A Little Bit of Evidence on the Natural Rate Hypothesis from the U.S.

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This paper presents some illustrative tests of the natural rate hypothesis along the lines suggested by Sargent [3]. A major problem in constructing a proper test of the natural rate hypothesis is that for any economy moving along in a stationary fashion under a set regime governing the responses of the policy authority, there exist multiple indistinguishable statistical models compatible with the time series observations. One of these models, if assumed invariant under changes in the policy regime, implies conclusions consistent with the natural rate hypothesis. But another one of the models, which is just as consistent with the data as is the first, delivers Keynesian policy implications if it is assumed invariant across policy regimes. Consequently, it seems impossible to test the natural rate hypothesis on the basis of time series observations on a single economy operating under a single fixed policy regime and fixed processes for other exogenous variables. One way to test the natural rate hypothesis is to find periods across which the policy regimes differ and to test for the invariance of alternative models across regimes. This is the approach taken here.

By way of motivating our test and summarizing the observational equivalence of alternative models, we state the following two representation facts.[1] Let y_t, m_t be a covariance stationary linearly indeterministic stochastic process. Here y_t is a goal variable like real GNP and m_t is a policy variable like the money supply.

Fact 1: There exists a representation (a model) for y_t, m_t of the form

(1a) $$y_t = \sum_{i=0}^{\infty} a_i m_{t-i} + \sum_{i=1}^{\infty} b_i y_{t-i} + \varepsilon_t$$

(1b) $$m_t = \sum_{i=1}^{\infty} c_i m_{t-i} + \sum_{i=1}^{\infty} d_i y_{t-i} + u_t$$

where the a_i, b_i, c_i, and d_i's are fixed numbers, where ε_t and u_s are serially uncorrelated random variables with mean zero and finite variances, and where $E\varepsilon_t u_s = 0$ for all integer t and s, so that u_s and ε_t are orthogonal at all lags.

Fact 2: For the same (y_t, m_t) process as described in Fact 1, there exists a representation (a model) of the form

(2a) $$y_t = \sum_{i=0}^{\infty} a_i'(m_{t-i} - E_{t-i-1} m_{t-i}) + \sum_{i=1}^{\infty} b_i' y_{t-i} + \varepsilon_t$$

(2b) $$m_t = \sum_{i=1}^{\infty} c_i m_{t-i} + \sum_{i=1}^{\infty} d_i y_{t-i} + u_t$$

where $E_{t-1} m_t$ is the linear least squares projection of m_t on m_{t-1}, $m_{t-2}, \ldots,$ y_{t-1}, $y_{t-2}, \ldots,$ and where the a_i', b_i' are fixed numbers, and the c_i, d_i, u_t and ε_t are identical with those in (1a) and (1b). Since the ε_t, u_t process in (1a)-(1b) and (2a)-(2b) are identical, model (1) is observational equivalent with model (a), i.e., the models fit equally well.

If model (1) is invariant under changes in the policy regime, Keynesian policy conclusions follow. In particular, the variance of y around a target y* is minimized by a feedback rule form of the form

$$a_0 m_t = -\sum_{i=1}^{\infty} a_i m_{t-i} - \sum_{i=1}^{\infty} b_i y_{t-i} + y^*.$$

Under the assumption that model (1) is invariant, the above feedback rule dominates Friedman's x-percent growth rule for money, which is a rule without feedback.

However, if model (2) is invariant under changes in the policy regime, the behavior of y_t is independent of the systematic part of the feedback rule for m. Any deterministic feedback rule delivers $E_{t-1}m_t = m_t$, and delivers a path for y_t described by

$$y_t = \sum_{i=1}^{\infty} b_i' y_{t-i} + \varepsilon_t,$$

which is independent of the parameters of the money supply feedback rule. Hence if (2) is invariant across policy regimes, we obtain the strong neutrality result that one deterministic feedback rule for m is as good as any other.

If one can identify distinct periods across which there occur changes in the policy regime, as summarized by the parameters of (1b), then it is possible to test the null hypothesis that (1a) or (2a) remains invariant across such periods. In this way, there is some hope of testing the critical invariance assumptions required to draw policy implications from a model like (1) or (2).

For quarterly data extending over the post-war period, we implemented the test, taking for y seasonally unadjusted real GNP in 1958 dollars, and as m alternatively seasonally unadjusted M1 (currency plus adjusted demand deposits) and seasonally adjusted M2 (currency plus deposits at commercial banks). For the pre-war period, we used monthly data on seasonally adjusted industrial production for y and seasonally adjusted M1 for m. For each period, we first regressed each series on a constant, trend, and seasonal dummies to remove obvious deterministic elements, and then treated the residuals from those regressions as our data.

We employed the periods[2] listed in Table 1. For the pre-war data (1920-1940), we split the period at January 1930, while for the post-war data, we split the period at 1964I. We then calculated the standard F-tests of the null hypothesis that equations (1b), (1a), and (2a), respectively were the same across the two periods. The F-statistics are reported in Table 2 while the corresponding marginal significance levels are reported in Table 3.

In each case, the null hypothesis that the "feedback rule" equation (1b) was invariant across periods for M1 is rejected at high confidence levels--the marginal significance level is less than .01 in each case. For M2 in the post-war period, there is less firm evidence that the feedback rule changed--the marginal significance level is .13. The precondition for our test--evidence of a break in the feedback rule (1b) for m--is met at least for M1.

The tests for invariance of (1a) and (2a) show the null hypothesis of stability of (1a) to be rejected at uniformly higher confidence levels than is the hypothesis of the stability of (1b). The marginal significance levels are such that a true believer in the invariance across regimes of (2a) would probably not find the evidence sufficiently negative to change his beliefs; a true believer in the invariance of (1a) would have to be rather more pigheaded in order not to have the evidence change his views.

Table 4 reports marginal significance levels for the null hypothesis that the residual variance remains constant across the sample splits. That hypothesis is of interest because rational expectations models like Lucas's [1973] predict a dependence between the residual variance in the money feedback rule and the slope parameters of the reduced form for output.

TABLE 1a

PERIODS

POST-WORLD WAR II

1. Whole Period

    (a) Period for regression (1b)

        from 1949/II to 1974/IV

    (b) Period for regressions (1a, 2a)

        from 1950/III to 1974/IV

2. First Subsample Period

    (a) Period for regression (1b)

        from 1949/II to 1963/IV

    (b) Period for regressions (1a, 2a)

        from 1950/III to 1963/IV

3. Second Subsample Period

    (a) Period for regression (1b)

        from 1964/II to 1974/IV

    (b) Period for regressions (1a, 2a)

        from 1965/V to 1974/IV

TABLE 1b

PRE-WAR PERIODS

PRE-WORLD WAR II

1. <u>Whole Period</u>

   (a) Period for regression (1b)

   from 1920/6 to 1940/12

   (b) Period for regressions (1a, 2a)

   from 1921/12 to 1940/12

2. <u>First Subsample Period</u>

   (a) Period for regression (1b)

   from 1920/6 to 1929/12

   (b) Period for regressions (1a, 2a)

   from 1921/12 to 1929/12

3. <u>Second Subsample Period</u>

   (a) Period for regression (1b)

   from 1930/1 to 1940/12

   (b) Period for regressions (1a, 2a)

   from 1930/11 to 1940/12

TABLE 2a

F-STATISTICS, 1974 REAL GNP-MONEY

Equation	M1	M2	Distribution of Test Statistic
1b	2.54	1.52	F(13, 78)
1a	2.58	2.47	F(19, 67)
2a	1.68	1.29	F(19, 67)

$F_{.05}(12, 70) = 1.89$, $F_{.01}(12, 70) = 2.45$

$F_{.05}(20, 65) = 1.73$, $F_{.01}(20, 65) = 2.18$

TABLE 2b

INDUSTRIAL PRODUCTION-MONEY

Equation	M1	Distribution of Test Statistic
1b	2.06	F(32, 183)
1a	1.74	F(43, 153)
2a	1.39	F(43, 153)

$F_{.05}(30, 150) = 1.54$, $F_{.01}(30, 150) = 1.83$
$F_{.05}(40, 150) = 1.47$, $F_{.01}(40, 150) = 1.72$

TABLE 3

MARGINAL SIGNIFICANCE LEVELS STRUCTURAL CHANGE IN COEFFICIENTS

	GNP-M1	M2
Equation (1b)	.00576	.1290
Equation (1a)	.00232	.00350
Equation (2a)	.06242	.21988

PREWAR DATA
INDUSTRIAL PRODUCTION -M1

Equation (1b)	.00163
Equation (1a)	.00779
Equation (2a)	.07615

These probabilities are the following:

$$P\{F \geq F^*\}$$

where F* = F-statistics in Table 1.

TABLE 4

MARGINAL SIGNIFICANCE LEVEL FOR TESTING HOMOGENEITY OF VARIANCES

Post-war

	M1	M2
Equation (1b)	.4387	.05101
Equation (1a)	.0216	.0873
Equation (2a)	.0694	.11943

INDUSTRIAL PRODUCTION -M1

Pre-war

Equation (1b)	.1958
Equation (1a)	.11957
Equation (2a)	.07696

FOOTNOTES

1. See Sargent [3] for proofs of these facts.

2. Neftci's thesis [2] contains a detailed study of the stability of (1b) across the post-war period.

REFERENCES

Lucas, Robert E., "Some International Evidence on Output-Inflation Tradeoffs," <u>American Economic Review</u>, June 1973.

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