Notes on Competition and the Distribution of Product

In macro the aggregate production function is typically used and it is assumed that the competitive equilibrium aggregates will satisfy it. Further, it is assumed that the rental price of a factor of production is equal to the partial derivative of the aggregate production function. The purpose of these notes is to provide some justifications for these assumptions.

Proposition: If factor markets are competitive, the competitive equilibrium output is the maximum output F(K,N). Further, factor payments exhaust total product; that is

$$Y' = r\,K + w\,N$$

where Y′ is the competitive equilibrium aggregate output, r is the equilibrium rental price of capital and w is the equilibrium rental price of labor.

Proof: Let $z' = \{z'_{kn}\}$ be a competitive equilibrium production plan. For all plant technologies,

$$(1) \qquad f_{kn} - r\,k - w\,n \leq 0.$$

If this were not the case there would be a profit opportunity, which is inconsistent with equilibrium. If the inequality is strict for some (k,n) type plant, z'_{kn} must be zero to be consistent with profit maximization (zero profit is better than a negative profit). Thus, for all (k,n)

$$(2) \qquad z'_{kn}\,(f_{kn} - r\,k - w\,n) = 0.$$

Summing (3) over all (k,n) gives

$$(3) \qquad Y' - r\,K - w\,N = 0.$$

This establishes the second part of the proposition, namely that total product is exhausted by payments to the factors of production.

Let $z = \{z_{kn}\}$ be a plan that *maximizes* output given K and N. Multiply (1) by z_{kn} and summing over all (k,n) gives

(4) $$Y - r\,K - w\,N \leq 0,$$

where $Y = F(K,N)$ is the maximal possible output given K and N. From (3) and (4),

(5) $$F(K,N) = Y \leq r\,K + w\,N = Y'.$$

Therefore, competitive equilibrium output Y' equals maximal feasible $F(K,N)$ as it is not possible to produce more than the maximum possible output.

Comment: The result that the competitive equilibrium output is maximal is much more general. There can be any number of inputs. There can be any number of output goods where maximal then means that the output of no good can be increased without a reduction in the output of some other good. In the jargon of economics, competition leads to efficient production. In general, absent externalities, competitive equilibria are efficient or Pareto optimal; that is, no other allocation makes all at least as well off and someone better off.

Differentiable aggregate production functions

Here the production function is assumed differentiable. Given there are constant returns to scale,

$$\lambda\,Y = F(\lambda\,K, \lambda\,N).$$

Differentiating both sides with respect to λ,

$$Y = K\frac{\partial F(\lambda K, \lambda N)}{\partial K} + N\frac{\partial F(\lambda K, \lambda N)}{\partial N}.$$

Setting $\lambda = 1$ and using the fact that profit maximization requires that marginal product be equal to rental prices,

$$Y = K\,r + N\,w.$$

This is another way to prove that, with constant returns to scale, payments to factors exhaust product.