Discussion Paper 143

Institute for Empirical Macroeconomics
Federal Reserve Bank of Minneapolis
90 Hennepin Avenue
Minneapolis, Minnesota 55480-0291

December 2005

# Optimal Welfare-to-Work Programs*

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ABSTRACT

A Welfare-to-Work (WTW) program is a mix of government expenditures on "passive" (unemployment insurance, social assistance) and "active" (job search monitoring, training, wage taxes/subsidies) labor market policies targeted to the unemployed. This paper provides a dynamic principal-agent framework suitable for analyzing the optimal sequence and duration of the different WTW policies, and the dynamic pattern of payments along the unemployment spell and of taxes/subsidies upon re-employment. First, we show that the optimal program endogenously generates an absorbing policy of last resort (that we call "social assistance") characterized by a constant lifetime payment and no active participation by the agent. Second, human capital depreciation is a necessary condition for policy transitions to be part of an optimal WTW program. Whenever training is not optimally provided, we show that the typical sequence of policies is quite simple: the program starts with standard unemployment insurance, then switches into monitored search and, finally, into social assistance. Only the presence of an optimal training activity may generate richer transition patterns. Third, the optimal benefits are generally decreasing or constant during unemployment, but they must increase after a successful spell of training. In a calibration exercise based on the U.S. labor market and on the evidence from several evaluation studies, we use our model to analyze quantitatively the features of the optimal WTW program for the U.S. economy. With respect to the existing U.S. system, the optimal WTW scheme delivers sizeable welfare gains, by providing more insurance to skilled workers and more incentives to unskilled workers.

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# 1 Introduction

Public expenditures on labor market policies targeted to the unemployed average 3% of GDP, across OECD countries (Martin, 2002). These policies face a delicate trade-off between providing unemployed workers with consumption insurance and re-employment incentives. In order to strike the right balance, governments use a wide range of policy instruments which, broadly speaking, can be divided into "passive" and "active" policies.

Two thirds of these expenditures are allocated to passive policies, mainly unemployment insurance and social assistance policies providing income support of last resort once unemployment benefits have expired. The remaining third is allocated to active policies, like job-search monitoring, training, and wage subsidies. Typically, job-search monitoring programs pair the unemployed worker with a public employee (the "caseworker") who verifies her job-search activity, and often helps with improving interviewing skills and selecting suitable job vacancies. Training programs tend to be one of three types: basic education (brush-up courses for individuals with poor literacy and numerical skills, preparation for high-school level diplomas), vocational training (classroom training in specific occupational skills) and temporary jobs (publicly funded employment in non-profit organizations or government agencies). Interestingly, the share of expenditures on active labor market programs has risen substantially over the past 10 years, and this type of government intervention is now a pivotal ingredient of social welfare policies.

It is instructive to consider how these various policy instruments have been assembled together in the United States and in the United Kingdom. In the United States, since 1935 there exists an unemployment insurance system with vast coverage which replaces on average 60% of the pre-displacement wage. Upon expiration of the unemployment compensation (usually after 26 weeks), other forms of financial support become available. The Temporary Assistance for Needy Families (TANF) program is the most notable example. With the Personal Responsibility and Work Opportunities Reconciliation Act (PRWORA) of 1996, the federal U.S. government limited the payment of TANF benefits to a maximum of 5 years–with tighter limits in several states–and imposed strict participation requirements to active labor market programs for welfare recipients. Broadly speaking, two types of programs have been put in place: "human capital development" programs where the emphasis is on training and skill formation, and "labor force attachment" programs, where the focus is on assistance and monitoring during job search. The Food Stamps program represents the main form of social assistance, once the TANF benefits have expired. Finally, the Earned Income Tax Credit, introduced by the federal government in 1975, represents today the major wage subsidy program for low-income workers (see Moffitt, 2003, for a survey of the U.S. welfare system).

An example of a more structured mix of passive and active policies is the U.K. "New Deal,"a mandatory program for all the unemployed workers introduced in 1998. Formally, the New Deal is organized in four sequential stages. Stage 1 consists of a standard unemployment insurance policy lasting 6 months for younger workers and 2 years for older workers. In stage 2 (the "gateway"), a personal adviser regularly meets with the worker to assist with and monitor her job search. If after four months the worker is still unemployed, she moves to stage 3 (the "options") where she enters training for up to one year. At the end of the training program, the worker enters stage 4 (the "follow-through"), another period of job-search assistance/monitoring. Also the U.K. welfare system contains an assistance policy of last resort, called Income Support. Finally, the U.K. system features a large earnings subsidy program for poor households, the Working Family Tax Credit (see Blundell and Meghir, 2001, for an overview of the U.K. system).

It is useful to note, at this point, two common features of the U.K. and the U.S. Welfare-to-Work programs, shared by many other countries as well. First, active policies tend to be implemented after an initial spell of unemployment insurance. Second, policies of pure social assistance (like Food Stamps) are usually ordered last in the sequence of policy interventions.

A Welfare-to-Work (WTW) program is precisely a government expenditure program that combines together passive and active policies, as in the U.S. and the U.K. examples. Clearly, every WTW program implicitly promises a certain level of ex-ante welfare to the unemployed agent. An *optimal* WTW program is a mix of policies that maximizes the expected discounted utility of the unemployed agent, subject to the constraint of not exceeding a given level of government expenditures. The fundamental trade-off to be solved in the design of an optimal WTW program is the one between offering consumption insurance and eliciting the right effort level in the search or training activity.

The first objective of this paper is to develop a theoretical framework suitable to study the key features of optimal WTW programs. Our point of departure is the classic setup–originated largely from the seminal article of Shavell and Weiss (1979)–where the optimal unemployment insurance contract is studied in the presence of a repeated moral hazard problem: the risk-neutral principal (planner/government) cannot observe the risk-averse unemployed agent's job search effort (hidden action).[1] Following the most recent contributions in the literature (Wang and Williamson, 1996; Hopenhayn and Nicolini, 1997; Zhao, 2001; Pavoni, 2003a), we build on the seminal insights of Spear and Srivastava (1987) and Abreu, Pearce and Stacchetti (1990) to exploit the recursive representation of the planner's problem where the expected discounted utility promised by the contract to the unemployed agent becomes a state variable.

We generalize this standard setup in two directions. First, following Pavoni (2003b), we let workers' wages and their job finding probabilities depend on *human capital* (skills) and allow human capital to depreciate along the unemployment spell. Human capital is our second key state variable in the recursive representation. Second, we enrich the economic environment of the standard setup with the introduction of a costly monitoring

[1] Bailey (1978), Flemming (1978) and more recently Fredricksson and Holmlund (2001) and Acemoglu and Shimer (1999) follow a different approach. They study economies where search effort is observable (i.e., there is no moral hazard) and model unemployment insurance as the optimal policy of a benevolent government that chooses the benefit *level* (not the entire time-path) associated to the welfare-maximizing labor market equilibrium. Recently, Saez (2002) and Laroque (2005) have analyzed the optimality of certain income maintenance programs in the U.S. and France within static models where the worker type is private information.

technology that can be used to observe the worker's search effort and of a costly skill accumulation technology that can be used to slow down depreciation and, possibly, increase the level of human capital.[2]

The introduction of human capital depreciation in the problem permits a better representation of labor market data along two important dimensions. First, since wages depend on human capital, in our economy workers experience wage losses upon separation, consistently with the findings of a vast set of empirical studies (Bartel and Borjas, 1981; Ruhm, 1991; Jacobson, LaLonde and Sullivan, 1993; for a survey, see Fallick, 1996). Note that in this literature, unemployment duration is often found to have an independent negative impact on re-employment earnings, beyond job-specific and occupational-specific skill losses (e.g., Addison and Portugal, 1989; Neal, 1995). Second, since we let the job-finding probability depend on human capital, search effort becomes less effective as the unemployment spell progresses, inducing negative duration dependence in the unemployment hazard–a common feature of the data, as reported by Machin and Manning (1999) in their survey. In particular, several studies (e.g., Blank, 1989, for welfare recipients; Bover, Arellano and Bentolila, 2002, for UI benefits recipients) continue finding a rapidly declining hazard even after explicitly controlling for unobserved heterogeneity. Moreover, the existence of human capital depreciation during unemployment represents a potential reason for training policies to be part of an optimal WTW program.[3]

As discussed earlier, job-search monitoring and training policies are key components of actual WTW schemes. The standard setup (e.g., Hopenhayn and Nicolini, 1997) is unable to address the optimality of such important policy instruments. Enriching the economic environment to include the monitoring and human capital accumulation technologies implies that, along the optimal contract, the planner may find it optimal to make use of them.

The second objective of the paper is to study quantitatively the features of the optimal WTW program for the typical welfare recipient in the U.S. economy and contrast them to the actual welfare system. There is an extended econometric literature on the evaluation of active labor market programs (for surveys, see LaLonde, 1995; Meyer, 1995; Heckman, LaLonde and Smith, 1999; Moffitt, 2003) which provides useful guidance on the effectiveness of different types of interventions, but cannot address the complex question of how to design a government program which combines various instruments and follows the worker throughout her non-employment experience. We use our framework in an attempt to answer this question for the U.S. labor market.

We calibrate the parameters of our model to match some key labor market statistics. In so doing, we exploit

[2]Extending Atkeson and Lucas (1995), Alvarez and Ayiagari (1995) study the optimal level of random monitoring of agents' reports on job offers and the long-run stationary distribution of consumption in a pure adverse-selection selection setup with temporary (one-period) job offers, and no human capital dynamics. Recently, Boone et al. (2002) have addressed the issue of the optimality of job-search monitoring within a structural search model of the labor market. They numerically compute the optimal level of monitoring in an economy where every worker is subject to the same monitoring probability. Therefore, they do not fully solve for the optimal contract where the use of each instrument is history-dependent, as we do. Their simpler framework has clearly some other advantages, for example they can study general equilibrium effects which we abstract from.

[3]Skill depreciation is also a central ingredient in a popular explanation of the comparative unemployment experience of the U.S. and Europe (e.g., Ljungqvist and Sargent, 1998).

information from the evaluation of several recent U.S. active labor market programs. Next, we solve numerically for the optimal program and, by simulation, derive the optimal sequence of policies, their duration, the pattern of optimal benefits, taxes and subsidies. Finally, we calculate the welfare gains for the worker and the budget savings for the government of shifting from the current scheme to the optimal scheme.

## 1.1 A Preview of the Main Results

The planner's recommendations on effort choices and on the use of the costly available (search monitoring and skill accumulations) technologies map naturally into four distinct policy instruments: 1) unemployment insurance, where the planner elicits positive search effort from the agent; 2) job search monitoring, where the planner can observe the search effort upon payment of a cost; 3) training, where the planner pays a cost to slow down human capital depreciation; and 4) social assistance, defined as an income-assistance program of "last resort" with the planner inducing zero search effort and simply insuring the worker. Moreover, as in Hopenhayn and Nicolini (1997), by choosing an agent's consumption during employment, the planner implicitly sets wage taxes and subsidies.

Our qualitative characterization of the optimal WTW program yields four main results. First, in absence of human capital depreciation, the optimal program does not contemplate switching between policy instruments: the program may select initially different policies for workers with different human capital levels, but each policy is absorbing within the unemployment spell.

Second, with human capital depreciation, when training is not chosen, the typical policy sequence in the optimal WTW program starts from unemployment insurance, switches into job-search monitoring, and then turns into social assistance, which remains the only absorbing policy. The faster the human capital depreciation, the more rapidly the optimal WTW program transits between policy-phases. When the optimal program involves training, the possible policy transitions of the program are richer, so we resort to numerical simulations for a full characterization.

Third, we can entirely describe the dynamics of the government transfers and taxes/subsidies associated to each policy phase of the program. Under social assistance and job-search monitoring, benefits are constant because of full insurance, while due to the presence of the incentive constraint they are decreasing both during unemployment insurance and after an unsuccessful spell of training. After a successful training phase, the optimal benefits must increase.

Fourth, the nature of re-employment wage taxes and/or subsidies is largely determined by competing forces which we explain intuitively. One fairly general result is that, as the optimal program approaches the monitoring phase, earnings subsidies become more and more likely, and during job-search monitoring the subsidy tends to increase with duration. This finding, which is linked to human capital depreciation, contrasts with the well-known result of Hopenhayn and Nicolini (1997) who uncover that, in an economy without human capital dynamics, taxes should rise with the length of the unemployment spell.

For expositional simplicity, in the benchmark model we make the usual assumption in this literature that the planner fully controls the consumption stream of the agent. This precludes the occurrence of self-insuring trades in the asset market. Recently, several authors have begun to analyze dynamic moral hazard models with hidden savings (Werning, 2002; Kocherlakota, 2003; Shimer and Werning, 2003; Abraham and Pavoni, 2004). In an extension of the benchmark model where the agent can anonymously access a credit market, we show that if the agent faces a no-borrowing constraint, the allocations induced by the same optimal contract can be implemented with the help of one simple additional instrument: a linear, time-invariant interest tax.

When we use our theoretical framework to study the optimality of the current U.S. welfare system, we find that, compared to the existing scheme, the optimal program should feature more job-search monitoring and less basic adult training. This result agrees with a vast empirical evaluation literature that finds adult training (for surveys, see Heckman, LaLonde and Smith, 1999; Heckman and Carneiro, 2002) rather ineffective, with few exceptions. Moreover, we find that the current WTW scheme should provide more insurance for skilled workers and more incentives for unskilled workers.

The size of the welfare gains of switching from the current scheme to the optimal program varies across skill type. High-skill workers who rejoin the employment ranks quickly have small gains, around 1% of lifetime consumption; however, low-skill workers with long durations can achieve gains beyond 8% of lifetime consumption. Also budget savings for the government are substantial: the optimal program would reduce public expenditures between 4% and 50%, depending on the worker's human capital level.

The rest of the paper is organized as follows. Section 2 presents the economic environment (available to the agent in autarky). Section 3 describes the contractual relationship between planner and agent, and presents the recursive formulation of the planner's problem. Section 4 characterizes the key features of the optimal WTW program. In Section 5 we analyze the implementation of the optimal contract with access to credit markets. Section 6 develops the quantitative analysis applied to the U.S. labor market. Section 7 concludes the paper.

# 2 The Economy

**Preferences:** Workers have period utility over consumption $c$ and effort $a$ given by $u(c) - a$. Preferences are time-separable and the future is discounted at rate $\beta \in (0,1)$. We impose that $c \geq 0$, and that $u(\cdot)$ is strictly increasing, strictly concave and smooth, with $\lim_{c \to \infty} u'(c) = 0$. A technical assumption that will prove useful in our characterization is that the first derivative of $u^{-1}$ is convex. This condition is satisfied by a wide range of utility functions, including the CRRA class with risk-aversion parameter greater than one half, and the entire

CARA class.[4]

**Employment status and effort:** The agent can be either unemployed ($z = z^u$) or employed ($z = z^e$). During unemployment, she can either search or train–these two activities are mutually exclusive within a period–and search/training effort can be either low or high, i.e., $a \in \{0, e\}$, with $e > 0$. Employment is assumed to be an absorbing state with zero disutility of effort $(a = 0)$.[5]

**Human capital:** Workers are endowed with a time-varying stock of human capital (skills) $h \geq 0$. During unemployment, if search or training fails, human capital depreciates geometrically at a constant rate $\delta \in [0, 1]$. During employment or after a successful spell of training, human capital accumulates. Let $y$ denote the outcome of the worker activity (search/train) during unemployment, with $y \in \{s, f\}$, where $s$ denotes "success", and $f$ denotes "failure". Then,

$$h^s = Ah^\alpha + (1-\delta)\,h, \text{ with } \alpha \in [0,1] \text{ and } A \geq 0, \tag{1}$$

$$h^f = (1-\delta)\,h. \tag{2}$$

**Production technology:** An employed worker of type $h$ produces output $\omega(h)$. We let $\omega(\cdot)$ be a continuous and increasing function, with $\omega(h) \in [0, \omega_{\max}]$ and $\omega(0) = 0$. During employment, human capital accumulates through the law of motion (1) at no cost (e.g., through learning-by-doing).

**Search technology:** During search, both effort $a$ and human capital $h$ affect the job finding probability of an unemployed worker. Denote the unemployment hazard rate as $\pi(h, a)$. We assume that $\pi(h, 0) \equiv 0$ and that $\pi(\cdot, e) \equiv \pi(\cdot) \in (0, 1)$ is continuous and increasing.

**Training technology:** The unemployed worker can choose to forego the search option and operate a training technology to accumulate human capital, upon payment of a cost $\kappa^{TR} > 0$. The training technology requires the unobservable worker's effort to succeed, but its outcome is stochastic: with probability $\theta(a)$, where $\theta(e) > \theta(0) = 0$, training is successful, and the worker's human capital next period accumulates following (1). Upon failure, human capital depreciates according to (2).

**Insurance and credit markets:** During unemployment the agent is subject to a random event she would like to (self-) insure against: the outcome $y$ of her search/training activity, which affects both her employment status and her level of human capital, thus her future income. In the baseline model, we study the optimal contract when the worker has no access to storage, insurance or credit markets–in particular we abstract from self-insurance. In Section 5, we will relax this assumption and explore how the same allocations implied by the optimal contract can be implemented when workers have anonymous access to the credit market.

[4] Newman (1995) highlights the role of the curvature of the first derivative of the cost function for the principal (the inverse of the agent's utility function) in a static moral hazard model of entrepreneurship.

[5] In the quantitative section we also study the case where during work the agent suffers a positive effort cost $a = e$.

It is useful to further discuss some of the assumptions we made in laying out our economic environment. The discrete effort choice is coherent with a long tradition in labor economics and macroeconomics which stresses the importance of fixed costs and the extensive margin in participation decisions (e.g., Cogan, 1981). The assumption that employment is an absorbing state is made, as in Hopenhayn and Nicolini (1997), to focus the analysis of the optimal dynamic contract to the unemployment experience.[6] The absence of effort cost and the presence of learning-by-doing on the job suffice to ensure that the value of employment always dominates the value of remaining unemployed so that no job offer is turned down. These two assumptions have no bearing on the qualitative characterization of the optimal WTW program during unemployment and can be relaxed, as long as employment remains dominant.

The monotonicity of $\pi$ in $h$ is a natural property, consistent with the overwhelming evidence that unemployment duration is longer for workers with lower pre-displacement wage (e.g., Meyer 1990). Strictly speaking, it is a "reduced form" that can be microfounded, for example, in a model where skilled workers have access to wider labor market opportunities, or in a model where productivity on the job is positively correlated with search skills. This monotonicity, together with skill depreciation, induces duration dependence in the unemployment hazard.[7]

Our specification of $\pi$ also displays complementarity between the stock of human capital $h$ and the search effort level $a$: increasing effort has a larger impact on the hazard rate the larger the level of $h$. Bover, Arellano and Bentolila (Table 4, 2002) document that the receipt of benefits significantly reduces the hazard rate of leaving unemployment and this effect tends to fade away as the unemployment spell progresses (and $h$ depreciates). It is logical to associate the receipt of benefits with a lower search effort, thus their finding provides a possible foundation for our complementarity assumption.

# 3 The Contractual Relationship

We now introduce a risk-neutral planner/government (principal) who faces an intertemporal budget constraint with a real interest factor equal to $\beta^{-1}$. At time $t = 0$, the planner offers the unemployed worker (agent) an insurance/credit contract that maximizes the expected discounted stream of net revenues (fiscal revenues minus expenditures) and guarantees the agent at least an expected discounted utility level $U_0$. The value of $U_0$ should be thought of as an exogenous parameter measuring the "generosity" of the welfare system (e.g., the outcome of voting).

[6] As long as the layoff rate is exogenous, the qualitative predictions of our model, within the same unemployment spell, are unchanged. Zhao (2001), and more recently Hopenhayn and Nicolini (2004), introduce a job-separation probability that is under agents' control and show that the optimal dynamic contract must also take the employment history into account.

[7] None of our analytical results relies on the *strict* monotonicity of $\pi$ in $h$. For example, in Proposition 6, we will provide a full analytical characterization of the optimal sequence of policies for the case where $\pi$ is constant (and only $\omega$ is affected by $h$).

**Information structure:** The planner can perfectly observe the employment status $z$, whether the unemployed worker is searching or training, and the outcome $y$ of the latter activity.[8] However, the agent's effort choice $a$ during both search and training is private information of the agent, so the planner faces a moral hazard problem.

A search-effort monitoring technology is available to the planner when the worker seeks job opportunities: upon payment of a cost $\kappa^{JM} > 0$, the job-search effort of the agent can be perfectly observed and enforced by the planner. The monitoring technology can be interpreted as the situation where the planner pays the services of a "caseworker" who monitors closely the search activity of the agent.[9] Such technology is unavailable during training.[10]

**Contract:** In each period $t$, the contract specifies transfers of resources to the worker, recommendations on search vs. training activities and on the search/training effort level to exert, and the choice of using the effort-monitoring technology, when search is suggested. The period $t$ components of the contract are contingent on all publicly observable histories up to $t$ and, whenever the monitoring technology is not used, search-effort recommendations must be incentive compatible. The training effort recommendations must always be incentive compatible. Moreover, at every $t$, we allow the planner to specify the contract contingent on the publicly observable realization $x_t \in [0, 1]$ of a uniform random variable $X_t$. We will explain later how this "randomization" may be used in the optimal contract to convexify the planner's problem and, thus, enhance welfare (see also Phelan and Townsend, 1991; and Phelan and Stacchetti, 2001).

**The components of the contract as policies of the Welfare-to-Work (WTW) program:** The combination of unmonitored search, monitored search, use of the training technology, together with the high and low effort recommendations configures six possible options. Notice first that the planner will never choose to pay the monitoring cost and suggest the minimal effort level. The reason is that, since $\pi(h, 0) = 0$, the observable realization of a successful search activity perfectly detects a deviation from the zero-effort recommendation at no additional cost.[11] Moreover, the planner will never choose to pay the training cost and suggest zero effort,

[8] Since the laws of motion for human capital dynamics are known and deterministic, it is enough knowing the pre-displacement wage, the occurrence and outcome of training and unemployment duration in order to recover the individual level of human capital $h$ at every point in the worker's history.

[9] Our monitoring technology could be interpreted in a more general way, as "random monitoring", with one additional assumption. Suppose that the government observes the agent's search effort only with some probability $q < 1$. When $u(0) = -\infty$, then if the government threatens to sanction the shirking worker by not paying any benefit, this random monitoring acts exactly as a perfect monitoring, since the worker will never want to be without consumption for a period. Such monitoring activity will be essentially identical to ours whenever the monitoring cost has a fixed component bounded away from zero.

[10] While certain elements of the learning effort, such as classroom attendance and homework, are easily verifiable, there are other key components, like attention, focus and concentration, that are extremely hard to be verified by an external party. It is reasonable to argue that learning is a far more complex activity than job search and, as such, fully monitoring the training effort is not possible.

[11] Put differently, the incentive-compatibility constraint associated to the zero effort recommendation is a trivial one, since the planner can punish without limits the worker upon finding a job, an outcome which is off the equilibrium induced by the optimal contract.

since $\theta(0) = 0$ and the cost would be wasted. As a result, the planner is left with four options, which we label "policy instruments" of the WTW program, and we index with $i$.

We denote as "Unemployment Insurance" ($i = UI$) the joint recommendation of search activity and positive search effort. When positive search effort is suggested together with the use of the monitoring technology, the policy will be labelled "Job-search Monitoring" ($i = JM$). The zero-effort recommendation in the search activity denotes the "Social Assistance" policy $(i = SA)$. A high-effort recommendation with the use of the training technology describes the "Training" option ($i = TR$). Finally, during employment, the difference between the wage and the planner's transfer defines implicitly the employment tax (if positive) or subsidy (if negative).

## 3.1 Recursive Formulation of the Planner's Problem

Appendix A describes the sequential formulation of the optimal contract and explains that, following the recursive contracts literature, the contract can be described through a state vector composed by the expected discounted utility $U$ promised to the agent by the continuation of the contract and the level of human capital $h$ of the worker.[12]

Exploiting this recursive representation, consider an unemployed worker who enters the period with state $(U, h)$. At the beginning of the period, the planner selects the optimal policy instrument $i(U, h)$ by solving

$$V(U, h) = \max_{i \in \{JM, SA, TR, UI\}} V^i(U, h), \tag{3}$$

where the function $V$ is the upper envelope of the values associated to the different policies. In choosing a particular policy, implicitly, the planner also chooses an effort recommendation $a(U, h)$, the transfer $c(U, h)$, and the continuation utilities $U^y(U, h)$ conditional on the outcome $y$ of the search/training activity. We describe these additional choices in the next section.

As anticipated, the planner in general may decide to use randomizations through $X$. In this case, the value function for the planner solves

$$\begin{aligned} \mathbf{V}(U, h) &= \int_0^1 \max_{U(x) \in D} V(U(x), h) dx, \\ s.t. &: \\ U &= \int_0^1 U(x)\, dx, \end{aligned} \tag{4}$$

where $V(U(x), h)$ is the upper envelope (3), and the integral constraint in (4) says that the planner is committed to keep his promises, hence he must deliver to the agent continuation utility $U$ in (ex-ante, with respect to the shock $x$) expected value terms.

[12]Strictly speaking, the employment status $z$ is also a state variable, but since employment is absorbing and presents no incentive problems, the contract is nontrivial only as long as the worker is unemployed and $z = z^u$. Therefore, in what follows, to lighten the notation, we omit $z$ from the state vector.

## 3.2 The Policies

We now describe in detail the planner problem during employment and during unemployment, under the four policies we considered.

**Employment state (wage tax/subsidy):** Consider an employed worker with states $(U, h)$. Since employment is an absorbing state without informational asymmetries, the planner simply solves

$$\begin{aligned} W(U,h) &= \max_{c,U^s} \omega(h) - c + \beta W(U^s, h^s) \\ s.t. &: \\ U &= u(c) + \beta U^s, \end{aligned} \tag{5}$$

where $h^s$ is obtained from the law of motion (1). The planner will provide full consumption smoothing for the agent, thus promised utility is constant over time. The promise-keeping constraint implies that in every period the optimal transfer $c^e$ is constant and satisfies $c^e(U) = u^{-1}((1-\beta)U)$. Therefore, the magnitude

$$\tau(U,h) = \omega(h) - c^e(U) \tag{6}$$

is the implicit tax (or subsidy, if negative) the government imposes on employed workers. State-contingent taxes and subsidies are a key component of an optimal WTW plan.

**Unemployment Insurance (UI):** When the worker is enrolled by the planner in the unemployment insurance scheme, the problem of the planner is

$$\begin{aligned} V^{UI}(U,h) &= \max_{c,U^f,U^s} -c + \beta\left[\pi(h)W(U^s,h^f) + (1-\pi(h))\mathbf{V}(U^f,h^f)\right] \\ s.t. &: \\ U &= u(c) - e + \beta\left[\pi(h)U^s + (1-\pi(h))U^f\right], \\ U &\geq u(c) + \beta U^f, \end{aligned} \tag{7}$$

where $h^f$ is generated through the law of motion (2). The pair $\left(U^s, U^f\right)$ are the lifetime utilities promised by the planner contingent on the outcome of search ($s$ denotes success and $f$ failure of the search activity). Given the observability of the employment status, the outcome of search is verifiable. For notational simplicity we have denoted $\pi(h,e)$ as $\pi(h)$. The first constraint describes the law of motion of the state variable $U$ (promise-keeping constraint), and the second constraint states that payments have to be incentive compatible. The expressions for $W$ and $\mathbf{V}$ are given by equations (5) and (4), respectively.

**Job Search Monitoring (JM):** The problem of the planner that chooses to monitor the search effort of the agent is

$$\begin{aligned}V^{JM}(U,h) &= \max_{c,U^f,U^s} -c-\kappa^{JM}+\beta\left[\pi(h)W(U^s,h^f)+(1-\pi(h))\mathbf{V}(U^f,h^f)\right]\\ s.t. &:\\ U &= u\left(c\right)-e+\beta\left[\pi(h)U^s+\left(1-\pi(h)\right)U^f\right].\end{aligned} \tag{8}$$

Notice the similarity between problem $(JM)$ and problem $(UI)$: the former is identical to $(UI)$ except for the fact that there is no incentive-compatibility constraint in exchange for the additional per period cost $\kappa^{JM}$. This cost can be interpreted as the salary of the government employee ("caseworker") who monitors and enforces the search activity of the unemployed agent, plus the additional administrative expenditures associated to this task.

**Training (TR):** The planner can opt to operate the costly stochastic training technology requiring the unobservable agent's effort as input. The planner's problem, when the worker is enrolled in training, is defined as

$$\begin{aligned}V^{TR}(U,h) &= \max_{c,U^s,U^f} -c-\kappa^{TR}+\beta\left[\theta\mathbf{V}(U^s,h^s)+(1-\theta)\mathbf{V}\left(U^f,h^f\right)\right]\\ s.t. &:\\ U &= u\left(c\right)-e+\beta\left[\theta U^s+(1-\theta)U^f\right],\\ U &\geq u(c)+\beta U^f,\end{aligned} \tag{9}$$

where we have simplified the notation for the success rate of training $\theta\left(e\right)$ as $\theta$. As usual, $\left(h^s,h^f\right)$ are given by (1) and (2).

The first natural interpretation of the training option is *formal training*. During formal training, workers improve their literacy and/or numerical skills (basic training), or learn some occupational-specific skills (vocational training) in the classroom. The cost $\kappa^{TR}$ is the per-period/per-head cost of administering the training course, and the probability $\theta$ denotes the likelihood of the worker passing the examination or attaining the degree in any given period. The second interpretation is that of a *transitional job*: the planner pays $\kappa^{TR}$ to the organization employing the worker for one period and providing her with some job-specific skills. At the end of the period, the worker is still unemployed, but with probability $\theta$ such on-the-job training was successful and human capital depreciation was contained.

**Social Assistance (SA):** In social assistance, the worker is "released" by the planner for the current period, in the sense that the planner does not demand high (search or training) effort, but simply transfers some income to the worker. The problem of the planner is

$$\begin{aligned}V^{SA}\left(U,h\right) &= \max_{c,U^f} -c+\beta\mathbf{V}(U^f,h^f)\\ s.t. &:\\ U &= u(c)+\beta U^f.\end{aligned}$$

The expression for $\mathbf{V}$ is given by equation (4) and the constraint describes how the promised utility $U$ can be delivered by a combination of current and future payments.

Below we will prove that if at any point during the contract the planner makes the "zero effort" recommendation, it is optimal to do so from that point onward: $SA$ is an absorbing policy. In light of this characterization, it is natural to think of $SA$ as a pure income-assistance program of last resort.

In what follows, it is convenient to state some basic properties of these value functions. By inspecting problem (5), it is easy to see that the value of employment has the following separable form:

$$W(U,h) = \frac{\Omega(h)}{1-\beta} - \frac{u^{-1}((1-\beta)U)}{1-\beta}, \tag{10}$$

where the first term, which can be computed recursively as

$$\Omega(h) = (1-\beta)\,\omega(h) + \beta\Omega(h^s),$$

is the discounted value of the stream of wages and the second term is the present value of the constant level of benefits guaranteed by the planner to an employed worker. It is easy to see that $W$ is a continuous function, which is decreasing and concave in $U$, and increasing in $h$.

By applying fairly standard results in dynamic programming, one obtains the same continuity, monotonicity and concavity properties for $\mathbf{V}$ as well, but two caveats are worth mentioning. First, monotonicity in $U$ is guaranteed whenever at $(U,h)$ the consumption level $c$ associated to the optimal program is positive (e.g., whenever $u(0) = -\infty$). Second, the concavity of $\mathbf{V}$ in $U$ is guaranteed because of the randomization in (4). In Appendix B (Proposition 0) we report the proof of all such properties. Finally, the properties of $\mathbf{V}$ are inherited by the value functions of each single policy $V^i$. In particular, all the problems defining policies $i \in \{JM, SA, TR, UI\}$ are also concave.

# 4 The Optimal WTW Program

We are now ready to study the key characteristics of an optimal WTW program. We begin with a discussion of the economics behind the choice among alternative policies, a useful step to understand two important preliminary results. First, we demonstrate that dynamics in human capital (accumulation and depreciation) are a necessary condition for the optimal WTW program to switch between policies. When an individual's human capital is constant, the optimal program involves only one policy stage, chosen optimally among JM, UI or SA, and no transition across policies (i.e., each policy is absorbing). Second, we show that even in the presence of human capital dynamics, social assistance maintains its absorbing nature.

We then turn to studying the optimal sequence of policies in the model with human capital dynamics. Here we heavily exploit the recursive formulation of the optimal contracting problem. By projecting the upper envelope (3) on the $(U,h)$ state space, we obtain a graphical representation of which policy is optimally implemented at every $(U, h)$ pair. The state space can then be divided into different connected areas, each corresponding to a specific policy whose value dominates all the others. A key step to this characterization is ranking the slopes of the value functions $V^i(U, h)$ for each policy $i$ with respect to its two arguments. Finally, we describe the dynamics of the government transfers associated to each policy phase of the program.

## 4.1 Choosing Among Policies: Economic Forces at Work

Within our model, there are several key economic forces that induce the planner to select one particular policy over the other three. For ease of exposition, we divide them into costs and returns. We begin by describing the costs and then move to the returns.

First of all, a planner who wants to implement $JM$ or $TR$ will have to incur certain direct expenses associated to the administration of the job search monitoring and training technologies (respectively, $\kappa^{JM}$ and $\kappa^{TR}$). The larger these costs are, the less attractive these two policies appear compared to $UI$ and $SA$.

Second, the planner must compensate the agent for her effort. Since $u$ is concave, the higher the promised utility $U$, the lower the marginal utility of consumption. Hence, the larger the payments of the planner must be to compensate the worker for the fixed disutility of the search/training effort cost $e$. This "wealth effect" makes $SA$ more attractive, compared to $UI, JM$ and $TR$, for high enough levels of $U$.

The third cost component is the cost induced by the presence of the incentive compatibility constraint. By using the promise-keeping constraint, the incentive compatibility constraint during unemployment insurance can be conveniently reformulated (independently of the unemployment benefit $c$), as

$$U^s - U^f \geq \frac{e}{\beta\pi(h)}. \tag{IC1}$$

The difference between the state-contingent utilities $U^s$ and $U^f$ is increasing as $h$ falls, through the hazard rate $\pi(h)$. Large utility dispersions are associated to large consumption dispersions. Since the agent is risk-averse, in order to compensate her for the wider spread of payments across states, the planner has to deliver a higher average transfer for any given promised utility $U$. In sum, incentive costs for the planner (i.e., resource costs of satisfying the incentive-compatibility restriction during $UI$) increase as human capital $h$ depreciates. Note that this cost is absent in $JM$ and is independent of $h$ in $TR$ (because $\theta$ does not depend on $h$).

Moreover, when the third derivative of $u^{-1}$ is positive, the cost of providing the utility lottery associated to (IC1), in terms of consumption payments of the planner, increases with $U$. Hence, the incentive costs (during both $UI$ and $TR$) increase with the level of promised utility $U$.[13]

[13] To understand this result, consider a static version of our model (hence, abstract from $h$). Let $g \equiv u^{-1}$ and let $C_i(U)$ be the cost for the planner of delivering utility $U$ under policy $i$. Then, it is easy to see that $C_{JM}(U) = \kappa^{JM} + g(U + e)$. In UI, from the (binding) IC and PK constraints,

$$U = -e + \pi u(c^s) + (1 - \pi) u\left(c^f\right) = u\left(c^f\right),$$

We now turn to the returns associated to each optimal policy. Social assistance has no direct returns for the planner. In $UI$ and $JM$, the returns to search in terms of job finding rate via $\pi(h)$, and earnings once employed via $\omega(h)$, are increasing in $h$. Since from (1) $h^s$ increases with $h$, the returns to human capital accumulation during training are of two types: a higher $h$ raises the effectiveness of future job search in $UI$ and $JM$ and it reduces the future incentive costs during $UI$; moreover, it increases earnings upon re-employment.

### 4.2 The "Absorbing" Nature of Social Assistance

Now, we are ready to state our first result. Consider a situation where, along some history, social assistance turns out to be optimal for the planner. Intuitively, given the absence of IC constraints, during SA the planner offers full insurance to the agent, and $U$ is constant. Because of depreciation, however, $h$ declines over time. As $h$ gets smaller, the incentive cost in $UI$ rises and the returns to search and training fall, hence any other alternative policy becomes less attractive compared to SA, which reinforces the optimality of SA. This mechanism delivers the following result.

**Proposition 1 (SA absorbing):** *SA is an absorbing policy. That is, if it is chosen at any period $t$, choosing it thereafter is optimal.*

**Proof:** *See Appendix B.*

Proposition 1 already establishes one key property of the optimal sequence of policies in a WTW program, as it rules out programs where incentive-provision or monitoring is offered after a spell of social assistance. As discussed in the Introduction, pure income support policies (like Food Stamps in the U.S.) tend to be interventions of last resort, in accordance with our result.

A consequence of Proposition 1 is that, because of its absorbing nature, and since $\pi(h,0)=0$, the equilibrium value of $SA$ does not depend on $h$ and can be written as

$$\hat{V}^{SA}(U) = -\frac{c^{SA}(U)}{1-\beta}, \tag{11}$$

where $c^{SA}(U) = u^{-1}((1-\beta)U)$ is the constant benefit paid to workers in $SA$ by the planner.

### 4.3 Human Capital Dynamics Are Necessary for Policy Transitions

In order to study the characteristics of the optimal program in absence of human capital dynamics, we consider the case where human capital is not a state variable (i.e., it remains always constant), thus $\pi(\cdot)$ and $\omega(\cdot)$ do

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which implies that the cost is $C_{UI}(U) = \pi c^s + (1-\pi)c^f = \pi g(U + \frac{e}{\pi}) + (1-\pi)g(U)$.

Differentiating with respect to $U$, we obtain that

$$C'_{UI}(U) - C'_{JM}(U) = \left[\pi g'(U + \frac{e}{\pi}) + (1-\pi)g'(U)\right] - g'(U+e)$$

which is positive for any $U$ if and only if $g'$ is convex (because $\pi(U+\frac{e}{\pi}) + (1-\pi)U = U+e$). Thus, under this condition, the cost of satisfying the IC constraint $[C_{UI}(U) - C_{JM}(U)]$ is increasing in $U$.

not depend on $h$ but $\pi, \omega > 0$. For example, this case occurs when $\delta = A = 0$. The planner's values of each policy are defined as in Section 3.2, without dependence on $h$. Since there is no human capital variation, there is no role for training policies.

In the next proposition, we show that the structure of an optimal WTW program in an economy without human capital depreciation is very sharp: each policy is absorbing.

**Proposition 2 (No human capital dynamics):** *If $\pi(\cdot)$ and $\omega(\cdot)$ do not depend on $h$, every policy (JM, SA, UI) is absorbing: if policy $i$ is chosen at the beginning of the program, choosing it thereafter is optimal. If, in addition, $\mathbf{V}$ is strictly concave, any optimal program must possess such absorbing characteristics.*

**Proof:** *See Appendix B.*

Consider the problem of a planner facing an agent with initial utility entitlement equal to a level $U_0$ where only one policy between $UI, JM$ and $SA$ is implemented with certainty.[14] For $U_0$ high enough, the search effort compensation cost is prohibitive and the planner will release the agent immediately into social assistance, which is absorbing.

Suppose now that $U_0$ is such that the planner decides to require the agent to supply positive search effort: the choice would be between either facing the $IC$ constraint or paying $\kappa^{JM}$ to monitor the agent's effort perfectly. As the utility entitlement falls, the $IC$ constraint becomes "cheaper" to satisfy, so for low enough initial levels of $U_0$, the planner will begin by enrolling the agent in $UI$, while for intermediate values of $U_0$ the planner will choose $JM$ as its initial policy. During $UI$, because of incentive compatibility, the state variable $U$ is decreasing which reinforces the optimality of $UI$ compared to the other available policies, since a smaller $U$ reduces the incentive costs. During $JM$ the agent is fully insured: since $\mathbf{V}$ is concave, keeping the promised utility constant over time and never switching out of $JM$ is always an optimal policy.

The same logic used in Proposition 2 can be used to rank the slopes of the value function with respect to $U$, a result that will be extended in the next section.

**Corollary to Proposition 2:** *The (negative) slopes of the value functions with respect to $U$ satisfy*

$$\hat{V}_U^{SA}(U) \geq V_U^{JM}(U) \geq V_U^{UI}(U),$$

*where the first inequality holds for any $U$, whereas the second inequality holds at the crossing point, i.e., at the unique (if any) $U$ where $V^{JM}(U) = V^{UI}(U)$.*

**Proof:** *See Appendix B.*

[14] In general at $U_0$ the optimal program might involve random assignment to different policies in period zero. The proof of Proposition 2 establishes that, in a stationary economy, period zero is the only period in which such random assignment might occur.

The value of unemployment insurance for the planner $V^{UI}$ falls more steeply than $JM$ with respect to $U$ because of the incentive cost, and $V^{JM}$ is steeper than $\hat{V}^{SA}$ because of the effort-compensation cost. These properties imply that the relevant value functions, if they cross, can cross at most once.

## 4.4 The Optimal WTW Program without Training

In this section we begin the characterization of the optimal WTW scheme in the presence of human capital dynamics. It is useful to start by abstracting from the training option and postpone its introduction to Section 4.5. Our ultimate aim is to answer questions such as: What is the optimal sequence of policies? What are the dynamics of unemployment benefits and taxes/subsidies upon re-employment, within each policy?

The recursive formulation naturally suggests the following two-step strategy to approach these questions. We will first derive a simple graphical representation of the regions within the $(U,h)$ state space where each policy arises as optimal, a result of its own independent interest. Next, by reading the $(U,h)$ state space as a phase diagram–whose dynamics are driven by the policy functions $U^{y}(U,h)$ describing the law of motion for the endogenous variable $U$, and by the exogenous laws of motion for human capital (1) and (2)–we will be able to recover the sequence of policies within the optimal WTW program.

Finally, the policy functions $c^{i}(U,h)$ and $c^{e}(U)$, together with the laws of motion for the two states, fully describe the optimal sequence of unemployment benefits, as well as wage taxes/subsidies during the optimal WTW program, since $\tau(U,h)=\omega(h)-c^{e}(U)$.

### 4.4.1 Representation in the (U,h) Space

In the Corollary to Proposition 2, we have established the relative slopes of the value functions with respect to $U$ in the stationary case. The next proposition establishes conditions under which the result remains true for the general case with human capital dynamics.

**Proposition 3 (Slopes of the value functions with respect to $U$)**: *Let $\eta(U,h)$ be the real number that solves*

$$\mathbf{V}_{U}(U,h)=-g'\left((1-\beta)\left(U+\eta(U,h)\right)\right),$$

*where $g\equiv u^{-1}$ is the inverse utility function. Assume that $\eta(U,h)$ is non-increasing in $U$ for every $h$. Then, the (negative) slopes of the value functions $V^{i}$ with respect to $U$ satisfy*

$$V_{U}^{UI}(U,h)\leq V_{U}^{JM}(U,h)\leq \hat{V}_{U}^{SA}(U)=-g'\left((1-\beta)U\right)\quad \textit{for all pairs } (U,h).$$

**Proof:** *See Appendix B.*

Given the convexity of $g'$, the required condition essentially establishes an upper bound on the curvature of $\mathbf{V}$. For example, in the stationary case with log utility, if the implemented policy is $SA$ then $\eta(U)=0$, if it is

$JM$ then $\eta(U) = e/(1-\beta)$, and if it is $UI$ then we can establish that $\eta(U) = \bar{\eta} > e/(1-\beta)$ (Pavoni, 2003a). From the definition of $\mathbf{V}$, it is therefore easy to see that, in this case, the condition of Proposition 3 is satisfied.

In the next proposition we establish an intuitive ranking on the slope of the value functions $V^i$ across the different policies $i = JM, SA, UI$, with respect to human capital $h$, under a somehow less restrictive assumption.

**Proposition 4 (Slopes of the value functions with respect to $h$)**: *Assume that the value function $\mathbf{V}$ is sub-modular and that both $\mathbf{V}$ and $V^i$ are differentiable in $h$. Then, their slopes with respect to $h$ satisfy*

$$V_h^{UI}(U,h) \geq V_h^{JM}(U,h) \geq \hat{V}_h^{SA}(U) = 0 \textit{ for all } U, h.$$

**Proof:** *See Appendix B.*

The logic of this proposition is that $V^{UI}$ is steeper than $V^{JM}$ because of the incentive cost and $V^{JM}$ is steeper than $\hat{V}^{SA}$ (invariant to $h$) because of the returns to search.

To understand the role of submodularity, recall that in the twice-differentiable case submodularity means $\mathbf{V}_{U,h}(U,h) \leq 0$. The shape of $\mathbf{V}$ is generated by two contrasting forces. First, "within-policy" there is a tendency toward *super*modularity: an increase in $h$ raises $\pi(h)$ and reduces the marginal cost of delivering a given level of utility $U$ since incentives are more easily provided. However, a high $h$ makes it more attractive to implement active policies like $JM$ or $UI$, and we saw that search-intensive policies have more negative slopes with respect to $U$. This "between-policy" force that tends to generate *sub*modularity of $\mathbf{V}$ hinges on the fact that the expected flow return $\pi(h)\omega(h)$ is an increasing function of $h$. The assumption in Proposition 4 holds whenever the second force dominates the first, for example, when $\pi(\cdot)$ is constant and the wage increases with skills, a case that will be analyzed in Proposition 6.[15]

Given our characterization of the relative slopes of $V^i$ with respect to $U$ and $h$, when the upper envelope $V(U,h) = \max_i V^i(U,h)$ is projected onto the $(U,h)$ space, as done in Figure 1, we obtain immediately the regions in the state space where each policy emerges as optimal. Note that these are well defined and connected regions. We start by interpreting Figure 1 as we move "horizontally" in the $(U,h)$ space, i.e., we let $U$ change for a given $h$. Next, we study the optimal policies as we move "vertically" through the diagram, i.e., we change $h$ for a given level of utility entitlement $U$.

**Moving horizontally (along $U$)**: Given any $h$, start from the highest utility level in the diagram. For high enough $U$, compensating the agent for the high effort is prohibitively costly, and $SA$ is optimal. Moving leftward, as we decrease $U$ the effort compensation cost falls and it becomes optimal to choose a program with high-effort requirement. For intermediate levels of $U$, the incentive cost is still high and the value of $JM$

[15] General conditions on the primitives for $\mathbf{V}$ to be submodular are hard to establish. One technical reason is that the nature of the *max* operator is to preserve *super*modularity, but not necessarily *sub*modularity (e.g., see Hopenhayn and Prescott, 1992).

dominates the value of $UI$. As we keep decreasing $U$, gradually the planner finds it more profitable to face the incentive cost rather than pay the fixed monitoring cost $\kappa^{JM}$, and $UI$ becomes optimal.

**Moving vertically (along $h$)**: Given any $U$, for high levels of $h$ (i.e., high $\omega(h)$ and $\pi(h)$), returns from search are high and incentive costs are low, so $UI$ is optimal. Moving downward, as $h$ falls incentive costs increase and the planner finds it optimal to pay the monitoring cost and implement $JM$. For very low levels of $h$, the returns to search are so low that the planner prefers to save the effort-compensation costs as well, and $SA$ is the optimal program.

#### 4.4.2 Optimal Sequence of Policies

The optimal sequence of policies is dictated by the evolution of the state vector $(U, h)$. Conditional on unemployment, $h$ declines monotonically, that is, whenever search is unsuccessful we have $h^f \leq h$. The evolution of $U$ depends on the specific policy. Formally, we have the following proposition:

**Proposition 5 (Optimal Policy Sequence):** *Assume that* $\mathbf{V}$ *satisfies all properties required in Propositions 3 and 4, and recall that* $\mathbf{V}$ *is a concave function. If* $\mathbf{V}$ *is either strictly concave or strictly submodular or both, then, if at some period* $t$ *the optimal program selects* $JM$*, then next period it is always optimal to either repeat* $JM$ *or switch to* $SA$*. In particular, an optimal WTW program never switches from* $JM$ *into* $UI$*.*

**Proof:** *See Appendix B.*

It is straightforward to show two properties of the dynamics of $U$. First, because of its absorbing nature and full insurance, during $SA$ the continuation utility $U$ is constant. Second, during $UI$, the utility entitlement $U^f$ promised by the planner to the unemployed worker when search fails tends to decline monotonically to satisfy the incentive constraint.

Third, perhaps surprisingly, during $JM$ the utility entitlement of the agent will *increase* (see Lemma A7 in Appendix B). There is a simple, almost mechanical, explanation for this behavior. As $h$ decreases along the unemployment spell, the optimal program approaches the social assistance option. Because of full insurance, the benefits $c$ are constant between $JM$ and $SA$, but once in social assistance, the agent will also save the search-effort cost $e$. Hence, the overall utility is higher in $SA$, and $U^f$ gradually increases during $JM$ as the implementation of $SA$ becomes more and more likely in the program.

Combining the dynamics of $h$ and $U$ with our graphical representation, we immediately obtain another key qualitative characteristic of an optimal WTW program: as soon as the program enters into the $JM$ region, it can either remain in $JM$ or switch to $SA$. In particular, a transition from $JM$ to $UI$ is never optimal because both human capital depreciation and the rise in promised utility during $JM$ worsen the incentive cost and make $UI$ less attractive over time.

Conditional on failure of search, the *typical sequence* of an optimal WTW program without training hence begins with $UI$ followed by $JM$ followed, in turn, by $SA$. Figure 1 illustrates a simulated individual history of an unemployed worker leading to this optimal sequence of policies.[16] Interestingly, this is also the typical observed sequence of actual programs in many countries: in general, the implementation of active policies starts after a spell of unemployment insurance.

Of course, with the same parameterization, for higher levels of initial promised utility $U_0$ (and/or lower levels of $h_0$) the optimal program will start in $JM$ and skip $UI$ altogether, and for even higher levels of $U_0$ (and/or even lower levels of $h_0$) it may start and end with $SA$. In contrast, for parameterizations where the monitoring cost $\kappa^{JM}$ is especially high, $JM$ may not arise as optimal at all and $UI$ will be directly followed by $SA$.

Part of our characterization of the state space and the optimal sequence of policies relies on restrictions on $\mathbf{V}$, which are endogenous.[17] Given the discussion following Proposition 4 on the submodularity of $\mathbf{V}$, it should be clear that one important special case of the model where such property is satisfied is an economy where the agent is subject only to wage depreciation during unemployment, while the hazard rate function $\pi(\cdot)$ is fixed at some constant $\pi > 0$. This important special case is also much easier to handle analytically, and we are able to provide a full characterization of the policy sequence by only imposing conditions on primitives of the model.

**Proposition 6 (No duration dependence):** *Assume that the hazard rate does not change with $h$ and that it is fixed at $\pi > 0$. Then, if at some period $t$ the optimal program selects $JM$, next period it is always optimal to either repeat $JM$ or switch to $SA$. In particular, an optimal WTW program never switches from $JM$ into $UI$.*

**Proof:** *See Appendix B.*

It may appear surprising, at first, that even though the hazard rate is constant, $JM$ should follow $UI$ since we argued that the rise in incentive costs as $h$ depreciates is associated to the fall in $\pi(h)$ and the corresponding widening of the utility spread in (IC1). There is, however, an additional reason why incentive costs go up as $h$ depreciates that survives as long as $\omega(\cdot)$ is increasing in $h$. In a multiperiod setting, the optimal incentive provision is shaped by the tension between intra- and inter-period consumption smoothing. The planner can improve intra-period consumption insurance (across employment states) by moving part of the punishment burden forward into the future. The emergence of $SA$, where $U^f$ cannot decline further, limits the use of the inter-period margin, forcing the $UI$ payments to be biased toward the static component of the incentives. This effective shortening of the time horizon during $UI$ makes the incentive cost higher for the planner, the lower is $h$.

[16] The parameterization of this example is exactly as in Section 6.1.

[17] It might be worthwhile mentioning that our numerical simulations have widely verified that the slopes of the value functions are those guaranteed by our assumptions on $\mathbf{V}$.

### 4.4.3 Optimal Benefits and Wage Taxes/Subsidies

We now turn to the qualitative characterization of the optimal sequence of the unemployment benefits and wage taxes/subsidies. We have the following result:

**Proposition 7 (Payments):** *(i) During unemployment insurance (UI), benefits are decreasing; (ii) during job search monitoring (JM), the benefits are constant and the wage tax (subsidy) is decreasing (increasing); (iii) during social assistance (SA) benefits are constant; (iv) if $\pi$ and $\omega$ do not depend on $h$ (fixed human capital case), during UI the wage tax is increasing over time, and during JM both the benefits and the wage tax/subsidy are constant.*

**Proof:** *See Appendix B.*

The need for incentive provision under UI implies that payments should decrease with unemployment duration. Benefits are constant in SA and JM because, within these policies, the absence of incentive problems allows the planner to implement full insurance.

The result on the structure of payments and taxes during UI in the model without human capital dynamics is a re-statement of Hopenhayn and Nicolini (1997) specialized to our environment. A direct consequence of *(iv)* is that wage subsidies are either paid at the beginning of the unemployment spell (for particular combinations of high $U_0$ and low $h_0$), or otherwise they will never be used: the government will never switch from a wage tax to a wage subsidy during the program. The presence of human capital depreciation changes predictions in two dimensions. First, the behavior of the wage tax during $UI$ becomes a quantitative issue, which will be discussed below. Second, since the expected gross wage $\omega(h)$ decreases during unemployment and $c^e$ is constant during $JM$, the wage tax $\tau(U,h) = \omega(h) - c^e(U,h)$ must now decrease and could become a subsidy.

In order to graphically illustrate the key features of the benefits paid across the various policies, it is useful to simulate the model. The bottom right panel of Figure 2 shows the path of wage depreciation. In particular, given the complexity of the algorithm, to reduce the grid points for $h$, we allow depreciation to be stochastic in the numerical simulations: upon realization of the outcome $y = f$, human capital either remains constant or falls by one grid point (the $h$ grid is spaced geometrically). The top left panel shows the behavior of the $UI$ benefits as a fraction of the initial wage, and the net wage (wage minus tax, or plus subsidy) that the unemployed worker would earn if she found a job in that period. The top right panel depicts the implied tax/subsidy, as a fraction of the current wage; the bottom left panel shows the dynamics of $U^f$ which are exactly those depicted on the state space in Figure 1.

As previously discussed, benefits (consumption during unemployment) decrease during $UI$ and remain constant throughout $JM$ and $SA$ because of consumption smoothing. The net wage (consumption during employment) first decreases and then tends to rise as $UI$ approaches $JM$. There are two main reasons for these dynamics. First, as human capital depreciates, expected utility dispersion must rise to satisfy the IC constraint ($IC1$) and part of this additional dispersion is generated in terms of a wider gap in current consumption across employment states. Second, as explained above, when $h$ depreciates, the effective time horizon of the $UI$ problem shortens and the planner must give incentives creating a wider spread between consumption across states. As a result, the planner starts using heavily wage subsidies in order to reward employment and to widen the difference between $UI$ payments and net wage upon job finding.[18]

When the worker enters $JM$, there is complete insurance also across employment and unemployment states, hence the net wage and unemployment benefits coincide and remain constant. Here, the behavior of the wage subsidy essentially mirrors that of the re-employment wage (and of human capital): a simple inspection of the bottom right and the top right panels shows that, indeed, once entered into $JM$ the wage subsidy increases if and only if human capital depreciates in order to keep consumption across states perfectly smooth. Thus, in the deterministic depreciation case, the subsidy rises monotonically.

## 4.5 The Optimal WTW Program with Training

We now allow the planner to access the training technology detailed in equations (9) and begin by discussing the typical region of the state space where training is the preferred policy instrument for the planner.

Formal training tends to emerge as optimal in the state space for low values of promised utility and for intermediate values of human capital. Since the training technology requires a positive input of effort and features an incentive constraint, we should expect either social assistance or job-search monitoring (or, possibly, both) to dominate training for high enough values of $U$ when compensating the training effort and satisfying the IC constraint is too costly for the planner. The existence of a fixed cost $\kappa^{TR}$ explains why $TR$ is too costly for low levels of human capital. Moreover, the fact that both $\pi(\cdot)$ and $\omega(\cdot)$ are bounded above implies that at high levels of $h$ the returns to training are too low to justify the training expenditure, so $TR$ will not be chosen for high enough values of human capital.

In Figure 3, we display two possible configurations of the training region. In the upper panel, we parameterized the training technology with a high fixed cost $\kappa^{TR}$, but also high returns; the training technology in the lower panel has instead low fixed cost and sharp decreasing returns, so the region where $TR$ is optimal shifts down in the space.[19]

It is evident that the model does not put tight restrictions on the position of the training phase in the optimal policy sequence. The two paths for $(U, h)$ plotted in the figure illustrate that in the case of the upper

[18] An advantage of allowing for stochastic depreciation in the numerical simulations is that we can choose a history of human capital where, for several periods (1 to 11), $h$ is constant, as assumed by Hopenhayn and Nicolini (1997), in order to illustrate that the features of the optimal contract they emphasize arise as a particular case of our setup: benefits, net wage, and $U_t^f$ never stop decreasing (and taxes never stop increasing) over this sub-period.

[19] For the upper panel, we have set $\kappa^{TR} = \$140$ and the return to training to 9.2%; for the lower panel, $\kappa^{TR} = 0$ and the return to training is 8.1%. In the data, we estimate the cost of training $\kappa^{TR} = \$162$ and its return to be 7%. See Section 6.1 for details.

panel, the typical sequence starts with $UI$, followed by $TR$ and then by $UI$ again; in the lower panel, $TR$ can be followed by $JM$.[20] We conclude that only a quantitative analysis, case by case, can shed light on the optimal policy sequence of a WTW program with training.

The histories of promised utility and human capital plotted in the two panels of Figure 3 highlight an interesting property of the optimal WTW program: after a successful spell of training, $U$ can rise in order to satisfy the incentive compatibility constraint, i.e., $U^s > U$. This increase in continuation utility is accompanied by human capital accumulation and the agent moves "north east" in the phase diagram, a result that has its counterpart in the dynamics of unemployment benefits, as stated in the following proposition.

**Proposition 8 (Payment under TR):** *Whenever the incentive compatibility constraint binds, benefits decrease when training fails and increase after a successful spell of training. When it does not bind, benefits are constant regardless of the training outcome.*

**Proof:** *See Appendix B.*

Therefore, during an optimal WTW program, benefits should genuinely rise if the verifiable outcome of $TR$ is positive as a reward for the worker's effort.

In Figure 4, we report the path of unemployment benefits and earnings subsidies upon re-employment for the same parametrization and the same history plotted in the lower panel of Figure 3. As for depreciation, we allow accumulation to be stochastic in these numerical simulations: upon realization of the outcome $y = s$, human capital either remains constant or rises by one grid point. The Figure shows that when $TR$ fails, in the first 4 periods of the training program, benefits decline, whereas when $TR$ is successful, in the next 10 periods, benefits increase rather steeply.

As for the tax/subsidy, when skills are rebuilt through successful training both the re-employment wage and the continuation utility $U^s$ promised by the WTW program tend to increase. The first force makes a tax upon re-employment more likely, while the second makes it less likely. More generally, the less effective the formal training technology is (e.g., small success probability $\theta$ and/or negligible human capital gain from training), the more likely the optimal wage tax (subsidy) is to decrease (rise) after a spell of successful training. For a given gain in the gross wage $\omega(h)$, a small value of $\theta$ will be associated with a higher value of $U^s$ and hence a higher promised consumption level upon re-employment. For given $U^s$, a small increase in human capital during training is associated to a low rise in $\omega(h)$, thus a larger subsidy is necessary to deliver the promised consumption level.

In our particular parameterization, after a successful spell of $TR$, human capital can either rise or remain constant. From Figure 4, one can see that when $h$ increases the re-employment tax rises, while when $h$ is constant

[20] Interestingly, one can also generate histories where $JM$ is chosen optimally both before and after $TR$, as in the mandatory sequence of the U.K. "New Deal" program discussed in the Introduction.

the tax falls, consistently with the discussion above on the forces shaping the optimal wage tax/subsidy during training.

# 5 A Simple Implementation with Access to Credit Markets

Throughout our analysis we have assumed that the agent cannot self-insure. In this section, we relax this assumption and allow the agent to save through credit markets at rate $R = \beta^{-1}$, but we maintain that she faces a no-borrowing constraint and starts without liquid assets. These two assumptions are not unreasonable for low-income workers on the welfare rolls. For example, Gruber (2001) documents that the median net financial wealth of the unemployed in the U.S. is zero. Crossley and Low (2005) arrive at the same finding for Canada where, in addition, 30% of the unemployed workers self-report to be unable to borrow.

Under these conditions, we demonstrate that with the help of an additional fiscal instrument, a linear interest rate tax, the planner can induce the agent not to save (by keeping her at the borrowing constraint) and, as a result, is able to fully control her consumption through the payments specified by the contract.

First of all, note that during $JM$ or $SA$ the optimal contract smooths the agent's consumption perfectly. It is easy to demonstrate that during a period of $UI$, for any level of human capital $h$, the payments of the optimal contract satisfy the following condition:

$$\frac{1}{u'(c_t)} = \pi(h) \frac{1}{u'\left(c^s_{t+1}\right)} + (1-\pi(h)) \frac{1}{u'\left(c^f_{t+1}\right)}, \tag{12}$$

where the superscripts $s$ and $f$ denote "success" and "failure" of search. During $TR$, this equation holds with $\theta$ in place of $\pi(h)$. From (12) and Jensen's inequality,

$$u'(c_t) \leq \pi(h) u'\left(c^s_{t+1}\right) + (1-\pi(h)) u'\left(c^f_{t+1}\right), \tag{13}$$

with strict inequality each time $c^s_{t+1} > c^f_{t+1}$ (a typical situation under $UI$ or $TR$ where the allocations must be incentive compatible and the planner cannot offer full insurance). This inequality shows that the optimal payment scheme always forces the agent to under-consume next period, compared to its individual optimum: the agent would then choose to *save* at time $t$ in order to increase consumption next period and equate current and expected marginal utility of consumption (Rogerson, 1985).

Therefore, the sole imposition of a borrowing constraint does not rule out a situation where individual consumption would diverge from the benefits paid by the planner–a failure of the optimal contract: the planner must prevent the agent from saving.

A simple linear tax $\tau^k$ on interests that restores equality in (13) is not enough to guarantee that the agent would not be willing to save. Indeed, the most profitable deviation for the agent is joint: the agent would reduce effort to zero and save at the same time. Because of the incentive constraint, typically $c^s_{t+1} > c^f_{t+1}$, hence a reduction in effort makes the consumption distribution shift toward the worst outcome, which in turn generates an additional incentive to save at $t$ to finance consumption at time $t+1$.

Consider instead a linear tax $\tau_t^k$ on the interests from savings that, for any $t$, satisfies the first equality below

$$u'(c_t) = \left(1-\tau_t^k\right) u'\left(c_{t+1}^f\right) > \left(1-\tau_t^k\right)\left[\pi(h)\, u'\left(c_{t+1}^s\right) + \left(1-\pi(h)\right) u'\left(c_{t+1}^f\right)\right].$$

The expression above demonstrates that an agent facing such tax is never willing to save, neither considering the joint deviation "save and shirk"(and the above Euler equation holds with equality), nor in equilibrium, i.e. when she follows the effort recommendations of the contract (and the above Euler equation holds with the strict inequality). The agent would always be willing to borrow, but because of the liquidity constraint, the planner maintains full control on her consumption stream and the optimal WTW contract characterized in the previous sections can still be implemented. In particular, it is easy to see that when utility is logarithmic in consumption, the planner can simply set a constant interest tax determined as

$$\tau^k = \max_t \left\{\frac{c_t - c_{t+1}^f}{c_t}\right\}.$$

In other words, the tax is proportional to the steepest slope of the unemployment benefits path along the optimal WTW program.

This scheme can also be adapted to situations where the initial (but observable) wealth is positive as long as it is not "too high". In these cases, the agent must be forced immediately toward the borrowing limit with an appropriately chosen initial transfer. Assuming that unemployment benefits cannot be negative, it is easy to show that the optimal WTW program can still be implemented for initial wealth levels up to the payment specified by the optimal WTW program at time $t=0$. In extreme cases, the optimal WTW program would require a *waiting period* without payment of benefits.[21]

Finally, note that this implementation mechanism is totally anonymous, since it does not require the planner to observe individual saving decisions, but it only demands control over the *aggregate volume* of savings.[22] We acknowledge that this implementation of the optimal contract where the agent's consumption is fully controlled by the government may not be too appealing for the design of an optimal taxation scheme in the aggregate economy (as in Golosov and Tsyvinski, 2003; Albanesi and Sleet, 2005; Kocherlakota, 2005). However, we think it has the advantage of being much simpler than those proposed in the existing literature and it fits well the case of low-income, low-wealth workers on the welfare rolls, which are the target of our study.

[21] Interestingly, in several states (e.g., Texas and California) UI benefits start to be paid one week after reporting the unemployment status. This rule corresponds exactly to a zero initial transfer to induce the agent to dissave. Our result echoes the finding by Wang and Williamson (2002) who conclude that when unemployed agents have enough self-insurance savings available, in the initial phase of the optimal contract benefits could be back-loaded.

[22] This requirement can be guaranteed, for example, by the presence of financial intermediaries which are allowed to maintain secrecy on the identities of the specific depositors, but whose aggregate volume of transactions is monitored for taxation proposes, and are required by the government to act as withholding agents, i.e., they deduct a withholding tax from all interest payments, and transfer the total revenues to the government. These institutional characteristics are actually present in some countries such as Italy and Sweden, for example.

# 6 Quantitative Analysis: Assessing the Optimality of Some U.S. Welfare Programs

In this section, we illustrate how our framework can be used for quantitative work. The first step is the calibration of the model to match some salient features of the U.S. labor market and of the current U.S. welfare system. We can then simulate histories of unemployed workers with different initial skill levels $h_0$ undergoing the existing welfare program and calculate the expected initial utility entitlement $\bar{U}_0(h_0)$ promised by the current system. Next, we solve for the optimal WTW program that delivers the same expected utility and contrast its features, in terms of optimal sequence of policies, time-path of benefits and structure of taxes/subsidies, to the current program. We then calculate the budget savings for the government associated to switching to the optimal WTW scheme.

## 6.1 Calibration

It is useful to divide the parameters of the model into four groups: first, the preference parameters $\{u(\cdot), \beta, e\}$; second, the labor market parameters $\{\omega(h), \delta, \pi(h)\}$; third, the parameters of JM and TR policies $\left\{\kappa^{JM}, \kappa^{TR}, \theta, A, \alpha\right\}$; fourth, the set of parameters characterizing the current U.S. welfare system $\left\{\bar{d}^{UI}, \bar{d}^{JM}, \bar{d}^{TR}, \bar{c}^{UI}, \bar{c}^{JM}, \bar{c}^{TR}, \bar{c}^{SA}, \bar{\tau}(h)\right\}$, i.e., observed durations $\left(\bar{d}^{i}\right)$ for each policy phase, observed payments $\left(\bar{c}^{i}\right)$, and the observed tax/subsidies $(\bar{\tau})$ in the actual U.S. scheme as a function of earnings.

The unit of time for our calibration is one month. We focus on the period 1991-1999, since the main source of our data on welfare-to-work policies refers to that period (see below). Next, we outline our calibration strategy, and in Table 1 we list the calibrated parameter values.

### 6.1.1 Demographics and Preferences

**Household and period of reference:** The typical individual on the welfare rolls is a 30-year-old single parent with two children. Roughly half of them hold a high-school diploma, and the other half are high-school dropouts (Moffitt, 2003). This is our household of reference in the exercise.

**Preferences:** We pick a value for the monthly discount factor $\beta = .9957$ in order to match an interest rate of 5% on a yearly basis, and use a logarithmic specification for the period-utility over consumption. We assume that the disutility of work effort equals the disutility of search/training effort $e$. To set a value for $e$, we follow a common practice in calibrated macroeconomic models. Suppose that the disutility of the fraction of time $n$ spent working/searching/training is also logarithmic, and let $\phi$ be the preference parameter measuring the relative weight on leisure versus consumption. From the static optimality condition of the agent and the observation that the labor share is 0.60, the consumption-income ratio is 0.75 and the fraction of time worked is

$n = 0.3$, one obtains $\phi = 1.78$ (Cooley, 1995). Then, the disutility of effort is $e = \phi [\ln(1) - \ln(1-n)] = 0.63$. This effort cost corresponds to roughly 1/4 of the utility associated to consuming a monthly wage of \$1,500.

### 6.1.2 Labor Market Parameters

**Wage function:** We assume a linear (monthly) wage function $\omega(h) = h$, where human capital $h$ is interpreted as efficiency units of labor in a competitive labor market. Thus, changes in human capital map directly into observable wage changes. We normalize $h$ so that one unit corresponds to \$100, e.g., $h = 15$ means a monthly wage of \$1,500.

**Wage depreciation:** The existing microeconomic estimates of wage depreciation span a wide range. At the high end, Keane and Wolpin (1997) within a structural model estimate an average annual skill depreciation rate of 23% (36% for white collars and 9.6% for blue collars). At the other end, the average annual earnings loss upon displacement computed by Jacobson, LaLonde and Sullivan (1993, Table 3) is roughly 10%. In the middle of the spectrum, Neal (1995, Table 3) reports that an additional week of unemployment reduces the re-employment wage by 0.37%, implying an annual decay rate of 17.5%. Addison and Portugal (1989) find that a rise in duration by 10% reduces post-displacement wages by 1%. Since the average duration of unemployment corresponds to 8.5 months in their sample, they implicitly estimate a yearly skill depreciation rate of 16%. Overall, choosing a value of 15% per year seems appropriate.

The numerical load required to solve for the optimal dynamic contract in our setup is quite heavy. Thus, to reduce the number of grid points for human capital, it is useful to allow depreciation to be stochastic: workers either keep their human capital level with probability $q^f$, or move down one step on the human capital grid, with probability $1 - q^f$. In order to have a constant expected depreciation rate for all levels of human capital, we set a geometrically spaced grid.[23]

**Hazard function:** To estimate the hazard function, we exploit information on the length of the current unemployment spell for workers who report to be unemployed in the Basic Monthly Current Population Survey (CPS). Within the window 1991-1999, the year from May 1995 to April 1996 witnessed a very stable unemployment rate, always between 5.5% and 5.7%. We choose these 12 months for our estimation in order to avoid issues of non-stationarity in the parameters. We restrict our sample to workers with at most a high-school degree, between 18 and 50 years old. Our estimation strategy follows closely the method outlined by Flinn (1986) and assumes a Weibull distribution with hazard function

$$\mathfrak{p}(t) = \alpha \lambda t^{\lambda - 1},$$

where $t$ is the length of the unemployment spell.

[23] If the grid is geometrically spaced at rate $\Delta$, and $x^f$ is the estimated monthly depreciation rate of human capital, $q^f$ solves $q^f = 1 - x^f/\Delta$.

The estimated hazard displays negative duration dependence $\left(\hat{\lambda} = 0.66\right)$, even after controlling for unobserved heterogeneity. The top panel of Figure 5 shows that the bulk of the drop in the job finding probability occurs in the first 12 months, after which the hazard keeps decaying more slowly. This qualitative feature is in line with the existing empirical evidence.[24] The bottom panel maps the hazard $\mathfrak{p}(t)$, as a function of duration, into a function of human capital $\pi(h)$, based on a depreciation rate of 15% per year and an initial earnings level of $1,500 ($h_0 = 15$), the average of our CPS sample. See Appendix C for more details on the estimation procedure.

#### 6.1.3 Parameters of JM and TR Technologies

The U.S. Department of Education and the U.S. Department of Health and Human Services jointly sponsored a large-scale evaluation of welfare-to-work policies, the *National Evaluation of Welfare-to-Work Strategies* (NEWWS), based on data pertaining to over 40,000 individuals followed for 5 years, (falling between 1991 and 1999, depending on the location), in 11 different locations. This study uses a rigorous research design based on random assignment of individuals to a treatment group and a control group, in each program.[25] This ensures that there are no systematic differences between background characteristics of people in the different groups.

In particular, three locations, Atlanta (GA), Grand Rapids (MI), and Riverside (CA) simultaneously operated, expressly for the evaluation, two different mandatory programs, a "labor force attachment" (LFA) program focused on job-search monitoring and a "human capital development" (HCD) program focused on training. In what follows, we only use data referring to these three locations for our calibration. Of the several adult training programs implemented (basic education, vocational training, post-secondary education, transitional jobs), due to data constraints we can only focus on "adult basic education" courses targeted to the achievement of a high-school diploma or an equivalent degree (GED).[26]

**Costs**: NEWWS (2001, Tables 13.2 and 13.3) documents in detail the operational costs of running each specific activity. An average across the three locations for the LFA programs reveals that the job-search monitoring cost is roughly $478 per month/per worker, and a similar average for the HCD programs shows that formal training costs $162 per month/per worker. These two numbers map exactly into our parameters $\kappa^{JM}$ and $\kappa^{TR}$, respectively.

**Success rate of training**: NEWWS (2002, Figure 3.3) reports that, on average, 20% of workers enrolled in

[24]For example, van den Berg and van Ours (1996) conclude that, in the U.S., the exit probability from unemployment falls by 30% after 3 months of unemployment. Blank (1989) finds that the bulk of the decline in the welfare hazard occurs before two years. For the U.K., Nickell (1979) finds a 50% decrease in the hazard rate after 15 months of unemployment, and van den Berg and van Ours (1994) report a decrease by over 30% after 6 months of unemployment.

[25]The control group was completely excluded from any program for at least 3 years. After 3 years, only 6% of the control group received some treatment.

[26]Data on the operational costs of each program are available by type of training, but detailed information on the success rate of training and, conditional on success, on its earnings and employment impact are only available for basic adult education.

adult education receive a high-school diploma or GED at the end of every month of enrollment, i.e., the average duration of training is roughly 5 months. It is natural to consider the achievement of these credentials as the successful outcome, hence we base our value for the success rate of training $\theta$ on this number. Interestingly, the success rate appears flat for a wide range of durations (2-18 months), exactly as in our specification, where it follows a stationary Poisson process.

**Return to training:** Among participants to basic training classes within HCD programs across our three locations, NEWWS (2002, Table 4.7) estimates that those who received a diploma earned 17% more than those who didn't per each quarter worked, three years after the treatment.[27] This value for the return of adult education is on the high end of the range of existing estimates. LaLonde (1995), and Heckman, LaLonde and Smith (1999) indicate that the typical return to public training is far below 10%.

This return to basic adult training contains both a pure wage gain and an impact on re-employment probabilities during the quarter. We need to isolate the first component to properly calibrate $A$. To this end, we use the fact that the NEWWS data also report the effect on annual earnings, besides quarterly earnings, and find it to be 46%. Comparing the quarterly and annual returns, we conclude that roughly 60% of the return is in terms of employment and 40% in terms of wages. This finding is in line with the existing literature: Jacobson, LaLonde and Sullivan (2003), in a study on the impact of classroom training on displaced worker, find that between 1/2 and 2/3 of the total earnings increase is associated with higher likelihood of becoming employed. In light of this evidence, we set $A$ to generate a permanent rise of 7% in monthly wages for workers going through successful training compared to the "untreated".

It remains to verify how a 7% change in $h$ affects the re-employment probability in the model, given our estimated hazard $\pi(h)$. To replicate the total earnings gain of 17%, the hazard rate should increase by approximately 10%. The effect on the hazard rate is nonlinear, given the shape of the Weibull, but it is simple to compute that based on our estimate, on average, augmenting $h$ by 7% moves up the hazard by 11%. Finally, given the inability to identify the curvature of the training technology from the available data, we assume a linear technology with $\alpha = 1$.[28]

### 6.1.4 Parameters of the Existing U.S. Welfare System

Several pieces of legislation over the years have built a network of federal and state government interventions in the U.S. labor market. Since in Appendix D, we list in detail the major components of the U.S. welfare system, here we keep the exposition to a minimum.

[27] To partially control for selection, we have used the estimate of the marginal effect of receiving a diploma conditional on the score of a literacy test (TALS) administered upon entering the program.

[28] To parsimoniously parameterize the stochastic training, we replicate what we do for the depreciation technology. We assume that workers can either keep their human capital level with probability $q^s$, or move up one step on the geometric human capital grid, with probability $1 - q^s$.

**Policy durations:** As they become unemployed, for the first 6 months workers are entitled to unemployment insurance. At the expiration of the UI phase, workers enter the Temporary Assistance for Needy Families (TANF) regime and are subject to mandatory active labor market programs. Based on the strictest time limits rules for TANF discussed in Appendix D, we assume that this regime lasts for 24 months. Broadly speaking, there exist two types of mandatory programs: Human Capital Development (HCD) programs where the emphasis is on training, and Labor Force Attachment (LFA) programs, where the focus is on job-search assistance and monitoring. Given the available information on the length of training and job monitoring, we model the HCD programs as a maximum of 18 months of training followed by a maximum of 6 months of assisted job search, and the LFA programs as a maximum of 18 months of monitored job search followed by a maximum of 6 months of basic education. We use these observed durations to calibrate $\left\{\bar{d}^{UI}, \bar{d}^{JM}, \bar{d}^{TR}\right\}$.

**Benefits:** During UI, workers are entitled to benefits comprising a replacement of 60% on their past earnings, plus food stamps. On a monthly basis, for a family of three, the maximum allotment of food stamps (averaged across Atlanta, Grand Rapids and Riverside) was $290 in the period under consideration (NEWWS 2001, Table 2.1). We also calculate that a typical household of three enrolled in the mandatory programs in any of these locations received average welfare payments (TANF benefits plus food stamps) of $740 per month independently of the search/training activity (NEWWS 2001, Table 2.1). When the TANF time limits are reached, we assume that workers receive only food stamps as social assistance. These values allow to calibrate $\left\{\bar{c}^{UI}, \bar{c}^{JM}, \bar{c}^{TR}\right\}$.

**Taxes/Subsidies:** In the event individuals find employment, they are subject to an unemployment tax (FUTA and state tax) at the flat rate of 0.8%. Workers' earnings are subsidized exactly as indicated by the Earned Income Tax Credit (EITC) legislation for a family of three in the period

**Table 1: Summary of the Parameterization**

| Parameter | Value | Moment to Match |
|---|---|---|
| $\beta$ | 0.9957 | Interest rate (Cooley, 1995) |
| $e$ | 0.630 | Fraction of time spent working (Cooley, 1995) |
| $\delta$ | 0.0135 | Monthly human capital depreciation (various sources) |
| $\pi(h)$ | see text | Unemployment hazard function (Basic CPS 1995-1996) |
| $\kappa^{JM}$ | $478 | Monthly cost of JSM (NEWWS, 2001) |
| $\kappa^{TR}$ | $162 | Monthly cost of TR (NEWWS, 2001) |
| $\theta$ | 0.20 | Fraction of workers in TR receiving degree (NEWWS, 2001) |
| $A$ | 1.07 | Wage gain upon successful TR (NEWWS, 2002) |
| $\bar{c}^{UI}$ | $0.60 \cdot \omega$ | UI Benefits (U.S. Department of Labor) |
| $\bar{c}^{JM}, \bar{c}^{TR}$ | $740 | TANF Benefits (NEWWS, 2001) |
| $\bar{c}^{SA}$ | $290 | Max allotment of Food Stamps (NEWWS, 2001) |
| $\bar{d}^{UI}$ | 6 | Duration of UI in months (U.S. Department of Labor) |
| $\bar{d}^{JM}$ | $LFA = 18, HCD = 6$ | Duration of JSM in months (NEWWS, 2001) |
| $\bar{d}^{TR}$ | $LFA = 6, HCD = 18$ | Duration of TR in months (NEWWS, 2001) |
| $\overline{\tau}(h)$ | see text | FUTA and EITC (Hotz and Scholtz, 2001) |

under consideration–see Appendix D for details. These two institutions determine jointly the function $\bar{\tau}(h)$.

Two key inputs of the normative analysis are the initial utility entitlement promised implicitly by the actual U.S. welfare program $\bar{U}_0(h_0)$, and the associated stream of public expenditures $\bar{V}(h_0)$. In computing these values, we follow closely the strategy suggested by Hopenhayn and Nicolini (1997). Since both employment and social assistance are absorbing states, by backward induction it is easy to reconstruct the initial expected utility entitlement a worker with initial human capital $h_0$ would receive under the current U.S. welfare system and the net government expenditures associated to this level of utility.[29] For example, the expenditures include the benefits and wage subsidies paid to the worker (during unemployment and employment, respectively), and the costs of operating training and job search monitoring programs for the durations specified above, net of the tax levied on earnings.

## 6.2 Results

We begin by studying the optimal sequence of policies and payments for two types of workers, a high-skilled worker with pre-displacement monthly wage of $1,500 ($h_0 = 15$) and a low-skilled worker with monthly wage of $600 ($h_0 = 6$). In what follows, when we refer to the "actual" system, we mean the LFA variant, and analyze how the planner would deliver the same utility entitlement implicit in the LFA version of the actual program. At the end of this section, we comment on how the results differ when we consider the HCD variant.[30] The average public expenditure per month (during the entire lifetime of the worker) implicit in the current system amounts to $215 for the high-skilled type and $247 for the low-skilled type, suggesting that low-skilled workers are more expensive, mainly due to their lower job-finding rates.

Figure 6 contrasts some key features of the optimal WTW program to the features of the actual U.S. welfare system for these two types of workers. Recall that in our simulations the evolution of $h$ is stochastic, and both payments and policy assignments depend on $h$. In order to provide a general idea of the main quantitative features of the optimal WTW program, for each type of worker we generate 10,000 histories of human capital shocks, and shocks to the training and search outcomes (success/failure). We then calculate sample averages of the optimal time-path of payments upon unemployment, taxes/subsidies upon employment, and compute the fraction of workers assigned to each policy since the initial displacement.

**Optimal sequence of policies:** The right hand side panels displays the fractions of workers assigned to the various policies at each unemployment duration. All high-skill workers start in $UI$, exactly like the actual

[29] In computing these values, we always let the worker optimally choose between high and low effort, except when she is in the job-search monitoring phase of the program, where high search effort is perfectly enforced.

[30] In the numerical solution, we discretize $h$ while we treat $U$ as a continuous variable. We assume that $h$ can take only values over a discrete set and choose a grid with 30 points geometrically spaced over the range $[0, 20]$. The grid for $U$ has 500 equidistant points in the interval $[100, 700]$. For given $h$, the value functions with respect to $U$ are computed using Chebyshev polynomials up to the 20th order. Details of the computation procedure are available upon request.

scheme. As human capital depreciates, workers begin to be gradually moved to $JM$, and the fraction of workers in $UI$ decreases while that in $JM$ steadily increases. After roughly 2 years of unemployment, the fraction of workers in $JM$ also begins to fall as the flow from $JM$ into $SA$ more than counterbalances that from $UI$ into $JM$. For sufficiently long durations, all unemployed workers end up in $SA$. The typical policy sequence of the optimal WTW program for a high-skilled worker is therefore UI-JM-SA. The median duration of each stage is 24 months for $UI$ and 8 months for $JM$. Recall that in the actual program, UI benefits are paid for 6 months and the combination of active policies (JM and TR) lasts for 24 months. Therefore, the median duration of the pre-SA phase of the optimal program (32 months) is remarkably close to the actual one, estimated at 30 months.

A striking feature of the optimal program for high-skill workers is that only a negligible fraction is ever assigned to basic training. In terms of the $(U, h)$ space, training appears only in a tiny region which skilled workers reach only under very few histories of human capital depreciation.

The unskilled workers start off in $UI$ as well, but a large fraction of them moves straight into $SA$, without any treatment through search monitoring or training. The reason for this lower transition rate across policies lies in the shape of the hazard function. As is clear from Figure 5, since for high levels of human capital the hazard is much steeper than for low levels, skill depreciation induces a bigger rise in the incentive cost for high-skill types, making $JM$ more attractive. Interestingly, when they transit across programs, low-skilled workers are assigned more often to training than their high-skill counterparts: after a first stage of $UI$, the optimal program mixes JM and, to a much less extent, TR. The median duration of $UI$ is 30 months; those workers who are subject to active policies during unemployment spend a median time of 4 months in $JM$ and 3 months in $TR$.

Overall, the optimal WTW program features longer unemployment insurance than the actual scheme and, for skilled workers, uses search monitoring quite heavily, in line with LFA programs in the U.S., whereas it makes only very limited use of basic education for the low-skill workers.[31] Simply put, it appears that $TR$ is too expensive and less effective compared to the other options available to the planner. Recall that we argued that our calibrated returns to basic education are on the high end of the existing estimates, thus the reason why $TR$ is barely chosen is not one of negative net returns of this policy per se (as often argued in the empirical evaluation literature), but rather one of opportunity cost for the planner of giving up more effective instruments.

**Unemployment benefits and wage subsidies:** The upper-middle panel shows that, for skilled workers, the average optimal replacement ratio for welfare benefits (the solid line) is more generous than the actual scheme (the dotted line). The optimal payments decrease smoothly from 95% to 75% of the pre-displacement wage, while the payments in the existing U.S. program never exceed 80% of the pre-displacement wage for our benchmark worker, and drop to 19% after 30 months. This higher benefit level of the optimal WTW program

[31] Given that our $TR$ technology is calibrated on basic adult education programs offering GED or high-school equivalent degrees, it is reasonable to obtain that only our low-skilled types (e.g., high-school dropouts) should be assigned to $TR$ during unemployment.

is entirely linked to the expected utility entitlement offered by the current system.[32] The fact that the optimal benefits decline so smoothly implies that the interest tax needed to implement the program when workers have access to credit markets would be only 0.9%.

In the upper-right panel, we compare the optimal and actual (the latter made up by FUTA and EITC) structure of earnings subsidies as a fraction of the re-employment wage. Note that the dependence of the actual subsidy on duration is not directly imposed by the legislation, but it occurs anyway through the depreciation of earnings power during the unemployment spell.[33] The optimal scheme traces quite closely the actual subsidy (around 15%) for the first year, but subsequently it increases much less with duration, so for long durations the optimal subsidy ends up being just 2/3 of the actual one. The presence of a subsidy in the optimal program is in sharp contrast with what Hopenhayn and Nicolini (1997) found for the stationary model without human capital depreciation. In Section 4.4 we discussed in detail the mechanism generating such result; in particular, recall that the subsidy rises fast in correspondence to the phase where $JM$ is used more intensively.

In the bottom-middle and bottom-right panels of Figure 6 we report benefits and subsidies for the low-skill workers with pre-displacement monthly earnings of \$600. The optimal sequence of benefits declines faster than for the skilled workers (from 94% to 66% in 5 years) due to the more intense use of incentive-based policies like $UI$ and $TR$, compared to $JM$. The current system appears to underinsure the unskilled worker at long durations–as for the high-skill type–but it provides too much insurance at short durations, with replacement ratios exceeding 100%. Once again, the optimal subsidy is substantially smaller than the actual one, by at least 10%.

To conclude, for skilled workers in our group, the current system exceeds in providing incentives through the static margin, by over-rewarding employment and over-punishing unemployment. In doing so, it does not provide enough insurance. For unskilled workers instead, the existing scheme under-provides incentives, especially at short durations.

**Budget savings and welfare gains:** The government budget savings are calculated by comparing the actual expenditures $\bar{V}_0(h_0)$ to the expenditures $\mathbf{V}\left(\bar{U}_0(h_0), h_0\right)$ that the planner would incur by delivering utility $\bar{U}_0(h_0)$ under the optimal program. The welfare gains are computed by comparing the actual utility entitlement of the current system $\bar{U}_0(h_0)$ with the level $U_0(h_0)$ that the planner can deliver by spending exactly as much as the actual program in the optimal scheme, i.e., $U_0(h_0)$ solves the equation

$$\mathbf{V}\left(U_0(h_0), h_0\right) = \bar{V}(h_0).$$

The welfare gain is then expressed in terms of fraction of lifetime consumption.

[32] Also Hopenhayn and Nicolini (1997) uncovered very high replacement rates, close to 100%, in the optimal UI scheme (with employment taxes) that offers the same discounted utility as the actual program.

[33] The tax/subsidy paid by the current U.S. scheme is computed by applying the rules for FUTA and EITC to each simulated history, and averaging out across histories. The calculation of the optimal tax/subsidy is similar, except that we excluded observations where the worker in $SA$ since it is an absorbing state with no re-employment possibility.

For our high-skill workers, the cost savings are of the order of 4% and the welfare gain is 0.3% of lifetime consumption. Such small welfare gain is due to the fact that skilled workers have very low unemployment durations (over 80% find a job within 6 months), so they quickly join the employment ranks and, upon employment, the actual wage subsidy is quite close to the optimal subsidy. For low-skill workers, instead, budget savings are just below 35% and the welfare gain of switching to the optimal program amounts to 5.8% of lifetime consumption, suggesting that the existing scheme could be substantially improved.

**Comparison with HCD programs:** When $\bar{U}_0$ is calibrated according to the HCD variant, the optimal WTW program presents very similar features, with slightly less generous payments in both states since the calculated initial utility entitlement under HCD is smaller than that computed under the LFA option. This is not surprising, given that training is not used much in the optimal WTW program. As a result, budget savings and welfare gains of shifting to the optimal WTW program are larger. For the skilled workers, cost savings reach 22% and welfare gains can be up to 1.5% of lifetime consumption; for the unskilled, we obtain budget savings beyond 50% and welfare gains around 8.8%.

If one wishes to draw some simple policy lessons from our analysis, perhaps the central findings are two. First, with respect to the choice among "active policy" instruments given the available technologies (i.e., costs and returns), programs focusing on search monitoring seem to be better designed than programs centered on basic adult education. Therefore, our analysis, which comes from a completely different angle, reinforces the typical findings of the evaluation literature (Heckman, LaLonde and Smith, 1999). Second, with respect to benefits and subsidies, a reformed system should offer more insurance to the skilled unemployed workers and provide more incentives for the unskilled.

# 7 Concluding Remarks

Welfare-to-Work programs combine passive and active labor market policies in an attempt to solve a complex trade-off between providing insurance to jobless workers and offering an incentive structure that will move them quickly among employment ranks.

In this paper we have provided a theoretical framework to study welfare-to-work programs from a pure normative standpoint. We have tried to tackle a large set of important questions, such as: What is the optimal sequence of policies in an optimal WTW program? And, how long should each policy stage be? What is the optimal level and dynamics of payments in each phase of the program? Should wages upon re-employment be taxed or subsidized? Our theoretical characterization offers sharp answers to some of these questions, but only general guidelines to other questions. In this latter case, we showed how a numerical analysis based on the calibrated model does an exhaustive job and can be used for quantitative policy analysis.

Overall, our work represents a first step toward a better understanding of the optimal design of WTW programs. As such, it has a number of limitations that future research on the topic should address.

By introducing a separation probability during employment, one can make both labor market states transient and characterize the stationary distribution of workers over the state vector $(U, h, z)$. This more general problem would also allow one to define an economy-wide government budget constraint and determine endogenously the initial utility entitlement $U_0$ associated to the self-financing optimal WTW program. Such a program would also allow one to analyze the optimal cross subsidization between workers with different skill levels. A richer model would also incorporate incentive problems during employment and assign a more prominent role to the firm in the analysis, as in Zhao (2001) and, more recently, Blanchard and Tirole (2004). Looking further ahead, equilibrium effects of large-scale programs should be considered in the analysis.

Some of the active policies we studied could, arguably, be modelled differently. For example, on-the-job training implicit in publicly funded transitional employment could be more effective than the basic classroom training we analyzed in the quantitative section (see Kirby et al., 2002, for a comprehensive evaluation of these "work-first" programs). Or, monitoring training effort at a cost may turn out to be effective. Further, the objective of the job-search monitoring program we modelled is exclusively that of enforcing search effort. Often, these programs also aim at improving workers' search effectiveness–for any given effort level–by providing counselling and explicit job placement services, an aspect we have neglected in our analysis. The challenge, as emphasized by Meyer (1995), is to obtain reliable data to separate these two aspects of job-search assistance policies. In the rare instances where the data allow it (e.g., van den Berg and van der Klaauw, 2001, for the Netherlands), the conclusion is that the aspect we have emphasized in this paper, monitoring, is far more effective.

Even though we compare quantitatively the optimal WTW program to the current U.S. welfare scheme, we have not questioned in depth the role of the government in providing insurance to the unemployed and credit for training. Theoretically, Abraham and Pavoni (2004) prove that in the presence of moral hazard due to hidden action (hence, differently from the hidden information case of Cole and Kocherlakota, 2001) the constrained-efficient allocations improve upon self-insurance even with hidden savings, which means that, at least qualitatively, in our economy there is scope for government-sponsored programs. Quantitatively, it remains to be established whether their welfare gains are sizeable. Some empirical studies hint that this could be the case. Gruber (1997) finds that unemployment compensation is a decisive help in consumption smoothing for the U.S. unemployed; Chapman, Crossley and Kim (2003) find that, in Canada, a significant fraction of job seekers are credit-constrained in the choice of acquiring private training.

A separate question is whether, in our environment, a simpler welfare scheme (i.e., not fully history dependent) would achieve similar gains. The pioneering work of Hansen and Imrohoroglu (1992) and more recently that of Wang and Williamson (2002) and Abdulkadiroglu, Kuruscu and Sahin (2002) tackles this issue in different environments.

Finally, one could choose to answer the set of questions we laid out from a different angle, i.e., adverse selection instead of moral hazard. Unobserved heterogeneity in "types" of workers (e.g., with high/low productivity, or high/low job finding rate) will induce the planner to optimally screen workers through a menu of alternative programs and payments. We defer all these considerations to ongoing and future research.

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# 8 Appendix A: Sequential Formulation

**History:** The world starts with an exogenously given initial condition $(z_0, h_0, y_0)$, where $z_0 \in \{z^e, z^u\}$ is the initial employment status of the worker, $h_0 \in H$ is its level of human capital, and $y_0 \in \{s, f\}$ is the outcome of the worker's activity. At the beginning of each period $t$, the outcome $x_t$ of a uniform $[0,1]$ random variable $X_t$ is publicly observed. The random variables $\{X_t\}$ are serially uncorrelated and independent of any choice made by the agent or the planner. Let $\sigma^t = \{z_0, h_0, y_0, x_0, ..., z_t, h_t, y_t, x_t\}$ be a history up to time $t$.

**Contract:** Let $\mathcal{W}(z_0, h_0, y_0) = \{\mathbf{c}, \mathbf{a}, \mathbf{d}, \mathbf{m}\} = \{c_t(\sigma^t), a_t(\sigma^t), d_t(\sigma^t), m_t(\sigma^t)\}_{t=0}^{\infty}$ to denote the contract, where

- $c_t(\sigma^t)$ is the transfer function, with $c_t(\sigma^t) \geq 0$ for any $\sigma^t$. Denote by $\mathbf{c}(x^\tau)$ the continuation plan of transfers after history $x^\tau$, i.e., $\mathbf{c}_t(x^\tau) = \{c_{t+n}(\sigma^{t+n})\}_{n=0}^{\infty} \ / \ \sigma^t$
- $a_t(\sigma^t)$ is the action (effort choice), where

$$a_t\left(\sigma^t\right) \in \begin{cases} \{0, e\} & \text{if } z_t = z^u, \\ e & \text{if } z_t = z^e, \end{cases}$$

  i.e., employment is defined as a state where the worker is productive and production requires the high effort level $e$. Denote by $\mathbf{a}_t(\sigma^\tau)$ the continuation plan of effort choices after node $\sigma^\tau$ and by $\mathbf{A}_t(\sigma^\tau)$ the set of all admissible continuation plans, after history $\sigma^\tau$.
- $d_t(\sigma^t)$ is the activity. If $z_t = z_u$, then $d_t(\sigma^t) \in \{search, train\}$. When $z_t = z_e$, $d_t(\sigma^t)$ equals to a singleton that we might call "work". Once again, $\mathbf{d}_t(\sigma^\tau)$ will denote the continuation plan of activities after node $\sigma^\tau$.
- $m_t(\sigma^t) \in \{0, 1\}$ is a dummy variable for the use of the search-effort monitoring technology, with $m_t(\sigma^\tau)$ denoting the continuation plan contingent on history $\sigma^\tau$.

Define the expected continuation utility promised in equilibrium by the contract $\mathcal{W}$ after history $\sigma^t$ as

$$U_t\left(\mathcal{W}; \sigma^t\right) = \mathbf{E}\left[\sum_{n=0}^{\infty} \beta^n u\left(c_{t+n}\left(\sigma^{t+n}\right)\right) - v_{z_{t+n}}\left(a_{t+n}\left(\sigma^{t+n}\right)\right) \mid \mathbf{a}_t\left(\sigma^t\right), \mathbf{d}_t\left(\sigma^t\right), \mathbf{m}_t\left(\sigma^t\right), \sigma^t\right].$$

we assume that $U_t(\mathcal{W}; \sigma^t)$ is well defined for all $(\mathcal{W}; \sigma^t)$.

**Incentive compatibility:** In our framework, $(z_t, h_t, y_t, x_t)$ is fully observable. We also assume that the activity $d_t$ (search, train, work) is observable and enforceable by the planner, hence "contractible". Because of the existence of the monitoring technology, at every node with $m_t(\sigma^t) = 1$, the effort chosen by the agent should be included in the set of contractible variables.

Define by $\mathbf{a}_t^{\mathbf{m}}(\sigma^t) \subset \mathbf{a}_t(\sigma^t)$ the sub-plan of actions which are not contractible under the monitoring plan $\mathbf{m}$. We then have that $a_t^m(\sigma^t) = a_t(\sigma^t)$ if and only if $m_t(\sigma^t) = 0$. In order to generate the sub-plan $\mathbf{a}^{\mathbf{m}}$ we simply delete the element $a_t(\sigma^t)$ from $\mathbf{a}$ whenever $m_t(\sigma^t) = 1$. We are now ready to define the set of incentive compatibility constraints. For all $\sigma^t$ we require

$$U_t\left(\mathbf{c}, \mathbf{a}, \mathbf{d}, \mathbf{m};\ \sigma^t\right) \geq U_t\left(\mathbf{c}, \hat{\mathbf{a}}, \mathbf{d}, \mathbf{m};\ \sigma^t\right), \tag{$IC(x^t)$}$$

where, $\hat{\mathbf{a}}_t(\sigma^t)$ can differ from $\mathbf{a}_t(\sigma^t)$ only on the non-contractible components $\mathbf{a}_t^{\mathbf{m}}(\sigma^t)$. Notice that in order to lighten notation, we have omitted the argument $(\sigma^t)$ from the continuation plans.

**Planner problem:** In the sequential representation of the contractual relationship, the planner solves

$$\begin{aligned} V^*\left(U_0, z_0, h_0, y_0\right) &= \sup_{\mathcal{W}} \mathbf{E}\left[\sum_{t=0}^{\infty} \beta^t \left(r\left(h_t, z_t, m_t\left(\sigma^t\right), d_t\left(\sigma^t\right)\right) - c_t\left(\sigma^t\right)\right) \mid \mathbf{a}, \mathbf{d}, \mathbf{m}, (z_0, h_0, y_0),\right] \\ s.t. &: \\ \int_0^1 U_0\left(\mathcal{W}; \sigma_0\right) dx_0 &\geq U_0 \text{ and } IC(\sigma^t) \text{ for all } \sigma^t \mid \sigma_0, \end{aligned}$$

where the return function during employment is $r\left(h_t, z_e, m_t\left(\sigma^t\right), d_t\left(\sigma^t\right)\right) = w\left(h_t\right)$, and during unemployment is $r\left(h_t, z_u, m_t\left(\sigma^t\right), d_t\left(\sigma^t\right)\right) = -\kappa\left(m_t\left(\sigma^t\right), d_t\left(\sigma^t\right)\right)$, with the costs given by $\kappa\left(0, search\right) = 0$, $\kappa\left(1, search\right) = \kappa^{JM} > 0$, $\kappa\left(0, train\right) = \kappa^{TR} > 0$, $\kappa\left(1, train\right) = +\infty$.

**Options of the contract during unemployment:** The table below represents all the admissible combinations of effort, activity and monitoring the planner can implement at every node. The entry $\times$ in a cell means that this option is never chosen by a welfare maximizing planner, whereas the entry $*$ denotes an option that can be optimal at some point during the contract.

| | $d_t = search$ | | $d_t = train$ | |
|---|---|---|---|---|
| | $m_t = 0$ | $m_t = 1$ | $m_t = 0$ | $m_t = 1$ |
| $a_t = e$ | $*$ (UI) | $*$ (JM) | $*$ (TR) | $\times$ |
| $a_t = 0$ | $*$ (SA) | $\times$ | $\times$ | $\times$ |

The last entry in the first line is due to the assumption that monitoring effort perfectly during training is prohibitively costly. The entries in the second line ($a_t = 0$) can be explained as follows. Choosing zero search effort and at the same time monitoring workers' effort is not optimal since in this case the moral hazard problem disappears: because $\pi\left(0, h_t\right) = 0$, $y_{t+1} = s$ is never an equilibrium outcome, the planner can implement $a_t = 0$ by threatening an infinite punishment, for example, no benefits, off the equilibrium (i.e., whenever $y_{t+1} = s$). Choosing zero effort during training is never optimal since, whenever $a = 0$, the training technology is ineffective, hence the planner will always prefer to implement search without monitoring which is cheaper and leads to the same outcome ($y = f$).

The planner can therefore restrict attention to the four remaining options labelled, respectively, Unemployment Insurance (UI), Job-search Monitoring (JM), Training (TR), and Social Assistance (SA), described in more detail in the main text.

**Recursive formulation:** The state space can be described as a correspondence $\Gamma\left(h, z\right)$ from all the pairs of human capital and employment status $(h, z) \in H \times \{z^e, z^u\}$ to the set of attainable workers' lifetime utility given by

$$\Gamma\left(h, z\right) = \left\{U :\ \exists\, \mathcal{W} \text{ satisfying } IC(\sigma^t)\ \forall \sigma^t \mid \sigma_0;\ \int_0^1 U_0\left(\mathcal{W}; \sigma_0\right) dx_0 = U,\ \ (h_0, z_0) = (h, z)\right\},$$

where we have omitted $y_0$ from the initial conditions since it is payoff irrelevant for both the agent and the planner.

A straightforward extension of the standard recursive-contracts methodology (e.g., Spear and Srivastava, 1987) delivers the recursive formulation of the principal-agent problem in terms of the triple $(U, h, z)$ we propose in the text. Below we will show that the functions solving the Bellman equation are bounded and continuous. Moreover, it is easy to show that by the Maximum Theorem the policy correspondence admits a (Borel) measurable selection. The usual verification theorem (SLP, Theorem 9.2) hence implies that the recursive formulation of the problem fully characterizes the optimal program.

# 9 Appendix B: Proofs

Some steps in the proofs of this Appendix are only sketched. The full-length proofs are contained in the companion Technical Appendix (Pavoni and Violante, 2005).

Proposition 0: properties of the value functions

Proposition 0 below states formally several technical properties of the value functions which have been briefly discussed in the main text.

**Proposition 0:** *(i) The functions* $W, \mathbf{V}$ *and* $V^i$ *are concave in* $U$ *(for all* $h$*). (ii) They are jointly continuous in* $(U,h)$ *and monotonically increasing in* $h$*. (iii) Let* $i^*(U,h)$ *be the implemented policy and* $c^*(U,h) > 0$ *be the optimal payment at* $(U,h)$ *. Then both* $\mathbf{V}$ *and* $V^i$ *are decreasing and continuously differentiable with respect to the first argument at* $(U,h)$ *with*

$$\mathbf{V}_U(U,h) = V_U^{i^*(U,h)}(U,h) = -\frac{1}{u'(c^*(U,h))}.$$

**Proof:** We can apply the well known results in Stokey, Lucas and Prescott (1989, thereafter, SLP) (chs. 4.2 and 9.2) to the Bellman operator $V^i$ associated to the problems defining the various policies $\{UI, JM, SA, TR\}$ and show that it preserves boundedness, continuity, concavity and monotonicity. This is all standard. The possibility of using randomized policy and utility assignments also permits one to 'convexify' the upper envelope function $V$ defined in (3) and to obtain concavity of the functions $V^i$ and $\mathbf{V}$ defined in (4). This is Lemma A1 in the Technical Appendix. Lemma A2 in the Technical Appendix shows that the integral in (4) preserves boundedness, continuity, and monotonicity. That $W$ in (10) is bounded, continuous, concave and monotone is obvious. Since the operator is a contraction, its fixed point will possess such properties as well. Finally, given concavity, differentiability of $V^i$ can now be shown by using standard arguments (SLP, Theorems 4.11-12). The conditions for $\mathbf{V}$ and $V^i$ displayed above follow immediately from the envelope theorem. See the Technical Appendix for a more extended proof. **Q.E.D.**

Proof of Proposition 1: SA absorbing

Although the result is very intuitive, the proof is non-trivial. The reason is that we must rule out situations where $SA$ is used at an early stage of the program for the sole purpose of allowing randomizations over policies with different implicit effort levels. The statement of Proposition 1 must therefore be re-formulated in probabilistic terms. Formally, we will show the following (equivalent, in probabilistic terms to the statement of Proposition 1).

**Proposition 1A**: *In order for a program to be strictly optimal at node* $(U, h_t)$, *it cannot be that at such node* $SA$ *is implemented with positive probability and, in period* $t+1$, *a different policy is implemented after* $SA$, *with positive probability.*

**Proof:** For the sake of contradiction, suppose (without loss of generality) that there is an optimal plan $\mathcal{W}$ implementing $SA$ in period $t$ almost surely for all $x_t$, and policy $i \neq SA$ in period $t+1$ for a positive measure of shocks $x_{t+1}$. The stated sequence ($SA$ followed by policy $i \neq SA$) cannot be part of an optimal program. We will show that the planner can provide the agent with the same expected utility as under $\mathcal{W}$ at a lower cost, by designing an alternative plan $\mathcal{W}'$ which uses a randomization between two branches. *Branch 1:* his branch is implemented with probability $\beta$ (i.e. equal to the discount factor). In this branch the new program $\mathcal{W}'$ implements exactly the same (randomization of) policies following $SA$ in the original program $\mathcal{W}$, and it delivers the same lifetime utility. *Branch 2:* With probability $(1-\beta)$, the planner implements $SA$ forever and transfers to the agent the same consumption level as that paid in the first period of the original plan $\mathcal{W}$ (recall that the $\mathcal{W}$ plan always implements $SA$ at time $t$).

By construction, the new plan $\mathcal{W}'$ provides the agent with the same ex-ante utility as under the original program $\mathcal{W}$. It turns out, though, that $\mathcal{W}'$ is also cost-reducing compared to $\mathcal{W}$, essentially because it uses policies $i \neq SA$ (e.g., $UI$ or $JM$) for higher levels of human capital ($h_t \geq h_{t+1}$), which relaxes the incentive compatibility (IC) constraint and increases expected returns to search. See the Technical Appendix for the detailed proof. **Q.E.D.**

Proof of Proposition 2: economy without human capital dynamics

*SA:* Proposition 1 shows that $SA$ is an absorbing policy for the general case.

*JM:* In order to show the absorbing property of $JM$, note that the first order conditions are $\mathbf{V}_U(U) = V_U^{JM}(U) = \mathbf{V}_U\left(U^f\right)$. This implies that setting $U^f = U$ is optimal. As a consequence, implementing the same policy, i.e. $JM$, every period is part of an optimal program. Clearly, whenever $\mathbf{V}$ is strictly concave this absorbing policy is the unique optimal one. Given the absence of the IC constraint, the absorbing nature of $JM$ implies that consumption remains constant once the worker enters $JM$.

*UI:* The proof that $UI$ is absorbing is quite involved. Let us start by stating the first-order and envelope conditions under $UI$:

$$\begin{aligned}
-\mathbf{V}_U(U) &= -V_U^{UI}(U) = \frac{1}{u'(c)}, \\
-\mathbf{V}_U\left(U^f\right) &= \frac{1}{u'(c)} - \mu\frac{\pi}{1-\pi}, \\
-W_U(U^s) &= \frac{1}{u'(c)} + \mu,
\end{aligned} \tag{14}$$

where $\mu \geq 0$ is the multiplier on the incentive compatibility constraint. It is useful to begin by stating the conditions under which the incentive constraint is binding and promised utility declines.

**Lemma A3:** *At any $U_0$ where $UI$ is optimal, we must have $U^f < U_0$. Moreover, if $V$ is strictly concave to the left of $U_0$, then it must be that $\mu > 0$.*

**Proof.** See Technical Appendix.

It is immediate from (14) that if the incentive compatibility constraint binds, then $U^f < U_0$. When it does not bind, then the planner can fully insure the worker upon success of search. The same result can then be obtained using the constraint (IC1) with strict inequality and the full-insurance version of the continuation utility $U^s = (1-\beta)u(c)$ into the promise-keeping constraint. Since each function $V^i$ is continuous by Proposition 0, if for different levels of utility different policies are preferred, the value functions must cross each other.

**Lemma A4:** *For every $U$ we have that $V_U^{SA}(U) \geq V_U^{UI}(U)$.*

**Proof.** See Technical Appendix.

This lemma is very intuitive: in $UI$, a rise in $U$ increases the incentive cost and the effort compensation cost, reducing $V^{UI}$, whereas $SA$ is a policy without IC problems and where effort is not required, so $V^{SA}$ is flatter with respect to $U$. Note that Lemma A3 and Lemma A4 imply that $SA$ will not follow $UI$: if $UI$ is optimal at $U_0$, then next period $U^f < U_0$ (by the first part of Lemma A3) which reinforces the optimality of $UI$ relative to $SA$ (by Lemma A4).

**Lemma A5:** *Let $U_0$ be such that $V^{JM}(U_0) = V^{UI}(U_0)$. Then, we have $V_U^{JM}(U_0) \geq V_U^{UI}(U_0)$ so $V^{UI}$ crosses $V^{JM}$ from above at most once.*

**Proof.** See Technical Appendix.

The intuition of the proof is the following. We know that $JM$ is an absorbing policy, so when the worker enters into $JM$, even though he is asked to always supply positive effort, he will never be subject to random consumption sequences. We also know that after implementing $UI$, an optimal program never uses policy $SA$. This implies that once the worker is assigned to policy $UI$, she will always be required to supply positive effort thereafter, as in $JM$. In addition to that, she will possibly face random consumption due to the IC constraint. Given our assumption that $1/u'$ is a convex function, this extra randomization on consumption will induce extra costs for the planner which will make $V^{UI}$ more negatively sloped than $V^{JM}$. This statement is formally shown in Lemma A6 of the TA. The proof of Lemma A5 heavily builds on that, and takes care of only 'off-the-optimum' behavior.

It remains to be shown that $UI$ cannot be followed by $JM$. By Lemma A3, during $UI$ we have that $U^f < U_0$ and by Lemma A5 if $UI$ dominates $JM$ at $U_0$, it still dominates $JM$ at $U^f < U_0$ which concludes the proof: the only policy that can optimally follow $UI$ is $UI$ itself. **Q.E.D.**

Proof of Corollary to Proposition 2: slopes

The ranking between $V^{UI}$ and $V^{JM}$ at the crossing point has been shown in Lemma A5. The proof that $\hat{V}_U^{SA}(U) \geq V_U^{JM}(U)$ is based on the idea that under $JM$ the agent is required to supply positive effort at least in the first period of $JM$, therefore the marginal cost for the planner of providing a given level of utility must be larger than that under $SA$, a policy with full insurance and $a = 0$ forever. See the Technical Appendix for details. **Q.E.D.**

Proof of Proposition 3: slopes wrt $U$ in the general case

That $\hat{V}_U^{SA}(U) \geq V_U^{JM}(U,h)$ can be easily derived in the general case with human capital dynamics by following exactly the same lines of proof proposed for the Corollary to Proposition 2, without relying on any additional assumption on the curvature of $\mathbf{V}$.

We now show the first inequality. Consider first the envelope condition in $UI$ at $(U,h)$, and let $\eta = \eta\left(U_{UI}^f, h^f\right)$ the quantity in the proposition for $h = h^f$ and $U = U_{UI}^f$. We have

$$\begin{aligned}
-V_U^{UI}(U,h) &= g'\left(U - \beta U_{UI}^f\right) \\
&= \pi(h)\, g'\left((1-\beta)\left[U_{UI}^f + \frac{e}{\beta\pi(h)}\right]\right) + (1-\pi(h))\left[-\mathbf{V}_U\left(U_{UI}^f, h^f\right)\right] \qquad (15)\\
&= \pi(h)\, g'\left((1-\beta)\left[U_{UI}^f + \frac{e}{\beta\pi(h)}\right]\right) + (1-\pi(h))\, g'\left((1-\beta)\left(U_{UI}^f + \eta\right)\right).
\end{aligned}$$

The first row is derived using the envelope condition and the promise-keeping constraint. The second row is obtained directly from the first order conditions under $UI$. The equality in the last line is due to the definition of $\eta$. Now, consider $JM$. We have

$$\begin{aligned}
-V_U^{JM}(U,h) &= g'\left(U + e - \beta\left[\pi(h)\, U_{JM}^s + (1-\pi(h))\, U_{JM}^f\right]\right) \\
&= -W_U\left(U_{JM}^s, h^f\right) = g'\left((1-\beta)\, U_{JM}^s\right) \qquad (16)\\
&= -\mathbf{V}_U\left(U_{JM}^f, h^f\right).
\end{aligned}$$

The equality in the first row uses the envelope condition and the promise-keeping constraint. The first and second equalities in the second row use the first order conditions under $JM$ and the shape of $W$, respectively. The equality in the last row uses again the first order conditions under $JM$. Now, if $-V_U^{UI}(U,h) \geq -V_U^{JM}(U,h)$ we are done. We must therefore rule out the possibility that $-V_U^{UI}(U,h) < -V_U^{JM}(U,h)$. For this purpose, assume that the last inequality is in fact true. We are looking for a contradiction. First of all, recall that because of incentive constraints we have $-V_U^{UI}(U,h) \geq -\mathbf{V}_U\left(U_{UI}^f, h^f\right)$. Hence, from (16) we get $-\mathbf{V}_U\left(U_{UI}^f, h^f\right) < -\mathbf{V}_U\left(U_{JM}^f, h^f\right)$ which, from the concavity of $\mathbf{V}$, implies that $U_{JM}^f \geq U_{UI}^f$. From our assumption on $\mathbf{V}$, and the convexity of $g'$ we get

$$-\mathbf{V}_U\left(U_{JM}^f, h^f\right) \leq g'\left((1-\beta)\left(U_{JM}^f + \eta\right)\right). \qquad (17)$$

Now, the convexity of $g'$ also implies that condition (15), together with the Jensen's inequality, yields

$$-V_U^{UI}(U,h) \geq g'\left((1-\beta)\left[U_{UI}^f + \frac{e}{\beta} + (1-\pi(h))\,\eta\right]\right).$$

Comparing this condition with the first line of (16), it is easy to see that whenever $U^f_{UI} + \frac{e}{\beta} + (1-\pi(h))\eta \geq U^f_{JM} + \eta$, we directly get a contradiction to the statement $-V^{UI}_U(U,h) < -V^{JM}_U(U,h)$, and we are done. Finally, assume instead the complementary inequality. Multiplying all by $\beta$ and rearranging, it becomes

$$\beta U^f_{UI} + e < \beta U^f_{JM} + \pi(h)\eta. \tag{18}$$

Now, since from (17) and the second line of (16) we have $U^s_{JM} \leq U^f_{JM} + \eta$, it must be true that

$$e - \beta\left[\pi(h) U^s_{JM} + (1-\pi(h)) U^f_{JM}\right] \geq e - \beta U^f_{JM} + \beta\pi(h)\eta.$$

In other words, if condition (18) is true, we get $e - \beta U^f_{JM} + \beta\pi(h)\eta > -\beta U^f_{UI}$. But comparing the first lines of (15) and (16), we get $-V^{UI}_U(U,h) \geq -V^{JM}_U(U,h)$, again a contradiction. **Q.E.D.**

Proof of Proposition 4: slopes wrt $h$ in the general case

That $\hat{V}^{SA}_h = 0$ is obvious since $\hat{V}^{SA}(U,h) = -\frac{u^{-1}((1-\beta)U)}{1-\beta}$. Since for $i = JM, UI$ the value functions $V^i$ are monotonically increasing in $h$, we just have to show the first inequality in the proposition. Fix $U_0$ and recall the planner's problem under $JM$. By the envelope theorem we have

$$\begin{aligned} V^{JM}_h(U_0,h) &= \pi'(h)\beta\left[W\left(U^s_{JM}, h^f\right) - \mathbf{V}\left(U^f_{JM}, h^f\right)\right] + \\ &\quad + \beta\left[\pi(h) W_h\left(U^s_{JM}, h^f\right) + (1-\pi(h))\mathbf{V}_h\left(U^f_{JM}, h^f\right)\right], \end{aligned} \tag{19}$$

where the subscript $JM$ indicates that $U^s$ and $U^f$ are the optimal choices under policy $JM$. Consider now the $UI$ policy. Clearly, if at $(U_0,h)$ the incentive compatibility under $UI$ is not binding, then $V^{UI}_h(U_0,h) = V^{JM}_h(U_0,h)$ as $UI$ and $JM$ solve essentially the same problem and $\kappa^{JM}$ does not depend on $h$. In what follows, we assume the incentive compatibility is binding. Substituting the incentive compatibility into the promise keeping constraint, differentiating the value function with respect to $h$, and using the envelope theorem, after rearranging we obtain

$$\begin{aligned} V^{UI}_h(U_0,h) &= \beta\pi'(h)\left[W\left(U^s_{UI}, h^f\right) - \mathbf{V}\left(U^f_{UI}, h^f\right)\right] - \beta\pi'(h) W_U\left(U^s_{UI}, h^f\right)\left(U^s_{UI} - U^f_{UI}\right) \\ &\quad + \beta\left[\pi(h) W_h\left(U^s_{UI}, h^f\right) + (1-\pi(h))\mathbf{V}_h\left(U^f_{UI}, h^f\right)\right], \end{aligned}$$

where we used the subscript $UI$ notation for the optimal choices under policy $UI$. Recall that from the $IC$ constraint, we have $U^s_{UI} - U^f_{UI} = \frac{e}{\pi(h)}$. We now show that the above expression is lower than the expression in (19). In light of (19), since $\beta\pi'(h) \geq 0$, the desired inequality $V^{UI}_h(U_0,h) \geq V^{JM}_h(U_0,h)$ will be shown for all $h$ if the following two conditions hold:

$$\pi(h) W_h\left(U^s_{JM}, h^f\right) + (1-\pi(h))\mathbf{V}_h\left(U^f_{JM}, h^f\right) \geq \pi(h) W_h\left(U^s_{UI}, h^f\right) + (1-\pi(h))\mathbf{V}_h\left(U^f_{UI}, h^f\right), \tag{20}$$

and

$$W\left(U^s_{UI}, h^f\right) - \mathbf{V}(U^f_{UI}, h^f) - W_U(U^s_{UI}, h^f)\left(U^s_{UI} - U^f_{UI}\right) \geq W\left(U^s_{JM}, h^f\right) - \mathbf{V}\left(U^f_{JM}, h^f\right). \tag{21}$$

We now show (20). First, the separable form of $W$ (displayed by condition (10) in the main text) implies that the derivative $W_h$ does not depend on $U^s$. Since $h^f$ is the same, $W_h$ is the same across the two policies, and can be omitted. Thus, we just have to show

$$\mathbf{V}_h\left(U^f_{UI}, h^f\right) \geq \mathbf{V}_h\left(U^f_{JM}, h^f\right).$$

Moreover, since $\mathbf{V}$ is submodular it suffices to show that $U^f_{JM} \geq U^f_{UI}$. From the first-order and envelope conditions, we get

$$\begin{aligned} W_U\left(U^s_{JM}, h^f\right) &= -\frac{1}{u'\left(c^{JM}\right)} = \mathbf{V}_U(U^f_{JM}, h^f), \\ W_U\left(U^s_{UI}, h^f\right) &< -\frac{1}{u'\left(c^{UI}\right)} < \mathbf{V}^f_U(U^f_{UI}, h^f), \end{aligned} \tag{22}$$

where $c^i$ is the optimal consumption under policy $i = UI, JM$. Now assume, for the sake of contradiction, that $U^f_{UI} > U^f_{JM}$. Since $\mathbf{V}$ is concave, it must be that

$$-\frac{1}{u'(c^{JM})} = \mathbf{V}_U(U^f_{JM}, h^f) \geq \mathbf{V}_U(U^f_{UI}, h^f) > -\frac{1}{u'(c^{UI})},$$

which implies $c^{JM} < c^{UI}$. Then, the first-order conditions (22) and the concavity of $W$ imply that $U^s_{UI} \geq U^s_{JM}$ which leads to a contradiction: payments and continuation utilities in $JM$ are lower than those under $UI$, and effort is required under both policies, therefore $UI$ and $JM$ cannot deliver to the agent the same promised utility $U_0$. As a consequence, the statement $U^f_{JM} \geq U^f_{UI}$ must be true and so does inequality (20).

We are now ready to show inequality (21). The first order conditions during $JM$ imply that $W_U(U^s_{JM}, h^f) = \mathbf{V}_U(U^f_{JM}, h^f)$. The concavity of both functions and the fact that $W$ is flatter than $\mathbf{V}$ for all $h$ implies that $U^s_{JM} \geq U^f_{JM}$. Since $W$ is decreasing in $U$ we have

$$W(U^f_{JM}, h^f) - \mathbf{V}(U^f_{JM}, h^f) \geq W(U^s_{JM}, h^f) - \mathbf{V}(U^f_{JM}, h^f).$$

From the above inequality, in order to show (21) it suffices to demonstrate that

$$W\left(U^s_{UI}, h^f\right) - \mathbf{V}(U^f_{UI}, h^f) - W_U(U^s_{UI}, h^f)\left(U^s_{UI} - U^f_{UI}\right) \geq W(U^f_{JM}, h^f) - \mathbf{V}(U^f_{JM}, h^f). \tag{23}$$

We claim that (23) must be true since $U^f_{JM} \geq U^f_{UI}$. The proof goes as follows. Rewrite first the above inequality as

$$W\left(U^s_{UI}, h^f\right) - W(U^f_{UI}, h^f) - W_U(U^s_{UI}, h^f)\left(U^s_{UI} - U^f_{UI}\right) + W(U^f_{UI}, h^f) - \mathbf{V}(U^f_{UI}, h^f) \geq W(U^f_{JM}, h^f) - \mathbf{V}(U^f_{JM}, h^f).$$

Now, the concavity of $W$ and the fact that $U^s_{UI} > U^f_{UI}$ imply that

$$W\left(U^s_{UI}, h^f\right) - W(U^f_{UI}, h^f) - W_U(U^s_{UI}, h^f)\left(U^s_{UI} - U^f_{UI}\right) \geq 0.$$

We are hence left to show that $W(U^f_{UI}, h^f) - \mathbf{V}(U^f_{UI}, h^f) \geq W(U^f_{JM}, h^f) - \mathbf{V}(U^f_{JM}, h^f)$, or–rearranging terms –that

$$\mathbf{V}(U^f_{JM}, h^f) - \mathbf{V}(U^f_{UI}, h^f) \geq W(U^f_{JM}, h^f) - W(U^f_{UI}, h^f).$$

But this inequality must be true since $U^f_{JM} \geq U^f_{UI}$ and $\mathbf{V}$ is steeper than $W$ for all $h$. **Q.E.D.**

Proof of Proposition 5: optimal policy sequence

We first describe the dynamics of $U$ during $JM$ :

**Lemma A7:** *Assume that* $\mathbf{V}$ *is submodular and recall that it is a concave function. If at* $(U, h)$ *either one of these properties (submodularity or concavity) is strict, and the implemented policy is* $JM$, *then* $U^f(x) \geq U$ *almost surely for all* $x \in [0,1]$.

**Proof.** The first order conditions under $JM$ imply

$$\mathbf{V}_U(U, h) = V^{JM}_U(U, h) = \mathbf{V}_U\left(U^f, h^f\right) = \mathbf{V}_U\left(U^f(x), h^f\right)$$

for (almost) all $x \in [0,1]$. Since $h^f \leq h$, sub-modularity implies that $\mathbf{V}_U\left(U, h^f\right) \geq \mathbf{V}_U(U, h)$. Since $\mathbf{V}$ is concave, if either one of the properties of $\mathbf{V}$ is strict at $(U, h)$, we have the desired result. **Q.E.D.**

Since $h^f \leq h$, the proof of the proposition is now trivial from the graphical representation and the utility dynamics of Lemma A7. The only important remark to make is that in this lemma not only have we established that $U^f \geq U$ in expected terms, but we have also shown that the continuation utility increases almost surely for all realizations of $X$. **Q.E.D.**

Proof of Proposition 6: no duration dependence

The line of proof is an extension to that adopted for Proposition 1 (SA absorbing). For the sake of contradiction, assume that at node $(U_t, h_t)$ there is an optimal plan $\mathcal{W}$ implementing $JM$ in period $t$ almost surely for all $x_t$, and $UI$ in period $t+1$ for a full measure of shocks. We allow for any random plan from period $t + 1onward$.

The idea of the proof is again that we can construct an alternative plan delivering the same ex-ante expected utility to the agent, and larger net returns for the planner. This plan can be generated as follows. First, construct a plan $\mathcal{W}'$ delivering the same ex-ante expected utility to the agent, and same net returns for the planner. Next, show that in the new plan the IC constraints are relaxed with respect to $\mathcal{W}$ in some states, thus costs can be further reduced and $\mathcal{W}$ cannot be optimal.

The program $\mathcal{W}'$ consists of two branches. *Branch 1:* with probability $\beta(1-\pi)$ the new plan $\mathcal{W}'$ starts with policy $UI$ at time $t$ and then continues by implementing a contingent plan which can be constructed *forward* from $\mathcal{W}$ as follows. Each time in the original plan we see $UI$ or $JM$ we retain $UI$ or $JM$, but whenever in the old plan we see $SA \to SA \to SA$... (notice that from Proposition 1 this is the only possibility) then we substitute this sequence with $JM \to SA \to SA$... *Branch 2:* with probability $1-\beta(1-\pi)$ $\mathcal{W}'$ starts with $JM$ at time $t$ and then, as for Branch 1, whenever in the original plan $\mathcal{W}$ one sees $SA \to SA \to SA$... then replace it with $JM \to SA \to SA$... Moreover, each time in the original plan we see $JM$ we retain $JM$. Finally, each time in the old plan we see $UI$, we replace it with $JM$.

Consider now the payments in plan $\mathcal{W}'$. Let $c_t$ be the payment made under $\mathcal{W}$ in period $t$ (notice that because of full insurance during $JM$, this must be the same for all $x_t$ and across states). In branch 1 we have the following. First, every time the new plan implements $UI$ the planner pays the agent exactly what was paid to her in the old plan during this policy at this node, both in case of success and of failure of search. Whenever $\mathcal{W}'$ implements $SA$ the agent gets exactly the same consumption as in the old plan at that node. The payments under $JM$ are as follows: whenever $JM$ is implemented in order to replace $JM$ in the old plan, the payments are again exactly the same as those in the old plan, in all states. When $JM$ is implemented (for one period) in order to replace the first period of $SA$ instead, then the transfers are those made in $SA$ in the old plan at that node. In branch 2, the new plan transfers $c_t$ to the agent in any period, history of shocks, and states. Her expected utility is $u(c_t)/(1-\beta(1-\pi))$.

It is now straightforward to check that the stated plan $\mathcal{W}'$ delivers the same ex-ante utility to the agent, and the same net return to the planner. We begin by showing the equivalence of the two programs with respect to the monitoring costs for the planner, in two steps.

*Step 1:* In branch 1, with weight $\beta(1-\pi)$, the only additional monitoring costs with respect to the continuation of the old plan $\mathcal{W}$ is borne whenever the latter contemplated $SA$ for the first time. Let $q$ be the probability of such event. Note now that this same additional cost occurs also in the second branch, which has weight $1-\beta(1-\pi)$. So, up to now, we conclude that the new plan yields additional costs with respect to the $t+1$-continuation of the old plan equal to $q\kappa^{JM}$.

*Step 2:* Recall that in the first period of program $\mathcal{W}$ the planner pays an initial monitoring cost $\kappa^{JM}$ with certainty. Consider now branch 2. If the plan always implemented $JM$, we would have a cost $\kappa_{JM}$ with probability $(1-\pi)$ every period, so we would get precisely a discounted present value of the monitoring costs equal to $\kappa^{JM}$, which is the initial cost of the old plan. But the cost in branch 2 is smaller since there is a tail, occurring with probability $q$, where the cost is not paid because the new plan picks $SA$ from then onward. The reduction in cost associated to this tail, compared to the unlimited implementation of $JM$, is $-q\frac{\kappa^{JM}}{1-\beta(1-\pi)}$ which, weighted with the probability of branch 2 occurring, yields exactly $-q\kappa^{JM}$, the excess cost of the new plan computed in *Step 1*, which proves that the expected discounted monitoring cost in the two programs must be the same.

The wage returns for the planner must also be the same. The new program implements positive search effort if and only if a positive effort was implemented in the original program for the same level of $h$, with exactly the same probabilities as in the old plan. In particular, recall that the new plan starts in period $t$ but it is constructed based in what happens in the original plan at date $t+1$. Implementing $JM$ for one more period whenever the old plan implemented $SA$ guarantees precisely that $a=0$ is chosen in the new plan only for values of human capital for which the old plan recommended $a=0$. The logic used above for the monitoring cost can then be easily extended to also show that - for each $h_t$ - the probability for which either $a=0$ or $a=e$ is implemented is the same between the two plans. This implies that the agent's effort cost is the same in the two plans as well.

Moreover, the payments are the same in the two programs. Branch 1 of the new plan can be thought of as the continuation of the old plan from time $t+1$ onward, since it has weight $\beta(1-\pi)$. It is easy to see that the total value of payments is exactly the same as that paid in the old plan from unemployment duration $t+1$ onward. In branch 2 payments are like in the old plan, so the discounted present value of payments in branch 2 equals $c_t/(1-\beta(1-\pi)) = c_t + \beta\pi\frac{c_t}{1-\beta}$, the consumption paid in the old plan during the first period of unemployment and upon employment. Precisely the missing part in order to get the total payment bill.

Finally, notice that under $\mathcal{W}'$ the incentive constraint in $UI$ is relaxed with respect to $\mathcal{W}$, since $\mathcal{W}'$ reduces utility $U^f$ upon failure of search. This is so since $\mathcal{W}'$ implements $JM$ instead of $SA$ for one more period at later dates, and $JM$ involves a positive effort cost. This allows the planner to further improve upon $\mathcal{W}'$ by reducing consumption dispersion (hence, average transfers). The Technical Appendix contains a fully detailed proof of this Proposition. **Q.E.D.**

Proof of Proposition 7: payments in UI, JM, and SA

*(i)* The behavior of the payments during $UI$ is easily derived from a set of first-order conditions analogous to (14), i.e.,

$$\begin{aligned}
-V_U^{UI}(U) &= \frac{1}{u'(c)}, \\
-\mathbf{V}_U\left(U^f, h^f\right) &= \frac{1}{u'(c)} - \mu\frac{\pi(h)}{1-\pi(h)}, \\
-W_U\left(U^s, h^f\right) &= \frac{1}{u'(c)} + \mu,
\end{aligned}$$

with $\mu \geq 0$. Since next period the envelope condition is

$$-\mathbf{V}_U\left(U^f, h^f\right) = \frac{1}{u'(c^f)}$$

where $c^f$ is the consumption payment in case of unsuccessful search, the result follows immediately from the strict concavity of $u$.

*(ii)* The first-order conditions and the envelope condition during $JM$ are

$$\begin{aligned}
-V_U^{JM}(U,h) &= \frac{1}{u'(c)}, \\
-\mathbf{V}_U\left(U^f, h^f\right) &= \frac{1}{u'(c)} = \frac{1}{u'(c^f)} \\
-W_U\left(U^s, h^f\right) &= \frac{1}{u'(c)} = \frac{1}{u'(c^s)},
\end{aligned}$$

hence unemployment payments and net wage $c^s$ are constant during this policy. The wage tax (subsidy) is defined as $\omega\left(h^f\right) - c^s$. Since $\omega\left(h^f\right)$ increases with $h^f$, and $h^f$ increases with $h$, the result is immediate.

*(iii)* It is obvious and the proof is omitted.

*(iv)* The shape of $W$ in (10) implies that the net wage $c_t^s$ satisfies $c_t^s = (1-\beta) U_t^s$, thus the tax in this case is simply $\omega - (1-\beta) U_t^s$. In $JM$ promised utility is constant, so the result follows easily. Consider $UI$. Recall that in the proof of Proposition 2 we have shown that in an economy without human capital dynamics $UI$ is absorbing and the incentive compatibility constraint is binding ($\mu > 0$). From Lemma A3, $U^f < U$. Rearranging the promise-keeping constraint and the incentive constraint over two consecutive periods, we get $U_{t+1}^s = U_{t+1}^f + \frac{e}{\beta\pi} < U_t^f + \frac{e}{\beta\pi} = U_t^s$. The desired result is now immediate since $U_{t+1}^s < U_t^s$. **Q.E.D.**

Proof of Proposition 8: payments in TR

The first-order conditions under training are

$$\begin{aligned} -V_U^{TR}(U,h) &= \frac{1}{u'(c)}, \\ -\mathbf{V}_U\left(U^f,h^f\right) &= \frac{1}{u'(c)} - \mu\frac{\theta}{1-\theta}, \\ -\mathbf{V}_U\left(U^s,h^s\right) &= \frac{1}{u'(c)} + \mu, \end{aligned}$$

with $\mu \geq 0$. From the next period envelope condition, we have

$$-\mathbf{V}_U\left(U^y,h^y\right) = \frac{1}{u'(c^y)} \qquad \text{for} \quad y = s, f$$

from which we easily obtain the desired result. **Q.E.D.**

# 10 Appendix C: Estimation of the Hazard Function

**Data Description:** From the Basic Monthly Current Population Survey (CPS), we selected every worker between 18-50-years-old with at most a high-school degree who reports to be unemployed during the months May 1995-April 1996. This initial sample includes 18,910 observations. However, part of these observations refer to the same individual and the same unemployment spell, since some of the households are interviewed for several consecutive months. In these cases, we selected only the most recent available record on the length of each unemployment spell. The final sample comprises $N = 15,100$ ongoing spells of unemployment. The median duration is 8 months, the 25th percentile is 3 months and the 75th percentile is 21 months. The mean monthly earnings for workers in our sample are just below \$1,500.

**Estimation:** The statistical problems with this type of data are two: right censoring and length bias. We follow Flinn (1986) in dealing with these two issues. We assume that the true distribution of unemployment spells is a Weibull with scaling parameter $\alpha$ and shape parameter $\lambda$. Let $F$ be the Weibull distribution function and $\Gamma$ the Gamma function. The log-likelihood function $\mathcal{L}_N$ for a size-N sample of right-censored, length-biased spells is then

$$\mathcal{L}_N(\alpha,\lambda) = \sum_{i=1}^{N} \ln\left[1 - F\left(t_i|\alpha,\lambda\right)\right] + N\ln\alpha + N\ln\lambda - N\ln\Gamma\left(\lambda^{-1}\right).$$

The estimation yields $\hat{\alpha} = 0.474$ (0.017) and $\hat{\lambda} = 0.661$ (0.027). The implied hazard function displays negative duration dependence, especially in the first 12 months of unemployment where the exit rate falls by half (top panel of Figure 5).

We divided the sample into subgroups based on gender and education, but we did not find any statistically significant evidence of heterogeneity across groups. We followed Flinn (1986) and introduced unobserved heterogeneity through a binomial distribution over the scale parameter $\alpha$, but once again the differences in the estimates of $\alpha$ for the two groups were not statistically significant. This finding is not surprising, given the homogeneity of our education group.

**From durations to human capital:** The estimated hazard is a function of unemployment duration, but in the model it is a function of human capital. In order to map duration into human capital all we need is the initial level of human capital (pre-displacement wage) and the monthly rate of human capital depreciation. Average monthly earnings of the workers in the CPS sample are $\$1,500$, thus we used an initial value of $h = 15$ and reconstructed human capital as a function of duration through the assumed depreciation rate.

Figure 5 (lower panel) plots the hazard as a function of human capital. Based on an annual depreciation rate of 15%, the market value of skills drops by half after roughly 50 months.

# 11 Appendix D: The U.S. Welfare System

In what follows, we describe the pivotal ingredients of the U.S. welfare system, which are then summarized into the "actual" U.S. WTW program of Section 6.1.4.

**Unemployment Insurance:** The unemployment insurance replacement ratio in the U.S. varies across states. The state-determined weekly benefits generally replace between 50% and 70% of the individual's last weekly pre-tax earnings. The regular state programs usually provide benefits up to 26 weeks. The permanent Federal-State Extended Benefits program, present in every State, extends coverage up to 13 additional weeks, for a combined maximum of 39 weeks. Weekly benefits under the extended program are identical to those of the regular program.[34]

**TANF:** The Temporary Assistance for Needy Families (TANF) program is the main cash assistance program for poor families with children under age 18 and at least one unemployed parent. It was implemented in 1996 as part of *The Personal Responsibility and Work Opportunity Reconciliation Act* (PRWORA) which, at the same time, eliminated all existing Federal assistance programs (the AFDC, in particular). The main innovations of the TANF program were three. First, the emphasis on encouraging self-sufficiency through work. TANF legislation specifies that, with few exceptions, recipients must participate to "work activities", such as un-subsidized or subsidized employment, on-the-job training, community service, job search, vocational training, or education directly related to work.

Second, the time limit to benefits: families with an adult who has received TANF assistance for a total of five years are not eligible for further cash aid over their lifetime. A number of states, however, have also imposed a shorter limit over fixed calendar intervals (e.g., 24 months over any given 5-year period). See Moffitt (2003) for a detailed description of the TANF program.

Third, financial incentives were created for states to run mandatory active labor market programs for workers on the TANF rolls. Generally speaking, U.S. states followed one of two alternative strategies. Some programs emphasized short-term job search monitoring (the Labor Force Attachment approach, LFA thereafter). Others emphasized longer-term skill-building activities and training (the Human Capital Development approach, HCD thereafter). The programs based on the LFA approach started each worker on job-search assistance activities (e.g., classroom instructions on resume preparation, preparation for specific job interviews, supervision of individual workers' search activity), and only later moved workers still on welfare into either basic education (e.g., brush-up courses in math and reading skills, preparation for GED or high-school completion courses), or college-level courses, or vocational training (e.g., occupational training courses in automotive repair, nursing, clerical work, computer programming, cosmetology), usually for fairly brief periods. The programs based on the HCD approach reverse the order of the policies, starting workers on education/training and moving them later (but only for a short period) onto job-search monitoring. See NEWWS (2001, Box 1.2) for a more detailed description.

**Food Stamp Program:** The Food Stamp program provides monthly coupons to eligible low-income families, which can be used to purchase food. Over 80% of TANF recipients also receive Food Stamps (DHHS,

[34]Extended programs can be activated when unemployment is "relatively high".

2004). Once TANF benefits expire, households remain virtually without any other form of benefits and have the right to the maximum allotment of food stamps.

**Unemployment Tax:** The Federal Government imposes a net payroll tax on employers (FUTA) of 0.8% on the first $7,000 of earnings paid annually to each employee.[35] States finance their welfare programs with an additional State Unemployment Tax. In 1996, the estimated national average tax rate as a fraction of total wages was 0.8% (House Ways and Means Committee, 1996).

**EITC:** The Federal Earned Income Tax Credit (EITC) is the major wage subsidy program in the United States. It is a refundable tax credit that supplements the earnings of low-income workers. It has a "trapezoid" structure as a function of annual earnings. In 1996, for a single-parent household with two children (the typical household on the welfare rolls), the subsidy rate was 40% up to $741 per month. In the range $741 − $967, the subsidy is fixed at $296. For monthly earnings over $967, workers start paying a tax rate of 21% over and above the $296 subsidy, until the break-even income such that the net subsidy is exactly zero, i.e., $2,377. See Hotz and Scholtz (2001, Table 1) for details.

[35] The current gross FUTA tax is 6.2%, but employers in states meeting certain requirements are eligible for a 5.4% tax credit.

## Figure 1

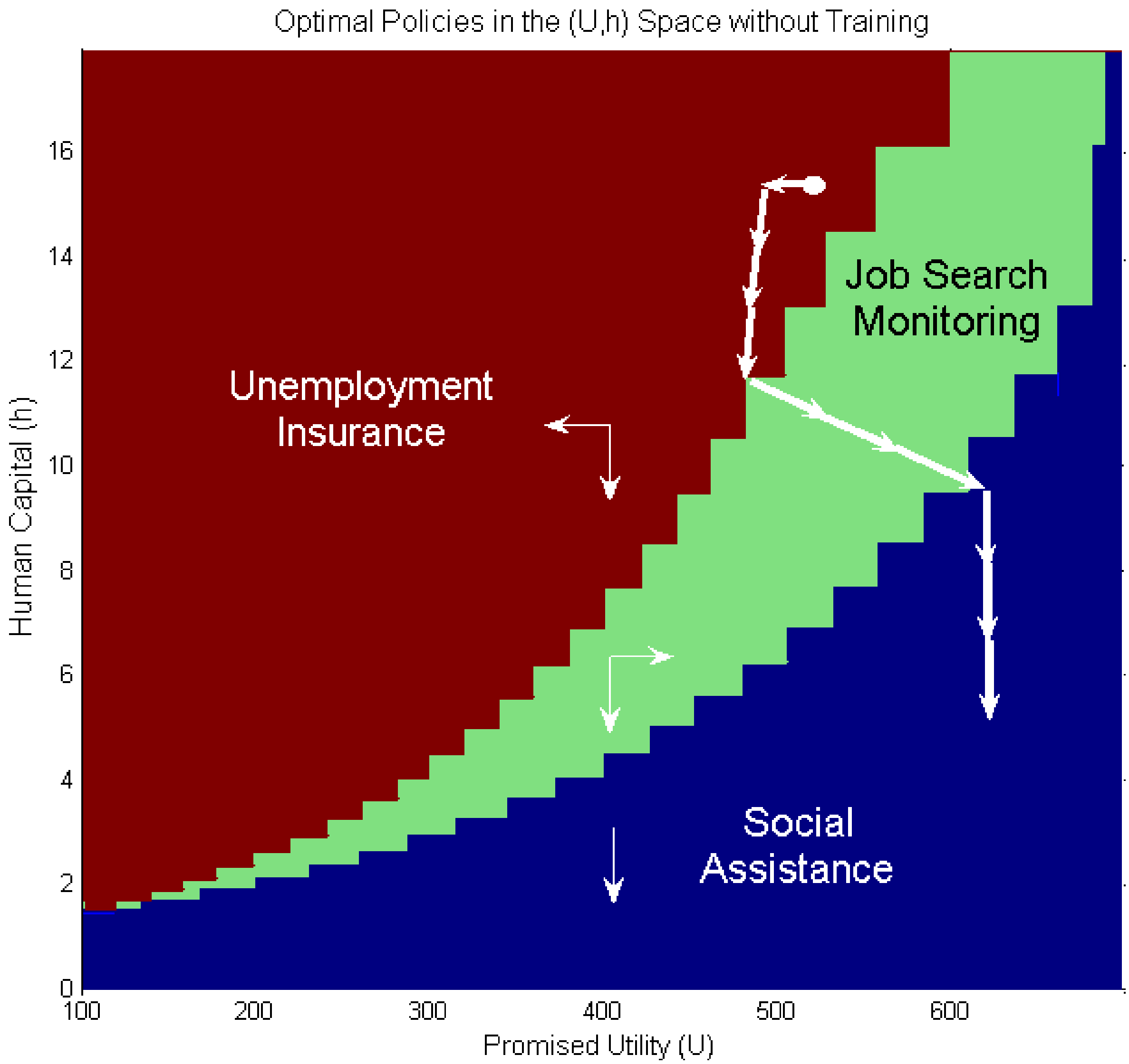



Figure 1: The policies of the optimal WTW program without training in the state space of human capital $h$ and promised utility $U$.

# Figure 2

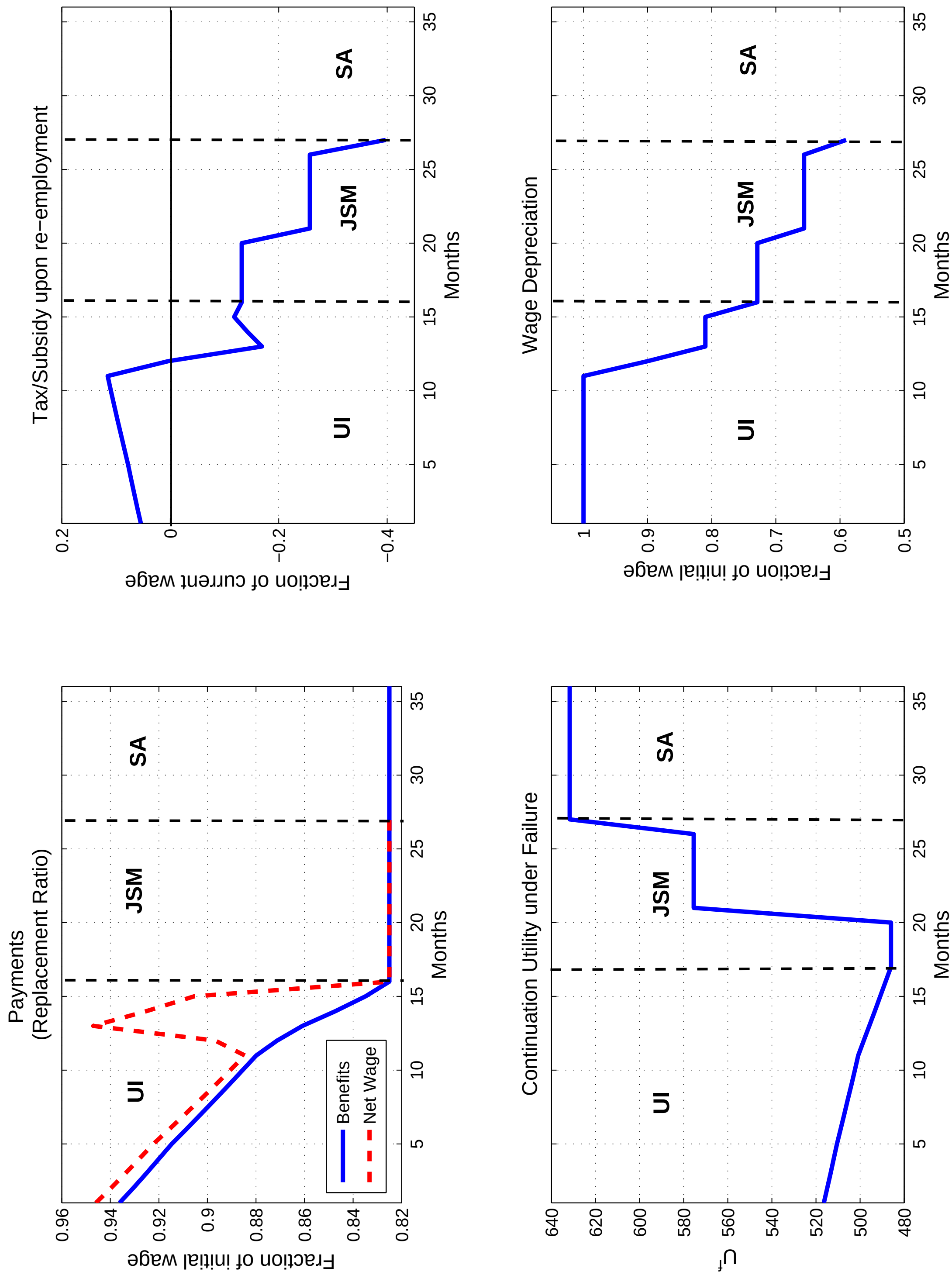



Figure 2: A representative history of the optimal WTW program without training policies.

# Figure 3

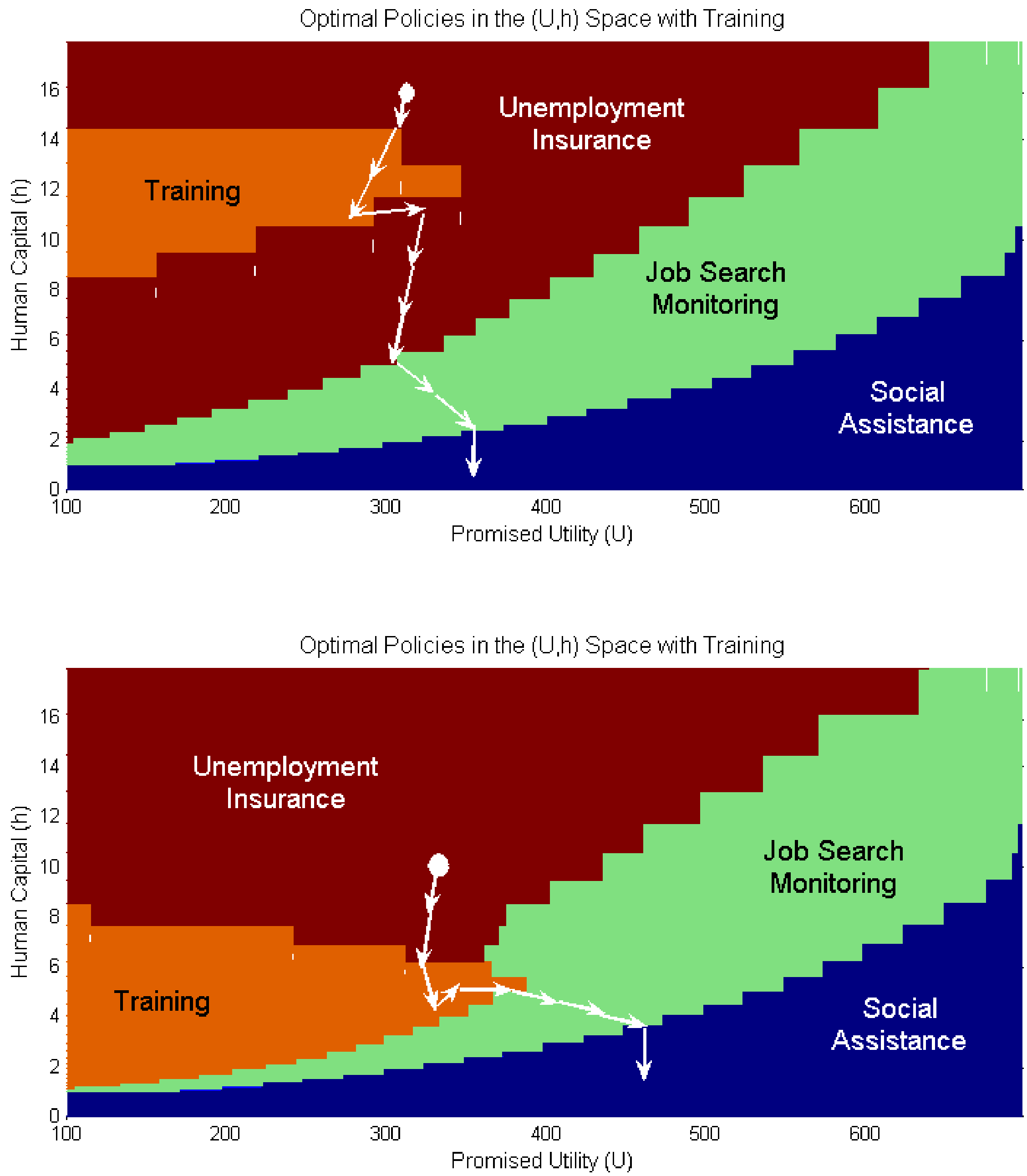



Figure 3: Two examples of the optimal WTW program with training in the state space of human capital $h$ and promised utility $U$.

## Figure 4

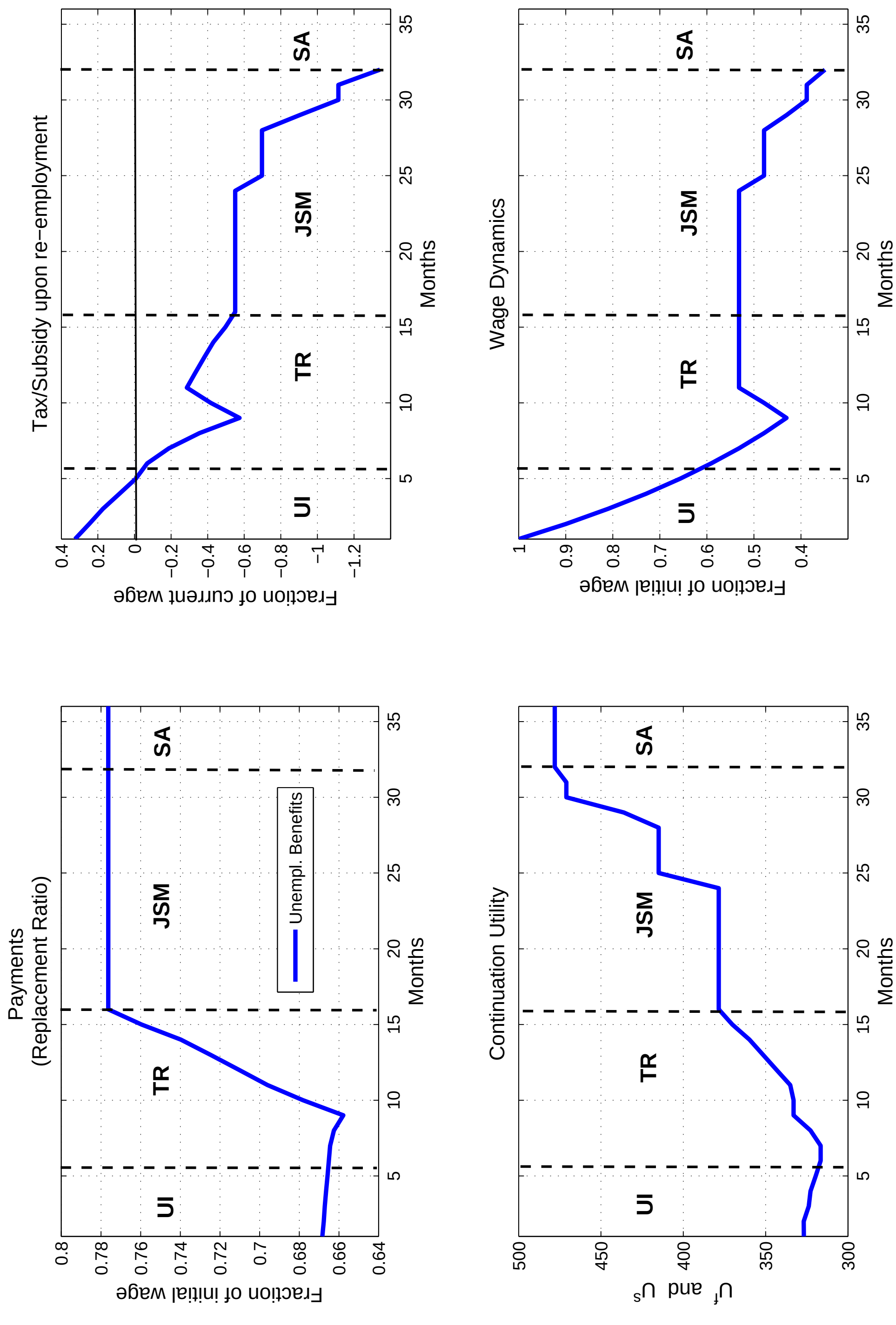
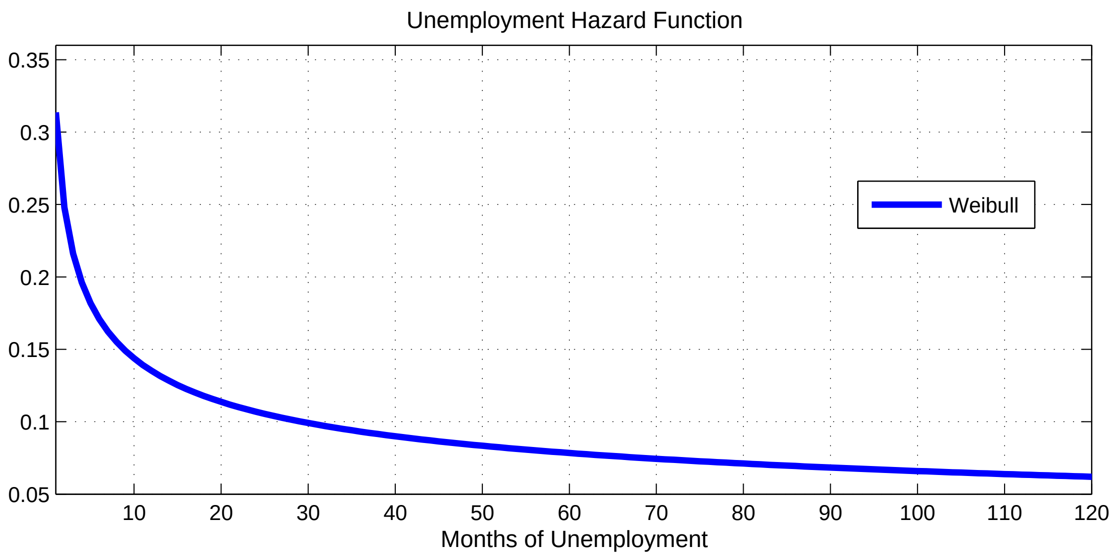



Figure 4: A representative history of the optimal WTW program with training.

## Figure 5



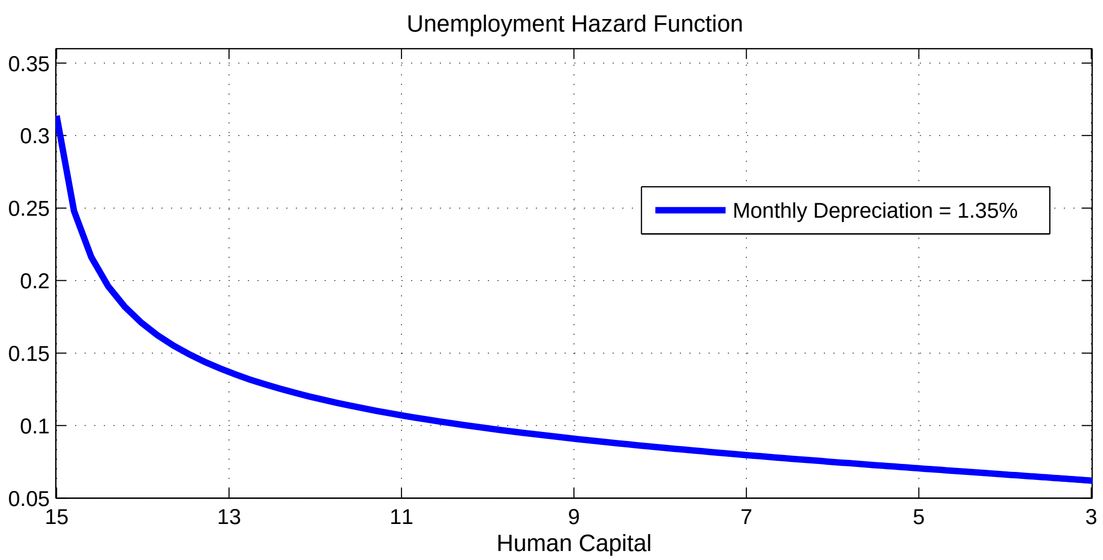




Figure 5: Exit rate from unemployment (Weibull hazard estimated on monthly CPS data May 1995-April 1996).

# Figure 6

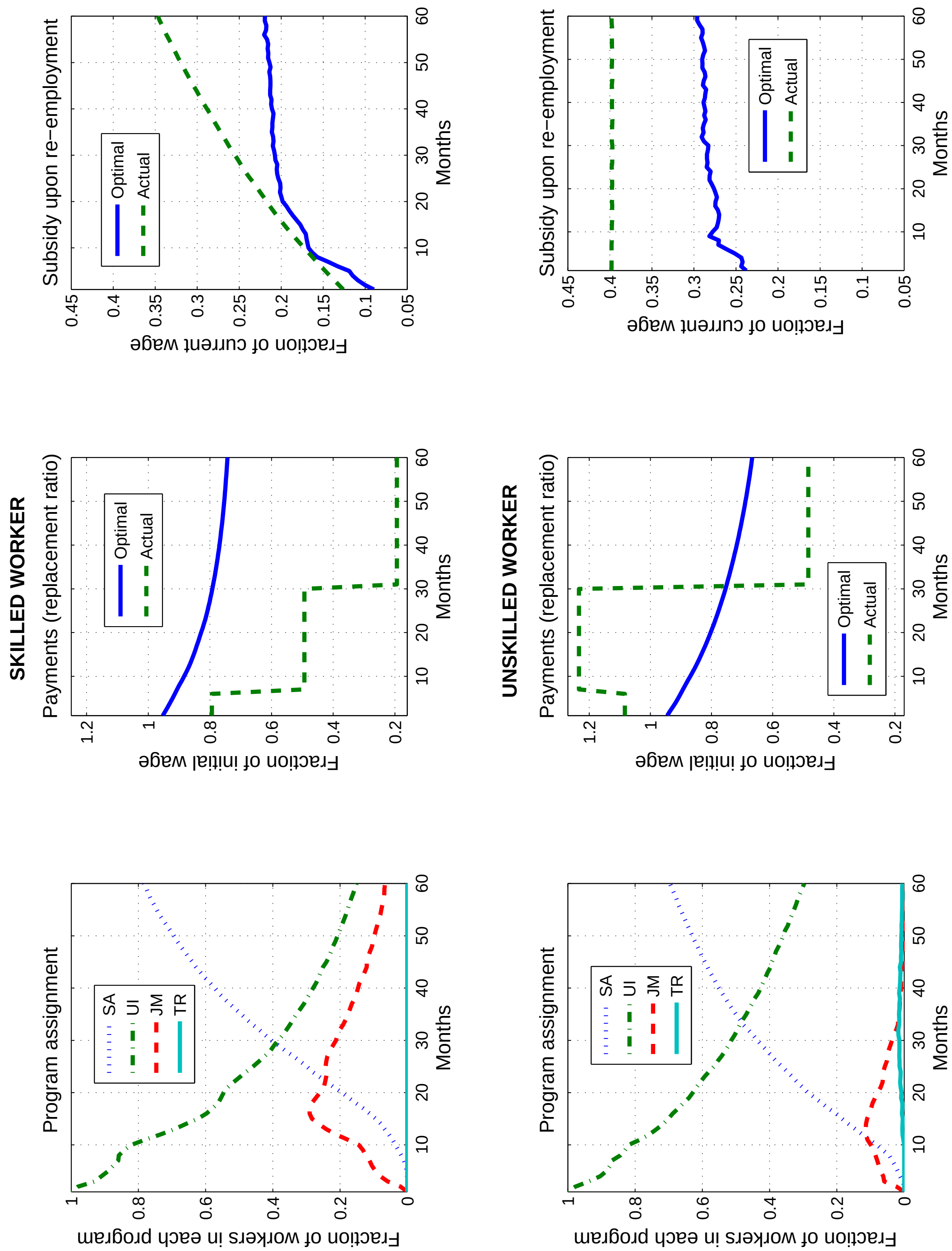



Figure 6: Features of the optimal WTW program compared to the actual U.S. welfare system for a high-skilled worker (monthly earnings of $1,500) and an low-skilled worker (monthly earnings of $600).