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SEIGNIORAGE AND THE WELFARE COST OF INFLATION: EVIDENCE FROM AN INTERTEMPORAL MODEL OF MONEY AND CONSUMPTION

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ABSTRACT

This paper empirically investigates the restrictions embodied in a popular dynamic monetary model for the cross relations between consumption, money holdings, inflation and assets' returns using quarterly data for the high-inflation economy in Israel, 1970-1988. The model considered includes money in agents' utility function. A set of the estimated parameters is used in the analysis to assess the model's quantitative implications for seigniorage and for the welfare costs of inflation. The estimates are found to account well for the observed stability over time of seigniorage in Israel and imply sizeable welfare costs of inflation.

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1. Introduction

A common feature of many high–inflation episodes is the lack of a strong positive association between the size of the budget deficit, government seigniorage revenue, and the rate of inflation. Consider for example the case of Israel in the 1980's. In spite of the increase in the rate of inflation from about 130 percent in 1980 to about 400 percent in 1984, the government deficit to GNP ratio showed a small increase, from 17 to 19 percent, and so did the seigniorage to GNP ratio, which increased only from 2.1 to 2.9 percent.[1] That this feature applies to the European hyperinflations of the 1920's was indicated by Sargent and Wallace (1973). With the exception of the last (extreme–inflation) observations, the data on seigniorage from these hyperinflations are generally without marked trends despite the rapid increase in inflation.[2]

These "stylized" facts have been used to question models that stress the role of seigniorage and of budget deficits in the inflationary process. Previous research has focused on two main explanations for these facts. First, models based on a Cagan–type semilogarithmic demand for money generally give rise to a Laffer curve and dual inflationary equilibria. These models imply that beyond a specific (revenue–maximizing) rate of inflation, seigniorage revenue decreases in response to increases in the rate of inflation [see e.g. Bruno and Fischer (1990) and Sargent and Wallace (1987)]. Thus, wide fluctuations in the rate of inflation need not be accompanied by noticeable movements in seigniorage revenue, and a given amount of seigniorage can be collected at either a high or a low rate of inflation. The second main explanation stresses the role of time–varying expectations of shifts in fiscal policy. Although the rate of inflation and the budget deficit increase together over time in anticipation of future monetization of the deficit, the anticipation of future increases in taxes gives rise to a negative correlation between inflation and the deficit.[3] Thus, changes over time in public's expectations of how high budget deficits will be closed in the future (e.g., through increases in taxes against money creation) give rise to various possible statistical links between deficits, seigniorage, and inflation.

In this paper we propose another explanation. We show that an empirically based parameterization of an optimizing model with money in the utility function is capable of accounting for the "stylized" facts embodied in time series for Israel, without resorting to an ad–hoc semilog demand for money or to expectations of future regime change. The analysis and results below provide a characterization of money demand and of the behavior of seigniorage that differs from those derived from models that directly postulate a semilog demand for real money balances. Moreover, we discuss the association between primitive parameters, such as the degree of risk aversion, and seigniorage revenue and report calculations of the welfare costs of different rates of inflation.

The first part of the paper deals with estimation, on quarterly time series for Israel, of the parameters of a model that treats consumption and money demand behavior as jointly arising from a single optimizing framework of a representative agent, as in modern monetary theory [see, e.g., Sidrauski (1967)].[4] To do so, we focus on the restrictions implied by the nonlinear Euler equations that characterize the first order conditions of optimization by a representative consumer, as in Hansen and Singleton (1982) and Eichenbaum, Hansen, and Singleton (1988). Thus, our research is related to recent work that has tested some of the implications of intertemporal monetary models using time series data [see e.g. Singleton (1985), Ogaki (1987), Poterba and Rotemberg (1987), Marshall (1988), and Finn, Hoffman, and Schlagenhauf (1990)]. While these investigations used data for the U.S., here we are particularly interested in exploring and testing the implications of an optimizing representative–consumer framework using data from an economy featuring wide fluctuations in inflation and in monetary aggregates such as Israel in the period 1970–1988. It is challenging for intertemporal models to attempt to account for observed consumption and money holdings behavior in this volatile environment, one in which there were relatively large costs and benefits associated with agents' decisions about how and when to shift purchasing power from one period to another.

After obtaining estimates for the key parameters, the second and main part of our work consists of comparing steady states of the model assuming different rates of inflation to determine whether the implied relation between seigniorage revenue and the rate of inflation conforms with the "stylized" facts and with the implications of a standard semilog money demand model. Using estimated and observable parameters, we find that seigniorage rises with the rate of inflation. However, although seigniorage revenue markedly increases when there is a shift from no inflation to an inflation rate of 10 percent per quarter, there are only negligible gains in seigniorage from increases in inflation beyond that rate. Our calculations indicate that seigniorage revenues in the 1980's were quite close to the maximal revenues (about 3 percent of GNP) that could be collected by the government. The simulated relation between seigniorage and the rate of inflation appears to more closely conform with the data than the Laffer curve that arises from a model based on a Cagan–type money demand.

In addition, we quantitatively assess the welfare losses associated with different steady state rates of inflation. We calculate the steady state welfare cost of a moderate inflation of 10 percent per year at 0.85 percent of GNP, which is more than double the 0.39 percent of GNP figure for the U.S. computed by Cooley and Hansen (1989).[5] The welfare cost of a rate of inflation of 168 percent per year, the average in Israel for the period

1980–84, reaches the sizeable figure of 4 percent of GNP.

The paper is organized as follows. Section 2 deduces the restrictions that are imposed on the data by a model that includes money in the utility function, and discusses some steady state implications of the model. Section 3 describes the estimation method, data, and results. Section 4 uses parameter estimates from the previous section along with observable parameters and with a set of auxiliary assumptions about a hypothetical steady state to determine the model's quantitative implications for the relation between seigniorage and the rate of inflation and for the welfare costs of inflation. Section 5 contains brief concluding remarks.

2. The Model

The economy is populated by infinitely lived families, with population growing at rate n. Each household maximizes expected discounted utility

$$E_0 \sum_{t=0}^{\infty} \beta^t U(m_t, c_t^*) \qquad , \tag{1}$$

where E_0 denotes expectations conditional on information available at time 0, β is a subjective discount factor, m denotes real money balances per capita,[6] c^* denotes consumption services per capita, and $U(\cdot)$ is a concave utility function that is increasing in both its arguments. Consumption services are assumed to be related to purchases according to the simple relation $c_t^* = c_t + \delta c_{t-1}$, where δ is a fixed parameter and c denotes actual purchases of consumer goods. Thus, consumption purchases at time t directly affect consumption services in both t and t+1.[7] In spite of the time separability of utility defined over consumption services and real money balances, the indirect utility function defined over consumption purchases and real money balances is temporally nonseparable.

Each household's budget constraint, in per capita real units, is given by

$$(2) \qquad b_t = b_{t-1}(1+r_{t-1})(1+n_t)^{-1} + m_{t-1}[(1+n_t)(1+\pi_t)]^{-1} + y_t - m_t - c_t,$$

where b_t , m_t , and c_t are respectively the real per capita values of one–period financial assets, money balances, and consumption chosen by the household for time t. n_t and π_t respectively denote population growth and the rate of inflation from t–1 to t, and the real interest factor $(1+r_{t-1})$ is equal to $(1+R_{t-1})/(1+\pi_t)$, where R_{t-1} denotes the nominal return on assets held from t–1 to t. y_t is real per capita income from other sources.

Substituting the budget constraint and the specification about the relation between consumption services and purchases into (1), differentiating with respect to b_t and m_t, and rearranging yields the following first order conditions for maximization of (1):

$$(3) \qquad \beta\, E_t\left[\frac{U_2(t+1)}{U_2(t)}\left[\frac{(1+r_t)}{(1+n_{t+1})} - \delta\right]\right] + \beta^2 \delta\, E_t\left[\frac{U_2(t+2)}{U_2(t)}\frac{(1+r_t)}{(1+n_{t+1})}\right] - 1 = 0$$

$$(4) \qquad \frac{U_1(t)}{U_2(t)} + \beta\, E_t\left[\frac{U_2(t+1)}{U_2(t)}\{[(1+n_{t+1})(1+\pi_{t+1})]^{-1} - \delta\}\right] +$$

$$\beta^2 \delta\, E_t\left[\frac{U_2(t+2)}{U_2(t)}[(1+n_{t+1})(1+\pi_{t+1})]^{-1}\right] - 1 = 0$$

where $U_i(t+s)$ is the marginal utility with respect to the i'th argument ($i = 1,2$) evaluated at time $t+s$ ($s = 0,1,2$).

Euler equation (3) is the standard condition for optimally allocating consumption between periods t and t+1. It equates the marginal utility cost of giving up one unit of consumption in period t to the expected utility gain from shifting that unit to consumption in the next period. This equation, in alternative versions, has been the focus of numerous recent empirical studies of consumption (e.g. Hansen and Singleton (1982)). Equation (4) equates the expected utility costs and benefits of reducing current period consumption by one unit and allocating that unit to money holdings and then to consumption in the next period. From an empirical perspective, both these equations can be used to derive the model's restrictions on the comovements of consumption, money holdings, inflation and assets' returns over time. Notice that in the special case in which the nominal return R_t is assumed to be kown at the start of the period and $\delta=0$, equations (3) and (4) can be combined to yield

$$\frac{U_1(t)}{U_2(t)} = \frac{R_t}{(1+R_t)} \quad ,$$

a nonstochastic relation between real money balances, consumption, and the nominal interest rate. This equation can be viewed as a conventional demand for money in implicit form (see Lucas (1986)). In our framework, however, equations (3) and (4) can not be combined to yield a nonstochastic relation.

In order to estimate the model and derive its implications for seigniorage and the welfare cost of inflation, we use the utility function

$$U(m_t,c_t^*) = \frac{\left[m_t^{\gamma}\ c_t^{*1-\gamma}\right]^{\theta} - 1}{\theta} \quad , \tag{5}$$

where γ is a preference parameter between zero and one, and θ is a preference parameter that is less than one[8]. The parameter $1-\theta$ represents both the coefficient of relative risk aversion and the inverse of the elasticity of intertemporal substitution. Accordingly, the marginal utilities appearing in equations (3) and (4) are expressed in terms of parameters and observables as follows:

$$U_1(t) = \gamma(m_t)^{\gamma\theta-1}(c_t+\delta c_{t-1})^{\theta(1-\gamma)}, \tag{6}$$

$$U_2(t) = (1-\gamma)(m_t)^{\theta\gamma}(c_t+\delta c_{t-1})^{\theta(1-\gamma)-1} . \tag{7}$$

When θ is equal to zero we attribute the marginal utilities in (6) and (7) to the log–utility specification $U(\cdot) = \gamma \log m_t + (1-\gamma)\log c_t^*$.

Using these specifications, we next turn to the implications of the model for seigniorage revenue and the welfare cost of inflation— implications which are derived by comparing steady states of the model assuming different rates of inflation. We assume that per capita consumption and real money balances grow in steady states at a constant rate $\phi > 0$, that population grows at a constant rate n , and that all real variables are invariant with respect to steady state changes in the rate of inflation.[9] Accordingly, equation (4) can be rearranged to yield a steady state "demand for money"

$$m = \frac{(\frac{\gamma}{1-\gamma})(1 + \frac{\delta}{(1+\phi)})c}{1 + \alpha_1 - \frac{\alpha_2}{(1+\pi)}}, \tag{8}$$

where $\alpha_1 \equiv \beta\delta(1+\phi)^{\theta-1}$, $\alpha_2 \equiv (1+n)^{-1}(1+\alpha_1)\beta(1+\phi)^{\theta-1}$, and c and π denote the steady state values of consumption per capita and rate of inflation. Being derived from an explicit optimizing model, steady state money demand is expressed as a function of primitive parameters which characterize preferences and technology.

We compare below the seigniorage implications of the foregoing specification against those of a Cagan demand for money given by

$$m = c\zeta\exp\{-\omega[\pi/(1+\pi)]\} ,$$

where ζ is a constant term and ω is a constant semi elasticity of money demand with respect to $\pi/(1+\pi)$.[10] This comparison is of interest because of the central role of this money demand function in most previous research on seigniorage under high inflation.

Assuming that the parameters in equation (8) are invariant with respect to steady state changes in the rate of inflation, we calculate from (8) the absolute value of the elasticity of money demand with respect to a steady state change in the inflation rate as

$$(9) \qquad \eta \equiv \left|\frac{\partial m}{\partial \pi}\frac{\pi}{m}\right| = [(1+\pi)(1+n)^{-1}(1+\phi)^{\theta-1}\beta^{-1} - 1]^{-1}\left(\frac{\pi}{1+\pi}\right).$$

According to the model, the inflation elasticity of money demand depends on the underlying parameters and on the rate of inflation; the exact form of this dependence is explored below using values of estimated parameters. The elasticity of the semilogarithmic demand for money with respect to $\pi/(1+\pi)$ is given by $\omega\pi/(1+\pi)$, and the elasticity with respect to π is $\omega\pi/[(1+\pi)^2]$.

In order to explore the present model's implications for seigniorage, notice that government's revenue from monetary base creation is given by

$$S_t = [(H_t - H_{t-1})/H_t][H_t/P_t],$$

where H is the monetary base. Seigniorage per–capita, denoted by $\hat{S}$, can be written as

$$\hat{S}_t = (1 - \frac{H_{t-1}}{H_t})h_t,$$

where h denotes the monetary base in real per–capita units. In the steady state equilibrium considered here the gross rate of change of the monetary base (H_t/H_{t-1}) is equal to $(1+n)(1+\phi)(1+\pi)$. Substituting for h_t the derived demand for real monetary base from equation (8), and dividing by GNP per capita we get the following expression for the ratio of seigniorage to GNP in steady state (denoted by SR: seigniorage ratio):

$$\text{(10)} \qquad SR = \left[1 - \frac{1}{(1+n)(1+\phi)(1+\pi)}\right]\left[\frac{(\frac{\gamma}{1-\gamma})(1 + \frac{\delta}{1+\phi})\psi\kappa}{1 + \alpha_1 - \frac{\alpha_2}{(1+\pi)}}\right],$$

where ψ is the ratio of consumption to GNP and κ is the inverse of the money supply multiplier. When the inflation rate accelerates there are two conflicting forces operating on SR: the inflation–tax rate increases but at the same time there is a decrease in the tax base (i.e., in the demand for real balances). A sufficient condition for an increasing SR with respect to π is that $[1 - \beta(1+\phi)^{\theta}] > 0$; a condition that is always met for configurations involving $\beta < 1$, $\phi \geq 0$, and $\theta \leq 0$.

For the Cagan specification of the demand for money, the steady state ratio of seigniorage to GNP is computed by replacing the second set of squared brackets in the right hand side of (10) with the expression $\kappa\psi\zeta\exp\{-\omega[\pi/(1+\pi)]\}$.

To calculate the welfare costs of various steady state levels of inflation we substitute equation (8) into (5) and compute the percentage decrease in consumption per capita that would generate the same welfare loss as that from moving from $\pi = 0$ to a given $\pi > 0$. This welfare loss, expressed as a percentage of GNP and denoted by WL, is given by [11]

$$\text{(11)} \qquad \mathrm{WL} = \psi\{\{(1 + \alpha_1 - \alpha_2[1+\pi]^{-1})/(1 + \alpha_1 - \alpha_2)\}^{\gamma} - 1\}\,.$$

3. Estimation

From equations (3) and (4), we define the disturbances of the model as

$$\text{(12)} \qquad d_{1t+2}(\sigma) = \beta\left[\frac{U_2(t+1)}{U_2(t)}\left[\frac{(1+r_t)}{(1+n_{t+1})} - \delta\right]\right] + \beta^2\delta\left[\frac{U_2(t+2)}{U_2(t)}\frac{(1+r_t)}{(1+n_{t+1})}\right] - 1$$

$$\text{(13)} \qquad d_{2t+2}(\sigma) = \frac{U_1(t)}{U_2(t)} + \beta\left[\frac{U_2(t+1)}{U_2(t)}\{[(1+n_{t+1})(1+\pi_{t+1})]^{-1} - \delta\}\right] +$$

$$\beta^2\delta\left[\frac{U_2(t+2)}{U_2(t)}[(1+n_{t+1})(1+\pi_{t+1})]^{-1}\right] - 1.$$

Substituting into these equations our parameterization of marginal utilities (i.e., equations (6) and (7)) delivers the two–equation system to be estimated, whose parameter vector is $\sigma = (\beta,\gamma,\theta,\delta)$. Notice that the Euler equations (3) and (4) imply the orthogonality conditions $E(d_{it+2}(\sigma_0)\cdot z_{jt}) = 0$, for $i = 1,2$, where z_{jt} is any variable that belongs to the information set at time t, and σ_0 is the true value of the parameter vector σ.

Based on these orthogonality conditions, we estimate the parameter vector by applying Hansen's (1982) Generalized Method of Moments (GMM) to quarterly data for Israel covering the period 1970:I to 1988:III. We impose the constraints that the weighting matrix is positive definite and that the disturbances follow a first–order moving average process (due to the presence of a two–period–ahead forecast error in the Euler equations).[12]

The aggregate time series used are as follows. Consumption is measured by total private consumption spending from the National Accounts. We also used a measure for purchases of nondurables and services as an alternative for the total measure. Money is defined as the standard M1 or alternatively as the monetary base. All nominal variables are deflated by the relevant consumption deflators, and per–capita measures are obtained by dividing aggregates by the existing population. The nominal interest rate is the quarterly lending rate charged by banks; results for the average nominal return on indexed government bonds are discussed in footnote 15. The inflation rate is measured by the percentage change in the relevant consumer price deflator.[13]

In estimating the model, we first used the following vector of instrumental variables: $z1_t' = [1, c_t/c_{t-1}, m_t/m_{t-1}, (1+r_{t-1})/(1+n_t)]$. With these four instruments and two equations, there are eight orthogonality conditions. Since there are four parameters to be estimated, there are four overidentifying restrictions. In addition, we explored the impact of allowing an additional lag of our instruments by using the vector $z2_t' = [z1_t', z1_{t-1}']$.

Results are displayed in Table 1. For each vector of instruments, we report four sets of estimates corresponding to two alternative definitions of consumption (total and nondurables plus services) and two alternative definitions of money (M1 and the monetary base). In each case we report the minimal value of the objective function J_T which, as shown by Hansen (1982), is a chi–square test statistic for the validity of the model's overidentifying restrictions.[14]

The parameter estimates for β and γ are economically meaningful and are quite similar, and large relative to their estimated standard errors, across the different systems that were estimated. Most estimated values of β are below unity and most estimates of γ are around 0.05. It turns out that the estimates for θ and δ do vary across the eight systems that were estimated. Although some such variation arises from the alternative time series used for consumption and money, the main differences are due to the choice of instruments. Most of the estimated values for θ are negative and range from a low of –5.6 to a high of 1.03. The former points to a high relative risk aversion coefficient and to a low intertemporal elasticity of substitution; the latter implies nonconcave utility. While the estimated values of δ under the z1 instrument vector are positive and range from 0.29 to 0.57 , the parameter estimates under the z2 instrument vector are negative.

The J_T statistics for the model estimated with total consumption are small relative to the degrees of freedom for the z1 instrument vector, but large relative to the degrees of freedom for the z2 instrument vector. An opposite pattern holds for estimates obtained under the nondurables plus services definition of consumption. In the case of four out of the eight estimated systems the J_T statistics indicate that the model's overidentifying restrictions are not rejected by the sample information at standard significance levels.[15] Overall, the extent to which the model's overidentifying restrictions are (or are not) rejected by the data depends on the definition of consumption and the choice of instruments. Hence, it is difficult to reach unambiguous conclusions regarding the empirical validity of the restrictions implied by the various specifications of the model implemented on the present sample.

4. Implications for Seigniorage and the Welfare Cost of Inflation

Based on the parameter estimates obtained in the previous section, we now explore the extent to which the model accounts for the observed stability of annual seigniorage in spite of large fluctuations in the annual rate of inflation. Then, we quantitatively assess the welfare cost of inflation. We do this by comparing, under the model's parameters, alternative hypothetical steady states under different rates of inflation.[16]

For our calculations of seigniorage and welfare cost of inflation we use the following parameter values:

$$\beta = 0.987 \mid \gamma = 0.05 \mid \psi = 0.61 \mid \mathrm{n} = 0.0058 \mid \phi = 0.008 ,$$

where the parameter values for β and γ are chosen from the estimates of the previous section and those for ψ , n , and ϕ correspond to the quarterly sample means of the share of consumption in GNP, the rate of change of population, and the rate of change of consumption per–capita, respectively. Since the econometric results indicate that the estimated risk aversion parameter θ is sensitive to the choice of instruments and data we experimented with three main values: –5.6, –1.5, and 0.0 (the latter corresponds to the case of log–utility). Similarly, our main calculations used $\delta = 0.3$, but we also checked the sensitivity of the results by using the alternative values $\delta = -0.3$, and $\delta = -0.7$.

Tables 2 and 3 report the results for seigniorage as a percentage of GNP, for the inflation rate elasticity of money demand, and for the welfare cost of inflation. Figure 1 depicts the implied seigniorage ratio for various rates of inflation and under three alternative values of the risk aversion parameter θ. There are four main features of these seigniorage calculations.

First, as evident from Tables 2 and 3, the ratio of seigniorage to GNP is an increasing function of the rate of inflation. That is, government can raise more revenue by increasing monetary base growth and inflation. This finding does not support the notion that inflation rates in Israel in the mid–eighties exceeded the revenue maximizing rate.

Second, although the gains to government from increasing inflation from zero to ten percent per–quarter are of about 1.5–2.0 percents of GNP, the gains from further increasing inflation are of a small order of magnitude. For example, shifting from a quarterly rate of inflation of 10 percent to 70 percent generally results in an increase in revenue of only one percent of GNP. As shown in Figure 1, for low rate of inflation SR markedly increases with increases in π, but then SR rapidly reaches an asymptote. It is this flatness of SR with respect to π that accounts in our model for the observed stability of the seigniorage to GNP ratio despite wide fluctuations in the rate of inflation. The calculated values for SR under mild and high inflation correspond well with the actual figures, generally between two to three percent of GNP, observed in Israel in the first half of the eighties.

Third, the results for the seigniorage ratio are not very sensitive to the values chosen for the θ and δ parameters — namely, those parameters which were not precisely estimated in the econometric work. Thus, the calculated values of SR under a quarterly rate of inflation of 28 percent (as between 1980 to 1984 on average) reported in Tables 2 and 3 range from a low of 2.4 percent of GNP to a high of about 3.0 percent. Notice that the higher the degree of relative risk aversion, the lower is the ratio of seigniorage to GNP (see Figure 1), and the lower is the elasticity of money demand with respect to steady state changes in the rate of inflation. Also, other things equal, lower values of δ result in lower values of SR.

Fourth, the model's implications for the relation between seigniorage and inflation markedly differ from those based on a Cagan semi–log demand for money. Figure 2 plots, for the period from 1980 to 1986, the actual data on seigniorage[17] along with the predictions of SR based on our model and a Cagan–type model. For the latter, we used a semi–elasticity of money demand of –5.0 which conforms well with estimates from previous empirical work on money demand in Israel and we normalized the constant term so as to give rise to the same SR for 1980 as our model's.[18] The simulation for SR under a semi–log demand for money indicates that the ratio of seigniorage to GNP should have decreased from 1981 to 1984, as inflation accelerated, and should have sharply increased thereafter. In contrast, the actual figures for SR (plotted with solid lines in Figure 2) indicate that it slightly increased from the early to mid–eighties and then decreased along with disinflation. In a broad sense, the relatively flat relation between SR and inflation that arises from the parameterization of our model (see Figure 2) matches the actual data more closely than the semi–log money demand alternative.[19]

Tables 2 and 3 also report values of the inflation–rate elasticity of money demand that are implied by the various configurations of the underlying parameters. Notice that this elasticity first increases with the rate of inflation, reaches a maximum, and then decreases with further increases in inflation. For high inflation rates such as in the mid–eighties, the calculated elasticity is of about –0.6, which conforms quite well with available empirical findings.[20] By virtue of the underlying microfoundations of the present model, it is possible to relate the inflation rate elasticity of money demand to a primitive parameter such as the degree of risk aversion. We find that the higher the degree of risk aversion, the lower is the inflation elasticity of money demand.

In order to provide some measure of the precision of the foregoing calculations for the seigniorage ratio and for the inflation rate elasticity of money demand, we computed simulated standard errors for these variables assuming randomly generated values of θ and

δ — namely, the two parameters that were quite imprecisely estimated in Table 1. The simulated standard errors are given in Table 4. We calculated them by using Monte Carlo methods to generate values for these two parameters using a normal distribution with means of $\theta = -5.6$ and $\delta = 0.3$ and standard errors of 1.262 and 0.1, respectively, (see Table 1) and 500 randomly generated observations. Other parameter values are set as in Tables 2 and 3. The simulated standard errors for the seigniorage ratio are quite low, and are generally no more than 10 percent of the value of SR. A similar finding holds for simulated standard errors of the inflation elasticity of money demand.

The last column of Tables 2 and 3 reports the welfare costs, as percents of GNP, associated with increasing inflation from zero to a positive rate. We use equation (11) to compute the decrease in per–capita consumption (expressed as percent of GNP) that would generate the same welfare loss as that from increasing inflation from zero to a given rate in the Tables. Notice that the welfare cost of inflation depends on the degree of risk aversion. Other things equal, the higher the degree of risk aversion, the lower is the welfare cost of inflation. From Table 2 we see that a shift from zero inflation to an annual rate of inflation of 10 percent (i.e., 2.41 percent per quarter) results in a loss in utility equivalent to about 1 percent of GNP. This is more than double the 0.39 percent of GNP figure computed by Cooley and Hansen (1989) for the U.S., and the 0.3 and 0.45 percents of GNP figures reported by Fischer (1981) and Lucas (1981) respectively.[21] The welfare cost of a rate of inflation of 168 percent per year (i.e., 28 percent per quarter), the average in Israel for the high–inflation period of 1980–84, reaches the sizeable figure of about 5 percent of GNP.[22]

5. Concluding Remarks

In this paper, we found that the steady state quantitative implications of a simple dynamic model of money in the utility function are generally compatible with the observed stability of seigniorage in Israel. That is, while inflation fluctuated in the sample between double digit figures to 500 percent per year, the ratio of seigniorage to GNP remained between 2 to 3 percent. Although changes in inflation were not accompanied by marked fluctuations in seigniorage, they had a strong impact on welfare in the steady state. Based on the model's estimated parameters, the steady state welfare cost of 10 percent inflation is about one percent of GNP, and the welfare cost of an inflation rate of 168 percent per year (the average in Israel between 1980 and 1984) is about 4 percent of GNP.

The analysis could be extended in several directions. First, our quantitative analysis of seigniorage and of the welfare cost of inflation was confined to steady states. It is well known that in episodes of high and volatile inflation, the actual levels of seigniorage revenue and of the welfare cost of inflation may well differ from steady state levels. Thus, caution is suggested in regarding our quantitative findings as definitive, as it would be desirable to extend the analysis to take into account transitional factors which give rise to these differences.

Second, it seems plausible that the calculation of welfare costs of inflation may depend on the extent to which the distortions induced by other taxes are affected by changes in the inflation tax. Some progress on this issue has been made recently by Cooley and Hansen (1990), who explore in the context of a real business cycle model how the distortions associated with the inflation tax compare with the distortions arising from taxes on labor and capital income and on consumption.

Third, the analysis could be extended to allow for potential nonneutralities of money and inflation both in and out of steady states. Previous research indicates that changes in the rate of inflation may affect the allocation of time between work and leisure as well as the profitability of capital accumulation. Explicitly taking into account these effects may have a nonnegligible impact on the calculations of seigniorage and of the welfare cost of inflation that are based on the assumption of neutrality.

FOOTNOTES

1 See Meridor (1988, Table 3).

2 For evidence of rather weak, and time–varying, statistical links between seigniorage, budget deficits, and inflation in low inflation industrial countries see King and Plosser (1985).

3 See Drazen and Helpman (1990); a very clear exposition of this result is provided by Blanchard and Fischer (1989, pp. 512–517). See also Bental and Eckstein (1990).

4 See also Sargent (1987, ch. 4) and Blanchard and Fischer (1989, ch. 4).

5 Fischer (1981) and Lucas (1981) calculated these costs for the U.S. at 0.3 and 0.45 percents of GNP respectively.

6 Although in the present paper we use a money in the utility function specification, we have also explored empirically, in previous work, a model with cash–in–advance constraints; see Eckstein and Leiderman (1988). On the functional equivalence of various specifications of the role of money, see Feenstra (1986).

7 See Singleton (1985) and Eichenbaum, Hansen, and Singleton (1988) for similar specifications.

8 This function is analogus to the one used in different nonmonetary contexts by Kydland and Prescott (1982), and Eichenbaum, Hansen, and Singleton (1988).

9 We thus assume the same neutrality or invariance property as in Sidrauski (1967). See McCallum (1990) for a discussion of conditions under which this neutrality feature holds.

10 For a derivation of a Cagan–type demand for money from utility maximization see Calvo and Leiderman (1990). Notice that the inflation variable enters as $\pi/(1+\pi)$ and not as π (as in many empirical studies).

11 Welfare calculations based on Cagan's demand for money generally measure the change in the area under the money demand function due a move from stable prices to a positive π.

12 In estimating the weighting matrix, we apply the modified Durbin procedure developed by Eichenbaum, Hansen, and Singleton (1988, Appendix B). We thank Masao Ogaki for providing us the GMM program which we used along with Gauss v. 1.49.

13 The quarterly lending rate is the interest rate most widely used in Israel as an indicator of conditions in the money market and of the stance of monetary policy. Other interest rates have typically moved together with movements in this rate. The data source for the consumption quantity and price variables is the National Accounts publication by the Israeli Bureau of Statistics. The data on monetary aggregates and asset returns are from the data bank of the Bank of Israel.

14 Time trend regressions (with correction for first order serial correlation) for the instrumental variables and the variables entering the Euler equations generally indicate lack of significant trends. This provides some indication of sample stationarity of these variables. The only exception is the $(1+r_{t-1})/(1+n_t)$ variable which has a trend coefficient of 0.0011 with a standard error of 0.00036.

15 The tables in the appendix provide evidence on the robustness of the results in relation to the asset return that is used in estimating the model: the interest rate used in Table 1 against the return on government indexed bonds. Since we had data on the latter only up until 1986:IV, we reestimated System 1 of Table 1 for this sample and compared the results with those with the alternative asset return. It turns out that the estimates of β and γ are quite insensitive to the asset return. Yet

for θ and δ we obtain somewhat lower estimates (in absolute value) when the return on government indexed bonds is used. This return also results in lower J_T statistics thus providing more supporting evidence for the overidentifying restrictions of the model than when the interest rate of Table 1 is used. All in all, we conclude that the results are not markedly sensitive to the choice of the asset return (among the two alternatives considered).

16 Clearly, there are limitations to comparisons restricted only to steady states. In many models the amount of seigniorage revenue that can be collected out of steady state markedly differs from that of a steady state. In future work, we plan to explore the implications of our framework for the dynamics of seigniorage out of steady state.

17 Since the discussion focuses on steady states, we express the figures on seigniorage as a five year moving average of the actual data reported by Meridor (1988, Table 3). That is, seigniorage at time t is the average of values from t–2 to t+2. This amounts to a smoothing of the seigniorage series.

18 Bruno (1986) also used in several of his calculations for seigniorage money demand semi elasticities of about –5.0. We have checked this number by estimating, with our data, a Cagan demand for money in Israel for the period 1970:III to 1988:III. The estimated semi elasticity is –5.04, with estimated standard error of 0.959.

19 As indicated in the Introduction, flatness of the seigniorage ratio with respect to changes in the rate of inflation is not unique to the case of Israel.

20 This value is quite close to the –0.5 elasticity of inventory (or transactions) models of the demand for money. In their study on money demand in Israel, Leiderman and Marom (1985) report a long run inflation rate elasticity of money demand of

–0.41 for the period October 1978 to December 1981, using a semi–log Cagan–type specification of money demand.

21 Cooley and Hansen (1989) studied the effects of the inflation tax in the context of a real business cycle model in which money is introduced via cash–in–advance constraints. Notice that in their framework monetary velocity is invariant with respect to changes in the rate of inflation. The welfare cost calculations of Fischer (1981) and Lucas (1981) are directly based on the area under the demand curve for money.

22 All these calculations apply to comparisons of steady states. A more comprehensive assessment of the welfare costs of inflation would have to take into account the distortions and costs imposed by inflation out of steady state.

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Table 1

Estimates of the Model Under Alternative Sets of Instruments
Israel, 1970:I - 1988:III

	System 1				System 2			
Parameters	CM	CNM	CB	CNB	CM	CNM	CB	CNB
β	0.995 (0.025)	0.976 (0.025)	1.019 (0.012)	1.011 (0.009)	0.998 (0.008)	0.968 (0.005)	0.944 (0.086)	0.973 (0.009)
γ	0.055 (0.003)	0.050 (0.002)	0.042 (0.001)	0.042 (0.001)	0.054 (0.001)	0.052 (0.001)	0.049 (0.002)	0.048 (0.001)
θ	-5.631 (1.262)	-5.383 (2.117)	-2.336 (0.624)	-1.001 (0.698)	-1.529 (0.417)	0.241 (0.141)	1.034 (0.452)	-0.866 (0.181)
δ	0.285 (0.088)	0.374 (0.122)	0.499 (0.230)	0.574 (0.397)	-0.006 (0.087)	-0.311 (0.054)	-0.753 (0.030)	-0.038 (0.060)
J_T	4.844 (0.304)	15.022 (0.005)	5.072 (0.280)	10.460 (0.033)	22.251 (0.014)	4.547 (0.919)	30.279 (0.001)	4.653 (0.913)
ρ(d1,d2)	0.783	0.746	0.346	0.176	0.590	0.611	0.918	0.516

Notes:

1. The data definitions are as follows. CM: aggregate consumption and M1 per-capita; CNM: aggregate consumption of nondurables and M1 per-capita; CB: aggregate consumption and monetary base per-capita; CNB: aggregate consumption of nondurables and monetary base per capita.

2. J_T is the value of the criterion quadratic function. Standard errors of estimates and probability values of J_T appear in parentheses. ρ(d1,d2) is the correlation between the estimated residuals, defined as in equations (12) and (13) in the text.

3. System 1 corresponds to the z1 instrument set discussed in the text and System 2 to the z2 set of instruments.

TABLE 2 - SEIGNIORAGE RATIO, MONEY DEMAND ELASTICITY, AND WELFARE COST OF INFLATION

π	θ = -5.6			θ = -1.5			θ = 0.0		
(Quarterly)	SR	η	WL	SR	η	WL	SR	η	WL
0.00	0.0064	0.00	0.0	0.0116	0.00	0.0000	0.0166	0.00	0.0000
0.0123	0.0104	0.14	0.0046	0.0116	0.23	0.0082	0.0216	0.31	0.0113
0.0241	0.0132	0.24	0.0085	0.0196	0.36	0.0144	0.0240	0.45	0.0193
0.05	0.0174	0.37	0.0153	0.0233	0.52	0.0246	0.0268	0.610	0.0317
0.10	0.0219	0.50	0.0250	0.0266	0.63	0.0375	0.0289	0.70	0.0467
0.15	0.0243	0.56	0.0318	0.0281	0.67	0.0461	0.0299	0.72	0.0560
0.20	0.0258	0.58	0.0370	0.0290	0.67	0.0525	0.0304	0.72	0.0630
0.28	0.0274	0.58	0.0434	0.0298	0.66	0.0600	0.0309	0.70	0.0710
0.32	0.0279	0.58	0.0459	0.0301	0.65	0.0630	0.0310	0.68	0.0741
0.50	0.0294	0.55	0.0543	0.0308	0.60	0.0724	0.0314	0.62	0.0841
0.70	0.0302	0.50	0.0602	0.0312	0.54	0.0789	0.0316	0.55	0.0910
9E+090	0.0325	0.00	0.1431	0.0323	0.00	0.1083	0.0322	0.00	0.1215

NOTES:

1. SR denotes seigniorage as a percentage of GNP, η denotes the elasticity of money demand with respect to inflation, and WL is the welfare cost of inflation as a percentage of GNP. See text for further explanations.
2. The figures in this table were calculated under the following parameter values:

   β = 0.987; δ = 0.3; and γ = 0.05.

TABLE 3 - SEIGNIORAGE RATIO, MONEY DEMAND ELASTICITY, AND WELFARE COST OF INFLATION: ADDITIONAL RESULTS

π (Quarterly)	SR	$\delta = -0.3$ η	WL	SR	$\delta = -0.7$ η	WL
0.00	0.0062	0.00	0.0000	0.0056	0.00	0.0000
0.0123	0.0100	0.14	0.0046	0.0091	0.14	0.0046
0.0241	0.0127	0.24	0.0085	0.0116	0.24	0.0085
0.05	0.0168	0.37	0.0153	0.0152	0.37	0.0153
0.10	0.0211	0.50	0.0250	0.0192	0.50	0.0250
0.15	0.0234	0.56	0.0318	0.0213	0.56	0.0320
0.20	0.0249	0.58	0.0370	0.0226	0.58	0.0370
0.28	0.0264	0.58	0.0434	0.0240	0.58	0.0434
0.32	0.0269	0.58	0.0459	0.0244	0.58	0.0459
0.50	0.0283	0.55	0.0543	0.0257	0.55	0.0543
0.70	0.0291	0.50	0.0602	0.0264	0.50	0.0602
9E + 090	0.0314	0.00	0.1430	0.0285	0.00	0.0873

Notes: 1. See notes to Table 2. Here we set $\beta = 0.987$; $\theta = -5.6$; and $\gamma = 0.05$.

TABLE 4 - SIMULATED STANDARD ERRORS FOR SR AND η

π (Quarterly)	St. Error for SR	St. Error for η
0.00	0.00062	0.0000
0.0123	0.00083	0.0148
0.0241	0.00091	0.0220
0.05	0.00092	0.0276
0.10	0.00080	0.0272
0.15	0.00067	0.0242
0.20	0.00057	0.0213
0.28	0.00046	0.0177
0.32	0.00041	0.0163
0.50	0.00029	0.0119
0.70	0.00021	0.0091
9E + 090	0.00011	0.0000

Notes: 1. See text for explanations.

APPENDIX

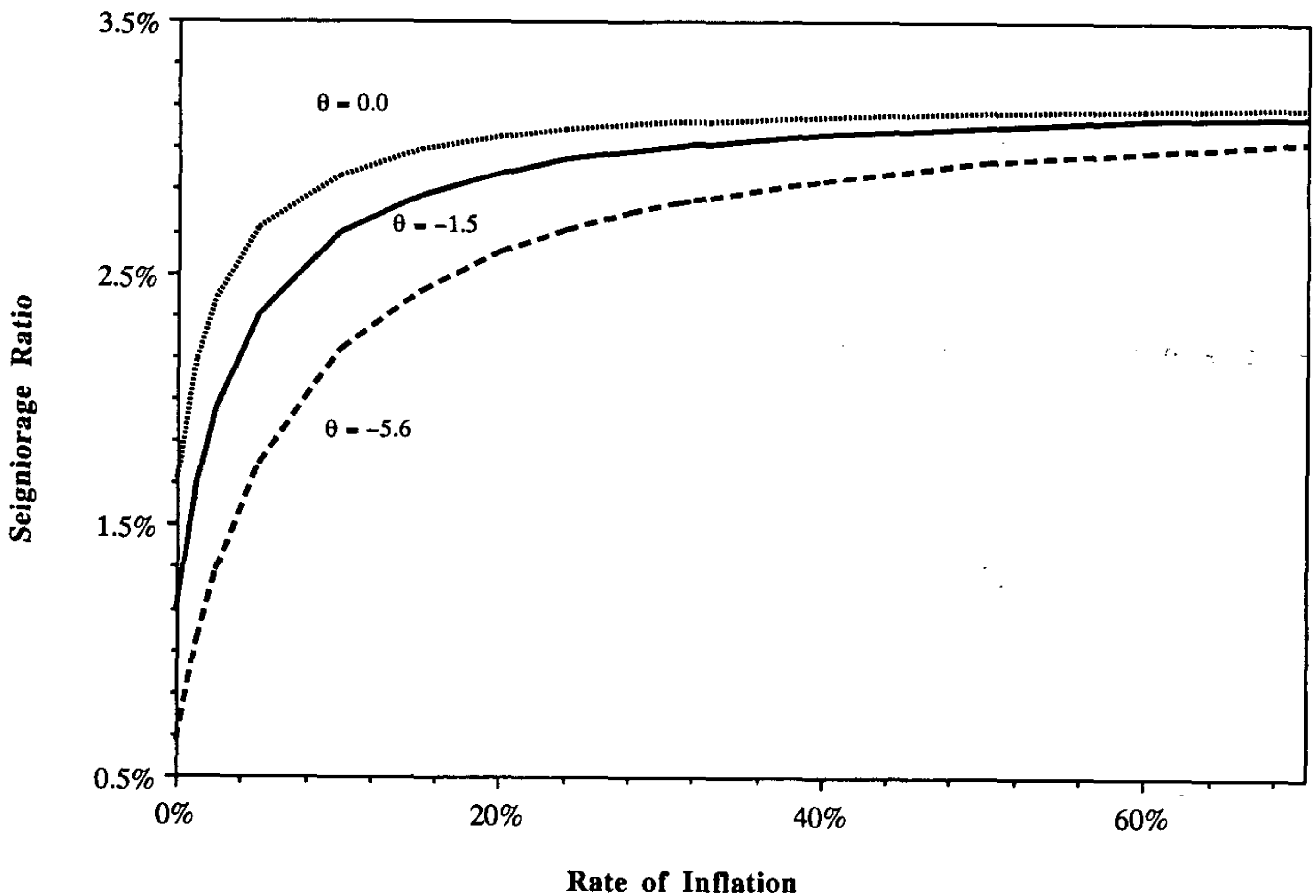
Estimates Under Alternative Asset Return
Israel, 1970:I - 1986:IV

Parameters	CM	CNM	CB	CNB
	Return: Bank's Lending Rate			
β	1.038 (0.019)	1.001 (0.014)	1.025 (0.014)	1.023 (0.010)
γ	0.055 (0.002)	0.052 (0.002)	0.042 (0.001)	0.042 (0.001)
θ	-4.270 (0.646)	-3.551 (1.202)	-2.609 (0.600)	-2.231 (0.919)
δ	0.233 (0.067)	0.388 (0.142)	0.510 (0.202)	0.587 (0.225)
J_T	7.472 (0.113)	10.931 (0.027)	4.811 (0.307)	7.591 (0.108)
ρ(d1,d2)	0.706	0.533	0.300	0.273
	Return: Yield on Government Indexed Bonds			
β	0.997 (0.004)	1.000 (0.005)	1.012 (0.004)	0.984 (0.007)
γ	0.049 (0.004)	0.043 (0.003)	0.041 (0.003)	0.034 (0.024)
θ	1.061 (0.058)	-1.143 (0.676)	-0.831 (0.377)	-0.096 (0.673)
δ	-0.394 (0.264)	0.149 (0.103)	0.150 (0.071)	0.239 (0.168)
J_T	3.669 (0.453)	3.985 (0.408)	3.998 (0.406)	4.071 (0.396)
ρ(d1,d2)	0.312	0.537	0.440	0.265

Notes: See notes to Table 1 in text. The instrument set Z1 was used in estimating the model. See also Footnote 15.

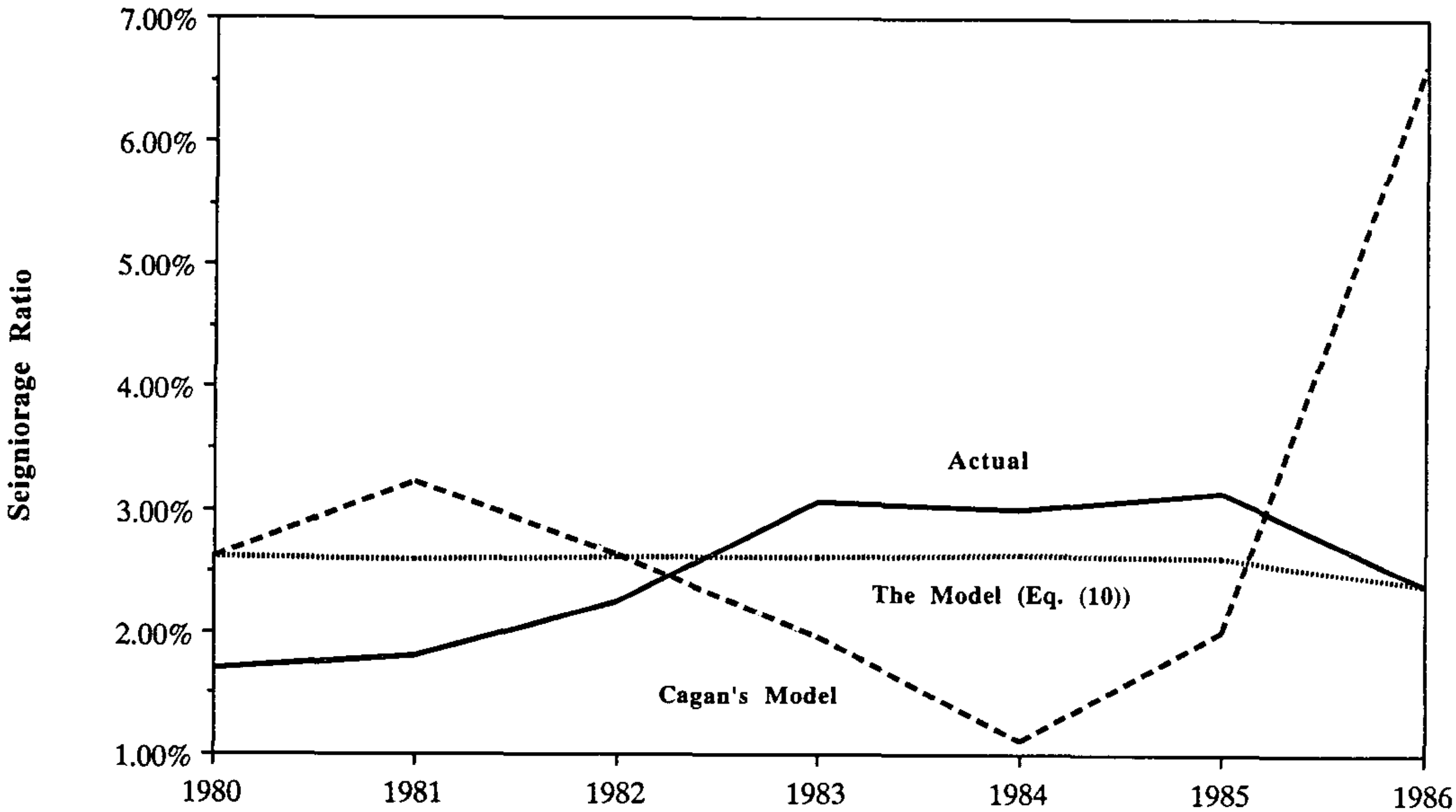
Figure 1 - Seigniorage as a Percentage of GNP

Under Various Rates of Inflation Rates and Degrees of Risk Aversion



Note: The plots are based on equation (10) in the text with the parameter values used in section 4

Figure 2 - Actual and Simulated Values of Seigniorage Ratio



Note: Actual figures correspond to a five year moving average of the seigniorage ratio.

See text for further explanations.