Financial Integration and Monetary Policy Coordination

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Abstract

Financial integration generates macroeconomic spillovers that may require international monetary policy coordination. We show that individual central banks may set nominal interest rates too low or too high relative to the cooperative outcome. We identify three sufficient statistics that determine whether the Nash equilibrium exhibits under-tightening or over-tightening: the output gap, sectoral differences in labor intensity, and the trade balance response to changes in nominal rates. Independently of the shocks hitting the economy, we find that under-tightening is possible during economic expansions or contractions. For large shocks, the gains from coordination can be substantial.

Central banks nearly everywhere feel accused of being on the back foot. The present danger, however, is not so much that current and planned moves will fail eventually to quell inflation. It is that they collectively go too far and drive the world economy into an unnecessarily harsh contraction...by simultaneously all going in the same direction, they risk reinforcing each other's policy impacts without taking that feedback loop into account.

Maurice Obstfeld, "Uncoordinated monetary policies risk a historic global slowdown," blog post, Peterson Institute, 09/12/2022.

1 Introduction

After a prolonged period of expansionary monetary policy, central banks around the world have shifted to a tightening cycle to tame rising inflation. However, the rapid pace and synchronous nature of the increase in interest rates have raised concerns that the unprecedented monetary tightening could lead to a severe economic downturn. In this context, there has been a renewed discussion on the necessity of cooperation to avert a global recession and achieve a soft landing (Obstfeld, 2022).[1]

Does cooperative monetary policy prescribe lower interest rates than those of the non-cooperative scenario? Or is it possible that countries may insufficiently tighten monetary policy relative to the social optimum? In a broader sense, what are the benefits from international coordination of monetary policy, and how do they depend on the degree of financial integration?

The study of international monetary policy cooperation has a long history in the international macro literature, dating back to Hamada (1976), and Canzoneri and Henderson (1991). In the context of the traditional Mundell-Flemming model, early studies argued that countries have incentives to weaken their currencies to gain a trade advantage, thus resulting in competitive devaluations and widespread inflation. By contrast, modern international macro-models with explicit microfoundations, as exemplified by Obstfeld and Rogoff (1995) and Corsetti and Pesenti (2001), predict that countries have incentives to appreciate their currencies to improve their terms of trade and extract more rents from foreign countries. From this perspective, dealing with the strategic manipulation of terms of

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[1]See the quote of the Peterson Institute blog post above and Figure 1 for the evolution of inflation and policy rates in advanced economies.

trade calls for cooperation towards more expansionary monetary policies.[2] Moreover, the gains from cooperation in this literature emerge purely from trade flows and are present even in the absence of financial flows.

In this paper, we approach the questions on international monetary coordination from a different, intertemporal perspective. Central to our model is the notion that monetary policy has effects on an intertemporal price—namely, the world real rate—and through this channel, a central bank's policy affects the ability of other central banks to achieve their output and inflation stability objectives. Our analysis builds on the analysis of international spillovers in Bianchi and Coulibaly (2021), where we show how countries use monetary policy to raise their net foreign asset position and reduce their vulnerability to liquidity traps. In general equilibrium, however, a lower world interest rate results in larger incentives for households' borrowing, driving the central bank to deviate further from its efficient level of output in an attempt to raise its net foreign asset position. Given the intertemporal nature of this mechanism, distinct from the static terms of trade manipulation, we refer to it as the *financial channel of international spillovers*. Here, we compare the Nash equilibrium where individual countries set their monetary policy optimally with the equilibrium under the cooperative monetary policy and provide a general characterization of whether the financial channel requires coordinating on a more expansionary or a more restrictive monetary policy.[3]

Our main result is that the Nash equilibrium may feature nominal rates that are too high (*over-tightening*) or too low (*under-tightening*) relative to the cooperative outcome. We elucidate how the outcome depends on a small set of sufficient statistics—specifically, the output gap, the difference in labor intensity across sectors, and the response of the trade balance to movements in the exchange rate. Our findings show that when the economy faces a recession, the Nash equilibrium displays over-tightening if non-tradables are more labor intensive than tradables and the trade balance expands in response to a devaluation. However, there are plausible constellations where interest rates are too low in the Nash equilibrium. For example, the Nash equilibrium displays under-tightening when the economy faces overheating, non-tradables are more labor intensive and the trade balance expands in response to a devaluation, or when the economy faces a recession, non-tradables are more labor intensive and the trade balance *contracts* in response to a devaluation.

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[2]From a quantitative standpoint, however, the consensus in the literature following Obstfeld and Rogoff (1995) is that the gains from cooperation due to this trade channel are negligible.

[3]Fornaro and Romei (2022) tackle the problem of monetary policy coordination from a similar perspective. We discuss in detail below how our framework and conclusions differ from theirs.

The general logic behind these results is that countries do not internalize how using monetary policy to steer capital flows affects the world real rate and how this, in turn, affects welfare abroad. Depending on the output gap and labor intensities, countries benefit from increases or decreases in the world real rate.

To focus on a concrete example, consider an economy facing a negative output gap where non-tradables are more labor intensive. To the extent that wages are rigid and inflation is costly, the central bank finds it optimal to expand monetary policy to help reduce the output gap (at the expense of higher inflation). In this scenario, we argue that a reallocation of employment from a low labor-intensive sector to a high labor-intensive sector helps mitigate inflation because the high labor-intensive sector has a lower elasticity of marginal cost with respect to output. Consequently, to the extent that the non-tradable sector is more labor intensive than the tradable sector, a shift in employment towards non-tradables would lead to an overall reduction in inflation.

In turn, the allocation of employment across sectors depends crucially on financial flows and the world real rate. If the world real rate is lower, households borrow more from abroad, resulting in higher consumption. In equilibrium, the higher demand for non-tradable goods leads to an increase in employment in the non-tradable sector (while employment in the tradable sector is independent of domestic demand conditions). Therefore, higher capital inflows result in relatively more employment in the non-tradable sector and help reduce inflation.

The final element is how monetary policy affects financial flows and the world real rate, a point that relates back to the classic Marshall-Lerner condition.[4] In the case where an exchange rate appreciation increases the trade deficit (i.e., the Marshall-Lerner condition holds), a central bank thus perceives that by raising interest rates, it can appreciate its exchange rate and generate a reallocation of employment towards non-tradables that helps to lower inflation.

However, when all countries attempt to run a trade deficit, this is self-defeating in general equilibrium. The result is that in response to a global recessionary shock, central banks end up with a nominal interest rate that is too high relative to the cooperative outcome, insofar as the Marshall-Lerner condition holds. That is, the Nash equilibrium displays under-tightening. However, if the Marshall-Lerner condition fails, central banks set an interest rate that is too low relative to the cooperative outcome in an attempt

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[4]The well-known Marshall-Lerner condition relates the change in the nominal exchange rate to the trade balance as a function of the elasticities of exports and imports. Following the convention, the Marshall-Lerner is said to hold when an appreciation of the exchange rate (or equivalently, an increase in the nominal interest rate) generates an increase in the trade deficit.

to generate a higher trade deficit. That is, in this case, the Nash equilibrium displays under-tightening.[5]

In sum, whether cooperation calls for lower or higher rates can be framed entirely in terms of the sign of the output gap, the sign of the product of the differences in labor intensity between the tradable sector and the non-tradable sector, and the response of the trade balance to a monetary expansion. The overall principle is that when central banks use monetary policy to steer capital flows, this ends up altering the world real interest rate in general equilibrium and lead to adverse welfare effects.

Our quantitative analysis shows that the differences between cooperative and non-cooperative equilibrium can be quite substantial. Although the welfare gains are modest for small shocks, they can quickly become substantial for moderately large shocks. For example, for shocks leading to an inflation gap of 3%, the difference in the level of output between cooperative and Nash equilibrium exceeds one percent.

In one extension, we allow for the anticipation of future shocks. In this case, we show that while the cooperative solution maintains zero inflation and zero output in response to the news shock, the Nash equilibrium exhibits either overheating and inflation or a recession and deflation. Moreover, the sign of the output gap, the differences in labor intensity, and the response of the trade balance to a monetary expansion remain the three key sufficient statistics as in our baseline analysis. These results also hold when we extend the model to allow for costly labor reallocation or oil price shocks.
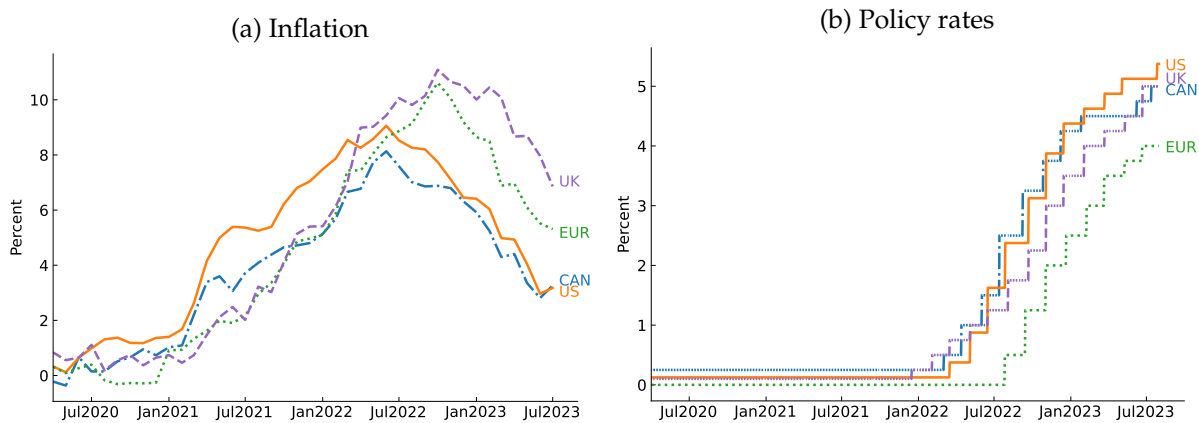
Figure 1: Synchronous Monetary Policy Tightening

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[5]When the economy has a positive output gap, the above conclusions reverse: That is, insofar as non-tradables are more labor intensive, central banks under-tighten if the Marshall-Lerner condition holds and over-tighten if the Marshall-Lerner condition fails. The logic is that when the labor market is overheated, central banks seek to run a trade surplus to reallocate labor away from non-tradables.

Related literature. Our paper belongs to a vast literature on international monetary policy coordination.[6] As mentioned above, a key theme in much of this literature is a terms of trade channel by which individual countries have incentives to manipulate their terms of trade in their favor at the expense of other countries. According to the optimal tariff argument, central banks generally over-tighten monetary policy relative to the socially optimal level, independently of the degree of financial integration. By contrast, we highlight a financial channel, involving an intertemporal price (i.e., the world real interest rate) and show that this generates the possibility of under-tightening.

Our paper is most closely related to that of Fornaro and Romei (2022), who also consider the scope for monetary policy cooperation in a two-sector New Keynesian model with tradables and non-tradables. They show how an increase in the preference for tradable goods leads to inflation and a negative output gap in equilibrium. Moreover, they find that cooperative monetary policy prescribes higher output levels relative to the Nash equilibrium. Our model differs from theirs by considering a more general structure with elastic labor supply, diminishing returns in labor, and non-unitary elasticities of substitution.[7] Our analysis shows that the Nash equilibrium may exhibit over-tightening or under-tightening and elucidates how this outcome depends on a set of sufficient statistics. Namely, we establish analytically that, independently of the shocks, whether cooperation calls for lower or higher rates depends on the degree of slack in the economy, the differences in labor intensities across sectors, and the response of the trade balance to a monetary expansion.[8]

Our paper is also related to the literature that examines the potential for international coordination in the context of various government policies. Chang (1990) and Kehoe (1987) study the coordination of fiscal policies when fiscal deficits in some countries make

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[6]For early contributions in the context of static Mundell-Flemming models, see Hamada (1976), Oudiz and Sachs (1984), Canzoneri and Gray (1985), and Canzoneri and Henderson (1991). For modern models with microfoundations, see Obstfeld and Rogoff (1995), Corsetti and Pesenti (2001, 2005), Tille, 2001; Obstfeld and Rogoff (2002) Clarida, Galı and Gertler (2002); Canzoneri, Cumby and Diba (2005); Devereux and Engel (2003); Benigno (2009); Egorov and Mukhin (2023); and Bodenstein, Corsetti and Guerrieri (2020).

[7]In their setup, a fixed endowment of hours implies that overheating cannot occur, linear production for non-tradables rules out inflation in non-tradables, and unitary elasticities of substitution imply that the trade balance always increases in response to a depreciation.

[8]As mentioned earlier, we also draw from our previous work on international spillovers (Bianchi and Coulibaly, 2021), which focuses on a prudential aspect of monetary policy. Another recent paper is Caldara, Ferrante, Iacoviello, Prestipino and Queralto (2023), which studies non-linear effects from monetary spillovers in a model with global banks. Previous work by Acharya and Bengui (2018), Eggertsson, Mehrotra, Singh and Summers (2016), Caballero, Farhi and Gourinchas (2021), and Fornaro and Romei (2019) studies the propagation of liquidity traps across countries but does not consider the scope for monetary policy cooperation. For the empirical literature on international monetary policy spillovers, see, for example, Rey (2013) and Kalemli-Ozcan (2019).

it more costly for others to finance their deficit (see also Azzimonti, de Francisco and Quadrini, 2014). In Halac and Yared (2018), governments exhibit present bias and fiscal rules are slacker under coordination. Obstfeld and Rogoff (1996) and Costinot, Lorenzoni and Werning (2014) consider the case for excluding capital controls when countries are large and have market power over the world interest rate.[9] In our case, countries are infinitesimal, and the case for coordination is grounded in a pecuniary externality, where the world interest rate influences monetary policy tradeoffs.

The key mechanism at play in our model is also related to the literature on aggregate demand externalities. In Schmitt-Grohé and Uribe (2016) and Farhi and Werning (2016), nominal rigidities and constraints on monetary policy create a rationale for capital controls. In our model, monetary policy faces no constraints, but inflation is costly, and divine coincidence fails, generating aggregate demand externalities. Crucially, the scope for monetary policy cooperation emerges because of the interaction between this aggregate demand externality and a pecuniary externality operating through the world real rate.

Finally, there has been an active recent literature on the rise of inflation following the COVID-19 pandemic and the connection with sectoral reallocation.[10] Besides our open economy focus, we also contribute to this literature by highlighting for the first time, to the best of our knowledge, the importance of differences in labor intensity across sectors for the determination of inflation and output.

Outline. Section 2 presents the model. Section 3 presents the Nash equilibrium and Section 4 presents the optimal monetary policy under cooperation. Section 5 presents extensions of the basic framework. Section 6 concludes.

2   Model

Time is discrete and infinite. We model the world economy as a continuum of identical small open economies indexed by $k \in [0, 1]$. There are two consumption goods, a tradable good and a non-tradable good. In each economy, these are produced using labor in a competitive market with nominally rigid wages. For simplicity, we focus on a deterministic environment.

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[9]Other recent examples are Clayton and Schaab (2022) on macroprudential policy with multinational banks and Chari, Nicolini and Teles (2023) on fiscal and trade policies in a multi-country business cycle model.

[10]See, for example, Rubbo (2023), Guerrieri, Lorenzoni, Straub and Werning (2021), di Giovanni, Kalemli-Özcan, Silva and Yildirim (2022, 2023), Baqaee and Farhi (2022), and Afrouzi and Bhattarai (2023).

We first describe the problem faced by households and firms in each economy k and then describe the competitive equilibrium. To avoid clutter in the notation, we do not index variables in each country by k. We will use $\{x_t\}$ to refer to the sequence $\{x_{k,t}\}_{t=0}^{\infty}$ for some variable x and country k.

2.1 Households

Each economy is populated by a continuum of identical households of measure one. Their preferences are given by

$$\sum_{t=0}^{\infty} \beta^t \left[U(c_t) - \kappa_t n_t + \frac{\chi}{2} (\pi_t - \overline{\pi}_t)^2 \right], \tag{1}$$

where $\beta \in (0,1)$ is the discount factor and U is a strictly increasing and concave utility function over a consumption good c_t, which is a composite of tradable consumption c_t^T and non-tradable consumption c_t^N, according to a Cobb-Douglass aggregator

$$c_t = \left(c_t^T \right)^{\phi^T} \left(c_t^N \right)^{\phi^N},$$

with $\phi^T \in (0,1)$ and $\phi^N = 1 - \phi^T$. For convenience, we use $u(c^T, c^N)$ to denote the utility as a function of the two consumption goods and $\sigma_t \equiv \frac{-c_t U''(c_t)}{U'(c_t)}$ to denote the inverse of the intertemporal elasticity of substitution. Households face a linear disutility from working given by κ. Aggregate hours n_t is the sum of hours worked in the tradable sector n_t^T and in the non-tradable sector n_t^N; that is, $n_t = n_t^T + n_t^N$. Implicit in the formulation is that labor is perfectly mobile across sectors, which, in turn, implies that in equilibrium the wage is equated in both sectors. In Section 5, we generalize preferences by allowing for imperfect labor mobility and a CES composite for consumption.

The last term in (1) represents a utility cost of inflation, which represents standard losses from price adjustments that emerge in models with costly price adjustments à la Rotemberg or staggered prices à la Calvo. This cost is assumed to be quadratic in the deviations of the inflation rate of the consumer price index $\pi_t \equiv P_t/P_{t-1} - 1$ from the central bank target $\overline{\pi}_t$. We denote by $\hat{\pi}_t \equiv \pi_t - \overline{\pi}_t$ the inflation gap. Given a unitary elasticity of substitution between tradables and non-tradables, the consumer price index P_t satisfies

$$P_t = \left(\frac{P_t^T}{\phi^T} \right)^{\phi^T} \left(\frac{P_t^N}{\phi^N} \right)^{\phi^N}, \tag{2}$$

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where P_t^N and P_t^T denote respectively the price of non-tradables and tradables in terms of the domestic currency.

We assume that the law of one price holds for the tradable good. Let us denote by P_{jt}^T the price of the tradable good in terms of the country j currency. Thus, it follows that $P_{kt}^T = P_{jt}^T e_{kt}^j$ for any pair of countries k and j, where e_{kt}^j is the nominal exchange rate defined as the price of the country j' currency in terms of the country k' currency.

In each period, households receive their labor income, $W_t(n_t^T + n_t^N)$. They also collect profits φ_t from domestic firms. Households have two assets available, a real international bond that pays R_t^* units of tradables and a nominal domestic bond that pays R_t in units of the domestic currency. These assets are referred to as b_t^* and b_t respectively. The budget constraint is, therefore, given by

$$P_t^T c_t^T + P_t^N c_t^N + \frac{b_{t+1}}{R_t} + \frac{P_t^T b_{t+1}^*}{R_t^*} = W_t(n_t^T + n_t^N) + \varphi_t + b_t + P_t^T b_t^*. \tag{3}$$

We assume that wages are rigid in period 0 at a given value W. For $t = 0$, households are off their labor supply, and hours worked are determined by firms' labor demand. For $t > 0$, we assume that wages are flexible.

The problem of the household consists of choosing a sequence of consumption $\{c_t^N, c_t^T\}_{t=0}^{\infty}$, asset positions $\{b_{t+1}, b_{t+1}^*\}_{t=0}^{\infty}$, and hours $\{n_t^T, n_t^N\}_{t=1}^{\infty}$, to maximize the expected present discounted value of utility (1), subject to (3) and taking as given profits $\{\varphi_t\}$, and prices $\{W_t, P_t^N, P_t^T, R_t, R_t^*\}_{t=0}^{\infty}$.

The optimality condition with respect to c_t^T and c_t^N equates the marginal rate of substitution between the two goods to the relative price. Given the Cobb-Douglas aggregator, households allocate a constant share of their expenditures to each good:

$$\phi^T P_t^N c_t^N = \phi^N P_t^T c_t^T. \tag{4}$$

The linearity of the disutility from working implies that for $t > 0$ (when wages are flexible), the wage in both sectors must satisfy

$$\frac{W_t}{P_t^N} = \frac{\kappa_t}{u_N(c_t^T, c_t^N)}, \tag{5}$$

where we use u_T and u_N to denote the respective partial derivatives. Finally, the optimality

condition with respect to asset holdings yields

$$u_T\left(c_t^T, c_t^N\right) = \beta R_t^* u_T\left(c_{t+1}^T, c_{t+1}^N\right) \tag{6}$$

$$R_t^* = R_t \frac{P_t^T}{P_{t+1}^T}. \tag{7}$$

Condition (6) is the Euler equation for the real bond. Condition (7) is a no-arbitrage condition that equates the return on real international bonds and domestic currency bonds, both of which are expressed in units of tradables.

2.2 Firms

There is a continuum of firms of measure one producing tradable goods and non-tradable goods. Output of the two goods $i = \{T, N\}$ is produced using labor with a production function F so that

$$y_t^i = F^i(h_t^i, A_t^i).$$

We assume an isoelastic production function such that $F^i(h_t^i, A_t^i) = A_t^i(h_t^i)^{\alpha^i}$. We refer to α^i as the *labor intensity* parameter.

Profits are given by $P_t^i F^T(h_t^i, A_t^i) - W_t h_t^i$. At the optimum, firms equate the marginal product of labor to the nominal wage in the two sectors:

$$P_t^T F_h^T(h_t^T, A_t^T) = W_t, \tag{8a}$$

$$P_t^N F_h^N(h_t^N, A_t^N) = W_t. \tag{8b}$$

Given competitive markets, the labor intensity equals the labor share for each sector in equilibrium. As we will see, differences in labor intensity across sectors, $\alpha^N - \alpha^T$, will play an important role in the analysis. We note that the fact that labor is the only factor of production or that the production function exhibits decreasing returns to scale is not restrictive.[11]

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[11]Adding a factor in fixed unit of supply with a flexible factor price does not alter allocations. In Section 5, we incorporate oil as an additional factor of production.

2.3 Monetary Policy

In each small open economy, there is a central bank that chooses nominal interest rates $\{R_t\}$. Because of the assumption that prices are flexible for $t > 0$, monetary policy is neutral starting from period 1. Therefore, we assume that monetary policy implements a strict inflation targeting regime such that $\pi_t = \bar{\pi}_t$ for $t > 0$. For $t = 0$, we will evaluate the optimal monetary policy, comparing the cooperative and non-cooperative outcomes.

2.4 Competitive Equilibrium

In each country, the market for non-tradable goods must clear. That is,

$$c_t^N = F^N(h_t^N, A_t^N). \tag{9}$$

At $t = 0$, households in each country supply hours in the tradable and non-tradable sectors to meet the demand by firms. For $t > 0$, the labor clears the labor market. That is, $n_t^T = h_t^T$ and $n_t^N = h_t^N$.

We assume without loss of generality that the bond denominated in domestic currency is only domestically traded in each country. Market clearing therefore implies

$$b_{t+1} = 0. \tag{10}$$

Finally, at the world level, real bonds are in zero net supply. To account for market clearing at the world level, we now explicitly index the policies of each country by k. We have that

$$\int b_{k,t+1}^* dk = 0. \tag{11}$$

We now define a competitive equilibrium in the global economy.

Definition 1 (Competitive Equilibrium). Given initial positions $b_{k,0}^*$, a sticky wage W, and a sequence of central bank policies $\{R_t\}$ in each country k, an equilibrium is a sequence of world real rates $\{R_t^*\}$, prices $\{P_t^T, P_t^N, W_t, e_{k,t}^j\}$ and allocations $\{c_t^T, c_t^N, h_t^T, h_t^N, b_{t+1}, b_{t+1}^*\}$ in each country k such that

(i) households optimize, and hence conditions (4), (6), (7) hold for all $t \geq 0$, and (5) holds for all $t \geq 1$;

(ii) firms optimize, implying (8a) and (8b) hold for all $t \geq 0$;

(iii) the law of one price holds for tradables: $P_{k,t}^T = P_{j,t}^T e_{k,t}^j$ for any country-pair k and j;

(iv) the market for non-tradables (9) and domestic bonds (10) clears; moreover, the labor market clears for $t \geq 1$;

(v) globally, the market for the real bond clears; that is, (11) holds.

If we combine the budget constraints of households and firms as well as market clearing conditions, we arrive at the country budget constraint for tradables, or the balance of payment condition:

$$c_t^T - F^T(h_t^T, A_t^T) = b_t^* - \frac{b_{t+1}^*}{R_t^*}, \tag{12}$$

which says that if a country runs a trade deficit, it accumulates net debt, and if it runs a trade surplus, it accumulates net external assets.

We assume that all countries start at $t = 0$ with zero net foreign asset position. To the extent that all countries follow the same policies, we can therefore restrict the analysis to symmetric competitive equilibrium.

2.5 Efficient Allocation, Output Gaps, and the Natural Wage

We conclude the description of the model by presenting the first-best allocation. We consider a benevolent social planner of the world economy who chooses allocations to maximize welfare, subject to a resource constraint. The planner's problem can be written as

$$\max_{\{h_t^N, h_t^T\}} \sum_{t=0}^{\infty} \beta^t \left[u \left(F^T(h_t^T, A_t^T), F^N(h_t^N, A_t^N) \right) - \kappa_t \left(h_t^T + h_t^N \right) \right].$$

First-order conditions with respect to tradable and non-tradable employment yield

$$F_h^T(h_t^T, A_t^T) \, u_T \left(F^T(h_t^T, A_t^T), F^N(h_t^N, A_t^N) \right) = \kappa_t, \tag{13}$$

$$F_h^N(h_t^T, A_t^N) \, u_N \left(F^T(h_t^T, A_t^T), F^N(h_t^N, A_t^N) \right) = \kappa_t. \tag{14}$$

Let us denote by $\bar{h}_t^T$ and $\bar{h}_t^N$ the employment levels in the two sectors in the first-best allocation. The following lemma shows that the ratio of employment levels can be expressed as the product of the relative weights in preferences and the relative labor intensities:

Lemma 1 (First-Best). *The optimal ratio of hours in the first-best allocation is given by*

$$\frac{\bar{h}_t^N}{\bar{h}_t^T} = \frac{\alpha^N \phi^N}{\alpha^T \phi^T}. \tag{15}$$

Proof. In Appendix A.1 □

We highlight that the first-best allocations coincide with those in a competitive equilibrium in a flexible wage version of our model. This can be seen by noting that if the nominal wage were flexible, we would arrive at (13) and (14) by combining firms' demand for labor (8a) and (8b) with households' labor supply decisions (5). This result will provide a clear benchmark for the normative analysis.

Output gaps. To characterize the central banks' tradeoff and to highlight the differences between the competitive equilibrium and the first-best allocation, we define a measure of *output gaps* as the deviations of employment relative to the first-best levels

$$\hat{h}_t^N \equiv \frac{h_t^N}{\bar{h}_t^N} - 1, \qquad \hat{h}_t^T \equiv \frac{h_t^T}{\bar{h}_t^T} - 1.$$

In addition, we define the labor wedges in the tradable and non-tradable sectors as

$$\tau_t^T \equiv F_h^T(h_t^T, A_t^T) u_T(c_t^T, c_t^N) - \kappa_t, \quad \tau_t^N \equiv F_h^N(h_t^N, A_t^N) u_N(c_t^T, c_t^N) - \kappa_t. \tag{16}$$

The assumption that good prices are flexible and the fact that wages are equalized across sectors owing to perfect labor mobility implies that in any competitive equilibrium,

$$F_h^T(h_t^T, A_t^T) u_T(c_t^T, c_t^N) = F_h^T(h_t^N, A_t^N) u_N(c_t^T, c_t^N), \tag{17}$$

and thus the labor wedges are equated. Accordingly, we use $\tau_t = \tau_t^T = \tau_t^N$. In addition, the next lemma shows that output gaps are equated in any *symmetric* equilibrium given any monetary policy.

Lemma 2. *In any symmetric competitive equilibrium, the output gaps in the tradable and non-tradable sectors are equalized $\hat{h}_t^T = \hat{h}_t^N = \hat{h}_t$. Moreover, the employment ratio h_0^N / h_0^T in the first-best allocations coincides with those in a competitive symmetric equilibrium for any monetary policy.*

Proof. In Appendix A.2 □

The natural wage. We define the *natural wage* as the nominal wage that would prevail in equilibrium if wages were flexible and the central bank stabilized inflation at $\bar{\pi}_t$. Denoting variables without a subscript as $t = -1$ variables, we can write the natural wage at date $t = 0$ as described in the following lemma.

Lemma 3 (Natural Wage). *The natural wage at date t is given by*

$$W_t^n = (1 + \bar{\pi}_t) P_{-1} \left[\prod_{i=T,N} \left(\alpha^i A_t^i \right)^{\phi^i} \left(\bar{h}_t^i \right)^{-(1-\alpha^i)\phi^i} \right]. \tag{18}$$

Proof. In Appendix A.3 □

Equation (18) characterizes the natural wage as a function of parameters for productivity, preferences, and the inflation target. In particular, the natural wage falls in period 0 when there is a decline in productivity for tradables or non-tradables, when there is a positive labor supply shock, or when there is a negative shock to the inflation target. In what follows, we assume that in period -1, the nominal wage is at its natural level, but not in period 0, when the nominal wage is fixed at an arbitrary value W.

Finally, we can define the wage gap $\hat{w}_0 \equiv \frac{W_0}{W_0^n} - 1$. It thus follows that when the market wage is above the natural wage, countries face a recession whereas when the market wage is below the natural wage, countries face overheating.

3 Monetary Policy in a Nash Equilibrium

This section studies non-cooperative monetary policy. We model the non-cooperative game as a Nash equilibrium where central banks choose their monetary policy to maximize their own welfare, taking as given monetary policy abroad.

3.1 Optimal Monetary Policy for a Single Country

We first study the individual problem of a central bank that takes as given $\{R_t^*\}$ and policies conducted in other countries. We distinguish between the problem for $t \geq 1$ when prices are flexible and $t = 0$ when wages are sticky.

3.1.1 Time t ≥ 1 Problem

Given that prices are flexible for $t \geq 1$, we can focus on a situation where the central bank sets monetary policy to implement $\pi_t = \bar{\pi}$ for all $t \geq 1$.[12] The lifetime welfare for a central bank with net foreign asset b_1^* in period 1 is given by

$$V_1(b_1^*) = \max_{\{c_t^T, c_t^N, h_t^T, h_t^N\}_{t=0}^{\infty}} \sum_{t=0}^{\infty} \beta^t \left[u(c_t^T, c_t^N) - \kappa_t(h_t^T + h_t^N) \right], \tag{19}$$

subject to (9), (13),(14), and

$$b_1^* = -\sum_{t=1}^{\infty} \frac{F^T(h_t^T, A_t^T) - c_t^T}{\prod_{j=1}^{t-1} R_j^*}.$$

The last constraint says that the present discounted value of future trade balances must be consistent with the initial level of debt.

3.1.2 Time t = 0 Problem

The central bank's policy choice in period 0 is the nominal interest rate. The central bank's objective is to choose an R_0 that maximizes the welfare of the domestic household subject to domestic allocations and prices consistent with a competitive equilibrium (given policies $\{R_{k,t}\}$ conducted in other countries). Notice that the continuation value for the central bank is given by (19).

Implementability constraints. Following a primal approach, we proceed to combine equilibrium conditions to express the problem in terms of allocations to derive the implementability constraints. First, combining the optimality condition of households (4) with the ones for firms (8a) and (8b), we arrive at an equation that determines the relative demand for hours in the two sectors:

$$\frac{h_0^N}{h_0^T} = \frac{\alpha^N \phi^N}{\alpha^T \phi^T} \left[1 - \frac{b_1^*}{R_0 F^T(h_0^T, A_0^T)} \right]. \tag{20}$$

A key implication of (20) is that when a country accumulates net foreign assets (or equivalently, runs a larger trade balance surplus), it will display in equilibrium a lower number

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[12]This is clearly without loss of generality when the central bank optimizes at $t > 0$. Moreover, we can also show that with commitment at $t = 0$, the central bank would also choose $\pi_t = \bar{\pi}$.

of hours in the non-tradable sector relative to those of the tradable sector. The logic is as follows: accumulation of net foreign assets implies lower available resources for consumption. Because preferences are homothetic, this means lower consumption for both tradables and non-tradables. As non-tradable goods are produced domestically, the decline in non-tradable consumption must be associated with lower hours worked in the non-tradable sector.

Second, the level of inflation can be expressed as

$$\frac{\widehat{\pi}_0}{1+\overline{\pi}_0} = \frac{W}{W_0^n}\left(\frac{h_0^T}{\overline{h}_0^T}\right)^{(1-\alpha^T)\phi^T}\left(\frac{h_0^N}{\overline{h}_0^N}\right)^{(1-\alpha^N)\phi^N} - 1. \tag{21}$$

This condition is an open economy version of the Phillips curve that relates employment in both sectors to inflation.[13] For a given wage, higher employment in tradables or non-tradables requires higher prices in the respective sectors, as can be seen from (8a) and (8b). Aggregating prices yields (21).

An important implication from (21) is that the labor intensities of the sectors play a crucial role in determining the extent to which higher employment in each sector raises inflation. To see this more clearly, we can totally differentiate firms' first-order conditions, and using that the wage is constant, we obtain

$$d\log P_t^i = \frac{1-\alpha^i}{\alpha^i}d\log y_t^i.$$

The higher is the labor intensity in each sector, the lower is the rise in prices needed to achieve a certain increase in output. Crucial for this result is that wages are sticky. Thus, if a good is more labor intensive, this means that firms can scale up production without significant raises in prices. As the curvature in the production function becomes lower, an increase in employment leads to a faster decline in the marginal product, thus necessitating a larger increase in prices to induce higher employment to be optimal for firms. Put differently, a higher labor intensity implies a lower elasticity of marginal cost. To our knowledge, this role of labor intensity in shaping the response of inflation to a monetary expansion is a channel that has not received attention in the literature.

Finally, in addition to (20) and (21), the central bank is also subject to the household intertemporal Euler equation (7).

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[13]To obtain (21), we use the definition of the price index (2) at dates $t = 0$ and $t = -1$ and combine it with firms' optimality (8a) and (8b), and the definition of the natural wage (18).

Lagrangian. We can then write the Lagrangian for the central bank problem as

$$u\left(F^T(h_0^T,A_0^T)-\frac{b_1^*}{R_0^*},F^N(h_0^N,A_0^N)\right)-\kappa_0(h_0^T+h_0^N)-\frac{\chi}{2}(\widehat{\pi}_0)^2+\beta V_1\left(b_1^*\right) \tag{22}$$

$$+\vartheta\left[\frac{\widehat{\pi}_0}{1+\overline{\pi}_0}-\frac{W}{W_0^n}\left(\frac{h_0^T}{\bar{h}_0^T}\right)^{(1-\alpha^T)\phi^T}\left(\frac{h_0^N}{\bar{h}_0^N}\right)^{(1-\alpha^N)\phi^N}+1\right]+\eta\left[\left(1-\frac{b_1^*}{R_0^*F^T(h_0^T,A_0^T)}\right)\frac{\alpha^N\phi^N}{\alpha^T\phi^T}\frac{h_0^T}{h_0^N}-1\right]$$

$$+\mu\left[u_T\left(F^T(h_0^T,A_0^T)-\frac{b_1^*}{R_0^*},F^N(h_0^N,A_0^N)\right)-\beta R_0^*u_T\left(\mathcal{C}^T(b_1^*),\mathcal{C}^N(b_1^*)\right)\right],$$

Two important observations from this problem are worth making. First, the only foreign variable that appears is the *world real rate*. The reason is that although foreign monetary policies can alter the exchange rate vis-à-vis the domestic country, the domestic central bank can alter these movements by varying the nominal rate. Because the presence of the world real rate reflects an intertemporal channel, we refer to it as the "financial channel."

Second, the trade balance not only changes resources today versus tomorrow but also affects the last two implementability constraints. The optimality condition with respect to b_1^* yields

$$\eta=\left[\delta_0-\phi^T+\sigma_0\phi^T\right]u_T(c_0^T,c_0^N)\mu, \tag{23}$$

where δ_0 is given by (A.7) in Appendix A.4 and satisfies $\delta_0>1$.

Condition (23) implies that the Lagrange multipliers on households' Euler equation (6) and households' intra-temporal allocation of hours worked (20) have the same sign. To understand why, suppose the central bank perceives a positive shadow value from raising the ratio of non-tradable employment to tradable employment (that is, $\eta>0$). Notice that if households were to borrow more, the increase in consumption would lead to higher demand for tradables and non-tradables. Higher demand for non-tradables implies higher hours employed in the non-tradable sector (while hours in the tradable sector are independent of domestic demand conditions). [14] Therefore, a higher level of borrowing would result in more hours in the non-tradable sector relative to those in the tradable sector (relaxing the constraints for the central bank). From the perspective of the central bank of the small open economy, this implies that a positive shadow value from higher

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[14]For given monetary policy, employment of tradables remains actually fixed. This is because tradable employment depends only on the wage in units of tradables, and the price of tradables in the small open economy is determined by the law of one price.

non-tradable to tradable hours is associated with a positive shadow value from higher household borrowing.

Optimality with respect to h_0^T and h_0^N delivers a *targeting rule* for the small open economy:[15]

$$\sum_{i=T,N} \delta_0^i \alpha^i \phi^i \, \tau_0 = (1+\psi_b b_1^*) \frac{\alpha^N \phi^N}{h_0^N} \sum_{i=T,N} \delta_0^i (1-\alpha^i) \phi^i \, \chi (1+\pi_0) \widehat{\pi}_0, \tag{24}$$

where δ_0^T and δ_0^N are positive coefficients defined in (A.10), (A.11) with $1+\psi_b b_1^* > 0$, and $\text{sign}(\psi_b) = \text{sign}(\alpha^N - \alpha^T)$ defined in (A.9). Recall also that τ_0 stands for the labor wedge.

Equation (24) equates the weighted average of the net marginal utility benefits from raising employment in both sectors to the marginal cost of higher prices. Relative to closed economy targeting rules, a novel consideration that emerges here is the trade balance. Depending on the difference in labor intensities $\alpha^N - \alpha^T$, a trade surplus can help reduce the marginal cost of inflation. We next delve into the incentives for an individual central bank to manage the trade balance.

Trade-balance management. When households borrow, they equate the marginal benefits of consuming today to the marginal costs of repaying tomorrow, as given by (6). However, by (20), a central bank also perceives that changes in international borrowing (and thus changes in the trade balance) affect the reallocation of hours worked across sectors, which in turn affects inflation. In particular, the perceived social marginal benefit of the reallocation of hours worked η across sectors is given by[16]

$$\eta_0 = \frac{\phi^N \phi^T}{\sum_i \delta_0^i \alpha^i \phi^i} (\alpha^N - \alpha^T) \chi \widehat{\pi}_0. \tag{25}$$

An important takeaway from condition (25) is that the sign of η (and thus μ) depends on the difference in labor intensity across sectors $\alpha^N - \alpha^T$ and the sign of the inflation gap. Assuming that non-tradables are more labor intensive ($\alpha^N > \alpha^T$), when the economy has high inflation, the central bank in the small open economy would like to reallocate labor towards the more labor-intensive sector (i.e., $\eta > 0$ and thus $\mu_0 > 0$)). As discussed above, when a sector is more labor intensive, prices respond relatively less to a change in production in that sector. Therefore, starting from a situation with high inflation, the central bank can achieve a reduction in inflation by shifting employment towards the more

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[15]See A.4 for a derivation.

[16]Expression (25) is obtained by combining (23) with optimality conditions for h_0^T and h_0^N and using (17).

labor-intensive sector. On the other hand, if the inflation gap is negative, the central bank internalizes that a reallocation of hours away from the more labor-intensive sector (non-tradables) towards the less labor-intensive sector (tradables) would help raise inflation towards the target and improve welfare.[17]

Condition (25) also implies that when the two sectors are equally labor intensive $\alpha^N = \alpha^T$, the central bank does not perceive any social benefit from changing the composition of hours between the tradable sector and non-tradable sector. It also therefore follows that households' borrowing choices are socially optimal, from the perspective of the central bank.

Given how the trade balance affects inflation, the key question then is how monetary policy shapes households' borrowing decisions. This is, in fact, a point related to the classic Marshall-Lerner condition which is said to hold when a depreciation of the exchange rate leads to an increase in the trade surplus. In our environment, we can derive the following *generalized* Marshall-Lerner condition:

Lemma 4 (Generalized Marshall-Lerner Condition). *In response to a domestic monetary expansion, the trade balance satisfies*

$$-\frac{db_1^*}{dR_0} > 0 \iff \sigma_0 > \widetilde{\sigma} \equiv 1 - \frac{\alpha^T}{\alpha^T \phi^T + \alpha^N \phi^N}.$$

Proof. In Appendix A.5 □

The lemma generalizes existing results in the literature to a situation with multi-sector production.[18] Whether an expansionary monetary policy expands the trade balance depends on the elasticities of substitution and labor intensities in the two sectors. If the tradable sector were an endowment, $\alpha^T = 0$, we would obtain the familiar result that the trade balance increases in response to a fall in the nominal rate (i.e., $db_1^*/dR_0 < 0$) if and only if the intertemporal elasticity of substitution was lower than the intra-temporal elasticity of substitution between tradables and non-tradables (which in this case is assumed to be one). [19] In our model with endogenous production in the tradable sector, the lower interest rate expands tradable output and thus is an additional force towards a

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[17]When $\alpha^N < \alpha^T$, the signs of both Lagrange multipliers are reverted.

[18]The classic Marshall-Lerner condition, derived originally in a partial equilibrium setting, posits that the trade surplus increases in response to a depreciation if the sum of the (static) elasticities of exports and imports to exchange rates exceed one. As is well understood, in a dynamic general equilibrium model, the effects depend on intertemporal considerations (see Bianchi and Coulibaly, 2023 for a decomposition).

[19]Much of the literature focuses on the Cole-Obstfeld parameterization with unitary elasticities of substitution where capital flows do not respond to changes in nominal rates.

trade surplus. Therefore, to obtain a decrease in net exports in response to a lower nominal interest rate, the intertemporal elasticity of substitution must be lower. In addition, it also follows that if $\alpha^T \geq \alpha^N$, a monetary expansion increases the trade surplus for *any* intertemporal elasticity of substitution. Intuitively, a higher α^T implies that tradable output responds more to an increase in the price of tradables (for a given wage), and through consumption smoothing, this means a higher trade surplus.

We highlight that the empirical literature does not offer conclusive evidence on whether an increase in the nominal interest rate leads to an increase in the trade deficit. For example, Boyd, Caporale and Smith (2001) argues that the Marshall-Lerner condition holds in the long run while Boehm, Levchenko and Pandalai-Nayar (2023) argues that it fails in the short run. Other studies such as Dong (2017) argue that whether the Marshall-Lerner condition holds or not depends on the precise methodology used.

Takeaway. To summarize, the key takeaway of this section is that by influencing the trade balance, the central bank can improve its output-inflation tradeoff when labor intensities differ between the tradable and non-tradable sectors. Moreover, whether the central bank would like to stimulate capital inflows or capital outflows depends on the sign of the inflation gap and the difference in labor intensities.

3.2   Nash Equilibrium

In the previous section, we characterized the optimal policy for the central bank of a small open economy for an arbitrary world real rate. We can now define a Nash equilibrium as the outcome when all central banks are simultaneously maximizing the welfare of their representative household and the market for the global real asset clears. Notice that because all countries are identical, we can restrict to symmetric Nash equilibrium.

We let $\mathcal{U}(R_0, R_0^*)$ denote the lifetime utility of the representative household in a competitive equilibrium where the central bank sets the nominal rate to R_0 and the world real rate is R_0^*. In addition, we let $\mathcal{R}^*(R_0)$ denote the equilibrium world real rate when all countries set R_0. We define the Nash equilibrium as follows:

Definition 2 (Nash Equilibrium)**.** The nominal interest rate in the Nash equilibrium is such that

$$R_0 = \text{argmax}_x \ \mathcal{U}(x, \mathcal{R}^*(R_0)).$$

That is, the Nash equilibrium corresponds to the outcome when every central bank is playing its best response to other central bank policies.

By symmetry, in any Nash equilibrium, there are no capital flows, and exchange rates are constant. Replacing $b_1^* = 0$ in the targeting rule (24) , we arrive at

$$\tau_0 = \chi \psi^{NE} \frac{1 + \pi_0}{h_0^N} \widehat{\pi}_0, \quad \text{with } \psi^{NE} \equiv \alpha^N \phi^N \frac{\sum_{i=T,N} \delta_0^i (1 - \alpha^i) \phi^i}{\sum_{i=T,N} \delta_0^i \alpha^i \phi^i}. \tag{26}$$

This expression reveals that under optimal policy, only one of two scenarios can emerge in the Nash equilibrium: either the economy is overheating ($\tau_0 < 0$ and $\widehat{h}_0 < 0$) and inflation is below target, or there is a recession ($\tau_0 > 0$ and $\widehat{h}_0 > 0$), and inflation is above target. To understand the intuition, consider the possibility that in a Nash equilibrium, there is a recession and inflation is below the target. In that case, by lowering the nominal interest rate and allowing for higher prices, the central bank can narrow the output gap and inflation gap. By the same token, if there is a positive output gap and inflation is above the target, it would be optimal to raise the policy rate, as this would help lower inflation and take output closer to the efficient level. From (26), it is also clear that if the inflation cost is zero, $\chi = 0$, central banks can implement the first-best allocation for any shocks.

4 Monetary Policy under Cooperation

We now analyze the optimal monetary policy under cooperation. The key question we will tackle is whether coordination calls for tighter or looser monetary policy relative to the Nash equilibrium.

We define the *optimal cooperative monetary policy* as the outcome of a planner's problem that chooses the interest rates on behalf of all countries to maximize average welfare. Because all countries are identical, the policy maximizes the welfare of any given country, and the nominal interest rate and the allocations are the same for all countries.[20]

4.1 Optimal Policy Problem

The problem of the global planner consists of choosing $\{h_0^N, h_0^T, \widehat{\pi}_0\}$ to maximize current utility. In contrast to the problem for a small open economy (22), the planner now internalizes that in equilibrium, the market for the global asset must clear, implying that $c_0^T = F^T(h_0^T, A_0^T)$.

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[20]One can interpret the cooperation regime as a monetary union.

We can write the associated Lagrangian as follows:

$$u\left(F^T(h_0^T, A_0^T), F^N(h_0^N, A_0^N)\right) - \kappa_0(h_0^T + h_0^N) - \frac{\chi}{2}(\widehat{\pi}_0)^2 \tag{27}$$

$$+ \vartheta \left[\frac{\widehat{\pi}_0}{1 + \overline{\pi}_0} - \frac{W}{W_0^n}\left(\frac{h_0^T}{\overline{h}_0^T}\right)^{(1-\alpha^T)\phi^T}\left(\frac{h_0^N}{\overline{h}_0^N}\right)^{(1-\alpha^N)\phi^N} + 1\right] + \eta\left[\frac{\alpha^N\phi^N}{\alpha^T\phi^T}\frac{h_0^T}{h_0^N} - 1\right].$$

Optimality with respect to h_0^T and h_0^N implies the following targeting rule[21]

$$\tau_0 = \chi\psi^{GP}\frac{1 + \pi_0}{h_0^N}\widehat{\pi}_0, \quad \text{with } \psi^{GP} = \frac{\psi^{NE}}{1 + (\alpha^N - \alpha^T)(\sigma_0 - \widetilde{\sigma})\Delta}. \tag{28}$$

with $\Delta \equiv \phi^T\phi^N[(\delta_0 - \phi^T + \sigma_0\phi^T)\sum_i \delta_0^i(1-\alpha^i)\phi^i]^{-1} > 0$.

Comparing equation (28) with (26) shows that independently of the shocks, whether the planner puts more weight on inflation or output than individual central banks depends on the product of two sufficient statistics, the difference in labor intensities, $\alpha^N - \alpha^T$ and the response of the trade balance to an expansionary policy—that is, the sign of $\sigma_0 - \widetilde{\sigma}$.

Notice that when labor intensities are equal across sectors, $\alpha^N = \alpha^T$, a change in the world real rate has no first-order effects on welfare, regardless of the sign of the output gap.[22] As a result, the planner and central banks in the Nash equilibrium put the same weight on output. The intuition for this result is that when the two sectors have the same labor intensity, the social and private marginal benefits of borrowing are aligned, as explained above. This can be seen more clearly by combining (23) and (25), which yields

$$u_T(c_0^T, c_0^N)\mu_0 = \Delta\frac{\alpha^N - \alpha^T}{\alpha^N}\tau_0 h_0^N, \tag{29}$$

where recall that $\Delta > 0$. It is immediate from this condition that $\mu_0 = 0$ when $\alpha^T = \alpha^N$ regardless of the value of the labor wedge.

Consider instead the case where $\alpha^N > \alpha^T$. If the economy faces positive inflation $\widehat{\pi}_0 > 0$—in which case it is also in a recession, $\widehat{h}_0 < 0$, as explained in Section 3.1—the central bank from every small open economy would like to relocate employment towards non-tradables and induce more household borrowing (i.e., $\eta > 0$ and $\mu > 0$). Insofar as the generalized Marshall-Lerner condition holds, this implies that central banks restrict monetary policy to attract capital inflows and run a trade deficit. This results in a larger

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[21]See Appendix A.7 for details.

[22]Crucial for this result is that countries are neither net borrowers nor net savers. If countries were asymmetric, there would be winners and losers.

output contraction relative to the global planner that internalizes that capital flows would be zero in equilibrium.

The next proposition leverages on this insight to formally compare the levels of employment in the Nash equilibrium and the cooperative equilibrium.

Proposition 1. *The output gaps in the Nash equilibrium $\widehat{h}_0^{NE}$ and in the cooperative equilibrium $\widehat{h}_0^{GP}$ have the same sign. Moreover, we have that*

$$\widehat{h}_0^{NE} > \widehat{h}_0^{GP} \iff (\alpha^N - \alpha^T)(\sigma_0 - \widetilde{\sigma})\widehat{w}_0 < 0. \tag{30}$$

Proof. In Appendix A.8 ☐

Turning to comparing the choice of nominal interest rates in the Nash equilibrium and in the cooperative equilibrium, we obtain the following

Corollary 1 (Sufficient statistics). *Denote by $\widehat{h}_0^{NE}$ the output gap in the Nash equilibrium. Then, we have that*

$$R_0^{NE} < R_0^{GP} \iff (\alpha^N - \alpha^T)(\sigma_0 - \widetilde{\sigma})\widehat{h}_0^{NE} > 0.$$

Proof. In Appendix A.9 ☐

The corollary highlights when the Nash equilibrium displays over-tightening ($R_0^{NE} > R_0^{GP}$) or under-tightening ($R_0^{NE} < R_0^{GP}$) depending on a set of sufficient statistics: the differences in labor intensity, the response of the trade balance to a monetary expansion and the sign of the output gap. In particular, when non-tradables are more labor intensive and the generalized Marshall-Lerner condition holds, we have over-tightening if the economy is in a recession (and under-tightening if the economy is overheated). Insofar as non-tradables are more labor intensive, the economy can also display under-tightening in a recession when the generalized Marshall-Lerner condition fails. Table 1 presents all cases.[23]

These results generalize and clarify results in the literature. In particular, Fornaro and Romei (2022) considers $\alpha^N = 1, \sigma_0 = 1$, and $\widehat{h}_0 \leq 0$. Thus, countries put too little weight on output and the Nash equilibrium displays over-tightening.[24] In Bianchi and Coulibaly (2021), $\alpha^T = 0, \chi = 0$, central banks seek to reduce their vulnerability to a liquidity trap by increasing their net foreign asset position. If the intertemporal elasticity of substitution is

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[23]As mentioned above, if $\alpha^T > \alpha^N$, the generalized Marshall-Lerner is always satisfied.

[24]Over-tightening also emerges in the game theoretic approach of Canzoneri and Henderson (1991) who postulate reduced-form relationship for output and inflation.

lower than the intratemporal elasticity, this implies that countries raise the interest rate too much relative to the cooperative outcome.

(a) Marshall-Lerner holds $\sigma > \widetilde{\sigma}$

	$\alpha^N > \alpha^T$	$\alpha^N < \alpha^T$
Recession $\widehat{\pi} > 0$	Over-tightening	Under tightening
Overheating $\widehat{\pi} < 0$	Under-tightening	Over-tightening

(b) Marshall-Lerner fails $\sigma < \widetilde{\sigma}$

	$\alpha^N > \alpha^T$	$\alpha^N < \alpha^T$
Recession $\widehat{\pi} > 0$	Under tightening	N/A
Overheating $\widehat{\pi} < 0$	Over-tightening	N/A

Table 1: When do we have over-tightening or under-tightening?

Illustration. Figure 2 presents an illustration of the cooperative and non-cooperative equilibrium. The x-axis shows the output gap (which recall is the same for tradables and non-tradables) and the y-axis shows the inflation gap. The downward sloping curves represent the targeting rules for the cooperative and non-cooperative equilibrium— respectively (26) and (28)—which we label inflation-output tradeoff (IO). The Phillips curve is represented by the green upward sloping curve and is common to the cooperative and non-cooperative equilibrium. This curve is given by (21) (using again that $\widehat{h}_0^T = \widehat{h}_0^N$). The intersection of the two curves represents the equilibrium.
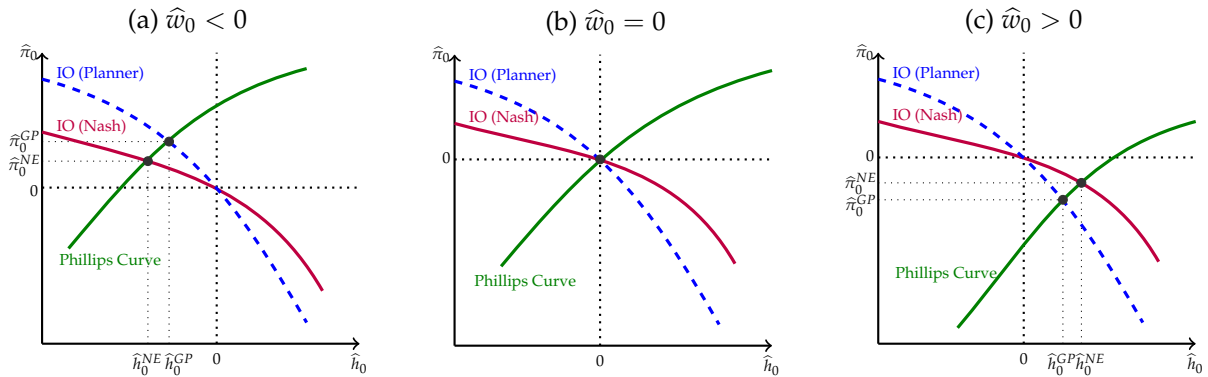

Figure 2: Nash equilibrium vs cooperative equilibrium for $\alpha^N > \alpha^T$ and $\sigma_0 > \widetilde{\sigma}$

Note: IO stands for Inflation-Output trade-off. IO (Nash) and IO (Planner) correspond respectively to (26) and (28). Phillips curve corresponds to (21) where we used $\widehat{h}_0^T = \widehat{h}_0^N = \widehat{h}_0$.

The plot considers the case where non-tradables are more labor intensive and the generalized Marshall-Lerner condition holds. That is, $\alpha^N > \alpha^T$ and $\sigma_0 > \tilde{\sigma}$. Notice that the slope of the inflation-output curve for the planner is steeper and intersects with the Nash at the ideal point (0,0). The figure displays three panels depending on the sign of the wage gap: negative wage gap (panel [a]), zero wage gap (panel [b]), and positive wage gap (panel [c]). Starting from the middle, we can see that the allocations under cooperative and non-cooperative monetary policy coincide and equal the first-best allocation. That is, the intersection of the two curves, goes through the ideal point (0,0). When the wage gap is negative (panel [a]), both economies feature a recession. Because the planner puts more weight on output and less weight on inflation, the planner allows for more inflation and faces a small recession. Finally, when the wage gap is positive (panel [c]), the planner allows for a larger inflation gap and reduces the degree of overheating in the labor market.

4.2   Inspecting the Mechanism

To delve deeper into the gains from coordination, we consider the dual formulation of the planner problem

$$\max_{R_0} \mathcal{U}(R_0, \mathcal{R}_0^*(R_0)).$$

where recall that $\mathcal{R}^*(R_0)$ denote the equilibrium world real rate when all countries set R_0 The optimality condition for the nominal rate for the planner yields

$$\frac{\partial \mathcal{U}(R_0, \mathcal{R}_0^*)}{\partial R_0} + \frac{d\mathcal{R}_0^*}{dR_0}\frac{\partial \mathcal{U}}{\partial R_0^*} = 0. \tag{31}$$

In contrast to the Nash equilibrium, where each country sets the nominal rate to maximize its own welfare, implying that $\frac{\partial \mathcal{U}_0}{\partial R_0} = 0$, the social planner instead realizes that changing nominal rates alters the real rate, and in turn, changes in the real rate affect welfare in other countries. The second term in (31) suggests that to understand how the planner would deviate from the non-cooperative equilibrium, we must take into account two crucial considerations: how welfare changes with R_0^* and how R_0^* changes with R_0. We proceed now to analyze these spillover effects.

Consider first the effects of an infinitesimal change in the world real rate. We have that evaluated at the Nash equilibrium, the welfare effects are given by

$$\left.\frac{\partial \mathcal{U}}{\partial R_0^*}\right|_{R_0^*=R_0^{*NE}} = -\frac{h_0^N}{\alpha^N \phi^N}\frac{\Delta}{R_0^*}(\alpha^N - \alpha^T)\tau_0. \tag{32}$$

This expression follows from an envelope condition.[25] It shows that the first-order effects of changes in the world real rate on welfare are determined by the output gap and the differences in labor intensity. In particular, welfare goes up when interest rates rise if the sign of the product of the output gap and the difference in labor intensity $\alpha^N - \alpha^T$ is positive. In a nutshell, countries benefit from lower real interest rates if they face a recession and non-tradables are more labor intensive (or if they face overheating and non-tradables are less labor intensive).

When individual countries set their monetary policy, they do not internalize the general equilibrium effects on the world real rate and how this affects welfare in other countries. Following the results from Lemma 4, we can infer how monetary policy affects the world real rate. Using the results of that lemma and market clearing $b_1^* = 0$, we obtain[26]

$$\sigma_0 \frac{d\mathcal{R}_0^*}{R_0^*} = (\sigma_0 - \widetilde{\sigma}) \frac{dR_0}{R_0}. \tag{33}$$

When $\sigma_0 > \widetilde{\sigma}$, a monetary policy expansion in one country raises its trade balance. When all countries simultaneously expand their monetary policy, the real rate must fall to clear the asset market.

Putting together (32) and (33), we can now trace the sign of the second term in the planner's optimality (31), in line with the results of Proposition 1 and Corollary 2. In sum, in a Nash equilibrium, central banks use monetary policy to steer capital flows and improve their output-inflation stability tradeoff. In general equilibrium, however, capital flows net out to zero, and the global economy ends up with a distorted inflation-output outcome. Whether the planner would choose a lower or higher nominal interest rate depends on whether individual central banks benefit from lower or higher real interest rates.

4.3 The Need for Cooperation

In this section, we delve into the importance of cooperation by inspecting how individual countries would unilaterally deviate from the coordinated solution.

Linearizing the equilibrium conditions for a single small open economy, we obtain the

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[25]Appendix A.6 provides the derivation. See also Bianchi and Coulibaly (2021).
[26]Equation (33) uses (6), (7), and (8a), (20), with market clearing for global assets $b_1^* = 0$.

following system:[27]

$$\widehat{b}^* = a_1 \left[-(\sigma - \widetilde{\sigma})\, \widehat{R} + \sigma\, \widehat{R}^* \right] \tag{MP}$$

$$\widehat{\pi} = \widehat{w} + \sum_{i=T,N} (1 - \alpha^i)\phi^i\, \widehat{h} + a_2 \left[(\sigma - \widetilde{\sigma}) \sum_{i=T,N} \alpha^i \phi^i\, \widehat{h} + \widehat{R}^* \right] \tag{AS}$$

with $\widehat{b}^* = \frac{b_1^*}{R_0^* F^T(\bar{h}_0^T, A_0^T)}$. Given a nominal rate $\widehat{R}$ and a world real rate $\widehat{R}^*$, the outcomes for $(\widehat{b}^*, \widehat{h}, \widehat{\pi})$, are fully determined by (MP), (AS), and[28]

$$a_2(\sigma - \widetilde{\sigma}) \sum_{i=T,N} \alpha^i \phi^i\, \widehat{h} = \widehat{b}^* - a_2 \widehat{R}^* \tag{34}$$

A second-order approximation of the objective function around the efficient allocation gives rise to the following welfare-based loss function:

$$\mathbb{L} \equiv \frac{1}{2} \left[\frac{\kappa \bar{h}^N}{\alpha^N \phi^N} \left(1 + (\sigma - 1) \sum_{i=T,N} \alpha^i \phi^i \right) \sum_{i=T,N} \alpha^i \phi^i \left(\widehat{h} \right)^2 + \chi\, (\widehat{\pi})^2 + (\delta - \phi^T + \sigma \phi^T) \phi^T \left(\widehat{b}^* \right)^2 \right] \tag{35}$$

Under this linear-quadratic setting, the problem of a central bank is to maximize (35) subject to the three conditions above. Figure 3 presents a graphical illustration. The circled lines in panels (b) and (d) represent the indifference curve, as given by (35), where we replace $\widehat{z}$ with (34). Notice that the slope of the indifference curves changes sign when the inflation gap or output gap changes sign. As the indifference curves get closer to the (0,0) point, the level of utility increases. The tangency point between AS and the indifference curve represents the optimal solution for an individual central bank.

Crucially, the curve (AS) depends on the world real rate, which is taken as given by individual central banks. To understand the incentives to deviate from the policy dictated by the global planner, consider the world aggregate supply:[29]

$$\widehat{\pi} = \widehat{w} + \sum_{i=T,N} (1 - \alpha^i)\phi^i\, \widehat{h}. \tag{AS_W}$$
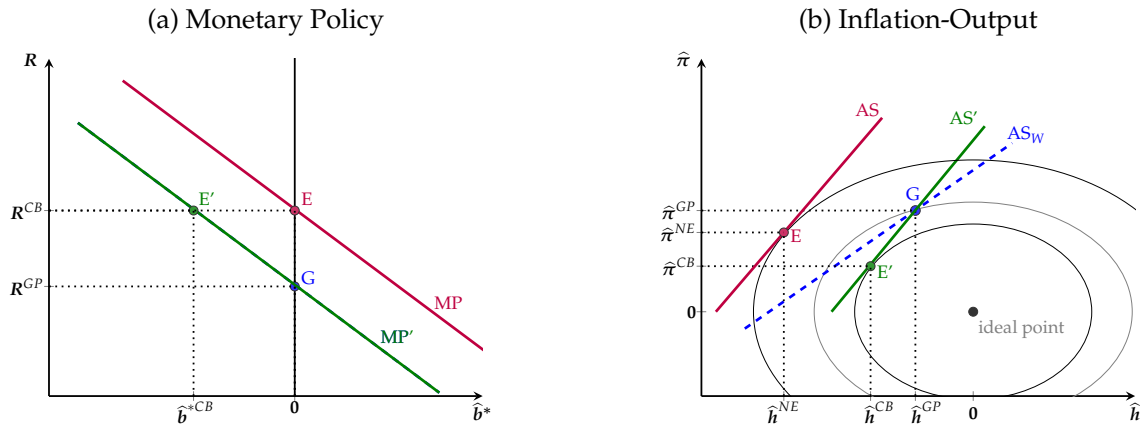
Because changes in output affect the world real interest rate, the aggregate supply faced by central banks is different from the one faced by the small open economy. Under the

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[27](MP) combines linearized (6) and (7) while (AS) combines linearized (20) and (21). Moreover, we have that $a_1 \equiv \sum_{i=T,N} \alpha^i \phi^i \left[(\delta - \alpha^T)(\delta + (\sigma - 1) \sum_{i=T,N} \alpha^i \phi^i) \right]^{-1} > 0$ and $a_2 \equiv \left[\delta + (\sigma - 1)(\phi^T + \alpha^N \phi^N) \right]^{-1} > 0$.

[28]Equation (34) is obtained by linearizing the Euler equation (6) and using (20).

[29]Equation (AS_W) follows directly from combining (34) with $\widehat{b}^* = 0$ and (AS).

Marshall-Lerner holds: $\sigma > \widetilde{\sigma}$

(a) Monetary Policy

(b) Inflation-Output

Marshall-Lerner fails: $\sigma < \widetilde{\sigma}$

(c) Monetary Policy
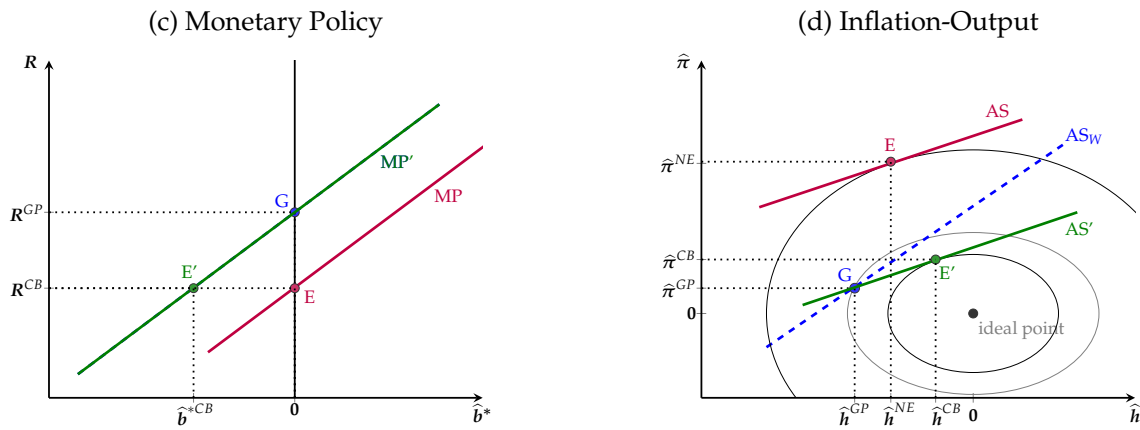
(d) Inflation-Output

Figure 3: The Need for Coordination

Note: The figure presents cases where $\widehat{\pi} > 0$ and parameters are such that $\alpha^N > \alpha^T$. The top (bottom) panels present the case of over-tightening (under-tightening).

assumption that non-tradables are more labor intensive $\alpha^N > \alpha^T$, the (AS$_W$) curve is flatter than (AS) when the generalized Marshall-Lerner condition holds (panel b) and steeper when it fails (panel [d]). The point G in this curve represents the point chosen by the global planner. The green, solid line represents the AS curve for an individual central bank that takes as given R^* and the point E′ represents the point chosen. As we can see in the Figure, the tangency point lies to the left (implying lower output and lower inflation) when the generalized Marshall-Lerner condition holds whereas it lies to the right (implying higher output and higher inflation) when the generalized Marshall-Lerner condition fails.

However, the point E' captures a situation where only an individual central bank deviates. When the generalized Marshall-Lerner condition holds, the higher nominal

interest rate for an individual central bank gives rise to a trade deficit (panel[a]). When all central banks deviate by raising the nominal interest rate, this implies that the world real rate must go up. Graphically, this means the curve MP in panel (a) shifts to the right until the point where $\widehat{b}_1^* = 0$. In addition, once the world real rate goes up, the AS curve faced by an individual central bank shifts up and to the left (panel [b]). As a result, the Nash equilibrium ends up at the point E further away from the ideal point. Compared to the cooperative outcome, economies in the Nash equilibrium have a larger recession and a lower inflation gap. On the other hand, when the generalized Marshall-Lerner condition fails, we can see in panel (d) that the Nash equilibrium ends up at a point with a smaller recession but a higher inflation gap.

4.4 Anticipated Shocks: A Case of Prudential Undertightening

Until now, we considered an economy that faces a sudden shock that creates an output-inflation tradeoff at $t=0$. In this section, we consider the possibility of a future shock. This extension allows us to examine a situation where central banks may be using monetary policy to affect their net foreign asset position and improve their output-inflation tradeoff in the future.

We assume that the economy is initially at period $t=-1$. The wage is rigid at a value W such that $W=W_{-1}^n$. We assume that agents suddenly anticipate a shock to the economy at period $t=0$.

Let us start with the analysis of the non-cooperative solution. The problem the central bank faces at $t=-1$ is analogous to the one described in (22), with the difference that now the continuation value is not the one associated with the flexible wage allocation. The individual central bank can still achieve the efficient allocation at $t=-1$, given that the shock will hit at $t=0$. However, the central bank perceives that by changing its net foreign asset position, it will improve the output-inflation tradeoff at $t=0$, when the shock hits. In particular, the central bank wants to run a trade deficit at $t=-1$ when a higher net foreign asset position at $t=0$ would help improve domestic policy tradeoff. By Lemma 4, this may require a monetary expansion or contraction, depending on the sign of $\sigma_{-1} - \widetilde{\sigma}$.

Under the assumption that $\alpha^N > \alpha^T$, the central bank would try to boost its net foreign asset position if the shock tomorrow led to a recession (and reduce its net foreign asset position if the shock tomorrow led to overheating). In turn, to the extent that $\sigma_{-1} > \widetilde{\sigma}$, the central bank would cut the nominal rate if the shock tomorrow led to a recession (and increase the nominal interest rate if the shock tomorrow led to overheating).

On the other hand, the anticipation of the shock has no effect on the optimal monetary policy under cooperation at period $t = -1$. That is, the planner sets the nominal rate to achieve the efficient allocation. Intuitively, the desire to accumulate net foreign asset position for individual countries is a zero-sum game. When central banks depart from the efficient allocation at $t = -1$, they end up worsening the allocation without any future gains.

These insights are summarized in the next proposition.

Proposition 2. *Consider $\widehat{w}_{-1} = 0$. Then,*

1. *the optimal monetary policy under cooperation features $\widehat{h}_{-1} = \widehat{\pi}_{-1} = 0$;*

2. *the Nash equilibrium features*

    i) $\widehat{h}_{-1} > 0$ and $\widehat{\pi}_{-1} > 0$ if $(\sigma_{-1} - \widetilde{\sigma})(\alpha^N - \alpha^T)\widehat{h}_0 > 0$

    ii) $\widehat{h}_{-1} < 0$ and $\widehat{\pi}_{-1} < 0$ if $(\sigma_{-1} - \widetilde{\sigma})(\alpha^N - \alpha^T)\widehat{h}_0 < 0$.

Proof. In Appendix A.10 □

A feature of our environment with anticipated shocks is that countries can now experience both overheating labor markets and high inflation. This is an interesting feature because a common characteristic of New Keynesian models is that the central bank faces unemployment and high inflation or overheating and low inflation.

The implications of cooperation for policy rates are summarized in the following corollary.

Corollary 2 (Prudential under-tightening). *Suppose countries anticipate a recession at $t = 0$. Then,*

$$R^{NE}_{-1} < R^{GP}_{-1} \iff (\alpha^N - \alpha^T)(\sigma_{-1} - \widetilde{\sigma})\widehat{h}^{NE}_{-1} > 0.$$

Proof. In Appendix A.11 □

Our sufficient statistic results therefore remain valid in the presence of anticipated shocks. That is, the extent to which there is over- or under-tightening depends on the product of the difference in labor intensity $\alpha^N - \alpha^T$, the response of the trade balance to a monetary expansion=, $\sigma_{-1} - \widetilde{\sigma}$, and the sign of the output gap.

The inefficiency of the non-cooperative outcome can be referred to as a problem of "prudential under-tightening." That is, by attempting to increase the future net foreign

asset position, with a prudential goal, central banks will conduct a monetary policy that inefficiently boosts output when there is an expected recession (and inefficiently depresses output when there is an expectation of overheating).

4.5 Quantitative Gains from Monetary Policy Coordination

We evaluate in this section the quantitative gains from monetary policy coordination. The time period is a year. We calibrate the economy using standard parameters from the literature. Households' utility function has the constant relative risk-aversion form $U(c_t) = \frac{c_t^{1-\sigma}}{1-\sigma}$.

Following Schmitt-Grohé and Uribe (2016), we set $\sigma = 5$, the labor intensity in the non-tradable sector to $\alpha^N = 0.75$ and the weight on tradable consumption in the CES function to $\phi^T = 0.26$. The labor intensity in the tradable sector is set to ensure an aggregate labor share of 2/3, which implies $\alpha^T = 0.43$.[30] Notice that given the calibrated parameters, $\sigma_0 = \sigma_{-1} > \tilde{\sigma}$. The discount factor β is set to 0.96, which ensures a steady-state value of 4% for the world real interest rate. We set the parameter governing the inflation cost χ to 10. With this value, we obtain, using the second-order approximation of the objective function (35) around a symmetric equilibrium, an effective weight on the output gap and inflation gap of 20% and 80%, respectively. These values are in the range of those considered in Woodford (2003) and Galí (2015).

As a proof of concept, we consider a negative shock to the disutility of labor κ_0. We assume the shock is anticipated at $t = -1$. In this way, we can evaluate the gains for coordination both the period the shock impacts and the period before. Notice that a shock to κ does not affect allocations in the Nash equilibrium for a given interest rate. However, it does affect the efficient allocation and thus the optimal monetary policy would respond to balance the output gap and inflation gap.

Figure 4 plots the output gap and the inflation rate under cooperation and in the Nash equilibrium in periods $t = -1$ and $t = 0$ and the welfare gains for a range of values of κ_0.

Let us discuss first the effects at $t = 0$ (panels [d] and [e]). If labor disutility falls at $t = 0$, the efficient level of output increases, which implies that the natural wage falls below the sticky wage and the economy faces an inefficiently low level of output given the initial monetary policy. In the Nash equilibrium, central banks respond by loosening monetary policy in order to mitigate the recession, and this policy gives rise to inflation.

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[30]The aggregate labor share is given by $\frac{W_t h_t^T + W_t h_t^N}{P_t^T y_t^T + P_t^N y_t^N} = \alpha^T \phi^T + \alpha^N \phi^N$.

Period t = −1

(a) Output gap

(b) Inflation gap

(c) Welfare Gains

Period t = 0
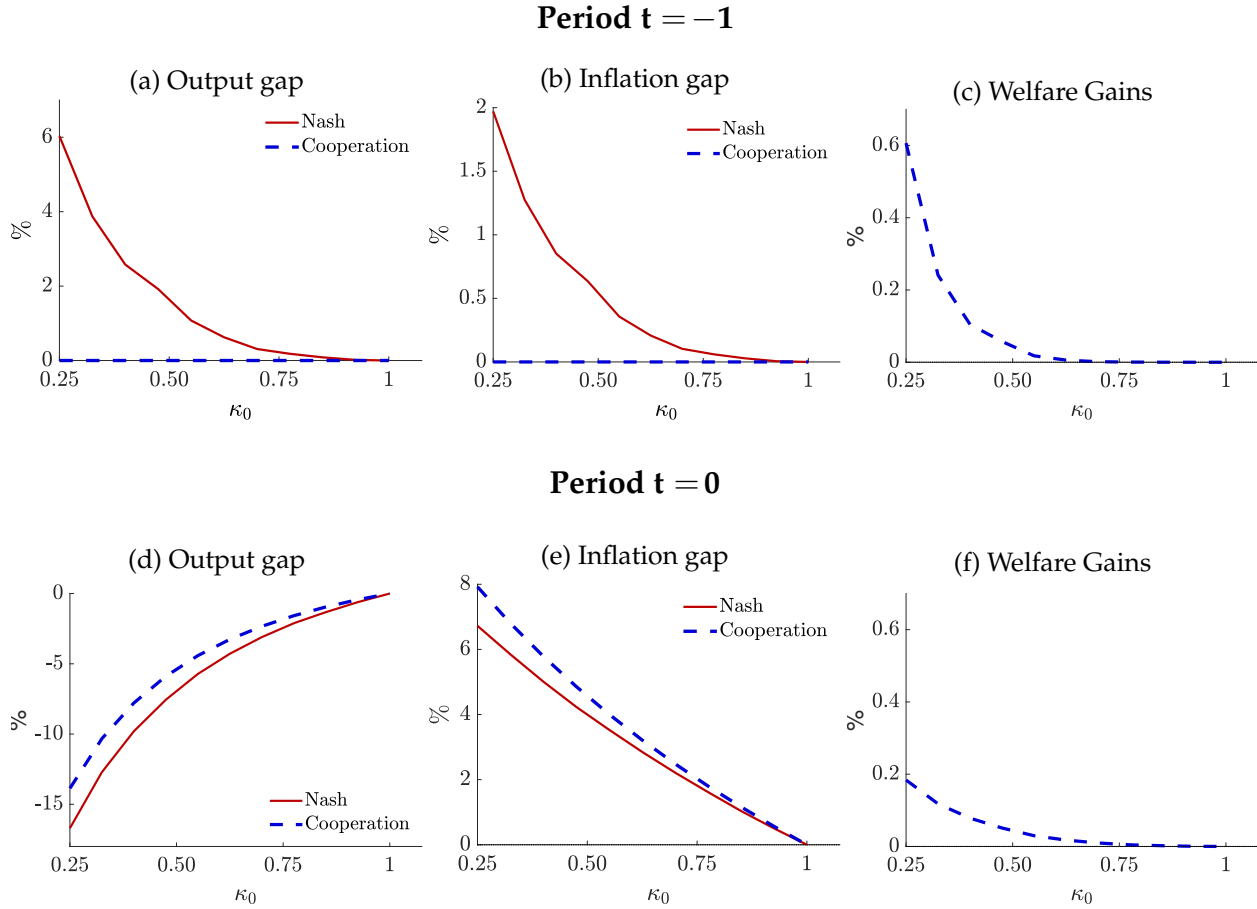

(d) Output gap

(e) Inflation gap

(f) Welfare Gains

Figure 4: Cooperation versus Nash Equilibrium

Note: The shock considered is a decrease in κ_0. The parameter values are $\alpha^N = 0.75$, $\alpha^T = 0.43$, $\phi^T = 0.26$, $\beta = 0.96$, $\chi = 10$, $\sigma = 5$. Under this parameterization, Marshall-Lerner holds. Welfare gains are measured in consumption equivalence in terms of current consumption.

Under the constellation of parameters considered, individual central banks do not lower interest rates sufficiently relative to the cooperative solution. As a result, countries face a deeper recession in the Nash equilibrium and a lower inflation rate. As shown in panels (c) and (b) of Figure 4, the difference in output gaps can reach about 2 percentage points, and the difference in inflation gaps can reach about 1.5 percentage points.

Let us now discuss the effects at $t = -1$. To be in a better position to manage the recession at $t = 0$, central banks seek to increase their trade surplus so as to have a higher NFA position. Under the value of σ considered, the generalized Marshall-Lerner condition holds, which implies that central banks cut the nominal rate at $t = -1$ in the Nash equilibrium, giving rise to an overheated labor market (panel [a]) and positive inflation (panel [b]). As the figures show, inflation can reach 2% and the output gap 6%. Meanwhile, as discussed above, under cooperation, the planner keeps policy rates unchanged and

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continues to stabilize the output gap and the inflation gap at $t = -1$.

Finally, we turn to analyze the welfare gains from cooperation. Panel (c) presents the percentage increase in consumption at $t = -1$ that would make households indifferent between remaining in the Nash equilibrium and moving to the cooperative equilibrium, assuming that at $t = 0$ the economy is in the cooperative equilibrium. Panel (f) presents the analogous consumption variation at $t = 0$. The key takeaway is that there are significant welfare gains from cooperation for moderately large shocks.

5   Extensions

The model presented allows to consider different configurations and applications. In this section, we show how our key results can be extended and generalized.

5.1   CES aggregate

In our baseline analysis, we consider a unitary elasticity of substitution between tradables and non-tradables. We now generalize the consumption of the composite to allow for a CES aggregator with elasticity $1/\gamma$.

Relative to our baseline analysis, the only difference is in the condition for the trade balance to increase in response to a monetary expansion. In particular, the Marshall-Lerner condition is such that the trade balance increases in response to a monetary expansion if and only if $\sigma_0 > \gamma\widetilde{\sigma}$. That is, the lower is the elasticity of substitution across goods, the lower is the elasticity of intertemporal substitution that delivers an increase in the trade balance in response to a monetary expansion. Intuitively, as the intra-temporal elasticity of substitution increases, a depreciation leads to larger expenditure switching from tradables towards non-tradables. Consuming fewer tradables therefore implies that more tradable output can be exported and that the trade balance increases.

5.2   Imperfect Labor Mobility

We now relax the assumption of perfect labor mobility. We assume that aggregate hours worked is a composite of hours worked in the tradable sector and in the non-tradable

sector according to the following CES aggregator:

$$n_t = \left[\frac{1}{2} \left(n_t^T \right)^{1+\frac{1}{\xi}} + \frac{1}{2} \left(n_t^N \right)^{1+\frac{1}{\xi}} \right]^{\frac{\xi}{\xi+1}}, \tag{36}$$

where $\xi \geq 0$ measures the degree of labor mobility—that is, how easy it is for a household to substitute hours worked in the tradable sector for hours worked in the non-tradable sector. When $\xi \to \infty$, there is perfect labor mobility, and the aggregate hours worked reduce to $2n_t = n_t^T + n_t^N$, as in Section 2. For $\xi = 0$, labor is perfectly immobile across sectors.

Given that hours worked in the tradable sector and in the non-tradable sector are not perfect substitutes, wages need not be equal across the two sectors. We denote by W^N and W^T the prevailing sticky wages at date $t = 0$ in the tradable and the non-tradable sector, respectively. The ratio of hours in a small open economy is given by

$$\frac{h_0^N}{h_0^T} = \frac{W^T}{W^N} \frac{\alpha^N \phi^N}{\alpha^T \phi^T} \left[1 - \frac{b_1^*}{R_0 F^T (h_0^T, A_0^T)} \right],$$

from which it follows that in any symmetric competitive equilibrium, the output gaps in the two sectors remain proportional. The optimal targeting rule in the Nash equilibrium and under cooperation continues to be given by (26) and (28), and the relative weights on the output gap continue to satisfy

$$\frac{\psi^{NE}}{\psi^{GP}} = 1 + (\alpha^N - \alpha^T)(\sigma_0 - \tilde{\sigma}) \Delta. \tag{37}$$

Equation (37) shows that relative to individual central banks in the Nash equilibrium, the global planner puts more weight on closing the output gaps if and only if the product of $\alpha^N - \alpha^T$ and $\sigma_0 - \tilde{\sigma}$ is positive. In other words, our key sufficient statistic result from Proposition 1 continues to hold.

5.3 Oil Shocks

Our baseline model assumes that labor is the sole factor of production. In this section, we incorporate oil as intermediate input and show how our results extend to this case.

We assume that households in each country are endowed with M_t units of oil, which are used as intermediate inputs for production and can be exchanged with the rest of the

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world without any trade costs. The endowment M_t is potentially time-varying and thus can give rise to "oil shocks."[31] The production functions, now given by $F^i(h_t^i, m_t^i, A_t^i)$, are differentiable, strictly increasing, concave, isoelastic with *intensity* parameters

$$\alpha^i \equiv \frac{d \log F^i(h_t^i, m_t^i, A_t^i)}{d \log h_t^i} \quad \text{and} \quad \zeta^i \equiv \frac{d \log F^i(h_t^i, m_t^i, A_t^i)}{d \log m_t^i}.$$

The aggregate demand for oil in the domestic country is $m_t = m_t^T + m_t^N$. The optimal policy problem of individual central banks in the Nash equilibrium and the problem under cooperation are presented in Appendix B.3. The optimal targeting rules are still given by (26) and (28), but now the relative weights on inflation are given by

$$\frac{\psi^{NE}}{\psi^{GP}} = 1 + (\alpha^N - \alpha^T)(\sigma_0 - \widetilde{\sigma}) \, \Delta_m,$$

with $\Delta_m > 0$ given by (A.49). The difference in labor intensity across sectors remains a key sufficient statistic, as in the baseline. On the other hand, the difference in the intensity of oil in production across sectors is irrelevant to whether central banks over- or under-tighten in the Nash equilibrium. The takeaway is that the relevant factor intensity is the one corresponding to the sticky price factor.

6 Conclusion

This paper developed a simple general theory of monetary policy coordination under financial integration. Instead of terms of trade externalities like those in the classic approach, we emphasize a pecuniary externality operating through the global capital market. Individual countries do not internalize how their monetary policy decisions affect the world real interest rate and alter the ability of foreign central banks to stabilize output and inflation.

We identify three sufficient statistics that determine whether the Nash equilibrium exhibits over-tightening or under-tightening: the output gap, sectoral differences in labor intensity, and the response of the trade balance to a nominal depreciation of the exchange. Our characterization is independent of the specific shocks driving the economy and provides general guidelines for concrete policy discussions on monetary policy coordination.

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[31] Auclert, Monnery, Rognlie and Straub (2023) show that coordinating on a tighter monetary policy is desirable from the perspective of oil importer countries to reduce their import prices. These terms of trade manipulation motives are absent in our setup.

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APPENDIX

A Proofs

A.1 Proof of Lemma 1

The proof follows directly from rearranging (13) and (14) and the specification of the utility function. □

A.2 Proof of Lemma 2

The mix of hours coincides with the competitive equilibrium. This follows from combining the optimality conditions of firms (8a) and (8b), with the optimality condition of households (4), we obtain

$$\frac{h_t^N}{h_t^T} = \frac{\alpha^N \phi^N}{\alpha^T \phi^T} \frac{c_t^T}{F^T(h_t^T, A_t^T)},$$ (A.1)

Combining (A.1) with $c_t^T = F^T(h_t^T, A_t^T)$ and (15), we obtain

$$\frac{h_t^N}{\bar{h}_t^N} = \frac{h_t^T}{\bar{h}_t^T}$$

Rearranging this equation and using $1 + \widehat{h}_t^i = \frac{h_t^i}{\bar{h}_t^i}$, we arrive at $\widehat{h}_t^N = \widehat{h}_t^T$. □

A.3 Proof of Lemma 3

We combine (2) with (8b) and (8a) to get

$$P_t = W_t \left[\prod_{i=T,N} \left(F_h \left(h_0^i, A_0^i \right) \right)^{-\phi^i} \right]$$

Assume flexible wage $h_t^i = \bar{h}_t^i$ and zero inflation gap $\frac{P_t}{P_{t-1}} - 1 = \bar{\pi}_t$, then we get

$$(1 + \bar{\pi}_t) P_{-1} = W_t^n \left[\prod_{i=T,N} \left(F_h \left(\bar{h}_0^i, A_0^i \right) \right)^{-\phi^i} \right]$$ (A.2)

Rearranging (A.2) we obtain (18). □

A.4 Derivation of (24)

The first-order conditions of the central bank's problem (22) with respect to h_0^N and h_0^T are respectively given by

$$\tau_0^N = \frac{\phi^N}{h_0^N}(1-\alpha^N)\chi(1+\pi_0)\widehat{\pi}_0 + \bar{\delta}_0^T\frac{\eta}{h_0^N} \tag{A.3}$$

$$\tau_0^T = \frac{\phi^T}{h_0^T}(1-\alpha^T)\chi(1+\pi_0)\widehat{\pi}_0 - \bar{\delta}_0^N\frac{\eta}{h_0^T} \tag{A.4}$$

where $\tau_0^i = F_h^i(h_0^i, A_0^i)u_i(c_0^T, c_0^N) - \kappa_0$ is defined in (16) and δ_0, $\bar{\delta}_0^T$ and $\bar{\delta}_0^N$ are given by

$$\bar{\delta}_0^T \equiv 1-\alpha^N+\alpha^N\frac{(\delta_0-1)+\sigma_0}{\delta_0-\phi^T+\sigma_0\phi^T} \tag{A.5}$$

$$\bar{\delta}_0^N \equiv 1-\alpha^T+\alpha^T\frac{(\delta_0-1)}{\delta_0-\phi^T+\sigma_0\phi^T}\frac{F^T(h_0^T, A_0^T)}{c_0^T} \tag{A.6}$$

$$\delta_0 \equiv 1+R_0^*c_0^T\left[\frac{1}{u_{T,1}}\frac{-du_T\left(\mathcal{C}^T(b_1^*), \mathcal{C}^N(b_1^*)\right)}{db_1^*}\right] \tag{A.7}$$

Note that $\bar{\delta}_0^T > 0$ and $\bar{\delta}_0^N > 0$ follows directly from $\delta_0 > 1$. Combining (A.3) and (A.4) and using (20), we arrive at

$$\sum_{i=T,N}\bar{\delta}_0^i\alpha^i\phi^i\,\tau_0 = \frac{\alpha^N\phi^N}{h_0^N}\sum_{i=T,N}\bar{\delta}_0^i(1-\alpha^i)\phi^i\chi(1+\pi_0)\widehat{\pi}_0. \tag{A.8}$$

Next, we isolate b_1^* in (A.8) by defining

$$\delta_b \equiv \alpha^T\frac{\delta_0-1}{\delta_0-\phi^T+\sigma_0\phi^T}\frac{1}{R_0^*c_0^T} \tag{A.9}$$

$$\delta_0^T \equiv 1-\alpha^N+\alpha^N\frac{\delta_0-1+\sigma_0}{\delta_0-\phi^T+\sigma_0\phi^T} = \bar{\delta}^T \tag{A.10}$$

$$\delta_0^N \equiv 1-\alpha^T+\alpha^T\frac{\delta_0-1}{\delta_0-\phi^T+\sigma_0\phi^T} = \bar{\delta}^N - \delta_b b_1^* \tag{A.11}$$

then, after very simple algebraic manipulation, rewrite (A.8) as

$$\sum_{i=T,N}\delta_0^i\alpha^i\phi^i\,\tau_0 = (1+\psi_b b_1^*)\frac{\alpha^N\phi^N}{h_0^N}\sum_{i=T,N}\delta_0^i(1-\alpha^i)\phi^i\chi(1+\pi_0)\widehat{\pi}_0.$$

where $1+\psi_b > 0$ follows from (A.8) and with

$$\psi_b = (\alpha^N - \alpha^T)\frac{\phi^T\phi^N\delta^N\delta^T}{\sum_i(1-\alpha^i)\phi^i\delta^i\sum_i\alpha^i\phi^i\bar{\delta}^i}\delta_b. \qquad \square$$

A.5 Proof of Lemma 4

The change in $\hat{z}_0$ induced by the change in R_0 can be decomposed as

$$\frac{db_1^*}{dR_0} = \frac{db_1^*}{dh_0^T}\frac{dh_0^T}{dR_0} \tag{A.12}$$

Using (6) and (20), we obtain

$$u_T\left(F^T(h_0^T, A_0^T) - \frac{b_1^*}{R_0^*}, F^N\left(\left(1 - \frac{b_1^*}{R_0^*F^T(h_0^T, A_0^T)}\right)\frac{\alpha^N\phi^N}{\alpha^T\phi^T}h_0^T, A_0^N\right)\right) = \beta R_0^* u_T\left(\mathcal{C}^T(b_1^*), \mathcal{C}^N(b_1^*)\right)$$

Totally differentiating this expression with respect to h_0^T and $\hat{z}_0$, we obtain

$$\left[1 + (\sigma_0 - 1)(\phi^T + \alpha^N\phi^N)\right]\frac{db_1^*}{R_0^*} - \left[\alpha^T + (\sigma_0 - 1)(\alpha^T\phi^T + \alpha^N\phi^N)\right]y_0^T\frac{dh_0^T}{h_0^T} = (1 - \delta_0)\frac{db_1^*}{R_0^*} \tag{A.13}$$

where we use $c_0^T u_{TT}(c_0^T, c_0^N) = -[1 + (\sigma_0 - 1)\phi^T]u_T(c_0^T, c_0^N)$ and $c_0^N u_{TN} = -(\sigma_0 - 1)\phi^N u_T$ and the definition of δ_0, that is (A.7).

Next, we use (8a) to express (7) as $R_0 = R_0^*\frac{W_1 F_h(h_0^T, A_0^T)}{F_h(h_1^T, A_1^T)W}$. Noting that $\frac{\kappa_1}{F_h(h_1, A_1^T)} = u_T(c_1^T, c_1^N)$, it is possible to differentiate this equation to obtain

$$\frac{dR_0}{R_0} = -(1 - \alpha^T)\frac{F^T(h_0^T, A_0^T)}{c_0^T}\frac{dh_0^T}{h_0^T} + (1 - \delta_0)\frac{db_1^*}{R_0^* c_0^T} \tag{A.14}$$

Finally, we substitute (A.13) and (A.14) into (A.12) and get

$$-\frac{db_1^*}{dR_0} = \frac{R_0^*}{R_0}c_0^T\frac{\alpha^T + (\sigma_0 - 1)(\alpha^T\phi^T + \alpha^N\phi^N)}{(\delta_0 - \alpha^T)[\delta_0 + (\sigma_0 - 1)(\phi^T + \alpha^N\phi^N)]}$$

The result in the lemma then follows directly from this expression. $\qquad\square$

A.6 Derivation of (32) and (29)

The value of the central bank's problem at date $t = 0$ is given by:

$$\begin{aligned}
V_0 &= \max_{h_0^T, h_0^N, \hat{\pi}_0, b_1^*} u\left(F^T(h_0^T, A_0^T) - \frac{b_1^*}{R_0^*}, F^N(h_0^N, A_0^N)\right) - \kappa_0(h_0^T + h_0^N) - \frac{\chi}{2}(\hat{\pi}_0)^2 + \beta V_1(b_1^*) \\
&\quad + \vartheta\left[\frac{\hat{\pi}_0}{1 + \overline{\pi}_0} - \frac{W}{W_0^n}\left(\frac{h_0^T}{\bar{h}_0^T}\right)^{(1-\alpha^T)\phi^T}\left(\frac{h_0^N}{\bar{h}_0^N}\right)^{(1-\alpha^N)\phi^N} + 1\right] + \eta\left[\left(1 - \frac{b_1^*}{R_0^*F^T(h_0^T, A_0^T)}\right)\frac{\bar{h}^N}{\bar{h}^T}\frac{h_0^T}{h_0^N} - 1\right] \\
&\quad + \mu\left[u_T\left(F^T(h_0^T, A_0^T) - \frac{b_1^*}{R_0^*}, F^N(h_0^N, A_0^N)\right) - \beta R_0^* u_T\left(\mathcal{C}^T(b_1^*), \mathcal{C}^N(b_1^*)\right)\right]
\end{aligned} \tag{A.15}$$

By the envelope theorem and using (6) and (23), we get

$$\left. \frac{dV_0}{dR_0^*} \right|_{R_0^* = R_0^{*NE}} = -\frac{1}{R_0^*} u_T(c_0^T, c_0^N) \mu \tag{A.16}$$

Combining (A.3) and (A.4), we get

$$\eta = \frac{\phi^T \phi^N}{\sum_i \delta_0^i \alpha^i \phi^i} (\alpha^N - \alpha^T) \chi (1 + \pi_0) \hat{\pi}_0 \tag{A.17}$$

Using the targeting rule (26) to express η in (A.17) in terms of $\widehat{h}_0^N$ and substituting into (23) we arrive at (29). Finally, (32) is obtained by substituting (29) into (A.16). □

A.7 Derivation of (28)

Combining the first-order conditions of the global planning problem (27) with respect to h_0^N and h_0^T along with (17) we arrive at

$$F_h^N(h_0^N, A_0^N) u_N(c_0^T, c_0^N) - \kappa_0 = \psi^{GP} \chi \frac{1 + \pi_0}{h_0^N} \hat{\pi}_0, \quad \text{with} \quad \psi^{GP} \equiv \alpha^N \phi^N \frac{\sum_i (1 - \alpha^i) \phi^i}{\sum_i \alpha^i \phi^i} \tag{A.18}$$

We then take the ratio ψ^{NE} / ψ^{GP} where ψ^{NE} is defined in (26) to obtain the expression of ψ^{GP} in (28). □

A.8 Proof of Proposition 1

Let $h_0^{N,NE}$ and $h_0^{N,GP}$ denote the level of non-tradable employment in the Nash equilibrium and in the cooperative equilibrium, and let us define

$$\mathcal{T}(h_0^N; \psi) \equiv \tau_0(h_0^N) - \chi \psi \frac{(1 + \bar{\pi}_0)^2}{h_0^N} (1 + \widehat{w}_0) \left(\frac{h_0^N}{\bar{h}_0^N} \right)^{\sum_i (1 - \alpha^i) \phi^i} \left[(1 + \widehat{w}_0) \left(\frac{h_0^N}{\bar{h}_0^N} \right)^{\sum_i (1 - \alpha^i) \phi^i} - 1 \right]$$

where we use $\widehat{h}_t^T = \widehat{h}_t^N$ to express the labor wedge τ_t (defined in (16)) as

$$\tau_t(h_t^N) \equiv F_h^N(h_t^N, A_t^N) u_N \left(F^T \left(\frac{\alpha^N \phi^T}{\alpha^N \phi^N} h_t^N, A_t^T \right), F^N \left(h_t^N, A_t^N \right) \right) - \kappa_t \tag{A.19}$$

Notice that $\mathcal{T}(h_0^{N,NE}; \psi^{NE}) = 0$ and $\mathcal{T}(h_0^{N,GP}; \psi^{GP}) = 0$. Moreover, we have that

$$\mathcal{T}\left(h_0^{N,NE}, \psi^{GP} \right) = \underbrace{\mathcal{T}(h_0^{N,NE}, \psi^{NE})}_{=0} + \frac{\psi^{GP}}{\psi^{NE}} \Delta (\sigma_0 - \widetilde{\sigma})(\alpha^N - \alpha^T) \tau_0(h_0^{N,NE})$$

by which $\mathcal{T}(h_0^{N,NE}, \psi^{GP}) < 0 \Leftrightarrow (\sigma_0 - \widetilde{\sigma})(\alpha^N - \alpha^T)\tau_0 < 0 \Leftrightarrow (\sigma_0 - \widetilde{\sigma})(\alpha^N - \alpha^T)\widehat{h}_0^{NE} > 0$. In addition, we have $\frac{d\mathcal{T}(h,\psi)}{dh} < 0$. Therefore, $\mathcal{T}(h_0^{N,NE}, \psi^{GP}) < 0 \Longleftrightarrow h_0^{N,GP} < h_0^{N,NE}$ and thus

$$\widehat{h}_0^{N,GP} < \widehat{h}_0^{N,NE} \Longleftrightarrow (\alpha^N - \alpha^T)(\sigma_0 - \widetilde{\sigma})\widehat{h}_0^{NE} > 0. \tag{A.20}$$

We then use the fact that $\widehat{w}_0 \cdot \widehat{h}_0^N \leq 0$ (with equality iff $\widehat{w}_0 = \widehat{h}_0^N = 0$) to obtain (30). $\qquad\square$

A.9 Proof of Corollary 1

Using (2) and (4) and market clearing (9) and (12), we get in any symmetric equilibrium

$$\frac{P_1}{P_0} = \left(\frac{F^T(h_0^T, A_0^T)}{F^N(h_0^N, A_0^N)} \right)^{-\phi^N} \left(\frac{F^T(\bar{h}_1^T, A_1^T)}{F^N(\bar{h}_1^N, A_1^N)} \right)^{\phi^N} \frac{P_1^T}{P_0^T} \tag{A.21}$$

We next use $P_1 = (1 + \bar{\pi}_1)P_0$ since monetary policy stabilizes prices for $t \geq 1$ and then substitute (A.21), (6) and (20) with $b_1^* = 0$ into (7) to arrive at

$$R_0 = \frac{1 + \bar{\pi}_1}{\beta \kappa_0} \left(\frac{A_0^T A_1^N}{A_0^N A_1^T} \right)^{\phi^N} \left(\frac{h_0^N}{\bar{h}_1^N} \right)^{(\alpha^T - \alpha^N)\phi^N} F_h\left(\bar{h}_1^T, A_1^T \right) u_T \left(F^T \left(\frac{\alpha^T \phi^T}{\alpha^N \phi^N} h_0^N \right), F^N \left(h_0^N \right) \right)$$

Totally differentiating this equation we obtain

$$\frac{dR_0}{dh_0^N} = -\sigma_0 \left[\alpha^T \phi^T + \alpha^N \phi^N \right] \frac{R_0}{h_0^N} < 0 \tag{A.22}$$

from which it follows that $R_0^{GP} > R_0^{NE} \Longleftrightarrow h_0^{GP} < h_0^{NE}$. Combined with (A.20) we get $R_0^{GP} > R_0^{NE} \Longleftrightarrow (\alpha^N - \alpha^T)(\sigma_0 - \widetilde{\sigma})\widehat{h}_0^{NE} > 0$. $\qquad\square$

A.10 Proof of Proposition 2

Given that the global planning problem is static, the solution to the problem date $t = -1$ (the targeting rule) is given by (A.18) where variables at $t = 0$ are replaced with variables at $t = -1$. Combining this rule with the Phillips curve (21) at $t = -1$,

$$\frac{\widehat{\pi}_{-1}}{1 + \bar{\pi}_{-1}} = \left(\frac{h_{-1}^N}{\bar{h}_{-1}^N} \right)^{\sum_i (1 - \alpha^i)\phi^i} - 1 \tag{A.23}$$

we arrive at

$$\tau(h_{-1}^N) - \chi\psi^{GP} \frac{(1 + \bar{\pi}_{-1})^2}{h_{-1}^N} \left(\frac{h_{-1}^N}{\bar{h}_{-1}^N} \right)^{\sum_i (1 - \alpha^i)\phi^i} \left[\left(\frac{h_{-1}^N}{\bar{h}_{-1}^N} \right)^{\sum_i (1 - \alpha^i)\phi^i} - 1 \right] = 0 \tag{A.24}$$

where $\tau(h^N_{-1})$ is the labor wedge at $t=-1$ given (A.19). Because $\tau(\bar{h}^N_{-1}) = 0$ by (14), we have that (A.24) implies $h^N_{-1} = \bar{h}^N_{-1}$.

Next, we turn to the solution under Nash. The Lagrangian associated with the central bank's problem at $t=-1$ is analogous to (22) where the continuation value $V_0(b_0)$ solves (A.15). Optimality condition for h^T_{-1} and h^N_{-1} combined with the envelope condition for $V_0(b_0)$ yields the following targeting rule

$$-\tau_{-1} + \chi\psi^{NE}\frac{1+\pi_{-1}}{h^N_{-1}}\widehat{\pi}_{-1} = \beta\Theta\left(\sigma_{-1} - \widetilde{\sigma}\right)u_T(c^T_0, c^N_0)\mu_0, \qquad (A.25)$$

with $\Theta > 0$ and where μ_0 satisfies (29). Combining (A.25) with (A.23) and (29), we arrive at

$$\mathcal{T}(h^N_{-1}) = -\beta\Theta\frac{\Delta}{\alpha^N}h^N_0(\sigma_{-1} - \widetilde{\sigma})(\alpha^N - \alpha^T)\tau_0 \qquad (A.26)$$

where

$$\mathcal{T}(h^N_{-1}) \equiv \tau(h^N_{-1}) - \chi\psi^{GP}\frac{(1+\bar{\pi}_0)^2}{h^N_{-1}}\left(\frac{h^N_{-1}}{\bar{h}^N_{-1}}\right)^{\sum_i(1-\alpha^i)\phi^i}\left[\left(\frac{h^N_{-1}}{\bar{h}^N_{-1}}\right)^{\sum_i(1-\alpha^i)\phi^i} - 1\right]$$

The left-hand side of (A.26), that is $\mathcal{T}(h^N_{-1})$, is decreasing in h^N_{-1} with $\mathcal{T}(\bar{h}^N_{-1}) = 0$. Therefore, $h^N_{-1} > \bar{h}^N_{-1}$ if and only if $(\sigma_{-1} - \widetilde{\sigma})(\alpha^N - \alpha^T)\widehat{h}^N_0 > 0$. Moreover, for $h^N_{-1} > \bar{h}^N_{-1}$, we have by (21) that $\widehat{\pi}_{-1} > 0$. Conversely when $h^N_{-1} < \bar{h}^N_{-1}$ we have that $\widehat{\pi}_{-1} < 0$. $\qquad \square$

A.11 Proof of Corollary 2

Suppose $\widehat{h}^N_0 < 0$. Note that in cooperation solution features $\widehat{h}^N_{-1} = 0$. By (A.26) the Nash equilibrium coincides with the cooperation solution if and only if $(\sigma_{-1} - \widetilde{\sigma})(\alpha^N - \alpha^T) = 0$. Furthermore, the Nash equilibrium features under-tightening $\widehat{h}^N_{-1} > 0$ if and only if $(\sigma_{-1} - \widetilde{\sigma})(\alpha^N - \alpha^T) > 0$ or equivalently if and only if $(\sigma_{-1} - \widetilde{\sigma})(\alpha^N - \alpha^T)\widehat{h}^N_{-1} > 0$. $\qquad \square$

B Proofs of Extensions

B.1 Elasticity of Substitution

This section extends the baseline model with CES aggregators. Households' preferences are still described by (1) where the consumption good c_t is now a composite of tradable consumption c_t^T and non-tradable consumption c_t^N, according to a CES aggregator

$$c_t = \left[\sum_{i \in \mathcal{S}} \phi^i \left(c_t^i \right)^{1-\gamma} \right]^{\frac{1}{1-\gamma}}$$

The budget constraint of households is identical to the one in the baseline model. The household's optimality condition with respect to c_t^T and c_t^N (4) is now given by

$$\frac{P_t^N}{P_t^T} = \frac{\phi^N}{\phi^T} \left(\frac{c_t^T}{c_t^N} \right)^{\gamma} \tag{A.27}$$

Using (A.27), we can express the share of expenditures in tradables $\tilde{\phi}_t^T \equiv P_t^T c_t^T / (P_t c_t)$ as $\tilde{\phi}_t^T = \phi^T (c_t^T / c_t)^{1-\gamma}$. and the share of expenditures in non-tradables is $\tilde{\phi}_t^N = 1 - \tilde{\phi}_t^T$. The remaining optimality conditions of the household's problem are (6), (7) (and (5) for $t > 0$) while for firms, (8a), (8b) continue to hold. Combining (A.27) with (8a) and (8b) we obtain

$$\frac{h_t^N}{h_t^T} = \frac{\alpha^N \tilde{\phi}_t^N}{\alpha^T \tilde{\phi}_t^T} \left[1 - \frac{b_1^*}{R_0^* F^T (h_0^T, A_0^T)} \right] \tag{A.28}$$

While using (13) and (14), the optimal ratio of hours in the first-best allocation becomes

$$\frac{\bar{h}_t^T}{\bar{h}_t^N} = \frac{\alpha^T \tilde{\phi}_t^T}{\alpha^N \tilde{\phi}_t^N}$$

which corresponds to the employment ratio in a competitive symmetric equilibrium for any monetary policy. Therefore, in any symmetric competitive equilibrium, the output gaps in the tradable and non-tradable sectors are proportional, and to a first-order

$$\widehat{h}_t^N = \Gamma \, \widehat{h}_t^T, \quad \text{where } \Gamma \equiv \frac{1 - \alpha^T + \alpha^T \gamma}{1 - \alpha^N + \alpha^N \gamma} > 0. \tag{A.29}$$

In the lemma below, we summarize the effects of monetary policy on the trade balance.

Lemma B.1 (Generalized Marshall-Lerner Condition). *The response of the trade balance to a domestic monetary expansion satisfies* $-\frac{db_1^*}{dR_0} > 0 \iff \sigma_0 > \gamma \tilde{\sigma}$ *where* $\tilde{\sigma} \equiv 1 - \frac{\alpha^T}{\alpha^T \phi^T + \Gamma \alpha^N \phi^N}$.

Proof. Proceeding similarly as in Appendix A.5 by combining (6), (7), (8a), (20) we get

$$\left[\delta_0 + (\sigma_0 - 1)(\alpha^T \phi^T + \Gamma \alpha^N \phi^N) \right] (\delta_0 - \alpha^T) db_1^* = -\frac{R_0^*}{R_0} c_0^T \left[\alpha^T + (\sigma_0 - \gamma)(\alpha^T \phi^T + \Gamma \alpha^N \phi^N) \right] dR_0$$

Thus $-\frac{db_1^*}{dR_0} > 0 \iff \alpha^T\gamma + (\sigma_0 - \gamma)(\alpha^T\phi^T + \Gamma\alpha^N\phi^N) > 0$. Defining $\tilde{\sigma} \equiv 1 - \frac{\alpha^T}{\alpha^T\phi^T + \Gamma\alpha^N\phi^N}$, we obtain that $-db_1^*/dR_0 > 0$ if and only if $\sigma_0 > \gamma\tilde{\sigma}$. $\qquad\square$

Note that, given preferences, the consumer price index P_t now satisfies

$$P_t = \left[\sum_{i\in\mathcal{S}} (\phi^i)^{\frac{1}{\gamma}} (P_t^i)^{1-\frac{1}{\gamma}}\right]^{\frac{\gamma}{\gamma-1}}$$

Thus, using the definition of the natural wage we can express the inflation gap as

$$\frac{\hat{\pi}_0}{1+\overline{\pi}_0} = \frac{W}{W_0^n}\left[\frac{\sum_{i=T,N}(\phi^i)^{\frac{1}{\gamma}}(F_h(h_t^i, A_t^i))^{\frac{1-\gamma}{\gamma}}}{\sum_{i=T,N}(\phi^i)^{\frac{1}{\gamma}}(F_h(\bar{h}_t^i, A_t^i))^{\frac{\gamma}{1-\gamma}}}\right]^{\frac{\gamma}{\gamma-1}} - 1 \qquad (A.30)$$

The Lagrangian associated with the central bank's problem can be written as follows

$$u\left((1-\hat{z}_0)A_0^T F^T(h_0^T), A_0^N F^N(h_0^N)\right) - \kappa_0(h_0^T+h_0^N) - \frac{\chi}{2}(\hat{\pi}_0)^2 + \beta V_1\left(R_0^* A_t^T F^T(h_0^T)\hat{z}_0\right)$$

$$+\vartheta\left[\frac{\hat{\pi}_0}{1+\overline{\pi}_0} - \frac{W}{W_0^n}\left(\frac{\sum_{i=T,N}(\phi^i)^{\frac{1}{\gamma}}(F_h(h_t^i, A_t^i))^{\frac{1-\gamma}{\gamma}}}{\sum_{i=T,N}(\phi^i)^{\frac{1}{\gamma}}(F_h(\bar{h}_t^i, A_t^i))^{\frac{\gamma}{1-\gamma}}}\right)^{\frac{\gamma}{\gamma-1}} + 1\right] + \eta\left[(1-\hat{z}_0)\frac{\alpha^N\tilde{\phi}_0^N}{\alpha^T\tilde{\phi}_0^T}\frac{h_0^T}{h_0^N} - 1\right]$$

$$+\mu\left[u_T\left((1-\hat{z}_0)A_0^T F^T(h_0^T), A_0^N F^N(h_0^N)\right) - \beta R_0^* u_T\left(C^T(b_1^*), C^N(b_1^*)\right)\right] \qquad (A.31)$$

The optimality condition for $\hat{z}_0$ yields $\eta_0 = \left[\delta_0 + \left(\sigma_0\gamma^{-1} - 1\right)\tilde{\phi}_0^T\right]u_T\left(c_0^T, c_0^N\right)\mu_0$ where δ_0 is given by (A.7). Using this equation and combining the first-order conditions for h_0^T and h_0^N, we obtain the following targeting rule in the Nash equilibrium (where $b_1^* = 0$):

$$\tau_0 = \chi\psi^{NE}\frac{1+\pi_0}{h_0^N}\hat{\pi}_0 \quad \text{with } \psi^{NE} = \alpha^N\phi^N\frac{\sum_{i=T,N}\delta_0^i(1-\alpha^i)\tilde{\phi}_0^i}{\sum_{i=T,N}\delta_0^i\alpha^i\tilde{\phi}_0^i} \qquad (A.32)$$

where $\tau_t^T = \tau_0^T = \tau_0$ is defined in (16), and $\delta_0^T > 0$ and $\delta_0^N > 0$ are given by

$$\delta_0^T \equiv 1 + \alpha^N(\gamma - 1) + \frac{(\sigma_0 - \gamma)\alpha^N\tilde{\phi}_0^N}{\delta_0 - \tilde{\phi}_0^T + \sigma_0\gamma^{-1}\tilde{\phi}_0^T}$$

$$\delta_0^N \equiv 1 + \alpha^T(\gamma - 1) - \alpha^T\frac{\gamma + (\sigma_0 - \gamma)\tilde{\phi}_0^T}{\delta_0 - \tilde{\phi}_0^T + \sigma_0\gamma^{-1}\tilde{\phi}_0^T}$$

To see why $\delta_0^T > 0$ and $\delta_0^N > 0$, notice that for $\sigma > \gamma$ this is trivial. For $\sigma < \gamma$, it can be shown that δ_0^T and δ_0^N are increasing in γ, and we have $\lim_{\gamma\to0}\delta_0^T > 0$ and $\lim_{\gamma\to0}\delta_0^N > 0$.

Under cooperation, the Lagrangian associated with the planner problem is given by

$$u\left((1-\hat{z}_0)A_0^T F^T(h_0^T), A_0^N F^N(h_0^N)\right) - \kappa_0(h_0^T+h_0^N) - \frac{\chi}{2}(\hat{\pi}_0)^2$$

$$+\vartheta\left[\frac{\hat{\pi}_0}{1+\overline{\pi}_0} - \frac{W}{W_0^n}\left(\frac{\sum_{i=T,N}(\phi^i)^{\frac{1}{\gamma}}(F_h(h_t^i, A_t^i))^{\frac{1-\gamma}{\gamma}}}{\sum_{i=T,N}(\phi^i)^{\frac{1}{\gamma}}(F_h(\bar{h}_t^i, A_t^i))^{\frac{\gamma}{1-\gamma}}}\right)^{\frac{\gamma}{\gamma-1}} + 1\right] + \eta\left[\frac{\alpha^N\tilde{\phi}_0^N}{\alpha^T\tilde{\phi}_0^T}\frac{h_0^T}{h_0^N} - 1\right]$$

The targeting rule, which combined the first-order condition, for h_0^T and h_0^N is given by

$$\tau_0 = \chi \psi^{GP} \frac{1+\pi_0}{h_0^N} \widehat{\pi}_0 \quad \text{with} \quad \psi^{GP} = \alpha^N \phi^N \frac{\sum_{i=T,N} \delta_x^i (1-\alpha^i) \tilde{\phi}_0^i}{\sum_{i=T,N} \delta_0^i \alpha^i \tilde{\phi}_0^i} \tag{A.33}$$

with $\delta_x^T = 1 + \alpha^N(\gamma-1)$ and $\delta_x^N = 1 + \alpha^T(\gamma-1)$. Taking the ratio between the relative weights in the targeting rules (A.32) and (A.33), we arrive at

$$\frac{\psi^{NE}}{\psi^{GP}} = 1+(\alpha^N - \alpha^T)(\sigma_0 - \tilde{\sigma}) \Delta, \quad \text{with} \quad \Delta \equiv \frac{\tilde{\phi}_0^T \tilde{\phi}_0^N}{(\delta_0 - \tilde{\phi}_0^T + \sigma_0 \gamma^{-1} \tilde{\phi}_0^T) \sum_i \delta_0^i (1-\alpha^i) \tilde{\phi}_0^i} > 0.$$

B.2 Imperfect Labor Mobility

In this section, we extend the baseline model with imperfect labor mobility. Households' preferences are given by (1) where aggregate hours worked n_t is now a composite of hours worked in the tradable sector and in the non-tradable sector according to:

$$n_t = \left[\frac{1}{2} \left(n_t^T \right)^{1+\frac{1}{\xi}} + \frac{1}{2} \left(n_t^N \right)^{1+\frac{1}{\xi}} \right]^{\frac{\xi}{\xi+1}}, \tag{A.34}$$

where $\xi \geq 0$ measures. The budget constraint of households is identical to the one in the baseline model. The household's optimality condition with respect to c_t^T and c_t^N is given by (4), while the optimal labor supply decisions for $t > 0$ (5) now satisfy

$$\frac{W_t^N}{P_t^N} = \frac{\kappa_t}{u_N(c_t^T, c_t^N)} \left(\frac{n_t^N}{n_t} \right)^{\frac{1}{\xi}}, \qquad \frac{W_t^T}{P_t^T} = \frac{\kappa_t}{u_T(c_t^T, c_t^N)} \left(\frac{n_t^T}{n_t} \right)^{\frac{1}{\xi}}. \tag{A.35}$$

where W_t^T and W_t^N are the nominal wages in the tradable and non-tradable sectors. The remaining optimality conditions of households are (6), (7). For firms, optimality conditions (8a), (8b) now become $P_t^i F_h(h_t^i, A_t^i) = W_t^i$ for $i = T, N$ which combined with (4) yields

$$\frac{h_0^N}{h_0^T} = \frac{W^T}{W^N} \frac{\alpha^N \phi^N}{\alpha^T \phi^T} \left[1 - \frac{b_1^*}{R_0 F^T(h_0^T, A_0^T)} \right], \tag{A.36}$$

while the employment ratio in the efficient allocation is

$$\frac{\bar{h}_0^N}{\bar{h}_0^T} = \left[\frac{\alpha^N \phi^N}{\alpha^T \phi^T} \right]^{\frac{\xi}{1+\xi}} \tag{A.37}$$

Before turning to the policy analysis under imperfect labor mobility, we find it useful to describe the natural wages at date $t = 0$ in the following lemma:

Lemma B.2 (Natural Wage). *The natural wages, $W_t^{N,n}$ and $W_t^{T,n}$, at date t satisfy*

$$\frac{W_t^{N,n}}{W_t^{T,n}} = \left(\frac{\alpha^N \phi^N}{\alpha^T \phi^T}\right)^{\frac{1}{\xi+1}} \tag{A.38}$$

$$\text{and} \quad W_t^{T,n} = (1+\overline{\pi}_t) P_{t-1} \left(\frac{W^{N,n}}{W_t^{T,n}}\right)^{\phi^N} \left[\prod_{i=T,N} \left(\alpha^i A_t^i\right)^{\phi^i} \left(\bar{h}_t^i\right)^{-(1-\alpha^i)\phi^i}\right] \tag{A.39}$$

Proof. By (A.36) and (A.39), we have that under flexible wage with symmetric monetary policy the relative wage must satisfy (A.38). Combining (2) with $P_t^i F_h(h_t^i, A_t^i) = W_t^i$, we get

$$P_t = (W_t^T)^{\phi^T} (W_t^N)^{\phi^N} \prod_{i=T,N} \left(F_h\left(h_0^i, A_0^i\right)\right)^{-\phi^i}$$

Assuming flexible wage $h_t^i = \bar{h}_t^i$ and zero inflation gap $\frac{P_t}{P_{t-1}} - 1 = \overline{\pi}_t$, we get (A.39). $\qquad\square$

As before, we assume that nominal wages are at their natural level at date $t = -1$, i.e. $W_{-1}^i = W_{-1}^{i,n}$ for $i = T, N$. At date $t = 0$, however, the nominal wages are fixed their previous level, i.e. $W_0^i = W_{-1}^i = W^i$.

The Lagrangian for the central bank problem is analogous to (22) where aggregate hours are now given by (A.34) (notice that the Phillips curve continues to be given by (21)) The optimality conditions for h_0^N and h_0^T continue to be given by (A.3) and (A.4) where the labor wedge in sector $i = T, N$ is now defined as

$$\tau_t^i \equiv F_h^i(h_t^i, A_t^i) u_i(c_t^T, c_t^N) - \kappa_t \left(\frac{h_t^i}{h_t}\right)^{\frac{1}{\xi}} \tag{A.40}$$

Because the nominal wages at $t = 0$ are fixed at their previous level, the ratio is still given by (A.38) which implies $\tau_0^T = \tau_0^N$. Thus, combining (A.3) and (A.4), we obtain in the Nash equilibrium where $b_1^* = 0$ that

$$\tau_0 = \chi \psi^{NE} \frac{1+\pi_0}{h_0^N} \widehat{\pi}_0, \quad \text{with } \psi^{NE} \equiv \alpha^N \phi^N \frac{\sum_{i=T,N} \delta_0^i (1-\alpha^i) \phi^i}{\sum_{i=T,N} \delta_0^i \alpha^i \phi^i} \tag{A.41}$$

Similar to the individual central bank problem, the Lagrangian associated with the global planner's problem is analogous to (27) with aggregate hours now given by (A.34). After combining the optimality conditions for h_0^T and h_0^N, we obtain

$$\tau_0 = \chi \psi^{GP} \frac{1+\pi_0}{h_0^N} \widehat{\pi}_0, \quad \text{with } \psi^{GP} \equiv \alpha^N \phi^N \frac{\sum_{i=T,N} (1-\alpha^i) \phi^i}{\sum_{i=T,N} \alpha^i \phi^i}$$

Taking the ratio of the relative weights we arrive at (37).

B.3 Oil Shock

This section extends the model to incorporate oil as an intermediate input. We assume that households receive an endowment of oil which is used by firms as inputs for production and can be exchanged with the rest of the world. The law of one price is assumed to hold in the market for oil, that is $P_{mt} = e_t P^*_{mt}$ where P_{mt} and P^*_{mt} are the domestic and the world price of oil, and e_t is the effective exchange rate. Combined with the law of one price for tradables, this implies that $\frac{P_{mt}}{P^T_t} = \frac{P^*_{mt}}{P^{T*}_t}$. We can thus express households' budget constraint as

$$P^T_t c^T_t + P^N_t c^N_t + \frac{b_{t+1}}{R_t} + \frac{P^T_t b^*_{t+1}}{R^*_t} = W_t(n^T_t + n^N_t) + \varphi_t + P_{mt}m^S_t + P_{mt}(M_t - m^S_t) + b_t + P^T_t b^*_t$$

where m^S_t is the domestic supply of oil and $M_t - m^S_t$ is the net export of oil. The production functions are given by $F^i(h^i_t, m^i_t, A^i_t)$ with α^i, and ζ^i denoting the intensity of labor and oil respectively. At the optimum, the demand for labor is analogous to (8a)-(8b) and given by $P^i_t F_h(h^i_t, m^i_t, A^i_t) = W_t$ for all $i \in \mathcal{S}$; while their demand for oil is given by

$$m^T_t = \frac{\zeta^T}{\alpha^T} \frac{W_t}{P_{mt}} h^T_t, \quad m^N_t = \frac{\zeta^N}{\alpha^N} \frac{W_t}{P_{mt}} h^N_t \tag{A.42}$$

The Lemma below describes the allocation of oil in any symmetric competitive equilibrium.

Lemma B.3. *In any symmetric competitive equilibrium, the allocation of intermediate oil inputs is efficient and given by*

$$m^N_t = \frac{\zeta^N \phi^N}{\sum_{i=T,N} \alpha^i_m \phi^i} M_t, \quad m^T_t = \frac{\zeta^T \phi^T}{\sum_{i=T,N} \alpha^i_m \phi^i} M_t \tag{A.43}$$

Proof. The proof combines the ratio of the two equations in (A.42) with (20), together with $\widehat{z}_0$ and market clearing for oil $m^T_t + m^N_t = M_t$. $\qquad\square$

Denoting by $\overline{m}^T_0$ and $\overline{m}^N_0$ the allocation in (A.43), the Lagrangian associated with the global planning problem can be expressed as

$$u\left(F^T(h^T_0, \overline{m}^T_0, A^T_0), F^N(h^N_0, \overline{m}^N_0, A^N_0)\right) - \kappa_0(h^T_0 + h^N_0) - \frac{\chi}{2}(\widehat{\pi}_0)^2$$

$$+ \vartheta\left[\frac{\widehat{\pi}_0}{1+\overline{\pi}_0} - \frac{W}{W^n_0}\left(\frac{F_h(\bar{h}^T_0, \overline{m}^T_0, A^T_0)}{F_h(h^T_0, \overline{m}^T_0, A^T_0)}\right)^{\phi^T}\left(\frac{F_h(\bar{h}^N_0, \overline{m}^N_0, A^N_0)}{F_h(h^N_0, \overline{m}^N_0, A^N_0)}\right)^{\phi^N} + 1\right] + \eta\left[\frac{\alpha^N \phi^N}{\alpha^T \phi^T} \frac{h^T_0}{h^N_0} - 1\right]$$

Notice that the allocation of oil is independent of policy. The targeting rule under cooperation, which combines the optimality condition for h^T_0 and h^N_0, is therefore identical to (28) and given by

$$\tau_0 = \chi \psi^{GP} \frac{1+\pi_0}{h^N_0} \widehat{\pi}_0 \quad \text{with } \psi^{GP} = \alpha^N \phi^N \frac{\sum_i (1-\alpha^i)\phi^i}{\sum_i \alpha^i \phi^i} \tag{A.44}$$

where

$$\tau_0 \equiv F_h^N(h_0^N, \overline{m}_0^N, A_0^N)\, u_N\left(F^T\left(\frac{\alpha^T \phi^T}{\alpha^N \phi^N} h_0^N, M_0 - \overline{m}_0^N, A_0^T\right), F^N\left(h_0^N, \overline{m}_0^N, A_0^N\right)\right) - \kappa_0$$

We now turn to deriving the targeting rule in the Nash equilibrium. Combining (A.42) with (20), the allocation of oil input across sectors in a small open economy is given by

$$m_t^T(b_{t+1}^*) = \frac{\zeta^T \phi^T (1 - \widehat{z}_t)}{\zeta^N \phi^N + \zeta^T \phi^T (1 - \widehat{z}_t)} m_t^S \quad \text{and} \quad m_t^N(b_{t+1}^*) = \frac{\zeta^N \phi^N}{\zeta^N \phi^N + \zeta^T \phi^T (1 - \widehat{z}_t)} m_t^S \quad \text{(A.45)}$$

with $z_t = \frac{b_1^*}{R_t^* F^T(h^T, m_t, A_t^T)}$. Using (A.45), we can express the Lagrangian associated with the central bank's problem as

$$u\left((1 - \widehat{z}_0) F^T(h_0^T, m^T(\widehat{z}_0), A_0^T), F^N(h_0^N, m^N(\widehat{z}_0), A_0^N)\right)$$

$$-\kappa_0(h_0^T + h_0^N) - \frac{\chi}{2}(\widehat{\pi}_0)^2 + \beta V_1\left(R_0^* F^T(h_0^T)\widehat{z}_0\right) + \eta\left[(1 - \widehat{z}_0)\frac{\alpha^N \phi^N}{\alpha^T \phi^T}\frac{h_0^T}{h_0^N} - 1\right]$$

$$+\vartheta_0\left[\frac{\widehat{\pi}_0}{1 + \overline{\pi}_0} - \frac{W}{W_0^n}\left(\frac{F_h(\bar{h}_0^T, \overline{m}_0^T, A_0^T)}{F_h(h_0^T, m^T(\widehat{z}_0), A_0^T)}\right)^{\phi^T}\left(\frac{F_h(\bar{h}_0^N, \overline{m}_0^N, A_0^N)}{F_h(h_0^N, m^N(\widehat{z}_0), A_0^N)}\right)^{\phi^N} + 1\right]$$

$$+\mu\left[u_T\left((1 - \widehat{z}_0) F^T(h_0^T, m^T(\widehat{z}_0), A_0^T), F^N(h_0^N, m^N(\widehat{z}_0), A_0^N)\right) - \beta R_0^* u_T\left(C^T(b_1^*), C^N(b_1^*)\right)\right]$$

The optimality condition with respect to $\widehat{z}_0$ is given by

$$\eta = \left[\delta_0^m + (\sigma_0 - 1)\phi^T\right] u_T(c_0^T, c_0^N)\mu \quad \text{(A.46)}$$

where on the competitive equilibrium path, δ_0^m is given by

$$\delta_0^m = \delta_0 + \frac{\zeta^T \phi^T \cdot \zeta^N \phi^N}{\zeta^T \phi^T + \zeta^N \phi^N} + \chi\left(\phi^T \frac{F_{hm}^T}{F_h^T} + \phi^N \frac{F_{hm}^N}{F_h^N}\right)(1 + \pi_0)\widehat{\pi}_0 \quad \text{(A.47)}$$

Notice by (A.45) and (A.43) that in the Nash equilibrium where $\widehat{z}_0 = 0$, the allocation of oil is optimal. Moreover, the optimality condition for h_0^N and h_0^T are akin to (A.3) and (A.4) where δ_0 is replaced with δ_0^m. As a result, the targeting rule in the Nash equilibrium is

$$\tau_0 = \chi \psi^{GP}\frac{1 + \pi_0}{h_0^N}\widehat{\pi}_0 \quad \text{with} \quad \psi^{NE} = \alpha^N \phi^N \frac{\sum_i \delta_0^i (1 - \alpha^i)\phi^i}{\sum_i \delta_0^i \alpha^i \phi^i} \quad \text{(A.48)}$$

where δ_0^T and δ_0^N satisfy (A.10) and (A.11) where δ_0 is replaced with δ_0^m. Taking the ratio of the relative weights on inflation in (A.44) and (A.48), we arrive at

$$\frac{\psi^{NE}}{\psi^{GP}} = 1 + (\alpha^N - \alpha^T)(\sigma_0 - \widetilde{\sigma})\,\Delta_m, \quad \text{with} \quad \Delta_m \equiv \frac{\phi^T \phi^N}{(\delta_0^m - \phi^T + \sigma_0 \phi^T)\sum_i \delta_0^i (1 - \alpha^i)\phi^i} > 0. \quad \text{(A.49)}$$