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The Social Discount Rate

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ABSTRACT

In welfare theory it is standard to pick the consumption stream that maximizes the welfare of the representative agent. We argue against this position, and show that a benevolent social planner will generally place a greater weight on future consumption than does the representative agent.

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Occasionally there have been those who have voiced discomfort with this prescription, especially those seeking to orient public policy more towards the future. Ramsey [1928], while acknowledging that private agents discount the future, argued that it was "ethically indefensible" for the government to do so. Pigou [1952], Allais [1947], and Solow [1974] have expressed similar views. For the most part, however, these views have either been ignored or dismissed as paternalistic by the majority of economists. Arrow and Kurz [1970] capture the feelings of the profession when they write:

> It is hard to see why the revealed preference of individuals should be disregarded in the realm of time, where it is accepted, broadly speaking, in evaluating current commodity flows. (p. 12)

In this paper we argue against current practice, and in favor of the more future-oriented vision. To immunize ourselves from charges of paternalism, we retain a standard utilitarian approach to evaluating social welfare. We show that social welfare corresponds to the agent's utility only under very special conditions. Instead, a benevolent social planner will generally place a greater weight on future consumption.

The main premise upon which we construct our argument is the observation that agents discount the past, as well as the future. If individuals discount both the past and the future, preferences change over time. The present period weighs more heavily in current decisions than in the decisions of any prior or subsequent period. This means that agents will regret their past consumption choices. We label this form of regret *retrospective time inconsistency*, since it involves disagreement over choices that were made in the past, as opposed to the standard form of time inconsistency which relates to future plans.

While our approach is not in keeping with current practice, it is not without precedent. In its essentials, our viewpoint was anticipated by Pigou [1952] in his classic work on welfare economics:

> The existence of preference for the present over equally certain future pleasures does not imply that any economic dissatisfaction would be suffered if future pleasures were substituted at full value for present ones. The non-satisfaction this year of a man's preference to consume this year rather than next year is balanced by

> the satisfaction of his preference next year to consume next year rather than this year. (p. 25)

Time varying tastes pose a challenge for welfare economics. We adopt the vision, familiar from the literature on time inconsistent preferences, that the agent at each point in time is a separate individual. We then follow Pigou [1952] and Schelling [1984] in arguing that all of these temporally distinct selves should be of concern to the social planner. The identification of an optimal policy turns on how the planner should weigh the perspectives of various periods. Any disagreement among the temporal selves will have to be resolved by some form of compromise. We apply the logic of static welfare theory to the dynamic case and formalize the compromise using a Bergsonian social welfare function.

In this framework the currently standard practice of maximizing the utility of a representative agent represents a form of dictatorship in which the perspective of the current self is given full weight in the social welfare function. We label this approach the *dictatorship of the present*. Our view of this dictatorship is that it has all the same ethical pros and cons as other forms of dictatorship. It certainly helps resolve questions in a quick and easy manner, but it may do so at the cost of doing tremendous and predictable harm to someone else in society, in this case the future selves. We see no normative reason to favor the present at the expense of the future. To paraphrase Arrow and Kurz: it is hard to see why dictatorship should be embraced in the realm of time, where it is rejected, broadly speaking, in evaluating current allocations.

Once we reject dictatorship, it follows immediately that the socially optimal plan places greater weight on future utility than does the representative agent. Any social welfare function that places weight on future perspectives, also places weight on perspectives that rank concerns of the future above those of the present. The optimal social discount rate is therefore lower than that of the representative agent. By extension, competitive equilibrium is myopic.

In section 2, we discuss the arguments for discounting the past and the future, and make the case for retrospective time inconsistency. Section 3 presents our criterion for social optimality. We characterize the social discount rate in terms of the agent's prospective and retrospective discount rates and the properties of the social welfare function. In general, the plan-

ner places greater weight on future consumption than does the agent. This bias toward the future is decreasing in the weight that the agent places on the past. For many parameterizations the planner actually places greater weight on the near future than on the present.

In section 4, we discuss a variety of issues raised by our approach. These include the policy implications and the feasibility of applying the theory in practice. We discuss how discounting the past sheds light on rational addiction and procrastination. We close this section by discussing the relationship to the literature.

In section 5, we apply our approach to the classical growth model of Ramsey. We provide a complete analysis for the special case of log utility with and without commitment by the planner. In both cases, there exist social welfare functions that support any steady state between the modified golden rule and the golden rule level of the capital stock. In the absence of commitment, we find a simple closed form representation of the social discount rate. As one important special case, if the private agent's forward-looking discount factor, backward-looking discount factor, and the social planner's discount factor are all equal, then the social discount rate is one third the private discount rate.

2 Disagreement in the Exponential Model

2.1 Retrospective Time Inconsistency

Consider an agent choosing how to allocate consumption and resources across the two periods. The agent is endowed with a certain amount of a raw material and a technology for transforming this material into period one and period two consumption. We assume that the production possibility set is convex. To keep things simple we suppose that the agent chooses consumption in period one to maximize a time additive utility function:

$$U_1(C_1, C_2) = u(C_1) + \beta u(C_2),$$

where C_t represents consumption in period $t = \{1, 2\}$, $u(\cdot)$ is the felicity from consumption within a period, and $\beta \in [0, 1]$ is the discount factor. To avoid later confusion, we will reserve the term "utility" for the welfare of the

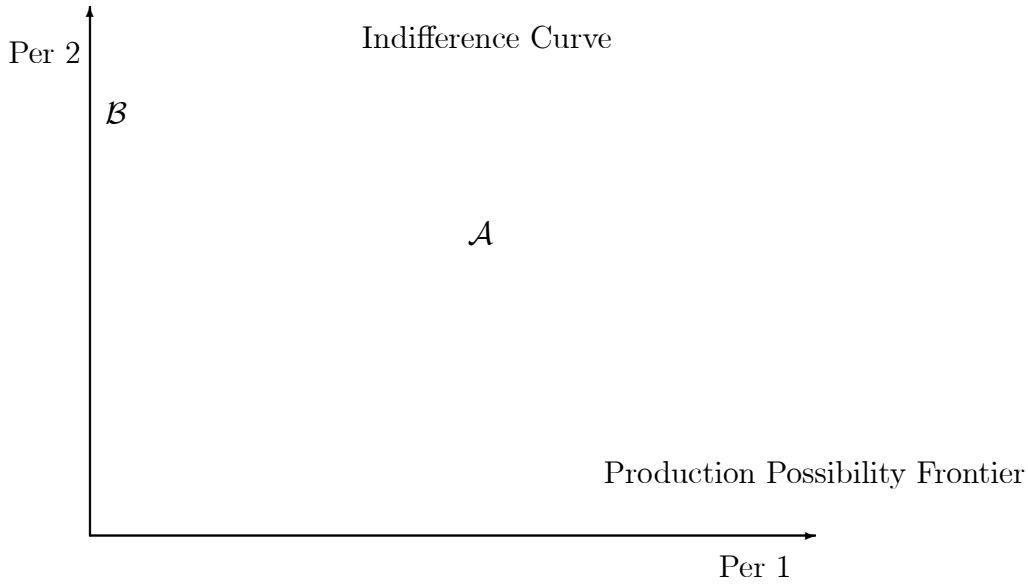

Figure 1: The Fisher Diagram

agent in period t, U_t, and the term "felicity" for the contribution to utility of consumption in period t in the absence of discounting, $u\left(C_t\right)$.

The solution to this problem is straightforward and can be found in many undergraduate textbooks. The optimal policy is to choose the point on the production possibility frontier associated with the highest indifference curve of the utility function U_1. This solution is represented by point $\mathcal{A}$ in the Fisher diagram (Figure 1).

What is not generally appreciated is that this solution only represents the optimal choice from the perspective of the first period, and does not generally maximize second period utility. To see this, let's proceed as is common in dynamic models and treat the past as sunk both for choice and for welfare. In this case second period utility is simply

$$U_2(C_1, C_2) = u(C_2). \tag{1}$$

Given this utility function, the second period indifference curves are horizon-

tal and the optimal choice of consumption from the second period perspective is point $\mathcal{B}$ in Figure 1. The agent's preference over consumption sequences changes over time.

The reason that tastes change in this example is that with this specification of utility the agent places zero weight on period 1 consumption in period 2. One might argue that this is because (1) is really only a partial representation of the individual's welfare. It represents that component of the agent's decision that is still "active" at the time of the period t decision, and ignores the past only because the past is sunk. As Deaton [1992] has emphasized, a complete specification of preferences must rank the entire consumption stream.

Even if the past enters period 2 welfare, the conditions under which tastes remain unchanged over time are very stringent. Consider the following specification of preferences in which the agent in period 2 cares about both period one and period two consumption

$$U_2(C_1, C_2) = \delta u(C_1) + u(C_2), \tag{2}$$

Here δ represents the "discount" factor applied to past felicity. We place the word discount in quotation marks because we want to allow for the possibility that δ is greater than one. Note (2) reduces to (1) when we restrict the domain of U_2 to current and future consumption.

From (2) it is easy to see that a necessary and sufficient condition for tastes to be unchanging over time, is that $\delta = \beta^{-1}$. In this case, U_2 is simply a monotonic transformations of U_1.[1]

If $\delta \neq \beta^{-1}$ preferences over consumption sequences change over time. A natural concomitant of $\delta \neq \beta^{-1}$ will be regret concerning past consumption choices. In the example above, the consumer regrets in period 2 the choice of $\mathcal{A}$ in period 1. In period 2, the consumer wishes that the choice had been closer to point $\mathcal{B}$. We see this as a form of time inconsistency that operates in a backward-looking rather than in a forward-looking manner and refer to the phenomenon as retrospective time inconsistency in order to distinguish it from the more familiar prospective time inconsistency.

The argument easily extends to a multi-period setting. Consider an infinite horizon, $t \in \{1, 2, \ldots\}$. We can represent the agent's utility over con-

[1]Deaton [1992] and Mas-Collel, Whinston and Green [1995] implicitly assume $\delta = \beta^{-1}$ when they assume that preferences over consumption sequences are time invariant.

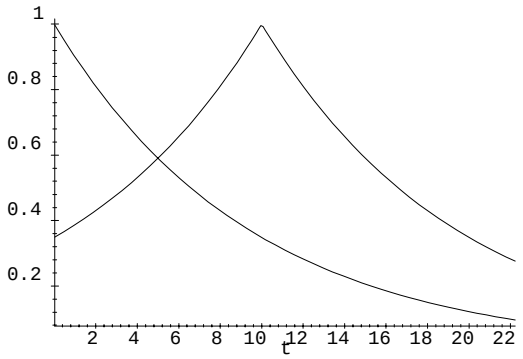

Figure 2:

sumption sequences by a function that discounts both the past and the future:

$$U_t(C_0, C_1, \ldots) = \sum_{m=1}^{t} \delta^m u(c_{t-m}) + u(c_t) + \sum_{n=1}^{\infty} \beta^n u(c_{t+n}), \tag{3}$$

Figure 2 illustrates both the discount factor of an individual in period zero and an individual in period ten under the assumption that $\delta = \beta = .9$. In period zero the agent places exponentially declining weights on future felicity. In particular, period zero felicity receives greater weight than period ten felicity. In period ten, the agent place exponentially declining weights on both past and future felicity, and the relative weights on period zero and period ten are reversed.

We can see from (3) that retrospective time inconsistency is perfectly consistent with exponential discounting and time consistency in the standard sense.[2] Given any dates $s' > s > t$, the marginal rate of substitution between consumption at s and s' is independent of the period of evaluation t:

$$\frac{\beta^{s'-s} u'(C_{s'})}{u'(C_s)}.$$

This property implies that optimal plans made to maximize (3) are time

[2]Strotz [1956] showed that exponential discounting is necessary and sufficient for time consistency.

consistent: a plan chosen to maximize utility in period t will remain optimal in all subsequent periods.

We also see from (3) that time consistency does not imply a preference ordering over sequences of consumption that is time invariant. Time consistency only requires that preferences over future consumption are unchanging. It does not rule out disagreements over the past. Retrospective time consistency requires $\delta = \beta^{-1}$.

The question whether choices today maximize utility from the perspective of all future time periods therefore boils down to how much agents care about the past. Is it reasonable that $\delta = \beta^{-1}$ or do preferences change over time? In the next section, we review the case for discounting the future and argue that the case for discounting the past is at least as persuasive.

2.2 The Argument for Discounting: Future and Past

Ever since Böhm-Bawerk [1959] first proposed discounting as one of the prime reasons for positive interest rates, there has been near unanimity among economists that individuals discount future felicity relative to present felicity.[3] The main argument in favor of discounting is empirical. It appears to be the revealed preference of individuals that current pleasure is preferred to future pleasure, and future pain to current pain. Introspection supports this view, as do market interest rates in excess of the growth rate of population and productivity, and so does the available experimental evidence from economics and psychology.[4]

Given that individuals discount the future, invariance requires that agents weigh the past more heavily than they do the present. In other words, it requires that a thirty year old cares more about an apple that was consumed at age three than about an apple to be consumed today. Moreover, the importance of that apple at age three relative to an apple today grows at an exponential rate as the consumer ages.

This implication of invariance is difficult to accept. Rather than viewing past consumption as progressively more important as the consumer ages, it is more plausible that agents discount past consumption. When asked

[3]For a rare statement of dissent see Becker and Stigler [1977].

[4]Although supportive of discounting, the experimental evidence does not necessarily support exponential disounting. See Ainslie [1992].

most people would prefer having completed an onerous task last week over having to perform one this evening; they would prefer a pleasurable meal this evening to one ten years ago; and they would prefer to be leaving on vacation to returning to work when their vacation is over.[5] All of these choices reflect discounting of the past.[6] Whenever one thinks, "I'm glad it is finally three o'clock and that experience is over," it reflects discounting of the past. The disagreeable experience is less painful when it is in the past than when it is being experienced in the present.

Beyond revealed preference, the strongest theoretical argument in favor of discounting, involves what Böhm-Bawerk referred to as the "brevity and uncertainty of human life." According to this view, people discount future felicity because they may not be around to enjoy it. An individual weighs pleasures at date t by the conditional probability of living until date t.

There is a parallel between mortality and forgetfulness. The past dies when we forget.[7] Do we value today meals eaten ten years ago, if we cannot even remember what we ate? How is our current utility reduced by past pains that we cannot even remember? Imperfect memory justifies discounting the past in much the same way that mortality justifies discounting the future, and just as mortality suggests that we discount the far off future more than the near future, imperfect memory suggests that we discount the far off past more than the recent past.

Finally, if preferences change over time and across individuals, does the weight on the past change in a similar manner? If, as commonly believed, the rich and educated are more forward looking, do they place relatively less weight on the past as invariance would require? If discounting is endogenous,

[5]This last statement assumes that the vacation is the pleasurable experience. For some families this may not be true. In this case the inequality is reversed, but the point is the same.

[6]There will, of course, be exceptions. There may be that particularly pleasurable occurrence which yields memories for years to come, memories so precious that one is glad that the experience was in the past so that the memories could be enjoyed over an extended period of time. Similar exceptions, however, apply to discounting the future. One may, for example, want to delay a particularly pleasurable event in order to savor the feelings of anticipation that it engenders (Loewenstein [1987]). The existence of a few exceptions, however, does not alter the general rule.

[7]The experimental evidence in psychology indicates that exponential decay of recall probabilities fits the data rather well, although a power function may perform better (See Crovitz and Schiffman [1974]).

as argued by Becker and Mulligan [1997], is not the weight on the past also endogenous? After all we spend considerable resources keeping the past alive, through history, stories, photographs, and diaries. If the weight on the past is endogenous, wouldn't it be a remarkable quirk of fate of this weight were exactly equal to the inverse of the discount factor?

We conclude that in almost any reasonable formulation, future selves weight current consumption less heavily than does the current self. The implication is that tastes change over time. We now consider the welfare implications of this observation.

3 Dynamic Welfare Theory

As with most economic theory, the roots of the dynamic case are to be found in a close examination of the static case. In static welfare theory we begin with a set of individuals with preferences defined on an appropriate commodity space. We add a social planner who has available a set of policy tools, each of which is identified with a particular consumption bundle for each individual in the economy. The social planner is then assumed to pick a policy that satisfies some set of ethical desiderata. The most universally accepted of these is Pareto optimality. Beyond Pareto optimality one might look to load on other properties directly, as in cooperative bargaining theory, or implicitly, as when maximizing some weighted social welfare function.

The case of a dynamic representative agent is no different. All that is needed is to identify the set of policy choices available to the social planner, the relevant set of individuals, their payoffs, and the nature of the social welfare criterion.

Identifying the set of policy choices is relatively straightforward. This set is usually given as part of the data of the problem or as the equilibrium outcome of some model. The set of individuals, as we have seen, is not a trivial matter. Even in representative agent models with exponential discounting, tastes may change over time. Whose preferences should the planner maximize? Instead of constructing a theory around a fixed answer to this question, we adopt a framework that is flexible enough that it allows for the possibility that the perspective of each point in time may affect welfare. We take the set of individuals at date t to be the set of temporally distinct selves dated $s \geq t$. We exclude the perspectives of past selves because there is no way for

the social planner to affect what is past.[8] Given this collection of individuals, the payoff function for the date t self is given by the U_t in equation (2).

This leaves only the determination of the welfare criterion. To ensure Pareto optimality at a minimum, we introduce a Bergsonian social welfare function. To simplify matters further we will adopt a linear (weighted utilitarian) specification.

Consider first the two period example with $\delta = 0$. In this case the Bergsonian social welfare function takes the form

$$S_1 = aU_1 + (1 - a)U_2.$$

Varying the choice of a from zero to one traces the production possibility frontier from $\mathcal{B}$ to $\mathcal{A}$. All of these points are Pareto optimal. Any movement from any one of them either violates the constraint or reduces welfare from the perspective of one of the periods.

The standard practice of maximizing the utility of representative agent is equivalent to setting $a = 1$. In this case, the period one self receives all of the weight in the social welfare function. This is a form of dictatorship, which we term the dictatorship of the present. As in the static case, we find dictatorship inherently troubling. It should also be noted that the dictatorship of the present yields point $\mathcal{A}$ which lies at the extreme right of the set of Pareto optima. All other choices of a produce outcomes that are more biased towards the future.

A preference for equal weights is implicit in Ramsey's [1928] contention that discounting the future was "ethically indefensible," and is also implied in the quotation from Pigou [1952] cited in the introduction.

In the infinite horizon case, the Bergsonian social welfare function becomes

$$S_t = \sum_{n=0}^{\infty} a_n U_{t+n}.$$

where a_n represents the weight on the representative individual's utility n periods hence. It remains to determine the weights a_n. Koopmans [1960] showed that some form of discounting is necessary with an infinite horizon

[8]The assumption that the social planner is concerned only with the present and future selves is not important for the qualitative results of the paper. Even if the social planner cares about the past selves, there is still a compromise to be made. All past selves value todays consumption over tomorrows, whereas all future selves value tomorrows over todays.

utility function is that function is to be sensitive to changes in utility at any given point in time. We therefore consider a specification in which the social planner discounts future utility by a factor $\alpha \in (0,1)$:

$$S_t = \sum_{n=0}^{\infty} \alpha^n U_{t+n}. \tag{4}$$

Note that the two cases described above arise as limits of (4).[9] The dictatorship of the present corresponds to $\alpha = 0$, whereas average utility maximization corresponds to $\alpha = 1$.

Proceeding with this formulation and substituting for U_t from (2) we arrive at:

$$S_t = \sum_{n=0}^{\infty} \alpha^n \left(\sum_{m=1}^{t+n} \delta^m u(c_{t+n-m}) + u(c_{t+n}) + \sum_{s=1}^{\infty} \beta^s u(c_{t+n+s}) \right) \tag{5}$$

As past selves and past consumption are sunk, maximizing S_t is equivalent to maximizing

$$\hat{S}_t = \sum_{n=0}^{\infty} \left[\left(\sum_{m=0}^{n} \alpha^m \beta^{n-m} + \sum_{m=1}^{\infty} \alpha^{n+m} \delta^m \right) u(c_{t+n}) \right] \tag{6}$$

Normalizing the weight on current felicity to one, it follows that the social planner discounts future consumption by the amount:

$$\begin{aligned} D_n &= \frac{\sum_{m=0}^{n} \alpha^m \beta^{n-m} + \sum_{m=1}^{\infty} \alpha^{n+m} \delta^m}{\sum_{m=0}^{\infty} \alpha^m \delta^m} \\ &= \frac{\alpha^{n+1} + \beta^{n+1}\alpha\delta - \beta^{n+1} - \alpha^{n+1}\delta\beta}{\alpha - \beta} \end{aligned} \tag{7}$$

D_n is the social discount factor. The remainder of this section is devoted to understanding the properties and implications of (7).

The standard practice of equating the social discount factor with the discount factor of the representative agent arises as the limit of (7). If $\delta = \beta^{-1}$, then preferences are time invariant and $D_n = \beta^n$. If $\alpha = 0$, we have the dictatorship of the present and again $D_n = \beta^n$.

[9] Other possibilities also exist. We could allow the weights to depend on the level of utility or on marginal utility.

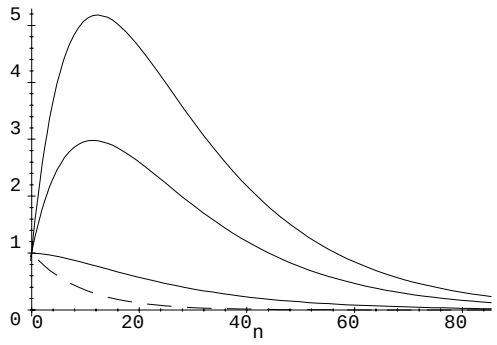

Figure 3: The Effect of δ on the Discount Factor.

More generally, if $\delta < \beta^{-1}$ and $\alpha > 0$, then $D_n > \beta^n$. The social planner places greater weight on future felicity than does the representative agent. This is the sense in which our analysis favors the future-oriented version over the standard analysis: if agents discount the past, then any social welfare function which places positive weight on the perspective of the future, is more future-oriented than the representative agent. By extension, competitive equilibrium is also myopic.

The comparative statics of (6) are straight forward. D_n is decreasing in δ. As δ increases the past becomes more important. Since earlier periods enter the past sooner, their weight increases, and D_n falls. Figure 3 depicts D_n for $\alpha = .95$, $\beta = .9$, and several choice of δ. The dashed line represents the exponential discount rate. Note that for several parameterizations the weight on the near future is actually greater than the weight on the present.

D_n is increasing in α. Figure 4 depicts D_n for $\beta = .9$, $\delta = .5$, and several values of α. The dashed line represents exponential discounting and corresponds to a choice of $\alpha = 0$. As α rises, future perspectives receive greater weight. The top line that increases monotonically and converges to a level just above 5 corresponds to $\alpha = 1$. Here the future felicity weighs more heavily than present felicity, because future felicity enters utility in periods before and after the consumption takes place, whereas current felicity only affects utility in the future.

Note that $\lim_{n\to\infty} D_{n+1}/D_n = \max\{\alpha, \beta\}$. The relative weights that the

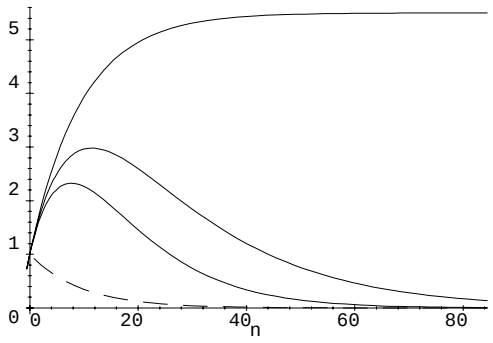

Figure 4: The Effect of α on the Social Discount Factor

social planner places on felicity in adjacent periods approaches the maximum of the planner's and the agent's discount factor. To understand this limit, note that there are two channels by which the social planner can care about felicity in the far future. First, the social planner may care about individuals int he near future who care about felicity in the far future. These individuals discount future felicity by β. Second, the social planner may care about individuals in the far future who care about felicity in the near future. The social planner discounts the utility of these individuals by α. Which channel is more important depends on which discount factor is larger.

One way to think about the social planner's problem is to consider a Rawlsian initial position. Which self shows up to represent the interests of the individual? When thinking about retirement, for example, does the individual take the perspective of an retired person receiving a pension or a young person saving? Our view is that all of these perspectives have some merit and that some form of compromise is natural.

4 Discussion

We begin with an number of observations concerning the practical implications of our approach and conclude with a discussion of the related literature.

4.1 Policy Implications

Our analysis has immediate implications for public policy: agents discount the future too much and therefore governments should promote future oriented policies. Theoretically, our analysis provides a foundation for what Malinvaud [1985] calls an Allais equilibrium (See also Allais [1947]). An Allais equilibrium is a situation in which individual preferences are retained for the choice between consumption goods at the same date, but not necessarily between consumption goods at different dates.

Relative to the market evaluation, our analysis tips the scales in favor of policies with short run costs and long run benefits. In the area of fiscal policy, it favors subsidies to capital accumulation. In the area of monetary policy, it favors low inflation. In the area of natural resource extraction, it favors conservation. In general, it favors investment and saving at the expense of consumption.

4.2 Procrastination

O'Donoghue and Rabin [1999a] model procrastination using time inconsistent preferences that exhibit a bias towards current felicity. They argue that procrastination occurs when a time inconsistent individual acts later than a time consistent individual would act, and that preponation occurs when a time inconsistent individual acts relatively sooner. They argue that procrastination is more likely when an agent incurs the costs of an action prior to the reward and that preponation is more likely when rewards precede costs.

Discounting past felicity provides an alternative characterization of procrastination and preponation. In this characterization, agents are time consistent, but they regret their past actions. Procrastination is associated with a wish that an action had been taken at an earlier date. Preponation is associated with a wish that an action had not been taken at an earlier date.

As in O'Donoghue and Rabin, procrastination will tend to occur when costs precede rewards. At the time that an act is postponed, these costs appear salient and the future rewards are discounted. In the future, the agent comes to regret this delay, because, if the action had been taken, the rewards which would now be received would be salient, and the costs which would have been incurred in the past would be discounted. In a similar manner, preponation is associated with rewards that precede costs.

4.3 Addiction

Becker and Stigler [1977] and Becker and Murphy [1988] model addiction as rational. Agents are time consistent and have stable felicity functions. They consciously choose to become addicts, at each point in time weighing the present value of costs and benefits of consuming the addictive substance.

There is a belief that, because addiction is the result of a rational choice in this framework, addicts are "happy" or at least better off than they would have been if the option to become addicted had never existed. This vision of the "happy addict" has caused some economists to question the usefulness of the framework. (O'Donoghue and Rabin [1999b])

This welfare calculation, however, assumes that agents are retrospectively time consistent. Harmful addiction is a case in which rewards precede costs. The addict decides that the benefits to current consumption of the addictive substance outweigh the future costs of increased addiction. As in the case of procrastination, agents who discount the past may come to regret their past behavior as these future costs become salient. Rational choice may maximize utility from the current perspective, but this does not imply that future utility is also maximized. Rational addicts need not be happy addicts.

4.4 Time Consistency of Social Plans

It would be hoped that the social planner's optimal policy would be time consistent. Unfortunately this is not generally the case. The discount rate implied by (7) is not generally exponential, so the planner's tastes change over time. The solution to the planner's dynamic optimization problem will therefore depend on the available commitment technology. We return to this point when we analyze optimal growth in the next section.

The time inconsistency arises because the planner places excessive weight on the near future, where excessive is taken to mean relative to exponential discounting. The planner wishes to postpone felicity slightly, but not indefinitely. The problem arises when the near future becomes the present and the planner wishes to postpone felicity again. The tendency if unchecked is to continually postpone happiness. The optimal plan will have to take this tendency into account.

It is interesting that the form of time inconsistency observed here is the opposite of that which arises under hyperbolic discounting. With hyperbolic

discounting, individuals value the present and the far future more highly than they do in the exponential model. Our planner values these periods less. If we applied our welfare analysis to the hyperbolic model, these two forces would tend to balance one another out, although not completely. The result might approximate preferences with exponential discounting.

4.5 Political Feasibility

Possibly the most powerful argument for the dictatorship of the present is positive rather than normative. If the social decision results from a democratic process, then the current self would indeed appear to be a dictator, since only the current self can vote. The dictatorship of the present may therefore arise as a type of constrained optimum. It may be the only criterion that is practically implementable.

The proper way to understand constrained optima, however, is to first understand the unconstrained problem, then to specify the nature of the constraints, and finally solve the constrained problem. Our contention in this paper is that the unconstrained problem is not well understood. How then can we be so sure of the solution to the constrained problem?

One might say that it simply impossible for people to took beyond their own present preferences. After all, by revealed preference, preferences and choice are one and the same thing. This conclusion, however, does not imply that the dictatorship of the present is the only feasible social welfare function. First, agents may be altruistic. Given any individual voter, there are many important political issues that mainly affect the welfare of others. In evaluating candidates on these issues the voter is in very much the same position as the social planner. To the extent that voters like candidates that support good policies even if these policies only affect others, the democratic process may produce the social optimum.

Moreover, once in office, candidates would do well to note that tastes change over time. If they wish to remain in office, they may not want to maximize constituents' current utility, but their utility at the time of the next election. This may lead to a slight forward bias in policy.[10]

Finally, there are many social decisions that lie outside of the democratic

[10]It may be possible to estimate the extent to which agents discount the past by observing how politicians voting patterns vary with their remaining term in office.

process. Governors of the Federal Reserve System and Federal judges are appointed for long terms partly to allow these decision makers to take a longer term perspective. Beyond government, many individuals, such as parents and teachers, often find themselves placed in situations where their decisions affect the welfare of others at various points in time. These individuals too may behave like our social planner.

4.6 On the Interpretation of the Utility Function

Until this point we have treated the utility function U_t as the welfare of the agent in period t, but we have been deliberately vague as to what we meant by this. For most of the argument all we need is that U_t represents the agent's ranking of consumption sequences in period t. This is enough to motivate retrospective time inconsistency and justify the social planner placing greater weight on the future. Although the precise weights that the planner places on different perspectives will require some notion of cardinal utility.

In order to better understand the welfare problem it is useful to consider how these rankings relate to experienced utility. This is essentially a question of why the future and the past matter to an agent in the present. As Loewenstein [1992] has pointed out, there are essentially two answers to this question, one due to Jevons and the other to Samuelson.[11] In order to avoid confusion we will use the words pleasure and happiness to refer to experienced utility, and contrast these with utility U_t and felicity u_t.

According to the Jevonian perspective the future and the past matter because they directly influence the agent's present happiness. Agents feelings today are influenced by memories of the past and anticipations of the future. In this view, U_t represents the pleasure that the agent experiences in period t. Future and past felicity are discounted in U_t because these experiences are less immediate than current ones.

Given the Jevonian perspective the social welfare function (6) counts felicity multiple times, but this is because felicity is experienced multiple times. Felicity at date t is first experienced as anticipation at dates $s < t$, then as part of current date t enjoyment, and finally as memory at dates $s > t$.

[11] See Caplin and Leahy [2000] for a more detailed discussion of these views and their relationship to welfare theory.

The weights on felicity in (6) reflect these multiple perspectives, as well as the weights that the social planner assigns to these perspectives. According to the Jevonian perspective, the dictatorship of the present is extremely myopic. It focuses solely on current pleasure and completely ignores all future pleasure.

Alternatively, the Samuelsonian perspective states that the pleasure that an agent experiences in period t is associated with the period t felicity $u(c_t)$. Only current consumption makes the agent happy. In order to come to grips with the fact that agents care about future consumption but receive utility only from current consumption, the Samuelsonian perspective divides the period t self into two components: a decision maker and a consumer. The consuming self experiences period t felicity. The decision making self essentially acts as an internal social planner who aggregates the preferences of the consuming selves of various periods. In this view, U_t represents the preferences of the internal social planner at date t.[12]

According to the Samuelsonian perspective, when $\delta \neq \beta^{-1}$ the social planner's rankings of consumption sequences changes over time, and the social welfare function (6) represents an aggregation of these rankings. Consumption at date t provides pleasure only at date t, but the weight placed on date t felicity reflects an average of the weights placed on date t felicity by internal social planners at all dates.

Given the Samuelsonian perspective, it would seem natural to assume that $\delta = 0$. Why should the decision making self care about past felicity, when decisions can only impact current and future felicity?

4.7 Related Literature

In the rare cases in which the past is explicitly considered or in which the perspectives of different temporal selves are considered, economists commonly assume that agents discount the past. Pigou [1957], Wolf [1970], and Page [1977] consider utility functions in which the past enters with diminishing weight. Pigou and Page note that this implies that tastes change over time.

In the literature on time inconsistency, Phelps and Pollak [1968] and Laibson [1996] discuss the welfare effects of changes in the savings rate in

[12]Kahneman, Wakker and Sarin [1997] argue that the external social planner should igore the perspective of the internal social planner and focus solely on the various consuming selves. The problem with this appraoch is that it entirely divorces intertemporal social choice from intertemporal private choice.

the presence of hyperbolic discounting. Each considers the effect of these changes on the utility of the various temporal selves. In each case, future selves are assumed to place no weight on past consumption. Neither note that this assumption implies that tastes change even if preferences are time consistent.

In an intergenerational context it is common to consider the weight that parents place on their children's utility, as well as the weight that children place on their parents utility. In Barro's [1974] model of imperfect altruism, for example, parents care about the utility of their children, but children do not care about the utility of their parents. This gives rise to a utility function for each generation in which future felicity is discounted exponentially, but past felicity is given no weight. Others have considered two-sided altruism (Abel [1987], Kimball [1987]). Here the common assumption is that $\delta < \beta^{-1}$ which gives rise to a discount factor kinked at the present as in Figure 2. The focus of these papers, however, is on the validity of Ricardian equivalence, not social welfare.

The literature contains several other arguments in support of a lower social discount rate. These arguments tend to focus on intergenerational conflicts. Many authors have argued that it is troublesome to rank the needs of the present generation above those of future generations (e.g. Ramsey [1928] and Solow [1974]). Others have argued for a reduced social discount rate based on the fact that some private decisions have external effects on future generations. One example is capital left after death (Pigou [1952]).

These arguments are really dynamic extensions of static welfare arguments. Shell [1971] has shown that the overlapping generations model can be reinterpreted as a static Arrow-Debreu model in which all agents trade simultaneously. The first argument is therefore no different than the static argument for income redistribution. The second argument rests on a missing market. In the example, it is the market for annuities. In contrast to these arguments, our story has no static counterpart. Time is essential for tastes to change.

These intergenerational arguments focus attention on long horizons. These issues may come into play over a span of 50 to 100 years, but they may be ignored over shorter horizons such as the business cycle. Hence there is a tendency in the literature to only discuss the issue of social discounting when considering long term issues such as global warming or resource extraction. The argument presented in this paper, in contrast, is operative in the short

run, as well as the long run. It applies as well to monetary policy as social security, as well to unemployment insurance as to the environment.

4.8 Overlapping Generations Models

While we have considered the welfare of an infinitely lived agent, our analysis has implications for the debate concerning welfare analysis in overlapping generations models. For concreteness consider a model with two-period lived agents who experience felicity $u(C)$ in the first period of life and $v(C)$ in the second. Two criteria have been proposed to evaluate welfare in these models. Both arose in response to the definition contained in Lucas [1972].

The first is an *ex ante* or unconditional measure of the agent's first period utility:

$$E\left\{u(C_t) + \beta v(C_{t+1})\right\}. \tag{8}$$

Muench [1997] has argued that a policy is Pareto optimal if no other policy improves (8). A second, *ex post* measure of the agent's welfare conditions on the first period state:

$$u(C_t) + \beta E_t v(C_{t+1}). \tag{9}$$

Peleg [1982] argues that a policy is Pareto optimal if no other policy improves (9) for every realization of the first period shock.

According to our analysis both of these criteria are incomplete. Each ignores the perspective of the agent in the second period of life. This preoccupation with the perspective of the young is the OLG version of the dictatorship of the present.

It is interesting that our analysis provides support for the approach originally taken by Lucas. Lucas defined a policy to be Pareto optimal if no other feasible policy raised both $u(C)$ and $v(C)$ in every state of the world. This definition considers the welfare of both the old and the young. If Lucas' condition is met then the policy maximizes utility in each period of life no matter how agents value present, past and future felicity. Lucas' definition is a version of our approach in which the welfare criterion is Pareto optimality and welfare is evaluated after uncertainty is resolved.

5 Socially Optimal Growth

In this section we consider the implications of retrospective time inconsistency for the Ramsey growth model. The setup is the standard one in growth theory, although it requires a changes in perspective from the previous sections. The analysis will be in continuous time, so we will shift from discount factors to discount rates. Since there is no overlap between the equations that precede this point and the equations that follow it, we will employ the standard notation of the Ramsey model even though it means reusing several variables (δ and α in particular).

At each point in time firms employ labor and capital and produce a consumption good with a constant returns to scale production function, $F(K_t, A_t L_t)$, where K_t represents the capital stock, A_t labor augmenting technological progress, and L_t the supply of labor at date t. Labor productivity A grows at a constant rate g, and the labor supply is constant and normalized to one. Capital depreciates at a constant rate δ and accumulates with investment. Given the constant returns to scale production function we can normalize all variables in terms of efficiency units of labor. Let $k = K/A$ and $f(k) = F(k, 1)$. Capital per effective unit of labor evolves according to

$$\dot{k} = f(k) - c - (\delta + g)k$$

where c is consumption per effective unit of labor.

We assume that the representative agent discounts the past at a rate γ and the future at a rate ρ. The agent's utility function at date t is therefore:

$$V_t = \int_0^t e^{\gamma(s-t)} u(c_s) ds + \int_t^\infty e^{\rho(t-s)} u(c_s) ds$$

The social planner discounts future utility at a rate α, and therefore maximizes:

$$\begin{aligned} W_r &= \int_r^\infty e^{\alpha(r-t)} V_t dt \\ &= \int_r^\infty e^{\alpha(r-t)} \left[\int_0^t e^{\gamma(s-t)} u(c_s) ds + \int_t^\infty e^{\rho(t-s)} u(c_s) ds \right] dt \end{aligned}$$

Since consumption in the past is sunk maximizing W_r is equivalent to maximizing $\hat{W}_r$ where:

$$\hat{W}_r = \int_r^\infty e^{\alpha(r-t)} \left[\int_r^t e^{\gamma(s-t)} u(c_s) ds + \int_t^\infty e^{\rho(t-s)} u(c_s) ds \right] dt$$

Reversing the order of integration we arrive at

$$\hat{W}_r = \int_r^\infty \left[\int_r^s e^{\alpha(r-t)} e^{\rho(t-s)} dt + \int_s^\infty e^{\alpha(r-t)} e^{\gamma(s-t)} dt \right] u(c_s) ds$$

Hence the planner weights felicity s periods in advance by

$$D(s) = \frac{\int_r^s e^{\alpha(r-t)} e^{\rho(t-s)} dt + \int_s^\infty e^{\alpha(r-t)} e^{\gamma(s-t)} dt}{\int_r^\infty e^{(\alpha+\gamma)(r-t)} dt}$$

Here the denominator normalizes $D(0) = 1$.

Again there is no reason to expect D to be exponential. Hence the planner's behavior will not generally be time consistent. We consider the planner's optimal policy in two cases. In the first, the planner can commit to carrying out the optimal plan from the period zero perspective. In the second, the planner cannot commit and must take into account the fact that future behavior will change.

5.1 Solution to the Model: Commitment

Suppose that the social planner can commit to a plan in period zero. It is immediate that[13]

$$\frac{\dot{c}}{c} = -\frac{u'(c)}{u''(c)c} \left(r + \frac{\dot{D}}{D} \right) \tag{10}$$

where

$$\frac{D'(s)}{D(s)} = \frac{-\rho \int_r^s e^{\alpha(r-t)} e^{\rho(t-s)} dt + \gamma \int_s^\infty e^{\alpha(r-t)} e^{\gamma(s-t)} dt}{\int_r^s e^{\alpha(r-t)} e^{\rho(t-s)} dt + \int_s^\infty e^{\alpha(r-t)} e^{\gamma(s-t)} dt}, \tag{11}$$

is the planner's discount rate on future felicity. Equation (10) together with the equation of motion for the capital stock then describe the evolution of the economy.

In two cases, this solution reduces to the standard solution. If, for example, $\gamma = -\rho$, then $\dot{D}/D = -\rho$. In this case, all of the temporal selves agree

[13] Equation (10) can be derived from a perturbation argument.

with the current self on the ranking of consumption flows. Also if $\alpha = \infty$, $\dot{D}/D = -\rho$. This is the dictatorship of the present.

What can we say in the more general case in which tastes change and the social planner puts weight on all perspectives? It is easy to show that $\lim_{s\to\infty} \dot{D}/D = -\min\{\alpha, \rho\}$. The limit behavior under commitment therefore depends on whether the representative agent discounts future felicity more than the social planner discounts future utility. If $\alpha \leq \rho$, then the resulting steady state level of k is the modified golden rule, $f'(k) = \rho + g + \delta$. If $\alpha = 1$, then the steady state is the golden rule, $f'(k) = g + \delta$. Regardless of the social discount rate, the steady state level of k will lie between these two points. Moreover there exists a choice of α that justifies each level of k between the modified golden rule and the golden rule.

In the case that $\alpha \leq \rho$, in which the optimal growth rate converges to the competitive growth rate, the optimal growth path, unlike the competitive economy, does not converge monotonically to the modified golden rule. If $\gamma > -\rho$, then $\dot{D}/D > -\rho$ at all finite horizons. This means that the social planner places greater weight on the future than does the private agent. The implication is that the social planner will overshoot the competitive equilibrium balanced growth path. If, for example, the economy begins at the modified golden rule, the planner will choose a growth rate of consumption that is greater than the competitive growth rate. To accomplish this, consumption will need to fall relative to the competitive equilibrium, investment will rise, and the growth rate will increase. After some time, since $\lim_{s\to\infty} \dot{D}/D = -\rho$ the planner's discount rate will fall, consumption growth will slow and become negative, and the economy will return to the modified golden rule.

5.2 Solution to the Model: No Commitment

If the social planner cannot commit to a plan, the planner must solve a complex maximization problem: choosing the best plan that will actually be followed. Pollak [1965] and Barro [1997] show that this problem is made easier if we consider log utility. In this case the sophisticated solution, in which the agent anticipates future changes in tastes, and the naive solution, in which the agent behaves in the present as if tastes will remain unchanged,

are the same.[14] Barro derives the Euler equation:[15]

$$\frac{\dot{c}_t}{c_t} = r_t + \frac{1}{\int_0^\infty D(s)ds} \tag{12}$$

In our case,

$$\frac{1}{\int_0^\infty D(s)ds} = -\frac{\alpha\rho}{\gamma+\alpha+\rho}$$

Let $d \equiv \frac{\alpha\rho}{\gamma+\alpha+\rho}$. Barro shows that d is a weighted average of the $D(s)$'s.

Whereas the model of behavior that gives rise to (12) is much more complex than in the commitment case, the resulting solution is much simpler. The social planner behaves as if the social discount rate were fixed at d. The dynamics of the model are the same as in the standard Ramsey model. Equation (12) and the equation of motion for the capital stock determine the evolution of the economy.

Again this solution reduces to the dictatorship of the present in a number of special cases. If $\gamma = -\rho$, then $d = \rho$ and we get the standard model. Also if $\alpha = \infty$, $d = \rho$.

More generally, if $\gamma > -\rho$ and $\alpha < \infty$, $d < \rho$. This means that if agents discount the past, the social planner places greater weight on the future than does the private agent. The economy converges to a point between the modified golden rule and the golden rule. It is easy to show that there are choices of α that support any such point. We call the level of k such that $f'(k) = d + g + \delta$ the *modified modified golden rule*. In the case in which agents care about the past and the present equally and the social planner discounts the future by the same amount as private agents we get

$$d = \rho/3$$

The social planner uses a discount rate that is one third of the representative agent's discount rate. Although difficult conceptually, the case without

[14]The intuition is straightforward. With log utility expenditure in each period is independent of intertemporal prices. Changes in the discount rate are like changes in intertemporal prices.

[15]Pollak solves a finite horizon model with a zero interest rate. Barro solves the infinite horizon model with time varying interest rates. Since, there are issues of the proper definition of consistent plans in infinite horizon settings, we follow Barro and take the solution in the infinite horizon case to be the limit of finite horizon models. This solution is valid if $\lim_{s\to\infty} D(s) = 0$.

commitment yields simple and intuitive results that can easily be applied to analyze optimal policy.

6 Conclusion

We formulate the problem of a social planner attempting to maximize welfare over time. Our analysis has immediate implications for public policy: agents discount the future too much and governments should promote future oriented policies. In the context of the growth model, we showed that competitive equilibrium outcomes imply a level of saving and hence a growth rate of the economy that is suboptimally slow. There is therefore justification for policies which promote growth such as subsides to saving and investment.

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