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FISCAL POLICY, SPECIALIZATION, AND TRADE IN THE TWO-SECTOR MODEL: THE RETURN OF RICARDO?

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ABSTRACT

This paper develops a two-sector neoclassical model of international trade with endogenous capital accumulation and intertemporal optimization. In contrast to the traditional "$2\times2\times2$" model, there is a Ricardian implication that countries specialize according to comparative advantage. Consequently, the theory predicts that government expenditure policies are unlikely to affect the established pattern of specialization and trade, but that changes in tax policies can result in a dramatic reorganization of world production. Further, the dynamic "$2\times2\times2$" model can explain many of the salient features of international trade that are problematic for the standard Heckscher-Ohlin-Samuelson model.

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"Reports of my death are greatly exaggerated."

— Mark Twain

1. Introduction

Neoclassical trade theory is widely viewed as theoretically elegant but empirically embarrassing. This view is based in large part on the fact that the predictions of the Heckscher-Ohlin-Samuelson (H-O-S) model are not borne out by the data. In the H-O-S model, aggregate quantities of each factor are fixed exogenously, and relative factor endowments are the primary determinants of specialization and trade. Extensive empirical tests have largely been unsupportive of these implications of the theory.[1] But neoclassical trade theory has its roots in the work of David Ricardo [1817]. In the Ricardian model, the pattern of specialization and trade depends on technical comparative advantage; consequently, this model predicts a tendency toward national specialization in production.

In this frankly reconstructionist paper, we present a neoclassical two sector, two factor, two country neoclassical model which differs from the standard H-O-S model in that it incorporates endogenous capital accumulation and intertemporal optimization. The predictions of this model regarding patterns of specialization and trade are markedly different from the H-O-S model, but are very much in the spirit of the traditional Ricardian model. That is, the equilibrium pattern of specialization and trade depends on comparative advantage, and there is a corresponding presumption of specialization. Based on the predictions of this model, we argue that neoclassical trade theory is <u>not</u> dead from an empirical point of view. On the contrary: this modern incarnation of neoclassical trade theory can potentially explain many phenomena that are inexplicable within the traditional H-O-S model, including many of those which prompted the development of the "new view" of trade theory, as synthesized by Helpman and Krugman [1985].

In the model developed in this paper, as in the standard "2x2x2" model, the two factors of production are capital and labor. There are two final goods in the economy: one sector produces a pure consumption good, and the other produces a good which can either be used to augment the capital stock or may be consumed. Both sectors produce according to neoclassical, constant returns to scale production functions, and both sectors require inputs of both factors. Individuals maximize the expected value of lifetime utility gained from consumption, thus, saving is endogenous.[2] The combination of reproducible capital, marginal costs which are independent of scale, and optimizing agents means that the long-run production possibility frontier is linear. Thus, countries specialize according to comparative advantage, as predicted by the simple Ricardian model. Further, the steady state "techniques of production"—quantities of labor and capital applied to the production process—are independent of demand conditions, including the structure of government purchases of goods and services. In the steady state, the rate of return to capital is equated to a fixed discount rate, and the autarky wage rate and the relative price of the two final goods are all invariant to the equilibrium division of output between the two final goods produced in the economy. Again, there emerges a strong parallel to the Ricardian single factor, fixed-coefficients model.

In our model, we find that tax policies—but not expenditure policies—emerge as important determinants of the long-run pattern of specialization and trade. In particular, with the capital good taken to be numeraire, our model generates the following predictions for the effects of fiscal policy interventions. First, permanent changes in government expenditures financed by debt or lump-sum taxes have no long-run effect on the autarky interest rate, wage rate, or relative price of the two final goods. In an open economy, changes in government expenditure are unlikely to affect the established pattern of specialization and trade. Second, permanent changes in taxes on output of the non-capital good (the pure consumption good) similarly do not affect the autarky interest rate, wage rate, or net-of-tax relative price of the two final goods. In an open economy, such a tax may lead to a reversal in the established pattern of specialization. Third, we find that permanent changes in taxes on output of the capital good sector <u>do</u> directly affect autarky factor returns and the net-of-tax relative price. As with taxation of the consumption-good sector, the established pattern of specialization and trade may shift in response to this tax. However, there is a second source of welfare loss associated with this tax that does not arise with taxation of the consumption good sector: producers choose socially inefficient techniques of production.

The paper is organized as follows. Section 2 presents the two-sector model of an autarky economy, and studies the determinants of the long-run factor returns, relative prices, and the structure of production. Section 3 studies the effects of a variety of fiscal interventions in the autarky economy. In section 4, we examine the determinants of specialization and trade in a small open economy, and investigate the effects of permanent changes in fiscal policy. Section 5 studies the general equilibrium of a two-country world. Section 6 takes up the challenge posed by Helpman and Krugman [1985]: that neoclassical models cannot explain central features of international trade, and that noncompetitive models are the only empirically relevant alternative. Section 7 concludes with a brief summary and a discussion of directions for future research.

2. The Model

Following the traditional approach of real trade theory, we study first an economy operating in autarky and then consider the effects of opening that economy to trade. In this model, as in the standard "2x2" model, there are two produced goods and two factors of production. Sector 1 produces a pure consumption good, which we call "food." This good is nonstorable and must therefore be consumed in the period in which it is produced. Sector 2 produces a durable good, "machines," which can be used as an investment good to augment the capital stocks in the two industries, or it can be consumed. The two factors of production are privately supplied labor and capital, and both factors are required for production of each of the two final goods produced by the economy. It is important for the predictions of this model that capital is reproducible, and that there is a single nonreproducible factor (labor). The economy is populated by a single representative agent.

Preferences: The representative individual in this economy receives utility from two goods: food and machines. Let ρ denote the representative individual's pure rate of time discount, and let $\beta \equiv [1+\rho]^{-1}$ denote his subjective discount factor. The aim of the individual is to maximize

$$U = \sum_{t=0}^{\infty} \beta^t u(C_{1t}, C_{2t}) \tag{1}$$

where C_{1t} denotes consumption of food and C_{2t} denotes consumption of machines.[3]

Production technology: The two final goods are produced according to production functions which exhibit constant returns to scale in both factors together, but decreasing returns in each factor separately. The production functions are given by:

$$Y_{1t} = F_1(K_{1t}, N_{1t}) \tag{2}$$

$$Y_{2t} = F_2(K_{2t}, N_{2t}) \tag{3}$$

where K_{jt}, N_{jt} denote capital and labor used in producing sector j output. The economy-wide capital stock is denoted by K_t:

$$K_t = K_{1t} + K_{2t} \tag{4}$$

Since the primary focus of the paper is on the determinants of specialization and trade in the steady state, we need not take a stand on the short-run degree of capital mobility across sectors and across countries. So long as capital can be moved in the long run—for example, by letting old capital equipment depreciate and placing new investment in a different location—the same steady state results will obtain.

Endowments: The representative agent allocates a fixed amount of time, $\bar{N}$, to market work each period.[4] This time is split between work in the two sectors:

$$N_{1t} + N_{2t} \leq \bar{N}. \tag{5}$$

Government: The government of this economy levies taxes, distributes transfers to private agents, and purchases output. Taxes may be lump-sum or may take the form of sector-specific distortionary taxes on output. The tax rate on output in sector j is denoted τ_j. Government purchases of the output of sector j are denoted G_j; government purchases do not yield utility to individuals, nor are they productive in the sense that they shift the production functions F_j.[5] P_t denotes the relative price of good 1 in terms of good 2, and T_t denotes transfers from the government to private individuals, denominated in terms of good 2. The government's budget constraint is:

$$P_tG_1 + G_2 + T_t = \tau_1P_tY_{1t} + \tau_2Y_{2t} \tag{6}$$

Resource constraints: Letting δ denote the rate of depreciation of capital, which we assume for simplicity to be identical across sectors (and, later, across countries as well), the resource constraints for this economy are:

$$Y_{1t} = C_{1t} + G_1 \tag{7}$$

$$Y_{2t} = C_{2t} + G_2 + [K_{1,t+1} - (1-\delta)K_{1t}] + [K_{2,t+1} - (1-\delta)K_{2t}]. \tag{8}$$

Output of the pure consumption good (good 1) is allocated either to private or government consumption. Output of the capital good may be used for private or government consumption, or to augment sectoral capital stocks.

2.1 Competitive equilibrium and the nonsubstitution theorem

Individuals and firms in this economy act competitively, i.e., they view themselves as too small to affect equilibrium prices. Firms maximize profits subject to the technological constraints and subject to the government's policy rules; individuals maximize lifetime utility subject to their time constraint and the government policy rules. The capital good (good 2) is chosen to be numeraire, thus all prices, wage rates, rental rates, etc., are expressed in units of good 2. The competitive equilibrium for this economy is a set of functions linking the endogenous variables (C_{jt}, K_{jt}, N_{jt}, Y_{jt}, P_t, etc.), to the exogenous variables G_j, τ_j, and T_{jt}, j=1,2. The competitive equilibrium for this economy is found by solving the system of first-order necessary conditions associated with the competitive equilibrium problem (see the Appendix). The key properties of the steady state of this economy are summarized in the first Proposition.

PROPOSITION 1: *Let $\mathcal{T}_j$ denote the minimal set of parameters that completely describes the production functions $F_j(K_j, N_j)$, j=1,2, and define the set $\varphi_1 \equiv \mathcal{T}_1 \cup \mathcal{T}_2 \cup \{\rho, \delta, \tau_1, \tau_2\}$. Let $\mathcal{P}$ denote the minimal set of parameters that completely describes the* *momentary utility function* $U(C_1,C_2)$, *and define the set* $\varphi_2 \equiv \mathcal{P} \cup \{G_1,G_2\}$. *The set* $\Psi \equiv \varphi_1 \cup \varphi_2$ *contains all the parameters of the model.*

(i) *In steady state equilibrium, the following are functions of the parameters of technology, tax rates, and the subjective discount rate (i.e., they are functions of* φ_1 *but not* φ_2*): the rate of return to capital in each sector; the wage rate; the relative price of one good in terms of the other; and capital-labor ratios in each sector.*

(ii) *Steady state quantities,* C_{jt}, I_{jt}, Y_{jt}, *depend on all the parameters of the model, i.e., the quantity variables are functions of* $\Psi = \varphi_1 \cup \varphi_2$.

(Proofs of the Propositions are contained in the Appendix.) Part (i) of Proposition 1 is closely related to the "nonsubstitution theorem" of Samuelson [1951], and later proven for increasingly general economies by Arrow [1951] and Mirrlees [1969]. The central results of this paper all stem from this first Proposition, so the basic insights behind the proof of this Proposition are discussed more fully below.

On a technical level, the important feature of the system of Euler equations which describes competitive equilibrium in this economy is that this system is block-recursive. The first block determines prices, factor returns, and capital-labor ratios as functions of φ_1, and the second block determines equilibrium quantities as functions of $\Psi \equiv \varphi_1 \cup \varphi_2$. The first block is itself recursive, and the first step in solving the model is to use the following familiar relationship to determine the marginal product of capital in sector 2:

$$(1-\tau_2)\ \frac{\partial F_2(K_2,N_2)}{\partial K_2} = \rho + \delta. \tag{9}$$

In neoclassical models with endogenous capital accumulation and individual optimization, the marginal product of capital in the capital-producing sector is equated to the "effective discount rate" $\rho+\delta$. Because of the homogeneity of the production functions, the marginal product of capital depends only on the capital-labor ratio. Thus (9) pins down $k_2 \equiv K_2/N_2$. In the Appendix we show that, having solved for the capital-labor ratio in sector 2, we can proceed sequentially to solve for the following as functions of φ_1 alone: the wage rate; the capital-labor ratio in sector 1; and the equilibrium rental rate. For example, the equilibrium wage rate equals the marginal product of labor in sector 2: $w_t = [(1-\tau_2)\partial F_2(K_{2t},N_{2t})/\partial N_{2t}]$. Since the capital-labor ratio in sector 2 is determined by (9), and since constant returns to scale in production means that the marginal product of labor depends only on the capital-labor ratio, we see that the wage rate depends only on τ_2 and the parameters of the sector 2 production function. Further, from (9) we see that the only policy variable that can affect the capital-labor ratio in sector 2 is the tax rate on sector 2 output. Because of the recursive structure of the model, it is also true that only changes in τ_2 can affect the capital-labor ratio in sector 1, the wage rate, or the rental rate. Changes in τ_1 and changes in government expenditure will not matter for these variables. We will use these results repeatedly to analyze fiscal interventions in closed and open economies.

2.2 The long-run PPF

The long-run production possibility frontier (PPF) is defined as the set of privately efficient production points given preferences, technology, endowments, and government policies. A useful expression for the long-run PPF can be developed as follows. Let $k_j \equiv K_j/N_j$ denote the capital-labor ratio in sector j, and define $f_j(k_j) \equiv F_j(k_j,1)$, j=1,2. Using these definitions and the labor constraint we have:

$$\bar{N} = N_1 + N_2 = \left[\frac{N_1}{Y_1}\right] Y_1 + \left[\frac{N_2}{Y_2}\right] Y_2 = [f_1(k_1)]^{-1} Y_1 + [f_2(k_2)]^{-1} Y_2. \quad (10)$$

The form of this expression recalls a similar expression from Jones's classic [1965] analysis of the standard two-sector model. Defining $a_{Nj} \equiv [f_j(k_j)]^{-1} = N_j/Y_j$ for j=1,2 as the "labor requirement coefficients" giving the number of units of labor required for the production of one unit of good j, we can rewrite (10) as:[6]

$$\bar{N} = a_{N1}Y_1 + a_{N2}Y_2 \ . \quad (11)$$

In Jones's analysis there is an analogous equation for the second factor, land, which is assumed to be nonreproducible and in fixed aggregate supply:

$$\bar{T} = a_{T1}Y_1 + a_{T2}Y_2 \quad .$$

Figure 1 plots the labor constraint and the land constraint in Jones's model. The set of feasible production points is given by the shaded area, and the PPF is the locus BAC. In the neoclassical model, however, the second factor—capital—is not fixed exogenously and it is reproducible. Because labor is the only nonreproducible factor, the only constraint that binds in the long run is the labor constraint. In the neoclassical model with endogenous capital accumulation, therefore, the set of feasible points is the triangle ODC, and the long-run PPF is just the labor constraint, DAC. Thus our second main finding is:

PROPOSITION 2. *The long-run PPF is linear in the two-sector neoclassical model with endogenous capital accumulation.*

Different points on the long-run PPF correspond to the same aggregate quantity of labor input, $\bar{N}=N_1+N_2$, but the aggregate quantity of capital input is different at each point unless the equilibrium capital-labor ratios are the same in each sector.

2.3 The short-run PPF

We define the short-run PPF as the set of efficient production points given a fixed aggregate stock of capital and fixed total labor input, but with both factors mobile across sectors—exactly the assumptions of the standard H-O-S model. Figure 2 shows an economy in steady state equilibrium at point A; the corresponding short-run PPF is the curve BAC. This short-run PPF will typically have the familiar "bowed-out" shape, which reflects (i) the fact that technologies differ across sectors, and (ii) in the short run (as in the H-O-S model) the fixed aggregate quantity of capital means that each sector faces an upward-sloping supply schedule for capital. The slope of the short-run PPF is the marginal rate of transformation of good 1 for good 2, holding fixed aggregate quantities of both factors. In competitive equilibrium, producers equate this marginal rate of transformation to the net-of-tax relative price of the two goods. The relationship between the short-run PPF, the long-run PPF, and the equilibrium relative price is spelled out in the next Proposition:

PROPOSITION 3. *The absolute value of slope of the short-run PPF at the point where it intersects the long-run PPF is the steady state net-of-tax relative price received by producers. The absolute value of the slope of the short-run PPF is greater than (less than) the absolute value of the slope of the long-run PPF if sector 1 is capital-intensive (labor-intensive).*

Figure 2 thus illustrates a situation in which sector 1 is the capital-intensive sector, so that points southeast (northwest) of A on the long-run PPF are associated with higher (lower) levels of the aggregate capital stock. Because the short-run PPF holds fixed the aggregate quantity of capital, the short-run PPF lies above the long PPF for all $Y_2 < Y_2^o$, and lies below the long-run PPF for all $Y_2 > Y_2^o$.

From Proposition 1 we know that steady state relative prices do not depend on the pattern of expenditure in the economy, i.e., they do not depend on which point on the long-run PPF represents the steady state. Each point on the long-run PPF is a potential steady state, and each has an associated short-run PPF. Thus the slope of the short-run PPF at the point where it crosses the long-run PPF must be the same everywhere along the long-run PPF, as drawn in Figure 3.

3. The Effects of Fiscal Policy

An important set of policy questions concern the domestic and international effects of governmental tax and expenditure policies. In the model of Section 2, the predictions concerning the response to policy changes are very different from the predictions of the H-O-S model. For some interventions, our model predicts no change at all in key macro variables; for other interventions, small changes in policy can induce large responses by private individuals. Because tax and expenditure policies affect the economy in very different ways, this section addresses in more detail the question of how the economy responds in autarky to changes in these policies.

3.1 A change in the pattern of government expenditure

To begin, suppose that government expenditure is zero for each of the two final goods in the economy. The economy's initial steady state equilibrium is given by point A in Figure 4. Now suppose that the government decides permanently to alter its purchases of one of the produced goods, financing this alteration in expenditure via lump-sum taxation. The effects of this intervention are summarized in the following Proposition.

PROPOSITION 4: *Permanent changes in government expenditure on either good, if financed by lump-sum taxes, have no effect on steady state wages, interest rates, relative prices, or capital-labor ratios. If both goods are normal goods in consumption, increases in government purchases of one good lead to an increase in the equilibrium output of that good, and a decrease in the equilibrium output of the other good.*

Figure 4 illustrates the effect of an increase in G_1 equal to ΔG_1. Because there is no change in equilibrium prices (which is a direct implication of Proposition 1, part (i)), consumers face only a pure wealth effect. If both goods are normal goods, consumption of both C_1 and C_2 decreases. Thus the new steady state is located along the (unchanged) long-run PPF southeast of the original equilibrium point, at a point like B. Output of good 1 rises, but by less than ΔG_1 (see the Appendix). A change in G_2 can be analyzed in a similar fashion: the new equilibrium point will be a point on the long-run PPF and, if both goods are normal, $0 < \Delta Y_2/\Delta G_2 < 1$, and $\Delta Y_1/\Delta G_2 < 0$.

3.2 Distortionary taxation of the consumption good sector

The previous section demonstrated that government expenditure policies do not affect steady state prices or factor returns, although they do affect steady state quantities. In this section and the next we study the effects of changes in distortionary taxation of output of the two final goods. Consider first a permanent increase in the tax rate on the output of sector 1 (food); suppose the tax rate rises from zero to τ_1. Suppose as well that all the proceeds from the tax are rebated in a lum-sum fashion, so that there is no direct wealth effect associated with the increase in tax rates. The steady state effects of this tax are summarized in the following Proposition:

PROPOSITION 5: *Imposition of a tax on sector 1 increases the price of sector 1 output by the full amount of the tax, leaving the net-of-tax price unchanged. Changes in τ_1 do not affect (i) the steady state net-of-tax return to capital; (ii) the net-of-tax wage rate, or (iii) capital-labor ratios.*

Figure 5 illustrates the effect of imposing the tax $\tau_1 > 0$. The economy was in equilibrium at point A before the tax was imposed. In the new equilibrium, point B, the relative price has risen by the full amount of the tax to $P' = P^o/(1-\tau_1)$. The net-of-tax relative price is unchanged at P^o, and factor returns are unaffected by the tax. Further, both sectors' capital-labor ratios are unchanged, as are the "labor input" coefficients, a_{Nj}. (Recall from the discussion of Section 2.1 that only taxation of sector 2 can affect capital-labor ratios and relative factor returns.)

Ricardo ([1817], Chapter 9, page 98) put forth a similar proposition; he predicted that a tax on corn would cause the price to rise by the full amount of the tax, for any smaller tax increase would lead the owners of productive factors to seek higher-paying employment:

"If the price of raw produce did not rise so as to compensate the cultivator for the tax, he would naturally quit a trade where his profits were reduced below the general level of profits; this would occasion a diminution of supply, until the unabated demand should have produced such a rise in the price of raw produce as to make the cultivation of it equally profitable with the investment of capital in any other trade."

His argument is still correct in the context of our two-sector neoclassical model with intertemporal optimization and endogenous capital accumulation: there is only one configuration of after-tax factor returns that will induce nonspecialization in this economy. Since the economy is operating in autarky, and because consumers will never choose to specialize in consumption, given our assumptions on preferences, competitive equilibrium requires nonspecialization in production.

The tax on output of the pure consumption good causes a welfare loss because it drives a wedge between the relative price faced by consumers and the relative factor cost of producing the two goods. Note that the new steady state production point (point B) is on the long-run PPF; since taxes on sector 1 do not interfere with efficient capital accumulation, the tax induces no inefficiency in the "techniques" used to produce the two goods. The welfare loss derives from the fact that the economy produces and consumes a suboptimal mix of goods as a result of the tax.

3.3 Distortionary taxation of the production of capital

Now, consider the effects of imposing a distortionary tax, τ_2, on the output of the capital-producing sector. The effects of this tax are qualitatively very different from a tax on the pure consumption good, and are summarized in the following Proposition:

PROPOSITION 6: *Imposition of a tax on the capital–producing sector affects steady state capital–labor ratios, factor returns, and relative prices. In addition, it shifts the long–run PPF so that the new long–run PPF lies everywhere below the initial long–run PPF. The steady state wage–rental ratio falls and capital–labor ratios fall in each sector. If the elasticity of substitution of capital for labor is equal across sectors, then the new long–run PPF is steeper (flatter) than the initial one if sector 1 is the capital–intensive sector (the labor–intensive sector).*

Figure 6 graphs a situation in which sector 1 is the capital–intensive sector, and the elasticity of substitution of capital for labor is the same across sectors. Thus the tax on sector 2 shifts the long–run PPF inward, and it becomes steeper. The net–of–tax price rises since good 1 is the capital–intensive good, and the new long–run equilibrium is at a point like point B.

The tax on the production of capital causes a welfare loss via two channels. First, as in the case of a tax on the pure consumption good, there is a welfare cost due to static inefficiency—the relative price paid by consumers differs from the relative factor cost of producing the two goods, given the choice of production technique. But there is a second cost due to the fact that the tax on the production of capital leads to inefficiency in capital accumulation; the tax induces substitution away from capital as an input into production, resulting in socially suboptimal capital accumulation. This second source of inefficiency does not arise with taxation of the pure consumption good, since taxation of the consumption good does not distort the process of capital accumulation.

This completes our analysis of the autarky economy. The remainder of this paper builds on this structure to study the determinants of specialization and trade and the effects of fiscal policies in open economies. The results for open economies follow in a straightforward way from our analysis of the closed economy, and in the remainder of the paper we proceed on a less formal level.

4. A Two-Sector Model of a Small Open Economy

The preceding model of a closed economy may be reinterpreted as a small open economy if the relative price is taken as exogenous, being determined in world markets. We assume that physical capital is internationally mobile so that, in the steady state, there is a single world rate of return earned by capital in all locations. From the closed-economy analysis we know that (i) there is a single relative price at which the small open economy will produce both goods—we will call this the autarky relative price; and (ii) at this relative price, it is an equilibrium for the small open economy to produce any combination of the two final goods, subject to the constraint that labor is fully employed.

In the discussion which follows, we will be exclusively concerned with the determinants of production and trade in the small open economy. Because of the Fisherian separation between production decisions and consumption/saving decisions which results from the assumption of a unified world capital market, we need not discuss the details of the preferences of the residents of the small open economy in order to completely describe the equilibrium patterns of production and trade in that economy.

4.1 Specialization in the small open economy

The world price facing the small open economy is unlikely to coincide with the autarky price, because this coincidence requires either (i) that relative technological opportunities and relative tax rates are the same in the small country and in the rest of the world, or (ii) that they differ in a way that maintains the equality of relative after-tax marginal products. The presumption must therefore be that the world price and the autarky price differ. Such a difference, however slight, means that the small open economy will specialize in production of one of the two final goods. In Figure 7 we have sketched the case in which there are no taxes in the small open economy, and the world relative price of good 1, P^W, exceeds the autarky relative price P. There is no tangency between the world price and any of the short-run PPF's at the point where the short-run and long-run PPF's intersect, as required for an equilibrium with nonspecialization. Thus the equilibrium production point is point A, at which the small open economy specializes in production of good 1.[7]

4.2 Fiscal policy in the small open economy

How do alterations in government expenditure and tax policies in the small open economy influence the country's pattern of specialization and trade? From Proposition 4, we know that expenditure policies do not affect the autarky relative price. Thus changes in government expenditure in the small open economy, if these are financed by debt or lump-sum taxation, will not alter the country's pattern of specialization or trade.[8]

Changes in taxes, on the other hand, can dramatically alter the pattern of specialization and trade in a small open economy. For example, suppose that the pre-tax situation is as sketched in Figure 7, in which the autarky *(gross-of-tax) relative price* P^O *is less than the world price,* P^W. The small open economy maximizes the value of GDP by specializing in production of good 1, producing the quantity Y_1^O and selling it at P^W. Now suppose that the government of the small economy imposes a tax on sector 1 in the amount τ_1. We know from the analysis in section 3 that the autarky relative price rises from P^O to $P' = P^O/(1-\tau_1)$. If $P' < P^W$, the small economy continues to specialize in good 1. The tax on output of good 1 is borne entirely by the internationally immobile labor force, since capital is mobile and must earn the world rate of return. So long as the tax increase on good 1 is small enough to leave the pattern of specialization unchanged, its only effect is to decrease the returns to labor.

If, however, the tax is large enough so that the post-tax autarky price exceeds the world price, $P' > P^W$, the small economy will cease production of good 1 and specialize in good 2. Thus distortionary taxes may induce the small open economy to specialize in production of a good other than the one for which it possesses comparative advantage in the strict Ricardian sense of possessing the relatively superior technology.[9]

4.3 Trade and "growth" in the small open economy

Recent research by Romer [1986], Lucas [1988], King and Rebelo [1990], and Grossman and Helpman [1990] has stressed increasing returns and/or human capital accumulation as central elements in the growth process. As such, they represent a potential explanation for the dramatic restructuring and "growth miracles" of some small open economies. Our two-sector neoclassical model incorporates neither increasing returns nor human capital, yet indicates that dramatic changes may nevertheless take place in the production structure of a small economy in response to apparently minor alterations in private incentives. For example, consider a small country largely engaged in agriculture. This economy may respond to increased openness to trade and/or an increase in relative after-tax rewards to manufacturing by undertaking a radical restructuring of production in the economy away from agriculture and toward manufacturing.

How fast will this occur? Our two-sector model, like the standard, one-sector neoclassical model, possesses a strong investment accelerator: starting from an initial position in which the capital stock is below its steady state level, the transition to the steady state is very rapid (see, for example, the analysis of King and Rebelo [1992]). Thus our model predicts that the economic restructuring of this hypothetical economy would be accompanied by a rapid accumulation of capital, if manufacturing is more capital-intensive than agriculture. Along the transition path, the economy may therefore appear to undergo a "growth miracle." Without denying the potential importance of endogenous growth and/or increasing returns, we note that this simple neoclassical model also predicts a potential for dramatic economic restructuring in response to modest changes in incentives.

5. The Two Country Model

This section analyzes the equilibrium of a two-country world economy, in which both countries are large enough to affect equilibrium prices.

5.1 Steady state considerations

The world comprises two countries, each of which are described by the model of Section 2. For this model to be compatible with "balanced" steady state growth (i.e., residents of one country do not eventually own all of the world's wealth), individuals in the two countries must have the same rate of subjective time preference, ρ; this assumption is maintained throughout.

5.2 Patterns of long-run specialization and trade

We are now ready to answer the most fundamental question which one can ask of trade theory: what are the international patterns of production and trade? In the two-country version of our model, at least one country must specialize, and both may do so. Each country exports the good in which it specializes, and imports the other. Letting unstarred variables denote the home country and starred variables denote the foreign country, the home country has comparative advantage in good 1 if it has the lower autarky relative price: $P < P^*$. From Proposition 1, we know that autarky prices depend only on the parameters of technology, taxes, and the rate of time preference. Thus comparative advantage depends on technological considerations, as in the Ricardian model, but also depends on the pattern of distortionary taxation.

It is straightforward to determine the pattern of specialization, once we have determined comparative advantage. So long as $P \neq P^*$, at least one country will completely specialize. This will happen no matter how small the differences in the tax rates or the production functions. For example, suppose that $P < P*$, so that the home country possesses comparative advantage in good 1. A possible equilibrium point is sketched in Figure 8, in which the home country is incompletely specialized, producing at point A in Panel I, and the foreign country is completely specialized in good 2, producing at point A in Panel II. Because the home country is incompletely specialized, the world price is given by the home country's autarky price: $P^W = P$. However, if both countries are completely specialized in production, the world price will lie between P and P*, and will be determined by demand considerations (panel III of Figure 8 shows the long-run world supply curve for good 1). This is the <u>only</u> case in which demand-side factors play a role in determining prices.

If $P = P^*$, the pattern of production and trade is indeterminate. In the absence of comparative advantage, it simply does not matter who produces what. While there is indeterminacy in the patterns of production trade, <u>world</u> production of each of the two goods is determinate. But with identical gross-of-tax relative prices in the two countries, the long-run pattern of production is not pinned down. Because capital and final goods are transportable across sectors and countries, it is a matter of indifference where any particular unit of a good is produced. However, this "knife-edge" situation in which autarky and world prices coincide is extremely unlikely. We are therefore left with a very strong prediction: in a neoclassical world in which capital is reproducible and is mobile in the long-run, there is a presumption of specialization.

5.3 <u>Open economy effects of expenditure policy</u>

How do changes in government expenditure affect the world pattern of specialization and trade? We know from Proposition 4 that shifts in the size and/or composition of government spending do not affect autarky prices. Since comparative advantage depends only on autarky prices, small changes in government expenditure will have no effect on the established pattern of specialization and trade. If, however, the change in government expenditure is large, it may have an effect on prices and the pattern of specialization. For example, suppose that the home country is incompletely specialized, and the foreign country is specialized in production of good 2, as sketched in Figure 8. Now, suppose that the government of the home country increases its purchases of good 1, and that at unchanged prices the world demand for good 1 exceeds the maximum quantity that can be produced by the home country, $\bar{Y}_1$. In the new equilibrium, the home country will specialize in production of good 1, the foreign country may be either completely or incompletely specialized in good 2, and the price of good 1 will rise. Although the change in government expenditure may alter the pattern of specialization, such a change will never reverse the existing pattern of specialization. In the context of this example, there is no pure expenditure policy that could lead the home country to specialize in good 2, while the foreign country specializes in good 1.

5.4 Open economy effects of tax policy

By contrast, the neoclassical model predicts that the pattern of specialization can be reversed by changes in tax policy. To explore the open-economy effects of taxation, assume that initially there are no distortionary taxes, and that the home country has comparative advantage in good 1, i.e., $P < P*$, where these refer to autarky prices. Let the pre-tax equilibrium be such that the home country produces both goods, and the foreign country specializes in production of good 2, as drawn in Panels I and II of Figure 8. The world supply curve for good 1 is drawn in Panel III. Because the home country is incompletely specialized, the world net-of-tax relative price is given by the home country's autarky relative price, $(P^W) = P$.

Now, suppose that the government of the home country imposes a tax, τ_1, on the production of good 1. We know (from Proposition 5) that the home country autarky price rises by the full amount of the tax: $P' = P/(1-\tau_1)$. There are two cases to consider, depending on whether the tax alters the world pattern of private comparative advantage. Suppose first that the home country retains private comparative advantage in good 1 after imposition of the tax, i.e., $P' = P/(1-\tau_1) < P*$. World output of sector 1 falls, although all of it is still produced by country 1, and world output of sector 2 increases. The world relative price increases to $(P^W)' = (P^W)/(1-\tau_1)$.

If, however, the tax increase is sufficiently large, or if the two countries were not too different before the imposition of the tax, the tax increase can alter the pattern of private comparative advantage, inducing a reversal in the pattern of specialization. If τ_1 is large enough so that $P' = P/(1-\tau_1) > P^*$, then the home country will cease production of good 1 altogether. In the new equilibrium the home country will specialize in production of good 2, and the foreign country will produce good 1 and perhaps some of good 2 as well. Clearly, this reversal of established patterns of specialization and trade will be more likely the more similar are private opportunities in the two countries before the change in tax policy.

Analysis of a tax on sector 2 proceeds in an analogous manner. However, taxation of sector 2 differs in two important respects from taxation of sector 1. First, we know that there is an additional welfare cost associated with the fact that the tax distorts capital accumulation. Second, we know that the tax affects autarky relative prices through its effects on equilibrium capital-labor ratios and factor returns.

The lesson from this section can be stated quite simply: the two-sector model with intertemporal optimization and endogenous capital accumulation predicts that government expenditure policies matter very little (if at all) for the long-run determination of specialization and trade. This theory predicts that tax policies matter a great deal, and that changes in tax policy can potentially be the source of dramatic, permanent shifts in the international pattern of specialization and trade.

6. The New View of International Trade

Recently, a new approach to trade theory has been advanced; Helpman and Krugman [1985] present a comprehensive treatment of this approach. With its twin assumptions of increasing returns to scale at the firm level and Chamberlinian monopolistic competition, this "new view" represents a radical departure from the neoclassical assumptions of constant returns to scale and perfect competition. This departure was motivated by a desire to explain features of the data viewed as inexplicable within the traditional framework. Helpman and Krugman ([1985], page 2) are explicit about the perceived failings of neoclassical theory, which they detail in a section entitled "Why we need a new theory of trade," as follows:

> "We can identify four major ways in which conventional trade theory seems to be inadequate in accounting for empirical observation: its apparent failure to explain the volume of trade, the composition of trade, the volume and role of intrafirm trade and direct foreign investment, and the welfare effects of trade liberalization."

Although Helpman and Krugman acknowledge the conceptual and technical difficulties inherent in models with increasing returns and imperfect competition, they view this approach as essential to understanding these stylized facts. In this section, we investigate the extent to which the inability of the H-O-S model to explain these empirical regularities stems from its assumption of the fixity of both factors of production. With the endogeneity of capital accumulation and long-run capital mobility, the concept of "factor endowments" no longer has any content—the "similarity" of countries is an endogenous feature of the model's equilibrium. Because of this, our neoclassical model with endogenous capital accumulation can potentially provide explanations for trade phenomena that are inexplicable within the H-O-S framework.[10]

Let us take the four "stylized facts" in turn. First, can we explain why similar countries experience large and growing volumes of trade? Our model predicts at least partial—and perhaps complete—specialization. Since individuals value many varieties of produced goods, they must necessarily trade in order to consume their preferred consumption basket. If the economies involved are growing over time, the volume of trade must expand over time as well. Thus the modern neoclassical model easily explains an increasing volume of trade as countries grow. Whether these trading partners are "similar" in terms of their capital-labor ratios depends on the form of the production functions and on distortionary taxes in the two countries. That is, the "similarity" or "dissimilarity" of countries is determined endogenously in the neoclassical model. If production functions are not too different for different goods, the requirement that the after-tax rate of return be equalized across countries provides a force leading to equilibrium capital-labor ratios that are similar across countries. If, in addition, economies transit from an initial position of autarky to a position of specialization as transportation technology and communication links improve, this model can explain explain growth over time in trade as a percentage of GNP.

The second criticism of traditional models is based on their inability to explain two-way trade in goods with similar "factor content." As discussed above, aggregate supplies of capital in the neoclassical model are endogenous, as are equilibrium choices of "factor content." It is certainly possible that, in equilibrium, producers in the two countries select similar capital-labor ratios to produce their respective goods. Unified capital markets and a tendency for technological "know-how" to diffuse across countries are two forces that might lead this to be the case. Combined with the presumption of specialization, this leads directly to the phenomenon of two-way trade in "similar" goods, where "similar" is defined in terms of factor content.

What about the large volume of trade attributable to intrafirm trade by multinationals, and the phenomenon of direct foreign investment? The neoclassical model, with its assumption of constant returns to scale production functions, is (trivially) consistent with a multitude of industrial structures. For example, it is consistent with the observation that, while both Korea and Taiwan have recently exhibited high growth rates, Korea is characterized by a small number of large firms, while Taiwan has a large number of very small firms.

The fourth difficulty with traditional models, as discussed by Helpman and Krugman, involves the view that trade liberalization often benefits all parties—something that the traditional H-O-S model does not predict. In the dynamic neoclassical model, trade barriers in the form of taxes, tariffs or quotas can lead to global welfare loss due to an inefficient world pattern of specialization and trade. Further, if taxes or tariffs affect the production of capital, there will be additional welfare losses stemming from suboptimal capital accumulation. Removing these sources of inefficiency could well leave everyone better off in the long run.

7. Summary and Conclusions

This paper has developed a two-sector neoclassical model of international trade with endogenous capital accumulation and intertemporal optimization. The predictions of this model concerning the determinants of specialization and trade have a decidedly Ricardian flavor. As a result of the nonsubstitution theorem which holds for this economy, minor differences either in production technologies or in relative tax rates lead at least one country to specialize in production. Correspondingly, small changes in private incentives may lead to large-scale reorganizations of industrial structure, and to apparent "growth miracles" as the economy transits to the new steady state. Our neoclassical model with optimization and endogenous capital accumulation predicts very different consequences arising from changes in expenditure policies versus tax policies. Changes in government expenditure are likely to leave unchanged the pattern of specialization and trade. In any case, pure expenditure policies cannot completely reverse established patterns. Changes in tax rates, on the other hand, <u>can</u> lead to a complete reversal in the world pattern of specialization.

Does anything weigh against the forces pushing the economy toward long-run specialization? After all, we do observe that some goods are produced in more than one location. One potential explanation is the simple nontradability of particular classes of goods. Another reason for nonspecialization is risk associated with the production process, as in Ruffin [1974a,b]. If there is country-specific randomness in the amount of output produced from a given level of input—due, for example, to technological shifts, weather, or random machine failures—then it is efficient to produce the same good in more than one location. The amount of "locational diversification" that is desirable would depend on the cross-country correlation of these shocks, and on the strength of comparative advantage in the absence of these shocks. Whether this effect is likely to be quantitatively important is an interesting subject for future research.

We confronted our model with the charges leveled by Helpman and Krugman—that received neoclassical theory based on the H–O–S model cannot explain salient features of international trade. We concluded that the 2x2x2 model with endogenous capital accumulation <u>can</u> potentially explain many of these phenomena, <u>without</u> departing from the classical assumptions of constant returns to scale and perfect competition.

Finally, since this paper has abstracted completely from growth considerations, a few words on this topic are warranted. If exogenous technical change is introduced in a way that permits steady state growth, then the economy can be transformed into a stationary economy that differs from the one studied in this paper only in that it has an altered discount factor (see Baxter [1988]). The analysis of this paper can therefore be reinterpreted as an economy in which the "engine of growth" is exogenous technical change. Another approach which is perhaps more appealing is to have an endogenous mechanism for growth. King and Rebelo [1990] study a two-sector model of a small open economy in which one sector produces a consumption/investment good and a second sector produces human capital. Both sectors require inputs of both goods, and capital is internationally mobile. King and Rebelo find that the findings of the present paper—that tax policies are important for the level and structure of economic activity—are translated in their setting into important effects of tax policies on steady state growth rates. Grossman and Helpman [1990] study a model with increasing returns to scale and endogenous "R&D" but without capital. They also find that policy can dramatically affect growth rates. Clearly, a fruitful path for future research is the further integration of capital theory and theories of endogenous growth into equilibrium models of the international economy.

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Footnotes

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[1]There is a voluminous literature on this subject, beginning with the seminal work of Leontief [1953]. Extensive references to this literature are contained in Leamer [1984], especially Chapter 2 (pages 45-59).

[2]This paper draws on, and is related to, several strands of existing literature. First, the model is related to paradigms developed by by Uzawa [1961,1963], Oniki and Uzawa [1965], Stiglitz [1970], Mussa [1978], and Manning and Markusen [1991]. Second, our results on the determinants of specialization in the absence of a government are related to previous analyses by Jones [1970], Jones and Ruffin [1975], Ethier and Ross [1971], Srinivasan and Bhagwati [1980], and Manning [1981]. Third, a version of the "nonsubstitution theorem" of Samuelson [1951], Arrow [1951], and Mirrlees [1969] applies in the two-sector neoclassical economy.

[3]Although this model assumes 100% depreciation for new "machines" allocated to consumption, Baxter [1991] studies a version of this model in which

consumer purchases of the output of the capital good sector become "consumer durables" which depreciate slowly over time, and yield a flow of services over their useful life. This modification does not alter any of the conclusions of the present paper concerning the determinants of specialization and trade, or the response of economies to permanent changes in fiscal policies.

[4]In this paper we have abstracted from the labor/leisure choice. The steady state properties of the model are not affected in any substantive way if substitution along the labor/leisure margin is permitted. This is because the central model element is the fixed, nonreproducible amount of time available to an individual. Variable leisure is, however, important for short-run dynamics, and this feature is incorporated into the model of Baxter [1991].

[5]See Baxter and King [1988,1990] for analyses of productive and utility-yielding government spending in the context of a one-sector, closed economy model.

[6]Jeremy Greenwood suggested this felicitous choice of notation and the analogy to Jones's work. I am embarrassed not to have thought of it myself.

[7]If, by chance, P^W does exactly equal P, the division of the small open economy's output between the two goods is indeterminate. When P^W=P, the world price line is tangent to the short-run PPF's at every point along the long-run PPF, i.e., every point along the long-run PPF is an equilibrium, since each yields the same level of GDP.

[8] Government expenditure policies may affect specialization and trade decisions in the presence of trade restrictions, in the presence of "domestic content" or "buy American" policies that apply to the government, or in the case in which government expenditures augment the public capital stock, thus shifting marginal product schedules for privately-owned capital and labor. The present analysis abstracts from these interesting alternative approaches to modeling government policy.

[9] The effects of a tax on sector 2 for a small open economy can be studied in a similar fashion. We therefore omit this case in the interest of conserving space and the reader's patience.

[10] In his [1981] "Treatise on the Family," Gary Becker conjectures with characteristic insight that issues involving capital accumulation lie at the heart of national specialization decisions: "I would argue, however, that differences in endowments are often only a proximate explanation of the gains from trade; the fundamental source of much of the gain is, as with households, the advantage of specialized investment and the division of labor." (Becker [1981], page 20). In the model presented in this paper, capital goods are not specialized by industry, as Becker had in mind, yet the presumption of specialization still obtains.

APPENDIX

This Appendix provides proofs of the Propositions stated in the text. These proofs involve manipulation of the first-order necessary conditions (presented below as equations (A1)-(A13)) which implicitly define the competitive equilibrium for the economy described in section 2. Additional notation for the utility-denominated shadow prices is as follows: ω_t denotes the wage rate; q_t denotes the (gross) rental rate; p_t denotes the price of good 1; and λ_t denotes the price of good 2. The capital good (good 2) is numeraire, so the relative price of good 1 in terms of good 2 is $P \equiv p/\lambda$. The wage rate in terms of good 2 is $w \equiv \omega/\lambda$. $\Omega_j \equiv (1-\tau_j)$, is the tax "wedge," j=1,2. Net investment in each sector is defined as $I_{jt} \equiv K_{j,t+1}-(1-\delta)K_{jt}$ for j=1,2; in the steady state, $I_j = \delta K_j$. As in the text, the capital-labor ratio in each sector is defined as $k_j \equiv K_j/N_j$, j=1,2. We define the production functions in terms of the capital/labor ratios as $f_j(k_j) \equiv F_j(k_j,1)$, j=1,2. Using this notation, the first-order necessary conditions are:

$$\partial u(C_{1t},C_{2t})/\partial C_{1t} - p_t = 0 \tag{A1}$$

$$\partial u(C_{1t},C_{2t})/\partial C_{2t} - \lambda_t = 0 \tag{A2}$$

$$-\omega_t + p_t\Omega_1[\partial F_1(K_{1t},N_{1t})/\partial N_{1t}] = 0 \tag{A3}$$

$$-\omega_t + \lambda_t\Omega_2[\partial F_2(K_{2t},N_{2t})/\partial N_{2t}] = 0 \tag{A4}$$

$$-q_t + p_t\Omega_1[\partial F_1(K_{1t},N_{1t})/\partial K_{1t}] + (1-\delta)\lambda_t = 0 \tag{A5}$$

$$-q_t + \lambda_t\Omega_2[\partial F_2(K_{2t},N_{2t})/\partial K_{2t}] + (1-\delta)\lambda_t = 0 \tag{A6}$$

$$\beta q_{t+1} - \lambda_t = 0 \tag{A7}$$

$$\bar{N} - N_{1t} - N_{2t} = 0 \tag{A8}$$

$$K_t - K_{1t} - K_{2t} = 0 \tag{A9}$$

$$Y_{1t} - C_{1t} - G_1 = 0 \tag{A10}$$

$$Y_{2t} + (1-\delta)K_{1t} + (1-\delta)K_{2t} - C_{2t} - G_2 - K_{t+1} = 0 \tag{A11}$$

together with the government budget constraint,

$$P_tG_1 + G_2 + T_t = P_t\tau_1Y_{1t} + \tau_2Y_{2t} \quad , \tag{A12}$$

and the "transversality condition":

$$\lim_{t\to\infty} \beta^t\lambda_tK_{t+1} = 0 \qquad . \tag{A13}$$

With time subscripts removed, these equations characterize the steady state of this economy. Since all of the Propositions are concerned with steady state effects, time subscripts are deleted in the proofs.

PROOF OF PROPOSITION 1. The key insight in proving this Proposition is to notice that the system of equations (A2)–(A11) is block-recursive. The first block of equations consists of equations (A3)–(A7) which can be solved for P, w, k_1, k_2, and the gross rental rate as functions of the set of parameters φ_1 (defined on page 6 of the text). In fact, the block (A3)–(A7) is itself recursive. The remaining Euler equations form a second block which determines equilibrium values of the quantity variables, given prices and capital–labor ratios as determined in the first block.

(i). Equation (A7) determines the steady-state ratio of the gross rental rate in terms of the numeraire (good 2) as follows:

$$Q \equiv q/\lambda = \beta^{-1} = 1+\rho. \tag{A14}$$

Thus Q just depends on $\rho \in \varphi_1$. Using this expression, we proceed to solve sequentially for k_2, w, k_1, and P, using equations (A3)–(A6). Letting primes (') denote the first derivative of a function, equation (A6) can be written as

$$\Omega_2[f_2'(k_2)] = \rho + \delta. \tag{A15}$$

Since all the parameters which implicitly determine k_2 via (A15) are elements of φ_1, we have $k_2 = k_2(\varphi_1)$. Next, equation (A4) determines w:

$$\Omega_2[f_2(k_2) - k_2 f_2'(k_2)] = w. \tag{A16}$$

Since $k_2=k_2(\varphi_1)$, it follows immediately that $w=w(\varphi_1)$. Next, we use the ratio of equation (A3) to equation (A5) to determine k_1:

$$\frac{P\Omega_1[f_1(k_1) - k_1 f_1'(k_1)]}{P\Omega_1 f_1'(k_1)} = \frac{[f_1(k_1) - k_1 f_1'(k_1)]}{f_1'(k_1)} = \frac{w(\varphi_1)}{\rho+\delta} \quad . \tag{A17}$$

Since δ and the parameters of f_1 are elements of φ_1, and since $w=w(\varphi_1)$, (A17) implies that $k_1 = k_1(\varphi_1)$. Finally, the relative price may be computed as the ratio of equation (A3) to equation (A4):

$$P = \frac{\Omega_2\ [\partial F_2(K_{2t},N_{2t})/\partial N_{2t}]}{\Omega_1\ [\partial F_1(K_{1t},N_{1t})/\partial N_{1t}]} = \frac{\Omega_2\ [f_2(k_2) - k_2 f_2'(k_2)]}{\Omega_1\ [f_1(k_1) - k_1 f_1'(k_1)]} \quad . \tag{A18}$$

or it may be computed as the ratio of equation (A5) to (A6):

$$P = \frac{\Omega_2\ [\partial F_2(K_{2t},N_{2t})/\partial K_{2t}]}{\Omega_1\ [\partial F_1(K_{1t},N_{1t})/\partial K_{1t}]} = \frac{\Omega_2\ f_2'(k_2)}{\Omega_1\ f_1'(k_1)} \quad . \tag{A19}$$

Since $k_j=k_j(\varphi_1)$, $j=1,2$, $P=P(\varphi_1)$.

(ii). In proving part (i), we solved the block (A3)–(A7) for k_1, k_2, w, P, and Q as functions of the set of parameters φ_1. Given the solution to the first block of the system, the rest of the Euler equations form a second block of equations which can be solved for N_1, N_2, C_1, C_2, Y_1, and Y_2 as functions of $\Psi \equiv \varphi_1 \cup \varphi_2$. Only in this second block of equations do we find the demand-side parameters (φ_2) which are important for the determination of quantities. Specifically, demand-side factors appear in (A1) and (A2) which involve the utility function, and in equations (A10) – (A12) in which government consumption appears.

PROOF OF PROPOSITION 2: The equation for the long-run PPF is $\bar{N} = a_{N1}Y_1 + a_{N2}Y_2$. The slope of the long-run PPF is given by:

$$\left[\frac{dY_2}{dY_1}\right]_{N_1+N_2=\bar{N}} = -\left[\frac{a_{N1}}{a_{N2}}\right]$$

To show that the long-run PPF is linear, it suffices to show that the a_{Nj} (the "techniques of production") are independent of the equilibrium quantities produced. Recall that $a_{Nj} = f_j'(k_j)^{-1}$, j=1,2. From Proposition 1 we know that the k_j are functions only of φ_1, and are thus independent of the composition of output between Y_1 and Y_2. Thus the a_{Nj} depend on φ_1 alone: the techniques of production do not change with alterations in the mix of output between goods 1 and 2.

PROOF OF PROPOSITION 3: The steady state net-of-tax price is given by $P(1-\tau_1)/(1-\tau_2) = P\Omega_1/\Omega_2$. The slope of the short-run PPF at the steady state point is the marginal rate of transformation between Y_1 and Y_2, holding fixed aggregate supplies of capital and labor. Differentiating the production functions, and using the fact that $dN_2 = - dN_1$, and $dK_2 = - dK_1$, we have:

$$\frac{dY_2}{dY_1} = -\frac{(\partial F_2/\partial K_2)dK_2 + (\partial F_2/\partial N_2)dN_2}{(\partial F_1/\partial K_1)dK_2 + (\partial F_1/\partial N_1)dN_2} \quad . \tag{A20}$$

Using (A18) and (A19), we obtain:

$$\frac{P\Omega_1}{\Omega_2} = \frac{(\partial F_2/\partial N_2)}{(\partial F_1/\partial N_1)} = \frac{(\partial F_2/\partial K_2)}{(\partial F_1/\partial K_1)} \; .$$

Substituting this expression for the net-of-tax relative price into (A20), we have $dY_2/dY_1 = -P\Omega_1/\Omega_2$, which proves the first part of this Proposition.

The absolute value of the slope of the long-run PPF is a_{N1}/a_{N2} (see Proposition 2) and the net-of-tax relative price of good 1 in terms of good 2 is given by equation (A20) above. Letting α_j denote labor's share in sector j output ($\alpha_j \equiv (\partial F_j/\partial N_j)N_j/Y_j$), and recalling that $a_{Nj} \equiv N_j/Y_j$, we may write:

$$\frac{P\Omega_1}{\Omega_2} = \frac{\alpha_2 a_{N1}}{\alpha_1 a_{N2}} \quad .$$

The ratio of the relative price, P, to the absolute value of the slope of the long PPF is therefore just

$$\frac{P\Omega_1/\Omega_2}{|\text{slope of long run PPF}|} = \frac{\alpha_2}{\alpha_1} \quad .$$

Thus the net-of-tax price line is steeper than the slope of the PPF if sector 1 is capital intensive, i.e., if labor's share in sector 1 (α_1) is less than labor's share in sector 2 (α_2), and conversely.

PROOF OF PROPOSITION 4: As shown in Proposition 1, the system of equations defining this economy's equilibrium is block-recursive. Part (i) of Proposition 1 established that variations in G_j, j=1,2, have no effect on k_j, w, or a_{Nj}. Thus we may take these variables as fixed in studying the effects of changes in government expenditure. Part (ii) of Proposition 1 showed that there is a second block of equations which determine the levels of the quantity variables in equilibrium. To prove the current Proposition, we therefore solve this second block for the equilibrium changes in the quantity variables as functions of the changes in G_j. It is most convenient to write this second block of equations as follows, letting U_j denote the partial derivative of $U(C_1,C_2)$ with respect to its j'th argument:

$$U_1(C_1,C_2) - PU_2(C_1,C_2) = 0 \tag{A21}$$

$$a_{N1}Y_1 + a_{N2}Y_2 = \bar{N}. \tag{A22}$$

$$C_1 + G_1 = Y_1 \tag{A23}$$

$$C_2 + G_2 + I_1 + I_2 = Y_2 \tag{A24}$$

$$I_1 = \delta_1\kappa_1Y_1 \qquad ; \text{ where } \kappa_1 \equiv K_1/Y_1 \tag{A25}$$

$$I_2 = \delta_2\kappa_2Y_2 \qquad ; \text{ where } \kappa_2 \equiv K_2/Y_2 \quad . \tag{A26}$$

Taking the total differential of these equations, we obtain:

$$[U_{11}(C_1,C_2) - PU_{21}(C_1,C_2)]dC_1 + [U_{12}(C_1,C_2) - PU_{22}(C_1,C_2)]dC_2 = 0 \tag{A27}$$

$$a_{N1}dY_1 + a_{N2}dY_2 = 0 \tag{A28}$$

$$dC_1 + dG_1 = dY_1 \tag{A29}$$

$$dC_2 + dG_2 + dI_1 + dI_2 = dY_2 \tag{A30}$$

$$dI_1 = \delta_1\kappa_1 dY_1 \tag{A31}$$

$$dI_2 = \delta_2\kappa_2 dY_2 \tag{A32}$$

Letting $\gamma_j \equiv [U_{1j}(C_1,C_2) - PU_{2j}(C_1,C_2)]$, j=1,2, equation (A27) may be written as $\gamma_1 dC_1 + \gamma_2 dC_2 = 0$. If both C_1 and C_2 are normal goods, consumption of both will rise in response to a pure wealth shock such as a change in G_j; so that $\gamma_1/\gamma_2<0$. Define $\Gamma \equiv -\gamma_1/\gamma_2 > 0$; $A \equiv a_{N2}/a_{N1}$, and $\eta \equiv [1-\delta\kappa_2+\delta\kappa_1 A+\Gamma A]$. Because $\delta\kappa_2$ must be less than one ($\delta\kappa_2$ is the share of output of sector 2 used as investment in sector 2) we have $\eta > 0$. Using these definitions, the solutions to equations (A27)-(A32) are obtained by straightforward algebra:

$$dY_1 = -A\left[\frac{1}{\eta}\right][-\Gamma dG_1 + dG_2]$$

$$dY_2 = \left[\frac{1}{\eta}\right][-\Gamma dG_1 + dG_2]$$

$$dC_1 = dY_1 - dG_1 = \left[\frac{A\Gamma - \eta}{\eta}\right]dG_1 - \left[\frac{A}{\eta}\right]dG_2$$

$$dC_2 = \Gamma dC_1 = \left[\frac{A\Gamma^2 - \Gamma\eta}{\eta}\right]dG_1 - \left[\frac{A\Gamma}{\eta}\right]dG_2$$

$$dI_1 = \delta\kappa_1 dY_1 = -\left[\frac{A\delta\kappa_1}{\eta}\right][-\Gamma dG_1 + dG_2]$$

$$dI_2 = \delta\kappa_2 dY_2 = \left[\frac{\delta\kappa_2}{\eta}\right][-\Gamma dG_1 + dG_2] \quad .$$

Evaluation of these expressions gives the desired comparative-steady-state results:

$dY_1/dG_1 > 0$; $dY_1/dG_2 < 0$

$dY_2/dG_1 < 0$; $dY_2/dG_2 > 0$;

$dC_j/dG_1 < 0$; $dC_j/dG_2 < 0$; $j=1,2$;

$dI_1/dG_1 > 0$; $dI_1/dG_2 < 0$

$dI_2/dG_1 < 0$; $dI_2/dG_2 > 0$.

Further, for sensible values of the depreciation rate and steady state capital-output ratios, it is straightforward to show that $0 < dY_j/dG_j < 1$, $j=1,2$. That is, output of a particular sector rises with increases in government expenditure in that sector, but less than one-for-one.

PROOF OF PROPOSITION 5: To evaluate the equilibrium effects of permanent changes in taxation, we return to the block of equations which determines k_1, k_2, w, and P. Letting a "hat" (ˆ) over a variable denote percentage deviation from the initial steady state, and letting ξ_{ijk} denote the elasticity of the marginal product of factor i with respect to factor j in sector k, we obtain the following. First, from equation (A15), we obtain:

$$\hat{k}_2 = -\hat{\Omega}_2/\xi_{KK2} \tag{A33}$$

Next, from equation (A16) we have:

$$\hat{w} = \hat{\Omega}_2\left[1 + \frac{\xi_{NN2}}{\xi_{KK2}}\right] \quad . \tag{A34}$$

Using (A3) and (A5), we obtain:

$$\hat{k}_1 = -\left[\frac{\hat{\Omega}_2}{\xi_{KK2}}\right]\left[\frac{\xi_{KK2}+\xi_{NN2}}{\xi_{KK1}+\xi_{NN1}}\right] = \hat{k}_2\left[\frac{\xi_{KK2}+\xi_{NN2}}{\xi_{KK1}+\xi_{NN1}}\right] . \tag{A35}$$

Finally, we use (A5), (A6), and (A14) to obtain:

$$\hat{P} = -\hat{\Omega}_1 + \hat{\Omega}_2\left[\frac{\xi_{KK1}}{\xi_{KK2}}\right]\left[\frac{\xi_{KK2}+\xi_{NN2}}{\xi_{KK1}+\xi_{NN1}}\right] . \tag{A36}$$

To analyze a change in τ_1, we note that the tax wedge for sector 1, Ω_1, enters only in equation (A36). Thus the tax on sector 1 output causes the gross-of-tax relative price of good 1 to increase by the full amount of the change in the tax wedge ($\hat{\Omega}_1 < 0$ for increases in τ_1). To examine the effect of the tax on the quantity side of the economy, we return to (A21)–(A26). Here we see that the relative price enters only in (A21) which implicitly determines the ratio of C_1/C_2. The equilibrium changes in quantities are computed by by differentiating this block of six equations with respect to P and the quantity variables, and solving the resulting system.

PROOF OF PROPOSITION 6: Equations (A33)–(A36) are used to evaluate the effects of a tax on sector 2, recalling that $\xi_{KKj} < 0$, $\xi_{NNj} < 0$, j=1,2, and $\xi_{KKj} = -\xi_{KNj}$, and $\xi_{NNj} = -\xi_{NKj}$, j=1,2. From (A33), an increase in τ_2 implies $\hat{k}_2 < 0$, since $\hat{\Omega}_2 < 0$. From (A35) we have $\hat{k}_1 < 0$ as well. From (A34) we have $\hat{w} < 0$, and from (A36), we have $\hat{P} > 0$. Because the tax on sector 2 affects producers' choices of capital/labor ratios, the tax will affect the a_{Nj} coefficients in the equation for the long-run PPF, as follows. Let α_j denote labor's share in sector j and let ζ_j denote the elasticity of substitution of capital for labor in sector j. The percentage change in a_{Nj} is $\hat{a}_{Nj} = -((1-\alpha_j)\zeta_j/\alpha_2)\hat{\Omega}_2$, j=1,2. Since $\hat{\Omega}_2 < 0$, $\hat{a}_{Nj} > 0$. The increase in

a_{N2} shifts down the Y_2-axis intercept of the PPF, $\bar{N}/a_{N2}$. If $\zeta_1=\zeta_2$, the change in the absolute value of the slope of the long-run PPF is $\hat{a}_{N1}-\hat{a}_{N2}$ = $(\alpha_1-\alpha_2)(\hat{\Omega}_2/\theta_{N2})$. Thus the long-run PPF becomes steeper if sector 1 is the capital-intensive sector, and becomes flatter if sector 2 is capital-intensive. We know that the post-tax PPF must lie everywhere below the pre-tax PPF because imposition of the tax alters the "techniques of production" (the a_{Nj} or, equivalently, the k_j). Since these techniques were available before imposition of the tax, but were not chosen by profit-maximizing producers, they must be inefficient. That is, given an amount of one good to be produced, fewer units of the other good can be produced with the post-tax techniques of production than with the pre-tax techniques.

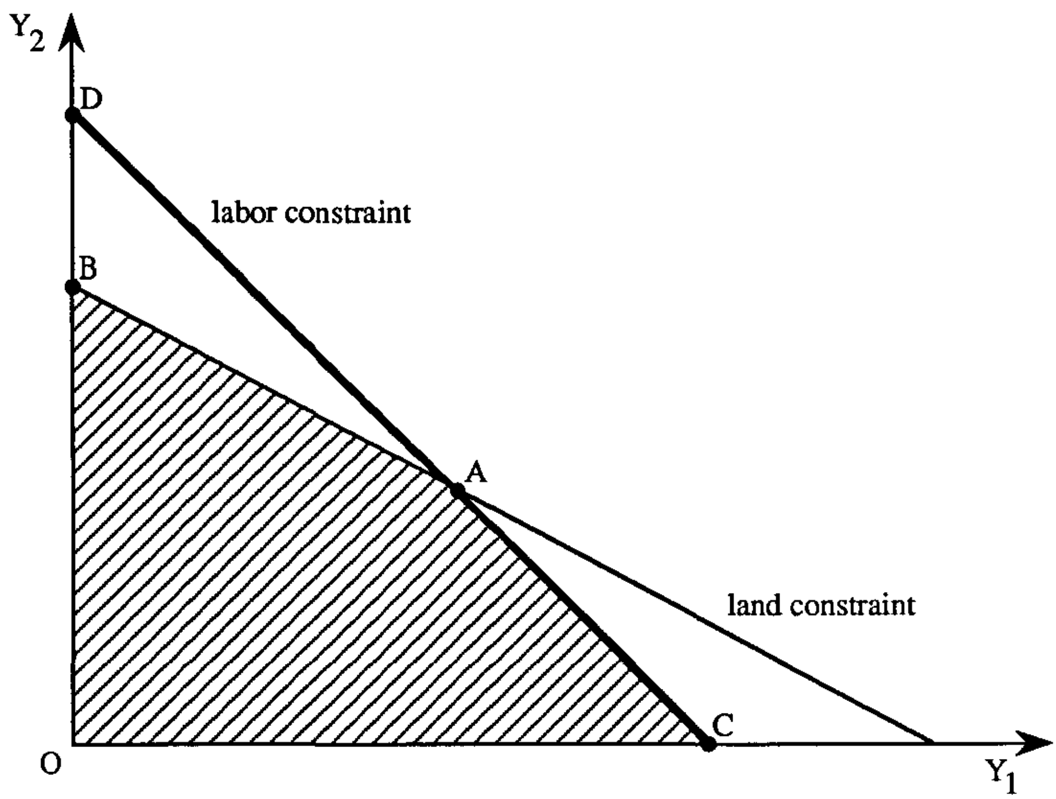


Fig. 1 PPF in the Jones Model

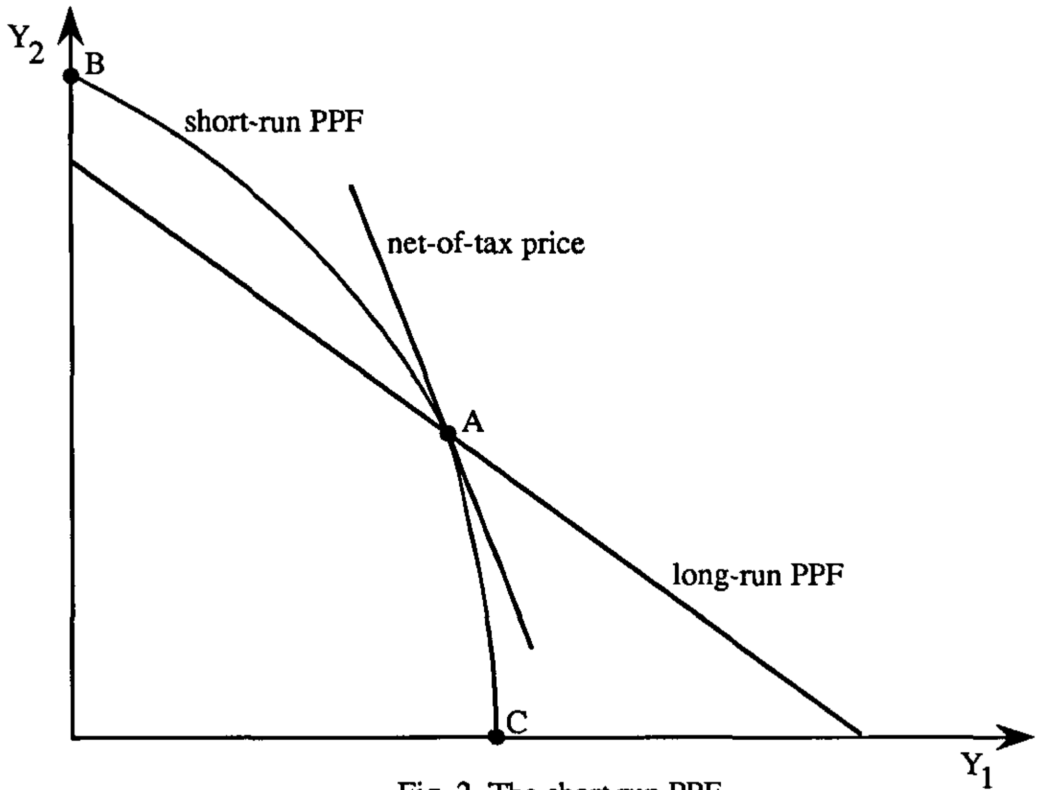


Fig. 2 The short run PPF

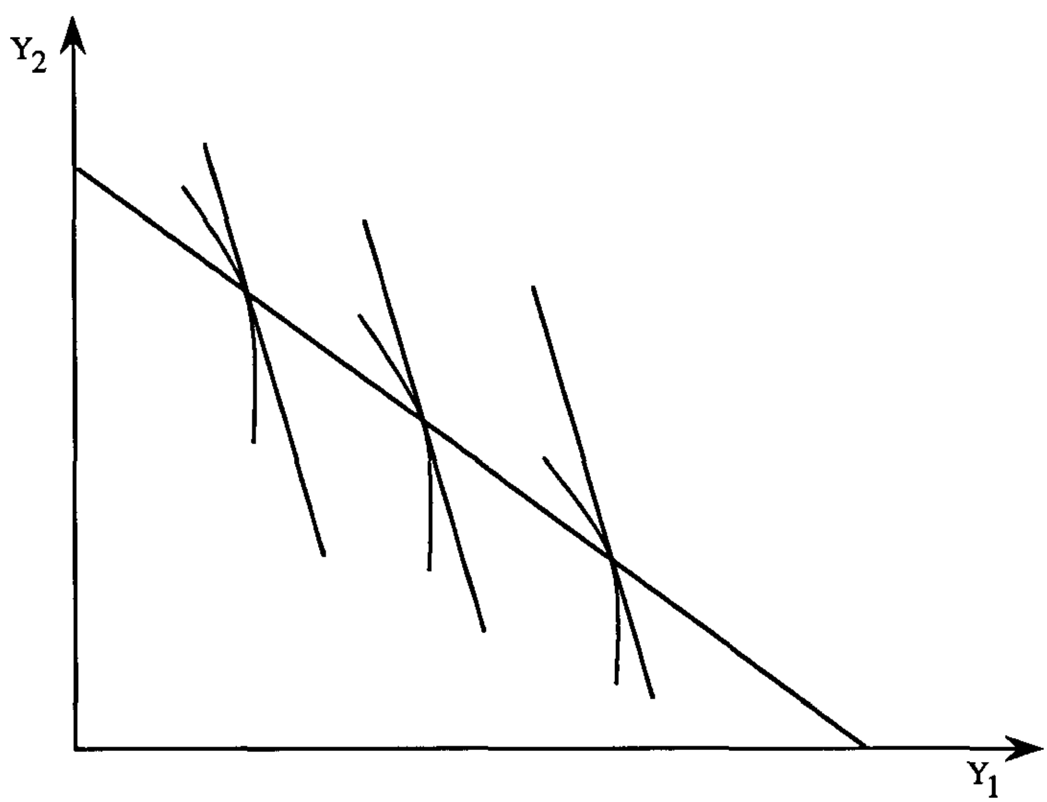


Fig. 3 The family of short run PPF's

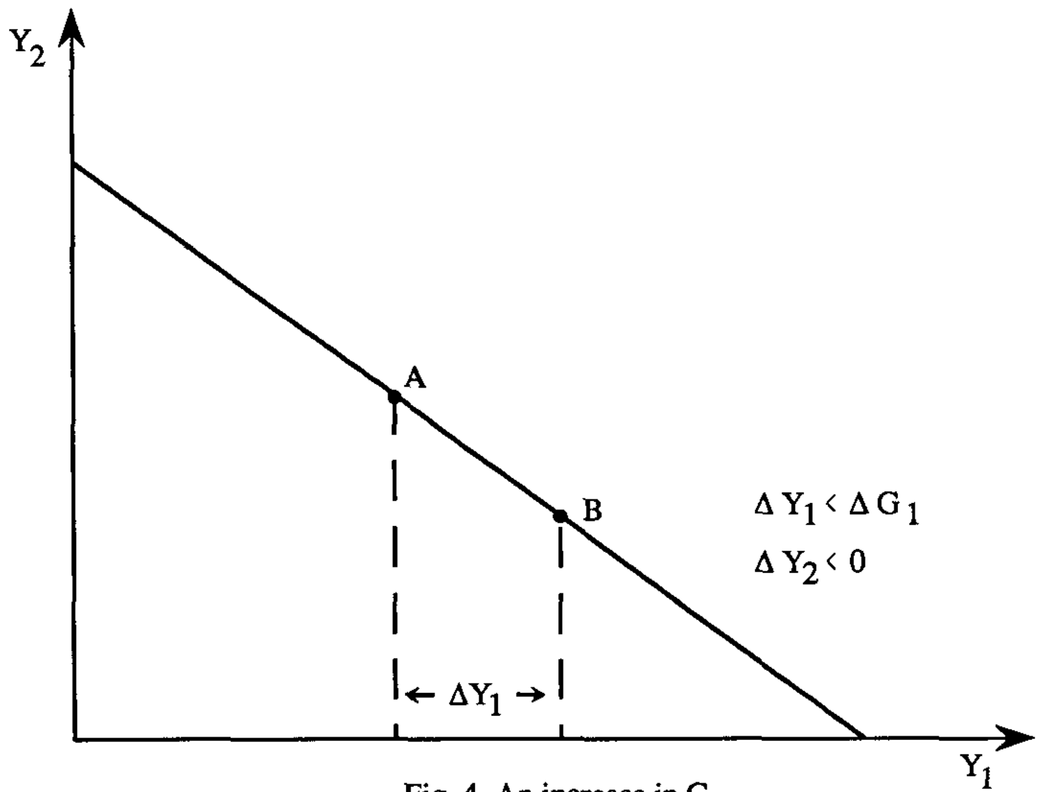


Fig. 4 An increase in G_1

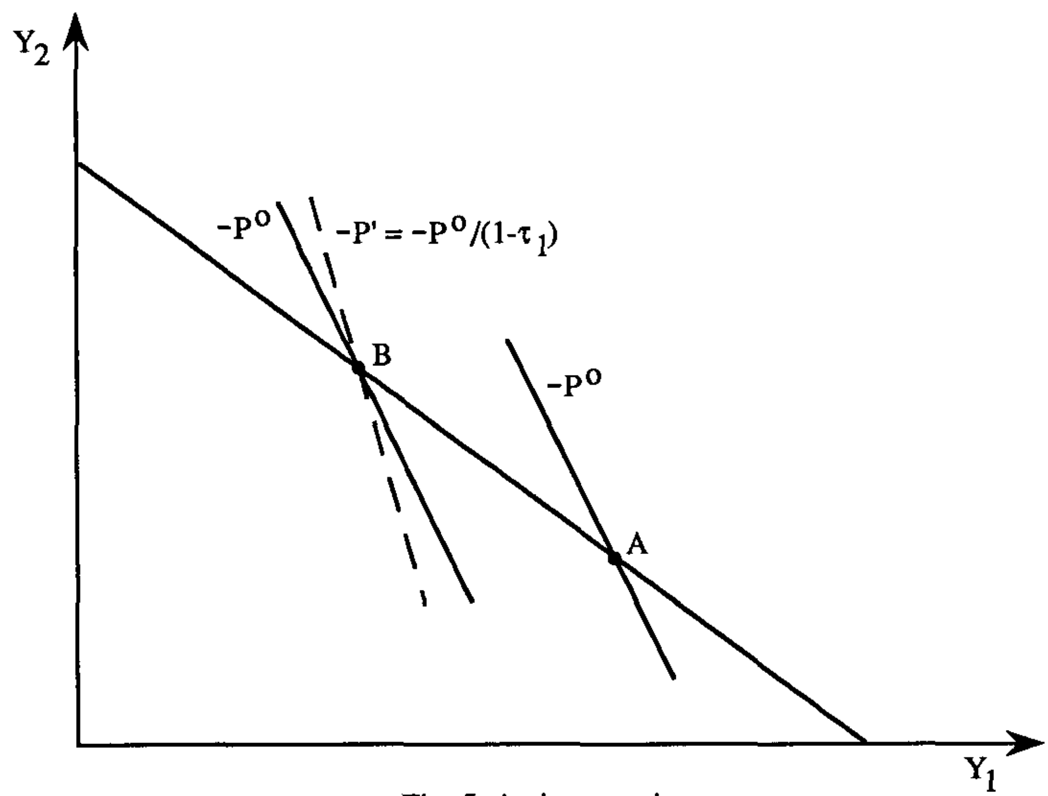


Fig. 5 An increase in τ_1

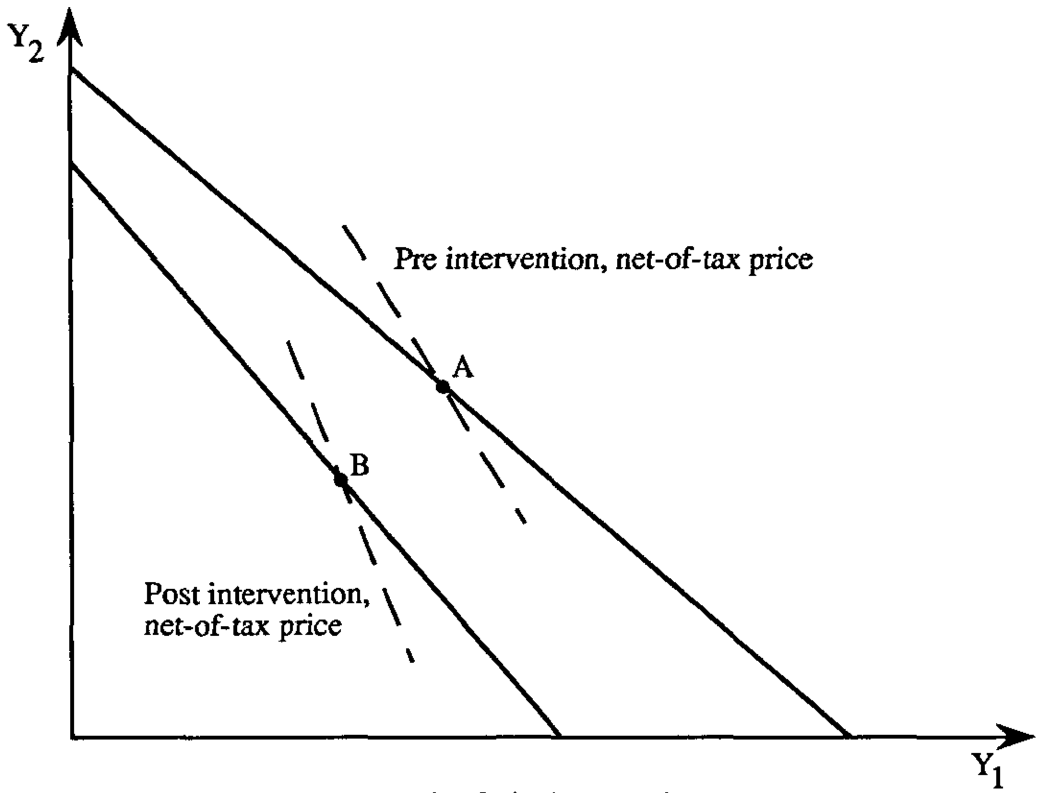


Fig. 6 An increase in τ_2

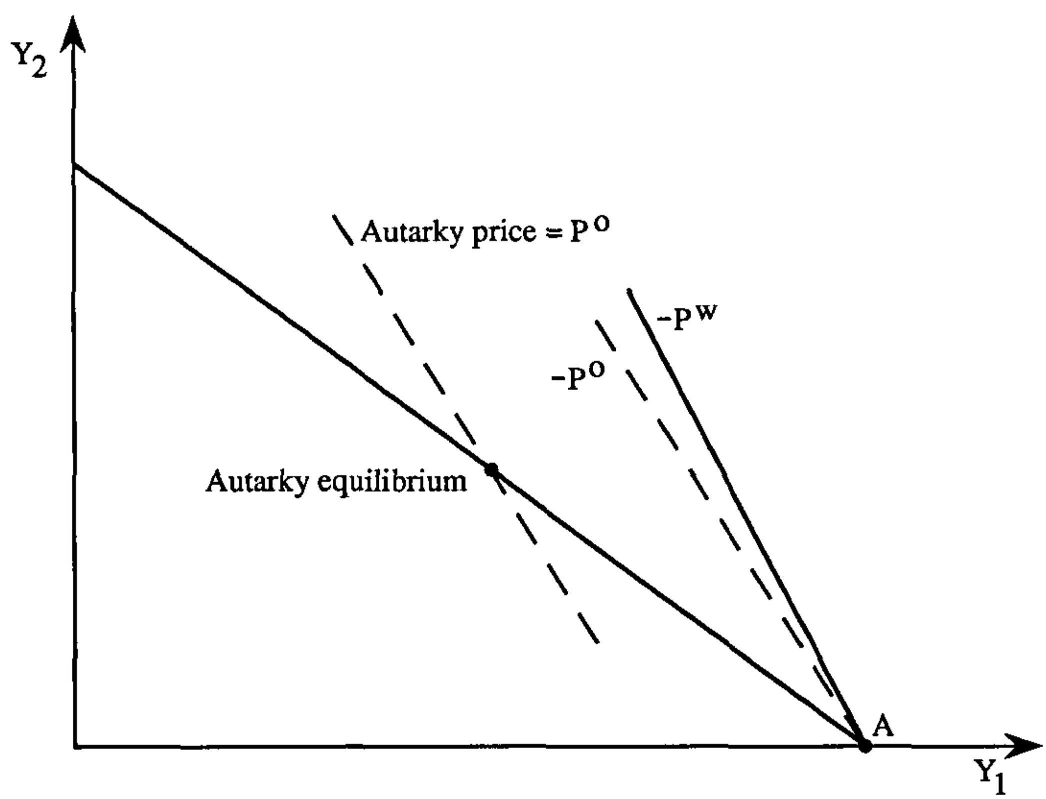
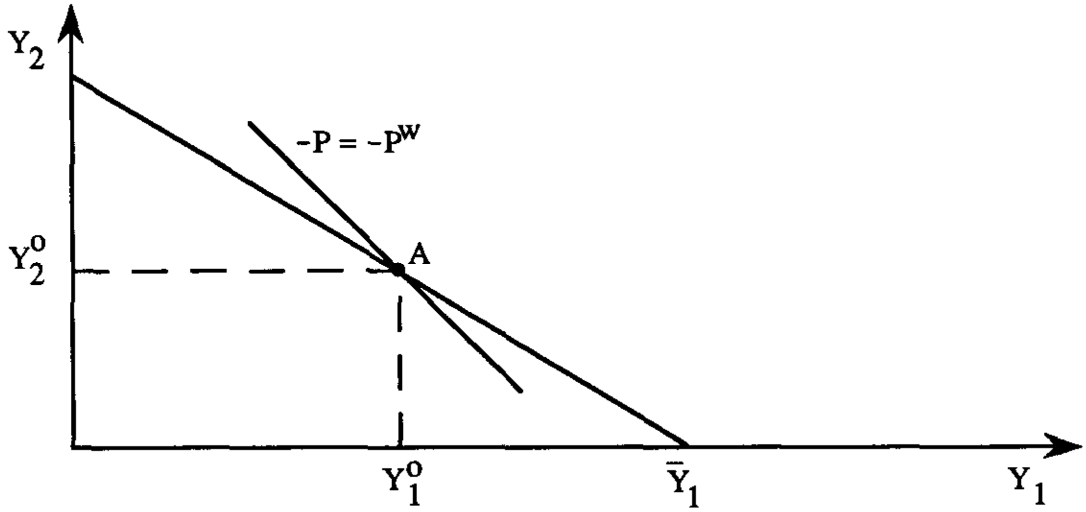
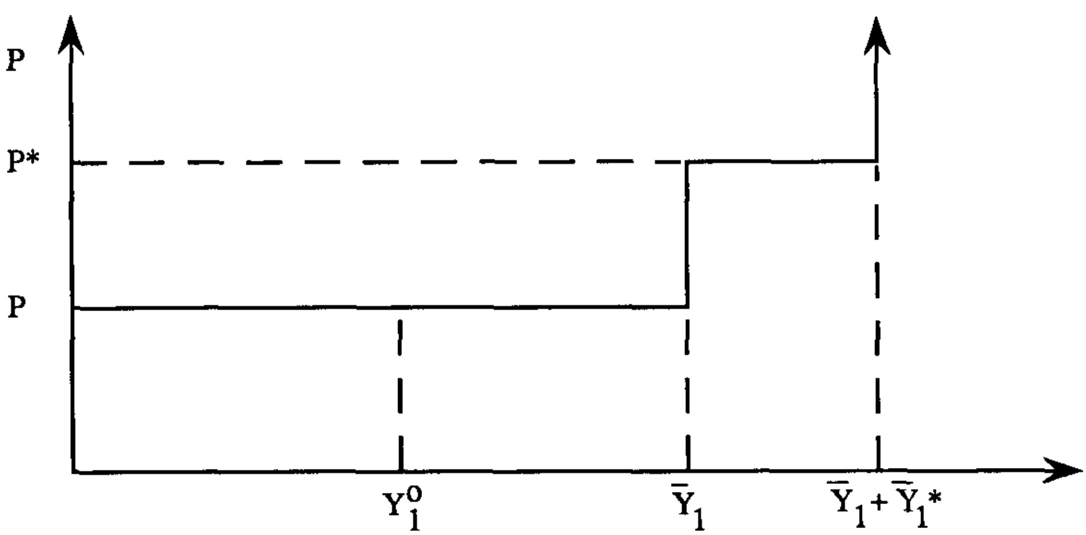


Fig. 7 Specialization in the small open economy



I: Home country

II: Foreign country



III: World supply of good 1

Fig. 8