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Federal Reserve Bank of Minneapolis
September 1972

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A Report on

PRESENT-VALUE PROCEDURES FOR CAPITAL INVESTMENT DECISION MAKING AT THE FEDERAL RESERVE BANK OF MINNEAPOLIS

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FOR CAPITAL INVESTMENT DECISION MAKING
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The present investigation of investment decision procedures for use by the Federal Reserve Bank of Minneapolis was initiated in 1971 largely at the urging of David M. Lilly, Chairman of the Board of Directors. Many staff members of the Bank have contributed their experiences and ideas in the process of the study. Primary responsibility for organizing and drafting the report was shared by Clarence W. Nelson and Ronald E. Kaatz of the Research Department staff.

THE FEDERAL RESERVE BANK OF MINNEAPOLIS
September 1972

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INTRODUCTION

In the course of daily business the Minneapolis Fed, like hundreds of other institutions, is continuously called upon to make capital spending decisions. Many of the decisions involve relatively small amounts of money, some fairly large amounts, and a few involve substantial sums. Some expenditure questions can be handled fairly easily. For example, the choice of a typewriter from among several competing models, all judged to be equal in quality and service, can usually be made by simply selecting the model with the lowest price. Other spending decisions are, of course, much more difficult.

Difficulties arise not simply because the decisions involve more money. Major problems are encountered when spending and the related benefits are spread over time. We then need an explicit way to measure the value of dollars to be received or spent at future dates, compared with dollars received or spent today. Present-value procedures (to be defined shortly) provide a way to do this. Even further complications must be met in dealing with the greater uncertainty associated with cash flows extending further into the future.

Such investment decision problems are well known and the professional literature on them is enormous. Economists, accountants, business finance experts, operations research specialists and others have addressed themselves to the issue. It would be rare for an individual to know nothing of interest rates and the concepts of the time value of money which underlie them.

But in spite of this general familiarity, great mystery and confusion surround investment decision-making procedures. This report is intended to examine these issues using everyday language. It will explain some of the rules commonly used; point out some of the limitations; establish a framework within which valid procedures can be selected and applied at a Federal Reserve Bank; and perhaps most importantly, begin developing a set of practical examples through which we can effectively accumulate and extend our experiences in systematic application of present-value procedures.

This draft report is in four parts: Part I discusses the rationale for using present-value procedures in government agency capital spending decisions and explores the difficult problem of selecting an appropriate discount rate for government agency decisions. Part II describes, in general terms, a set of recommended procedures for applying present-value concepts to Federal Reserve capital spending decisions. Part III illustrates these general procedures by discussing a real case example. Part IV provides for a continuing collection of case applications which should illustrate the successes and limitations of present-value procedures applied to a widening spectrum of typical Federal Reserve Bank situations. One small case has been provided as a starter for Part IV.

Our ultimate design, then, is to begin a "loose-leaf" service on the subject of capital spending decision procedures -- all of its contents subject to change and improvement -- developed by and for Federal Reserve Banks.

PART I. RATIONALE FOR PRESENT-VALUE PROCEDURES IN CAPITAL INVESTMENT

Before setting down a rationale for the use of present-value procedures, we will explain the objectives behind it all and some of the terms and concepts to be used. At this point, discussion will be limited to sketching a few basic ideas. These ideas will be explained in more detail in Part II although it is not our intent to write yet another treatise on the mathematics of finance or on economic theory. The more interested reader may find it useful to refer to the abundant literature.

A. Basic Operating Objectives

In the course of its operations, the Federal Reserve System receives income from various sources (principally interest earned on government securities), provides a wide variety of services to the public and the financial community, and remits to the Treasury Department all income not used in providing these services. A basic operating objective of the Federal Reserve, somewhat parallel to the profit motive of private business firms, is to keep expenditures to a minimum (or equivalently, to remit as much to the Treasury as possible) while still providing the appropriate kind and level of services. Two classes of decisions are normally involved: (1) choosing what services we undertake to provide and (2) choosing the minimum cost way to provide these services.

As it turns out, the objective of cost minimization (or guaranteeing a maximum payment to the Treasury over the years) requires in principle that present-value procedures be used in our capital spending decisions. The reasoning further suggests that the discount rate used in our present-value calculations be something like the Treasury's average cost of borrowing -- a relatively low, "safe" rate of interest. However, there is another, more fundamental rationale for using present-value procedures based on the effects our decisions have on the economy as a whole. This second analysis calls for using a discount rate much higher than the safe rate in order to achieve socially superior investment decisions.

B. Definitions and Conventions

1. Definition of Present Value

Given a choice between receiving \$100 today or receiving it one year from today, the rational man, we assume, will elect to receive it today. Given the choice between \$100 today or \$103 a year from now the decision is less clear. Some will choose one way, some the other. Suppose for a particular individual the payoff next year is successively increased until it reaches \$105 at which point he first decides to give up the \$100 now and wait for the \$105. Then in some sense this man gives equal value to \$100 now versus \$105 in a year. To him the "present value" of \$105 to be received in one year is \$100. Presumably then the same man would value \$210 to be received in a year at \$200 today. Extending this reasoning we discover we can calculate for this man the present value of any amount to be received in one year simply by dividing it by 1.05. He requires 5 percent more money to wait one year so his annual discount (interest) rate is said to be 5 percent. His one-year discount ratio (1.05) is equal to one plus his discount rate (expressed as a decimal), i.e., equals 1 + .05 = 1.05.

Now suppose, to consider a simplified example, he is offered \$200 to be paid two years hence -- what is the present value of this offer? We found above that the one-year discount ratio from next year to the present is 1.05. If this one-year discount ratio can be used between any pair of successive years then we can discount the \$200 first from two years hence back to one year hence by dividing by 1.05...

$$\$200 \div 1.05 = \$190.48 = \text{value one year from now}$$

then discount this value to the present

$$\$190.48 \div 1.05 = \$181.41 = \text{the present value}$$

Combining the arithmetic...

The present value of \$200 received two years hence = $\dfrac{\$200}{(1.05)^2}$ = \$181.41

The general result of this reasoning is that the present value of any number of dollars, say \$R, to be received n years from now, discounted at i percent per year is...

$$\text{Present value} = \frac{\$R}{(1+i)^n}$$

2. Other Conventions

In this paper we speak of capital investment or capital spending decisions as those that entail flows of costs (expenditures) and benefits (receipts) taking place over some span of time. Decisions to buy a coin sorting machine, to lease computer equipment, or to sponsor an employee training program, are examples of capital investment decisions.

An investment project involving costs and benefits can be characterized by a sequence of numerical net cash flows over a period of time. The relevant time period, called the "project life" or "project horizon," is divided into convenient subperiods -- months, quarters, years. Generally then, expenditures and receipts will occur during each subperiod. A net cash flow for any subperiod can be computed by subtracting that period's expenditures from receipts. We will adopt the convention here that receipts or inflows be designated as positive (+) numbers and expenditures or outflows as negative (-) numbers. If for the first subperiod receipts are ten dollars (+$10) and expenditures two hundred dollars (-$200), then the net cash flow for that subperiod would be a negative one hundred and ninety dollars (+$10) + (-$200) = (-$190).

The premise supporting use of some form of time-weighted decision procedure is that "future money" is less valuable in some sense than "current money." On these grounds, future flows of money should be given correspondingly less weight in any decision than flows closer to the present date. More specifically, we will argue that future cash flows ought to be discounted back to the present date using present-value procedures before summing up the net cash flows for use as an indicator of the project's worth. The rationale for these kinds of procedures has nothing to do with "risk" or "uncertainty;" that is a separate issue to be tackled later. The procedures would apply equally well to projects for which future cash flows were known with certainty.

To speak in terms of projected "cash flows" suggests projects more likely to be found in the realm of private profit-seeking businesses than activities of Federal Reserve Banks or other public agencies. Yet the considerable body of literature that has by now emerged on optimal methods for investment decision making by public institutions agrees that, here too, correct procedures require discounting future costs and benefits at some appropriate rate of interest. That in turn suggests the need to make some effort to place dollar valuations on the benefits expected to flow from Federal Reserve Bank activities. Whether in practice many Federal Reserve investment projects can adequately be reduced to measurement by numerical cash flows is an important question -- one that will be determined by experience. While it is true that accurate measurement of benefits would provide the ideal basic information for decision making, present-value procedures can be modified to suit circumstances in which only cost projections are available.

C. Rationale for Present-Value Procedures

Considerably differing views exist on the question of what interest rate (conceptually) is appropriate for discounting, as well as how one would determine, in practice, the correct numerical value of that rate. We will deal here only summarily with the theoretical rationale for use of discounting procedures and the choice of discount rates. It seems important to spend most of our time in this report examining ways in which existing procedures can be used in practical applications at a Federal Reserve Bank to assure sound financial stewardship. A bit of justification, however, is important: because the literature is rather complex -- often confounding -- we do not attempt a balanced survey.

As outlined by section A, rationales for using present-value procedures in government agency decisions exist on two levels.[1/] The one we shall consider first establishes simply that, as long as some safe alternative earning rate exists at which we can place our idle funds, then failure to use present-value procedures (with that safe rate of discount) risks a poorer return of funds to the Treasury over time than would be possible if we use present-value procedures.

1. Financial Efficiency Arguments for Present Valuing at Some Safe Alternative Interest Rate

We'll proceed with this first-level rationale by illustrative example, though it should be clear the argument can be generalized. Consider the following annual net cash flow series from investment in project X:

[1/] We might remind the reader at this point that our discussion is aimed at establishing appropriate rules for government sector decisions. For a private firm present-value procedures are equally important, but in order to achieve "profit maximation" the proper choice for a discount rate is, as most standard texts discuss in detail, the firm's cost of capital.

-\$100,000; \$30,000; \$80,000

which is intended to represent an initial outlay of \$100,000, followed at the end of the first year by receipt of \$30,000, and at the end of the second year by receipt of \$80,000. If we ignore the time delay in returning the \$100,000 investment, then project X promises to return over its life \$10,000 more than we put into it, and may seem to be an attractive prospect for investment. But it's not attractive if an alternative safe earning rate of, say, 6 percent is available. This can be seen if we apply the present-value procedure as follows:

- *COMPUTE THE PRESENT VALUE OF THE CASH FLOW SERIES AT 6 PERCENT INTEREST*

  Discount each of the flows in the series back to the present time before summing them algebraically to obtain the present value of the series. We take the initial outflow as the "present," so discounting doesn't affect it. The flow at the end of the first year is divided by 1.06; that at the end of the second year by $(1.06)^2$.

$$PV\ @\ 6\% = -100{,}000 + \frac{30{,}000}{(1.06)} + \frac{80{,}000}{(1.06)^2}$$

$$= -100{,}000 + 28{,}300 + 71{,}200$$

$$= -500$$

- *EXAMINE THE PRESENT VALUE OF THE PROJECT; IF IT IS POSITIVE, GO AHEAD WITH THE INVESTMENT, IF IT IS NEGATIVE REJECT THE INVESTMENT*

With that quick preview of the textbook present-value procedure (which will be discussed in more detail in Part II of this report) our example project X, with flows,

-\$100,000; \$30,000; \$80,000

is rejected.

The correctness of this decision from a purely financial point of view can also be demonstrated by computing the amount of cash each alternative would have accumulated at the end -- rather than the beginning -- of the two-year horizon (a process sometimes referred to as a "future-value" computation). We assumed an alternative safe earning rate of 6 percent.

Compare the two options of investing in project X or depositing the $100,000 at the safe interest rate for the same period of time (two years):

	Cash flows at end of year...		
Option I: Invest in Project X	**0**	**1**	**2**
An initial outlay of $100,000 will	-$100,000		
return $30,000 at end of year 1...		$30,000	
and $80,000 at end of year 2...			$80,000
while the $30,000 received at end of year 1 can be deposited at safe 6 percent and convert to $31,800 at the end of year 2...			$31,800
which gives us a total cash on hand at end of year 2 of...			$111,800

Option II: Place funds at safe 6 percent	**0**	**1**	**2**
$100,000 placed at 6 percent will show	-$100,000		
... a balance at end of year 1 of...		$106,000	
and a balance at end of year 2 of...			$112,360

Comparison shows our financial statement would be $560 poorer at the end of year 2 by going ahead with project X than by depositing the funds at a safe 6 percent. That is, the future value of project X at 6 percent interest is -$560. The message is fairly clear on the face of this evidence that project X is earning a return less than 6 percent. We could return more money to the Treasury (in future value) by putting the $100,000 in the bank than by going ahead with project X.

Our final example is project Y, with the following pattern of cash flows:

-$100,000; $95,000; $12,000

If we ignore the time pattern of flows (which is what failing to use present-value procedures essentially leads us to do) project Y seems inferior even to project X, since Y's inflows exceed outflows by only $7,000 compared with net inflow of $10,000 for X.

But if we compute the present value of the flows at 6 percent for project Y, we get

$$PV\ @\ 6\% = -100{,}000 + \frac{95{,}000}{(1.06)} + \frac{12{,}000}{(1.06)^2}$$

$$= -100{,}000 + 89{,}623 + 10{,}680$$

$$= +303$$

The present value of project Y is positive and the decision rule used above tells us to proceed with the project. We can also calculate our second-year financial statement as we did for the two earlier options:

	end of year		
Option III: Invest in Project Y	0	1	2
An initial outlay of $100,000...	-$100,000		
will return $95,000 at end of year 1...		→$95,000	
and $12,000 at end of year 2...			→$12,000
while the $95,000 received at the end of year 1 can be deposited at 6 percent and convert to $100,700 at the end of year 2...			→$100,700
resulting in total cash on hand end of year 2 of...			$112,700

The final year cash total of $112,700 is $340 greater than the $112,360 we would have accumulated placing our initial funds at 6 percent. That is, the future value of project Y at 6 percent is $340. The outcome of the present-value decision rule likewise signals that project Y promises to earn at a greater-than-6-percent rate and we would in fact return more to the Treasury by investing in Y than we could by depositing the $100,000 to earn 6 percent.

In summary then, one simple but impressive reason for using present-value procedures in choosing among investment alternatives is that you may be throwing away money if you don't. In Part II we will discuss cases for which an estimate of revenues is not possible and the choice among alternative ways to accomplish the same project is made on the basis of minimum outlays (in present-value terms). The time pattern of outlays is crucial there too. Use of present-value procedures can help you avoid the prospect of selecting an alternative on the basis of minimum overall outlay while you are, in fact, penalizing your ultimate financial position because of differences you did not take into account in the timing of outlays. But the case for government agencies using present-value procedures is now generally made and discussed at a different level; we turn there next.

2. The Optimal Social Rate of Discount -- A Second Rationale

The second rationale -- the cumbersome details of which appear in Appendix A -- establishes that something higher than the safe interest rate is required if we wish to achieve a more "socially optimal" allocation of the nation's economic resources between the private sector and government. Most recent literature on the subject agrees we ought to be using a rate that represents what society gives up when we (government) command resources into an investment project -- a rate representing the "opportunity cost" to society of our decision. If we use a lower rate then, we would be drawing resources into government sector activities that return less to society than they would if invested by the private sector. If we use a higher rate, the resulting misallocation will be in the other direction with government failing to provide some services which it ought to. That's simple enough in principle, but the problem of determining the appropriate rate of "opportunity cost" is formidable, and no unambiguous estimate is available.

The practical upshot, though, whether we can pin down "the" optimal rate or not, is that the rate we should be using is higher than the rate on safe, liquid earning assets discussed in the preceding section. And in general, for any given agenda of projects the higher the discount rate, the greater the number of projects that will be rejected under a present-value decision rule.

The Federal Reserve Bank of Minneapolis has accepted in principle the use of present-value procedures in investment decisions. Choice of a unique best rate of interest is, we recognize, an unanswered question. A theoretical approach to the problem is sketched out in Appendix A.

In general we consider other decision rules based on such concepts as pay-back period, average return on investment, and internal rate of return, to be inferior to properly applied present-value decision rules.

The argument developed in Appendix A concludes that the appropriate rate for discounting projected cash flows lies somewhere in the range 7 percent to 11 percent, and that it is really not possible to pin down a single best value. Consistent with that finding, the decision procedure recommended in Part II is based on use of a range of discount rates and does not rely on choice of a single precisely estimated rate.

While we won't spell out the reasoning in any detail here, the range derived is based on estimates of the rates corporations in particular, and the private sector in general, earn on their marginal investments. Marginal investments are the ones the private sector would choose first to "give up" when government agency spending decisions channel resources away from them. That, of course, is why those rates represent the "social opportunity cost" of our decisions.

A key premise of the argument is that the economy is fully employed. In other words when we make an investment decision we draw from the private sector the full extent of the resources used.

One reason for ambiguity of the opportunity cost of any specific government investment decision is that, generally, each specific government investment may affect private-sector resource use differently. Particular industries with marginal capital returns either very much higher or very much lower than the corporate aggregate figure may be hit differentially. Regional differences in impact, too, could occur. As a consequence, the "true" opportunity cost almost certainly varies with each government investment decision.

Furthermore, when we abandon the full-employment premise, the true rate of opportunity cost would tend to vary with the state of the economy -- with the rate effectively lower during times of greater slack and underutilized labor and production facilities and higher during periods of resource pressures. Our general prescription during periods of high resource pressures, high price pressures and high interest rates is to pay somewhat more attention to comparative present values calculated at rates toward the upper end of the 7 percent to 11 percent range. In periods of slack, we would emphasize values toward the lower end of the range. The practical meaning of this practice will become more clear after you've read Part II.

D. Risk

One last matter we wish to take up in the opening part of this report is proper treatment of risk. Risk as used here is essentially the chance that a particular investment will return less than some specified rate of return. One contention regarding risk that frequently appears in the literature is that government investment decisions ought to use a safe rate (e.g., the Treasury borrowing rate) for discounting purposes. Another is that prospective investments ought to be sorted into "risk classes" with higher discount rates used for riskier classes of investment. We believe both of these procedures to be incorrect. In fact they could prove perverse to the social objective of optimal allocation of resources. Risk must be considered, but not by adjusting the discount rate. Rather, it should be incorporated into procedures used to calculate the expected cash flow data.

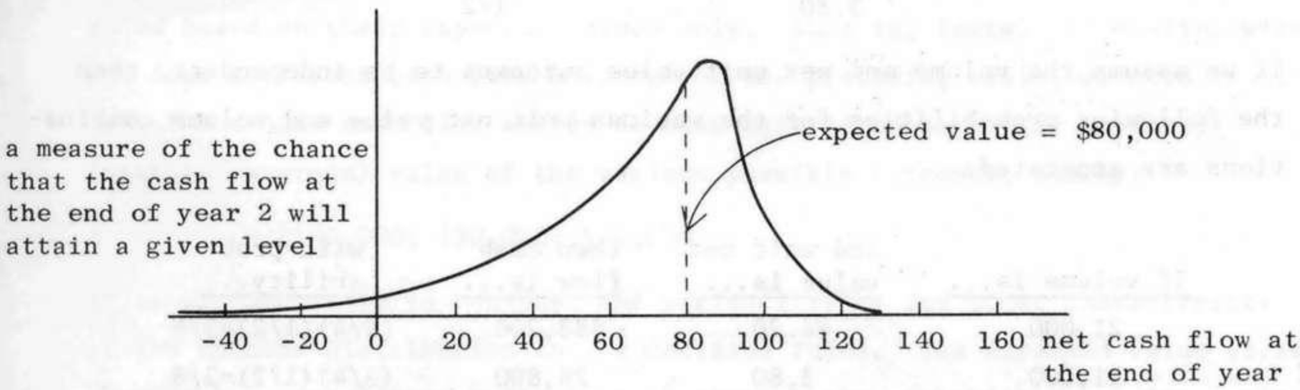
Uncertainty about the future clearly enters any effort to make projections of cash flows. And the more distant the time period, the more uncertain projections are likely to be. A projected cash flow series will usually be built from projections of various cost, price, and quantity elements including: expected volume of output of an operation, unit labor requirements, wage rates, prices, maintenance costs, power requirements, and scrap, trade-in or resale values of the raw materials and production equipment. The values of each element could trace a variety of alternative paths over the future. We do not know in advance which of the many possible paths will be taken in the final outcome.

In principle we can accommodate some measure of uncertainty by properly structuring the projections. Future cash flows, or the elements that comprise them, can be thought of in terms of a distribution of possible outcomes with a probability of occurrence assigned to each. This procedure is described in standard references. We'll simply illustrate the idea by considering the earlier simple example with the following cash flows:

-\$100,000; \$30,000; \$80,000

To single out just one quantity, the projected cash inflow of \$80,000 at the end of year 2 is in any practical case uncertain. One possible model of our uncertainty would be that \$80,000 merely represents the expected value (the mean) of a distribution of possible outcomes:

FIGURE I.1



Cash flows projected for other time periods are also uncertain (though presumably those closer to the present have less variance). Discounting the expected values for the cash flows, which is what we did earlier, will generate a single present value at any given discount rate. If we could take fully into account the probabilities of getting cash flows different from the expected values (and that's a tricky matter since they are not in general independent), then we could generate a distribution of present values that would reflect our uncertainty about cash flow projections.

A mechanically simpler approach is to express our uncertainty by specifying a small number of possible outcomes. Each outcome is assigned a probability of occurrence in such a way that the total of the probabilities equals 1. For an example of this, take the second-year inflow of $80,000. Suppose that value to be derived from (a) a volume estimate and (b) an estimated net value per unit. We judge we may produce 21,000 units, but there is some chance output could be lower. We may choose to express our uncertainty about volume of output in the following way:

Production Volume	Probability of that Outcome
21,000	3/4
17,000	1/4

Similarly, say we are uncertain about the net value per unit as follows:

Net Value Per Unit	Probability of that Outcome
$4.20	1/2
3.80	1/2

If we assume the volume and net unit value outcomes to be independent, then the following probabilities for the various unit net value and volume combinations are generated:

If volume is...	and unit net value is...	then cash flow is...	with probability...
21,000	$4.20	$88,200	(3/4)(1/2)=3/8
21,000	3.80	79,800	(3/4)(1/2)=3/8
17,000	4.20	71,400	(1/4)(1/2)=1/8
17,000	3.80	64,600	(1/4)(1/2)=1/8

The expected value of the cash flow at the end of year 2 is the weighted sum of each possible outcome -- weighted by the associated probability of occurrence.

$$\begin{aligned} \text{Expected value} = \;& \$88{,}200 \times 3/8 \\ +\;& 79{,}800 \times 3/8 \\ +\;& 71{,}400 \times 1/8 \\ +\;& 64{,}600 \times 1/8 \\ =\;& \$80{,}000 \end{aligned}$$

Assuming at this point that the first two cash flow figures in our example (-$100,000; $30,000) are known with certainty, we now have available a probability distribution of cash flow series embodying the expressed uncertainties over the final cash flow:

Cash flow Outcome ($1,000)	Probability of that Outcome	Present Value @ 6% ($1,000)
-100, 30, 88.2	3 out of 8	6.8
-100, 30, 79.8	3 out of 8	-0.7
-100, 30, 71.4	1 out of 8	-8.2
-100, 30, 64.6	1 out of 8	-14.2

We can then compute a "discrete" probability distribution of present values at any given discount rate as has been done at 6 percent in the third column above. This distribution of present values does convey useful information about uncertainty and risk, however, the main purpose of considering alternative outcomes and their associated probabilities is to arrive at the expected cash flow outcome. Then, it is sufficient to define present-value decision rules based on these expected values only. That is, instead of dealing with the four different cash flow outcomes of the preceding table, we need examine only the single cash flow series we have determined to be the expected (weighted average) value of the various possible outcomes, namely:

- \$100,000; \$30,000; \$80,000

It is not necessary to consider the variability or any other characteristic of the outcome distribution in the decision rules. The expected value series already has uncertainty built into it through the probabilities we've assigned to the various constituent outcomes.

PART II. GENERAL DESCRIPTION OF PRESENT-VALUE PROCEDURES IN CAPITAL INVESTMENT DECISION MAKING

The objective of Part II is to describe present-value procedures and explain how they can be used to provide information useful in investment decision making. It will be clear from the examples that these procedures are merely tools of analysis; that they may prove to be more or less useful in a particular application depending on the nature of the decision-making problem. We will take up first what we call the standard case involving problems for which dollar values can be assigned both to the stream of future benefits expected to flow from the project and also to expected cost flows. The second class of cases considered will be those for which dollar valuation of the expected benefits is not possible.

A. The Standard Case with Both Receipts and Expenditures Known

We start with a hypothetical example to illustrate the steps involved in organizing and analyzing data in the standard case. Suppose our bank is considering establishment of a new banking service for which a charge will be made. It has determined two possible levels of activity. The first level, call it alternative A, will produce estimable revenues over a period of years and will generate particular costs for acquisition of equipment, additional labor and so on. The second (higher) level of activity, alternative B, would generate greater revenue but would also require greater outlays for equipment, labor and supplies to handle the greater activity. The question is which, if either, alternative should the bank choose?

1. Constructing Net Cash Flow Streams

Four general steps are involved in developing the necessary net cash flow data: (1) define the project horizon and time units to be used, (2) estimate expenditure flows, (3) estimate receipts flows and (4) compute the resulting net cash flow streams. All four steps require judgment as to what facets of the problem can (can't) be adequately quantified and incorporated into net cash flow data. These decisions largely determine whether useful information will come out of the present-value analysis.

• *Project Horizon and Time Units*

A number of factors usually influence choice of the project horizon. The expected life of key processing equipment or conventional replacement practices might indicate an appropriate planning cycle. Equipment lease periods may play a part. For our example we assume analysts have decided, after considering the factors above and other relevant information, that revenue and cost streams throughout the foreseeable future will follow a reasonably predictable three-year cycle and that it is adequate for this problem to simply examine one three-year cycle. If there is doubt on this point the analysis can be extended to cover additional three-year cycles in order to determine whether the conclusions would be altered. There are few if any general rules to guide the analyst in the choice of the project planning horizon.

Having chosen a three-year horizon, it remains to select an appropriate time unit. Sometimes the nature of the project will suggest an obvious choice, sometimes not. Suppose in our example that elements of cash flows (billings, equipment rental payments, wage payments, etc.) occur variously at quarterly, monthly, semimonthly, biweekly and weekly intervals. Some considerations in choice of a time unit are:

(1) If we choose the week as the time unit, the important monthly and quarterly data must arbitrarily be interpolated to derive weekly figures. Then too, three years (the project horizon) of weekly data involves over 150 time observation intervals; in general, it is more costly to handle large amounts of data.

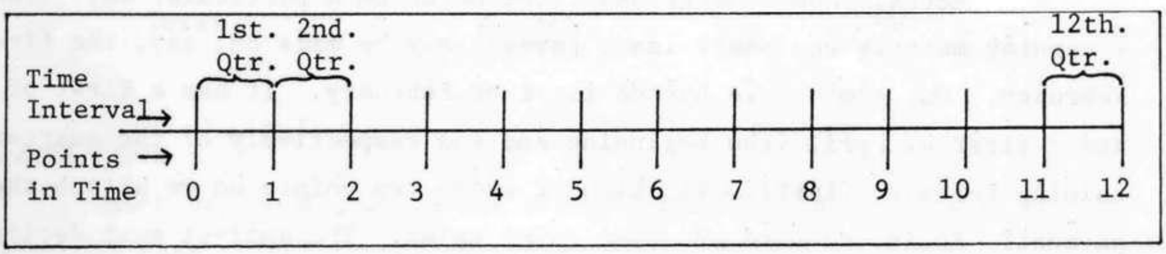
(2) If we choose the month as the time unit, semimonthly data could easily be aggregated to provide monthly figures though weekly and biweekly data would cause some aggregation problems. Quarterly data must still be interpolated to derive equivalent monthly figures. With a monthly time unit the total number of data intervals is reduced to 36 (three years of monthly data), not an unwieldly number.

(3) A time unit of one quarter eliminates the need to interpolate data. Twelve intervals (three years of quarterly data) is certainly manageable. The question becomes, is this too few observations? Will quarterly data mask important features of the problem or would

all the essential information still be revealed? Again, there are few general rules to guide the analyst. From our experience 12 observations is not obviously too few. Doubt can be resolved in any particular case by extending the analysis to the next finer time unit to determine whether the conclusions differ. For our example we assume quarterly data are acceptable.

(4) From the standpoint of computational requirements, time units greater than one quarter also could be used; half years, annual data, even a unit measuring the full three years is not mechanically impossible. The main point of a cash flow analysis, however, is to properly account for the importance of differential timing of cash flows. To choose a time unit so large that all flows are clustered at a few points widely spaced in time conceals the very information we are trying to exploit.

One final point on the matter of time scale. Our example involves 12 quarterly time intervals. We will adopt the convention of numbering points in time (rather than intervals) as illustrated below.



Let the letter t be our index of points in time so that t = 0 indicates the beginning of the first quarter of the project horizon, t = 1 denotes the end of the first quarter and the beginning of the second, and so on through t = 12 which is the end of the twelfth quarter and also the end of the project horizon for this problem.

• *Expenditure Flows*

Our investment decision will involve accepting one or the other of two mutually exclusive courses of action, or rejecting both alternatives. One alternative is to establish a certain (lower) level of bank service (alternative A). The second is to establish a higher level of bank service (alternative B). Analysis of both alternatives must be based on the time scale described above. The task before us now is to estimate the sequence of quarterly cash outflows (expenditures) for each alternative.

Cash flow data, as the name suggests, include only monetary transactions in which the ownership of funds is transferred to or from the individual or corporation making the decision. Accounting artifices, such as depreciation accounts or reserves for possible future losses, have no part in a cash flow analysis.[2/]

Constructing a cash flow stream requires that we forecast future events -- a process involving uncertainty. The procedure discussed at the end of Part I could be helpful. Basically that involves estimating two or more realistic alternative levels for expenditure outcomes for each time period, assigning probabilities of occurrence to each outcome, and calculating weighted averages ("expected values") for each period. For present-value analysis it is important that the cash flow estimates represent the expected (i.e., mean or weighted average) outcomes, not simply an ideal or favorable outcome.

Actual cash flow transactions occur on a particular day. For example, a regular monthly equipment lease payment may be made on, say, the first of February. Our time scale has no first of February. It has a first of January and a first of April (the beginning and end respectively of the quarter containing February first). To which of these two points do we attach the lease payment? Again, no hard and fast rules exist. The analyst must decide what procedures are best suited to the problem at hand. If the time unit chosen is sufficiently small the problem will be a minor one. If the time unit is larger (relative to the total project horizon especially) then more attention must be given to this issue.

In our example the major large outlays for equipment and other factors are assumed to be assignable to the point $t = 0$. By $t = 2$ and beyond the outflows have settled down to reflect recurring operating expenses and regular cash payments for lease contracts. The estimated expenditure stream includes a small, steady increase due to anticipated increases in labor and

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[2/] Except insofar as they may affect cash flows, as depreciation reduces income tax payments. We assume here no such situation exists and that, in fact, either there is no income tax or that we have decided a before-tax analysis is appropriate to the present problem.

other costs. Table II.1 gives assumed cash outflow data for each alternative. The notation E^A denotes the expenditure flows for alternative A and similarly for E^B.

TABLE II.1

Expenditure Cash Flows

t	E^A	E^B
0	$-1350	$-2175
1	-200	-310
2	-260	-390
3	-262	-394
4	-265	-398
5	-265	-403
6	-267	-409
7	-269	-412
8	-270	-413
9	-274	-419
10	-276	-419
11	-278	-420
12	-280	-423
Total	$-4516	$-6985

● *Receipts Flows*

Much of the discussion of expenditure flows applies also to receipts. Only actual cash transfers are recorded. Thus, unlike some accounting practices, an inflow is not recorded at the time of sale unless cash is received at that time. Forecast cash inflows also must represent the expected (i.e., mean or average) outcomes not high or low extremes. Again, no general rules exist for assigning intraperiod cash receipts to a particular value of end-of-period time (t). Common sense may suggest a procedure. If the time unit was chosen adequately small, no major problems will be encountered.

The letter R will be used to denote the cash receipts flows with the appropriate superscript (A or B) to denote the particular alternative. In our example we assume no inflows will be received until t = 2.

We must also recognize the scrap value of equipment at the end of the cycle. Scrap values should represent estimates of actual amounts which could be obtained by sale on the used equipment market, or the best attainable alternative. Scrap value is not determined by arbitrary depreciation procedures used in accounting practices. The estimated stream of future receipts is given in Table II.2.

TABLE II.2

Receipts Cash Flows

$\underline{t}$	$\underline{R^A}$	$\underline{R^B}$
0	$ 0	$ 0
1	0	0
2	400	600
3	403	605
4	407	611
5	408	618
6	411	625
7	414	630
8	416	633
9	421	640
10	424	642
11	427	645
12	580	999
Total	$4711	$7248

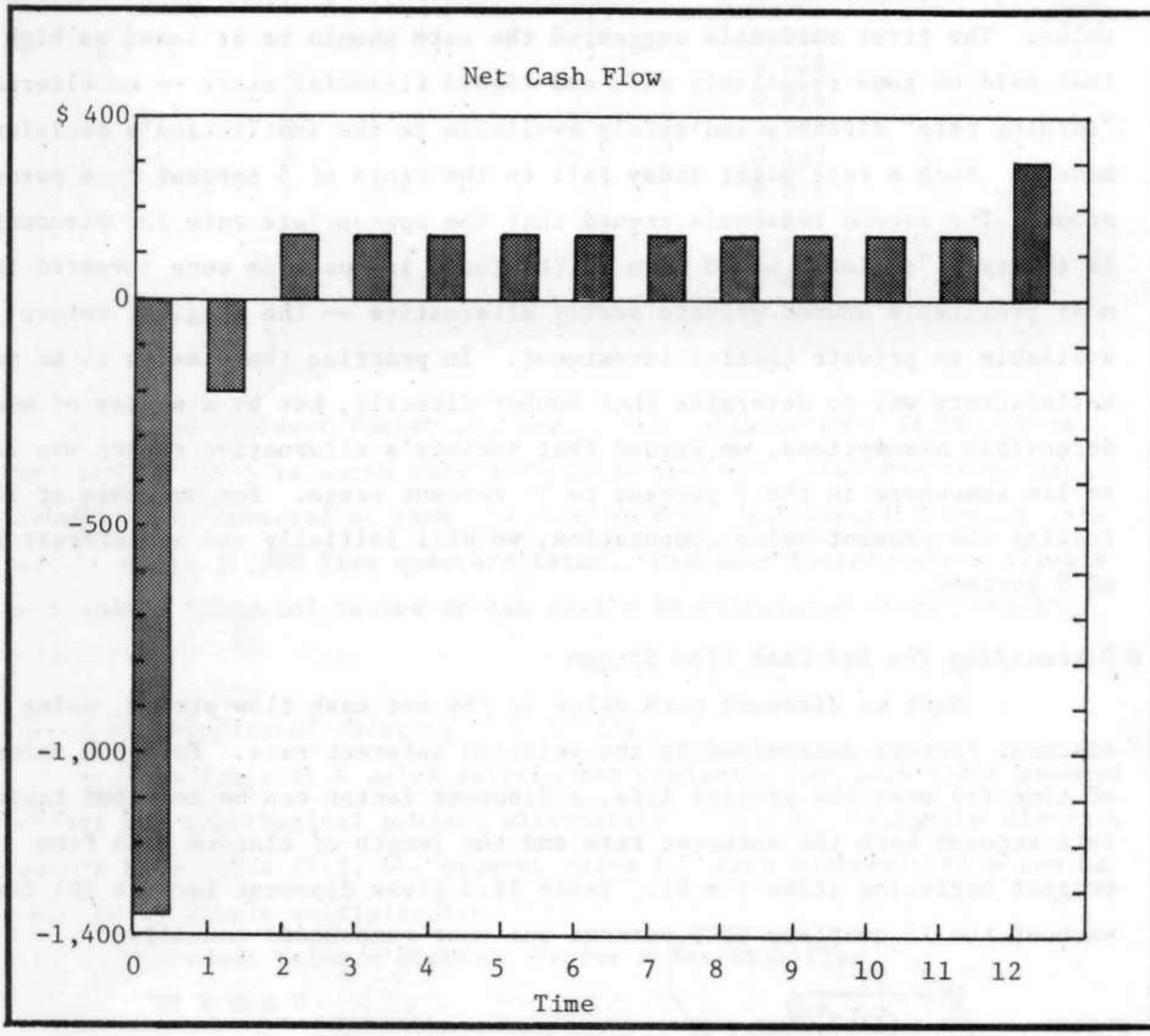
• *Net Cash Flow Stream*

The net cash flow stream (denoted by the letter F) is simply the sum of the receipt stream and the expenditure stream, taking account of our agreement to specify expenditures as negative numbers. If for a particular value of t we have receipts of $300 and expenditures of $250, the net cash flow for that value of t is...

$$(+\$300) + (-\$250) = (+\$50)$$

Problems amenable to cash flow analysis frequently involve large expenditures (outflows) with little or no offsetting revenues in the early periods. Start-up costs may reflect large initial outlays for plant and equipment, intensive early planning and development requirements, low initial productivity and so on. Later periods in the horizon typically produce a more or less steady stream of positive net cash flows (inflows) as income earned on the larger initial investment. In many cases, including the present example, much of the large initial investment outlay can be bunched at t = 0, the very beginning of the project horizon. There may or may not be additional net outflows in other periods. Heavy equipment installation or plant construction typically extends over several early periods. The net cash flows are shown in Table II.4. The net cash flow for alternative A is graphed in Figure II.1 and is typical of the time pattern for B and for many other practical problems.

FIGURE II.1



2. Analysis of Net Cash Flow Streams

- *Selecting the Appropriate Interest Rate*

The next step in the standard case investment decision process is selection of an appropriate interest rate to be used in computing present values for each net cash flow stream. Two rationales for choosing interest rates were discussed in Part I. Neither generates an unambiguous numerical value. The first rationale suggested the rate should be at least as high as that paid on some relatively safe and liquid financial asset -- an alternative "earning rate" directly and safely available to the institution's decision makers. Such a rate might today fall in the range of 5 percent to 6 percent per annum. The second rationale argued that the appropriate rate for discounting is the rate "society" would earn if the funds in question were invested in the most profitable unused private sector alternative -- the marginal return available to private capital investment. In practice there seems to be no satisfactory way to determine that number directly, but by a series of modestly defensible assumptions, we argued that society's alternative return was likely to lie somewhere in the 7 percent to 11 percent range. For purposes of illustrating the present-value computation, we will initially use an interest rate of 8 percent.

- *Discounting the Net Cash Flow Stream*

Next we discount each value in the net cash flow stream, using discount factors determined by the selected interest rate. For each value of time (t) over the project life, a discount factor can be computed taking into account both the interest rate and the length of elapsed time from project beginning (time t = 0). Table II.3 gives discount factors (D) for each of the 12 quarters at 8 percent per year compounded annually:

$$D = \frac{1}{(1+r)^{t/4}}$$

where r is the interest rate.

TABLE II.3

Quarterly Discount Factors at an 8% Annual Rate

t	Discount Factor (D_t)
0	1.000
1	0.981
2	0.962
3	0.944
4	0.926
5	0.908
6	0.891
7	0.874
8	0.857
9	0.841
10	0.825
11	0.809
12	0.794

The discount factor at, say, t = 5 tells us that $1,000 to be received at t = 5 is worth only $908 to us at t = 0. Put the other way around: $908 invested at time t = 0 at an 8 percent annual interest rate will be worth $1,000 five quarters later. Discount factors are available in standard financial tables or can easily be calculated by electronic computers.

- *Computing the Present Value of the Project*

In Table II.4 we've entered the projected net cash flows assumed earlier for hypothetical project alternatives A and B. Using the discount factors from Table II.3, the present value for each quarterly flow can be computed by simple multiplication:

Present Value = Discount Factor x Net Cash Flow

$PV = D \times F$

TABLE II.4

Present Values Discounted at 8%
(Dollars)

t	D	Alt. A		Alt. B	
		F^A	PV^A	F^B	PV^B
0	1.000	-1350	-1350	-2175	-2175
1	.981	-200	-196	-310	-304
2	.962	140	135	210	202
3	.944	141	133	211	199
4	.926	142	131	213	197
5	.908	143	130	215	195
6	.891	144	128	216	192
7	.874	145	127	218	191
8	.857	146	125	220	189
9	.841	147	124	221	186
10	.825	148	122	223	184
11	.809	149	121	225	182
12	.794	300	238	576	457
Total		195	-32	263	-105

The present value for each project is simply the sum of the individual present values over all values of t. For project A, as we can read on Table II.4, the present value discounted at 8 percent is -\$32. For project B the present value is -\$105. For each alternative the present value is less than the sum of the net cash flows taken without discounting. The simple sums (totals of the F's) are also in a technical sense present values. They are the present values one would obtain using an interest rate of zero percent. For both cash flows in Table II.4, the higher the interest rate chosen for discounting, the lower the resultant present value. This will be the case for most cash flow streams.[3/]

[3/] It can happen for rather strange patterns of cash flows that, within certain ranges of the discount rate, the present value will increase if the interest rate is increased a little. A cash flow with many alternating net inflows and outflows may have this property. We will encounter few cash flows of this type.

• *Simple Decision Rules Based on Present-Value Computations*

We will discuss two fairly standard decision rules by which alternatives can be judged on the basis of their present values at a given interest rate. The first applies to any project, whether or not alternatives are being considered. These are cases of "accept-reject" decisions where the alternatives are either to adopt a contemplated project or not to adopt it.

> Rule I: If the present value of a project is greater than zero, adopt it. If the present value of a project is less than or equal to zero, do not adopt it.

The second rule applies in the case of multiple alternatives which are mutually exclusive, that is, only one of the several alternatives can be selected. The hypothetical example given above is of this type; select either project A or B. If we adopt one we can't adopt the other.

> Rule II: If two or more mutually exclusive alternatives have present values greater than zero, adopt that alternative with the greatest present value.

Applying rule I to our hypothetical problem we would conclude that neither alternative should be selected. Both have negative present values discounted at 8 percent. We might note that although alternative B appeared superior to A in terms of the simple sums of net flows ($263 for B compared with $195 for A), after discounting at 8 percent the ordering is reversed (-$105 for B compared with -$32 for A).

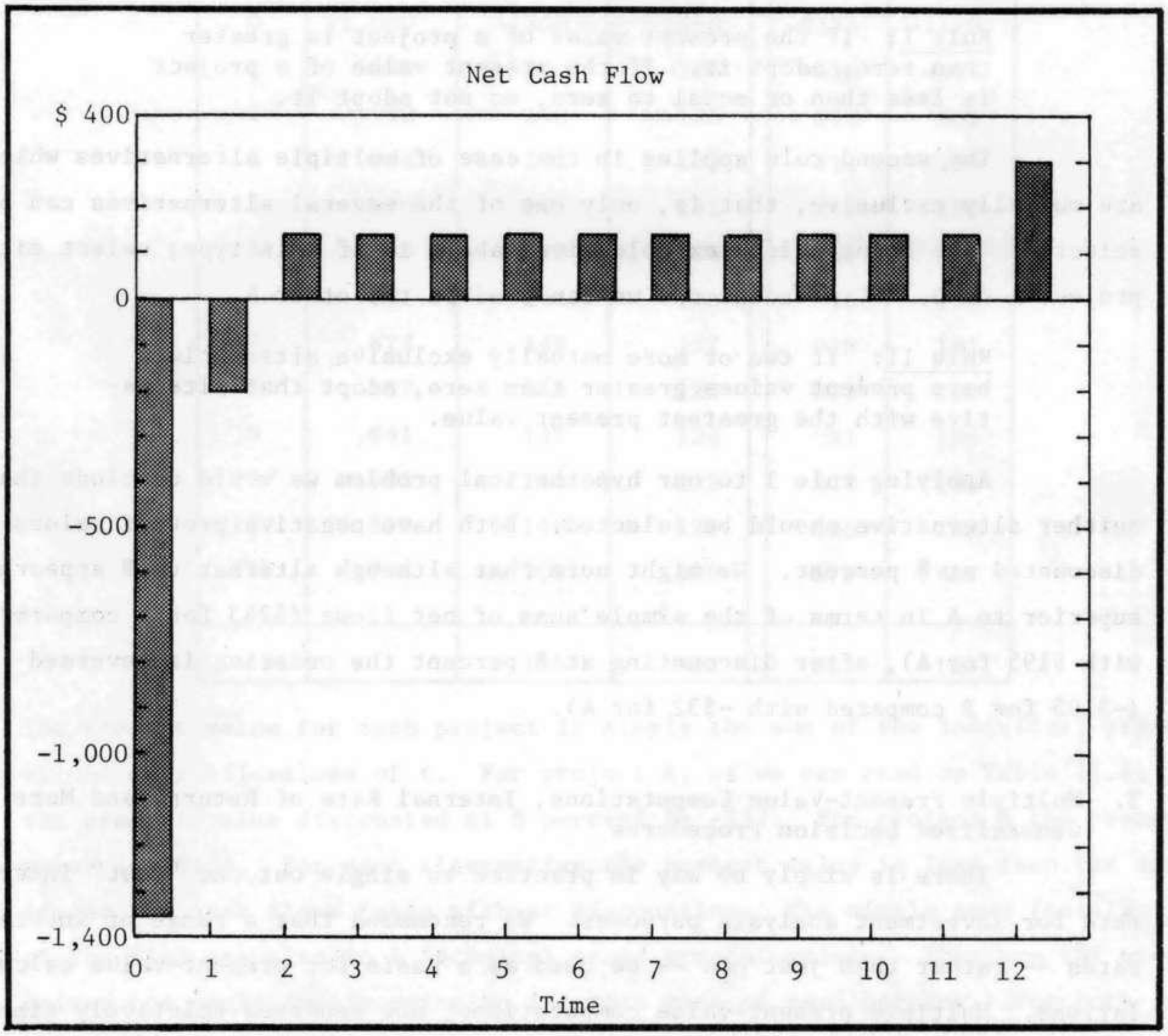
3. Multiple Present-Value Computations, Internal Rate of Return, and More Generalized Decision Procedures

There is simply no way in practice to single out one "best" interest rate for investment analysis purposes. We recommend that a range of interest rates -- rather than just one -- be used as a basis for present-value calculations. Multiple present-value computations, now rendered relatively simple through computer programs, will considerably enhance the information provided to the decision maker. We suggest that present-value information for each project be prepared as a continuous graph over a reasonably wide range of interest rates, say 0 to 16 or 20 percent, and that the graphs so prepared be the basis for displaying present-value information about investment alternatives. The next subsection explains such a procedure.

• *Present-Value Properties Over a Range of Interest Rates*

In our prior example, alternative A had a series of net cash flows as represented in the following diagram:

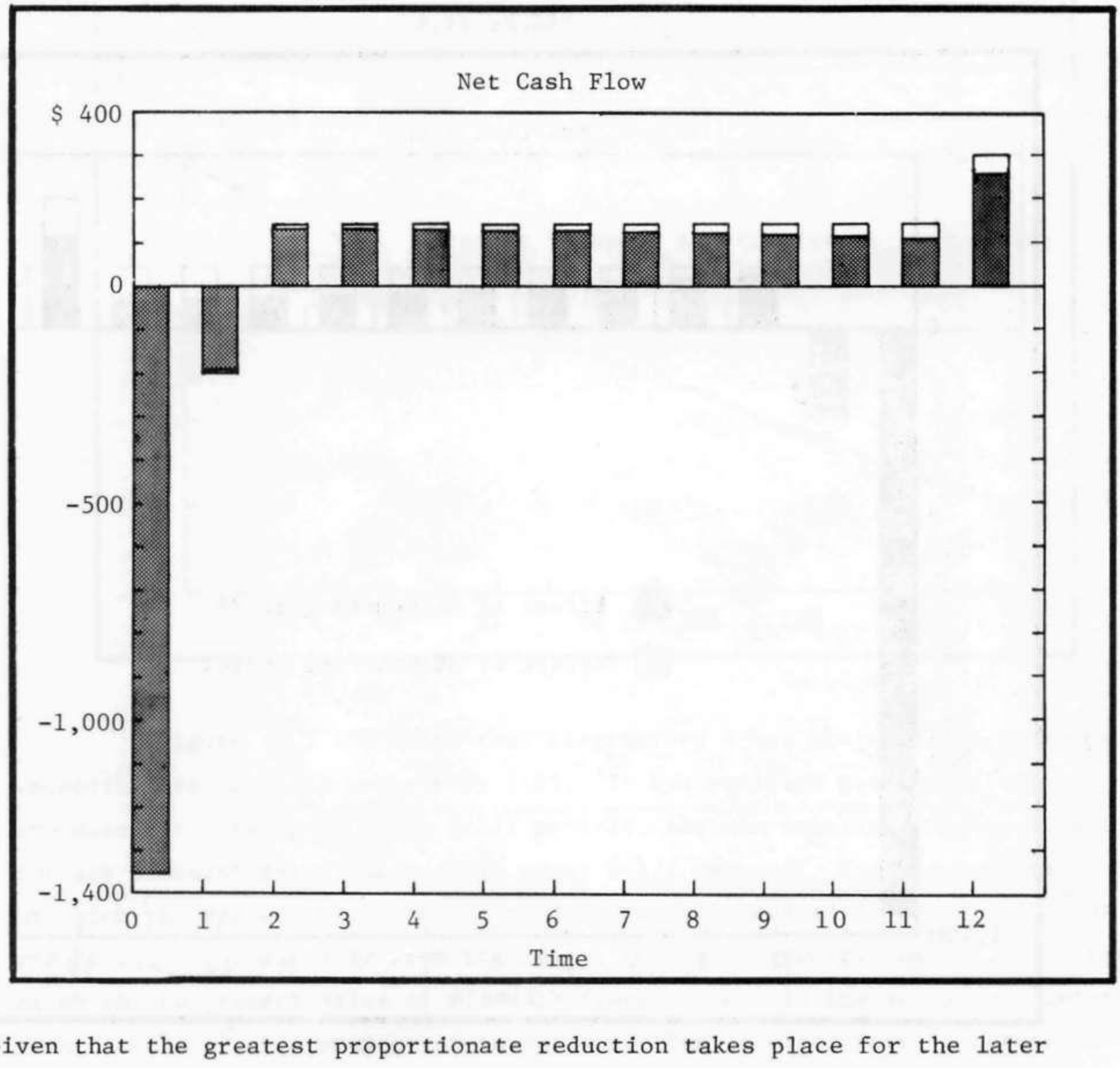
FIGURE II.2



The actual quarterly net cash flows for each value of time t can be thought of as present values for the special case in which the discount rate is equal to zero. To put it another way, not discounting at all is the same thing as discounting with the interest rate set equal to zero. Thus, present value for the project (at a zero rate of discount) can be viewed as the algebraic sum of the heights of the bars in the above diagram -- a sum we calculated in Table II.5 to be $195.

Discounting the flows to time t = 0 at, say, a 4 percent rather than zero percent annual rate can be viewed as a process that has no effect on the length of the bar at t = 0, reduces the length of the bar at t = 1 by about 1 percent, the bar at t = 2 by about 2 percent, and so on. This process is shown graphically in Figure II.3:

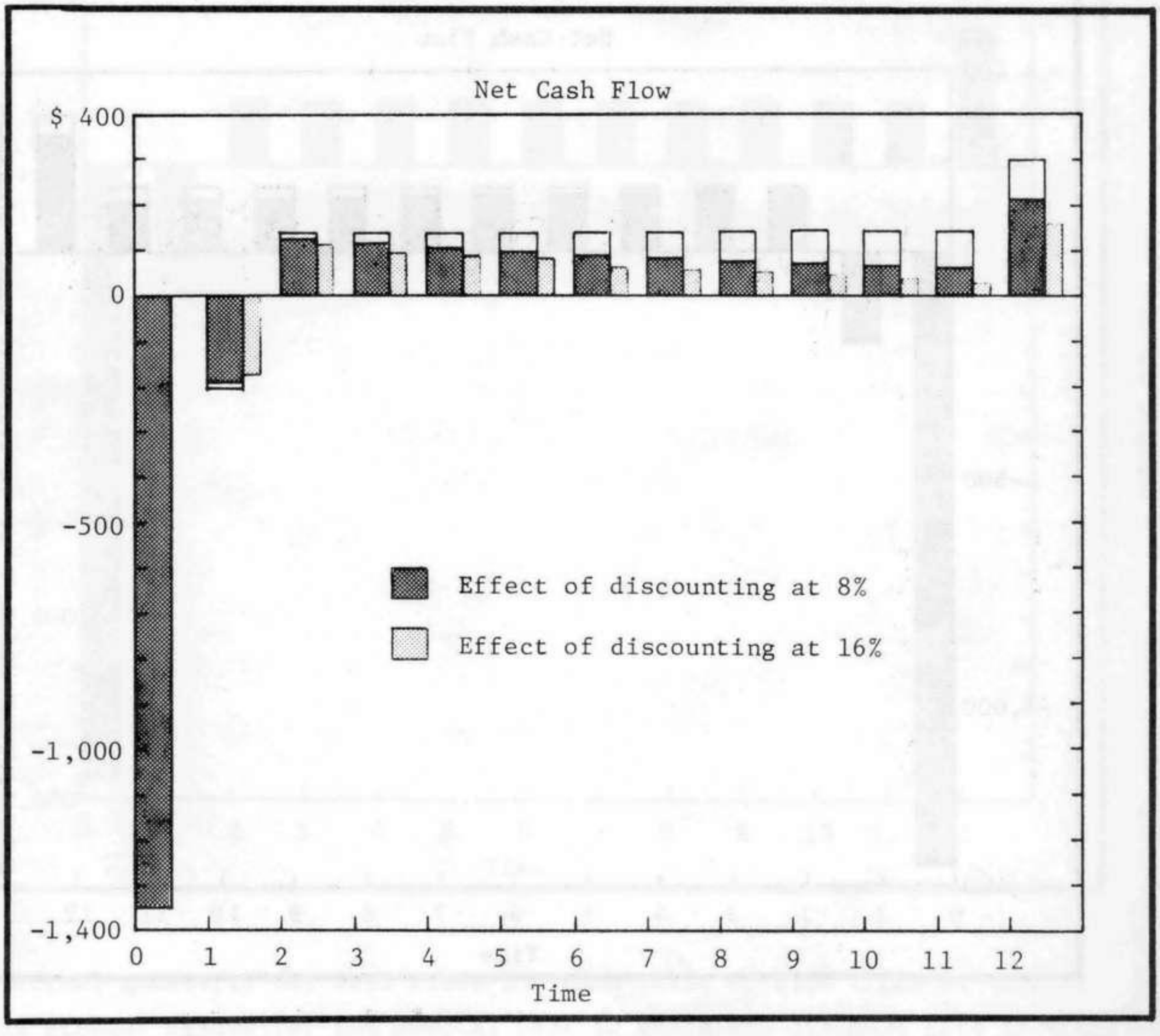
FIGURE II.3



Given that the greatest proportionate reduction takes place for the later points in time where positive flows predominate, the impact of discounting on the project, of course, is to reduce its present value. In contrast to the \$195 present value we got at a zero rate of discount, we have only a \$74 present value at a 4 percent rate of discount.

For cash flow patterns of the sort we have been looking at, successively raising the selected discount rate will so shrink the contribution of the later, positive inflows, that the overall project present value would eventually be reduced to zero and then become negative. At an 8 percent annual rate of discount, alternative A has a present value of -$32; and higher interest rates would result only in larger negative values (see Figure II.4).

FIGURE II.4



We can summarize all of this information on one graph by plotting present value on the vertical axis against selected interest rates used for discounting on the horizontal axis. Using computer programs, present values can be computed for, say, integer values (i.e., 0%, 1%, 2%, 3%, 4%,...) over any range of rates desired. The points so plotted can then be connected by straight lines to give a full picture of the behavior of the present value of the alternatives as a function of interest rates over the range selected. This we've done in Figure II.5 for project A using rates from zero percent to 16 percent.

FIGURE II.5

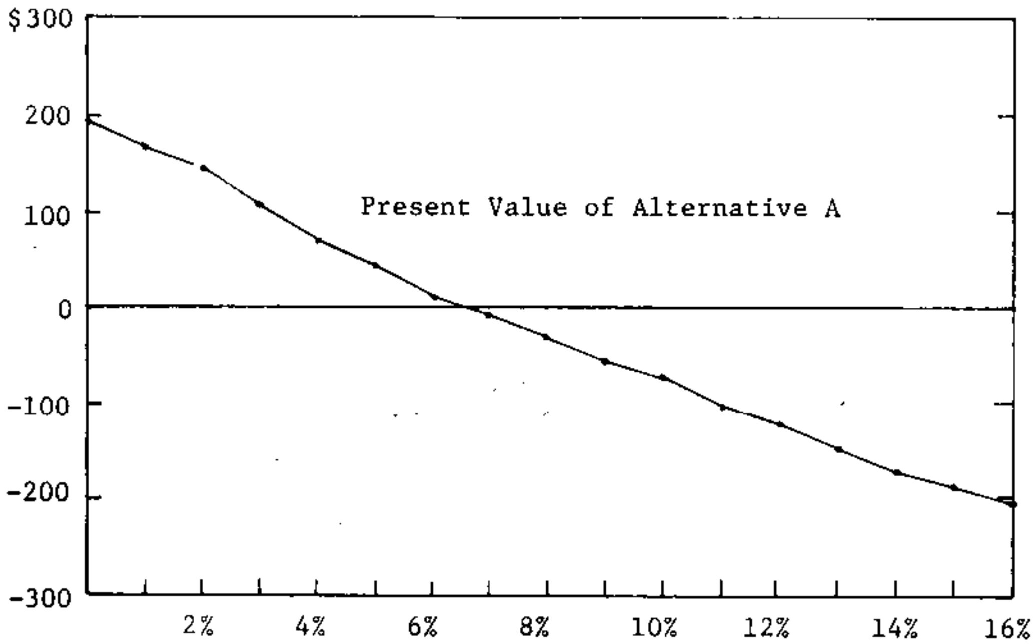
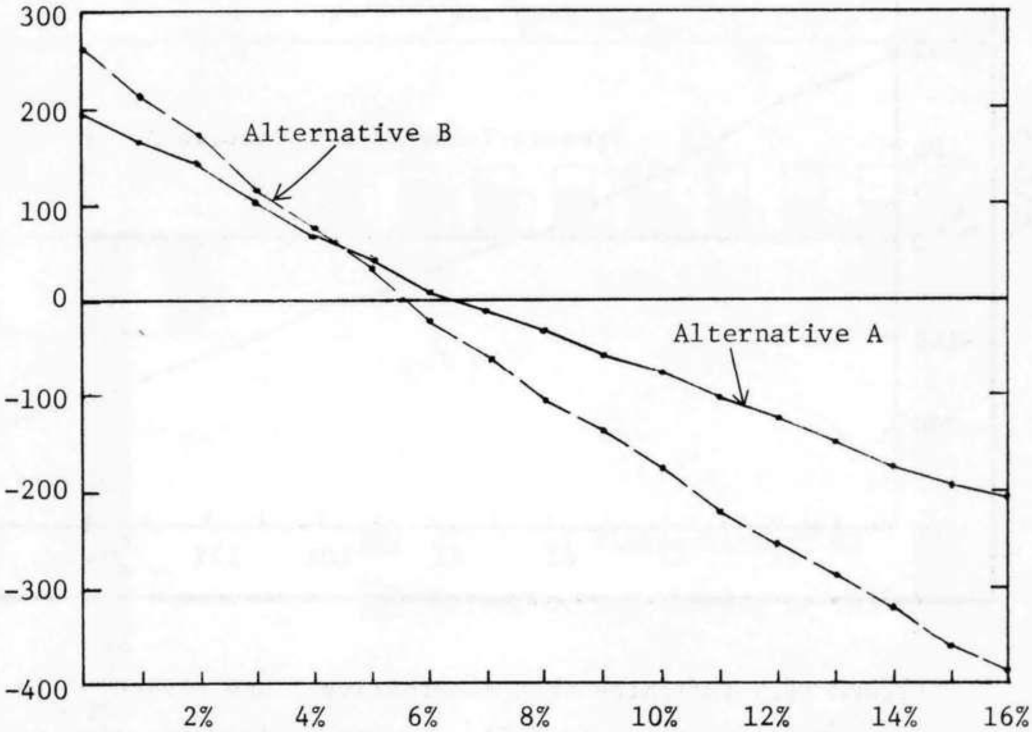


Figure II.5 indicates that alternative A has project-life revenues exceeding project-life outlays by $195. It has positive present values at any discount rate up to about 6-1/2 percent, and has negative present values for any discount rates larger than about 6-1/2 percent. The interest rate at which the present value of the project becomes zero (just under 7 percent) can be estimated directly from the graph, and has a special meaning. The rate at which the present value of net cash flows is zero is the so-called "internal rate of return" of a project or investment. The internal rate of return is often used as an index of profitability and there are various suggested investment criteria that build their decision rules around this concept. Present-value rules, however, are more generally applicable. We will use present value as the basic criteria in our procedure. The internal rate of return will serve as an additional piece of information, falling out of the calculations.

• *Comparing Projects Using a Range of Interest Rates*

Now let's return to our earlier example and plot the "present-value curve" for each of the two alternatives on the same graph. This we do in Figure II.6.

FIGURE II.6



These curves contain a considerable amount of analytic information that should be of use to the decision maker. It's immediately obvious from the graphs that, in the absence of any consideration of the "time value" of money, alternative B with its greater excess of revenues over outlays would be preferred. And in fact at low levels of interest rates -- out to about 3-1/2 percent per annum -- alternative B still has a higher present value and would, under our decision rules, be selected.

But at any higher discount rate, alternative A has the larger present value and so would be preferred over B. This crossing of present-value curves is not uncommon and in such cases the choice of the most "appropriate" interest rate for discounting purposes may become quite critical in helping to judge which project is superior. If 6 percent per year is chosen as the discount rate then alternative A is still attractive (has a positive present value) but alternative B is no longer acceptable under the conventional decision rule because its present value is negative (slightly). At an interest rate of 8 percent, neither project has a positive present value, and neither project would be acceptable as measured by present-value criteria.

Based on the argument in the Appendix, we recommend the range 7 percent to 11 percent as the most significant region for present-value comparisons. That could mean, in some cases, application of the decision "rules" would be ambiguous. We reemphasize our earlier point that present-value computations provide some, but never all, of the information necessary to sound decision making. So even if the present value comes out negative over most if not all of the 7 percent to 11 percent range, it would not be conclusive that the decision must be to reject the project. There are always intangibles associated with any major decision. In the case of alternative A we might imagine it represents a new line of activity with some significant longer-term promise which we have no way of incorporating into the cash flow data. Or perhaps it represents an avenue for staff development that we believe to be important but can't assign specific dollar values to today. So we might still choose to go ahead with alternative A. And our present-value calculations may still be useful as measures of the "opportunity cost" that the intangibles in the project must be judged to outweigh.

In another situation, we may weigh two alternatives that both happen to have positive present values and decide to go with the lower present-value alternative. We may reason, for example, that it represents purchase from a different equipment manufacturer that we believe should be encouraged in the interest of improving competition in the suppliers' market. And again, we can read from our graphs what this choice is costing us by observing the difference between the two present values in the relevant range of interest rates.

Finally we may note that alternative A has an internal rate of return of just under 7 percent which is modestly greater than that calculated for B (about 5-1/2 percent). Perhaps that information may give us some additional feeling for what's involved in the selection or rejection of one or the other alternative.

B. The Case in Which Benefits (Receipts) Cannot be Measured

In the standard case considered in section A both the expenditure (cost) stream and the revenue (benefits) stream were known. For public institutions, including the Federal Reserve, it is difficult or even impossible to accurately measure in monetary terms the social value of many of the activities undertaken. Some public agencies do attempt to assign benefit values to their projects. This may involve very subjective and arbitrary decisions of values affecting the social worth of say a bridge, a sewer system, a recreational site and so on. Generally, we recommend that an attempt be made to estimate benefits, so that decision-making procedures applicable to the standard case can be used. If, however, it is not possible to make a sufficiently accurate estimate of benefits, cash flow analysis procedures are still available which provide useful information for decision making. We will discuss two situations of this kind.

1. Choose from Two or More Alternatives by Minimizing Present Value of the Cost Stream (Benefits Assumed to be the Same for Each)

A typical problem of this kind is the situation in which we have been instructed (say by Congress or the Board of Governors) to engage in an activity. The problem is to select the most efficient (e.g., least cost) means to accomplish the objectives of the activity. Another common situation arises when management has determined, based on analysis at a broader level, that an objective is appropriate; that even though we can't measure the benefit stream, it is surely large enough in their judgment so that the net cash flow stream does have a positive present value at relevant discount rates.

More generally, we include here any situation in which it can be assumed that the alternatives all have benefit streams with the same present value. The time patterns of the benefit streams need not be identical so long as the present values are the same. A common example of this type is the lease-buy decision. A decision may have been made to acquire, say, specific computer equipment. The question is whether to buy or to lease, and if lease, which of several lease options to elect. A rule applicable to these cases is:

> Rule III. For projects assumed to have future benefit streams with the same present value, accept the alternative for which the present value of the cost stream is a minimum, provided that the present value of benefits is judged to be at least as great as the present value of costs.

To illustrate this situation we will consider an example in which the problem is to determine on what basis to acquire terminal equipment for access to a commercial computer timesharing service, to be used mainly by the Research Department. All of the basic principles, problems, and procedures regarding time scale, constructing cash flows, and so on apply here.

The objective of the acquisition is to facilitate economic research involving the use of large econometric models. We assume that this objective has already been judged by management to be worth pursuing, and that various alternative approaches to providing larger and faster computer facilities have been considered, including (a) acquisition of a larger Bank computer, (b) using various computer service bureaus on a walk-in basis, and (c) using the Board of Governor's computer by transmitting jobs and receiving the output by mail. On the basis of an initial review, which need not have included any present-value procedures, the decision has been made that commercial timesharing services offer the most efficient and flexible approach. Computer terminal equipment must now be installed, and the question is, on what basis?

The inquiry is based on a two-year project horizon. Three alternatives are open to us through agreements with the vendor.

(1) Purchase the terminal, pay cash in the full amount of $16,700 at time $t = 0$.

(2) Purchase the terminal, pay for it in 24 equal monthly installments, with an initial down payment of $1,700 at $t = 0$.

(3) Lease for 12 months at $750 per month, renew the lease for the second year.

For the two purchase options it is necessary to estimate the "scrap" value of the equipment at the end of the two-year horizon. This is essentially the price we expect the equipment can be sold for on the used market at time t = 24, less all transaction costs. This scrap value then appears as an inflow and as such has a positive (+) sign attached to it. Table II.5 gives the monthly cash flows for each alternative. Notice that there are 24 monthly intervals so we have 25 time points (t = 0,1,...,24). All lease or purchase payments are assumed to be made at the beginning of the month. Thus, cash flow for the lease option is zero at the end of the 24th month (t = 24) but equal to the scrap value ($3,340) for both purchase plans.

TABLE II.5

Monthly Cash Flows

Time	Purchase, Pay Cash	Purchase, 24 Pay	Lease
0	$-16,700	$-2406	$-750
1	0	-706	-750
2	0	-706	-750
3	0	-706	-750
4	0	-706	-750
5	0	-706	-750
6	0	-706	-750
7	0	-706	-750
8	0	-706	-750
9	0	-706	-750
10	0	-706	-750
11	0	-706	-750
12	0	-706	-750
13	0	-706	-750
14	0	-706	-750
15	0	-706	-750
16	0	-706	-750
17	0	-706	-750
18	0	-706	-750
19	0	-706	-750
20	0	-706	-750
21	0	-706	-750
22	0	-706	-750
23	0	-706	-750
24	+3340	+3340	0

Multiple discount rate analysis is now applied to these alternatives. Table II.6 gives the resulting present values for selected discount rates.

TABLE II.6

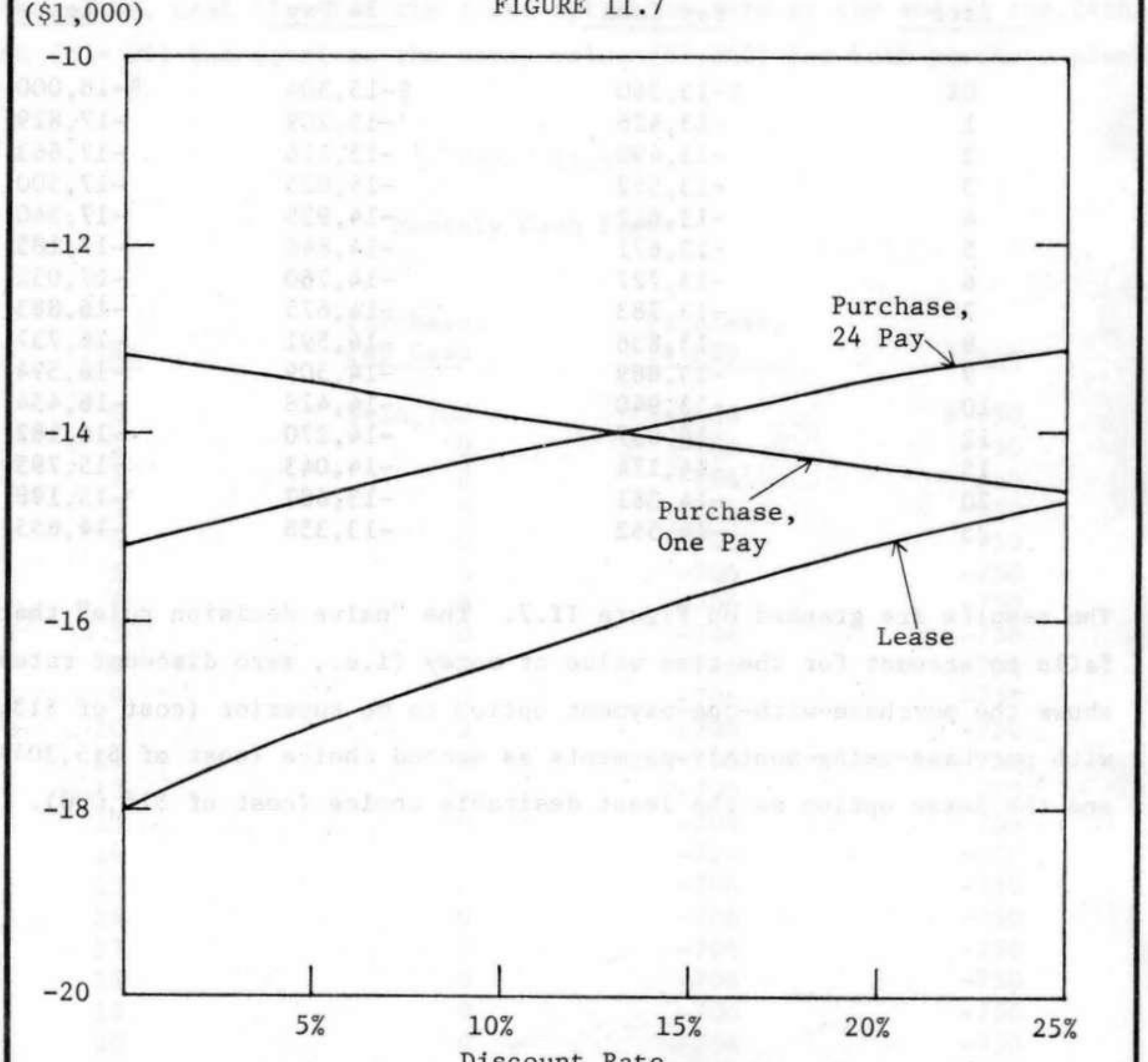
Present Values
(at selected discount rates)

Discount Rate	Purchase, Pay Cash	Purchase, 24 Pay	Lease
0%	$-13,360	$-15,304	$-18,000
1	-13,426	-15,209	-17,829
2	-13,490	-15,116	-17,663
3	-13,552	-15,025	-17,500
4	-13,612	-14,935	-17,340
5	-13,671	-14,846	-17,185
6	-13,727	-14,760	-17,032
7	-13,783	-14,675	-16,883
8	-13,836	-14,591	-16,737
9	-13,889	-14,509	-16,594
10	-13,940	-14,428	-16,454
12	-14,037	-14,270	-16,182
15	-14,174	-14,043	-15,795
20	-14,381	-13,687	-15,198
25	-14,562	-13,358	-14,655

The results are graphed on Figure II.7. The "naive decision rule" that fails to account for the time value of money (i.e., zero discount rate) shows the purchase-with-one-payment option to be superior (cost of $13,360), with purchase-using-monthly-payments as second choice (cost of $15,304), and the lease option as the least desirable choice (cost of $18,000).

Over the 7 percent to 11 percent range the same ranking holds but the difference in present value of cost is narrowing. For discount rates greater than about 14 percent, the option to purchase on time payments becomes more desirable than the one-pay purchase (that ranking holds out to at least a 25 percent discount rate although the lease option closes rapidly on the one-pay purchase option).

FIGURE II.7



So we decide to buy the machine and pay cash, right? Not necessarily. The three cash flows were constructed by considering only actual known cash payments and an estimate of the scrap value of the terminal. No other aspects of the problem were explicitly incorporated into the cash flows and the present-value analysis can provide no information on these excluded factors. There are important additional considerations. One is the desirability of maintaining a flexible position: retaining the option to dispose of the terminal at low cost in the event early experience with the facility fails to meet advance expectations, or in case our needs change.

We mean to emphasize by this discussion that present-value analysis represents merely a tool in decision making, not an all-conclusive key. So it is quite possible in the face of the present-value data generated by our example to arrive at the judgment that the benefits of greater flexibility offered by the lease option will be sufficient to offset the cost differential measured by the factors present in the present-value analysis. Still we can get useful quantitative information from the present-value table. At an 8 percent discount rate the lease option costs $16,737 while the least expensive option, purchase for cash, costs only $13,836. The difference in cost -- $2,901 -- represents a minimum value which all factors not explicitly represented in the cash flow streams ought to exceed in order to support our choice of the lease option.

2. Accept or Reject an Investment Opportunity for Which Benefits Are Not Known

We will illustrate this kind of decision problem by modifying the computer terminal example used above. Suppose this time we have just one alternative, the one-pay purchase option, and the choice is to accept or reject acquisition of a terminal. Benefits available from use of the terminal cannot accurately enough be estimated to apply the standard decision rules. A rule applicable to this situation is:

> Rule IV: If the present value of the future benefit stream is judged to be at least as great as the present value of the cost stream then accept the project; reject the project otherwise.

Again, the same cash flow stream is constructed. The same multi-discount rate present-value computations are performed. The present value of costs at 8 percent is $13,836. Although we don't know the present value of benefits, if we judge it to be at least $13,836 then Rule IV says go ahead, accept the project. Not all decisions involving unmeasurable benefits can be resolved by framing the question in this manner. But Rule IV provides additional information which may be of some value.

There are other ways to manipulate cash flow data to provide information that may be useful in a given problem. The analyst will need to exercise ingenuity. One computation that may be helpful is to compute an equivalent equal-payment monthly benefit stream which corresponds to the known expense flows. For the present example, this monthly annuity amount for 24 months is $626. If management can determine that the value of the benefits is at least equal to $626 per month over the project life, then, again, a decision to accept the project can be made even though the exact value of benefits is not known.

PART III. AN EXAMPLE ILLUSTRATING PRESENT-VALUE PROCEDURES

In April 1971 the Board of Governors issued broad guidelines pertaining to currency and coin services provided by Federal Reserve Banks. Among other provisions the statement set forth the Board's position that Reserve Banks should provide wrapped coin to all member banks at such time as facilities permit. Volume of wrapped coin at the Minneapolis Fed is expected to increase from about 5 million rolls per year at present to about 14 million rolls per year when the service is fully established in 1973. Later this year an extensive study will be conducted to choose from among a number of alternative systems of coin wrapping equipment and to decide on the purchase or lease question. The illustration presented here is based on this coin wrapping question. Fewer alternatives are considered here than will be treated in the subsequent study and a number of minor but bothersome considerations which must be dealt with later are ignored in this illustration. Nevertheless, the general approach developed here is a realistic one in response to a realistic problem. The subsequent study will probably be based on a similar framework, augmented to treat the alternatives ignored here.

A. Description of the Problem

Although several equipment manufacturers have submitted proposals to provide coin wrapping equipment, this illustration will consider only one brand of equipment. A choice must be made between leasing or purchasing the machine system. Also considered here will be the question of how long to keep the equipment. Generally the longer an investment can be kept earning revenue the more favorable will be the net cash flow stream it generates. Usually offsetting this favorable tendency is the sometimes substantial increase in maintenance cost associated with old equipment, the decrease in productivity due to more frequent equipment failures, and finally obsolescence which results from technological improvements available in new equipment models. A thorough analysis of optimal equipment replacement cycles is very difficult and won't be attempted here. Some information on the question will be provided simply by running the cash flow analysis for both a three-year life and a five-year life.

B. Constructing the Net Cash Flow Stream

1. Time Units and Horizon

Quarterly data will be used for this illustration. Monthly data are available and would add to the precision of the information to be generated, but the results of this exercise will show that quarterly data are probably precise enough.

Two time horizons will be examined.

(i) Three years (t = 0,1,2,...,12)

(ii) Five years (t = 0,1,2,...,20)

2. Expenditures Flows

The following cash outflows must be considered:

(1) Labor cost. Six operators will be assigned full time to the coin wrapping facility along with one-half of one supervisor's time. Total cost for salaries and benefits are estimated to be $50,500 in the first year of operation and are projected to increase by 5 percent per year in each of the following years.

Year	Labor Cost
1	$50,500
2	53,025
3	55,676
4	58,460
5	61,383

Labor costs are assumed to be paid at the end of each quarter.

(2) Equipment maintenance contract. If the machine systems are purchased, an equipment maintenance contract will be signed with the vendor. The cost for labor is $1,560 per quarter with no charge for the first quarter. Parts are supplied at no cost to the Bank during the first year. It is estimated that in subsequent years parts costs will be:

Year	Parts Cost
1	$ 0
2	2,000
3	2,400
4	3,200
5	4,800

Equipment maintenance costs are assumed to be paid at the end of each quarter.

(3) Building maintenance. The policy of the Bank regarding building costs for wrapped coin service is to recover building maintenance costs but not the original building investment. A rate of $5.60 per square foot per year is charged for this purpose in the present building and will be used here. Maintenance costs for the new building are not yet determined. The coin wrapping operation will require 2,100 square feet. The cost is $2,940 per quarter and has been held constant over the five-year horizon. Building maintenance costs are assumed to be paid at the end of each quarter.

(4) Supplies. Supplies include mostly paper and bags. Historical records indicate supplies cost to be about 12¢ per 100 rolls of wrapped coin (12¢ per 100 wraps). This cost has been very stable -- even declining a bit in recent years -- and is not projected to increase during the coming five years.

Volume of wraps for this example is projected to be 14,000,000 in the first year of operation, and is expected to increase by 5 percent per year in each of the four subsequent years.

Year	Volume	Supplies Cost
1	14,000,000	$16,800
2	14,700,000	17,640
3	15,434,000	18,524
4	16,207,000	19,448
5	17,017,000	20,420

Supplies costs are assumed to be paid at the end of each quarter.

(5) Purchase price. Twelve coin wrapping systems are required at a total purchase price of $97,443. In addition three scales are required at a cost of $9,000. The entire purchase amount is payable at t = 0.

(6) Lease contract. The quarterly lease cost is $10,866 and is fixed by contract for the first three years. It has been assumed the lease price would be the same in both the fourth and fifth years also. The lease contract includes equipment maintenance -- both parts and labor. Scales, at $9,000, must still be bought if the lease option is elected. Lease payments are assumed to be due at the beginning of each quarter.

3. Revenue Flows

The Minneapolis Federal Reserve Bank has established a price of \$.85 per 100 wraps to be charged member banks. With the expected volume, annual revenue will be:

Year	Revenue
1	\$119,000
2	124,950
3	131,189
4	137,760
5	144,645

Revenue is assumed to be received at the end of each quarter.

Additional inflows are generated by scrap value of equipment at the end of the time horizon. Scrap values are assumed to be:

TABLE III.1

	Wrapping Equipment	Scales	Total Scrap Value
Three-Year Horizon	\$1,000	\$3,000	\$4,000
Five-Year Horizon	0	\$3,000	\$3,000

4. Net Cash Flows

Using this information, and converting to quarterly time units, the basic cash flow data are developed as shown in Table III.2. Net cash flow data for each of the alternatives are constructed in Tables III.3 through III.6. The four alternatives considered are:

Alternative A: Purchase the coin wrapping equipment with a three-year horizon.

Alternative B: Lease the coin wrapping equipment with a three-year horizon.

Alternative C: Purchase the coin wrapping equipment with a five-year horizon.

Alternative D: Lease the coin wrapping equipment with a five-year horizon.

TABLE III.2

Qtr.	I Volume (Wraps)	II Labor Cost	III Maint. Contract (Labor)	IV Maint. (Parts)	V Supplies Cost I*0.0012	VI Building Maint. Cost	VII Revenue .0085*I
1	3,500,000	$12,625	$ 0	$ 0	$4,200	$2,940	$29,750
2	3,500,000	12,625	1,560	0	4,200	2,940	29,750
3	3,500,000	12,625	1,560	0	4,200	2,940	29,750
4	3,500,000	12,625	1,560	0	4,200	2,940	29,750
5	3,675,000	13,256	1,560	500	4,410	2,940	31,238
6	3,675,000	13,256	1,560	500	4,410	2,940	31,238
7	3,675,000	13,256	1,560	500	4,410	2,940	31,238
8	3,675,000	13,256	1,560	500	4,410	2,940	31,238
9	3,859,000	13,919	1,560	600	4,631	2,940	32,802
10	3,859,000	13,919	1,560	600	4,631	2,940	32,802
11	3,859,000	13,919	1,560	600	4,631	2,940	32,802
12	3,859,000	13,919	1,560	600	4,631	2,940	32,802
13	4,052,000	14,615	1,950	800	4,862	2,940	34,442
14	4,052,000	14,615	1,950	800	4,862	2,940	34,442
15	4,052,000	14,615	1,950	800	4,862	2,940	34,442
16	4,052,000	14,615	1,950	800	4,862	2,940	34,442
17	4,254,000	15,346	1,950	1,200	5,105	2,940	36,159
18	4,254,000	15,346	1,950	1,200	5,105	2,940	36,159
19	4,254,000	15,346	1,950	1,200	5,105	2,940	36,159
20	4,254,000	15,346	1,950	1,200	5,105	2,940	36,159

TABLE III.3

Alternative A: Purchase, 3-Year Horizon

t	E_t^A	R_t^A	F_t^A
0	$-106,443	$ 0	$-106,443
1	-19,765	29,750	9,985
2	-21,325	29,750	8,425
3	-21,325	29,750	8,425
4	-21,325	29,750	8,425
5	-22,666	31,238	8,572
6	-22,666	31,238	8,572
7	-22,666	31,238	8,572
8	-22,666	31,238	8,572
9	-23,650	32,802	9,152
10	-23,650	32,802	9,152
11	-23,650	32,802	9,152
12	-23,650	36,802	13,152

** Purchase price at t = 0

** Revenue and expenses at the end of the quarter

** Purchase scales at t = 0; $9,000

** Expenses = labor
+ equipment maintenance
+ building maintenance
+ supplies
+ purchase price

** Scrap = $4,000 at t = 12

TABLE III.4

Alternative B: Lease, 3-Year Horizon

t	E_t^B	R_t^B	F_t^B
0	$-19,866	$ 0	$-19,866
1	-30,631	29,750	-881
2	-30,631	29,750	-881
3	-30,631	29,750	-881
4	-30,631	29,750	-881
5	-31,472	31,238	-234
6	-31,472	31,238	-234
7	-31,472	31,238	-234
8	-31,472	31,238	-234
9	-32,356	32,802	446
10	-32,356	32,802	446
11	-32,356	32,802	446
12	-21,490	35,802	14,312

** Lease payments at the beginning of the quarter

** Revenue and expenses at the end of the quarter

** Purchase scales at t = 0; $9,000

** Expenses = Labor
+ Building maintenance
+ Supplies
+ Lease price

** Scrap = $3,000 at t = 12

TABLE III.5

Alternative C: Purchase, 5-Year Horizon

t	E_t^C	R_t^C	F_t^C
0	$-106,443	$ 0	$-106,443
1	-19,765	29,750	9,985
2	-21,325	29,750	8,425
3	-21,325	29,750	8,425
4	-21,325	29,750	8,425
5	-22,666	31,238	8,572
6	-22,666	31,238	8,572
7	-22,666	31,238	8,572
8	-22,666	31,238	8,572
9	-23,650	32,802	9,152
10	-23,650	32,802	9,152
11	-23,650	32,802	9,152
12	-23,650	32,802	9,152
13	-25,167	34,442	9,275
14	-25,167	34,442	9,275
15	-25,167	34,442	9,275
16	-25,167	34,442	9,275
17	-26,541	36,159	9,618
18	-26,541	36,159	9,618
19	-26,541	36,159	9,618
20	-26,541	39,159	12,618

** Purchase price at t = 0

** Revenue and expenses at the end of the quarter

** Purchase scales at t = 0; $9,000

** Expenses = Labor
+ Equipment maintenance
+ Building maintenance
+ Supplies
+ Purchase price

** Scrap = $3,000 at t = 20

TABLE III.6

Alternative D: Lease, 5-Year Horizon

t	E_t^D	R_t^D	F_t^D
0	$-19,866	$ 0	$-19,866
1	-30,631	29,750	-881
2	-30,631	29,750	-881
3	-30,631	29,750	-881
4	-30,631	29,750	-881
5	-31,472	31,238	-234
6	-31,472	31,238	-234
7	-31,472	31,238	-234
8	-31,472	31,238	-234
9	-32,356	32,802	446
10	-32,356	32,802	446
11	-32,356	32,802	446
12	-32,356	32,802	446
13	-33,283	34,442	1,159
14	-33,283	34,442	1,159
15	-33,283	34,442	1,159
16	-33,283	34,442	1,159
17	-34,257	36,159	1,902
18	-34,257	36,159	1,902
19	-34,257	36,159	1,902
20	-23,391	39,159	15,768

** Lease payments at the beginning of the quarter

** Revenue and expenses at the end of the quarter

** Purchase scales at t = 0; $9,000

** Expenses = Labor
+ Building maintenance
+ Supplies
+ Lease price

** Scrap = $3,000 at t = 20

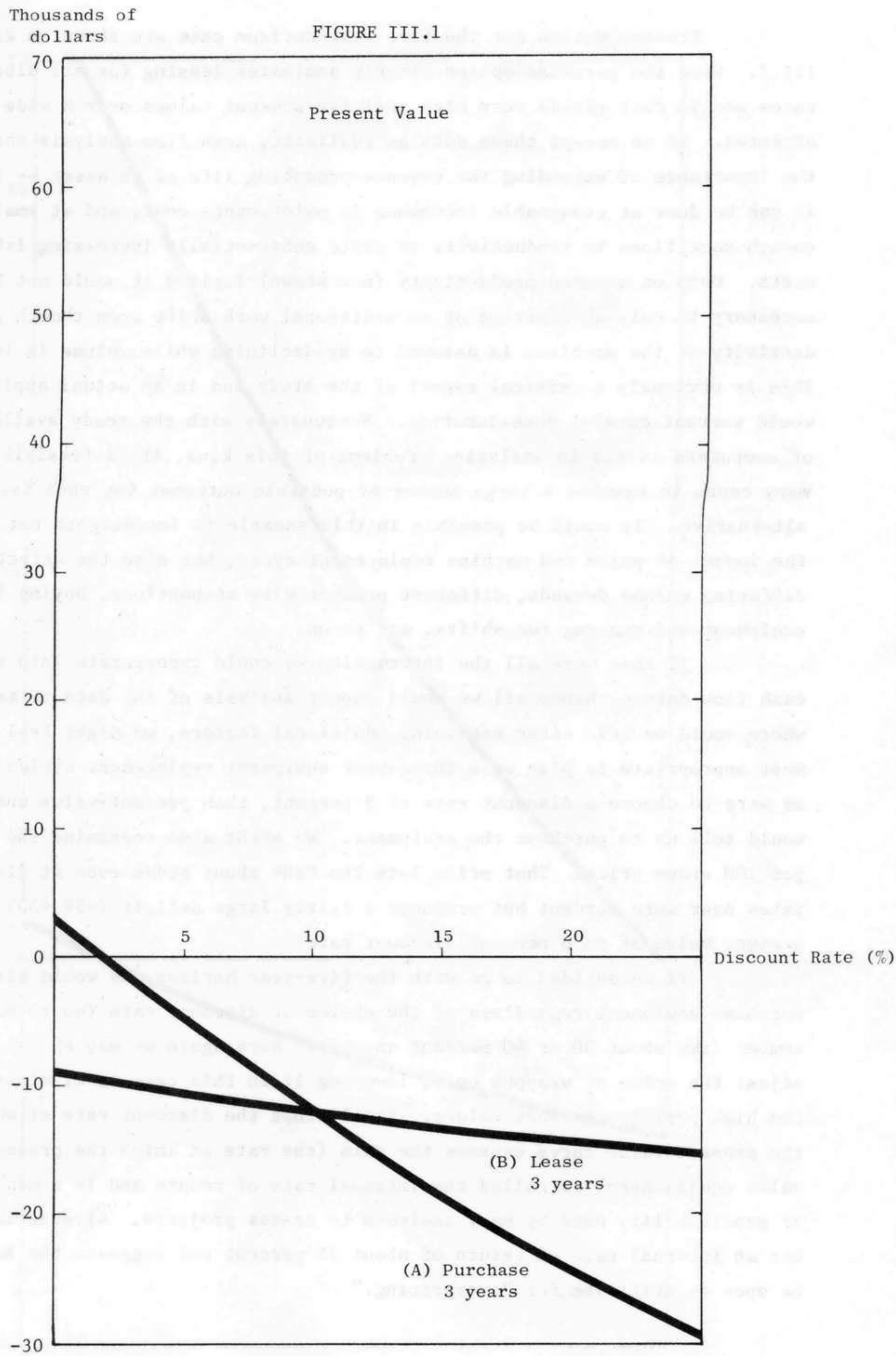
C. Interpretation of Cash Flow Data

Present values for each alternative are given in Table III.7. Consider first the three-year horizon. Present-value curves for both three-year alternatives are shown in Figure III.1. At a zero discount rate (in effect ignoring the time distribution of cash flows), it appears clearly better to purchase. The present value of cash flows is +$3,713 compared with -$8,676 for the lease option. As the choice of discount rate increases, preference for the purchase option decreases until the rate reaches just under 10 percent where the present-value criterion shifts in favor of the lease option. At any discount rate greater than about 2 percent both options produce negative present values.

Another way in which cash flow analysis can be used in this instance is to determine the price we should charge for wrapped coin in order that the present value of cash flows at our selected discount rate is just equal to zero. This is a more general way to define a break-even point than simply assuming a zero discount rate. The problem framed in this way becomes: choose the alternative which minimizes the price per 100 wraps that we must charge in order to yield a zero present value of cash flows discounted at, say, 8 percent. From Figure III.1 we see that at an 8 percent discount rate, the purchase option is superior to the lease option but the present value is less than zero. If the cash flow computations were repeated using a higher price per 100 wraps, both of the present-value curves would be raised on the graph but, given the particular data in this example, they would closely maintain their relative positions. It is not difficult in principle to determine the price necessary to produce the desired zero present value.

If the selected discount rate were greater than about 10 percent the same methods could be employed with the result that the lease option would permit the lowest price necessary to achieve a zero present value.

FIGURE III.1


Thousands of dollars
Present Value
70
60
50
40
30
20
10
0
-10
-20
-30
5
10
15
20
Discount Rate (%)
(B) Lease
3 years
(A) Purchase
3 years

Present values for the five-year horizon case are shown on Figure III.2. Here the purchase option clearly dominates leasing for all discount rates and in fact yields very high positive present values over a wide range of rates. If we accept these data as realistic, cash flow analysis shows the importance of extending the revenue-producing life of an asset -- if it can be done at reasonable increases in maintenance cost, and at small enough sacrifices in productivity to avoid substantially increasing labor costs. Data on assumed productivity (not shown) implied it would not be necessary to rely on overtime or an additional work shift even though productivity of the machines is assumed to be declining while volume is increasing. This is obviously a critical aspect of the study and in an actual application would warrant careful consideration. Fortunately with the ready availability of computers to aid in analyzing problems of this kind, it is feasible in many cases to examine a large number of possible outcomes for each basic alternative. It would be possible in this example to investigate not only the impact of price and machine replacement cycle, but also the effect of differing volume demands, different productivity assumptions, buying less equipment and running two shifts, and so on.

If this were all the information we could incorporate into the cash flow data -- hence all we could expect analysis of the data to tell us -- where would we be? After examining additional factors, we might feel it most appropriate to plan on a three-year equipment replacement cycle; if we were to choose a discount rate of 8 percent, then present-value analysis would tell us to purchase the equipment. We might also reexamine the 85¢ per 100 wraps price. That price lets the Bank about break even at discount rates near zero percent but produces a fairly large deficit (-$9,430) in present value at an 8 percent discount rate.

If we decided to go with the five-year horizon, we would clearly purchase equipment regardless of the choice of discount rate (up to some high number like about 30 or 40 percent anyway). Here again we may choose to adjust the price of wrapped coin, lowering it in this case to eliminate the high positive present values. Recall that the discount rate at which the present-value curve crosses the axis (the rate at which the present value equals zero) is called the internal rate of return and is a measure of profitability used by many analysts to assess projects. Alternative C has an internal rate of return of about 25 percent and suggests the Bank may be open to criticism for "overpricing."

FIGURE III.2

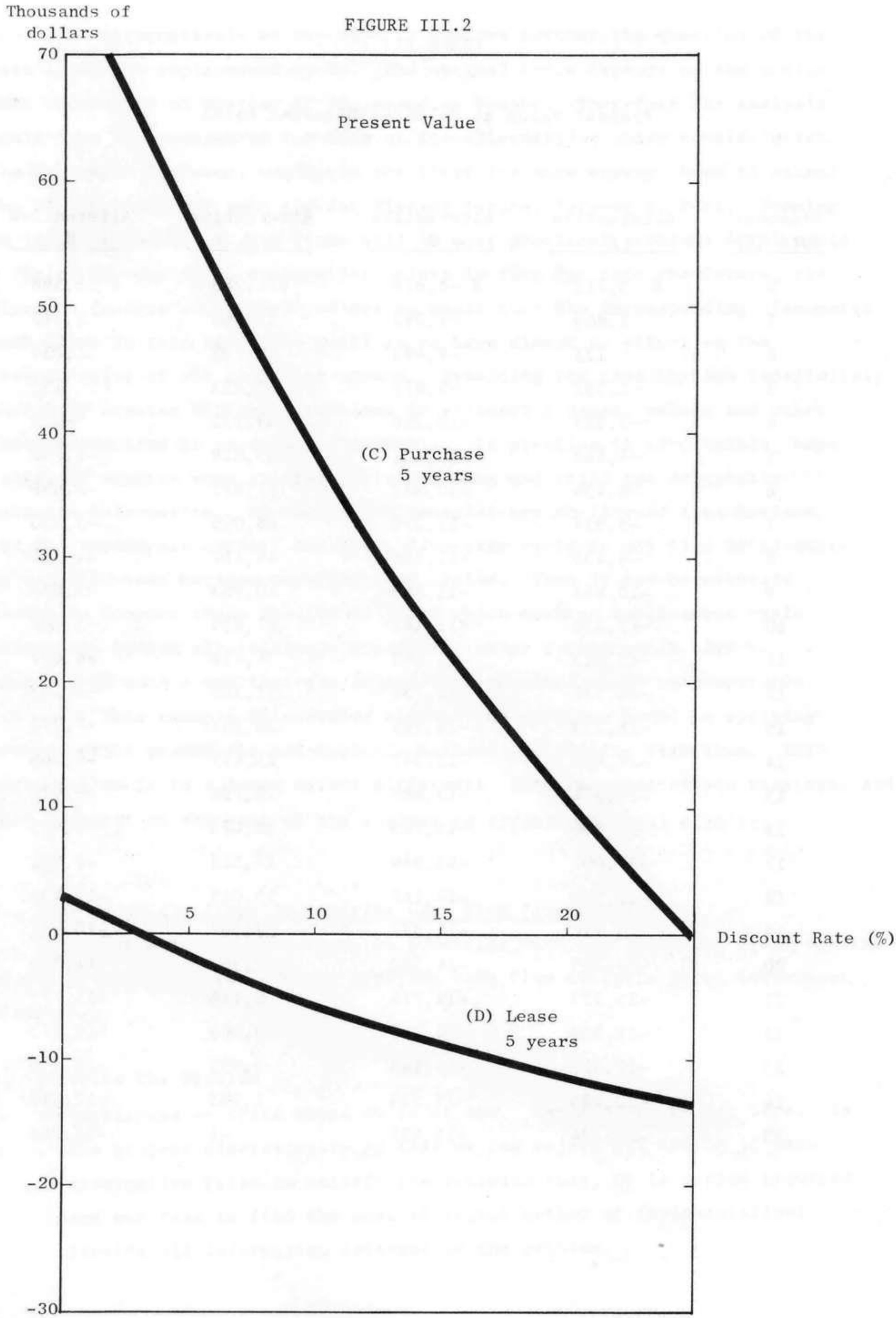

Thousands of dollars
Present Value
70
60
50
40
30
20
10
0
-10
-20
-30
5
10
15
20
Discount Rate (%)
(C) Purchase 5 years
(D) Lease 5 years

TABLE III.7

Present Value at Selected Discount Rates

Discount Rate (%)	Alternative A	Alternative B	Alternative C	Alternative D
0	$ 3,713	$ -8,676	$78,285	$ 3,568
1	1,889	-9,092	73,388	2,379
2	121	-9,493	68,705	1,256
3	-1,592	-9,877	64,223	195
4	-3,255	-10,246	59,931	-808
5	-4,868	-10,601	55,819	-1,758
6	-6,434	-10,942	51,877	-2,658
7	-7,954	-11,270	48,095	-3,510
8	-9,430	-11,586	44,467	-4,317
9	-10,864	-11,889	40,983	-5,083
10	-12,258	-12,182	37,636	-5,809
11	-13,613	-12,463	34,419	-6,499
12	-14,931	-12,734	31,326	-7,153
13	-16,213	-12,995	28,351	-7,775
14	-17,460	-13,247	25,487	-8,366
15	-18,674	-13,489	22,729	-8,928
16	-19,856	-13,723	20,073	-9,463
17	-21,007	-13,949	17,512	-9,971
18	-22,128	-14,167	15,043	-10,456
19	-23,220	-14,377	12,662	-10,917
20	-24,285	-14,580	10,364	-11,356
21	-25,323	-14,776	8,146	-11,775
22	-26,335	-14,965	6,003	-12,175
23	-27,322	-15,148	3,933	-12,556
24	-28,284	-15,324	1,933	-12,919
25	-29,224	-15,495	-1	-13,266

Alternatively we may wish to explore further the question of the best equipment replacement cycle. The optimal cycle depends on the equipment chosen and on whether it is leased or bought. Therefore the analysis would have to be repeated for each of the alternatives under consideration. The framework for study would, in the first instance anyway, have to extend the time horizon off into the far distant future, forever in fact. Summing an infinite number of cash flows will in most practical problems still yield a finite present value because for points in time far into the future, the discount factors -- $1/(1+r)^t$ -- are so small that the corresponding discounted cash flows in turn become so small as to have almost no effect on the present value of the cash flow stream. Extending the time horizon indefinitely obviously creates difficult problems in estimating costs, volume and other factors required to perform the analysis. In practice it is possible, hopefully, to examine some shorter period of time and still get acceptably accurate information. We could, for example, try an 18-year time horizon for the three-year cycle. And for a five-year cycle we may find it adequate to use a 20-year horizon covering four cycles. Then it may be accurate enough to compare these results to judge which machine replacement cycle offers the better alternative. Hopefully, other cycles could also be examined in such a way that the information obtained would be comparable.

This example illustrates some of the problems faced in applying present-value procedures and suggests methods for dealing with them. But each problem is to a large extent different. Each requires custom treatment and good judgment on the part of the analyst to effectively deal with it.

D. <u>Summary of the Steps in Applying Cash Flow Procedures</u>

For purposes of review, we summarize with the following brief outline of procedures to be followed in applying cash flow analysis to an investment prospect.

1. Describe the Problem
   - Background -- state where we're at now. Explain how we got here. Is the project discretionary so that we can reject all action if each alternative fails to satisfy the decision rule, or is action required and our task to find the most efficient method of implementation? Provide all information relevant to the problem.

- Define the objectives to be accomplished and constraints, if any, imposed on potential actions.
- Specify the framework for analysis and the decision rules that will be used. Indicate the kinds of information that can (will) be incorporated into the cash flow data and what information must be excluded from the data. Some situations may not call for cash flow analysis at all if, for example, the dollar values are small, or if the timing of cash flows is concentrated in a relatively short time span, or if reliable data are unobtainable.

2. Specify Alternatives

Outline the alternatives available to accomplish the objectives. This could start with a long list including options we are quite certain will not prove optimal -- some may appear even a little absurd. The purpose is to be certain every possible approach is given at least meager consideration. Why? Because (1) on rare occasions an oddball approach may be best (2) it may be good for us to stretch our imagination -- to keep us from falling into a set of routine solution types (3) it may not be so obvious to decision makers unfamiliar with the problem that certain of the alternatives are prima facie not optimal -- it will be necessary to outline these approaches and show that they come out second best. At this point very little, if any, cash flow data are necessary. That makes it easier to deal with a long list of alternatives.

3. Reduce Alternatives to a Smaller Set for Detailed Analysis

Run through the long list of alternatives explaining why certain of them are not feasible. Some may not satisfy constraints imposed, some may be clearly dominated by other alternatives in the set. The smaller set that survives this filtering process is the set that will undergo detailed cash flow analysis.

4. Estimate Cash Flow Data for Each Surviving Alternative

- Select time horizon and time units.
- Estimate expenditure flows for each alternative.
- Estimate revenue flows (if possible).
- Compute net cash flows (if possible).

5. Analyze the Net Cash Flow Streams

- Compute the present values.
- Prepare tables and graphs for analysis.
- Examine the results for information.
- Summarize the information obtained from the cash flow analysis.

6. Recommend a Decision

Within the broad framework specified for the problem in step 1, analyze the cash flow information along with other information not incorporated into the cash flow data. Recommend a decision.

● Define the objectives to be accomplished and [illegible] impose on potential actions.

● Specify the framework for analysis and the criteria that will be used. Indicate the kinds of information that [illegible] excluded from the data. Some situations may not call for cash flow analysis at all if, for example, the dollar values [illegible] short time [illegible].

2. Specify Alternatives

Outline the alternatives available to accomplish the objectives. This could start with a long list including options we are quite certain will not prove optimal — some may appear even a little absurd. The purpose is to be certain every possible approach is given at least a [illegible] consideration. Why? Because (1) on rare occasions an oddball approach may be best (2) it may be good for us to stretch our imagination — to keep us from falling into a set of routine solution types (3) it may not be so obvious to decision makers unfamiliar with the problem that certain of the alternatives are prima facie not optimal — it will be necessary to outline these approaches and show that they come out second best. At this point very little, if any, cash flow data are necessary. That makes it easier to deal with a long list of alternatives.

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4. Estimate Cash Flow Data for Each Surviving Alternative

● Select time horizon and time units.

● Estimate expenditure flows for each alternative.

● Estimate revenue flows (if possible).

● Compute net cash flows (if possible).

PART IV. CASE APPLICATIONS

Part I of this report discussed the theoretical rationale for use of cash flow analysis procedures in investment decision making. Parts II and III illustrated the basic procedures by applying them to simplified problems. The purpose of Part IV is to present more detailed case applications of decision problems faced by the Bank. A primary goal will be to establish the extent to which the kinds of investment decisions confronting the Bank can effectively be handled through cash flow analysis. It is one thing to discuss ideal decision-making techniques. It is another matter to shape the particular investment problems of a given institution into a form required by the decision technique; and still a further problem to collect accurate data required for application of the methods. Few Bank decision problems will easily yield accurate data on benefits. A certain amount of cost data can be determined for many decision problems. Only by experience will we learn the extent to which cash flow analysis can provide useful information for our decision problems. Experience and sound managerial judgment will be required to determine in each case how much of the total information relevant to the problem can accurately be captured in the cash flow data, and how much is outside the ability of mechanical techniques to correctly assess.

Part IV will hopefully evolve into an expanded loose-leaf collection of cases to which new and unique or otherwise noteworthy applications can be added.

Case I. Bank Automobiles; Lease or Buy

A. Description of the Problem

Based on experience and studies over past years, Bank management has made various decisions relating to automobile transportation for staff conducting Bank business. It has been decided, for example, that employees shall be encouraged to use Bank-owned cars for business. Studies have shown it preferable to trade cars when they are two years old or accumulate 30,000 miles.

The Bank maintains 11 cars for general use by staff. Since a good deal of travel involves long-distance highway driving, full-size cars -- with features and accessories chosen to promote safety and comfort -- are preferred. We will not reexamine these decisions. Instead we will decide the best (least cost in present value) means to acquire the automobiles -- whether to purchase outright or to lease. Recent past data will be used to avoid the problem of predicting future auto prices and trade-in values.

Fortunately the Bank has kept extensive records on automobile expenses from which cash flow data can be constructed. The problem will be framed in terms of lease or buy one car.

B. Constructing the Net Cash Flow Stream

1. Time Units and Horizon

Monthly data will be used. The time horizon is 24 months ($t = 0,1,\ldots,24$).

2. Expenditure Flows

Based on 1,250 miles per month (30,000 miles over a 24-month period).

(1) Operating costs and maintenance

Operating Costs	\$ 73.00 per month
In-House Maintenance	68.00 per month
	\$141.00 per month

(2) Purchase Price and Scrap Value

Purchase Price at t = 0	$3,510
Scrap Value at t = 24	1,977

(3) Lease price is $117 per month on a 24-month basis at about 30,000 miles. The Bank must still insure and maintain the car.

Using this information the alternative cash outflows have been constructed (Table IV.1). Assume operating costs and maintenance expenses are paid at the end of the month. Lease payments are made at the beginning of the month. Present value of costs for both options are given in Table IV.2 with selected discount rates, and plotted in Figure IV.1.

C. Interpretation of Cash Flow Data

Present value of costs for the buy alternative is only slightly affected by changes in the discount rate over the range considered. This is because the great bulk (over 70 percent) of the total expense is incurred at t = 0 which is unaffected by discounting. Lease option present values follow a more typical pattern with declining present value of costs as the discount rate is increased. At 8 percent it is clearly superior to buy; the present values are $4,941 for purchase and $5,616 for lease. It isn't until the discount rate exceeds 23 percent that the lease option becomes more desirable according to the present value of costs criterion.

D. A Computational Shortcut

The decision in this particular problem centers on the difference in the present value of cash outflows between two mutually exclusive alternatives (just another way of saying select the alternative with the lowest present value of cost). In this case it is acceptable to net out cost factors that are common to both alternatives. When originally framing the problem it could have been argued correctly that since operating costs and maintenance expenses are exactly the same in either alternative, they can be ignored. The resulting cash flows would have been as shown in Table IV.3.

The present values for these cash flows (not shown) would each have been less negative than those in Table IV.2, but the two present-value lines in the graph would have maintained the same relative positions, still crossing at just over a 23 percent discount rate. The decision would have been the same as one based on complete expenditure data.

Netting out common costs in this manner saves some work and is appropriate when only the difference in the present value of cost streams is of interest. Some information is lost when using computational shortcuts, however. The present value of costs for a particular alternative no longer measures total costs. Hence it is not useful as a guide in asking the question: is the present value of benefits (which are assumed not to be measurable) at least as great as the measured present value of costs? In problems presenting more than two alternatives, terms netted out must be present in exactly the same amount and time pattern in each alternative cash flow. Attempts at shortcut cash flow construction expose the analysis to additional risk of mistaken judgment in framing the problem and also limit the possibility of augmenting the decision exercise by easily adding newly discovered alternatives which may not share the common term previously netted out.

When it is impossible to measure particular costs accurately, netting out cash flows is justifiable. In those cases the cash flow analysis framework will have to be designed to permit use of a decision rule which does not depend on the unknown common cost elements.

TABLE IV.1

CASH OUTFLOWS

Time	Lease	Buy
0	$ -117	$-3,510
1	-258	-141
2	-258	-141
3	-258	-141
4	-258	-141
5	-258	-141
6	-258	-141
7	-258	-141
8	-258	-141
9	-258	-141
10	-258	-141
11	-258	-141
12	-258	-141
13	-258	-141
14	-258	-141
15	-258	-141
16	-258	-141
17	-258	-141
18	-258	-141
19	-258	-141
20	-258	-141
21	-258	-141
22	-258	-141
23	-258	-141
24	0	+1,836

TABLE IV.2

PRESENT VALUE OF CASH OUTFLOWS AT VARIOUS DISCOUNT RATES

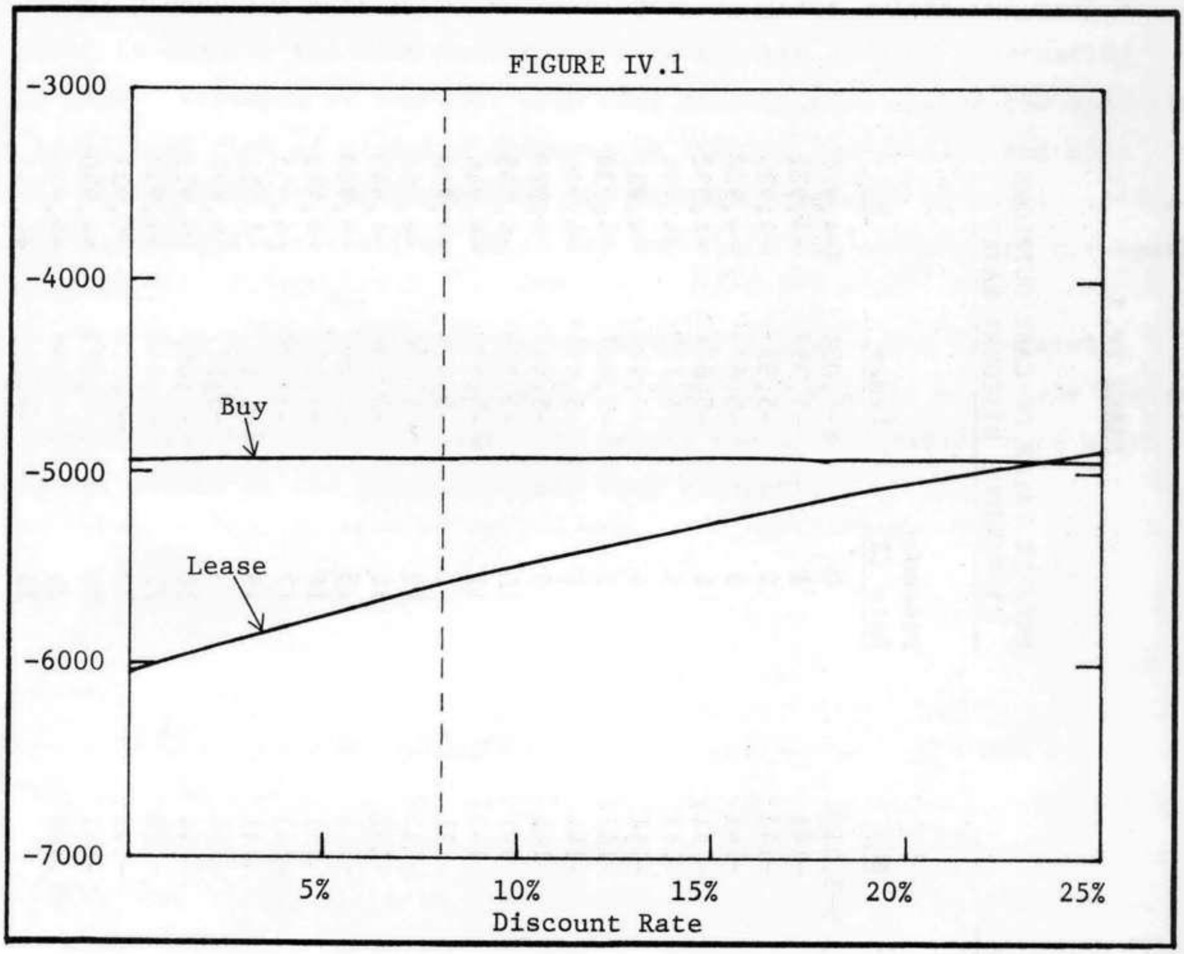
Discount Rate (%)	Lease	Buy
0	$-6,051	$-4,917
1	-5,992	-4,921
2	-5,935	-4,925
3	-5,879	-4,928
4	-5,824	-4,932
5	-5,770	-4,934
6	-4,718	-4,937
7	-5,667	-4,939
8	-5,616	-4,941
9	-5,567	-4,993
10	-5,519	-4,945
11	-5,472	-4,946
12	-5,426	-4,948
13	-5,380	-4,948
14	-5,336	-4,949
15	-5,292	-4,950
16	-5,250	-4,951
17	-5,208	-4,951
18	-5,167	-4,951
19	-5,127	-4,951
20	-5,087	-4,951
21	-5,048	-4,951
22	-5,010	-4,951
23	-4,973	-4,950
24	-4,936	-4,950
25	-4,900	-4,949

TABLE IV.3

SHORTCUT CASH OUTFLOWS

Time	Lease	Buy
0	$-117	$-3,510
1	-117	0
2	-117	0
3	-117	0
4	-117	0
5	-117	0
6	-117	0
7	-117	0
8	-117	0
9	-117	0
10	-117	0
11	-117	0
12	-117	0
13	-117	0
14	-117	0
15	-117	0
16	-117	0
17	-117	0
18	-117	0
19	-117	0
20	-117	0
21	-117	0
22	-117	0
23	-117	0
24	0	+1,977

FIGURE IV.1



APPENDIX A

THEORETICAL ARGUMENT UNDERLYING CHOICE OF AN INTEREST RATE

APPENDIX A

THEORETICAL ARGUMENT UNDERLYING CHOICE OF AN INTEREST RATE

The purpose of this appendix is to discuss some theory behind choice of an interest rate for use in government sector present-value computations. Ideally, the construction presented here[4/] would enable one to examine financial markets data at any point in time, and to compute an appropriate numerical rate for use in the decision process. Practically, however, the data actually available are sufficiently incomplete or equivocal so that only a neighborhood or range of rates can be pinned down at best.

A. A Simplified Markets Model of the Economy

At an abstract level we can consider an economy with three sectors: government, business, and households (the public-at-large). From conventional economic theory two basic determinants of the interest rate in ideal markets are posited.

The first is a "rate of time preference" that summarizes the household sector's willingness, in aggregate, to trade income today for income in the future. Specifically it may be assumed that for each individual there is some amount of dollars, R, such that he or she will give up 100 dollars today in return for the payment of 100 + R dollars with certainty at a specific future date. If the date is one year from today, then R represents an annual interest rate. Higher rates of R paid by business or government are assumed to induce households to increase the amount they lend and to reduce the amount of their income going into consumption -- which is another way of

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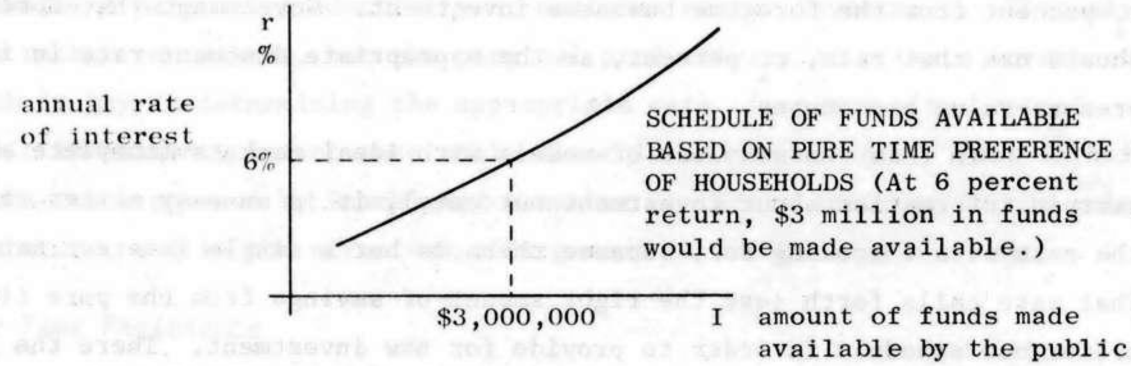
[4/] The argument developed here is largely based on the work of Baumol: "On the Social Rate of Discount," American Economic Review, 1968, p. 788, and "On the Discount Rate for Public Projects," The Analysis and Evaluation of Public Expenditures: The PPB System, Joint Economic Committee Print, GPO, 1969, p. 489. Also see Hirshleifer and Shapiro, "The Treatment of Risk and Uncertainty," same volume, p. 505. Additional commentary on Baumol's AER paper appears in American Economic Review, 1969, pp. 909-930. For more theoretical treatment see Sandmo and Dreze, "Discount Rates for Public Investment in Closed and Open Economies," Economica, 1971, p. 395, and Sandmo, "Discount Rates for Public Investment under Uncertainty," International Economic Review, 1972, p. 287.

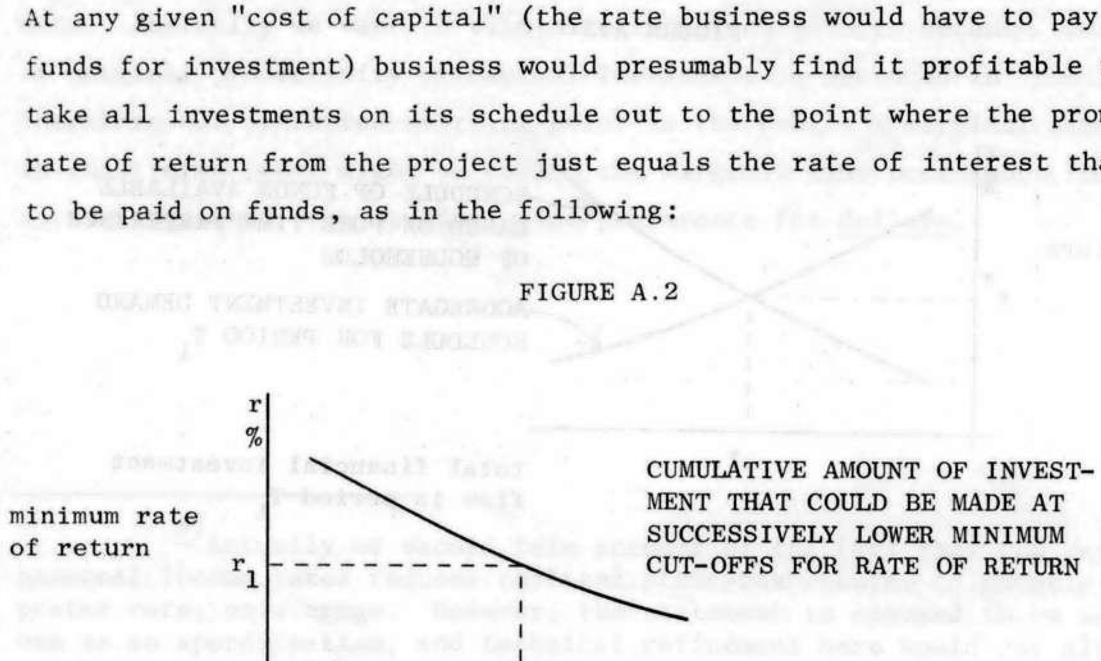
saying the public is willing to trade away some consumption today in favor of known increases in its consumption tomorrow (next year). In aggregate then, the amount of funds households are willing to lend to business and government gets larger as the rate of return becomes higher. The relationship can be graphed as follows:

FIGURE A.1



The second determinant of the interest rate is the "productivity of investment capital" schedule assumed to face the business sector as a technological fact of life. This model assumes that business has determined prospective rates of return on an agenda of possible, though untried investment projects and has catalogued these in decreasing order of rate of return. At any given "cost of capital" (the rate business would have to pay to acquire funds for investment) business would presumably find it profitable to undertake all investments on its schedule out to the point where the promised rate of return from the project just equals the rate of interest that needs to be paid on funds, as in the following:

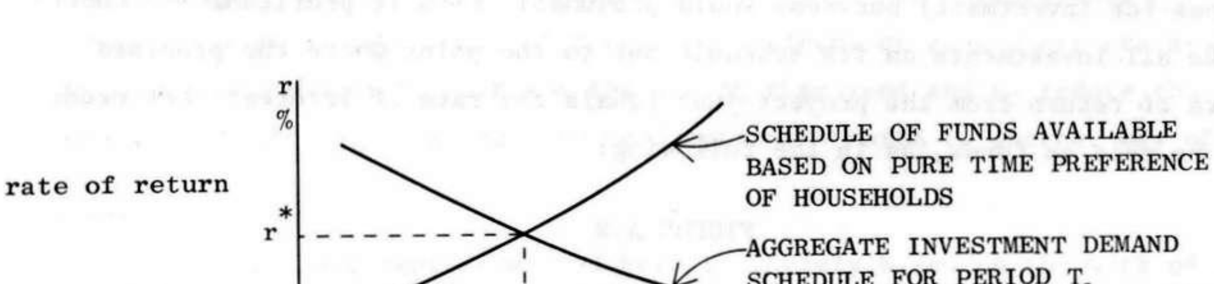
FIGURE A.2



This graph, which may also be termed an aggregate investment demand schedule, in theory gives us the opportunity cost we seek for use as the "optimal social rate of discount." Suppose the private sector is about to make the marginal investment that brings it to an aggregate I_1 dollars. If all resources are fully employed, which is a major assumption we make here, our decision as a government authority to invest in some particular undertaking will cause the marginal private sector decision not to be made. Under those conditions, society is in effect being asked to give up a return of r_1 percent from the foregone business investment. Government therefore should use that rate, r_1 percent, as the appropriate discount rate in its present-value procedures.

In this the simplest of models with ideal markets (complete and certain information about investment outcomes), it is an easy matter to find the rate we are looking for, because there is but a single interest rate. That rate calls forth just the right amount of savings from the pure time preference schedule in order to provide for new investment. There the market bringing together savings and investment clears at an interest rate, say r^*, which equals the marginal time preference of the public as well as the marginal return on private investment. In that sort of world all we would need do to determine the rate of social opportunity cost would be to observe the market interest rate.

FIGURE A.3



B. Risk, Corporate Income Tax, and Other Real Life "Impediments"

A slightly more realistic model emerges from attempts to accommodate some real life institutional features that, when introduced into the above simple model, act to "drive a wedge" between the public's marginal time preference rate and the business sector's marginal productivity of investment.

So, if we were to start with an estimate of the public's marginal time preference rate and were able to construct reasonable approximations of the impact of major institutional "wedges," then we would end up with an estimate of marginal productivity of business sector investment. That may seem rather roundabout, but the marginal productivity of private investment, which is key in determining the appropriate rate of opportunity cost of public investment, is not observable directly in the markets. That in brief is the track pursued by Baumol and a number of recent articles in the relevant literature, and is the structure of argument to be sketched out next.

- *Pure Time Preference*

Our starting point, then, is the premise that the public's pure time preference rate is fairly well reflected in the rate on safe, liquid financial assets, e.g., U.S. government securities.[5/] We'll take the 10-year bond rate as an appropriate basis for comparison (to give us a term more nearly like that of durable capital commitments) and adjust it to approximately eliminate any apparent "inflation premium" built into our observable market data. Basically we want to eliminate inflation effects because the rate of marginal productivity of capital investment is measured in "real" terms. Therefore our preferred starting point is the public's marginal time preference in real terms (what might be called the marginal time preference for goods as distinguished from marginal time preference for dollars).

[5/] Actually we should take account of the fact that the existence of personal income taxes reduces realized after-tax returns to roughly 4/5 of their pretax rate, on average. However, the statement is assumed to be a reasonable one as an approximation, and technical refinement here would not alter our conclusions.

We can observe that long-term U.S. government bond yields ranged between 5 and 7 percent over the past five years. How much of that was inflation premium we cannot say with precision, but we think it reasonable to argue that most of the increase that occurred since 1966 was inflation premium. (See Figure A.4.) In consequence we think that, in round numbers, 4 percent to 5 percent per annum reasonably brackets the public's "real" marginal rate of time preference. For purposes of computation we'll start with 4 1/2 percent.

● *Risk*

By that token the public would require a 4 1/2 percent rate of return in order to give up the use of its money to a business corporation with certainty of full return. But in real life an investment payoff isn't certain. There's always the risk a project will return less than expected or even that a company will go bankrupt. Therefore the public requires a "risk primium" over and above its pure time preference rate as inducement to provide equity funds to individual firms in the business sector.

While our model of the economy neatly posits a single risk premium, the task of measuring that risk premium by observation of real life market rates is something else again. Differences between corporate bond rates and government bond rates provide some information though other market characteristics must also be taken into consideration. A reasonable guess would seem to place the risk premium in the 2 to 3 percent per annum range. We shall take 2 1/2 percent for computation purposes.

Next we proceed to add this so-determined risk premium rate to the estimated marginal rate of time preference. Then 7 percent (= 4 1/2 percent + 2 1/2 percent) would be the necessary minimum return on investment an individual corporation could set out to earn and still expect to obtain funds from the public -- and that would imply a marginal productivity of investment of about 7 percent.

The notion of "risk" as we've used it here has a fairly simple statistical probability model underlying it. It says that even if a firm's decisions, accurately calculated, lead it to expect a rate of return of, say, 7 percent, that figure is somehow only the most probable or average return it can expect. What the firm really faces is a distribution of various outcomes as in Figure A.5 where the height of the curve at a particular value for rate of return is proportional to the probability of getting that rate of return.

FIGURE A.4
Illustrative Market Rate Data
and a Possible Interpretation of "Inflation Premium"

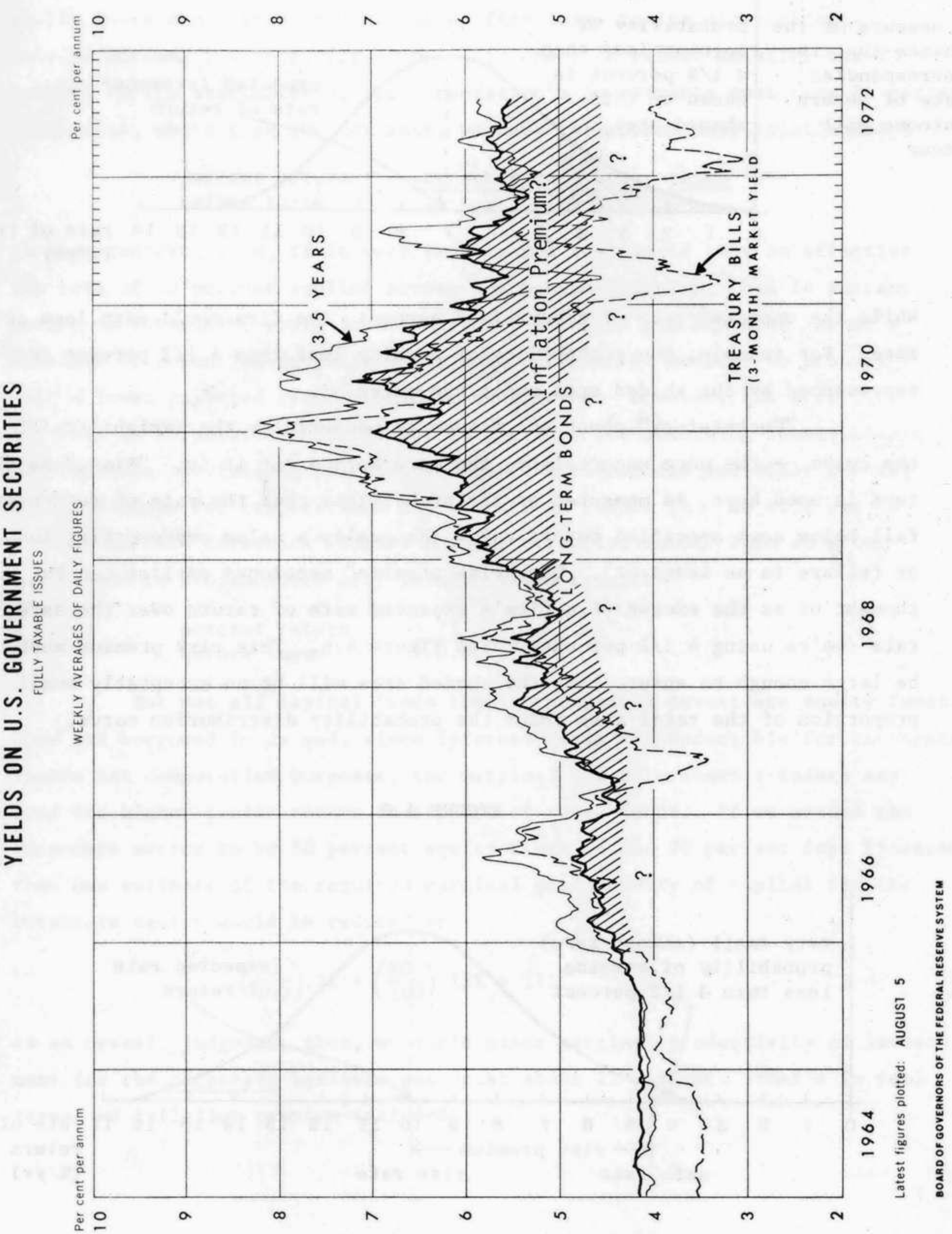
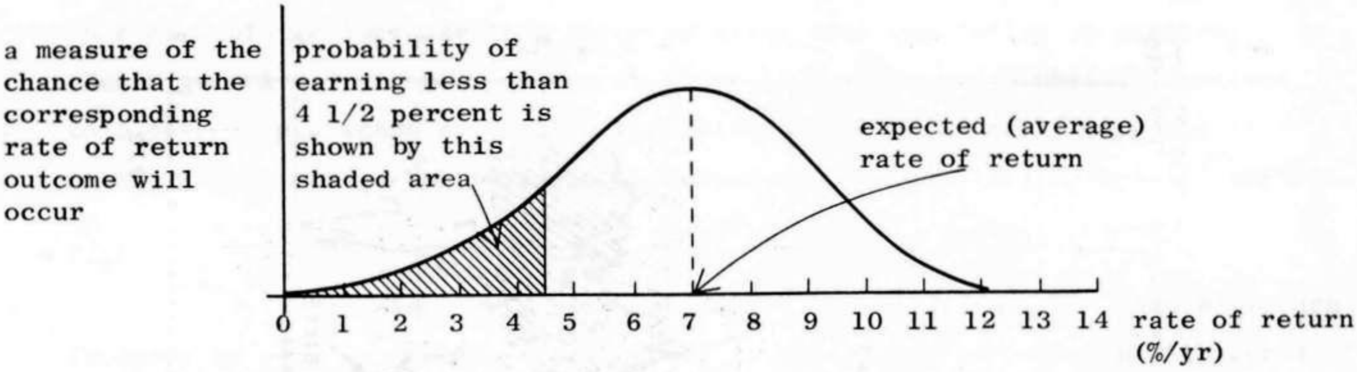


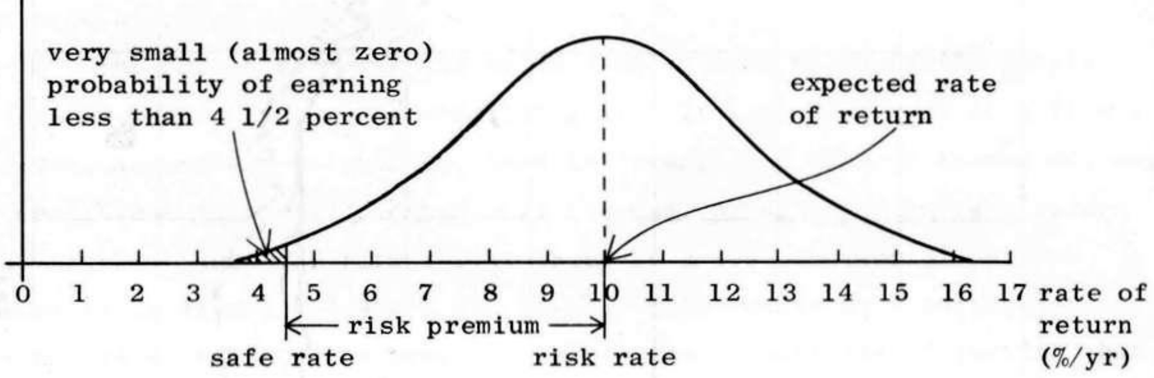
FIGURE A.5



While the expected rate of return is 7 percent, the firm could earn less or more. For example, the probability of earning less than 4 1/2 percent is represented by the shaded area under the curve.

"Uncertainty" about the future is measured by the variability of the curve -- the more uncertainty, the more spread out it is. "Risk," as the term is used here, is measured by the probability that the rate of return will fall below some specified cutoff value (generally a value representing loss or failure to an investor). The "risk premium" mentioned earlier can be thought of as the excess of a firm's expected rate of return over the safe rate (we're using 4 1/2 percent). See Figure A.6. This risk premium must be large enough to ensure that the shaded area will be an acceptably small proportion of the total area under the probability distribution curve.

FIGURE A.6



• *Corporate Income Tax*

The effect of a corporate income tax when introduced into the simple model is to require any corporation to earn a higher marginal return on its investments than it must yield after taxes to its stockholders. At a 50 percent tax rate, for example, in order to return an after-tax 7 percent to its stockholders, the corporation's investments must earn 14 percent. In general, where t is the tax rate, we have the approximate relationship:

$$\text{percent return before taxes} = \frac{\text{percent return after taxes}}{1-t}$$

In that context, then, if it were true in the real world that an effective tax rate of 50 percent applied across the business sector, then 14 percent return on investment would indeed mark the point in the business sector's schedule of potential projects where a cutoff must be made -- no project with a lower expected return would be undertaken. However, the effective tax rate on corporate income is not 50 percent, but something lower. Rapid amortization provisions, percentage depletion allowances and other special tax provisions for corporations all operate to reduce it. We will use 40 percent as our "horseback" figure for the effective rate. Then an after-tax return to stockholders of 7 percent would require

$$\text{percent return before taxes} = \frac{7\%}{1-0.4} = 12\%$$

But not all capital funds that corporations invest are equity funds. Some are borrowed funds and, since interest costs are deductible for corporate income tax computation purposes, the marginal tax rate doesn't induce any need for higher pretax return in support of those funds. If we assume the corporate sector to be 80 percent equity financed and 20 percent debt financed then our estimate of the required marginal productivity of capital for the corporate sector would be reduced to

$$\left(\frac{20}{100}\right) 7\% + \left(\frac{80}{100}\right) 12\% = 11\%$$

As an overall judgment, then, we would place marginal productivity of investment for the <u>corporate</u> business sector at about 11 percent. That's in real terms, no inflation premium included.

• *Sector Effects*

The actual opportunity cost of a particular government investment decision depends on where specifically resources are drawn from. Resources commanded by government at full employment could, in principle, be drawn from corporate producers, noncorporate producers, or from the consumer sector.[6/] Suppose all the resources are drawn from the corporate part of the business sector. Then the 11 percent figure for the marginal return on corporate capital suggested in the preceding section would seem a reasonable measure of the opportunity cost of government sector investment. But suppose all of the resources were drawn instead from the noncorporate part of the business sector. With no distorting effect from the corporate income tax to interfere, marginal investment return may be taken to be close to the 7 percent "risk rate" estimate made earlier. Finally, to the extent that government sector decisions to invest divert resources from consumption, then the appropriate opportunity cost would seem to be the 4 1/2 percent "safe rate" -- roughly the public's time preference rate for goods.

Each specific government sector investment could in fact affect sector resource use differently -- particular industries with marginal returns either very much higher or very much lower than the corporate aggregate figure could be differentially hit. Regional differences in impact, too, would occur. (That exercise really brings us into a more complicated model than we have been describing.) As a consequence, some authors point out, the "true" opportunity cost probably varies with each government investment decision. Furthermore, if we abandon our full-employment premise, the true rate of opportunity cost would tend to vary with the state of the economy -- with the rate effectively lower during times of greater slack and underutilized labor and plant.

We lean toward using an economy-wide rate for opportunity cost. The suggested refinement of tracing subsector, industry, or regional impact seems beyond any foreseeable technical competence and may perhaps even be conceptually in error. An economy-wide rate would presumably be some weighted average of the sector marginal rates developed above. Without wishing to

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[6/] Where the resources come from will depend to some extent on how government finances its expenditures; financing can be done either by taxing or by borrowing.

defend rigorously the weights chosen (which are used by Baumol) we generate the following average rate for the record:

70/100	corporate sector	@	11%
20/100	noncorporate sector	@	7%
10/100	consumption	@	4.5%
	overall economy average	=	9.5%

If we're fully confident in our estimation procedure and in the logic of using a single economy-wide rate for opportunity cost, then the 9.5 percent figure we've just computed would appear to be the rate we would choose for calculating present values. That ought to be adjusted upward, however, by our estimate of inflation premium at the time of planning, on the grounds that our projected cash flows are nominal values and so have an inflation premium built into them. Given the usual relationships between trends in market rates and the business cycle, the net effect of this procedure ought to lead us to be relatively more liberal in our investment in times of "slack" in the economy -- which seems to make sense.

Of course, we aren't all that confident in any point estimate we are able to generate. So our recommended present-value analysis procedure involves no single discount rate, but rather examination of "what goes on" with present values over a range of discount rates. Given the preceding rate discussion, to be sure, our major attention will focus on what happens over the 7 percent to 11 percent range.

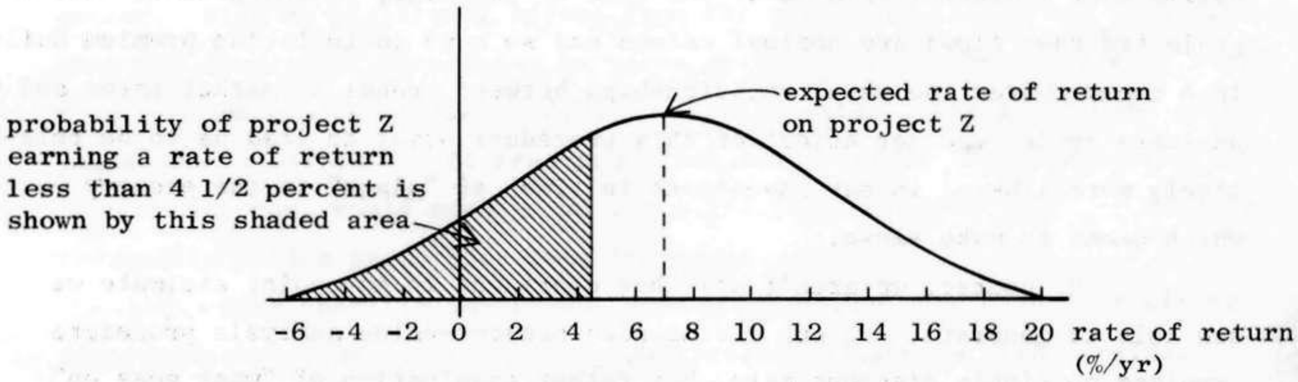
C. Risk and the Issue of "Risk Adjustment"

One last set of matters we wish to take up is the proper treatment of risk. We want to deal with a couple of contentions frequently appearing in the literature on the matter of adjusting the data or the rates for "risk." One contention is that government investment decisions ought to use a safe rate (or the Treasury borrowing rate) for discounting purposes. The other is that prospective investments ought to be sorted into "risk classes" with higher discount rates used for riskier classes of investment.

• *Social Risk, Private Risk*

A case for government use of a safe (risk-free) rate in present-value calculations argues that because of the very large number of projects undertaken by government, the "risk pooling" effect makes achievement of the aggregate expected value of all its returns taken together virtually certain, or "riskless." The argument is made that the social risk in any particular government investment undertaking is less than the private risk to an individual firm undertaking a comparable project. The model that generates this conclusion can be illustrated fairly simply. It assumes that uncertainty about any individual investment project confronts an investor with a distribution of possible rate-of-return outcomes. For example:

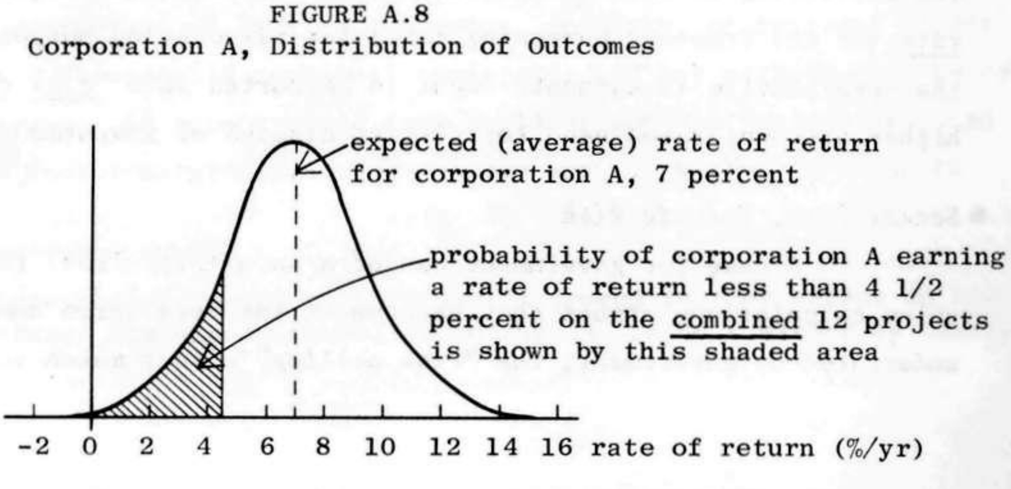
FIGURE A.7
Project Z, Distribution of Outcomes



Project Z is expected to earn 7 percent, but it could do very much better or very much worse.

Now consider corporation A, organized to undertake a dozen different and independent projects, all of which have an identical distribution of outcomes to that of Project Z (i.e., for each project the mean or expected value of outcomes is the same and the spread of outcomes around the mean is the same). The statistical consequence of combining the several projects in one firm and looking at the aggregate outcome is to reduce the spread of combined outcomes leaving the mean value unchanged at 7 percent as follows:

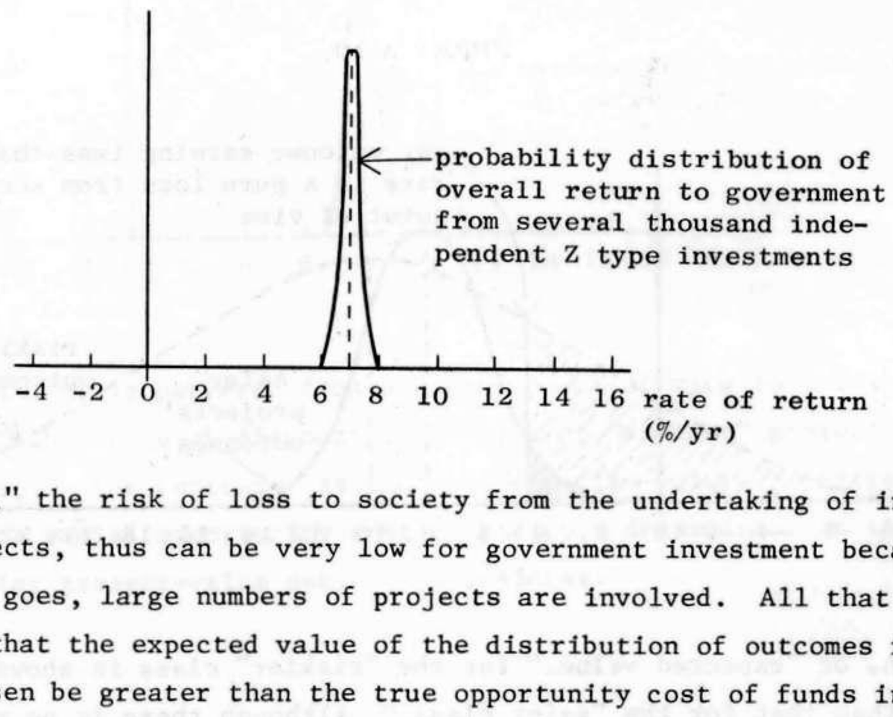
FIGURE A.8
Corporation A, Distribution of Outcomes



Corporation A, by pooling risks over several projects has cut the risk to the investor from what it would have been in any of the projects taken individually. "Risk" here is defined by implication as the probability of earning less than some specified rate of return. For example, the shaded areas in the above diagrams represent outcomes earning less than 4.5 percent; say that funds "cost" corporation A that much and so to earn less would be to go broke as a corporation.

The statistical "law of large numbers" guarantees that if government were to undertake many thousands of independent projects, each with a spread of rate of return outcomes identical to that of project Z, the combined outcome with virtual certainty would be very close to the expected value of 7 percent.

FIGURE A.9



"Social risk," the risk of loss to society from the undertaking of individually "risky" projects, thus can be very low for government investment because, so the argument goes, large numbers of projects are involved. All that is required is that the expected value of the distribution of outcomes for those projects chosen be greater than the true opportunity cost of funds involved. A critical point here, however, is that while the "private risk" of an individual firm's decisions may be appreciable (as illustrated in Figure A.8), the "social risk" of private decisions about Z type projects within the business sector as a whole is as low as that for government decisions -- again because from society's point of view very large numbers of projects are involved. Thus there are no grounds from a "social risk" standpoint for treating government decisions on a different risk footing from that applied to individual private firm decisions.

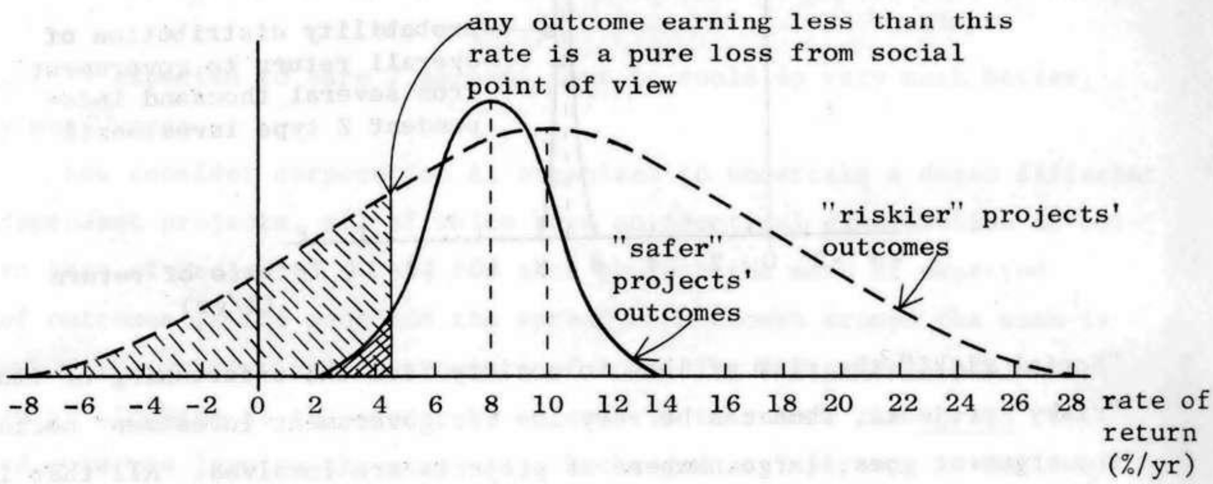
And the point was established earlier that the proper "footing" to be used in social decisions is the opportunity cost to society of those decisions -- and, in choosing interest rate measures, that clearly is not the riskless, safe rate.

● *Risk Classes and Differing Project Riskiness*

Should investment projects be classified according to inherent "riskiness" and higher discount rates applied in computing present value for "riskier" classes of projects? Our answer here is an unequivocal "no." The function of the discount rate, in effect, is to measure the <u>opportunity cost</u> of an investment which is neither affected nor altered by the particular properties of the individual investment project under consideration.

Presumably a "riskier" class of projects is one for which the chance of falling below the "loss point" in terms of earned return is larger than that for some standard set of projects, as in the following diagram:

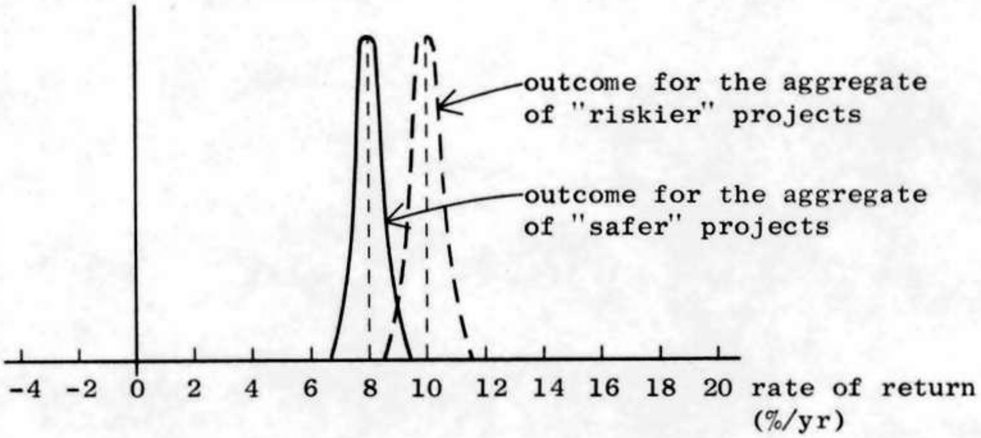
FIGURE A.10



The mean, or "expected value," for the "riskier" class is shown here to be larger than that for the "safer class." Although there is no reason to assume this relationship, in general it's likely to be true for projects seriously considered by businessmen. The point is that a class of projects may be "riskier" (and necessarily more uncertain in the sense of having a greater range or spread of individual outcomes) yet, from a social point of view, when a large number of such projects is undertaken, uncertainty about the aggregate outcome drops toward zero and with it risk drops as well. The "spread" of expected outcomes collapses around the mean value as a large number of projects is pooled. To fit that model of "riskiness," the projects must be independent and the distribution of outcomes as well as the expected mean value must be accurately known.

For the case illustrated on the above diagram, once the "pooling" effect of large numbers has whittled away the uncertainty (spread) of outcome, society would be better off from investment in the "riskier" class of projects because they will return 10 percent per year with virtual certainty whereas the "safer" projects will return only 8 percent per year as in the following:

FIGURE A.11



Thus it could prove perverse from a social point of view to insist on application of a higher rate of discount to classes of "riskier" projects. What is required, on the contrary, is that the expected values be correctly assessed and that the social opportunity cost be the sole determinant of the interest rate used for present-value decision procedures.