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Pablo Andrés Neumeyer
Universidad Torcuato Di Tella

Juan Pablo Nicolini
Federal Reserve Bank of Minneapolis and Universidad Torcuato Di Tella

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The Incredible Taylor Principle*

Pablo Andrés Neumeyer

Universidad Torcuato Di Tella

pan@utdt.edu

Juan Pablo Nicolini

Federal Reserve Bank of Minneapolis and Universidad Torcuato Di Tella

juanpa@utdt.edu

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Abstract

This note addresses the role of the Taylor principle to solve the indeterminacy of equilibria in economies in which the monetary authority follows an interest rate rule. We first study the role of imposing two additional ad-hoc restrictions on the definition of equilibrium. Imposing the equilibrium to be locally unique never delivers a unique outcome. Imposing the equilibrium to be bounded, renders the outcome unique only if the inflation target is the Friedman rule. Second, we show that the Taylor principle is strongly time inconsistent - in a sense we make very precise - and that policies that implement the Friedman rule are the only sustainable policies.

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1 Introduction

This note addresses the role of the Taylor principle as a device to solve the indeterminacy of equilibria in economies in which the monetary authority follows an interest rate rule.

Sargent and Wallace (1975) showed that in models with rational expectations, when the monetary authority targets a nominal interest rate, there is a continuum of equilibrium price level paths indexed by the initial price level. A proposed solution to this indeterminacy problem is an interest rate feedback rule that follows the Taylor principle. It is commonly argued that this principle, together with additional restrictions on the conditions required for an allocation to be an equilibrium, solves the indeterminacy problem.

Cochrane (2011) challenges on two grounds the logic supporting the uniqueness of the price level under the Taylor principle. First, he argues that the additional restrictions are ad-hoc, not based on economic theory, and that in some circumstances, they do not render the equilibrium unique. We go beyond his analysis and show that none of the additional restrictions renders the equilibrium unique, except in very exceptional circumstances that do not resemble the way monetary policy is executed in modern economies. Second, he also criticizes the Taylor principle by arguing that even if one were to accept the ad-hoc restrictions, determinacy of the nominal equilibrium is achieved by "threats to blow up the world". We formalize this second criticism by studying the credibility of the Taylor principle.

Regarding our first contribution, we study two restrictions to equilibrium outcomes. We first explore the role of restricting equilibria to be locally unique, a criterion used by Woodford (2003). We show that this restriction does rule out some equilibrium outcomes if the target for the nominal interest rate is strictly positive. However, in no case does this restriction alone render the equilibrium unique. Second, we explore the role of restricting equilibria to exhibit bounded inflation, a restriction discussed at length by Cochrane (2011). We show that this restriction always rules out some outcomes. However, only if the target for the nominal interest rate is zero every period–the Friedman rule–is a unique equilibrium obtained. For an inflation target above the Friedman rule to be a unique equilibrium, it is necessary to assume the equilibrium inflation is bounded, equilibria are locally unique, and the Taylor principle holds.

Our main contribution is to show that the Taylor principle involves following policies that no rational policy maker will choose to implement. We find that the Taylor principle is strongly time inconsistent, in the sense that there are histories after which the planner would find it optimal to switch to any policy that delivers bounded inflation. We then study sustainable equilibria as defined in Chari and Kehoe (1990). We show that policies that implement the Friedman rule are the only sustainable policies. The two results combined imply that the Taylor principle is not a sustainable policy.

As we study the incentives of future governments to change their policy rules, we naturally consider monetary rules in which the parameters can change over time. This formulation of interest policy reveals that this generalization of the Taylor principle imposes almost no restrictions on the nominal interest rate's reaction to deviations from the inflation target in the near future. The strong reaction of interest rates to deviation from the target required for uniqueness needs to hold only after some arbitrarily distant future. In our view, this casts further doubt on the the Taylor principle's role in restricting the set of nominal equilibria, as well as on the ability to identify the restrictions estimated monetary rules impose on the equilibrium set.

The paper is structured as follows. Section 2 presents the monetary model and establishes our results regarding the determinacy of the price level. Then, in section 3, we show that interest rules that follow the Taylor principle are strongly time inconsistent and that the only sustainable interest rate rule is the Friedman rule. Section 4 concludes and discusses the applicability of our results to more general environments. We argue that the price level determinacy and strong time inconsistency result extends to models with nominal rigidities. However, the unique sustainable outcome result would not extend to those environments. Overall, our results reinforce Cochrane's (2011) critical view of the Taylor principle.

2 A Simple Monetary Model

We analyze the interplay between the monetary authority's commitment to certain interest rate rules and the determinacy of the competitive equilibrium in a bare bones monetary model in which inflation is socially costly. We do so by introducing money into the utility function in Cochrane's (2011) frictionless model.[1] This allows us to evaluate the welfare properties of the nominal equilibrium, which we show is identical to the one in Cochrane (2011) and Woodford (2003).

Consider a monetary economy with a representative agent. Preferences are described by

$$U_t = \sum_{j=0}^{\infty} \beta^j \left[u\left(C_{t+j}\right) + H\left(M_{t+j}/P_{t+j}\right)\right], \tag{1}$$

where C_t is period t, consumption; M_t is end of period money balances; P_t is the price level in period t, $u : R_+ \to R$ is increasing, concave, and differentiable; $H : R_+ \to R$, is concave, increasing and satisfying $\lim_{m\to 0} \frac{\partial H}{\partial m}(m) = \frac{\partial u}{\partial C}(Y)$; and there is a finite $\bar{m}$ such that for all $m \geq \bar{m} : \frac{\partial H}{\partial m}(m) = 0$. Assumptions on H ensure that optimal real money balances go to zero as the nominal interest rate goes to infinity and that there is satiation in money balances.[2]

The household's present value budget constraint in period t is

$$\sum_{j=0}^{\infty} q_{t,t+j}\left[C_{t+j} + (1 - Q_{t+j,t+j+1})m_{t+j}\right] = \frac{M_{t-1} + B_{t-1}}{P_t} + \sum_{j=0}^{\infty} q_{t,t+j}\left(Y + T_{t+j}\right), \tag{2}$$

where Y is a constant non-storable endowment, $m_t \equiv M_t/P_t$ is real money balances, B_t is one-period claims on a unit of money delivered at date $t+1$, T_t is lump sum transfers, $q_{t,t+j} \equiv \prod_{r=0}^{j-1}\left(1 + r_{t+r}\right)^{-1}$ is the price of a unit of consumption in period $t+j$ in terms of consumption in period t, and $Q_{t,t+1} \equiv (1 + i_t)^{-1}$ is the price of a unit of money in period $t+1$ in terms of money in period t.

For a given sequence of prices, initial holdings of money (M_{-1}), and bonds (B_{-1}), the consumer's problem is to maximize utility (1) subject to the budget constraint (2) for $t = 0$.

[1] We follow the notation in Cochrane (2011).

[2] All our results hold for a broad set of preferences, for which the Friedman rule is optimal, characterized in Alvarez, Kehoe, and Neumeyer (2004).

We assume a Ricardian fiscal policy in which the government sets transfers, T_t, so that the government's present value intertemporal budget constraint is satisfied for any initial price level. Without loss of generality, since transfers are lump sum, we assume a balanced budget, so that real government debt, $(M_{t-1} + B_{t-1})/P_t$, is constant. Transfers must be set to

$$T_t = (1 - Q_{t,t+1})\, m_t - (1 - \beta)\, \frac{M_{-1} + B_{-1}}{P_0}. \tag{3}$$

The monetary authority follows the interest rate rule

$$1 + i_t = \Phi(\pi_t, t),$$

where $\pi_t = P_t/P_{t-1} - 1$. Observe that if nominal interest rates are negative, there is an arbitrage that would allow households to attain infinite wealth. Therefore, we restrict nominal interest rates to be non-negative: $\Phi(\pi_t, t) \geq 1$.

In addition, we restrict our analysis to policy rules $\Phi(\pi_t - \bar{\pi}_t, \pi_{t-1}; \phi_t, \bar{\pi}_t, \bar{\pi}_{t+1})$, which satisfy

$$\Phi(\cdot) = \begin{cases} \max\left\{\beta^{-1}\left[1 + \bar{\pi}_{t+1} + \phi_t(\pi_t - \bar{\pi}_t)\right], 1\right\} & \text{for } t = 0 \text{ and } \forall t > 0 \text{ such that} \\ & \forall s \leq t : \Phi(\pi_s, s) > 1 \\ 1 & \text{if } \Phi(\pi_{t-1}, t-1) = 1 \text{ and } t > 0. \end{cases} \tag{4}$$

The rule is specified by two parameters per period, the target for inflation next period, $\bar{\pi}_{t+1}$, and the the policy rate's response to current inflation, ϕ_t. Note that the initial target for inflation, $\bar{\pi}_0$, is a predetermined variable. We allow the parameters to be time dependent to explore the incentives future governments may have to change the rule.

We assume that the inflation target is bounded between the one associated with the Friedman rule and a finite number so that $\beta - 1 \leq \bar{\pi}_t \leq \bar{\pi}^{\max} < \infty$ and with the feedback parameter is non-negative and finite, $0 \leq \phi_t < \infty$. The rule implies that if the interest rate hits the zero bound after the initial period, it remains there.[3]

The family of interest rate rules described by equation (4) encompasses many of the cases considered in the literature. It is also general enough, in the sense that it can implement very broad classes of equilibria, as we show below.

It is quite standard in the literature to assume that $\{\bar{\pi}_{t+1+s}, \phi_{t+s}\}_{s=0}^{\infty} = \{\bar{\pi}, \phi\}_{s=0}^{\infty}$ for all t and that the Taylor principle, $\phi > 1$, is satisfied, so that $\lim_{t\to\infty} \phi^t \to \infty$. In our more general set-up, we will define the *generalized Taylor principle* as the condition

$$\lim_{T\to\infty} \Pi_{t=0}^{T} \phi_t \to \infty. \tag{5}$$

Notice that a necessary condition for the generalized Taylor principle to hold is that ϕ_t must be strictly larger than zero for all periods.

[3]This simplifies the analysis, as we do not have to consider dynamics in which after interest rates hit the zero bound inflation rises above $\beta - 1$ and restarts the dynamical system. This could happen because either there is a high inflation target or a feedback parameter below one at some t after the zero bound is hit. In the case usually considered in the literature, for example in Benhabib, Schmitt-Grohe, and Uribe (2001), with a constant inflation target $\bar{\pi}$ and a constant feedback parameter $\phi > 1$, the zero bound on nominal interest rates is an absorbing value.

2.1 Definition and Characterization of Equilibria.

Definition 1. **(Competitive Equilibrium)** Given initial values $\{P_{-1}, M_{-1}, B_{-1}, \bar{\pi}_0\}$ and any interest rate rule $\Phi(\pi_t, t)$ as defined in equation (4), the quantities and prices $\mathcal{E} = \{c_t, M_t, q_{t,t+1}, Q_{t,t+1}, \pi_t\}_{t=0}^{\infty}$ are a *competitive equilibrium* if, and only if,

i $\{c_t, M_t\}_{t=0}^{\infty}$ solve the household's problem; that is, for given prices and initial conditions the allocation maximizes equation (1) subject to equation (2) at $t = 0$;

ii transfers T_t are set so that the fiscal policy is Ricardian; that is, the conditions in equation (3) are satisfied for for all $t \geq 0$;

iii the resource constraints $C_t = Y$ are satisfied for all $t \geq 0$.

The definition of equilibrium implies that an inflation sequence and an allocation are a competitive equilibrium if, and only if, the fiscal rule in equation (3) and the following conditions are satisfied for $t = 0, 1, \ldots$:

$$\pi_{t+1} = \beta\Phi(\pi_t, t) - 1 \tag{6a}$$

$$\frac{\frac{\partial H}{\partial m}(m_t)}{\frac{\partial u}{\partial C}(C_t)} = \frac{\Phi(\pi_t, t) - 1}{\Phi(\pi_t, t)} \tag{6b}$$

$$C_t = Y \tag{6c}$$

$$q_{t,t+1} = \beta \tag{6d}$$

$$Q_{t,t+1}^{-1} = \Phi(\pi_t, t). \tag{6e}$$

Equilibrium inflation sequences are the solution to the first equation, which can be solved independently of the rest. Equation (6a), the Fisher equation $1 + i_t = \beta^{-1}(1 + \pi_{t+1})$, results from the interest rate rule and the household's Euler equation. The price level is then determined using the definition $P_t = (1 + \pi_t) P_{t-1}$ and the given value of P_{-1}.

The money demand is recovered from the first order conditions, equation (6b), and it will be convenient to write it as

$$m(1 + i_t) \equiv H_M^{-1}\left(\frac{(1 + i) - 1}{1 + i}\right), \tag{7}$$

taking $\frac{\partial u}{\partial C}(Y)$ as a constant. Assumptions on preferences imply that m is decreasing in $1 + i$, and it attains a maximum at the Friedman rule with $m(1) = \bar{m}$ and its minimum when the interest rate tends to infinity, $\lim_{i\to\infty} m(1 + i) = 0$.

The money market equilibrium embedded in equation (7) plays no role in the determination of the nominal equilibrium, which is exactly the same as the one in Cochrane (2011) and Woodford (2003). Given the interest rate set by the monetary authority and the equilibrium price level, equation (7) determines the money supply the central bank will accommodate through open market operations in order to peg the interest rate.

The real interest rate is the inverse of the discount factor, and nominal interest rates are given by the interest rate rule Φ. Resource balance, equation (6c), and the fiscal rule, equation (3), ensure allocations are budget feasible.

We now discuss the mapping between policy rules, pairs $\{\phi_{t+1}, \bar{\pi}_t\}_{t=0}^{\infty}$, and equilibrium outcomes. Under the interest rule described in equation (4) and the equilibrium conditions equation (6a), an inflation sequence is an equilibrium if, and only if, it satisfies equation (8):

$$\pi_1 - \bar{\pi}_1 = \max\left[\phi_0\left(\pi_0 - \bar{\pi}_0\right), \beta - 1 - \bar{\pi}_0\right] \tag{8a}$$

$$\pi_t - \bar{\pi}_t = \left(\prod_{j=0}^{t-1}\phi_j\right)\left(\pi_0 - \bar{\pi}_0\right) \qquad \text{if } t > 1 \text{ and } \forall s \le t : \pi_s > \beta - 1 \tag{8b}$$

$$\pi_t = \beta - 1 \qquad \text{if } t > 1 \text{ and } \pi_{t-1} = \beta - 1. \tag{8c}$$

The equilibrium conditions for inflation when the monetary authority pegs the interest rates following the feedback rule in equation (4) are a dynamical system without a boundary condition. The nominal equilibrium is indeterminate in the sense that there is a family of equilibrium inflation sequences indexed by the initial inflation π_0, as explained in Sargent and Wallace (1975).

We find it convenient to partially characterize the set of equilibria when the policy rule satisfies the generalized Taylor principle.

Lemma 1. Assume the interest rate policy rule satisfies the generalized Taylor principle in equation (5). Then,

1. $\pi_0 > \bar{\pi}_0 \Rightarrow \lim_{t\to\infty} \pi_t = \infty$;
2. $\pi_0 = \bar{\pi}_0 \Rightarrow \pi_t = \overline{\pi}_t$ for all $t \ge 0$;
3. $\pi_0 < \bar{\pi}_0 \Rightarrow \forall t : \pi_t \le \bar{\pi}_t$ and $\exists t^{ZB} : \pi_t = \beta - 1$ for all $t \ge t^{ZB}$.

Proof. Recall first that if the generalized Taylor principle holds, $\phi_t > 0$ for all t, so $\left(\prod_{j=1}^{t}\phi_{j-1}\right) > 0$ for all t. The proof of points 1, 2 and the first part of 3 follow directly from equation (8). Point 1 uses the fact that the inflation target is bounded below by $\beta - 1$. To prove the second part of 3, notice that $\pi_0 - \bar{\pi}_0 < 0$ implies $\lim_{t\to\infty}\left(\prod_{j=1}^{t}\phi_{j-1}\right)\left(\pi_0 - \bar{\pi}_0\right) \to -\infty$, while $\beta - 1 - \bar{\pi}_t$ is bounded below by $\beta - 1 - \bar{\pi}^{\max}$. It follows that there exists a t^{ZB} such that $\max\left[\left(\prod_{j=1}^{t}\phi_{j-1}\right)\left(\pi_0 - \bar{\pi}_0\right), \beta - 1 - \bar{\pi}_t\right] = \beta - 1 - \bar{\pi}_t$ for all $t \ge t^{ZB}$. As the zero bound is an absorbing state for the interest rate rule in equation (4), $\pi_t = \beta - 1$ for all $t \ge t^{ZB}$. ∎

Lemma 1 shows that if equilibrium inflation is higher than the target at time zero, then inflation grows without bound. If it is equal to the target in the first period, it will be equal to its target every period, while if it is lower than the target, it eventually reaches its lower bound in finite time. This discussion highlights why the generalized Taylor principle in itself, does not reduce the set of equilibria, as emphasized in Cochrane (2011). The generalized Taylor principle, however, implies that a sub-class of the equilibria involves unbounded inflation sequences. A first attempt at refining the equilibrium concept, discussed at length in Cochrane (2011), is to impose the ad-hoc restriction that equilibrium sequences be bounded. Below, we show that this restriction is not enough to obtain a unique equilibrium, unless the target for inflation is consistent with the Friedman rule of zero interest rates every period. Lemma 2 summarizes these results from Cochrane (2011).

Lemma 2. The equilibrium price level is unique if, and only if, the following three conditions hold:

i The generalized *Taylor principle* $\lim_{t\to\infty}\prod_{j=0}^{t}\phi_j = \infty$ holds.

ii Inflation sequences are required to be *bounded*; that is, $\sup_{t\in\mathbb{N}}|\pi_t| < \infty$.

iii The inflation target is the *Friedman rule*, $\bar{\pi}_t = \beta - 1$ for all t.

Proof. Assume that $\bar{\pi} = \beta - 1$ and $\lim_{t\to\infty}\prod_{j=0}^{t}\phi_j = \infty$. Then, equation (8) implies that if $\pi_0 = \beta - 1$, then $\pi_t = \beta - 1$ for all t. If $\pi_0 > \bar{\pi}$, the term $(\pi_0 - \bar{\pi})\prod_{j=0}^{t}\phi_j$ diverges as $t \to \infty$, and the sequence implied by equation (8) is not bounded. If $\pi_0 < \bar{\pi}$, the zero lower bound is violated. Therefore, the characterized equilibrium is unique. To prove that uniqueness implies $\lim_{t\to\infty}\prod_{j=0}^{t}\phi_j = \infty$, note that inspection of equation (8) shows that if $\prod_{j=0}^{t}\phi_j < \infty$ for all t, any $\pi_0 \in (\bar{\pi}, \infty)$ is a bounded equilibrium. To prove that uniqueness implies $\bar{\pi} = \beta - 1$, observe that if $\bar{\pi} > \beta - 1$ and $\lim_{t\to\infty}\prod_{j=0}^{t}\phi_j = \infty$, then any $\pi_0 \in (\beta - 1, \bar{\pi})$ is an equilibrium with bounded inflation as inflation will converge to $\beta - 1$. If $\lim_{t\to\infty}\prod_{j=0}^{t}\phi_j < \infty$, inflation will remain bounded for any $\pi_0 \in (\beta - 1, \bar{\pi})$. □

An alternative criterion, popularized by Woodford (2003), is to consider only equilibria that are locally unique. We first define local uniqueness and then show that the equilibria with $\pi_0 \geq \bar{\pi}_0$ in lemma 1 are locally unique, while those with $\pi_0 < \bar{\pi}_0$ are not.

Let $d(\pi, \pi') = \sup_t\{\pi_t - \pi'_t\}$ be the distance between two points in $\mathbb{R}^\infty$, where $\pi = \{\pi_t\}_{t=0}^{\infty}$, $\pi' = \{\pi'_t\}_{t=0}^{\infty}$. Define the set $\mathbb{P}(\pi, \epsilon) = \{\pi' \in \mathbb{R}^\infty : d(\pi, \pi') < \epsilon \text{ for some } \epsilon > 0\}$. An equilibrium inflation sequence, π, is locally unique if there exists an $\epsilon > 0$ such that the equilibrium is the unique equilibrium in the set $\mathbb{P}(\pi, \epsilon)$.

Lemma 3. Assume the interest rate policy rule satisfies the generalized Taylor principle in equation (5). Then,

i all equilibria with $\pi_0 \geq \bar{\pi}_0$ are locally unique;

ii all equilibria with $\pi_0 < \bar{\pi}_0$ are not locally unique.

Proof. The proof proceeds in three steps. Step 1 considers cases with π_0 at or above the initial target, step 2 the case with initial inflation at the target, and step 3 paths that end up at the Friedman rule.

Step 1. Consider equilibria with $\pi_0 \geq \bar{\pi}_0$. Let π, π' be two equilibrium inflation sequences with $\pi'_0 = \pi_0 + \delta$ for some $\delta > 0$. Equation (8b) implies that $\pi'_t - \pi_t = \left(\prod_{j=0}^{t-1}\phi_j\right)\delta$. Under the generalized Taylor principle, $\lim_{t\to\infty}|\pi'_t - \pi_t| = \infty$. This implies that in this case, $\nexists\ \epsilon < \infty : d(\pi, \pi') < \epsilon$. All the equilibria with $\pi_0 > \bar{\pi}_0$ are locally unique. For $\pi_0 = \bar{\pi}_0$ we have to verify that there are no other equilibria in an open ball close to $\bar{\pi}$ that start below $\bar{\pi}_0$.

Step 2. For the case of $\pi_0 = \bar{\pi}_0$ and $\pi'_0 = \bar{\pi}_0 - \delta$, assume π' reaches $\pi'_T = \beta - 1$ at time T so that that $\pi_T - \pi'_t = \bar{\pi}_T - (\beta - 1) = \left(\prod_{j=0}^{T-1}\phi_j\right)\delta$. For $\epsilon = 1/2\left(\prod_{j=0}^{T-1}\phi_j\right)\delta$, for all δ, the distance $d(\pi, \pi') > \epsilon$. Combining this result with step 1, we conclude that the path $\pi = \bar{\pi}$ is locally unique.

Step 3. Finally, consider the case $\pi_0 = \pi'_0 + \delta < \bar{\pi}_0$. All these sequences hit $\beta - 1$ in finite time and stay there. Let π' reach the zero bound at time T, so that $\pi_t - \pi'_t = \left(\prod_{j=0}^{t-1}\phi_j\right)\delta$ for $t \leq T$. This implies that for all $t \leq T$ and $\epsilon > 0$, there is a

$\delta > 0$ such that $\sup \left[\left(\prod_{j=0}^{t-1} \phi_j \right) \delta \right] < \epsilon$. For $t > T$, $\pi_t - \pi'_t < \pi_T - (\beta - 1) = \left(\prod_{j=0}^{T-1} \phi_j \right) \delta$. Therefore, for every ϵ there is a δ such that $d(\pi, \pi') < \epsilon$, so π is not locally unique. ■

The difference in the local uniqueness of the inflation paths that start at a value weakly larger, or strictly smaller, than the critical value $\bar{\pi}_0$ is that the former diverge as time evolves, while the latter converge to the lower bound in finite time. In the second case, there are always initial conditions that are close enough for their subsequent paths to be very close until the zero lower bound is hit. The inflation target is locally unique because paths with $\pi_0 > \bar{\pi}_0$ diverge to infinity, and paths with $\pi_0 < \bar{\pi}_0$ converge to the Friedman rule. Clearly, if the target is always the Friedman rule, then all equilibria are locally unique.

In summary, the requirement that inflation ought to be bounded does reduce the set of equilibria. However, it uniquely selects the inflation target only when it is the Friedman rule every period. In contrast, the requirement that the equilibrium be locally unique never uniquely selects the inflation target equilibrium. Moreover, if the target is the Friedman rule every period, then the requirement does not reduce the set of equilibria.

We close the section on the characterization of equilibria by pointing out that the Taylor principle imposes almost no restrictions on short-run monetary policy. By considering monetary rules in which the parameters can change over time, it becomes evident that the only restriction that the Taylor principle imposes on the interest rate's response to inflation (which we assumed to be non-negative) is that it be strictly positive for an arbitrarily large but finite amount of time, as we argue in remark 1.[4]

Remark 1. The *generalized Taylor principle* implies that uniqueness of the equilibrium under the conditions in lemma 2 is independent of the policy rule in the near future and depends exclusively on the policy rule implemented after some arbitrary period of time.

Remark 1 follows from the fact that $\lim_{t\to\infty} \prod_{j=0}^{t} \phi_j = \left(\prod_{j=0}^{T-1} \phi_j \right) \lim_{t\to\infty} \left(\prod_{j=T}^{t} \phi_j \right)$, which implies that $\lim_{t\to\infty} \prod_{j=0}^{t} \phi_j = \infty$ if, and only if, $\lim_{t\to\infty} \prod_{j=T}^{t} \phi_j = \infty$ for some $T > 0$. It follows that as long as $\phi_t > 0$, the generalized Taylor principle is independent of the values ϕ_t for $t \leq T$.

3 On the Credibility of Interest Rate rules

In this section, we explore the credibility of interest rate rules. First we discuss the mapping from policies to welfare, and then we discuss the time inconsistency of the Taylor principle. Finally, we show that the Friedman rule is the only sustainable interest rate policy.

We assume social welfare corresponds to that of the representative agent Consider a sequence of equilibrium inflation rates, satisfying equation (8) under the interest rate rule in equation (4), and the resulting equilibrium interest rates. Associated with these interest rates, there is an equilibrium allocation of real money balances given by

[4]Experimenting with determinacy issues under Taylor rules in continuous time models, Guillermo Calvo arrived to a similar conclusion (private communication).

$m_t = m(1+i_t)$, defined in equation (7). For a sequence of equilibrium interest rates, welfare is given by the indirect utility function

$$V_t = \sum_{j=0}^{\infty} \beta^j \left[u(Y) + H\left(m(1+i_{t+j})\right)\right] \tag{9}$$
$$= u(Y) + H\left(m(1+i_t)\right) + \beta V_{t+1}.$$

Welfare is independent of initial wealth because of the assumption that fiscal policy is Ricardian; that is, equation (3). Assumptions on preferences imply the indirect utility function $H\left(m(1+i_t)\right)$ is decreasing in $1+i$. It attains its minimum when $i \to \infty$ and its maximum at the Friedman rule when $i = 0$. That is, $\partial H\left(m(1+i)\right)/\partial(1+i) < 0$, and

$$\lim_{i\to\infty} H\left(m(1+i)\right) \leq H\left(m(1+i)\right) \leq H\left(m(1)\right) \text{ for all } 1+i \in [1,\infty).$$

In order to prove the main results, it is convenient to define histories as sequences of interest rate policy parameters and equilibrium inflation outcomes. Specifically, use $\Pi = \{\Pi_t\}_{t=0}^{\infty}$ to denote an interest rate policy history, where $\Pi_t = (\bar{\pi}_{t+1}, \phi_t)$ is the interest rate policy at time t.

The discussion in lemma 1 makes clear that we can index any equilibrium under policy Π by π_0. Let $h_t(\pi_0, \Pi) = (\Pi_t, \pi_t - \bar{\pi}_t, \pi_{t-1})$ be the history at t under policy Π when initial equilibrium inflation was π_0. Past inflation is included in h_t as an indicator of whether the economy was at the zero lower bound for the nominal interest rate in period $t-1$. We define the history up to time t as $h^t = \{h_0, h_1, \ldots, h_t\}$.

We can then write the interest rate rule in equation (4) as $\Phi(h_t)$, a function of the history at t. This enables us to write the equilibrium value of welfare in equation (9) as a function of histories. The welfare associated with a history that starts at π_0 under policy Π is

$$V(h_t) = u(Y) + H\left(\Phi(h_t)\right) + \beta V(h_{t+1}). \tag{10}$$

It is convenient to characterize the welfare associated with each initial equilibrium inflation under a policy that satisfies the generalized Taylor principle.

Lemma 4. Consider the policy rule in equation (4) with feedback parameters satisfying the generalized Taylor principle, $\lim_{t\to\infty} \prod_{j=0}^{t} \phi_j \to \infty$. Then, equilibrium welfare is characterized by the following:

1. If $\pi_0 > \overline{\pi}_0$, then $\lim_{t\to\infty} (1-\beta)V(h_t) = [u(Y) + H(0)]$.
2. If $\pi_0 < \overline{\pi}_0$, then $\lim_{t\to\infty} (1-\beta)V(h_t) = [u(Y) + H(\overline{m})]$.
3. If $\pi_0 = \overline{\pi}_0$, then $V(h_t) = \sum_{s=0}^{\infty} \beta^s [u(Y) + H\left(m(\beta^{-1}(1+\overline{\pi}_{t+s+1})\right)]$.

Proof. The lemma follows directly from Lemma 1 and the properties of the indirect utility function $H\left(m(1+i_t)\right)$. □

Lemma 4 establishes that under an interest rate feedback rule satisfying the generalized Taylor principle, welfare will converge to its minimum value if equilibrium initial inflation is above its target, will converge to its maximum if equilibrium initial inflation is below its target, and it will correspond to the welfare associated with the sequence of inflation targets if equilibrium initial inflation is equal to its target.

3.1 Strong Time Inconsistency

In order to grasp the social costs of commitment to an interest rate rule that satisfies the (generalized) Taylor principle, we introduce the concept of *strong time inconsistency*. Time consistency arises when at any given future period $\hat{t}$, the policy that is optimal from the view point of period $\hat{t}$ dominates the policy rule adopted at time zero. Our use of the qualifier "strong" indicates a requirement that the social planner may find it optimal to switch to *any* rule within a class of rules. In this way, to be strongly time inconsistent, a rule adopted at time zero ought to perform worse than all policy rules within that previously specified class of rules, once re-evaluated at some time $\hat{t}$.

The strength of the definition depends on the broadness of the class of policies that the rule is compared with at time $\hat{t}$. In what follows, we argue that the generalized Taylor principle is strongly time inconsistent when compared with a class of policies that implement equilibria that exhibit bounded inflation. This is so for ***any*** finite bound established for inflation.

Next, we define the set of bounded equilibria and a complete set of policies that support it.

Definition 2. The ***set of bounded equilibria*** $\mathcal{E}(\hat{\pi}, \hat{t})$ is the set of all possible competitive equilibria such that $\pi_t \leq \hat{\pi}$ for all $t \geq \hat{t}$.

We now define a *complete class of policy rules* in reference to the set of bounded equilibria $\mathcal{E}(\hat{\pi}, \hat{t})$ as a set of policies, $\widehat{\Pi}\left(\hat{\pi}, \hat{t}\right)$, that is general enough for every element in $\mathcal{E}\left(\hat{\pi}, \hat{t}\right)$ to be an equilibrium given a policy $\Pi \in \widehat{\Pi}\left(\hat{\pi}, \hat{t}\right)$ and restrictive enough to exclude policies with equilibria that do not belong to $\mathcal{E}(\hat{\pi}, \hat{t})$. At the same time, it allows us to disregard redundant policies $\Pi' \notin \widehat{\Pi}\left(\hat{\pi}, \hat{t}\right)$ that also induce bounded equilibria.

Definition 3. A set of policies $\widehat{\Pi}\left(\hat{\pi}, \hat{t}\right)$ initialized at $\hat{t}$ is a ***complete class of policies for a given*** $\mathcal{E}\left(\hat{\pi}, \hat{t}\right)$ if, and only if,

- i for all sequences $\{\pi_{t+1}\}_{t=\hat{t}}^{\infty} \in \mathcal{E}\left(\hat{\pi}, \hat{t}\right) : \exists\ \Pi \in \widehat{\Pi}\left(\hat{\pi}, \hat{t}\right)$ such that $\{\pi_{t+1}\}_{t=\hat{t}}^{\infty}$ is a competitive equilibrium given policy Π, and
- ii $\Pi \in \widehat{\Pi}\left(\hat{\pi}, \hat{t}\right) \Rightarrow$ any equilibrium $\{\pi_{t+1}\}_{t=\hat{t}}^{\infty} \in \mathcal{E}\left(\hat{\pi}, \hat{t}\right)$.

Remark 2. The Sargent and Wallace (1975) interest rate rules are a familiar example of a complete class of policies in which interest rates do not depend on the state of the economy. In the case of policies of the form $\{\Pi_t = (\bar{\pi}_{t+1}, 0)\}_{t=\hat{t}}^{\infty}$, with $\bar{\pi}_{t+1} \in [\beta - 1, \hat{\pi}]$, the monetary authority can generate any equilibrium bounded inflation sequence $\{\pi_{t+1}\}_{t=0}^{\infty} = \{\bar{\pi}_{t+1}\}_{t=\hat{t}}^{\infty}$ by setting $i_t = \beta^{-1}(1 + \bar{\pi}_{t+1}) - 1$.

We say that a policy rule Π is *strongly* time inconsistent in reference to a set of equilibria bounded by $\hat{\pi}$ if there is a history $h^{\hat{t}}$ such that for all $t > \hat{t}$, the welfare attained by switching to any policy in the set $\widehat{\Pi}\left(\hat{\pi}, \hat{t}\right)$ is higher than at the continuation equilibrium under the policy Π. We can write this formally as follows:

Definition 4. A policy rule $\Pi = \{\Pi_t = (\bar{\pi}, \phi)\}_{t=0}^{\infty}$ is ***strongly time inconsistent*** in reference to a set of bounded equilibria $\mathcal{E}(\hat{\pi}, \hat{t})$ if there exists a history $h^{\hat{t}}$ such that for all $\Pi' \in \widehat{\Pi}(\hat{\pi}, \hat{t}) : V\left(h_{\hat{t}}\left(\pi_0, \Pi\right)\right) < V\left(h_{\hat{t}}\left(\pi_{\hat{t}}, \Pi'\right)\right)$.

We will show that policies that follow the Taylor principle are strongly time inconsistent. Before delving into this issue, observe that the welfare associated with equilibria in the set $\mathcal{E}(\hat{\pi}, \hat{t})$ is bounded below by the upper bound on inflation. This is because the period utility from real money balances, $H(m(\beta^{-1}(1+\pi_{t+1}))$, is a decreasing function of inflation, and therefore its value at the upper inflation bound, $H(m(\beta^{-1}(1+\hat{\pi}))$, implies a lower bound on welfare, as stated in lemma 5.

Lemma 5. For any value of $\hat{\pi}$, if a policy $\Pi' \in \widehat{\Pi}\left(\hat{\pi}, \hat{t}\right)$, then $V\left(h_t\left(\pi_{\hat{t}}, \Pi'\right)\right) \geq \underline{V}(\hat{\pi})$ for all $\pi_{\hat{t}}$ and for all $t \geq \hat{t}$, with $\underline{V}(\hat{\pi}) \equiv \frac{u(Y)+H\left(m\left(\beta^{-1}(1+\hat{\pi})\right)\right)}{1-\beta}$.

Lemma 5 states that for any arbitrary bound on inflation $\hat{\pi}$, a policy that starts at $\hat{t}$ with initial condition $\pi_{\hat{t}}$ and belongs to the complete class of policies for $\mathcal{E}\left(\hat{\pi}, \hat{t}\right)$ has a lower welfare bound $\underline{V}(\hat{\pi})$. Lemma 5 uses the second property of complete policies in definition 3.

The results in lemma 4 imply that if $\pi_0 > \bar{\pi}_0$, policies that satisfy the (generalized) Taylor principle eventually will have a lower welfare than all the policies in the set $\hat{\Pi}(\hat{\pi})$ because they will cross the boundary in lemma 5 from above.

The following proposition states that a policy that follows the Taylor principle is strongly time inconsistent.

Proposition 1. Assume the interest rate rule Π satisfies the generalized Taylor principle. Then, for any value of $\hat{\pi}$, the interest rate rule Π is strongly time inconsistent in reference to the set of bounded equilibria $\mathcal{E}\left(\widehat{\pi}, \hat{t}\right)$.

Proof. Let $\hat{\pi}$ be any finite bound on the inflation rate, and let $\bar{\pi}_0$ be the inflation target at time zero. As Π satisfies the generalized Taylor principle, lemma 4 implies that for any initial value for inflation $\pi_0 > \overline{\pi}_0$, $\lim_{t\to\infty} V(h_t(\pi_0, \Pi)) = \frac{u(Y)+H(0)}{1-\beta}$. As $H(0) < H(\widehat{\pi})$, it follows that there exists a $\hat{t}$ such that for all $t \geq \hat{t}$ and for all $\Pi' \in \widehat{\Pi}(\hat{\pi}, \hat{t})$,

$$V\left(h_t\left(\pi_0, \Pi\right)\right) < \underline{V}(\hat{\pi}) = \frac{u(Y) + H(\widehat{\pi})}{1-\beta} \leq V\left(h_t\left(\pi_{\hat{t}}, \Pi'\right)\right),$$

where the last inequality follows from lemma 5. ∎

The intuition behind the proposition is that for $\pi_0 > \bar{\pi}_0$, the (generalized) Taylor principle eventually commits the monetary authority to ever-increasing rates of inflation, which make money costlier to use as time goes by. Any policy that keeps inflation under some arbitrary upper bound makes money eventually cheaper to use than under a policy that follows the Taylor principle, and hence, compared to that policy, it improves welfare.

The definition we are using of a competitive equilibrium assumes that agents have perfect foresight and that they take the government's policies as given. However, we know from proposition 1 that the government has strong incentives to abandon a policy that adheres to the (generalized) Taylor principle. This conflict calls for an equilibrium concept in which private agents understand the government's incentives and correctly anticipate its actions, while the government understands this situation when it makes its own policy decisions. This equilibrium concept is discussed next.

3.2 Sustainable Interest Rate rules

We now look at sustainable interest rate rules using the equilibrium concept in Chari and Kehoe (1990)[5]. Sustainable interest rate rules induce competitive equilibrium allocations that are optimal choices for a benevolent planner for every possible future history.

For each period $t \geq 0$, at the beginning of the period, the monetary authority chooses the policy rule parameters $\mathcal{P}_t = (\bar{\pi}_{t+1}, \phi_t)$, given the history h_{t-1}, to solve the problem

$$V(h_{t-1}) = u(Y) + \max_{\bar{\pi}_{t+1}, \phi_t} \left[H\left(\Phi\left(\pi_{t-1}, \pi_t - \bar{\pi}_t; \bar{\pi}_{t+1}, \phi_t, \bar{\pi}_t\right)\right) + \beta V(h_t) \right], \tag{11}$$

subject to equation (8). This choice is the *policy rule* $\mathcal{P}(h_{t-1}) = (\bar{\pi}_{t+1}, \phi_t)$. Future histories are induced by $h^t = \left\{h^{t-1}, h_t\right\}$, with $h_t = \left(\mathcal{P}(h_{t-1}), \pi_t - \bar{\pi}_t, \pi_{t-1}\right)$ and equilibrium inflation given by equation (8).

Definition 5. A *sustainable equilibrium* is a sequence of inflation rates and policy rules $\{\pi_t, \Pi(h_{t-1})\}_{t=0}^{\infty}$ such that
(i) the inflation rates $\{\pi_t\}_{t=0}^{\infty}$ are a competitive equilibrium (solution of equation (8));
(ii) the policy rules $\{\Pi(h_{t-1})\}_{t=0}^{\infty}$ solve the government's problem (equation (11)) for all possible future histories $\{h_t\}_{t=0}^{\infty}$ starting from h_{-1}.

A sustainable equilibrium differs from an equilibrium under commitment, as the latter requires the sequence of policies $\{\bar{\pi}_{t+1}, \phi_t\}_{t=0}^{\infty}$ to be optimal only from the perspective of date $t = 0$, given h_{-1}, while the former requires policies to be optimal for all possible future histories.

Proposition 2. A policy rule that implements the Friedman rule, $\pi_t = \beta - 1$ for all t, is the only sustainable equilibrium

Proof. If the policy rule implements the Friedman rule for all t, then H attains its maximum for every t, and welfare attains the first best. Any policy rule resulting in a positive nominal interest rate for some t reduces the utility of equilibrium real money balances and therefore it does not solve the government's optimization problem at that t. Hence, it cannot be sustainable. ■

Remark 3. The Sargent-Wallace rule of a fixed interest rate with $\phi_t = 0$ and $\bar{\pi}_{t+1} = \beta - 1$ for all t is a sustainable equilibrium that implements the first best. The equilibrium price level at date 0 is indeterminate, as inflation is determined only for $t \geq 1$.

4 Conclusion

We extended Cochrane's (2011) critical view of the Taylor principle as a device to reduce the set of nominal equilibria in economies in which the monetary authority follows an interest rate rule.

First, we showed that in general, adding the typical ad-hoc restrictions to the definition of an equilibrium does not render the equilibrium unique, even if the interest

[5]In this application of the idea of sustainable plans, it will not be necessary to make allocations a function of history.

rate rule satisfies the Taylor principle. The only case in which the Taylor principle does pin down a unique equilibrium is when the inflation rate is required to be bounded and the target for inflation is consistent with the Friedman rule of zero nominal interest rates every period.

Second, we showed that an interest rate rule that follows the Taylor principle is strongly time inconsistent, in the sense that there are histories under which a social planner would abandon the rule in favor of any policy that guarantees a bounded inflation rate, no matter how high the bound is allowed to be. We also showed that the Friedman rule is the unique sustainable equilibrium.

We deliberately chose a bare bones monetary model in which only anticipated inflation is costly. In this way, we could prove our results in the simplest possible way. But our determinacy results and the time incosistency results extends straightforwardly to other environments that do not exhibit neutrality of money, like those that allow for frictions in the setting of prices, or segmented markets. These features bring real determinacy problems and additional costs of inflation, so the same logic applies.

The uniqueness of the sustainable equilibrium, on the other hand, does hinge on the assumption that money is neutral. Sensible variations of the model with nominal rigidities will typically have multiple sustainable outcomes. But the fact that the Taylor principle is strongly time inconsistent implies that no sustainable outcome will be implemented with it.

In our analysis, we did not consider sunspot equilibria in which the price level is indexed by a non-fundamental random variable. However, as in our model sunspots have no welfare effects, they do not change our results. Sunspot equilibria would matter in models with frictions. The time inconsistency results would extend to these models, but in general, the uniqueness of the sustainable equilibrium will not.

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