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LIQUIDITY AND REAL ACTIVITY IN A SIMPLE OPEN ECONOMY MODEL

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ABSTRACT

We examine nominal and real exchange rates, interest rates, prices, and evolutions of real variables in a two-country, monetary general-equilibrium model that includes a financial sector and shocks to technologies and money growth rates. Qualitative properties of the model are provided and moment predictions from a calibrated version of the model are compared to moments of time series drawn from actual economies. We focus on international monetary shock transmissions, and effects of monetary innovations on nominal and real exchange rates and nominal interest rates.

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1. INTRODUCTION

Research conducted on equilibrium explanations of business cycle phenomena using real models has recently been extended to add international dimensions by, among others, Backus, Kehoe, and Kydland (1989), Baxter (1988), Schlagenhauf (1990), and Stockman and Tesar (1990). The analysis in this paper differs in that we assign a role for money in performing transactions and incorporate a financial sector. We investigate real and nominal exchange rates, interest rates, prices, and the evolutions of real economic variables in a two-country, monetary-general-equilibrium model in which agents face shocks to each country's technology and monetary growth rate. A focal point of the paper is the predictive content of a general equilibrium monetary model for nominal and real exchange rate dynamics.

Since the environment that we consider includes shocks to each country's money growth rate, it is important to construct a model which properly accounts for effects of monetary shocks on interest rates and output levels. A prevalent view is that positive shocks to money lead to reductions in nominal interest rates and increases in output. Indeed, empirical support for this view is reported by Christiano and Eichenbaum (1991), who find that in U.S. data the nominal federal funds rate is negatively correlated with various money supply measures and with real output.

Standard monetary equilibrium models of business cycle phenomena generally fail to predict negative correlations of nominal interest rates with money and output. For example, the basic cash-in-advance model of Cooley and Hansen (1989), shopping-time models of Kydland (1989) and den Haan (1990), and the transactions cost model of Marshall (1987) all have the feature that with positive persistence in money growth, positive monetary innovations lead to increases in the nominal interest rate and reductions in output. This feature derives from monetary shocks exerting influences on interest rates exclusively through an anticipated inflation effect. Positive shocks to money growth in these models lead to an increase in expected inflation and, for standard Fisherian reasons, nominal interest rates rise.

Recently, monetary general equilibrium models have been constructed by Lucas (1990), Fuerst (1990), Christiano (1990), and Christiano and Eichenbaum (1991) which, in addition to anticipated inflation effects, also allow for liquidity effects on nominal interest rates and output of unanticipated monetary shocks. The liquidity effect of a surprise increase in money growth drives nominal interest rates down, to induce agents who borrow to absorb additional money, leading to increased output as the cost to firms of borrowing to finance input acquistions falls. The net effect of an unanticipated increase in money growth on interest rates and output in these models depends on the relative magnitudes of anticipated inflation and liquidity effects.

The model in this paper is a straightforward extension of the closed economy models of Lucas (1990) and Fuerst (1990), which allows for both anticipated inflation and liquidity effects of unanticipated monetary shocks. The infinitely-lived two-country model economy that we examine is inhabited by multi-member households facing stochastic shocks to each country's technology and monetary growth rate. Different members of a representative household for each country trade in spatially and sometimes informationally distinct markets within each period, but meet to pool resources and information at each period's completion.

A typical household in each country consists of a shopper, firm member, worker, and financial intermediary as in Fuerst (1990). Within a period, the different household members trade separately in goods, labor, and financial markets. Shoppers face domestic and foreign cash-in-advance constraints on purchases of goods, while firms face cash-in-advance constraints on input acquisitions. Cash required for a firm's input acquisitions is obtained by borrowing from a financial intermediary who supplies cash derived from household deposits and monetary injections.

Intermediaries in each country receive injections of their home currency which are then disproportionately absorbed by home-country firms. The effect of a domestic monetary injection on domestic nominal interest rates depends on the relative strengths of liquidity and anticipated inflation effects. Changes in the nominal interest rate represent changes in the cost to firms of borrowing funds to acquire inputs and therefore lead to effects on domestic employment and output.

A domestic monetary injection also serves to change the domestic price level which alters desired consumption of domestically produced goods by households in each country. With nonseparability of leisure and consumptions of domestic and foreign goods in agents' utility functions, changes in desired consumptions of domestically produced goods influences the marginal utilities of leisure and consumption of foreign produced goods. Consequently, foreign prices and anticipated foreign inflation are affected by domestic money shocks giving rise to effects on foreign real activity. Thus, monetary shocks are transmitted in our model in a way that differs from the traditional Mundell-Fleming model. In our model, all prices are flexible.

The paper is organized as follows. Section 2 describes the model. Trading opportunities and information sets of agents are described along with preferences, technologies, and shocks. The choice problem confronting a typical household is displayed, optimality conditions are interpreted, and general equilibrium for the world economy is described. In order to quantitatively analyze the model, we assign values to the deep structural parameters and discuss our choices of values in section 3. Section 4 presents quantitative results and discusses the model's properties. Section 5 concludes.

2. A TWO-COUNTRY MONETARY MODEL

We analyze a general equilibrium, two-country, monetary model with each country, domestic and foreign, inhabited by a multi-member representative household. For each country, the representative household consists of a shopper, worker, financial intermediary, and firm manager.

Within each period the domestic household members perform the following functions. The shopper travels to domestic and foreign goods markets, the firm manager trades in the domestic goods, labor, and financial markets, the worker trades in the domestic labor market, the intermediary trades in the domestic financial market, and the household trades in a currency-exchange market. Similar tasks are performed by foreign household members. In any market, agents are atomistic and cannot tell who is or is not a member of their own households. Since objectives and constraints of households in the two countries are isomorphic, we provide details only for the domestic household's objective and trading opportunities.

At the beginning of a period, say t, the domestic and foreign households trade currencies brought forward from last period in an exchange market at a nominal exchange rate e_t, expressed in units of domestic currency per unit of foreign currency. The domestic household enters the exchange market with beginning nominal wealth A_t^D derived from domestic currency units brought forward from last period. A_t^D is allocated between a period t domestic currency balance $M_{D,t}^D$ and a foreign currency balance $M_{F,t}^D$ at the going exchange rate e_t.[1] Thus,

$$(1) \qquad A_t^D = M_{D,t}^D + e_t M_{F,t}^D.$$

The domestic household divides $M^D_{D,t}$ by sending $N^D_{D,t}$ to the domestic financial market with the Intermediary member of the household as a deposit, and the remaining $M^D_{D,t} - N^D_{D,t}$ to the domestic goods market with the shopper. The shopper is also provided with the household's foreign currency balance $M^D_{F,t}$ for use in purchasing goods produced in the foreign country. The shopper purchases $C^D_{D,t}$ units of the domestically produced good subject to the cash-in-advance (hereafter, CIA) constraint:[2]

$$(2) \qquad P^D_t C^D_{D,t} = M^D_{D,t} - N^D_{D,t}$$

The domestic shopper also purchases $C^D_{F,t}$ units of the foreign produced good subject to the CIA constraint:

$$(3) \qquad P^F_t C^D_{F,t} = M^D_{F,t}$$

With the constraints specified as equalities, the shopper returns home with goods, but no leftover cash.

Upon arrival at the financial market, the domestic intermediary receives a monetary injection X^D_t which is deposited on the household's behalf. The typical domestic intermediary then holds $N^D_{D,t} + X^D_t$ in cash which it lends to domestic firms who agree to pay, at the end of the period, the gross domestic loan rate $(1+i^D_{L,t})$ times the amount borrowed. Thus, loanable cash supplied to the financial market is:[3]

$$(4) \qquad \hat{\mathcal{L}}^D_{s,t} = N^D_{D,t} + X^D_t \ .$$

At the end of the period the intermediary pays out the gross domestic deposit rate $(1+i^D_{D,t})$

times deposits of $(N_{D,t}^{D}+X_t^{D})$.

The firm manager possesses the household's capital stock, K_t^D, and production technology, hires workers, and invests. Prior to hiring inputs, the firm must borrow domestic currency from an intermediary to finance input acquisitions. After borrowing an amount $\hat{\mathfrak{L}}_{D,t}^{D}$ of cash, the manager hires H_t^D units of labor at the domestic wage W_t^D and purchases $K_{t+1}^D-(1-\delta^D)K_t^D$ units of capital to add to the household's stock. The fixed rate of capital depreciation is denoted by δ^D. Consumption and capital goods are indistinguishable in the domestic goods market and therefore sell at a common price P_t^D.

As input acquisitions must be entirely cash financed, the typical domestic firm manager faces a "loan-in-advance" (hereafter, LIA) constraint given by:

$$(5) \qquad \hat{\mathfrak{L}}_{D,t}^{D} = W_t^D H_t^D + P_t^D(K_{t+1}^D-(1-\delta^D)K_t^D)$$

After combining labor and capital with the production technology, output is taken to the goods market where domestic and foreign shoppers purchase units of current consumption. Cash receipts from goods market sales cannot be used to purchase units of the good in the current period.

The domestic household's worker trades only in the domestic labor market, supplying $\tilde{H}_t^D$ units of labor at the market wage W_t^D. At the end of the current period, the worker returns home with nominal wage receipts

$$(6) \qquad W_t^D \tilde{H}_t^D$$

After the close of goods market trading, the firm manager passes by the financial market to pay its loan obligation $(1+i_{L,t}^{D})\hat{\mathfrak{L}}_{D,t}^{D}$, and returns home with capital, and cash profits of:

(7) $P_t^D Y_t^D - (1+i_{L,t}^D)\hat{\mathfrak{L}}_{D,t}^D$,

where Y_t^D is real domestic output.

Each domestic intermediary receives loan repayments $(1+i_{L,t}^D)(N_{D,t}^D + X_t^D)$ and pays a gross deposit return $(1+i_{D,t}^D)(N_{D,t}^D + X_t^D)$. The typical household's intermediary member returns home at the end of the period with the household's deposit return, $(1+i_{D,t}^D)(N_{D,t}^D + X_t^D)$, plus any cash derived from intermediary activities, $(1+i_{L,t}^D)(N_{D,t}^D + X_t^D) - (1+i_{D,t}^D)(N_{D,t}^D + X_t^D)$. Thus, the intermediary member brings home a cash balance of:

(8) $(1+i_{L,t}^D)(N_{D,t}^D + X_t^D)$

Upon reuniting, the household pools currencies and consumes goods acquired in the current period. Combining cash brough home by the household's worker in (6), the firm manager in (7), and the intermediary in (8), the domestic household's end-of-period nominal wealth can be described by:

(9) $A_{t+1}^D = W_t^D \bar{H}_t^D + P_t^D Y_t^D - (1+i_{L,t}^D)\hat{\mathfrak{L}}_{D,t}^D + (1+i_{L,t}^D)(N_{D,t}^D + X_t^D)$.

Preferences

Each of the identical, infinitely-lived domestic households orders sequences of consumptions $\{C_{D,t}^D\}$, $\{C_{F,t}^D\}$, and leisure $\{L_t^D\}$ according to the utility function

(10) $E_t \sum_{j=0}^{\infty} (\beta_D)^j \mu(C_{D,t+j}^D, C_{F,t+j}^D, L_{t+j}^D), \qquad 0<\beta_D<1$,

where E_t represents the expectation operator conditional on all information dated t and earlier. Total domestic household leisure for period t+j is $L_{t+j}^D = 1-\bar{H}_{t+j}^D$, where the normalized household time endowment is unity, and $\bar{H}_{t+1}^{D}$ is the worker's labor supply, $0 \le \bar{H}_{t+1}^{D} \le 1$.

Results of our simulations are based on the following specifications of period utility functions, *for the domestic household in (11) and for the foreign household in (12):*

$$(11) \qquad \mu(C_{D,t}^{D}, C_{F,t}^{D}, L_t^{D}) = \frac{\left[(C_{D,t}^{D})^{\phi^D(1-\gamma^D)} (C_{F,t}^{D})^{(1-\phi^D)(1-\gamma^D)} (L_t^{D})^{\gamma^D}\right]^{\psi^D}}{\psi^D} \qquad \text{for } \psi^D \neq 0$$

$$= \phi^D(1-\gamma^D)\log C_{D,t}^{D} + (1-\phi^D)(1-\gamma^D)\log C_{F,t}^{D} + \gamma^D \log L_t^{D} \qquad \text{for } \psi^D = 0$$

$$(12) \qquad \mu(C_{F,t}^{F}, C_{D,t}^{F}, L_t^{F}) = \frac{\left[(C_{F,t}^{F})^{\phi^F(1-\gamma^F)} (C_{D,t}^{F})^{(1-\phi^F)(1-\gamma^F)} (L_t^{F})^{\gamma^F}\right]^{\psi^F}}{\psi^F} \qquad \text{for } \psi^F \neq 0$$

$$= \phi^F(1-\gamma^F)\log C_{F,t}^{F} + (1-\phi^F)(1-\gamma^F)\log C_{D,t}^{F} + \gamma^F \log L_t^{F} \qquad \text{for } \psi^F = 0$$

where ϕ^D, γ^D, ϕ^F, and γ^F are each between zero and unity.

Production Technologies

Firms in the domestic country combine inputs to produce output according to:

$$(13) \qquad Y_t^D = f^D(K_t^D, Z_t^D H_t^D),$$

where the exogenous shock to domestic labor productivity, Z_t^D, is the sum of a deterministic trend and random deviations about that trend. In particular:

$$(14) \qquad Z_t^D = \exp(\mu^D t + \theta_t^D)$$

Foreign firms utilize the technology

$$(15) \qquad Y_t^F = f^F(K_t^F, Z_t^F H_t^F), \qquad Z_t^F = \exp(\mu^F t + e_t^F).$$

Results of our simulations are based on the following specifications of production functions:

$$f^D(K_t^D, Z_t^D H_t^D) = (K_t^D)^{\alpha^D}(Z_t^D H_t^D)^{1-\alpha^D},\ 0<\alpha^D<1,$$
$$f^F(K_t^F, Z_t^F H_t^F) = (K_t^F)^{\alpha^F}(Z_t^F H_t^F)^{1-\alpha^F},\ 0<\alpha^F<1.$$

Exogenous Shocks

Shocks to labor productivities, θ_t^D and θ_t^F, evolve according to a bivariate autoregression:

$$(16)\quad \begin{bmatrix}\theta_t^D\\ \theta_t^F\end{bmatrix} = \begin{bmatrix}T_{11} & T_{12}\\ T_{21} & T_{22}\end{bmatrix}\begin{bmatrix}\theta_{t-1}^D\\ \theta_{t-1}^F\end{bmatrix} + \begin{bmatrix}\varepsilon_{\theta^D,t}\\ \varepsilon_{\theta^F,t}\end{bmatrix},$$

where $\varepsilon_{\theta^D,t}$, $\varepsilon_{\theta^F,t}$ are serially independent with covariance matrix given by $V_\theta = \begin{bmatrix}V_{11}^\theta & V_{12}^\theta\\ V_{21}^\theta & V_{22}^\theta\end{bmatrix}$. This specification allows for contemporaneous correlation between domestic and foreign productivity shocks.

The exogenous money growth rates are $x_t^D = \dfrac{X_t^D}{M_{D,t}^s}$ and $x_t^F = \dfrac{X_t^F}{M_{F,t}^s}$, where $M_{D,t}^s$ and $M_{F,t}^s$ are aggregate stocks of domestic and foreign currencies, and $X_t^D = M_{D,t+1}^s - M_{D,t}^s$, $X_t^F = M_{F,t+1}^s - M_{F,t}^s$. Money growth rates evolve according to the bivariate autoregression

$$(17)\quad \begin{bmatrix}x_t^D\\ x_t^F\end{bmatrix} = \begin{bmatrix}M_{11} & M_{12}\\ M_{21} & M_{22}\end{bmatrix}\begin{bmatrix}x_{t-1}^D\\ x_{t-1}^F\end{bmatrix} + \begin{bmatrix}\varepsilon_{x^D,t}\\ \varepsilon_{x^F,t}\end{bmatrix},$$

where $\varepsilon_{x^D,t}$, $\varepsilon_{x^F,t}$ are serially independent with covariance matrix given by $V_x = \begin{bmatrix}V_{11}^x & V_{12}^x\\ V_{21}^x & V_{22}^x\end{bmatrix}$.

The State of the System

In what follows, the state of the world economy in period t is described by values taken by $M^s_{D,t}$, $M^s_{F,t}$, K^D_t, K^F_t, and s_t. $s_t = [\theta^D_t, \theta^F_t, x^D_t, x^F_t]'$ is a vector of period t shock realizations. The representative household in each country begins a period, say t, with knowledge of the beginning-of-period values of capital and money stocks. During the period, the current shocks, s_t, are realized. Shocks to the system form a Markov process with transition function $\Phi(s_t, ds_{t+1})$.

Removing Growth

Prior to a full specification of the household's choice problems it is useful to convert to a nongrowing economy. To do so, define the following transformed variables:

$$c^D_{D,t} = C^D_{D,t}\exp(-\mu^D t) \qquad k^D_{t+1} = K^D_{t+1}\exp(-\mu^D t) \qquad c^F_{D,t} = C^F_{D,t}\exp(-\mu^D t)$$

$$(18) \qquad y^D_t = Y^D_t\exp(-\mu^D t) \qquad c^F_{F,t} = C^F_{F,t}\exp(-\mu^F t) \qquad c^D_{F,t} = C^D_{F,t}\exp(-\mu^F t)$$

$$k^F_{t+1} = K^F_{t+1}\exp(-\mu^F t) \qquad y^F_t = Y^F_t\exp(-\mu^F t) \qquad \bar{p}^D_t = P^D_t\exp(\mu^D t) \qquad \bar{p}^F_t = P^F_t\exp(\mu^F t)$$

These transformations serve to eliminate the deterministic real trends in each variable. In addition, it is useful to scale all nominal variables by the corresponding total money stocks ($M^s_{D,t}$ and $M^s_{F,t}$). Thus, we define scaled nominal variables as

$$m^D_{D,t} = \frac{M^D_{D,t}}{M^s_{D,t}},\ n^D_{D,t} = \frac{N^D_{D,t}}{M^s_{D,t}},\ w^D_t = \frac{W_{tD}}{M^s_{D,t}},\ \mathcal{L}^D_{D,t} = \frac{\hat{\mathcal{L}}^D_{D,t}}{M^s_{D,t}},\ p^D_t = \frac{\bar{p}_{tD}}{M^s_{D,t}},\ x^D_t = \frac{X^D_t}{M^s_{D,t}},\ n^F_{D,t} = \frac{N^F_{D,t}}{M^s_{F,t}},$$

$$(19) \qquad m^F_{F,t} = \frac{M^F_{F,t}}{M^s_{F,t}},\ n^F_{F,t} = \frac{N^F_{F,t}}{M^s_{F,t}},\ w^F_t = \frac{W^F_t}{M^s_{F,t}},\ \mathcal{L}^F_{F,t} = \frac{\hat{\mathcal{L}}^F_{F,t}}{M^s_{F,t}},\ p^F_t = \frac{\bar{p}^F_t}{M^s_{F,t}},\ x^F_t = \frac{X^F_t}{M^s_{F,t}},\ n^D_{F,t} = \frac{N^D_{F,t}}{M^s_{D,t}},$$

$$e^*_t = e_t\frac{M^s_{F,t}}{M^s_{D,t}},\ a^D_t = \frac{A^D_t}{M^s_{D,t}},\ a^F_t = \frac{A^F_t}{M^s_{F,t}}.$$

The Domestic Household's Problem and General Equilibrium

We now consider the problem facing a household stated in terms of transformed and scaled variables. The problems facing a domestic and foreign household are isomorphic. We therefore focus on domestic household choices to provide economic intuition behind the optimality conditions.

The representative domestic household's objective is to maximize utility

$$(20) \quad E_t \sum_{j=0}^{\infty} (\beta_D^*)^j \mu(c_{D,t+j}^D, c_{F,t+j}^D, 1-\bar{H}_{t+j}^D),$$

where $\beta_D^* = \beta_D \exp[\phi^D(1-\gamma^D)\psi^D\mu^D + (1-\phi^D)(1-\gamma^D)\psi^D\mu^F]$,

subject to

$$(21) \quad L_t^D = 1-\bar{H}_t^D$$

$$(22) \quad y_t^D = f_D^*(k_t^D, H_t^D, \theta_t^D)$$

$$(23) \quad p_t^D c_{D,t}^D = m_{D,t}^D - n_{D,t}^D$$

$$(24) \quad p_t^F c_{F,t}^D = m_{F,t}^D$$

$$(25) \quad \mathcal{L}_{D,t}^D = w_t^D H_t^D + p_t^D(k_{t+1}^D - (1-\delta_D^*)k_t^D), \qquad (1-\delta_D^*) = (1-\delta^D)\exp(-\mu^D)$$

$$(26) \quad \mathcal{L}_{s,t}^D = n_{D,t}^D + x_t^D$$

$$(27) \quad a_{t+1}^D = \frac{1}{1+x_t^D}\left\{w_t^D \bar{H}_t^D + p_t^D f_D^*(k_t^D, H_t^D, \theta_t^D) - (1+i_{L,t}^D)\mathcal{L}_{D,t}^D + (1+i_{L,t}^D)(n_{D,t}^D + x_t^D)\right\}$$

$$(28) \quad a_t^D = m_{D,t}^D + e_t^* m_{F,t}^D$$

Restricting attention to stationary rational expectations equilibria in which prices, monetary shocks, and decision rules are fixed functions of the state of the system, we drop time subscripts in what follows. With the exception of s' denoting current shocks, primes are used below to denote next-period values of variables, while non-primed variables represent current period values.

The Full-Information Setting

We begin by considering the domestic household's choice problem when all current decisions are made with full information (FI) concerning current shock realizations, s'. This information assumptions is standard in basic, open economy cash-in-advance models in the literature following the work in Lucas (1982). With full contemporaneous information, the household begins a period with observations of beginning money stocks M_D^* and M_F^*, beginning capital stocks k^D and k^F, and current-period shock realizations s'.

Let $J^D(a^D,k^D,s')$ represent the value function corresponding to the domestic household's problem.[4] J^D satisfies the functional equation

$$(29)\quad J^D(a^D,k^D,s') = \underset{\{m_F^D,n_D^D,\tilde{H}^D,k^{D\prime},\mathcal{L}_D^D\}}{\text{MAX}} \mu(c_D^D,c_F^D,1-\tilde{H}^D) + \beta_D^* \int J^D(a^{D\prime},k^{D\prime},s'')\Phi(s',ds'')$$

where $a^{D\prime}$ is given by

$$(30)\quad a^{D\prime} = \frac{1}{1+x^D}\Big\{w^D\tilde{H}^D + p^D f_D^*(k^D,H^D,\theta^D) - (1+i_L^D)\mathcal{L}_D^D + (1+i_L^D)(n_D^D+x^D)\Big\}.$$

We use the binding CIA and LIA constraints to eliminate c_D^D, c_F^D and H^D as separate decision variables. We also eliminate m_D^D as a separate choice variable since $m_D^D = a^D - e\cdot m_F^D$ from (28), a^D is predetermined, and e^* is taken as given by the household. Thus, choice of m_F^D is sufficient to determine the household's choice of m_D^D.

First order conditions for the domestic household's problem with respect to choices of

m_F^D, n_D^D, $\bar{H}^D$, $k^{D\prime}$, and $\mathcal{L}_D^D$ are:

$$(31) \quad -\mu_{c_D^D}(t) \cdot \frac{e^*}{p^D} + \mu_{c_F^D}(t) \cdot \frac{1}{p^F} = 0$$

$$(32) \quad -\mu_{c_D^D}(t) \cdot \frac{1}{p^D} + \beta_D^* \int J_1^D(a^{D\prime},k^{D\prime},s'')(1+i_L^D)\frac{1}{1+x^D}\,\Phi(s',ds'') = 0$$

$$(33) \quad \mu_{\bar{H}^D}(t) + \beta_D^* \int J_1^D(a^{D\prime},k^{D\prime},s'')\frac{w^D}{1+x^D}\,\Phi(s',ds'') = 0$$

$$(34) \quad -\beta_D^* \int J_1^D(a^{D\prime},k^{D\prime},s'')\frac{p^D}{1+x^D} \cdot \frac{p^D}{w^D}\, f^*_{H^D}(k^D,H^D,s')\Phi(s',ds'')$$

$$+ \beta_D^* \int J_3^D(a^{D\prime},k^{D\prime},s'')\,\Phi(s',ds'') = 0$$

$$(35) \quad \beta_D^* \int J_1^D(a^{D\prime},k^{D\prime},s'')\frac{1}{1+x^D}\left[\frac{p^D}{w^D} f^*_{H^D}(k^D,H^D,s') - (1+i_L^D)\right]\Phi(s',ds'') = 0$$

and the following envelope conditions apply

$$(36) \quad J_1^D(a^D,k^D,s') = \mu_{c_D^D}(t) \cdot \frac{1}{p^D}$$

$$(37) \quad J_3^D(a^D,k^D,s') = \beta_D^* \int J_1^D(a^{D\prime},k^{D\prime},s'')\frac{p^D}{1+x^D}\Big[f^*_{k^D}(k^D,H^D,s')$$

$$+ \frac{p^D}{w^D}(1-\delta_D^*) f^*_{H^D}(k^D,H^D,s')\Big]\,\Phi(s',ds'').$$

where, for example, $\mu_{c_D^D}(t)$ denotes the marginal utility of current, period t, domestic consumption of the domestically produced goods.

In a stationary equilibrium for the world economy, domestic and foreign decision rules will be fixed functions of the state of the system. An equilibrium consists of p^D, p^F, w^D, w^F, i_D^D, i_D^F, i_L^D, i_L^F, and e^* along with value function J^D for the domestic household (and J^F for the foreign household) satisfying (29) (and the foreign household's analog to (29)).

Decision rules associated with the household's problems satisfy the following market clearing conditions:

$m_F^D + m_F^F = 1$, $m_D^F + m_D^D = 1$, $H^D = \bar{H}^D$, $H^F = \bar{H}^F$, $k^{D'} = f_D^*(k^D, H^D, s') - c_D^D - c_D^F + (1-\delta_D^*)k^D$, $k^{F'} = f_F^*(k^F, H^F, s') - c_F^F - c_F^D + (1-\delta_F^*)k^F$, $\mathcal{L}_D^D = \mathcal{L}_s^D$, $\mathcal{L}_D^F = \mathcal{L}_s^F$.

For the domestic household, combining (31)-(35) with the envelope conditions, we can rewrite the conditions governing choices of m_F^D, n_D^D, $\bar{H}^D$, $k^{D'}$, and $\mathcal{L}_D^D$ as:

(38) $$-\mu_{c_D^D}(t) \cdot \frac{1}{p^D} e^* + \mu_{c_F^D}(t) \cdot \frac{1}{p^F} = 0$$

(39) $$-\mu_{c_D^D}(t)\frac{1}{p^D} + \beta_D^* \int \mu_{c_D^D}(t+1)\frac{1}{p^{D'}}(1+i_L^D)\frac{1}{1+x^D}\Phi(s', ds'') = 0$$

(40) $$\mu_{\bar{H}^D}(t) + \beta_D^* \int \mu_{c_D^D}(t+1)\frac{p^D}{p^{D'}}\frac{w^D}{p^D}\frac{1}{1+x^D}\Phi(s', ds'') = 0$$

(41) $$\beta_D^* \int -\mu_{c_D^D}(t+1)\frac{1}{p^{D'}}\frac{p^D}{1+x^D}\frac{p^D}{w^D}f_{H^D}^*(k^D, H^D, s')\ \Phi(s', ds'')$$

$$+ (\beta_D^*)^2 \int \mu_{c_D^D}(t+2)\frac{1}{p^{D''}}\frac{1}{1+x^{D'}}p^{D'}\left[f_{k^D}(k^{D'}, H^{D'}, s'') + \frac{p^{D'}}{w^{D'}}(1-\delta_D^*)f_{H^D}^*(k^{D'}, H^{D'}, s'')\right]\Phi(s', ds'') = 0$$

(42) $$\frac{p^D}{w^D}f_{H^D}^*(k^D, H^D, s') - (1+i_L^D) = 0,$$

Condition (38) is associated with the household's current choice of foreign and domestic currency balances. Suppose that the household were to increase m_F^D by one foreign currency unit while also cutting its domestic currency balance, m_D^D. According to the CIA constraint on foreign goods purchases, the unit increase in m_F^D enables the household to increase consumption of the foreign good by an amount $\frac{1}{p^F}$. The utility benefit of this increase in consumption is given by $\mu_{c_F^D}(t) \cdot \frac{1}{p^F}$. On the cost side, note that to increase m_F^D by one unit the household reduces its domestic balance m_D^D by an amount e^* given the nominal exchange rate. With e^* fewer units of domestic cash, the household will cut consumption of the domestic good by an amount $\frac{e^*}{p^D}$ given the CIA constraint on domestic consumption purchases. The utility cost of this change in consumption is $\mu_{c_D^D}(t) \cdot \frac{1}{p^D} e^*$. Thus, according to (38), the household chooses currency holdings up to where the utility cost of altering its holdings further equals the utility benefit.

Condition (39) is associated with the household's current deposit decision. Suppose that the household were to increase n_D^D by one unit of domestic currency. Then, given the CIA constraint on domestic good purchases, the household cuts current consumption of the domestic good by $\frac{1}{p^D}$ generating a current utility cost of $\mu_{c_D^D}(t)\frac{1}{p^D}$. On the benefit side, the increase in deposits means that the intermediary will loan an additional unit of currency and will therefore bring home $(1+i_L^D)$ additional units of domestic currency. As a result, $(1+i_L^D)\frac{1}{1+x^D}$ extra units of domestic cash will be available to the household at the beginning of next period which can be used to purchase $\frac{1}{p^{D'}}(1+i_L^D)\frac{1}{1+x^D}$ additional units of the domestic good for consumption. The discounted expected utility benefit generated from the additional unit of current deposits can therefore be expressed as $\beta_D^* \int \mu_{c_D^D}(t+1)\frac{1}{p^{D'}}(1+i_L^D)\frac{1}{1+x^D}\Phi(s', ds'')$.

Thus, according to (39), the household chooses to deposit up to where the marginal cost and expected marginal benefit associated with deposited currency are balanced.

According to (40), labor is supplied by the household up to where the utility cost of a marginal reduction in leisure due to increasing labor supply further, $\mu_{H^D}(t)$, equals the discounted expected future benefit. The benefit from a marginal increase in current labor supply is the discounted expected utility value of extra domestic consumption next period that can be purchased with the $\frac{w^D}{1+x^D}$ units of cash made available for next period by additional current wage receipts.

Condition (41) is associated with the firm's investment decision. Suppose that the firm were to increase $k^{D'}$ by one unit and, for a given amount of borrowing, cuts labor employment by $\frac{p^D}{w^D}$ in accord with the LIA constraint. The firm's current revenues would then decline by $p^D \frac{p^D}{w^D} f^{\bullet}_{H^D}(k^D, H^D, s')$, which implies less cash for the household to carry into next period. With next period's beginning domestic cash balance reduced by $\frac{p^D}{1+x^D} \frac{p^D}{w^D} f^{\bullet}_{H^D}(k^D, H^D, s')$, the household, according to its CIA constraint, will reduce consumption of the domestic good next period. Thus, the discounted expected utility cost associated with a current unit increase in $k^{D'}$ can be expressed as the first term in (41). On the benefit side, the additional unit of capital acquired in the current period becomes productive next period. In addition, given the unit increase in $k^{D'}$ and the firm's LIA constraint, the firm will increase labor demand by $\frac{p^{D'}}{w^{D'}}(1-\delta^{\bullet}_D)$ next period for given amounts of $k^{D''}$ and $\mathfrak{L}^{D'}_D$. Revenues next period will therefore be increased and, consequently, domestic cash available to the household for purchases of domestic consumption goods in the beginning of the period following next period will increase by

$\frac{1}{1+x^{D'}}p^{D'}\left[f^{*}_{k^D}(k^{D'},H^{D'},s'') + \frac{p^{D'}}{w^{D'}}(1-\delta^{*}_{D})f^{*}_{H^D}(k^{D'},H^{D'},s'')\right]$. The discounted expected utility value of the implied extra consumption two periods forward is given by the second term in (41). Hence, according to (41), the firm purchases units of capital $k^{D'}$ up to where the resulting discounted expected future utility benefit and cost to the household are equal.

Condition (42) corresponds to the firm's loan choice. An additional unit of domestic currency from loan proceeds allows the firm to hire $\frac{1}{w^D}$ extra units of labor, according to the firm's LIA constraint. The extra labor then provides the household with $\frac{p^D}{w^D}f^{*}_{H^D}(k^D,H^D,s')$ additional units of domestic currency from goods market revenue which measures the benefit from extra firm borrowing in terms of additional cash that can be carried into next period. The marginal cost of an extra unit of borrowed currency is $(1+i_L^D)$, the loan obligation that must be paid by the firm at the end of the current period. According to (42), the firm will borrow up to where the marginal benefit and marginal cost in terms of end-of-period cash are equal.

Given the optimality conditions governing household choices in the FI setting above, it is useful to consider the effects on households' choices of a monetary shock. Any temporary monetary shock in either country will be neutral. For example, when a transitory domestic monetary injection is received by domestic intermediaries, the fully informed households choose currency balances and deposits such that there are equiproportionate increases in cash balances used by shoppers in the domestic goods market and by domestic firms. The result is equiproportionate increases in current and future levels of the nominal exchange rate and domestic wages and prices. Domestic inflation, nominal and real interest rates, and all foreign variables are unaffected. For both countries, current and future levels of consumption, investment, employment, and output are all unchanged.

In contrast, with persistence in money growth rates, current money shocks are nonneutral. The contemporaneous effect of a positive shock to domestic money growth is an increase in both the nominal exchange rate and domestic nominal interest rate, while domestic employment and output respond negatively. To understand the forces acting on interest rates and output, note that a positive current monetary injection leads to an upward revision of expected domestic inflation. For standard Fisherian reasons, with a relatively small effect on the real interest rate, the upward revision of anticipated inflation leads to an increase in domestic nominal interest rates.[5] Since the nominal rate of interest is the rate at which firms borrow, increases in the domestic nominal rate leads to reduced investment and labor demand by domestic firms.

In addition, as the optimality condition governing household labor supply in (40) indicates, higher expected domestic inflation reduces domestic labor supply. Intuitively, since current wage receipts cannot be used to purchase units of consumption until next period, higher expected inflation reduces the expected real reward to supplying work effort. Thus, domestic employment, investment, and output fall while the domestic nominal interest rate increases in response to a domestic monetary injection when money growth is persistent.

A domestic money injection, with persistent money growth, also serves to change the domestic price level which, in turn, alters desired consumption of domestically produced goods by households in each country. When the arguments in the utility functions are nonseparable, i.e., ψ^D in (11) and ψ^F in (12) are nonzero, changes in desired consumptions of domestically produced goods influences the marginal utilities of leisure and consumption of foreign produced goods for households in both countries. As a result, foreign prices and anticipated foreign inflation are affected giving rise to effects on foreign real activity of a domestic monetary shock.

In addition, if there are spillover effects between money growth shocks (nonzero values of M_{12} and M_{21} in (17)) or if money innovations across countries are correlated (nonzero off diagonals V^{*}_{12} and V^{*}_{21}), then when one country realizes a monetary shock, the other country's money growth is shocked as well. Consequently, with persistence in money growth in each country, anticipated inflation effects of monetary shocks of the type discussed above will also lead to effects on real activity in each country.

Given a strong Fisherian connection in the FI setting between expected inflation and nominal interest rates, a persistent shock to money growth in a country causes the nominal interest rate in that country to rise and output to fall. Such a result is, however, inconsistent with a prevailing view that positive monetary innovations lead to nominal interest rate reductions and to increases in output. This observation leads us to consider alternative variants of the model which correspond to alternative assumptions about information available to agents when making current period choices.

The Sluggish-Portfolio Setting

The second variant of the model that we consider assumes that household portfolio decisions are made prior to observing current realizations of shocks to the system, s'. We refer to this as the sluggish-portfolio (SP) setting, which corresponds to closed-economy analyses by Christiano (1990), Fuerst (1990), and Lucas (1990). In particular, assume in the SP setting that the domestic (foreign) household chooses currency balances and deposits m^D_D, m^D_F, and n^D_D (m^F_F,m^F_D, and n^F_F) prior to observing s'. All other decisions are assumed to be made under full contemporaneous information. The information assumptions employed here are intended to capture the idea that continuous portfolio reallocation is costly for households. The households' problems, equilibrium, and interpretations of the optimality conditions are similar to what appear above for the FI setting. The exception is that now s, last periods' shock realizations, rather than the current realizations s′, will be in household information sets when currency balances and deposits are chosen.

The motivation for considering the SP setting is to allow for liquidity effects of monetary injections on interest rates and output. Given slow portfolio adjustment, nominal interest rates depend not only on Fisherian fundamentals, but also on liquidity effects.[6] The effect of a monetary shock on interest rates and output levels is ambiguous, depending on the relative magnitudes of anticipated inflation and liquidity effects. With a sufficiently large liquidity effect following a domestic monetary injection then, even with higher anticipated domestic inflation, the domestic nominal interest rate can fall. By reducing the cost to domestic firms of borrowing to finance input acquisitions, a nominal rate decrease will lead to increased domestic employment, investment, and output.

Also of interest is the effect of introducing a portfolio rigidity on exchange rates. In the FI setting, period t currency balances and deposits are chosen after agents observe contemporaneous shock realizations, s′. The nominal exchange rate is consequently a function of s′ as are domestic and foreign price levels in the FI setting. In contrast, in the SP setting period t currency balances and deposits are chosen before observing s′. The nominal exchange rate will therefore be a function of s, last period's shock realizations. As a result of the difference in timing of portfolio choices across the FI and SP variants of the model, predictions for nominal and real exchange rate volatilities will differ. In addition, an implication of allowing for portfolio rigidities is that the model implies that goods prices adjust more rapidly to shocks than does the nominal exchange rate. We discuss this implication further, and a possible remedy, in sections 4 and 5.

By allowing for liquidity effects, the SP setting allows for positive money shocks in a country to generate decreases in nominal interest rates and output increases. However, if liquidity effects are outweighed by anticipated inflation effects, then a positive monetary injection in a country will lead to nominal interest rate increases and decreases in output in that country. Indeed, in the closed economy context, Christiano (1990) finds that anticipated inflation effects of monetary injections do outweigh liquidity effects when a sluggish portfolio model is quantitatively evaluated using empirically plausible parameter values. This leads us to consider a third variant of the model.

The Sluggish-Capital Setting

The third variant of the model that we consider will be referred to as the sluggish-capital (SC) setting based on the closed economy analysis of Christiano (1990). In the SC setting, in addition to the portfolio rigidity assumed above, firms are assumed to make investment decisions prior to observing contemporaneous shock realizations. This assumption captures the notion that capital investment plans take time to formulate and execute. The household's problems, equilibrium, and interpretations of optimality conditions are similar to what appear above for the FI setting. The exceptions now are that s, rather than s', will be in household information sets when currency balances, deposits, and $k^{D'}$ and $k^{F'}$ are chosen. The investment rigidity that is assumed in the SC setting precludes firms from using monetary injections, borrowed from intermediaries, to contemporaneously finance capital good purchases. Consequently, positive monetary injections in a country have the potential of generating larger positive employment and output responses than in the FI or SP settings.

3. NUMERICAL SOLUTION AND PARAMETERIZATION OF THE MODEL

Solving each variant of the model involves combining first order and envelope conditions for the domestic household's problem with analogous conditions for the foreign household and equilibrium conditions to obtain policy functions satisfying equilibrium and household optimality conditions. Since it is not possible to find closed-form solutions, we solve the models using a linear-quadratic approximation procedure which involves: (i) linearization of the optimality conditions, with equilibrium conditions imposed, by taking a first-order Taylor series expansion about the model's nonstochastic steady-state; (ii) conjecturing decision rules for choice variables that are linear in the state variables and shocks; (iii) determining coefficient values for the linear decision rules using a method of undetermined coefficients. Details of the approximation procedure that we use are provided in the Appendix.

To investigate quantitative properties of the model, we must assign values to the following parameters: β_j, ϕ^j, γ^j, ψ^j, α^j, δ^j, μ^j, x^j, for j=D,F, and the parameters of the bivariate productivity and money growth shock processes. Parameter values used in our baseline simulations are summarized in Table 1.

The parameters ψ^D and ψ^F determine curvatures of domestic and foreign utility functions. We set ψ^D and ψ^F to zero, yielding log utility for each country. Also, to examine the effects of introducing risk aversion, we provide results for cases where ψ^D and ψ^F are each set to -1. Discount rates β_D and β_F are each set to .99. When log utility is assumed, these values for discount rates imply that agents in both countries face a steady state real interest rate of one percent per quarter. A quarterly real rate of one percent is close to the average rate of return on capital over the last century in the United States.

The share of domestic (foreign) goods in the domestic (foreign) household's total consumption bundle is governed by $\phi^D(\phi^F)$. Estimating these shares from data is difficult. We proceed by setting ϕ^D and ϕ^F each equal to .50, values employed by Stockman and Tesar (1990). We also report results using alternative values to check for sensitivity of the results to values taken by ϕ^D and ϕ^F.

On the technology side, θ^D and θ^F are simply scale variables which we set to unity. The depreciation rate of capital is set equal to ten percent per annum for each country. Thus, we set $\delta^D = \delta^F = .025$. These values are used by Stockman and Tesar (1990), and are close to the average depreciation rate of capital for the U.S. over a sample period 1972:1–1989:4.[7] We also set the average growth rates of technologies, μ^D and μ^F, each equal to .0041, the average growth rate in per capita real GNP from U.S. data.

The stochastic processes that we assume govern shocks to technologies and money growth rates for the two countries are specified in (16) and (17). Parameters of the bivariate productivity shock process are based on results of Backus, Kehoe, and Kydland (1991)(BKK). BKK estimate parameters of a bivariate productivity shock process using Solow residuals for the United States (the domestic country) and an aggregate of six major European countries (the foreign country). Based on their estimates we set

$$T = \begin{bmatrix} .892 & .062 \\ .156 & .906 \end{bmatrix},$$

where the standard deviation of domestic and foreign productivity innovations are, respectively, .01359 and .01261. The correlation between innovations is .263.[8]

Values for parameters of the bivariate money-growth process are taken from an estimated autoregression. For the domestic money measure, U.S. monetary base data were used. For the foreign money measure, we constructed a foreign monetary base variable using data for Canada, Germany, Japan, and the United Kingdom.[9] The foreign base measure was created by converting each country's base into pounds using average 1980 exchange rates. Exchange rates were held constant so that the foreign money measure reflects only monetary base changes. Based on our estimate of (17) over the sample period 1972:1–1989:4 we set

$$M = \begin{bmatrix} .592 & .007 \\ .098 & .695 \end{bmatrix} .$$

The standard deviation of domestic and foreign money growth innovations are, respectively, .00397 and .00623. The covariance between the innovations is .00025. In addition, based on our estimates of (17), mean money growth for the U.S. (the domestic country) and for the composite foreign sector (the foreign country) are, respectively, .0181 and .0163.

It remains to specify the technology parameters α^D and α^F, and preference parameters γ^D and γ^F. The technology that we have specified for each country has a Cobb-Douglas form of capital-labor substitution. This conforms with the relatively constant share of output accruing to labor observed in the U.S., in spite of large secular real wage increases. Labor's share for the domestic and foreign countries are governed by $1-\alpha^D$ and $1-\alpha^F$, respectively. The preference parameters γ^D and γ^F govern shares of period utilities accounted for by leisure in each country.

We have chosen to treat the domestic and foreign countries symmetrically for most of the simulations by setting γ^D and γ^F and α^D and α^F. We report sensitivity of the results, however, to alternative settings of these parameters. We have chosen, for baseline simulations, to set $1-\alpha^D = 1-\alpha^F = .65$ and $\gamma^D = \gamma^F = .76$. These are standard values used in closed economy real business cycle models. The .65 value for labor's share in each country conforms to postwar data for the U.S. economy. Also, given the form of the utility functions that we employ and parameter values assigned above, a .76 value for leisure share parameters is consistent with a steady state allocation in each country of roughly twenty six percent of nonsleeping time endowments to market activity.

Each variant of the model shares a common nonstochastic steady state. Table 2 displays the models' implications, given the share and other parameter values assigned above, for steady state ratios of capital to output, and leisure to market activity, denoted in the table by $\frac{K^D}{Y^D}\left(\frac{K^F}{Y^F}\right)$ and $\frac{1-H^D}{H^D}\left(\frac{1-H^F}{H^F}\right)$ for the domestic (foreign) country. With symmetric parameter settings across countries of the model, these steady state ratios are the same for both countries. For comparison with the model's implications, to serve as a first-moment check, Table 2 also provides sample averages of the capital to output ratio and the ratio of leisure to market activity for data drawn from the U.S. economy for the sample period 1972:1–1989:4.[10] As the first two rows of Table 2 indicate, the steady state capital to output and leisure to market activity ratios implied by the models are quite close to the U.S. sample counterparts when the baseline parameter values in Table 1 are used. The remaining rows of Table 2 are presented to provide information on sensitivity of the model's steady state to alternative parameter values.

4. QUANTITATIVE RESULTS

Using parameter values assigned above, along with decision rules obtained from the approximation procedure used to solve the models, we simulate the model economies. To quantitatively evaluate the models, we begin by considering the models' implications for contemporaneous responses of variables to monetary shocks. Longer-term responses to monetary shocks are then analyzed, followed by comparisons of volatilities of variables implied by the models with volatilities found in data from actual economies.

Contemporaneous Responses of Variables to a Domestic Money Shock

Table 3 reports contemporaneous responses of select variables to a one percent shock to the domestic money growth rate for each model. Baseline parameter values in Table 1 were used to obtain the contemporaneous responses with the exception of the money growth process. For money growth, we used parameter values from estimated univariate autoregressions for domestic and foreign money growth. This is done to abstract at first from the positive contemporaneous correlation between money growth rate shocks in the two countries and the spillover effects in money growth rates found in our estimated bivariate money growth process. We can then identify contemporaneous responses of the models' variables due solely to a domestic money growth rate shock.

By spillover effects, we mean that shocks to either country's money growth influences the other country's money growth rate via nonzero off diagonal elements M_{12} and M_{21} in equation (17). Using univariate money growth processes amounts to setting $M_{12} = M_{21} = 0$, and $V^{*}_{12} = V^{*}_{21} = 0$ in the money-growth-innovations covariance matrix. According to our univariate estimates, we set M_{11} =.595 and M_{22} = .705. The estimated standard deviations of domestic and foreign money growth innovations are, respectively, V^{x}_{11} = .00394 and V^{x}_{22} = .00551.

The first row of Table 3 shows that in the FI model a one-percent increase in domestic money growth, by increasing anticipated domestic inflation, raises the domestic nominal interest rate. Since an increase in the nominal rate raises the borrowing cost to firms of acquiring inputs, labor demand falls. Also, the increase in anticipated domestic inflation acts as a tax on work effort and leads to a decline in labor supply. Thus, in equilibrium, domestic employment and output fall.

Foreign output and the foreign price level are unaffected by the domestic money shock in the FI model, when log utility is assumed (row 1). The domestic money shock does not influence anticipated foreign inflation and therefore does not influence foreign output since the money growth processes in the two countries are not connected. While domestic prices are driven up, the foreign price level is unchanged. Higher domestic prices leads to stronger demands for domestic currency to accommodate planned domestic consumption expenditures which, in turn, drives the nominal exchange rate up. The higher nominal exchange rate in conjunction with higher domestic prices and no change in foreign prices leads to a decline in the real exchange rate.

Now consider the SP model where nominal interest rates and levels of employment and output are subject to liquidity effects as well as anticipated inflation effects. As row (2) of Table 3 shows, the negative effect of increased liquidity on the domestic nominal interest rate is outweighed in equilibrium by the positive effect of higher anticipated inflation induced by the domestic money growth shock. In addition, due to the dominant anticipated inflation effect, labor demand is lower on net as is labor supply which leads to the reduction in domestic employment and output shown in the table.

Aside from its failure to display dominant liquidity effects, the SP model also carries with it adverse implications for nominal and real exchange rate responses to money shocks. In the SP model, the nominal exchange rate does not respond contemporaneously to a domestic money shock while the real exchange rate declines by a large amount relative to the FI model. The lack of any contemporaneous nominal exchange rate response is due to the sluggish portfolio assumption that agents have made shopping balance and deposit decisions prior to observing current shock realizations. Since households arrange their domestic and foreign shopping balances by trading in the currency exchange market, the nominal exchange rate is contemporaneously unresponsive to current shocks. However, since domestic and foreign price levels are functions of current shocks, relative prices and the real exchange rate respond in the period of the shock. Thus, modifying a basic FI cash-in-advance model to include a Lucas-Fuerst type of portfolio rigidity as we have done leads to the unappealing implication that nominal exchange rates are slower to adjust than prices to monetary and technological shocks.

For the SC model, the portfolio rigidity of the SP environment is augmented by the assumption that firms' investment choices are temporarily inflexible. As a result of investment inflexibility, there is a stronger positive effect of a domestic money infusion on domestic labor demand relative to the SP model since investment is temporarily unable to respond. As row (3) of Table 3 shows, in the SC model a positive domestic money growth shock leads to a decline in the domestic nominal interest rate as the liquidity effect dominates the effect of higher anticipated inflation. Also, the positive effect on labor demand outweighs any negative anticipated inflation effect on labor supply and, in equilibrium, domestic employment and output rise. Again, given the assumed portfolio rigidity, the nominal exchange rate is unaffected contemporaneously by a domestic money injection while relative price levels and the real exchange rate adjust.

When log utility is assumed, the foreign country is unaffected by a domestic monetary injection according to the first three rows of Table 3. However, as the last three rows of the table reveal, with increased curvature in the utility functions, domestic monetary shocks are transmitted to the foreign country. The reason for international transmission of monetary shocks when $\psi^D = \psi^F = -1$ is that domestic and foreign marginal utilities of consumption levels and leisure are no longer separable. For example, as domestic consumption of domestically produced goods, c_D^D, and domestic leisure, $1-\bar{H}^D$, respond to a domestic money injection, the marginal utility of domestic consumption of foreign produced goods is influenced. This leads to a change in desired domestic consumption of foreign produced goods which, in turn, influences foreign prices and real activity.

To see international effects of a domestic money shock under risk aversion consider, first, the FI model in row (4) of Table 3. As the domestic price level is driven up in response to the positive money growth rate shock, domestic and foreign households cut back on consumption of the domestically produced good. As a result, for households in each country the marginal utility of consumption of foreign produced goods rises and households respond by increasing consumption of foreign produced goods, e.g. c_F^D rises. Stronger demand for foreign goods serves to drive the foreign price level up. With positive persistence in domestic money growth, the foreign price effects are anticipated to persist so that the current domestic money injection leads to higher anticipated foreign inflation. Then, due to anticipated foreign inflation effects on foreign labor supply and demand, foreign employment and output decline in response to a positive domestic money shock.

Next, consider the SP model in row (5). As the domestic money infusion drives the current domestic price level up, current domestic and foreign consumptions of domestically produced goods fall. This reflects the CIA constraints since domestic prices are higher and shopping balances cannot respond to the money shock by assumption. As c_D^D and c_D^F fall, the marginal utility of consumption of foreign produced goods rises for households in each country. Therefore, desired levels of the consumption of foreign-produced goods rise. Also, in the foreign country, the anticipated foreign inflation effects identified above for the FI model lead to reductions in foreign employment and output in equilibrium. As the foreign price level rises, domestic and foreign households are forced by their CIA constraints and predetermined foreign shopping balances to cut back on consumption of foreign produced goods. As a consequence, c_F^D and Y^F fall in response to the domestic money shock.

For the SC model in row (6), liquidity effects in the domestic country dominate anticipated inflation effects in equilibrium leading to a domestic nominal interest rate reduction, increased domestic output, and lower domestic prices. As domestic prices fall, domestic and foreign consumption of domestically produced goods rise given predetermined shopping balances and households CIA constraints. This causes the marginal utility of consumption of foreign produced goods to fall in each country. Households therefore desire to consume fewer foreign produced goods which puts downward pressure on foreign prices. Also, anticipated foreign inflation falls which leads to increased foreign employment and output. As foreign output rises, the foreign price level falls. Given predetermined shopping balances and households' CIA constraints on foreign goods purchases, the decline in foreign prices leads to increased consumption of foreign produced goods by domestic and foreign households.

The results above provide information about the models' dynamic properties when univariate processes are used to characterize money growth in the two countries. Table 4 provides contemporaneous responses of variables in the models to a domestic money growth shock when we use our estimated bivariate process to characterize money growth rates. In other words, we allow for spillover effects in money growth rates and a positive contemporaneous correlation between money growth rate shocks in the two countries.

Now, a current positive shock to domestic money growth is accompanied by a positive shock to foreign money growth, with persistence in money growth in each country. Comparing the responses in Tables 3 and 4 reveals that qualitatively the results are the same, and quantitatively the results do not differ much. The main difference is that when the bivariate process is employed, a domestic money growth shock is accompanied by responses in foreign output and consumption of foreign produced goods even when log utility is assumed. The reason for this is, of course, the positive contemporaneous correlation between domestic and foreign money growth shocks and positive spillover effects in money growth rates. Consequently, a domestic money growth shock is accompanied by a foreign money growth shock, and these shocks have persistent effects on money growth for each country. Thus, even when log utility is assumed, a domestic money shock is accompanied by a foreign money shock which leads to anticipated foreign inflation effects.

Contemporaneous and Longer-Term Responses of Variables to a Domestic Monetary Impulse

To provide a view of contemporaneous and longer-term effects of monetary shocks on variables in the models, impulse responses are provided in Figure 1. The figure displays contemporaneous and lagged responses for select variables for each model to a one standard-deviation domestic money growth shock. Assuming that a period in the model is one-quarter, the shock occurs in quarter 10. The two countries are in nonstochastic steady state prior to the shock.

The impulse responses are based on parameter values used for rows (4)-(6) of Table 3, where we set $\psi^D = \psi^F = -1$ and assume univariate money growth processes. This allows us to study the models' dynamic properties in the absence of money growth spillovers and contemporaneous money growth shock correlation between the two countries.

There are four key features of the impulse responses. First, in all models the nominal exchange rate rises in response to the positive domestic money growth shock. However, the response in the SP and SC models lags the money shock by one period reflecting the portfolio rigidity in these models. The persistence in the nominal and the real exchange rate responses to the domestic money growth shock for all models reflects persistence in domestic money growth.

Second, for the FI and SP models the real exchange rate falls in the period of the money shock as the contemporaneous positive domestic price response is large relative to the foreign price and nominal exchange rate responses. Following the contemporaneous responses, prices and exchange rates gradually return to their steady state values. For the SC model, the contemporaneous domestic price response is again relatively large. However, domestic and foreign price levels fall in the period the shock occurs reflecting increased domestic and foreign output levels due to the dominant domestic liquidity effect. As a result of the relatively large contemporaneous domestic price response, the real exchange rate rises in the period of the shock. Following the period of the shock to domestic money growth, the domestic price level surges above its steady state value and then resembles the longer term responses in the other models by gradually falling to steady state. The surge in the domestic price level in the period after the shock followed by a gradual decline to the steady state level results in real exchange rate overshooting. That is, after an initial rise, the real exchange rate falls below its steady state value in the period following the shock and then gradually rises back to its steady state value.

The third feature of the impulse responses worth noting is that the dominant liquidity effect on the domestic nominal interest rate in the SC model lacks persistence. The dominant liquidity effect exists only in the period of the money shock.

Fourth, note the inverse responses of foreign (domestic) goods consumption and foreign (domestic) prices for the SP and SC models. This feature reflects the effects of the CIA constraints given that shopping balances in these models are, by assumption, not contemporaneously responsive to shocks. Thus, when the price level in a country changes in response to the domestic money shock, consumption of that country's output by households responds in the opposite direction of the price movement.

Volatilities of Variables and Cross-Correlations with Domestic Output

We now consider volatilities of variables implied by the models compared to volatilities found in data drawn from actual economies. We also consider cross correlations of variables with domestic output. These comparisons are displayed in Table 5. For comparability, moments are calculated for actual and simulated time series that have been Hodrick-Prescott filtered. For each variable listed in Table 5, the first row of numbers are actual data moments from the sample period 1972:1–1989:4. Moments calculated for the models are from simulations based on baseline parameter values reported in Table 1, including parameters for the bivariate money growth process, with ψ^D and ψ^F each set to -1.

Consider, first, volatilities of nominal and real exchange rates. For comparison of the models' implications with actual data we report in Table 5 properties of trade weighted exchange rates.[11] Dynamic properties of trade weighted as well as country-specific nominal and real exchange rates found in actual data are provided separately in Table 6. As Table 6 reveals, standard deviations of measured nominal and real exchange rates are high relative to the standard deviation of U.S. output. Exchange rates are also positively autocorrelated and in general are negatively correlated with contemporaneous U.S. output.

The standard deviations of trade weighted nominal and real exchange rates in the data are, respectively, 7.319 and 6.951. However, as Table 5 shows, nominal and real exchange rate volatilities implied by the models are much lower. This results from the restriction in the models that households exchange currencies solely for the purpose of accumulating balances to use for purchases of consumption goods. Allowing for other motives for holding foreign cash balances would obviously help to increase the exchange rate volatilities implied by the models. It will be useful to allow for physical capital mobility and deposit mobility across countries, which provide monetary and technological risk sharing opportunities, as well as motives for holding foreign currency other than solely for consumption purposes.

Another noteworthy feature of the exchange rate volatilities implied by the models is that nominal exchange rate volatility falls and real exchange rate volatility rises when the rigidities of the SP and SC models are introduced. This reflects the lack of contemporaneous response of nominal exchange rates to current shocks once the Lucas-Fuerst type of portfolio rigidity is introduced.

The standard deviation of domestic employment relative to output reported for the models fall short of what is found in the data. The result for the SC model comes closest to the data moment since the assumed investment rigidity leads to greater effects of shocks on employment than in the other models. The SC model also generates much more domestic nominal interest rate volatility than found in the data and in the FI and SP models. As firms are precluded in the SC model from altering investment in the period of a shock, the effect of shocks on employment and nominal interest rates are magnified, leading to increased volatilities for these variables relative to the other models.

With respect to the cross correlations of variables with domestic output reported in Table 5, there are two notable features. First, the detrended domestic price level is negatively correlated with domestic output both in the data and for all the models. The negative correlation in the models reflects that the importance of monetary shocks for the dynamics is small relative to the importance of technology shocks.[12] The second notable feature of the cross correlations with output is that the nominal exchange rate is negatively correlated with the contemporaneous value of domestic output in the data and each of the models. However, the real exchange rate is negatively correlated with contemporaneous domestic output in the data while a positive correlation is implied by each of the models. Relative to the data, the models imply stronger negative contemporaneous correlations between domestic prices and output, and between the nominal exchange rate and domestic output. In conjunction with the weaker negative contemporaneous foreign price level and domestic output correlation in the models relative to the data, the result is that the models imply positive contemporaneous correlations between the real exchange rate and domestic output.

The dynamic properties of the models indicate that, at least along the international dimension of exchange rate behavior, the models considered here do not perform well quantitatively. Nominal and real exchange rate volatilities implied by the models are far below what are found in the data. The models also imply, counterfactually, that the real exchange rate is positively correlated with contemporaneous domestic output. Finally, the portfolio rigidity in the SP and SC models implies that the nominal exchange rate responds to shocks with a lag while relative price levels and, hence, the real exchange rate respond more rapidly.

5. CONCLUSION

We have assessed the qualitative properties and quantitative performance of an open economy cash-in-advance model with and without portfolio rigidities of the type introduced by Lucas (1990) and Fuerst (1990). Overall, we find that a basic open economy, monetary general equilibrium model with money introduced via cash-in-advance constraints (our FI model) fails to allow for liquidity effects of monetary shocks and, along an international dimension, implies nominal and real exchange rate volatilities that are counterfactually too low. Modifying a basic cash-in-advance open economy model to allow for liquidity effects by incorporating portfolio rigidities (our SP and SC models) does not help along the international dimension of exchange rate dynamics and, in fact, hurts. By assuming that households choose foreign and domestic currency balances for a period prior to observing current shock realizations, the modified models imply that nominal exchange rates are slower in responding to shocks than domestic and foreign prices. The models that we have considered in this paper require modification.

There seem to us to be two modifications to the open economy cash-in-advance model with portfolio rigidities analyzed in this paper that will be useful to explore. The first is to allow for risk sharing across countries which has been shut down in the models examined in this paper. A first step would be to allow for deposit and physical capital mobility. The second modification is to alter the timing of shopping balances and foreign currency choices relative to the timing assumed in the models in this paper. As an example, suppose that prior to observing current period shock realizations, households allocate beginning domestic and foreign currency holdings between shopping balances to use for consumption purchases and balances to take to the foreign exchange market. Further, suppose that households then trade currencies after observing current shocks to acquire domestic and foreign cash for purchases or rental of domestic and foreign capital, or for deposits in foreign and domestic intermediaries. As a result, the implications of the SP and SC model in this paper that nominal exchange rates are slower to respond to shocks than prices is overturned. In particular, currency balances for consumption purchases are determined prior to households observing current shocks, thereby placing a degree of sluggishness in the responses of goods prices to current shocks. The nominal exchange rate, however, fully reflects contemporaneous shock realizations. In addition, since households in one country choose to hold currency of the other country for purposes other than simply consumption expenditures, the modifications above could lead to higher exchange rate volatilities than implied by the simple models that we have analyzed above. Our current research involves incorporating modifications to a basic open economy monetary model, such as those outlined above.

ENDNOTES

1. We adopt the convention of using a subscript to refer to the country of origin of a good or money balance and a superscript to denote the residence of the agent choosing the variable.

2. Typically such a constraint, along with the other CIA and loan-in-advance constraints which follow, is specified as a weak inequality allowing for the possibility that an agent may choose not to spend all cash. Since our simulations will be performed using parameter values for which agents drive their cash constraints to bind as equalities, we work with binding constraints in the model to ease exposition.

3. As long as the loan rate is nonnegative in equilibrium, intermediaries will lend all their cash to firms. If the loan rate is zero, intermediaries will be indifferent between lending and storing cash, in which case assuming that they lend does not sacrifice generality.

4. Strictly, the value function depends on all state variables, and not just a^D, k^D, and s′ as written. To make equations less cumbersome we have included explicitly only the dependencies that are necessary to expose the household's optimality conditions.

5. The domestic (foreign) nominal interest rate is governed by the so-called "Fisherian fundamentals"—the real interest rate and the expected rate of inflation. To see this formally, note that (39) implies

$$(*) \qquad \frac{\mu_{c_D^D}(t)/p_t^D}{\beta_D^* E_t \mu_{c_D^D}(t+1)/p_{t+1}^D} = \frac{(1+i_{Lt}^D)}{1+x_t^D} = R_t^D,$$

where R_t^D denotes the domestic nominal rate of interest and E_t is the expectations operator conditional on information dated t and earlier. According to (*), the domestic household's choices equate the nominal rate with the relative marginal utility values of a unit of domestic currency in periods t and t+1.

Next, note that according to the standard condition for optimal intertemporal consumption allocation,

$$(**) \qquad \frac{\mu_{c_D^D}}{\beta_D^* E_t \mu_{c_D^D}(t+1)} = r_t,$$

where r_t is the real rate of interest. Combining (*) and (**) gives

$$(***) \quad R_t^D = E_t\left[\frac{\mu_{c_D^D}(t)}{\beta_D^* \mu_{c_D^D}(t+1)} \frac{p_{t+1}^D}{p_t^D}\right] = r_t E_t \frac{p_{t+1}^D}{p_t^D} + \mathrm{cov}\left[\frac{\mu_{c_D^D}(t)}{\beta_D^* \mu_{c_D^D}(t+1)}, \frac{p_{t+1}^D}{p_t^D}\right],$$

where use has been made of the relation $E_t x_t y_t = E_t x_t E_t y_t + \mathrm{cov}(x_t, y_t)$ for two random variables x_t, y_t.

According to (***), if the covariance term is small enough to ignore, the domestic nominal interest rate is approximately the sum of the real rate and the expected gross rate of inflation--the so-called "Fisherian fundamentals." Thus, the FI setting contains a strong connection between nominal rates of interest and Fisherian fundamentals.

6. To see the liquidity effects formally, consider the condition for optimal choice of domestic deposits, $n_{D,t}^D$, for the SP setting. Since the domestic household now makes its deposit choice prior to observing current shock realizations, the optimality condition governing choice of $n_{D,t}^D$, analogous to (39) in the FI setting, is

$$(+) \qquad E_{t-1}\mu_{c_D^D}(t)\frac{1}{p_t^D} = E_{t-1}\beta_D^* \frac{(1+i_{L,t}^D)}{1+x_t^D}\mu_{c_D^D}(t+1)\frac{1}{p_{t+1}^D}.$$

For comparability with the FI setting, we can alternatively express the condition above by first defining $\Lambda_t^D = \frac{(1+i_{L,t}^D)}{1+x_t^D} E_t \beta_D^* \mu_{c_D^D}(t+1)\frac{1}{p_{t+1}^D} - \mu_{c_D^D}(t)\frac{1}{p_t^D}$, where $E_{t-1}\Lambda_t^D = 0$ follows from (+).

Solving for $\frac{(1+i_{L,t}^D)}{1+x_t^D} = R_t^D$ gives

$$(++) \quad R_t^D = \frac{\Lambda_t^D + \mu_{c_D^D}(t)\frac{1}{p_t^D}}{\beta_D^* E_t \mu_{c_D^D}(t+1)\frac{1}{p_{t+1}^D}}.$$

The new variable, Λ_t^D, is what Christiano (1990) and Fuerst (1990) have termed a liquidity effect. Here, the liquidity effect measures a relative valuation of domestic currency in the domestic consumption-goods and financial markets. If, say, Λ_t^D is positive then the domestic financial market can be thought of as relatively illiquid. That is, relative to what would arise under full information, the domestic nominal interest rate turns out to be high. If domestic shoppers were able to do so, they would act by increasing deposits which would provide more loans to domestic firms. Shoppers cannot do so, however, since deposits are predetermined before current shocks are realized.

In the FI setting, shoppers and firms act with equal information and $\Lambda_t^D = 0$ for all t. There are no liquidity effects in the FI setting. In the SP setting, however, Λ_t^D is only zero on average. Thus, the strong connection between nominal interest rates and Fisherian fundamentals that holds in the FI setting will hold only on average in the SP setting due to liquidity effects.

7. Using per capita capital stock and investment series, the sample average rate of depreciation on capital for the U.S. is $\delta = .020$. This average is of $1 - \frac{(K_{t+1} - I_t)}{K_t}$, where K_t is the per capita capital stock and I_t is per capita investment. The capital stock data are for the flexible exchange rate period 1972:1–1989:4 and have been provided to us by John Musgrave.

8. The values for the variance–covariance matrix are different from those reported in Backus, Kehoe, and Kydland. This is due to the fact that the technology shock enters the production function differently in the two papers. We have adjusted their variance–covariance estimate so as to be consistent with the specification employed in this paper. Backus, Kehoe, and Kydland's sample period is 1969:2–1987:3.

9. The monetary base data for the United States are defined as monetary base adjusted for reserve ratio changes and are taken from the CITIBASE data tape. The monetary base data for other countries are from International Financial Statistics and are defined as reserve money, seasonally adjusted.

10. The U.S. data used for Table 2 are per capita capital stock and investment series identified in note 7 above, and per capita hours–worked data of Hansen (1984) updated to 1989:4.

11. Trade weights are defined in note (9) of Table 6.

12. Although not reported, we find that when the variances V_{11}^x and V_{22}^x of money growth rate innovations are set to zero, output volatilities implied by the models do not differ substantially from those reported in Table 5. Monetary shocks have a relatively small influence on output volatilities in the models.

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APPENDIX

The Numerical Solution Procedure

This appendix provides an overview of the solution procedure that we employ for the models in the text. For more details, see Christiano (1990). The first step in finding a numerical competitive equilibrium solution is to derive a nonstochastic steady state solution of the model. To determine steady state values of variables, the first order conditions for the domestic household (equations (38)–(42)) and the analagous conditions for the foreign household, with equilibrium conditions imposed, must be evaluated at the nonstochastic steady state.

Using the utility function specified in (11) and substituting from the first order condition governing choice of $\mathcal{L}^D_{D,t}$ into the condition governing choice of deposits $n^D_{D,t}$, the domestic household's first order conditions evaluated in nonstochastic steady state are:

(A1) $$m^D_{ft}:\ \frac{\phi^D}{1-\phi^D}\cdot\frac{c^D_{Fs}}{c^D_{Ds}} = \frac{p^D_s}{p^f_s}\cdot\frac{(1-m^f_{fs})}{(1-m^d_{ds})}$$

(A2) $$n^D_{D,t}:\ (1+r^D) = \frac{\exp(u^D)}{1+x^D}\cdot\frac{p^D_s}{w^D_s}f_{H^D_t}$$

(A3) $$\bar{H}^D_t:\ \frac{\gamma^D}{\phi^D(1-\gamma^D)}\cdot\frac{c^D_{Ds}}{1-H^D_s} = \beta^D\frac{w^D_s}{p^D_s}\cdot\frac{\exp(u^D)}{1+x^D_s}\cdot(1+r^D)$$

(A4) $$k^D_{t+1}:\ (r^D+\delta^D) = \frac{w^D_s}{p^D_s}\cdot\frac{f_{K^D}}{f_{H^D_t}}$$

where a variable with a subscript s denotes the steady state value of that variable, and where

$$(1+r^D) = \exp[\mu^D(1-\phi^D\psi^D(1-\gamma^D))]\cdot\exp[-\mu^F(1-\psi^D)\phi^D(1-\gamma^D)]/\beta_D$$

$$f_{H^D_t} = (1-\alpha^D)\exp[(1-\alpha^D)\theta^D-\alpha^D u^D]\left(\frac{H^D_s}{k^D_s}\right)^{-\alpha^D}$$

$$f_{k^D} = \alpha^D\left(\exp(\theta^D+u^D)\cdot\left(\frac{H^D_s}{k^D_s}\right)\right)^{1-\alpha^D}$$

Use has been made in (A1) of the equilibrium steady state exchange rate,

$$e_s = \frac{1 - m_{Ds}^D}{1 - m_{fs}^D}.$$

Steady state values of the endogenous variables must satisfy the domestic and foreign households' first order conditions, equilibrium conditions, and the CIA and LIA constraints. The models' steady state is therefore obtained by solving a nonlinear system of equations. This is done numerically using a nonlinear simultaneous equations routine in GAUSS, which employs a quasi-Newton method for finding the zeros of a system of nonlinear equations.

Once steady state values of variables are determined, the first order conditions for the domestic and foreign households, with equilibrium conditions imposed, are linearized by taking first-order Taylor series expansions about the steady state. Denote a variable expressed in deviation-from-steady-state form with a caret, e.g. $\hat{n}_{D,t}^D = n_{D,t}^D - n_{D,s}^D$. Each linearized first order conditions will in general be a function of

$$\hat{m}_{dt}^d,\ \hat{m}_{dt+1}^d,\ \hat{m}_{ft}^f,\ \hat{m}_{ft+1}^f,\ \hat{n}_{dt}^d,\ \hat{n}_{dt+1}^d,\ \hat{n}_{ft}^f,\ \hat{n}_{ft+1}^f,\ \hat{K}_t^D,\ \hat{K}_{t+1}^D,\ \hat{K}_{t+2}^D,\ \hat{K}_t^f,\ \hat{K}_{t+1}^f,\ \hat{K}_{t+2}^f,\ \hat{H}_t^D,\ \hat{H}_{t+1}^D,\ \hat{H}_t^f,\ \hat{H}_{t+1}^f,\ \hat{s}_t,\ \hat{s}_{t+1},$$

where $s_t = [\theta_t^D, \theta_t^F, x_t^D, x_t^F]'$.

The expected value of each linearized first order condition, conditioned on information available to the household when the decision is made, is equal to zero. The conditioning set of variables depends on the information structure that is assumed. For example, if domestic and foreign agents make decisions on deposits and money holdings prior to the realization of the current state, the conditioning information set for the domestic and foreign deposit first order conditions includes s_{t-1}, but not s_t.

Decision rules are conjectured for the choice variables that are linear in the state variables, expressed as deviations from their steady state values. The conjectured linear decision rules for the domestic household under the SP (Lucas-Fuerst) information structure are, for example:

(A5)
$$\hat{m}^D_{Ft} = m^D_{F1} \cdot \hat{k}^D_t + m^D_{F2} \cdot \hat{k}^F_t + m^D_{F3} \cdot \hat{s}_{t-1}$$
$$\hat{n}^D_{Dt} = n^D_{D1} \cdot \hat{k}^D_t + n^D_{D2} \cdot \hat{k}^F_t + n^D_{D3} \cdot \hat{s}_{t-1}$$
$$\hat{H}^D_t = H^D_1 \cdot \hat{k}^D_t + H^D_2 \cdot \hat{k}^F_t + H^D_3 \hat{s}_{t-1} + H^D_4 \hat{s}_t$$
$$\hat{k}^D_{t+1} = k^D_1 \cdot \hat{k}^D_t + k^D_2 \cdot \hat{k}^F_t + k^D_3 \hat{s}_{t-1} + k^D_4 \hat{s}_t$$

where m^D_{Fi}, N^D_{Di}, H^D_i, k^D_i are undetermined scalars for i=1,2 and 1 x 4 vectors of undetermined coefficients for i=3,4. Decision rules for the foreign household are analogous to the domestic decision rules in (A5).

For the SP information structure, there are sixty-four undetermined decision rule coefficients. To determine values of these coefficients we use (A5) and the foreign analog to (A5), along with the conditional expectations of the linearized first order conditions, and solve for the undetermined coefficients subject to transversality conditions. Once the decision rule coefficients are known, artificial time series data can be generated from (A5), the foreign analog to (A5), and parameters of the exogenous shock processes.

TABLE I

BASELINE MODEL PARAMETERS

Parameter	Domestic Country	Foreign Country
α	.350	.350
δ	.025	.025
θ	1.000	1.000
β	.9926	.9926
ψ	0 (or −1.0)	0 (or −1.0)
ϕ	.500	.500
γ	.760	.760
μ	.0041	.0041
χ	.0181	.0163
T_{11}	.892	
T_{12}	.062	
T_{21}		.156
T_{22}		.906
$\sigma_{\varepsilon\theta^D}$	.01359	
$\sigma_{\varepsilon\theta^F}$		.01261
$\sigma_{\varepsilon\theta^D,\varepsilon\theta^F}$	.0191	.0191
M_{11}	.592	
M_{12}	.007	
M_{21}		.098
M_{22}		.695
σ_{xD}	.00397	
σ_{xF}		.00623
σ_{xDxF}	.00025	.00025

TABLE 2

CAPITAL-TO-OUTPUT AND LEISURE-TO-MARKET ACTIVITY RATIOS

	PARAMETERS[a]					STEADY STATE OF THE MODELS (FI, SP, SC)	
Rows	Utility $\psi^D = \psi^F$	Deprec. $\delta^D = \delta^F$	Consump. shares $\phi^D = \phi^F$	Capital shares $\alpha^D = \alpha^F$	Leisure shares $\gamma^D = \gamma^F$	$\frac{K^D}{Y^D} = \frac{K^F}{Y^F}$	$\frac{1-H^D}{H^D} = \frac{1-H^F}{H^F}$
(1)	0	.025	.5	.35	.76	10.527	3.776
(2)	−1	.025	.5	.35	.76	10.205	3.824
(3)	0	.025	.5	.45	.76	13.559	3.987
(4)	0	.025	.5	.35	.65	10.527	2.214
(5)	0	.025	.7	.35	.76	10.527	3.776

Rows	ψ^D	ψ^F	δ^D	δ^F	ϕ^D	ϕ^F	α^D	α^F	γ^D	γ^F	$\frac{K^D}{Y^D}$	$\frac{K^F}{Y^F}$	$\frac{1-H^D}{H^D}$	$\frac{1-H^F}{H^F}$
(6)	0	0	.025	.025	.7	.5	.35	.35	.76	.76	10.528	10.546	3.776	3.760
(7)	0	0	.025	.025	.5	.5	.35	.30	.76	.76	10.527	9.037	3.766	3.670

Sample Averages for U.S. Data[b], 1972:1–1989:4 $\frac{K}{Y} = 10.30$ $\frac{1-H}{H} = 3.69$

Rows (1) and (2) show results from baseline parameter values in Table 1.

Row (3) shows that increasing capital's share drives the capital to output ratios up relative to baseline results.

Row (4) shows that decreasing leisure shares reduces leisure to market activity ratios relative to baseline results.

Row (5) shows that increasing the share parameter on home produced consumption in each country's composite consumption has no effect on the steady state ratios relative to benchmark results.

Row (6) shows the effects of increasing the share parameter ϕ^D while leaving all other parameter values at their baseline settings.

Row (7) shows the effects of decreasing the foreign capital share α^F while leaving all other parameter values at their baseline settings.

a Parameters not listed in the table are set at the baseline values provided in Table 1.
b Data Sources are provided in endnotes 7 and 10.

TABLE 3

CONTEMPORANEOUS IMPACT OF A DOMESTIC MONEY GROWTH SHOCK: No Correlation Between Domestic and Foreign Money Growth Shocks, and No Spillovers

PARAMETERS[a]							CONTEMPORANEOUS RESPONSES TO A ONE PERCENT DOMESTIC MONEY GROWTH SHOCK[b]							
Utility $\psi^D = \psi^F$	Deprec. $\delta^D = \delta^F$	Consump. shares $\phi^D = \phi^F$	Capital shares $\alpha^D = \alpha^F$	Leisure shares $\gamma^D = \gamma^F$	Row	Model	$e(x^D)$	$re(x^D)$	$R^D(x^D)$	$Y^D(x^D)$	$Y^F(x^D)$	$c_D^D(x^D)$	$c_F^D(x^D)$	$H^D(x^D)$
					(1)	FI	1.059	−0.383	0.159	−0.587	0.000	−0.221	0.000	−0.121
0	.025	.5	.35	.76	(2)	SP	0.000	−1.418	0.157	−0.769	0.000	−1.057	0.000	−0.122
					(3)	SC	0.000	0.893	−3.221	1.333	0.000	0.667	0.000	0.151
					(4)	FI	0.974	−0.439	0.176	−0.540	−0.023	−0.238	0.012	−0.112
−1	.025	.5	.35	.76	(5)	SP	0.000	−1.389	0.136	−0.799	−0.127	−1.053	−0.034	−0.129
					(6)	SC	0.000	0.712	−3.143	1.174	0.130	0.587	0.065	0.132

a Univariate money growth processes were used. Aside from parameters of the money growth processes all other parameter values are from Table 1.

b Responses above are in the period of a one percent increase in domestic money growth, where

$e(x^D) = \frac{d\log e}{d\varepsilon_{x^D}}$ is the percentage change in the nominal exchange rate; $re(x^D) = \frac{d\log\left(e \cdot \frac{P^F}{P^D}\right)}{d\varepsilon_{x^D}}$ is the percentage change in the real exchange rate;

$R^D(x^D) = \frac{dR^D}{d\varepsilon_{x^D}}$ is the percentage point change in the domestic nominal interest rate; $Y^D(x^D) = \frac{d\log Y^D}{d\varepsilon_{x^D}}$ is the percentage change in domestic real output;

$Y^F(x^D) = \frac{d\log Y^F}{d\varepsilon_{x^D}}$ is the percentage change in foreign real output; $c_D^D(x^D) = \frac{d\log c_D^D}{d\varepsilon_{x^D}}$ is the percentage change in domestic consumption of domestically produced goods;

$c_F^D(x^D) = \frac{d\log c_F^D}{d\varepsilon_{x^D}}$ is the percentage change in domestic consumption of foreign produced goods; $H^D(x^D) = \frac{d\log H^D}{d\varepsilon_{x^D}}$ is the percentage change in domestic employment.

All derivatives are evaluated in nonstochastic steady state.

TABLE 4

CONTEMPORANEOUS IMPACT OF A DOMESTIC MONEY GROWTH SHOCK:
Positive Contemporaneous Correlation Between Domestic and Foreign Money Growth Shocks, and Positive Spillovers

PARAMETERS[a]							CONTEMPORANEOUS RESPONSES TO A ONE PERCENT DOMESTIC MONEY GROWTH SHOCK[b]							
Utility $\psi^D = \psi^F$	Deprec. $\delta^D = \delta^F$	Consump. shares $\phi^D - \phi^F$	Capital shares $\alpha^D = \alpha^F$	Leisure shares $\gamma^D = \gamma^F$	Row	Model	$e(x^D)$	$re(x^D)$	$R^D(x^D)$	$Y^D(x^D)$	$Y^F(x^D)$	$c_D^D(x^D)$	$c_F^D(x^D)$	$H^D(x^D)$
					(1)	FI	1.071	-0.355	0.179	-0.768	-0.010	-0.268	-0.003	-0.1227
0	.025	.5	.35	.76	(2)	SP	0.000	-1.413	0.157	-0.771	-0.010	-0.324	-0.004	-0.1229
					(3)	SC	0.000	0.905	-3.223	1.337	-0.013	0.669	-0.006	0.2128
					(4)	FI	0.982	-0.415	0.195	-0.689	-0.037	-0.293	0.012	-0.1135
-1	.025	.5	.35	.76	(5)	SP	0.000	-1.384	0.136	-0.801	-0.137	-1.054	-0.038	-0.1229
					(6)	SC	0.000	0.721	-3.144	1.176	0.119	0.588	0.059	0.1906

a Baseline parameter values in Table 1 were used, including parameters of the bivariate money growth process.

b See note b of Table 3.

TABLE 5

Moments for Select Variables:
Baseline Parameter Values with $\psi^D = \psi^F = -1$*

Variable	MODEL	Standard Deviation	Autocorrelation			Cross Correlations with Domestic Output				
			1	2	3	-4	-1	0	1	4
Domestic Output (y^D)	Data	1.974	0.872	0.679	0.543	0.196	0.872	1.000	0.872	0.196
	FI	1.294	0.675	0.405	0.207	0.064	0.675	1.000	0.675	0.064
	SP	1.299	0.659	0.394	0.199	0.057	0.659	1.000	0.659	0.057
	SC	1.673	0.444	0.245	0.117	0.028	0.444	1.000	0.444	0.028
Foreign Output (y^F)	Data	1.201	0.679	0.440	0.225	0.128	0.455	0.503	0.444	0.062
	FI	1.134	0.634	0.350	0.135	−0.087	0.152	0.262	0.277	0.165
	SP	1.132	0.614	0.336	0.129	−0.093	0.152	0.281	0.289	0.169
	SC	1.385	0.376	0.202	0.059	−0.064	0.062	0.220	0.155	0.112
Domestic Consumption (c^D)	Data	0.502	0.882	0.731	0.589	0.593	0.850	0.783	0.617	0.075
	FI	0.621	0.683	0.407	0.200	−0.001	0.489	0.731	0.556	0.136
	SP	0.781	0.425	0.239	0.104	−0.039	0.329	0.693	0.510	0.112
	SC	0.881	0.165	0.054	0.003	−0.073	0.082	0.737	0.340	0.102
Domestic Investment (I^D)	Data	3.344	0.909	0.740	0.538	0.324	0.880	0.953	0.862	0.242
	FI	2.376	0.636	0.358	0.156	0.057	0.605	0.907	0.593	0.025
	SP	2.122	0.651	0.367	0.156	0.091	0.756	0.768	0.501	0.005
	SC	1.658	0.649	0.374	0.171	0.201	0.907	0.435	0.226	−0.054
Domestic Employment (H^D)	Data	0.909	0.911	0.741	0.536	0.059	0.735	0.921	0.941	0.504
	FI	0.265	0.558	0.271	0.062	−0.001	0.489	0.731	0.389	0.136
	SP	0.292	0.484	0.231	0.038	0.023	0.351	0.613	0.356	0.001
	SC	0.604	0.021	−0.046	−0.061	−0.105	0.085	0.784	0.301	0.064

Domestic Price Level (p^D)	Data	1.157	0.934	0.814	0.663	−0.614	−0.762	−0.710	−0.599	−0.115
	FI	1.358	0.637	0.369	0.174	−0.074	−0.626	−0.927	−0.614	−0.019
	SP	1.356	0.624	0.359	0.167	−0.072	−0.617	−0.929	−0.595	−0.042
	SC	1.386	0.020	−0.046	−0.061	0.157	−0.244	−0.965	−0.421	−0.019
Foreign Price Level (p^F)	Data	1.334	0.908	0.785	0.586	−0.544	−0.745	−0.669	−0.544	−0.046
	FI	1.468	0.575	0.279	0.064	0.087	−0.158	−0.268	−0.263	−0.125
	SP	1.459	0.559	0.267	0.057	0.096	−0.161	−0.290	−0.268	−0.125
	SC	1.869	0.199	0.091	−0.011	0.064	−0.031	−0.200	−0.132	−0.096
Domestic Nominal Interest Rate (i_L^D)	Data	0.388	0.762	0.484	0.364	−0.590	0.054	0.306	0.408	0.405
	FI	0.404	0.594	0.309	0.099	0.041	0.126	0.189	0.111	−0.005
	SP	0.393	0.494	0.261	0.083	0.009	0.102	0.343	0.207	−0.006
	SC	1.790	−0.035	−0.068	−0.086	0.135	0.268	−0.546	−0.157	−0.068
Nominal Exchange Rate (e^*)	Data	7.319	0.828	0.632	0.509	0.217	−0.221	−0.253	−0.315	−0.415
	FI	1.110	0.506	0.224	0.026	−0.100	−0.281	−0.411	−0.205	0.048
	SP	0.787	0.520	0.238	0.044	−0.147	−0.462	−0.243	−0.102	0.093
	SC	0.792	0.532	0.254	0.056	−0.140	−0.440	−0.180	−0.067	0.099
Real Exchange Rate $\left(e * \frac{p^F}{p^D}\right)$	Data	6.951	0.807	0.595	0.472	−0.065	−0.259	−0.263	−0.298	−0.384
	FI	0.593	0.582	0.312	0.123	0.198	0.484	0.635	0.333	−0.100
	SP	1.356	0.071	0.041	−0.032	0.090	0.172	0.458	0.242	−0.031
	SC	2.233	−0.012	−0.015	−0.047	−0.010	0.039	0.613	0.239	0.001

a Results in this table are based on data that have been logged and then Hodrick-Prescott filtered. Actual percent standard deviations are reported for all variables except c^D, I^D, H^D where the standard deviations have been divided by the standard deviation of domestic output. Nominal interest rates are expressed at a gross quarterly rate prior to logging and filtering. Data for domestic variables, except employment, are from the CITIBASE tape defined as: Y^D(GNP82), c^D(GCS82 + GCN82), I^D(GIF82), p^D(GD), i_L^D(FYGN3). Employment data are based on Hansen's (1984) series updated to 1989:4. Foreign variables are constructed by combining data for Canada, Germany, Japan, and the United Kingdom from the OECD data bank. Foreign output is the pound value of real GNP for each country evaluated at 1980 exchange rates. The foreign price level is the weighted average of the GNP deflators for each country, where the weights are the 1980 share of the total pound value of GNP for the four countries. The exchange rate series are traded weighted data defined in Table 6.

TABLE 6

PROPERTIES OF EXCHANGE RATES: 1972:1–1989:4[a]

COUNTRY	STANDARD DEVIATION (percent)	AUTOCORRELATION 1	AUTOCORRELATION 2	AUTOCORRELATION 3	CROSS CORRELATIONS WITH U.S. OUTPUT AT LEADS AND LAGS -4	-1	0	1	4
	NOMINAL EXCHANGE RATES								
Canada	2.829	.749	.523	.379	.151	.306	.333	.318	.378
Germany	9.831	.807	.588	.461	−.013	−.153	−.184	−.268	−.411
Japan	9.831	.892	.631	.430	−.148	−.444	−.432	−.395	−.227
United Kingdom	9.549	.844	.664	.542	.242	−.058	−.143	−.257	−.577
Trade Weighted	7.319	.828	.632	.509	.217	−.221	−.253	−.315	−.415
	REAL EXCHANGE RATES								
Canada	2.937	.779	.566	.417	−.029	.261	.358	.415	.607
Germany	9.668	.799	.569	.440	−.095	−.221	−.231	−.291	−.362
Japan	9.673	.795	.569	.341	−.171	−.429	−.410	−.364	−.181
United Kingdom	8.705	.817	.615	.500	.165	−.057	−.103	−.190	−.519
Trade Weighted	6.951	.807	.595	.472	−.065	−.259	−.263	−.298	−.348
	Standard Deviation of Real U.S. GNP 1.974								

a Results in this table are based on data that have been first logged and then Hodrick–Prescott filtered. Data are from the International Monetary Fund's International Financial Statistics. Exchange rates are series ae and the price levels are the consumer indexes as expressed by series 64. The trade weight index is the weighted index of spot rates for Canada, Germany, Japan, and the U.K., where the weights are 1980 trade shares.

FIGURE 1: RESPONSES TO A ONE STANDARD DEVIATION SHOCK TO DOMESTIC MONEY GROWTH IN PERIOD 10*

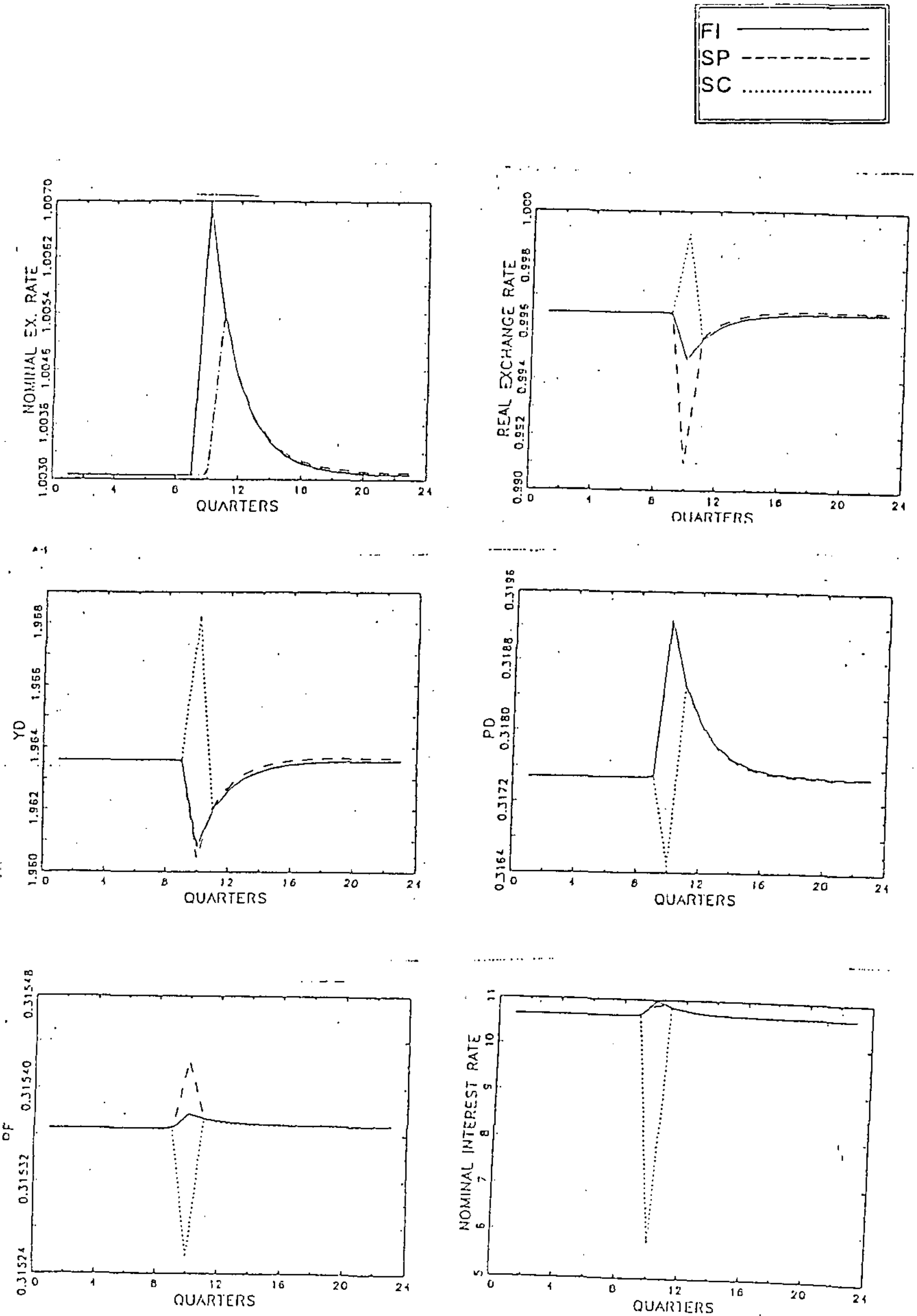
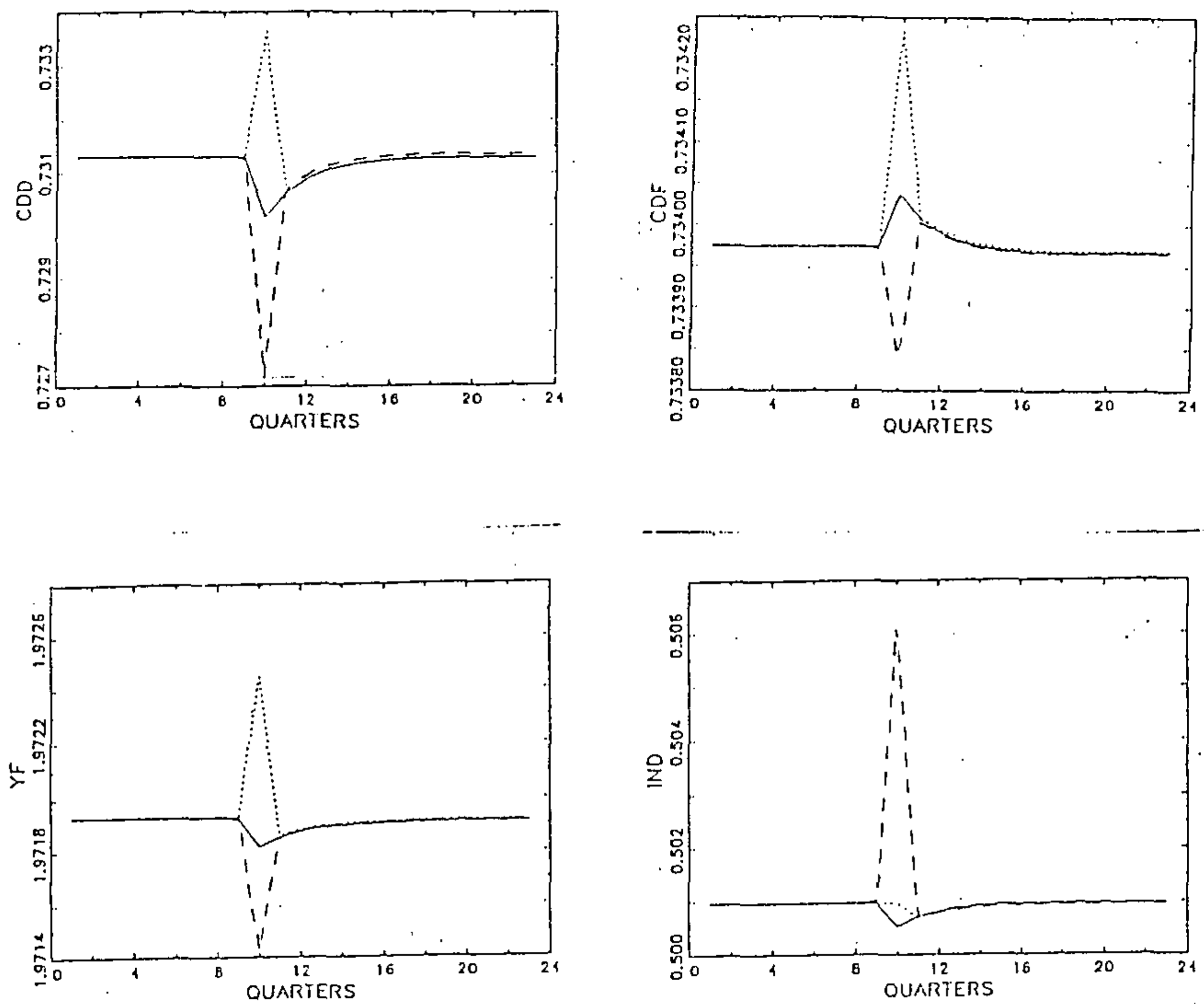


FIG. 1 IMPULSE RESPONSES



a. Responses have been calculated using baseline parameter values from Table 1, with $\psi^D = \psi^F = -1$, except for parameters of the money growth process. Univariate money growth processes were used, with $M_{11} = .595$, $M_{12} = M_{21} = 0$, $M_{22} = .705$, and

$$V^x_{11} = .00394,\ V^x_{12} = V^x_{21} = 0,\ V^x_{22} = .00551\ .$$