharing conditions for the following zero-trade-balance conditions for Country $i \in \{1, 2\}$:

$$\begin{aligned} 0 \; = \; & P^{q_1}\left(Q^1 - q_1^1\right)\widetilde{P_t^{q_1}} + P^{q_1}Q^1\widetilde{Q_t^1} - P^{q_1}q_1^1\widetilde{q_{1,t}^1} - P^{q_2}q_2^1\widetilde{P_t^{q_2}} - P^{q_2}q_2^1\widetilde{q_{2,t}^1} - P^{q_3}q_3^1\widetilde{P_t^{q_3}} \\ & - P^{q_3}q_3^1\widetilde{q_{3,t}^1} + P^{x_1}\left(X_1^1 - x_1^1\right)\widetilde{P_t^{x_1}} + P^{x_1}X_1^1\widetilde{X_{1,t}^1} - P^{x_1}x_1^1\widetilde{x_{1,t}^1} + P^{x_2}\left(X_2^1 - x_2^1\right)\widetilde{P_t^{x_2}} \\ & + P^{x_2}X_2^1\widetilde{X_{2,t}^1} - P^{x_2}x_2^1\widetilde{x_{2,t}^1} + X_3^1\widetilde{X_{3,t}^1} - x_3^1\widetilde{x_{3,t}^1}, \end{aligned} \tag{27}$$

$$\begin{aligned} 0 \; = \; & -P^{q_1}q_1^2\widetilde{P_t^{q_1}} - P^{q_1}q_1^2\widetilde{q_{1,t}^2} + P^{q_2}\left(Q^2 - q_2^2\right)\widetilde{P_t^{q_2}} + P^{q_2}Q^2\widetilde{Q_t^2} - P^{q_2}q_2^2\widetilde{q_{2,t}^2} - P^{q_3}q_3^2\widetilde{P_t^{q_3}} \\ & - P^{q_3}q_3^2\widetilde{q_{3,t}^2} + P^{x_1}\left(X_1^2 - x_1^2\right)\widetilde{P_t^{x_1}} + P^{x_1}X_1^2\widetilde{X_{1,t}^2} - P^{x_1}x_1^2\widetilde{x_{1,t}^2} + P^{x_2}\left(X_2^2 - x_2^2\right)\widetilde{P_t^{x_2}} \\ & + P^{x_2}X_2^2\widetilde{X_{2,t}^2} - P^{x_2}x_2^2\widetilde{x_{2,t}^2} + X_3^2\widetilde{X_{3,t}^2} - x_3^2\widetilde{x_{3,t}^2}. \end{aligned} \tag{28}$$

Again, we have a system of 59 equations and 58 variables, without any endogenous state variables or expectation equations.

B.2 Computation of the steady-state equilibrium

Variables remain constant in steady-state, so we suppress time subscripts. We assume that bond holdings are zero, that is, $B^1 = B^2 = B^3 = 0$. We normalize the price of primary commodity X_3 to one, $P^{x_3} = 1$, and iterate on the prices of intermediate goods and primary commodities $[P^{q_1}, P^{q_2}, P^{q_3}, P^{x_1}, P^{x_2}]$.

Given a guess for the vector $[P^{q_1}, P^{q_2}, P^{q_3}, P^{x_1}, P^{x_2}]$, we can compute the other prices and allocations in the economy. We start with Country 1. From the cost-minimization problem of the firms, perfect competition implies that the prices of the final good P^{c_1}, intermediate good P^{q_1}, and primary commodities P^{x_1}, P^{x_2}, and P^{x_3} are equal to their respective marginal costs:

$$
\begin{aligned}
P^{c_1} &= \left(Z^1\right)^{-1}\left(\frac{P^{q_1}}{\alpha_1^1}\right)^{\alpha_1^1}\left(\frac{P^{q_2}}{\alpha_2^1}\right)^{\alpha_2^1}\left(\frac{P^{q_3}}{\alpha_3^1}\right)^{\alpha_3^1}\left(\frac{W^1}{\alpha_4^1}\right)^{\alpha_4^1}, && (29)\\
P^{q_1} &= \left(Z^1\right)^{-1}\left(\frac{P^{x_1}}{\beta_1^1}\right)^{\beta_1^1}\left(\frac{P^{x_2}}{\beta_2^1}\right)^{\beta_2^1}\left(\frac{P^{x_3}}{\beta_3^1}\right)^{\beta_3^1}\left(\frac{W^1}{\beta_4^1}\right)^{\beta_4^1}, && (30)\\
P^{x_1} &= \left(Z^1\right)^{-1}\left[\left(1-\phi_1^1\right)\left(P^{e_1^1}\right)^{1-\sigma_{x_1}^1}+\phi_1^1\left(W^1\right)^{1-\sigma_{x_1}^1}\right]^{\frac{1}{1-\sigma_{x_1}^1}}, && (31)\\
P^{x_2} &= \left(Z^1\right)^{-1}\left[\left(1-\phi_2^1\right)\left(P^{e_2^1}\right)^{1-\sigma_{x_2}^1}+\phi_2^1\left(W^1\right)^{1-\sigma_{x_2}^1}\right]^{\frac{1}{1-\sigma_{x_2}^1}}, && (32)\\
P^{x_3} &= \left(Z^1\right)^{-1}\left[\left(1-\phi_3^1\right)\left(P^{e_3^1}\right)^{1-\sigma_{x_3}^1}+\phi_3^1\left(W^1\right)^{1-\sigma_{x_3}^1}\right]^{\frac{1}{1-\sigma_{x_3}^1}}. && (33)
\end{aligned}
$$

Given the vector of prices for the tradable goods, we use equation (30) to solve for the wage W^1. With the wage and price of intermediate goods, we solve for the price of the final good P^{c_1} using equation (29), and for the price of the endowments of primary commodities using equations (31)–(33).

Next, we compute the allocations. With the assumption that $B^1 = 0$ in steady-state, consumption C^1 is directly determined by the budget constraint:

$$
C^1 = \frac{W^1}{P^{c_1}}\overline{N}^1 + \frac{P^{e_1^1}}{P^{c_1}}e_1^1 + \frac{P^{e_2^1}}{P^{c_1}}e_2^1 + \frac{P^{e_3^1}}{P^{c_1}}e_3^1. \tag{34}
$$

With prices and total consumption, we can use the optimality conditions in the final good sector to compute its input choices:

$$
\begin{aligned}
q_1^1 &= \alpha_1^1\frac{P^{c_1}}{P^{q_1}}C^1, && (35)\\
q_2^1 &= \alpha_2^1\frac{P^{c_1}}{P^{q_2}}C^1, && (36)\\
q_3^1 &= \alpha_3^1\frac{P^{c_1}}{P^{q_3}}C^1, && (37)\\
n_c^1 &= \alpha_4^1\frac{W^1}{P^{q_1}}C^1. && (38)
\end{aligned}
$$

Given that the supply of the endowment of natural resources is fixed, we solve for the production of primary commodities X_j^1 and their labor inputs using the respective optimality conditions in the primary commodity sector $j = 1, 2, 3$:

$$
\begin{aligned}
X_j^1 &= \frac{e_j^1}{1-\phi_j^1}\left(\frac{P^{e_j^1}}{P^{x_j}}\right)^{\sigma_{x_j}^1}\left(Z^1\right)^{1-\sigma_{x_j}^1}, && (39)\\
n_{x_j}^1 &= \phi_j^1\left(\frac{P^{x_j}}{W^1}\right)^{\sigma_{x_j}^1}\left(Z^1\right)^{\sigma_{x_j}^1-1}X_j^1. && (40)
\end{aligned}
$$

The labor input in the production of the intermediate-good sector Q^1 is determined by the market-clearing condition for labor in Country 1:

$$
n_q^1 = \overline{N}^1 - \left(n_c^1 + n_{x_1}^1 + n_{x_2}^1 + n_{x_3}^1\right). \tag{41}
$$

Finally, we solve for the production of the intermediate good Q^1 and its inputs of primary commodities x_1^1, x_2^1, and x_3^1, using the optimality conditions in the intermediate-good sector:

$$
\begin{aligned}
Q^1 &= \frac{W^1}{P^{q_1}}\frac{n_q^1}{\beta_4^1}, && (42)\\
x_1^1 &= \beta_1^1\frac{P^{q_1}}{P^{x_1}}Q^1, && (43)\\
x_2^1 &= \beta_2^1\frac{P^{q_1}}{P^{x_2}}Q^1, && (44)\\
x_3^1 &= \beta_3^1\frac{P^{q_1}}{P^{x_3}}Q^1. && (45)
\end{aligned}
$$

Given the vector of prices for the tradable goods, we use the same procedure as above to compute the allocations and prices in Countries 2 and 3, noting that Country 3 receives exogenous endowments of primary commodities. After computing the productions of primary commodities and intermediate goods in each country, and their demand in the production of intermediate and final goods, we can check whether their market-clearing conditions are satisfied. The algorithm iterates on the prices of the tradable goods until they do.

B.3 Chain-weighted real GDP and productivity shocks

In this appendix, we show that the chain-weighted real GDP in Country 1 is proportional to its productivity shock up to a first-order approximation. The same applies to Country 2.

To simplify the exposition, we define:

$$
\begin{aligned}
\mathrm{GDP}^{1}_{P_{t_1}Y_{t_2}} &= \mathrm{GDP}^{c,1}_{P_{t_1}Y_{t_2}} + \mathrm{GDP}^{q,1}_{P_{t_1}Y_{t_2}} + \mathrm{GDP}^{x_1,1}_{P_{t_1}Y_{t_2}} + \mathrm{GDP}^{x_2,1}_{P_{t_1}Y_{t_2}} + \mathrm{GDP}^{x_3,1}_{P_{t_1}Y_{t_2}} && (46)\\
\mathrm{GDP}^{c,1}_{P_{t_1}Y_{t_2}} &= P^{c_1}_{t_1} C^{1}_{t_2} - P^{q_1}_{t_1} q^{1}_{1,t_2} - P^{q_2}_{t_1} q^{1}_{2,t_2} - P^{q_3}_{t_1} q^{1}_{3,t_2} && (47)\\
\mathrm{GDP}^{q,1}_{P_{t_1}Y_{t_2}} &= P^{q_1}_{t_1} C^{1}_{t_2} - P^{x_1}_{t_1} x^{1}_{1,t_2} - P^{x_2}_{t_1} x^{1}_{2,t_2} - P^{x_3}_{t_1} x^{1}_{3,t_2} && (48)\\
\mathrm{GDP}^{x,1}_{P_{t_1}Y_{t_2}} &= P^{x_1}_{t_1} X^{1}_{1,t_2}, && (49)\\
\mathrm{GDP}^{x,2}_{P_{t_1}Y_{t_2}} &= P^{x_2}_{t_1} X^{1}_{2,t_2}, && (50)\\
\mathrm{GDP}^{x,3}_{P_{t_1}Y_{t_2}} &= P^{x_3}_{t_1} X^{1}_{3,t_2}. && (51)
\end{aligned}
$$

$\mathrm{GDP}^{1}_{P_{t_1}Y_{t_2}}$ is a measure of value added in Country 1. It is defined as the sum of value added in the final-good, intermediate-good, and primary-commodity sectors using prices from period t_1 and quantities from period t_2. For example, nominal GDP in Country 1 in period t is equal to $\mathrm{GDP}^{1}_{P_tY_t}$.

Let RGDP^1 denote the chain-weighted real GDP in Country 1, the measure of real GDP reported in the data. It evolves according to:

$$
\frac{\mathrm{RGDP}^1_t}{\mathrm{RGDP}^1_{t-1}} = \left(\frac{\mathrm{GDP}^1_{P_tY_t}}{\mathrm{GDP}^1_{P_tY_{t-1}}}\right)^{\frac{1}{2}} \times \left(\frac{\mathrm{GDP}^1_{P_{t-1}Y_t}}{\mathrm{GDP}^1_{P_{t-1}Y_{t-1}}}\right)^{\frac{1}{2}}. \tag{52}
$$

Taking a first-order approximation of equation (52) around the steady-state, we reach:

$$
2\left(\widetilde{\mathrm{RGDP}}^1_t - \widetilde{\mathrm{RGDP}}^1_{t-1}\right) = \widetilde{\mathrm{GDP}}^1_{P_tY_t} - \widetilde{\mathrm{GDP}}^1_{P_tY_{t-1}} + \widetilde{\mathrm{GDP}}^1_{P_{t-1}Y_t} - \widetilde{\mathrm{GDP}}^1_{P_{t-1}Y_{t-1}}, \tag{53}
$$

where $\widetilde{X_t}$ denotes the log-deviation of variable X from its steady-state level in period t.

Using equations (46)–(51), each term in the right-hand-side of equation (53) can be decomposed into:

$$\begin{aligned}
GDP^1\widetilde{GDP^1_{P_{t_1}Y_{t_2}}} &= GDP^{c,1}\widetilde{GDP^{c,1}_{P_{t_1}Y_{t_2}}} + GDP^{q,1}\widetilde{GDP^{q,1}_{P_{t_1}Y_{t_2}}} + GDP^{x_1,1}\widetilde{GDP^{x_1,1}_{P_{t_1}Y_{t_2}}} \\
&\quad + GDP^{x_2,1}\widetilde{GDP^{x_2,1}_{P_{t_1}Y_{t_2}}} + GDP^{x_3,1}\widetilde{GDP^{x_3,1}_{P_{t_1}Y_{t_2}}}, \qquad (54)
\end{aligned}$$

$$\begin{aligned}
\frac{GDP^{c,1}}{P^{c_1}C^1}\widetilde{GDP^{c,1}_{P_{t_1}Y_{t_2}}} &= \widetilde{P^{c_1}_{t_1}} + \widetilde{C^1_{t_2}} - \frac{P^{q_1}q^1_1}{P^{c_1}C^1}\widetilde{P^{q_1}_{t_1}} - \frac{P^{q_1}q^1_1}{P^{c_1}C^1}\widetilde{q^1_{1,t_2}} - \frac{P^{q_2}q^1_2}{P^{c_1}C^1}\widetilde{P^{q_2}_{t_1}} - \frac{P^{q_2}q^1_2}{P^{c_1}C^1}\widetilde{q^1_{2,t_2}} \\
&\quad - \frac{P^{q_3}q^1_3}{P^{c_1}C^1}\widetilde{P^{q_3}_{t_1}} - \frac{P^{q_3}q^1_3}{P^{c_1}C^1}\widetilde{q^1_{3,t_2}}, \qquad (55)
\end{aligned}$$

$$\begin{aligned}
\frac{GDP^{q,1}}{P^{q_1}Q^1}\widetilde{GDP^{q,1}_{P_{t_1}Y_{t_2}}} &= \widetilde{P^{q_1}_{t_1}} + \widetilde{Q^1_{t_2}} - \frac{P^{x_1}x^1_1}{P^{q_1}Q^1}\widetilde{P^{x_1}_{t_1}} - \frac{P^{x_1}x^1_1}{P^{q_1}Q^1}\widetilde{x^1_{1,t_2}} - \frac{P^{x_2}x^1_2}{P^{q_1}Q^1}\widetilde{P^{x_2}_{t_1}} - \frac{P^{x_2}x^1_2}{P^{q_1}Q^1}\widetilde{x^1_{2,t_2}} \\
&\quad - \frac{P^{x_3}x^1_3}{P^{q_1}Q^1}\widetilde{P^{x_3}_{t_1}} - \frac{P^{x_3}x^1_3}{P^{q_1}Q^1}\widetilde{x^1_{3,t_2}}, \qquad (56)
\end{aligned}$$

$$\frac{GDP^{x_1,1}}{P^{x_1}X^1_1}\widetilde{GDP^{x_1,1}_{P_{t_1}Y_{t_2}}} = \widetilde{P^{x_1}_{t_1}} + \widetilde{X^1_{1,t_2}}, \qquad (57)$$

$$\frac{GDP^{x_2,1}}{P^{x_2}X^1_2}\widetilde{GDP^{x_2,1}_{P_{t_1}Y_{t_2}}} = \widetilde{P^{x_2}_{t_1}} + \widetilde{X^1_{2,t_2}}, \qquad (58)$$

$$\frac{GDP^{x_3,1}}{P^{x_1}X^1_3}\widetilde{GDP^{x_3,1}_{P_{t_1}Y_{t_2}}} = \widetilde{P^{x_3}_{t_1}} + \widetilde{X^1_{3,t_2}}. \qquad (59)$$

where variables without time subscript, such as $GDP^{c,1}$, denote their steady-state levels. Our goal is to simplify the equations above using the equilibrium equations described in

Section B. Note that:

$$
\begin{aligned}
\mathrm{GDP}^{1}\left(\widetilde{\mathrm{GDP}^{1}_{P_{t_1}Y_{t_2}}}-\widetilde{\mathrm{GDP}^{1}_{P_{t_1}Y_{t_1}}}\right) &= \mathrm{GDP}^{c,1}\left(\widetilde{\mathrm{GDP}^{c,1}_{P_{t_1}Y_{t_2}}}-\widetilde{\mathrm{GDP}^{c,1}_{P_{t_1}Y_{t_2}}}\right) \\
&\quad +\mathrm{GDP}^{q,1}\left(\widetilde{\mathrm{GDP}^{q,1}_{P_{t_1}Y_{t_2}}}-\widetilde{\mathrm{GDP}^{q,1}_{P_{t_1}Y_{t_1}}}\right) \\
&\quad +\mathrm{GDP}^{x_1,1}\left(\widetilde{\mathrm{GDP}^{x,1}_{P_{t_1}Y_{t_2}}}-\widetilde{\mathrm{GDP}^{x,1}_{P_{t_1}Y_{t_1}}}\right) \\
&\quad +\mathrm{GDP}^{x_1,1}\left(\widetilde{\mathrm{GDP}^{x,1}_{P_{t_1}Y_{t_2}}}-\widetilde{\mathrm{GDP}^{x,1}_{P_{t_1}Y_{t_1}}}\right) \\
&\quad +\mathrm{GDP}^{x_1,1}\left(\widetilde{\mathrm{GDP}^{x,1}_{P_{t_1}Y_{t_2}}}-\widetilde{\mathrm{GDP}^{x,1}_{P_{t_1}Y_{t_1}}}\right), \qquad (60)
\end{aligned}
$$

$$
\begin{aligned}
\frac{\mathrm{GDP}^{c,1}}{P^{c_1}C^{1}}\left(\widetilde{\mathrm{GDP}^{c,1}_{P_{t_1}Y_{t_2}}}-\widetilde{\mathrm{GDP}^{c,1}_{P_{t_1}Y_{t_1}}}\right) &= \left(\widetilde{C^{1}_{t_2}}-\widetilde{C^{1}_{t_1}}\right)-\alpha^{1}_{1}\left(\widetilde{q^{1}_{1,t_2}}-\widetilde{q^{1}_{1,t_1}}\right)-\alpha^{1}_{2}\left(\widetilde{q^{1}_{2,t_2}}-\widetilde{q^{1}_{2,t_1}}\right) \\
&\quad -\alpha^{1}_{3}\left(\widetilde{q^{1}_{3,t_2}}-\widetilde{q^{1}_{3,t_1}}\right), \qquad (61) \\
\frac{\mathrm{GDP}^{q,1}}{P^{q_1}Q^{1}}\left(\widetilde{\mathrm{GDP}^{q,1}_{P_{t_1}Y_{t_2}}}-\widetilde{\mathrm{GDP}^{q,1}_{P_{t_1}Y_{t_1}}}\right) &= \left(\widetilde{Q^{1}_{t_2}}-\widetilde{Q^{1}_{t_1}}\right)-\beta^{1}_{1}\left(\widetilde{x^{1}_{1,t_2}}-\widetilde{x^{1}_{1,t_1}}\right)-\beta^{1}_{2}\left(\widetilde{x^{1}_{2,t_2}}-\widetilde{x^{1}_{2,t_1}}\right) \\
&\quad -\beta^{1}_{3}\left(\widetilde{x^{1}_{3,t_2}}-\widetilde{x^{1}_{3,t_1}}\right), \qquad (62) \\
\mathrm{GDP}^{x_1,1}\left(\widetilde{\mathrm{GDP}^{x_1,1}_{P_{t_1}Y_{t_2}}}-\widetilde{\mathrm{GDP}^{x_1,1}_{P_{t_1}Y_{t_1}}}\right) &= P^{x_1}X^{1}_{1}\left(\widetilde{X^{1}_{1,t_2}}-\widetilde{X^{1}_{1,t_1}}\right), \qquad (63) \\
\mathrm{GDP}^{x_2,1}\left(\widetilde{\mathrm{GDP}^{x_2,1}_{P_{t_1}Y_{t_2}}}-\widetilde{\mathrm{GDP}^{x_2,1}_{P_{t_1}Y_{t_1}}}\right) &= P^{x_2}X^{1}_{2}\left(\widetilde{X^{1}_{2,t_2}}-\widetilde{X^{2}_{1,t_1}}\right), \qquad (64) \\
\mathrm{GDP}^{x_3,1}\left(\widetilde{\mathrm{GDP}^{x_3,1}_{P_{t_1}Y_{t_2}}}-\widetilde{\mathrm{GDP}^{x_3,1}_{P_{t_1}Y_{t_1}}}\right) &= P^{x_3}X^{1}_{3}\left(\widetilde{X^{1}_{3,t_2}}-\widetilde{X^{3}_{1,t_1}}\right). \qquad (65)
\end{aligned}
$$

Using equations (3), (8), and (13), we can replace equations (61)–(65) by:

$$
\begin{aligned}
\mathrm{GDP}^{c,1}\left(\widetilde{\mathrm{GDP}^{c,1}_{P_{t_1}Y_{t_2}}}-\widetilde{\mathrm{GDP}^{c,1}_{P_{t_1}Y_{t_1}}}\right) &= P^{c_1}C^{1}\left(\widetilde{Z^{1}_{t_2}}-\widetilde{Z^{1}_{t_1}}\right)+W^{1}n^{1}_{c}\left(\widetilde{n^{1}_{c,t_2}}-\widetilde{n^{1}_{c,t_1}}\right), \qquad (66) \\
\mathrm{GDP}^{q,1}\left(\widetilde{\mathrm{GDP}^{q,1}_{P_{t_1}Y_{t_2}}}-\widetilde{\mathrm{GDP}^{q,1}_{P_{t_1}Y_{t_1}}}\right) &= P^{q_1}Q^{1}\left(\widetilde{Z^{1}_{t_2}}-\widetilde{Z^{1}_{t_1}}\right)+W^{1}n^{1}_{q}\left(\widetilde{n^{1}_{q,t_2}}-\widetilde{n^{1}_{q,t_1}}\right), \qquad (67) \\
\mathrm{GDP}^{x_1,1}\left(\widetilde{\mathrm{GDP}^{x_1,1}_{P_{t_1}Y_{t_2}}}-\widetilde{\mathrm{GDP}^{x_1,1}_{P_{t_1}Y_{t_1}}}\right) &= P^{x_1}X^{1}_{1}\left(\widetilde{Z^{1}_{t_2}}-\widetilde{Z^{1}_{t_1}}\right)+W^{1}n^{1}_{x_1}\left(\widetilde{n^{1}_{x_1,t_2}}-\widetilde{n^{1}_{x_1,t_1}}\right), \qquad (68) \\
\mathrm{GDP}^{x_2,1}\left(\widetilde{\mathrm{GDP}^{x_2,1}_{P_{t_1}Y_{t_2}}}-\widetilde{\mathrm{GDP}^{x_2,1}_{P_{t_1}Y_{t_1}}}\right) &= P^{x_2}X^{1}_{2}\left(\widetilde{Z^{1}_{t_2}}-\widetilde{Z^{1}_{t_1}}\right)+W^{1}n^{1}_{x_2}\left(\widetilde{n^{1}_{x_2,t_2}}-\widetilde{n^{1}_{x_2,t_1}}\right), \qquad (69) \\
\mathrm{GDP}^{x_3,1}\left(\widetilde{\mathrm{GDP}^{x_3,1}_{P_{t_1}Y_{t_2}}}-\widetilde{\mathrm{GDP}^{x_3,1}_{P_{t_1}Y_{t_1}}}\right) &= P^{x_3}X^{1}_{3}\left(\widetilde{Z^{1}_{t_2}}-\widetilde{Z^{1}_{t_1}}\right)+W^{1}n^{1}_{x_3}\left(\widetilde{n^{1}_{x_3,t_2}}-\widetilde{n^{1}_{x_3,t_1}}\right). \qquad (70)
\end{aligned}
$$

Using equations (66)–(70) in equation (60), the fact that $GDP^1 = P^{c_1}C^1$, and noting that equation (16) must be satisfied in equilibrium, we reach:

$$\widetilde{GDP^1_{P_{t_1}Y_{t_2}}} - \widetilde{GDP^1_{P_{t_1}Y_{t_1}}} = \frac{P^{c_1}C^1 + P^{q_1}Q^1 + P^{x_1}X^1_1 + P^{x_2}X^1_2 + P^{x_3}X^1_3}{P^{c_1}C^1}\left(\widetilde{Z^1_{t_2}} - \widetilde{Z^1_{t_1}}\right). \tag{71}$$

It is trivial to check that $\widetilde{GDP^1_{P_{t_1}Y_{t_2}}} - \widetilde{GDP^1_{P_{t_1}Y_{t_1}}} = \widetilde{GDP^1_{P_{t_2}Y_{t_2}}} - \widetilde{GDP^1_{P_{t_2}Y_{t_1}}}$, so we reach our final result:

$$\widetilde{RGDP^1_t} - \widetilde{RGDP}^1_{t-1} = \frac{P^{c_1}C^1 + P^{q_1}Q^1 + P^{x_1}X^1_1 + P^{x_2}X^1_2 + P^{x_3}X^1_3}{P^{c_1}C^1}\left(\widetilde{Z^1_{t_2}} - \widetilde{Z^1_{t_1}}\right). \tag{72}$$

Equation (72) shows that shocks to primary commodities in Country 3, the rest of the world, have no effect on Country 1's chain-weigthed real GDP up to a first-order approximation.

B.4 Calibration: endowment distribution

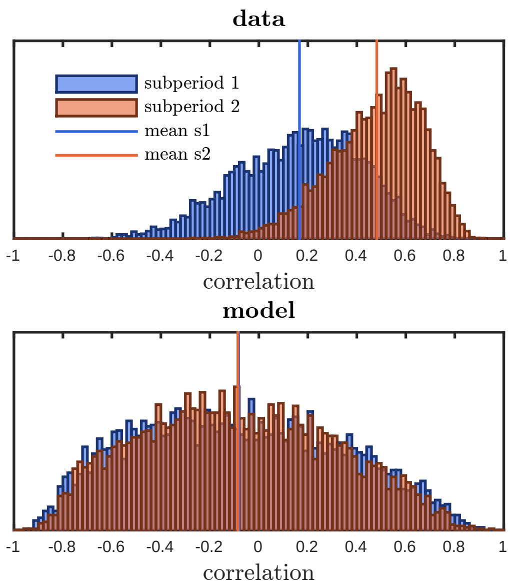
Table 1: Endowment distribution of benchmark model

USA (Country 1)	**Japan (Country 2)**	**Rest of the World (Country 3)**
$N_1 = 4.5$	$N_2 = 2.0$	$N_3 = 93.5$
$e_{11} = 1.00$	$e_{21} = 0.01$	$X_{31} = 1.08$
$e_{12} = 0.24$	$e_{22} = 0.14$	$X_{32} = 0.60$
$e_{13} = 1.80$	$e_{23} = 0.13$	$X_{33} = 0.36$

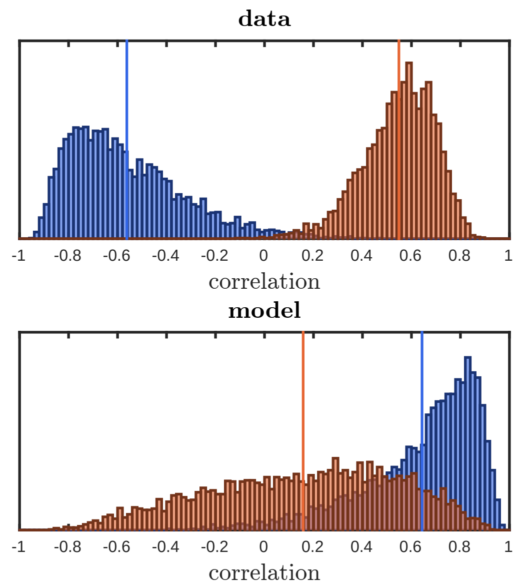
B.5 Simulation results

Figure 2: Benchmark model: correlation between commodity prices

(a) Price of energy P^{x_1} and agriculture P^{x_2}



(b) Price of energy P^{x_1} and metals P^{x_3}



(c) Price of agriculture P^{x_2} and metals P^{x_3}

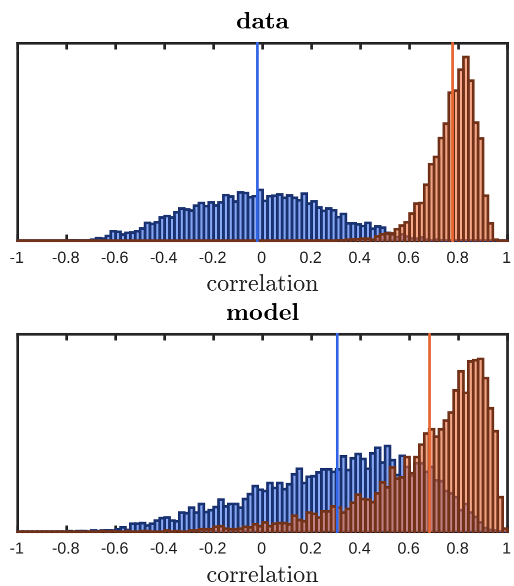


Figure 3: Reducing the share of primary commodities

(a) Real exchange rate and relative consumption

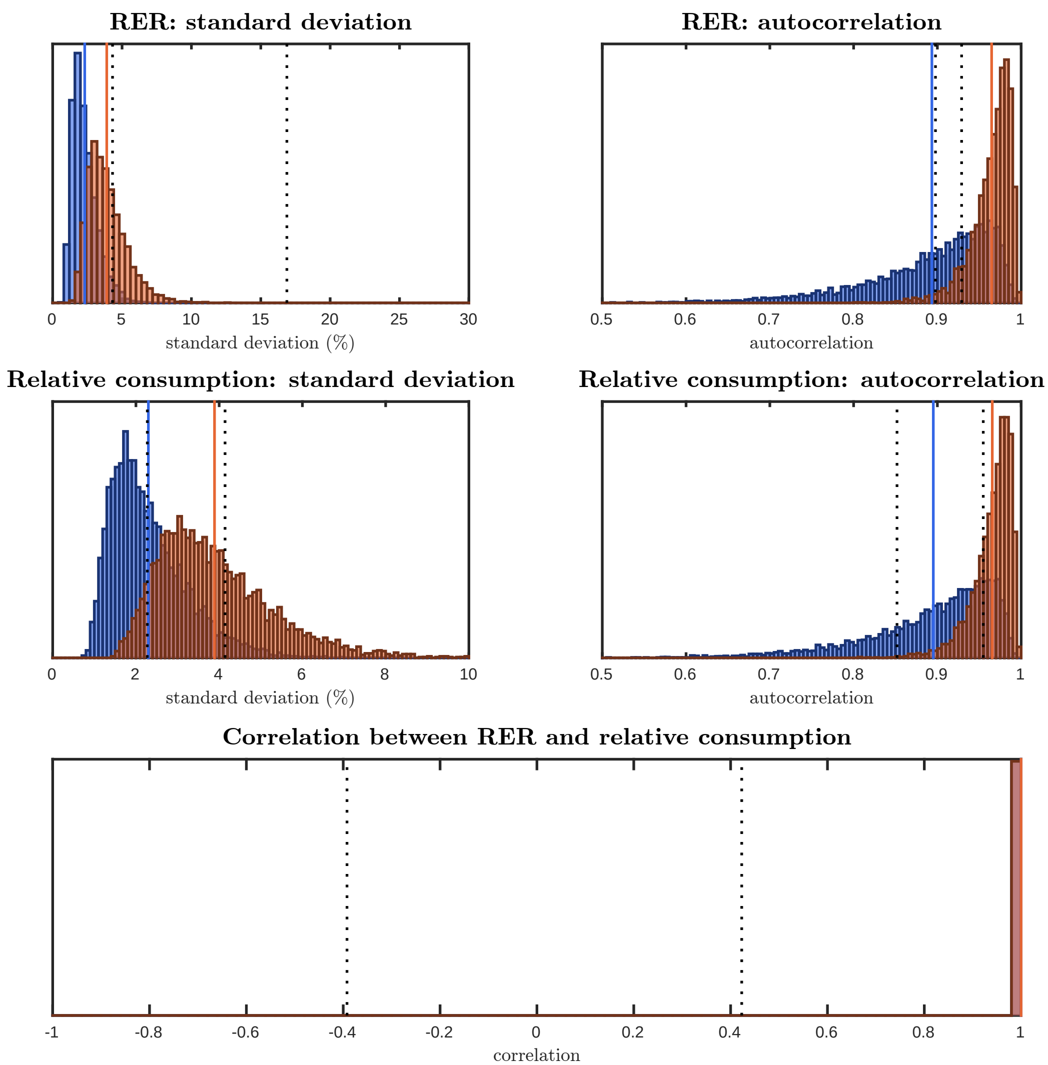


Figure 3: Reducing the share of primary commodities

(b) Primary commodity prices

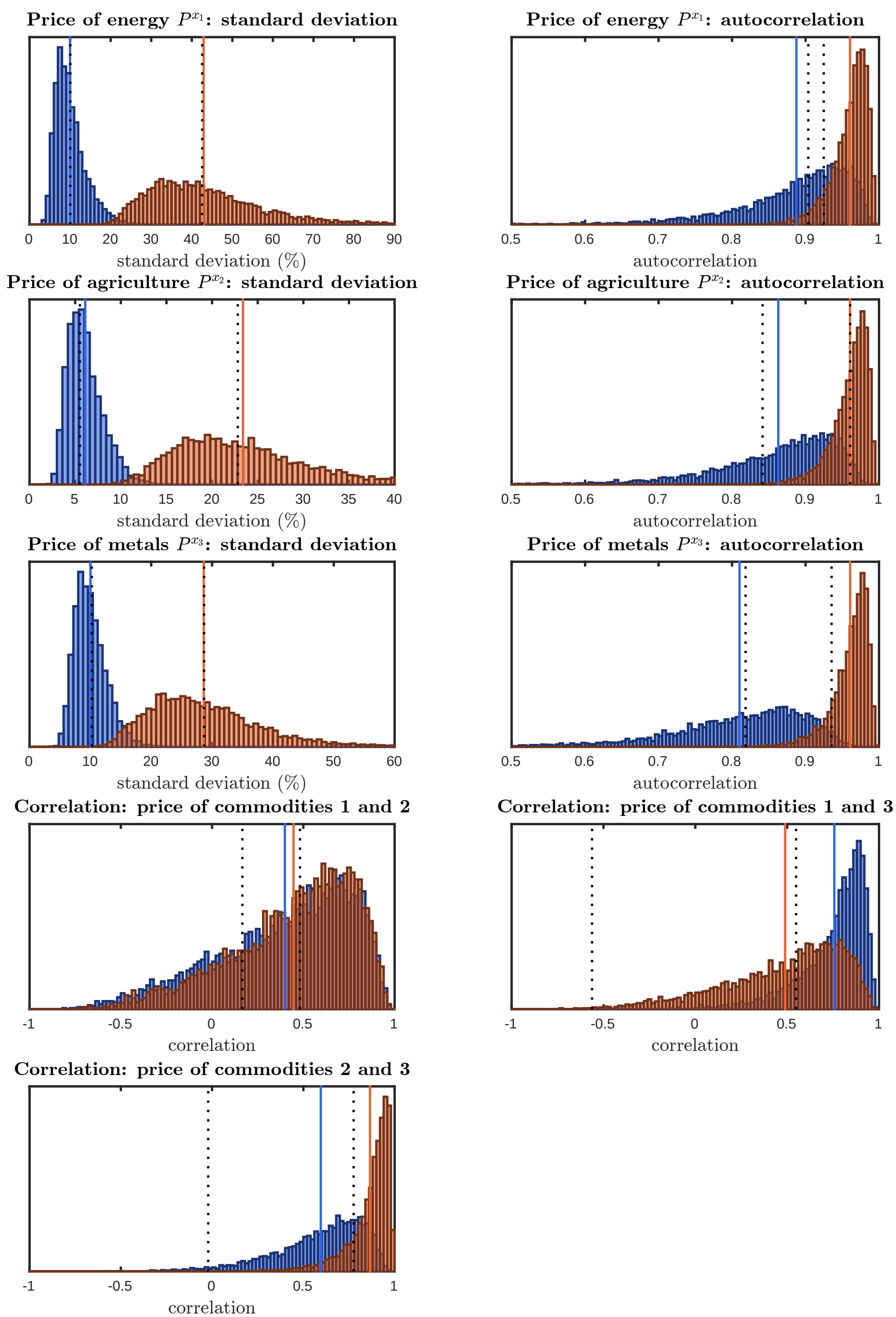


Figure 4: Homogeneous commodity sectors in countries 1 and 2

(a) Real exchange rate and relative consumption

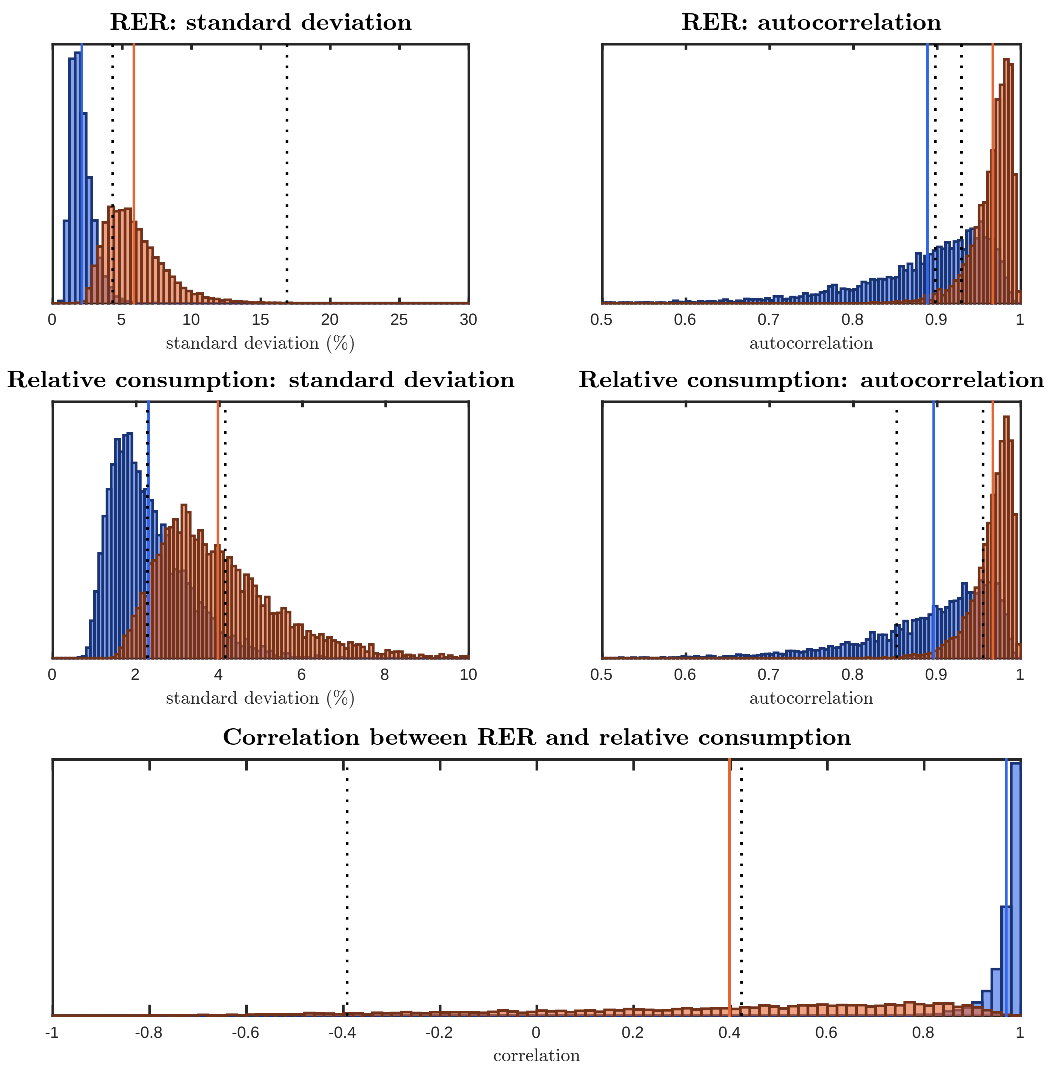


Figure 4: Homogeneous commodity sectors in countries 1 and 2

(b) Primary commodity prices

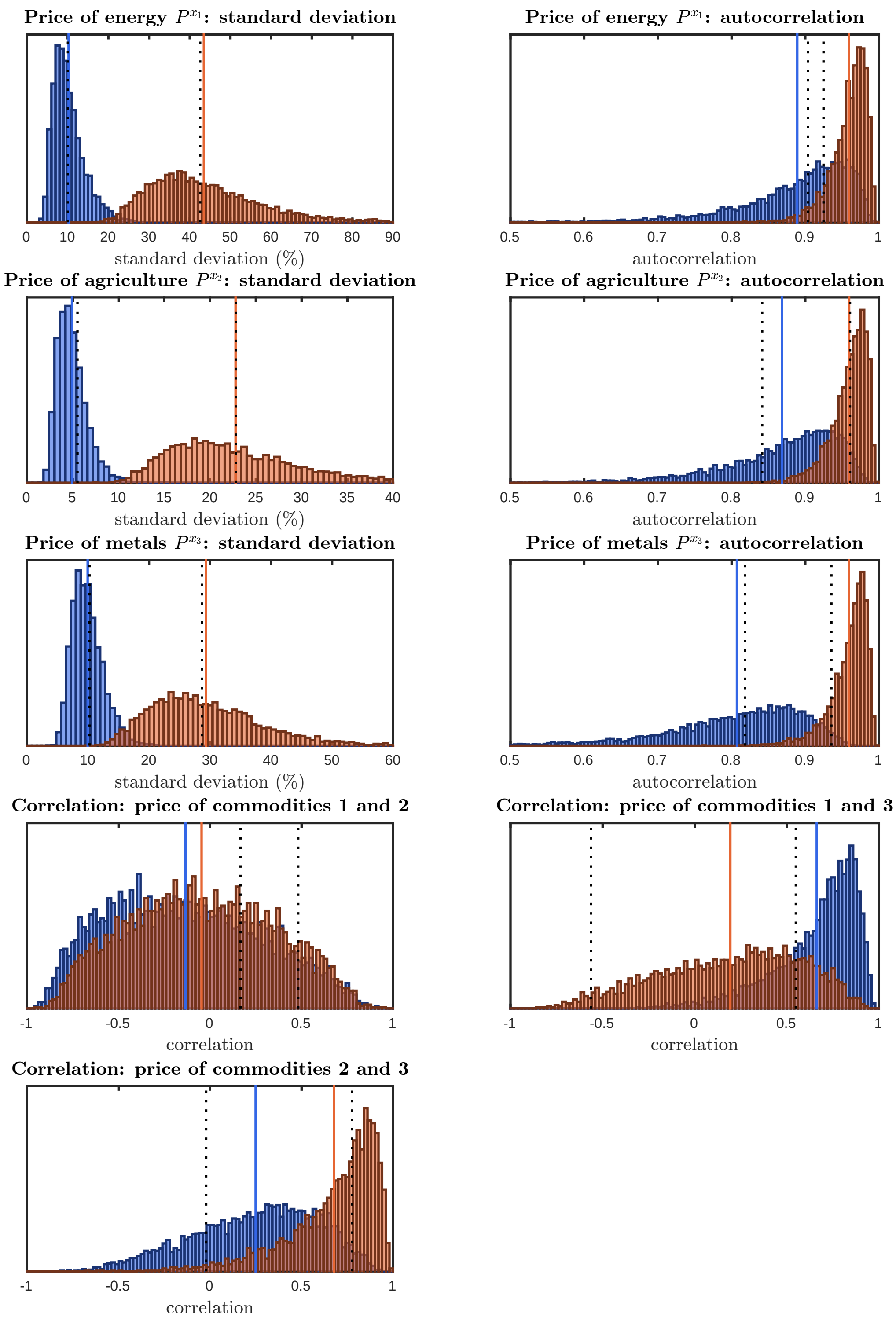


Figure 5: Heterogeneous commodity sectors in countries 1 and 2

(a) Real exchange rate and relative consumption

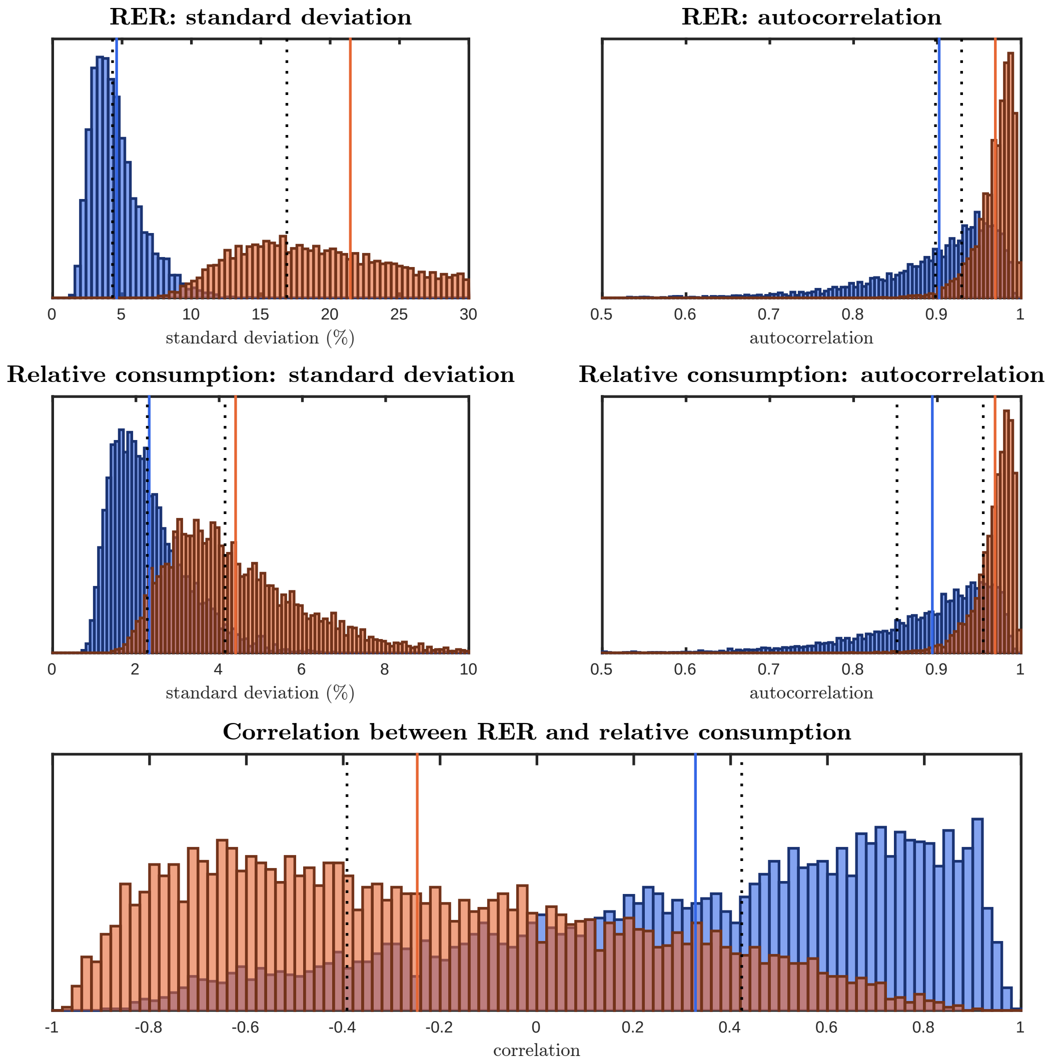


Figure 5: Heterogeneous commodity sectors in countries 1 and 2

(b) Primary commodity prices

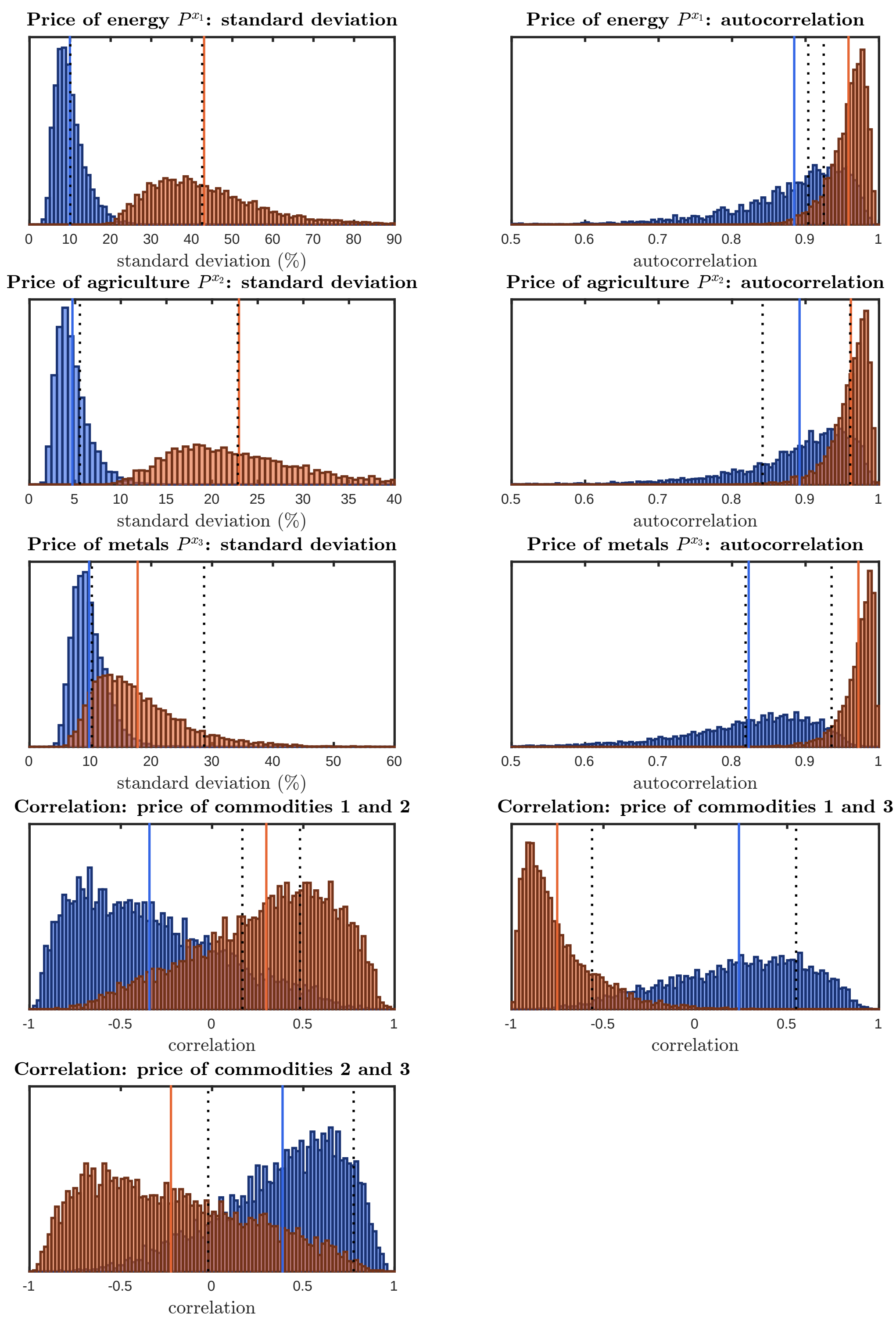


Figure 6: Lower elasticity of substitution in commodity production ($\sigma_x = 0.65$)

(a) Real exchange rate and relative consumption

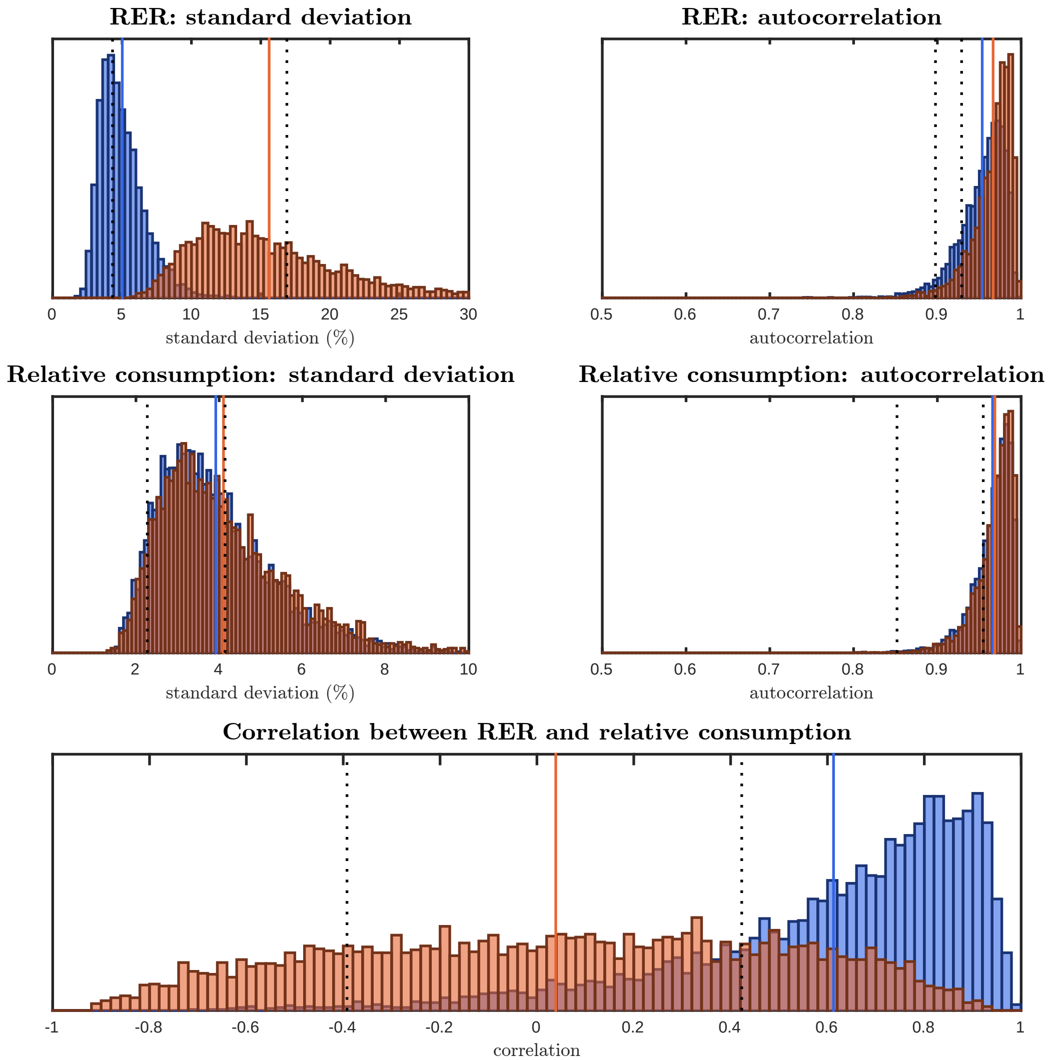


Figure 6: Lower elasticity of substitution in commodity production ($\sigma_x = 0.65$)

(b) Primary commodity prices

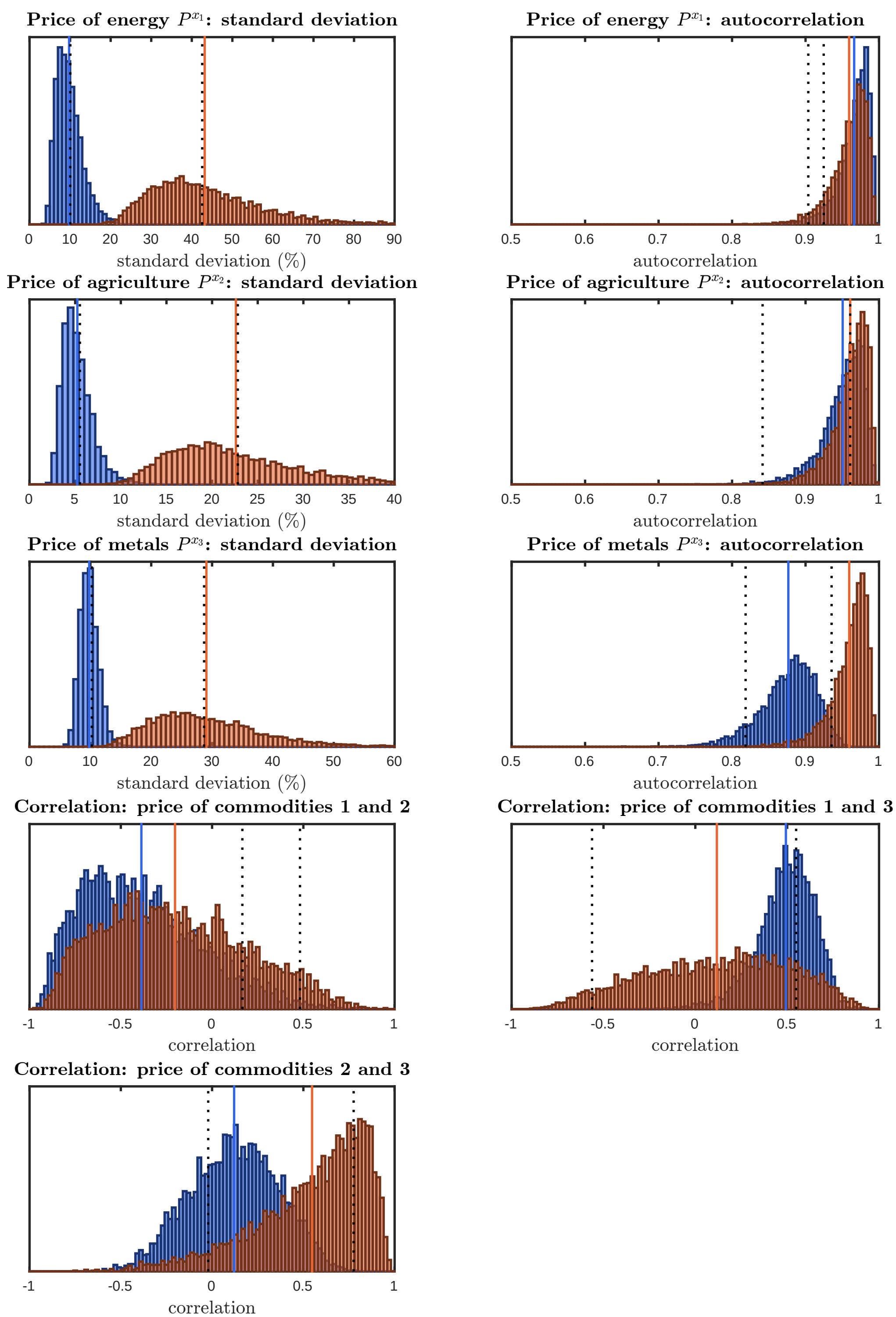


Figure 7: Complete markets

(a) Real exchange rate and relative consumption

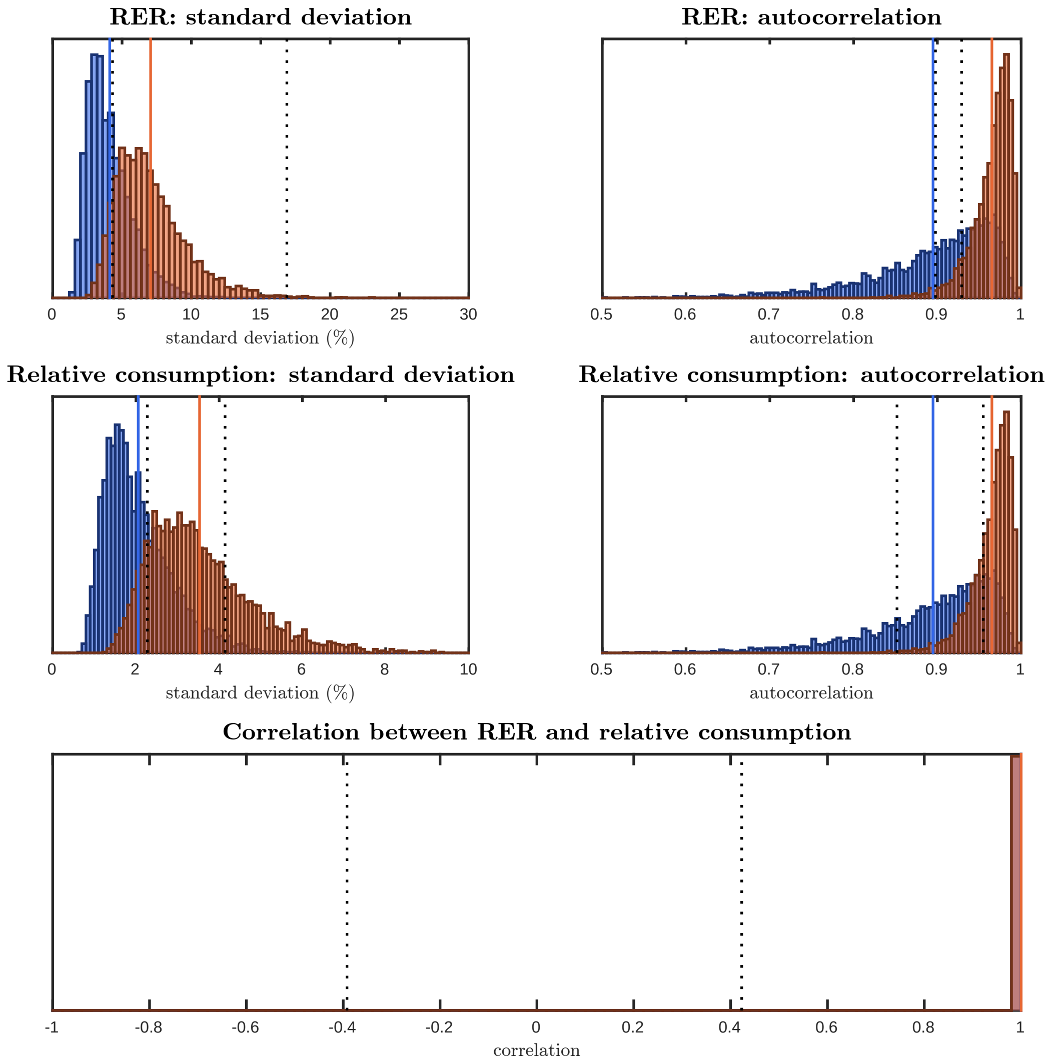


Figure 7: Complete markets

(b) Primary commodity prices

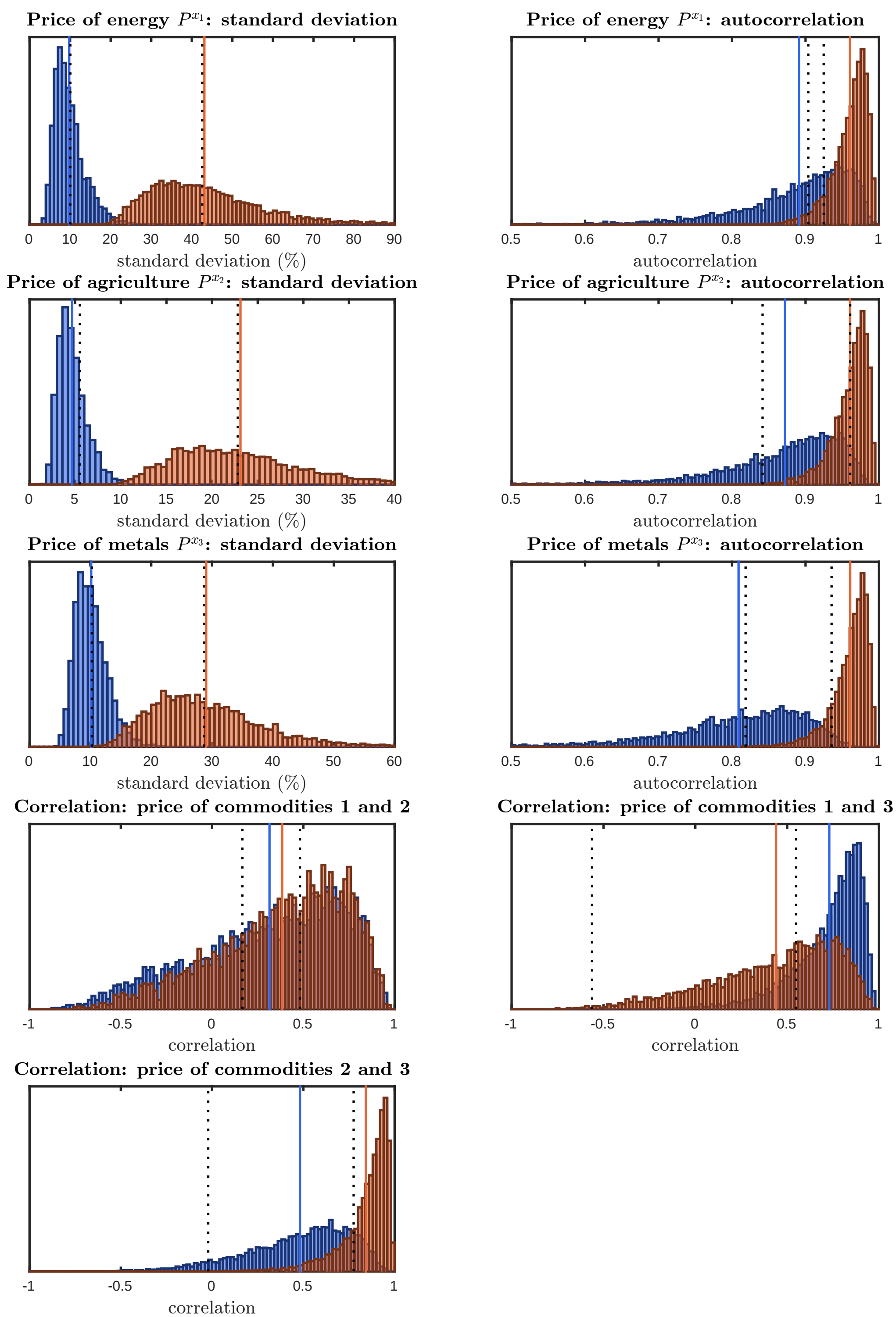


Figure 8: Financial autarky

(a) Real exchange rate and relative consumption

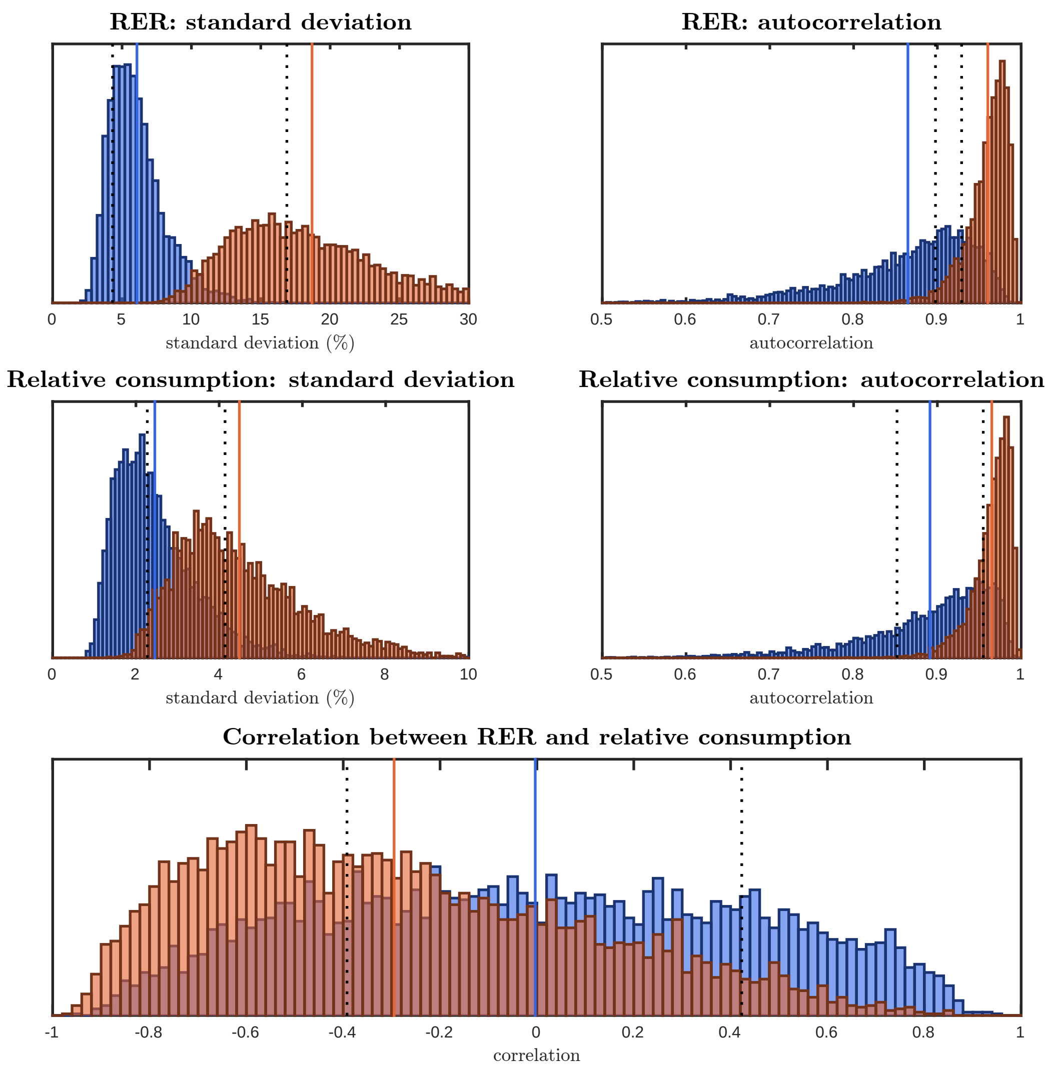


Figure 8: Financial autarky

(b) Primary commodity prices

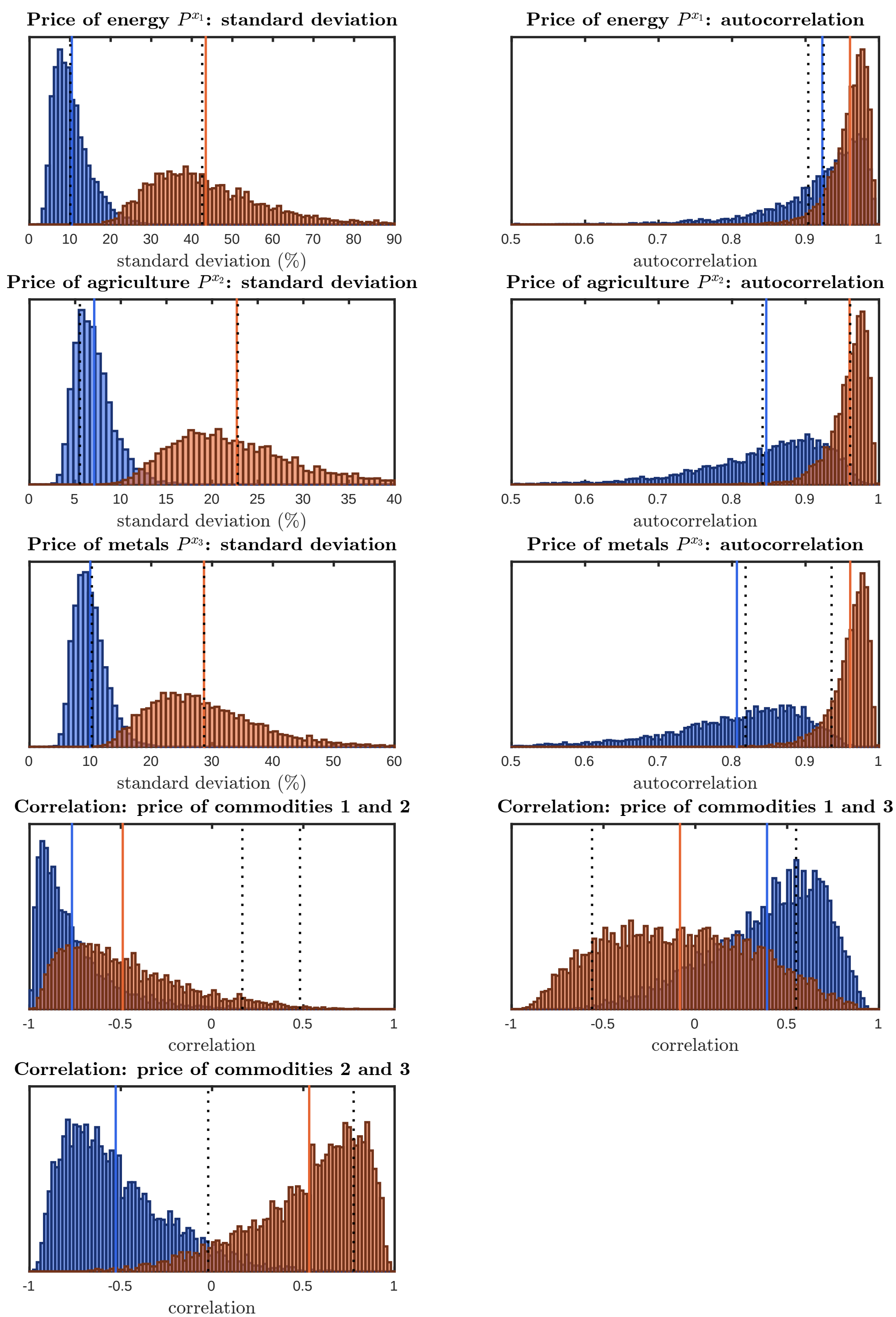


Figure 9: Lower persistence of commodity shocks ($\rho = 0.95$)

(a) Real exchange rate and relative consumption

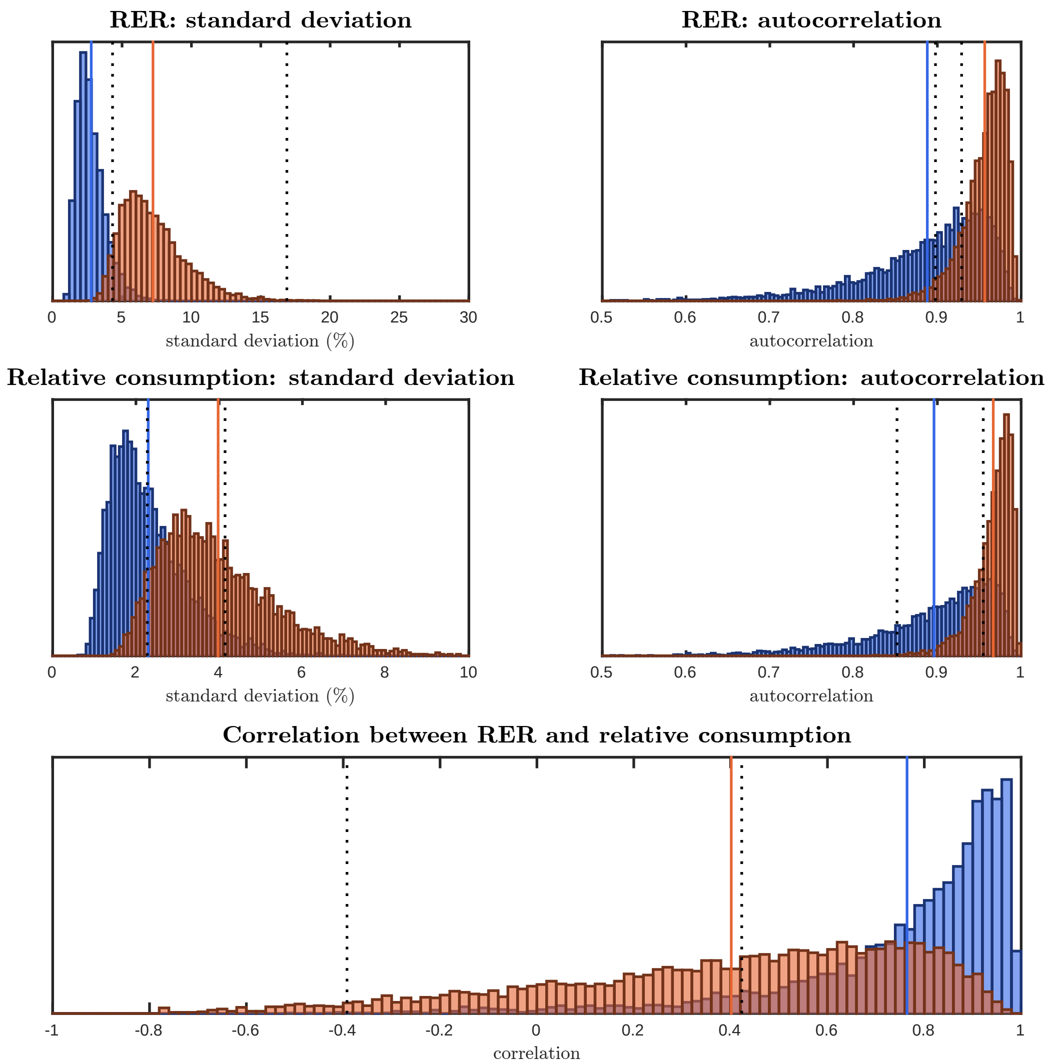


Figure 9: Lower persistence of commodity shocks ($\rho = 0.95$)

(b) Primary commodity prices

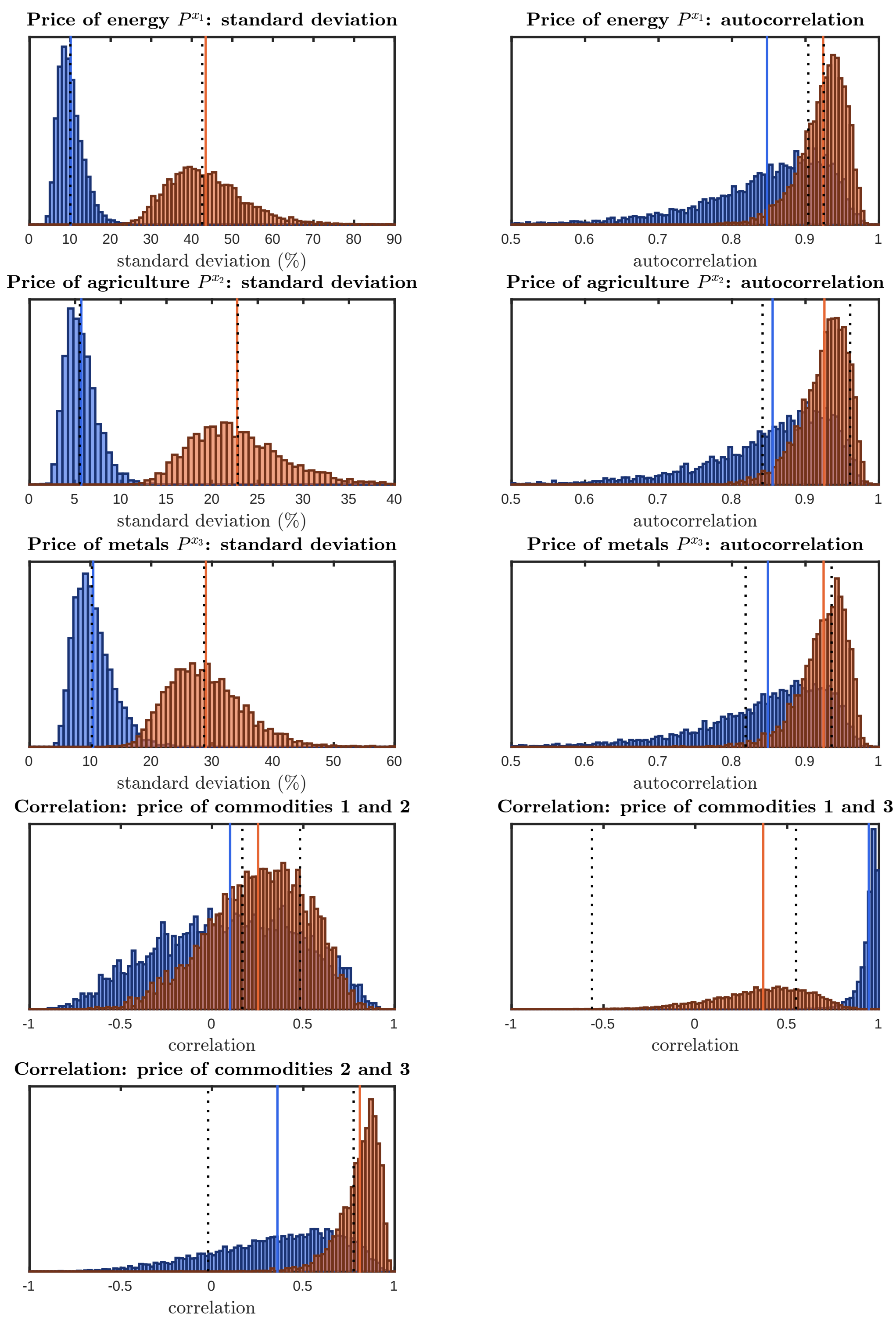


Figure 10: Higher persistence of commodity shocks ($\rho = 0.995$)

(a) Real exchange rate and relative consumption

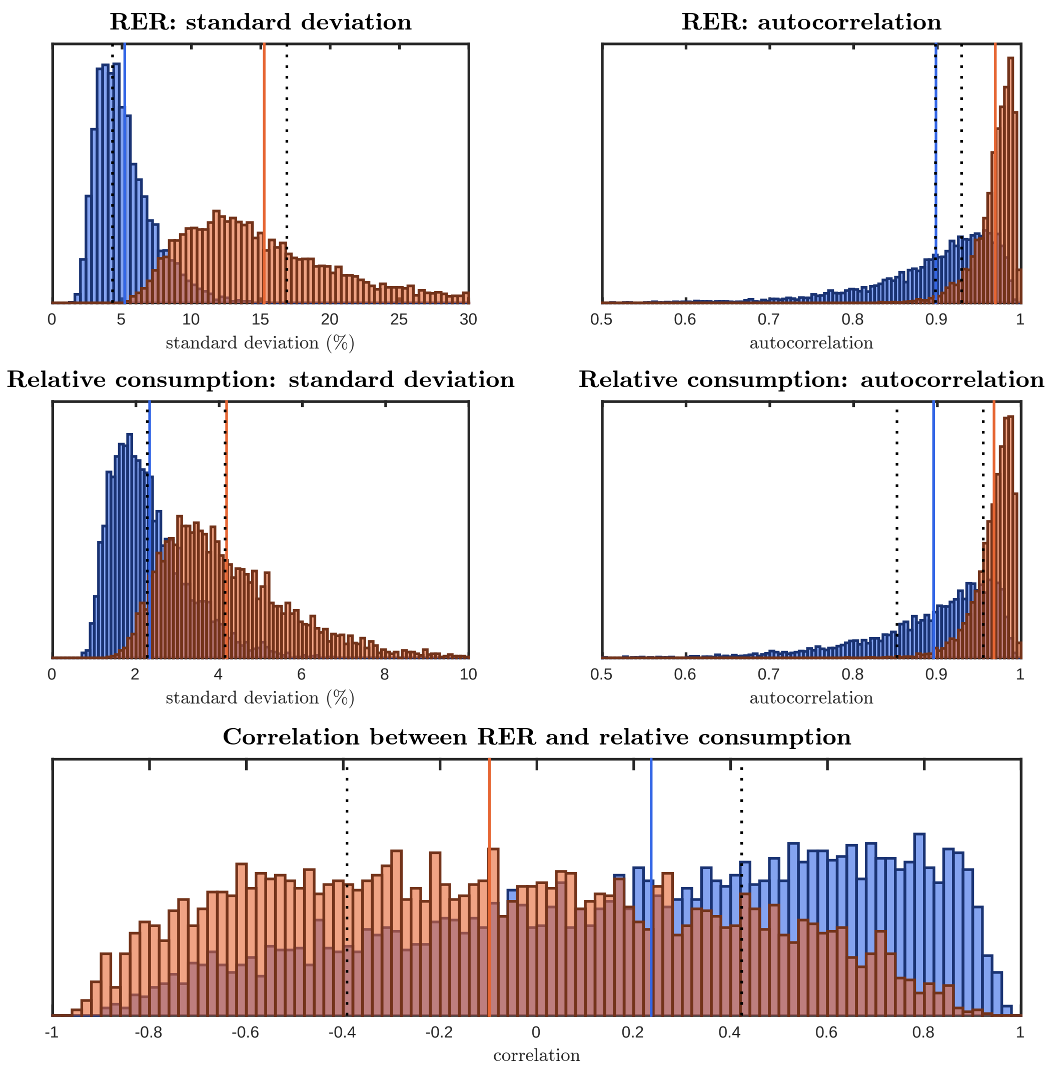


Figure 10: Higher persistence of commodity shocks ($\rho = 0.995$)

(b) Primary commodity prices