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LIQUIDITY AND REAL ACTIVITY IN THREE MONETARY MODELS

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ABSTRACT

This paper investigates interest rate determination and evolutions of nominal and real variables in alternative monetary, general equilibrium models. Three approaches to characterizing monetary transactions services are utilized: a cash-in-advance approach, in which agents face cash constraints on goods purchases; a transaction-cost approach, in which goods are sacrificed in transactions; and a shopping-time approach, in which leisure is sacrificed in transactions. Models which employ these approaches are used to examine liquidity effects of monetary innovations on interest rates and real activity.

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1. Introduction

This paper investigates interest rate determination and evolutions of nominal and real economic variables in alternative monetary, general equilibrium models. We use three approaches to characterizing monetary transactions services: a cash-in-advance approach, in which agents face cash constraints on goods purchases; a transaction-cost approach, in which agents sacrifice real resources to effect transactions; a shopping-time approach, in which agents sacrifice leisure in transactions. Transactions services performed by money give rise to agents' desire to hold currency despite it being dominated in pecuniary return by other assets. In each model, agents face stochastic shocks to technology and the rate of money growth. We utilize the representative household construct of Lucas (1990) to abstract from effects on the distribution of wealth that arise from monetary innovations. The multi-member representative household consists of members who trade in spatially and sometimes informationally distinct markets within each period, but meet to pool resources and information at each period's completion.

A cash-in-advance model is properly viewed as a special case of the transaction-cost or shopping-time model that we analyze. In a cash-in-advance model, agents are required to hold real balances in advance of trading that are at least just sufficient for real transactions. If the cash-in-advance constraint is nonbinding, then there is no cost, in terms of goods or leisure, of increasing the volume of transactions with a given real money balance. If the constraint binds, however, there is an infinite marginal cost of increasing transactions. Relative to a cash-in-advance model, the transaction-cost and shopping-time models that we analyze introduce curvature into the marginal transaction cost, either in terms of goods or leisure, of increasing the volume of real transactions to be executed with a given real money balance.

The focus of our analysis is on interest rate and output effects of monetary injections. We examine implications of each of the three models for the theory of interest rate determination espoused by Irving Fisher, and for liquidity effects of monetary injections. According to Fisher's theory, nominal interest rates are determined by anticipated inflation and the real rate of interest—the so-called "Fisherian fundamentals"—and monetary injections raise nominal interest rates via an anticipated inflation effect. As Lucas (1990) points out, however: "A long line of econometric research from Sargent (1973) through Hansen and Singleton (1983) has failed to confirm a relationship between short-term interest rates and their Fisherian fundamentals, real interest rate movements and expected inflation." Moreover, a prevalent view is that positive innovations in money coincide with reductions in nominal interest rates and increases in output. Indeed, empirical support for this view is reported recently by Christiano and Eichenbaum (1991), who find that in U.S. data, the nominal federal funds rate is negatively correlated with various money supply measures and with real output.

One possible way to reconcile observations with theory is to modify a basic cash-in-advance model as in Lucas (1990) and Fuerst (1992). The Lucas-Fuerst modification assumes that it is prohibitively costly for households to continuously adjust portfolios in response to new information, including information on monetary innovations, and that monetary injections show up first in financial markets. In the models of Lucas and Fuerst, as financial intermediaries receive newly injected cash, a negative influence is exerted on nominal interest rates as firms must be enticed to absorb the currency by a reduction in the cost of borrowing. This negative influence of monetary injections on nominal rates is termed the "liquidity effect" which arises for entirely non-Fisherian reasons. In addition, since injections are distributed asymmetrically across different types of agents, monetary innovations alter real quantities chosen by agents. Cash-in-advance models with and without the Lucas-Fuerst modification are analyzed in recent work by Christiano (1991) and Christiano and Eichenbaum (1991, 1992).

In this paper, a Lucas-Fuerst portfolio rigidity is allowed as a modification to basic cash-in-advance, transaction-cost, and shopping-time models to determine consequences of allowing for liquidity effects. Qualitative features of each model, with and without a portfolio rigidity, and quantitative implications for the evolutions of interest rates and other nominal and real variables are investigated. The quantitative analysis is performed by numerically solving and simulating each model using empirically plausible parameter values. Quantitative evaluation of the models involves comparing properties of simulated series from the models with properties of time series drawn from the U.S. economy.

The paper proceeds as follows. Section 2 describes each monetary model. Trading opportunities and information sets of various members of a representative household are set out along with a description of preferences, technology, and exogenous shocks to the system. Section 3 provides the choice problem confronting a household for each model, interprets optimality conditions, describes general equilibrium, and investigates interest rate and real effects of monetary shocks for each model. Section 4 assigns parameter values used in simulations and discusses chosen values. Section 5 provides quantitative results and discusses properties of each model. Section 6 concludes.

2. Three Monetary Models

We analyze three monetary models utilizing the representative household construct employed by Lucas (1990) and Fuerst (1992) to abstract from effects on the distribution of wealth that arise from monetary innovations while allowing different agents to face different trading opportunities. The multi-member representative household consists of members who trade in spatially and sometimes informationally distinct markets within each period, but meet to pool resources and information at the end of each period. Post-trade wealth positions of different types of agents need not be accounted for since all per capita wealth resides with the household at each period's completion.

Trading Opportunities

The representative household consists of a shopper, worker, financial intermediary, and firm manager. Within each period household members have different tasks to perform in markets for goods, labor, and money. In any market, agents cannot tell who is or is not a member of their own households. At the beginning of period t, the household holds a money balance M_t which it divides by sending N_t with the intermediary to the financial market and the remaining M_t-N_t to the goods market with the shopper. After dividing M_t, the shopper goes to the goods market, the worker to the labor market, and the intermediary and manager to the financial market. Upon arrival at the financial market all intermediaries receive a lump-sum monetary injection X_t. An intermediary then is able to lend N_t+X_t to firms who agree to pay, at the end of the period, the gross loan interest rate $(1+i_t^L)$ times the amount borrowed. Loanable cash supplied to the financial market by an intermediary is[1]

(1) $\mathfrak{L}_t^s = N_t + X_t$

The firm manager possesses the household's production technology and capital stock, K_t, hires workers, and invests. Prior to hiring inputs, the manager borrows cash to finance input acquisitions. After borrowing an amount $\mathfrak{L}_t^d$ of cash, the manager hires H_t units of labor at nominal wage W_t and purchases K_{t+1} - $(1-\delta)K_t$ units of capital at a price P_t to add to the household's stock. We assume that consumption and capital goods are indistinguishable and therefore sell at a common price P_t. The fixed rate of capital depreciation is denoted by δ. After combining labor and capital with the production technology, output is taken to the goods market where household shoppers purchase current consumption. The typical shopper purchases C_t units of the good using a cash balance of M_t-N_t.

The household's worker supplies $\tilde{H}_t$ units of labor at the market wage W_t. Nominal wage receipts of the worker are

(2) $W_t\tilde{H}_t$

After the close of goods market trading, the firm manager takes cash revenue from goods market sales, passes by the financial market to pay its loan obligation $(1+i_t^L)\mathcal{L}_t^d$, and returns home with capital, and cash profits given by

(3) $$P_tY_t + \mathcal{L}_t^d - W_tH_t - P_t(K_{t+1} - (1-\delta)K_t) - (1+i_t^L)\mathcal{L}_t^d,$$

where Y_t is real output.

Each intermediary receives loan repayments $(1+i_t^L)\ (N_t + X_t)$ and pays a gross return $(1+i_t^d)N_t$ on deposits. A typical intermediary member returns home at the end of the period with the household's deposit return, $(1+i_t^d)N_t$, plus any cash derived from intermediary activities, $(1+i_t^L)\ (N_t + X_t) - (1+i_t^d)N_t$. Thus, the intermediary brings home a cash balance

(4) $$(1+i_t^L)\ (N_t + X_t)$$

Upon reuniting, the household consumes goods and pools cash for use at the beginning of the next period. To identify the evolution of cash balances through time for the household, it is necessary to describe how we model households' desire to use cash in transactions. We consider three approaches which we label: a cash-in-advance (CIA) approach; a transaction-cost (TC) approach; and a shopping-time (ST) approach.

Cash-In-Advance Economy

The CIA approach assumes that the shopper faces a cash-in-advance constraint on purchases of the consumption good, and the firm manager faces a loan-in-advance constraint on input acquisitions. The shopper's cash-in-advance constraint is[2]

(5) $$P_tC_t = M_t - N_t.$$

With the constraint expressed as an equality, the shopper returns home with goods, but no leftover cash. The firm's loan-in-advance constraint is given by

(6) $$\mathcal{L}_t^d = W_tH_t + P_t(K_{t+1} - (1-\delta)K_t).$$

Combining cash brought home at the end of the period by the worker in (2), the firm manager in (3), and the intermediary in (4), the household's beginning cash balance next period is given by

(7) $$M_{t+1} = W_t\tilde{H}_t + P_tY_t - (1+i_t^L)\mathcal{L}_t^d + (1+i_t^L)(N_t+X_t)$$

Transaction-Cost Economy

The TC approach assumes that the household sacrifices units of the good to effect transactions and that these transaction costs are declining in the amount of cash used for a given volume of transactions. Specifically, real transactions costs, denominated in units of the good, are assumed to be $T_t^S = T^S\left(C_t, \frac{M_t - N_t}{P_t}\right)$ for the household shopper, and by $T_t^F = T^F\left[\frac{W_t}{P_t}H_t + K_{t+1} - (1-\delta)K_t, \frac{\mathcal{L}_t^d}{P_t}\right]$ for the firm. The functions T^S, T^F are assumed to each be homogeneous of degree 1, nonnegative, and twice continuously differentiable, with $T_1^i > 0$, $T_2^i < 0$, $T_{11}^i > 0$, $T_{22}^i > 0$, $T_{12}^i < 0$, for $i = S,F$. We incorporate transaction costs in a simple fashion by specifying that the required nominal expenditure for the shopper to acquire C_t units of net consumption is $P_t(C_t + T_t^S)$ given a cash balance of M_t - N_t. Similarly, the required nominal expenditure for the firm to acquire current labor inputs and invest is $P_t\left[\frac{W_t}{P_t}H_t + K_{t+1} - (1-\delta)K_t + T_t^F\right]$ given a cash balance of $\mathcal{L}_t^d$. Adding any leftover cash brought home at the end of the period by the shopper to cash brought home by the firm, the worker in (2), and the intermediary in (4), M_{t+1} is given by

(8) $$M_{t+1} = M_t - N_t - P_t(C_t + T_t^S) + P_tY_t + \mathcal{L}_t^d - W_tH_t - P_t(K_{t+1} - (1-\delta)K_t + T_t^F) - (1+i_t^L)\mathcal{L}_t^d + W_t\tilde{H}_t + (1+i_t^L)(N_t + X_t)$$

Shopping-Time Economy

The ST approach assumes that the household sacrifices leisure in transactions and that the leisure cost decreases in the amount of cash used for a given volume of transactions. In particular, total household leisure is given by

(9) $$L_t = 1 - \tilde{H}_t - g^S\left(C_t, \frac{M_t - N_t}{P_t}\right) - g^F\left[\frac{W_t}{P_t}H_t + K_{t+1} - (1-\delta)K_t, \frac{\mathcal{L}_t^d}{P_t}\right],$$

where the normalized household time endowment is unity, $\tilde{H}_t$ is the worker's labor supply, $g^s(.,.)$ represents the shopper's time devoted to consumption transactions, and $g^F(.,.)$ represents the firm's time devoted to acquiring inputs. The shopping time functions g^S, g^F are assumed to each be homogeneous of degree zero, nonnegative, and twice continuously differentiable, with $g_1^i > 0$, $g_2^i < 0$, $g_{11}^i > 0$, $g_{22}^i > 0$, $g_{12}^i < 0$, for $i = S,F$. For the shopper to bring home C_t units of the good, the required cash expenditure is P_tC_t, and the required time expenditure is $g^S\left(C_t, \frac{M_t - N_t}{P_t}\right)$ when the shopper uses cash balance M_t-N_t. For the firm manager to acquire $K_{t+1} - (1-\delta)K_t$ units of capital and H_t units of labor, the required cash expenditure is $P_t\left[\frac{W_t}{P_t}H_t + K_{t+1} - (1-\delta)K_t\right]$, and the time expenditure is $g^F\left[\frac{W_t}{P_t}H_t + K_{t+1} - (1-\delta)K_t, \frac{\mathcal{L}_t^d}{P_t}\right]$. Adding any leftover cash brought home by the shopper to cash brought home by the firm, the worker in (2), and the intermediary in (4), M_{t+1} is given by

$$(10) \qquad M_{t+1} = M_t - N_t - P_tC_t + P_tY_t + \mathcal{L}_t^d - W_tH_t - P_t(K_{t+1} - (1-\delta)K_t) - (1+i_t^L)\mathcal{L}_t^d + W_t\tilde{H}_t + (1+i_t^L)(N_t + X_t).$$

Preferences and Production Technology

Each household orders sequences of consumption $\{C_{t+j}\}$ and leisure $\{L_{t+j}\}$, according to the utility function

$$E_t\sum_{j=0}^{\infty}\beta^j\mu(C_{t+j}, L_{t+j}), \qquad 0 < \beta < 1.$$

E_t represents the expectation operator conditional on information dated t and earlier. Total household leisure for period t+j is:

$$(11) \qquad L_{t+j} = 1 - \tilde{H}_{t+j}, \text{ for the CIA and TC models,}$$

$$(12) \qquad L_{t+j} = 1 - \tilde{H}_{t+j} - g^s\left(C_{t+j}, \frac{M_{t+j} - N_{t+j}}{P_{t+j}}\right) - g^F\left[\frac{W_{t+j}}{P_{t+j}}H_{t+j} + K_{t+j+1} - (1-\delta)K_{t+j}, \frac{\mathcal{L}_{t+j}^d}{P_{t+j}}\right] \text{ for the ST model.}$$

For each model, we adopt the following specification of the period utility function:

$$\mu(c_t, L_t) = \begin{cases} \left[C_t^{1-\gamma}, L_t^{\gamma}\right]^{\Psi}/\Psi & \text{for } \Psi \neq 0 \\ (1-\gamma)\log(C_t) + \gamma\log L_t & \text{for } \Psi \neq 0, \quad 0 < \gamma < 1. \end{cases}$$

Firm managers in each model combine inputs to produce output according to the following technology:

$$(13) \qquad Y_t = f(K_t, Z_tH_t), \qquad Z_t = \exp(\mu t + \theta_t)$$

where the exogenous productivity shock, Z_t, is the sum of a deterministic trend and random deviations about trend. The law of motion governing the evolution of θ_t is specified below. We adopt a Cobb-Douglas specification of the production function: $f(K_t, Z_tH_t) = K_t^{\alpha}(Z_tH_t)^{1-\alpha}$, $0 < \alpha < 1$.

Transaction-Cost and Shopping-Time Functions

For the TC model, we use the transaction-cost specifications:

$$T_t^S = T^S\left(C_t, \frac{M_t - N_t}{P_t}\right) = A^S (C_t)^{d^S} \left(\frac{M_t - N_t}{P_t}\right)^{1-d^S}, \; d^S > 1$$

$$T_t^F = T^F\left(\frac{W_t}{P_t}H_t + K_{t+1} - (1-\delta)K_t, \frac{\mathcal{L}_t^d}{P_t}\right) = A^F \left(\frac{W_t}{P_t}H_t + K_{t+1} - (1-\delta)K_t\right)^{d^F} \left(\frac{\mathcal{L}_t^d}{P_t}\right)^{1-d^F}, \; d^F > 1$$

These transaction cost functional forms are similar to the form used in Marshall (1987).

For the ST model we use the shopping-time specifications:

$$g^S\left(C_t, \frac{M_t - N_t}{P_t}\right) = Q^S \cdot (C_t)^{q^S} \left(\frac{M_t - N_t}{P_t}\right)^{1-q^S}, \; q^S > 1$$

$$g^F\left(\frac{W_t}{P_t}H_t + K_{t+1} - (1-\delta)K_t, \frac{\mathcal{L}_t^d}{P_t}\right) = Q^F \cdot \left(\frac{W_t}{P_t}H_t + K_{t+1} - (1-\delta)K_t\right)^{q^F} \left(\frac{\mathcal{L}_t^d}{P_t}\right)^{1-q^F}, \; q^F > 1.$$

These functions correspond to a Cobb-Douglas technology in which shopping time and real money balances are combined to produce services needed to purchase goods and services. Similar shopping time specifications are found in Kydland (1989), and Den Haan (1990). For the shopping time functions to perform a mapping into the nomalized time constraint [0,1], we require that the scaling factors Q^S and Q^F grow at a rate exp(-μt).

<u>**Exogenous Shocks and The Economy's State**</u>

We assume the following specifications for the evolutions of exogenous shocks to productivity, θ_t, and the rate of growth of money, $x_t \equiv \frac{X_t}{M_t^a}$, where M_t^a is the per capita aggregate money stock:

(14) $\theta_t = (1 - \rho_\theta)\theta + \rho_\theta\theta_{t-1} + \varepsilon_{\theta,t}$

(15) $x_t = (1 - \rho_x)x + \rho_x x_{t-1} + \varepsilon_{x,t}$.

$\varepsilon_{\theta,t}$ and $\varepsilon_{x,t}$ are mutually uncorrelated at all leads and lags, and are uncorrelated with θ_{t-j}, x_{t-j}, $j > 0$. θ and x represent the mean values of θ_t and x_t, respectively.

The state of the economy in period t for each model is represented by values taken by M_t, M_t^a, K_t, κ_t and s_t. The household's beginning-of-period money and capital stocks are denoted by M_t and K_t, respectively. M_t^a represents the per capita aggregate money stock and κ_t represents the per capita aggregate capital stock. s_t is a vector containing productivity and money-growth innovations. The representative household begins a period with knowledge of its beginning-of-period capital and money stocks, and the beginning-of-period per capita values of the capital and money stocks. During the period, the current shocks, s_t, are realized. Shocks to the system form a Markov process with transition function $\Phi(s_{t+1}|s_t)$.

3. THE HOUSEHOLD'S PROBLEM, EQUILIBRIUM, AND EFFECTS OF MONEY SHOCKS

Two variants of the problem facing a typical household in each model will be considered, corresponding to alternative information assumptions. In one variant, referred to as the full-information (FI) setting, all decisions are made with full contemporaneous information. In this setting, nominal interest rates are determined by Fisherian fundamentals. In the second variant, referred to as the sluggish-portfolio (SP) setting, the household chooses its division of beginning cash into shopping and deposit balances prior to observing current shock realizations. All other decisions are made with full contemporaneous information. The SP assumption, intended to capture the notion that continuous portfolio reallocation is costly for households, is employed elsewhere in cash-in-advance models by Fuerst (1992), Lucas (1990), Christiano (1991), and Christiano and Eichenbaum (1991, 1992). In the SP setting, liquidity effects not present in the FI environment are introduced into interest rate determination due to sluggish nominal portfolio adjustments in response to shocks.

For each model, we state the household's problem, describe conditions necessary for general equilibrium, and interpret optimality conditions. To economize on space, we provide details for the FI variant of each model and, when appropriate, note the effects of introducing the sluggish-portfolio assumption. The household's problem, equilibrium, and interpretations of optimality conditions for the SP variant of each model parallel what appear below for the FI variant. We also discuss effects of monetary innovations on interest rates and real activities implied by each variant of each model.

The Full-Information, Cash-In-Advance Model

In the full-information variant of the cash-in-advance model, the household maximizes utility

$$(16) \qquad E_t \sum_{j=0}^{\infty} \beta^j \mu(C_{t+j}, 1 - \tilde{H}_{t+j}) \ ,$$

subject to

$$(17) \qquad L_t = 1 - \tilde{H}_t$$

(18) $\quad Y_t = f(K_t, Z_t H_t)$

(19) $\quad P_t C_t = M_t - N_t$

(20) $\quad \mathcal{L}_t^d = P_t(K_{t+1} - (1-\delta)K_t) + W_t H_t$

(21) $\quad \mathcal{L}_t^S = N_t + X_t$

(22) $\quad M_{t+1} = W_t H_t + P_t f(K_t, Z_t H_t) - (1+i_t^L)\mathcal{L}_t^d + (1+i_t^L)(N_t + X_t)$

Equation (17) gives household leisure; (18) is the production function; (19) is the shopper's CIA constraint; (20) is the firm's loan-in-advance constraint; (21) is loan supply; (22) gives the household's ending cash balance from (7). The household takes P_t, W_t and i_t^L as given. The household begins the current period with observations of its beginning cash balance, M_t, beginning capital stock, K_t, the aggregate per-capita capital and money stocks, κ_t and M_t^a, and current shock realizations, s_t.

Let $V(M_t, M_t^a, K_t, \kappa_t, s_t)$ represent the value function for the household's problem, satisfying the functional equation

(23) $$V(M_t, M_t^a, K_t, \kappa_t, s_t) = \max_{K_{t+1}, \tilde{H}_t, \mathcal{L}_t^d, N_t} \left\{ \mu(C_t, 1-\tilde{H}_t) + \beta \int V(M_{t+1}, M_{t+1}^a, K_{t+1}, \kappa_{t+1}, s_{t+1})\, \Phi(s_{t+1} | s_t) \right\},$$

where $N_t \in [0, M_t]$, $M_{t+1} = W_t \tilde{H}_t + P_t f(K_t, Z_t H_t) - (1+i_t^L)\mathcal{L}_t^d + (1+i_t^L)(N_t + X_t)$, and using binding cash and loan-in-advance constraints, $C_t = \dfrac{M_t - N_t}{P_t}$ and $H_t = \dfrac{\mathcal{L}_t^d - P_t(K_{t+1} - (1-\delta)K_t)}{W_t}$.

An equilibrium consists of price, wage, and interest rate functions of the state such that the representative household holds the per capita aggregate money and capital stocks, supplies labor that a typical firm demands, demands via the firm manager's loan choice the cash supplied by a typical intermediary, and absorbs what is produced by consuming and investing. That is, in equilibrium P_t, W_t, and i_t^L along with value function V satisfy (23), and decision rules which solve the household's problem satisfy the following market-clearing and aggregate-consistency conditions: $f(K_t, Z_t H_t) = C_t + K_{t+1} - (1-\delta)K_t$, $\tilde{H}_t = H_t$, $\mathcal{L}_t^d = \mathcal{L}_t^S$, $K_t = \kappa_t$, and $M_t = M_t^a$.

Combining first-order and envelope conditions for the problem, household choices of K_{t+1}, $\tilde{H}_t$, $\mathfrak{L}_t^d$, and N_t satisfy:

(24) $$-\beta\int\mu_c(t+1)\frac{P_t}{P_{t+1}}\frac{P_t}{W_t}f_H(t)\Phi(s_{t+1}|s_t) + \beta^2\int\frac{P_{t+1}}{P_{t+2}}\left[f_K(t+1) + \frac{P_{t+1}}{W_{t+1}}(1-\delta)f_H(t+1)\right]\Phi(s_{t+1}|s_t) = 0$$

(25) $$-\mu_L(t) + \beta\int\mu_c(t+1)\frac{P_t}{P_{t+1}}\frac{W_t}{P_t}\Phi(s_{t+1}|s_t) = 0$$

(26) $$\frac{P_t}{W_t}f_H(t) - (1+i_t^L) = 0$$

(27) $$-\mu_c(t)\frac{1}{P_t} + \beta\int\mu_c(t+1)\frac{1}{P_{t+1}}(1+i_t^L)\Phi(s_{t+1}|s_t) = 0\ ,$$

where $\mu_c(t) = \dfrac{\partial\mu(c_t,L_t)}{\partial c_t}$, $\mu_L(t) = \dfrac{\partial\mu(c_t,L_t)}{\partial L_t}$, $f_K(t) = \dfrac{\partial f(K_t,Z_tH_t)}{\partial K_t}$, and $f_H(t) = \dfrac{\partial f(K_t,Z_tH_t)}{\partial H_t}$.

Condition (24) is associated with the firm's investment decision. To identify margins relevant for the investment decision, consider a unit increase in K_{t+1} accompanied by a reduction in current labor employment of $\frac{P_t}{W_t}$ units in accord with the firm's loan-in-advance constraint for a given amount of borrowing. The employment reduction reduces current revenue by $P_t\left(\frac{P_t}{W_t}\right)f_H(t)$ which implies less cash for the household to carry into next period. Since the utility cost of a unit reduction in next period's beginning cash balance is $-\mu_c(t+1)\frac{1}{P_{t+1}}$, the discounted expected utility cost to the household of the current revenue reduction is given by the first term in (24). On the benefit side is the discounted expected utility gain from a unit increase in K_{t+1}. With K_{t+1} increasing by one unit, for given amounts of borrowing and K_{t+2} next period, labor demand must rise by $\frac{P_{t+1}}{W_{t+1}}(1-\delta)$ according to the firm's loan-in-advance constraint. With more labor and productive capital next period, the firm's revenue will rise by $P_{t+1}\left[f_K(t+1) + \frac{P_{t+1}}{W_{t+1}}(1-\delta)f_H(t+1)\right]$ which implies additional beginning cash two periods ahead. The discounted expected utility value of additional consumption afforded by the additional cash is given by the second term in (24). According to (24), the firm purchases capital K_{t+1} up to where the discounted expected future marginal benefit and cost are balanced.

According to (25), labor is supplied by the household's worker up to where the utility cost of a marginal reduction in leisure due to increasing labor supply further, $-\mu_L(t)$, equals the discounted expected future benefit. The benefit is the utility value of extra consumption next period that can be financed by additional current wage receipts. If the worker increases labor supply by one unit in the current period, then W_t units of cash are made available for next period. This currency can be used to purchase $\frac{W_t}{P_{t+1}}$ extra units of consumption next period, the discounted expected utility value of which is the second term in (25). It is important to note that in the CIA model, current wage receipts cannot be used to finance current goods market expenditures. As a consequence, anticipated inflation acts as a tax on current labor supply. According to (25), for a given current real wage and planned future consumption and leisure, an increase in anticipated inflation serves to increase current desired leisure and decrease current labor supply. Intuitively, higher anticipated inflation reduces the reward to current work effort by eroding the purchasing power of nominal wage receipts.

Condition (26) governs the firm manager's borrowing decision. For a given investment decision, an additional dollar of loan proceeds allows $\frac{1}{W_t}$ extra units of labor to be hired according to the firm's loan-in-advance constraint. The extra labor then provides $P_t\left(\frac{1}{W_t}\right)f_H(t)$ additional dollars of revenue from goods market sales which can be carried into next period. The marginal cost of an extra dollar of current firm borrowing is $(1+i_t^L)$, the amount that must be repaid at the end of the period. According to (26), the firm borrows to where the marginal benefit and marginal cost in terms of end-of-period cash are equal.

An alternative way to express (26) is $f_H(t) = \frac{W_t}{P_t}(1+i_t^L)$. From this, note that for a given real wage, an increase in the marginal cost of borrowing to finance input acquisitions, $(1+i_t^L)$, reduces current labor demand.

Condition (27) is associated with the household's current deposit decision. A unit increase in deposited currency, according to the cash-in-advance constraint, implies that current consumption is reduced by $\frac{1}{P_t}$ units, the utility value of which is the first term in (27). On the benefit side, a unit increase in deposits means that the household's intermediary lends an additional unit of currency and, consequently, brings home $(1+i_t^L)$ additional units of currency at the end of the period. This provides additional cash for next period when each additional unit of currency can be used to increase consumption by $\frac{1}{P_{t+1}}$ units. The second term in (27) captures the discounted expected utility benefit from extra consumption next period afforded by cash derived from an additional unit of current deposits. According to (27), the household deposits to where the marginal cost and expected marginal benefit are equated.

Interest Rates and Effects of Money Shocks in the Cash-In-Advance Model

Since an additional unit of deposited currency returns $(1+i_t^L)$ units of currency for use next period, the full-information gross nominal interest rate is $R_t^{FI} = (1+i_t^L)$. According to condition (27), we have that

$$(28) \qquad R_t^{FI} = \frac{\mu_C(t)/P_t}{\beta E_t(\mu_C(t+1)/P_{t+1})},$$

where E_t denotes the expectation operator conditioned on information dated t and earlier. In words, according to (28), the household equates the one-period nominal interest rate with the appropriately discounted relative marginal utility values of currency in periods t and t+1. The marginal utility value of a unit of currency in a period, say t, is the additional consumption, $\frac{1}{P_t}$, that can be acquired per unit of currency times the marginal utility of consumption in the period. From the standard efficiency

condition for intertemporal consumption allocation, the real interest rate, r_t, is

$$(29) \qquad r_t = \frac{\mu_C(t)}{\beta E_t \mu_C(t+1)},$$

Combining (28) and (29) gives

$$(30) \qquad R_t^{FI} = E_t\left[\frac{\mu_C(t)}{\beta\mu_C(t+1)} \cdot \frac{P_{t+1}}{P_t}\right],$$

which displays that the nominal interest rate depends on Fisherian fundamentals—the real rate and the expected gross rate of inflation—in the full-information CIA model.

Contemporaneous effects of a monetary shock on household choices depend on the degree of persistence in the money growth rate. In the full-information setting, with no persistence in the money growth rate, a monetary shock will be neutral. When a transitory monetary injection is received by intermediaries, the fully informed households choose currency balances and deposits such that there are equiproportionate increases in cash balances used by shoppers in the goods market and by firms. The result is equiproportionate increases in current and future wages and prices. Inflation, nominal and real interest rates, as well as current and future levels of consumption, investment, employment and output are all unchanged.

In contrast, with persistence in the money growth rate, current money shocks are nonneutral. The contemporaneous effect of a positive shock to money growth is an increase to the nominal interest rate and reductions in employment and output. To understand the forces acting on interest rates and real activity note that a positive current money growth shock leads to an upward revision of expected inflation. For standard Fisherian reasons, with a relatively small effect on the real interest rate, the upward revision of expected inflation leads to an increase in the nominal rate. In turn, since an increase in the nominal rate increases the cost to firms of borrowing to finance input acquisitions, labor demand and investment fall. In addition, according to condition (25) governing labor supply, an increase in expected inflation serves to reduce current labor supply as the reward to current work effort is eroded. The result is that employment and output respond negatively to a positive current money growth shock while the nominal rate rises. However, as mentioned earlier, such a result is inconsistent at least with the conventional view that positive monetary innovations coincide with decreases in nominal rates and increased output.

Now consider the effects of introducing the sluggish-portfolio assumption into the cash-in-advance model, according to which the household's deposit choice is made prior to observing current shock realizations. In this setting, per unit of currency deposited in the current period, the household expects to receive $E_{t-1}(1+i_t^L)$ units of currency for use next period. Thus, the sluggish-portfolio nominal interest rate can be expressed as

$$R_t^{sp} = E_{t-1}(1+i_t^L). \tag{31}$$

The condition governing the deposit choice in the sluggish-portfolio setting, analagous to (27) in the full-information setting, is

$$E_{t-1}\mu(t)\frac{1}{P_t} = \beta E_{t-1}\mu_c(t+1)\frac{1}{P_{t+1}}(1+i_t^L) \ . \tag{32}$$

Since deposits are chosen prior to observing current shocks, the expectation is conditioned on information dated t-1 and earlier.

For comparison of the sluggish-portfolio nominal rate with the nominal rate under full-information in (30), it is useful to express (32) alternatively as

$$R_t^{sp} = \frac{\Lambda_t + \mu_C(t)/P_t}{\beta E_t(\mu_C(t+1)/P_{t+1})}, \text{ where } \Lambda_t = R_t^{sp}\beta E_t\frac{\mu_C(t+1)}{P_{t+1}} - \frac{\mu_C(t)}{P_t}, \text{ and } E_{t-1}\Lambda_t = 0. \tag{33}$$

Λ_t is what Fuerst and Christiano refer to as a liquidity effect, measuring a relative valuation of currency in the consumption-goods and financial markets. Whenever Λ_t is negative, for example, the financial market can be thought of as relatively liquid in that the nominal rate turns out to be low relative to what would arise under full information. If Λ_t is negative, the household would act, if it were able to do so, by decreasing deposits in order to cut back on currency to be loaned to firms. However, this is not possible in the sluggish-portfolio setting since deposits are predetermined.

The strong Fisherian connection between the nominal rate and fundamentals that holds under full information holds only on average in the sluggish-portfolio setting, since $E\Lambda_t = 0$, but not in each period. The household's sluggish adjustment of deposits to new information serves to sever the strong relation between the nominal rate and fundamentals found in the full-information setting. In addition, liquidity effects have important implications for the effects of money shocks on real activity. If a positive current money growth shock induces a sufficiently large decline in Λ_t in (33), the nominal interest rate can fall even though anticipated inflation increases. Then, by reducing the cost to firms of borrowing to finance input acquisitions, a decrease in the nominal rate exerts upward pressure on employment, investment, and output. If, however, anticipated inflation effects outweigh the liquidity effects, a positive money growth shock will lead to an increase in the nominal rate and declines in employment and output. The effects of a money growth shock on the nominal rate and real activity are ambiguous, depending on relative magnitudes of expected inflation and liquidity effects. Relative magnitudes of these competing effects on interest rates and real activities are quantitatively assessed when we simulate the model using empirically reasonable parameter values. Before turning to the quantitative evaluation, we first consider the transaction-cost and shopping-time alternatives to the cash-in-advance model.

<u>The Full-Information, Transaction-Cost Model</u>

In the transaction-cost economy, the household shopper uses a nominal balance $M_t - N_t$ and spends $P_t(C_t + T_t^S)$ units of currency to obtain C_t units of the good for consumption. The shopper's transaction cost denominated in units of the good is $T_t^S = T^S\left[C_t, \frac{M_t - N_t}{P_t}\right]$. The firm manager spends $P_t\left[\frac{W_t}{P_t}H_t + K_{t+1} - (1-\delta)K_t + T_t^F\right]$ currency units to obtain H_t labor units and $K_{t+1} - (1-\delta)K_t$ units of capital. The firm's transaction cost denominated in units of the good is

$$T_t^F = T_t^F\left[\frac{W_t}{P_t}H_t + K_{t+1} - (1-\delta)K_t, \frac{\mathcal{L}_t^d}{P_t}\right].$$

The household's problem is to maximize utility, $E_t\sum_{j=0}^{\infty}\beta^j\mu(C_{t+j}, 1-\tilde{H}_{t+j})$, subject to: $L_t = 1-\tilde{H}_t$; $Y_t = f(K_t, Z_t H_t)$;

$\mathcal{L}_t^S = N_t + X_t$; and, from (8), the household's nominal wealth evolution

$$M_{t+1} = M_t - N_t - P_t(C_t + T_t^S) + P_t Y_t + \mathcal{L}_t^d - W_t H_t - P_t(K_{t+1} - (1-\delta)K_t + T_t^F)$$

$$- (1+i_t^L)\mathfrak{L}_t^d + W_t\tilde{H}_t + (1+i_t^L)(N_t+X_t) \ .$$

Let $V(M_t,M_t^a,K_t,\kappa_t,s_t)$ now be the value function corresponding to the household's problem in the transaction-cost model, satisfying

$$V(M_t,M_t^a,K_t,\kappa_t,s_t) = \max_{N_t,\mathfrak{L}_t^d,K_{t+1},\tilde{H}_t,H_t,C_t} \left\{ \mu(C_t,1-\tilde{H}_t) + \beta\int V(M_{t+1},M_{t+1}^a,K_{t+1},\kappa_{t+1},s_{t+1})\Phi(s_{t+1}|s_t) \right\} \ , \tag{34}$$

where $N_t \in [0,M_t]$. An equilibrium consists of price, wage, and interest rate functions of the state, along with value function V which satisfies (34), and decision rules which solve the household's problem and satisfy the following market-clearing and aggregate-consistency conditions: $\mathfrak{L}_t^d = \mathfrak{L}_t^S$, $\tilde{H}_t = H_t$, $f(K_t,Z_tH_t) = C_t + K_{t+1} - (1-\delta)K_t + T_t^s + T_t^F$, $K_t = \kappa_t$, $M_t^a = M_t$.

Combining first-order and envelope conditions for the household's problem, decisions on N_t, $\mathfrak{L}_t^d$, K_{t+1}, $\tilde{H}_t$, H_t , and C_t satisfy:

$$(1+i_t^L) - (1-T_2^S(t)) = 0 \tag{35}$$

$$(1+i_t^L) - (1-T_2^F(t)) = 0 \tag{36}$$

$$-\mu_c(t)\left[\frac{1+T_1^F(t)}{1+T_1^S(t)}\right] + \beta\int\mu_c(t+1)\frac{1}{1+T_1^S(t+1)}\left\{f_K(t+1) + (1-\delta)(1+T_1^F(t+1))\right\}\Phi(s_{t+1}|s_t) = 0 \tag{37}$$

$$-\mu_L(t) + \mu_c(t)\frac{W_t}{P_t(1+T_1^S(t))} = 0 \tag{38}$$

$$f_H(t) - \frac{W_t}{P_t}(1+T_1^F(t)) = 0 \tag{39}$$

$$\mu_c(t)\frac{1-T_2^S(t)}{P_t(1+T_1^S(t))} - \beta\int\mu_c(t+1)\frac{1}{P_{t+1}}\left[\frac{1-T_2^S(t+1)}{1+T_1^S(t+1)}\right](1+i_t^L)\Phi(s_{t+1}|s_t) = 0 \ , \tag{40}$$

where $T_j^i(t)$ is the partial derivative of transaction-cost function $T^i(t)$, i=S,F , with respect to its jth argument, j=1,2.

Conditions (35) and (36) govern the household's deposit decision and firm manager's loan demand decision. According to these conditions, the household allocates currency between the shopper and firm so that the marginal transaction value of currency to the shopper and firm, given respectively by $T_2^S(t)$ and $T_2^F(t)$, are equated.

Condition (37) is associated with the firm's investment decision. To identify the relevant margins, consider a unit increase in K_{t+1} and corresponding alteration in the household's consumption plan with end-of-period currency balances M_{t+1} and M_{t+2} left unaffected. Accounting for the marginal transaction-cost effect on the firm due to increased capital purchases, the firm spends an additional $P_t(1+T_1^F(t))$ units of currency in the current period. For the ending cash balance M_{t+1} to be unaffected, and since the effective nominal cost of a unit of current consumption is $P_t(1+T_1^S(t))$, current consumption must fall by $\left[\frac{1+T_1^F(t)}{1+T_1^S(t)}\right]$ units. The utility value of this consumption decline is the first term in (37). On the benefit side, note that a unit increase in K_{t+1} increases the firm's revenue next period for two reasons. First, for a given end-of-period capital stock next period, K_{t+2}, investment next period falls by $(1-\delta)$ units. Accounting for the transaction cost effect on the firm of less capital purchases, this implies that next period's revenue will rise by $P_{t+1}(1-\delta)(1+T_1^F(t+1))$. Second, the currently acquired extra unit of capital becomes productive next period causing revenue next period to rise by $P_{t+1}f_K(t+1)$. Given the two sources of revenue increase, in order for next period's ending cash balance M_{t+2} to be unaffected the household can increase consumption by an amount

$P_{t+1}\left\{f_K(t+1) + (1-\delta)(1+T_1^F(t+1))\right\} \cdot \frac{1}{P_{t+1}(1+T_1^S(t+1))}$. The discounted expected future utility benefit associated with a current increase in investment is therefore given by the second term in (37) which the household equates with the first, marginal cost, term.

According to (38) governing the worker's labor supply, the intratemporal marginal rate of substitution between consumption and leisure is equated to the transaction-cost adjusted real wage $\frac{W_t}{P_t(1+T_1^S(t))}$. The nominal wage is deflated by the effective, or transaction-cost adjusted, price of a unit of current consumption to determine the current consumption purchasing power of the nominal wage. On the demand side of the labor market, according to condition (39), the firm hires labor to where the marginal product equals the transaction-cost adjusted real wage to the firm $\frac{W_t}{P_t}(1+T_1^F(t))$. The real cost of hiring an additional unit of labor is the real wage, $\frac{W_t}{P_t}$, along with the increase in real transaction costs for the firm, $\frac{W_t}{P_t}T_1^F(t)$.

There are two noteworthy differences in determinants of labor market activity between the transaction-cost and cash-in-advance models. First, recall that in the cash-in-advance model current wage receipts cannot be used to finance current expenditures and, consequently, inflation acts as a tax on work effort by eroding the future purchasing power of current nominal wage receipts. In contrast, in the transaction-cost model current wages can be used for current expenditures and, according to (38), labor supply depends only on current values of variables. Any effect of a change in anticipated inflation on labor supply will arise only through effects on current values of variables in (38). Second, recall that firms in the cash-in-advance model face a loan-in-advance constraint on input acquisitions and the nominal interest rate therefore influences labor demand. Anticipated inflation and liquidity effects operate on labor demand by changing the nominal interest cost of acquiring labor. In contrast, in the transaction-cost model, labor demand does not directly depend on the nominal rate. According to (39), anticipated inflation or liquidity effects on labor demand will arise only by way of effects on the transaction-cost adjusted real wage and the marginal product of labor.

The remaining optimality condition to consider is (40), associated with the household's consumption plan. To identify the relevant margins, suppose that the household were to allocate an additional unit of currency to the shopper, financed by a decrease in current deposits. Accounting for the transaction cost reduction for the shopper implied by the shopping balance increase, the shopper effectively is provided with an additional $1 - T_2^S(t)$ of nominal consumption purchasing power to devote to current nominal consumption expenditure. Since the effective, or transaction-cost adjusted, price per unit of consumption is $P_t(1+T_1^S(t))$, the extra currency can be used to increase current real consumption by $\frac{1-T_2^S(t)}{P_t(1+T_1^S(t))}$ units, the utility value of which is the first term in (40). On the cost side, the unit decrease in current deposits implies that next period's beginning cash balance falls by $(1+i_t^L)$ units, reflecting the foregone deposit return. Since the marginal utility value of a unit of currency next period is $\mu_c(t+1)\frac{1-T_2^S(t+1)}{P_{t+1}(1+T_1^S(t+1))}$, the reduction in next period's beginning cash balance generates a discounted, expected utility cost given by the second term in (40). According to (40), the household balances the marginal cost associated with perturbations from its optimal consumption plan, with the marginal benefit.

Interest Rates and Effects of Money Shocks in the Transaction-Cost Model

In the full-information version of the model, the gross nominal interest rate is $\tilde{R}_t^{FI} = (1 + i_t^L)$. Using conditions (35) and (36) governing deposit and loan demand decisions, we can express the full-information nominal rate as[3]

$$\tilde{R}_t^{FI} = \left[1 - T_2^S\left(C_t, \frac{M_t - N_t}{P_t}\right)\right] = \left[1 - T_2^F\left(\frac{W_t}{P_t}H_t + K_{t-1} - (1-\delta)K_t, \frac{\mathcal{L}_t^d}{P_t}\right)\right] . \tag{41}$$

The equalities in (41) identify the household allocates currency to equate the marginal transaction value of liquidity to the shopper with the marginal value to the firm.

To identify effects of money shocks in the full-information model, first consider the case of zero persistence in the money growth rate. In this case, a current monetary injection is neutral. Current and future wages and prices increase equiproportionately, real variables and the nominal interest are unaffected. As a purely transitory money injection increases the price level, the fully informed household responds by reducing deposits so that real balances allocated to the shopper and supplied by intermediaries to firms are unchanged. Given pre-shock desired levels of transactions by the shopper and firm, the marginal transaction value of real balances to the shopper, $T_2^S(t)$, and to the firm, $T_2^F(t)$, remain at pre-shock levels. Since $T_2^S(t)$ and $T_2^F(t)$ are unaffected by a transitory money shock, the nominal interest rate is unaffected. Employment and output also do not change since there is no change in the transaction-cost-adjusted real wage facing the worker, $\dfrac{W_t}{P_t(1 + T_1^S(t))}$, or in the adjusted real wage facing the firm, $\dfrac{W_t}{P_t}(1 + T_1^F(t))$.

Now consider effects of introducing the sluggish-portfolio assumption into the model. The household makes its decision on the firm's loan demand with full current information and, as above, equates the nominal loan rate with the marginal transaction value of liquidity to the firm. The deposit decision, however, is now made prior to observing current shocks. The condition governing the deposit choice, analogous to (35) of the full-information model, is

$$E_{t-1}\frac{\mu_c(t)}{P_t(1 + T_t^S(t))}\left\{(1 + i_t^L) - (1 - T_2^S(t))\right\} = 0 ,$$

where the expectation is conditioned on information dated t-1 and earlier to reflect that current deposits are chosen prior to observing current shocks. To see the influence of introducing the sluggish portfolio assumption on the contemporaneous responses of variables to money shocks, again consider first a purely transitory positive shock to the money growth rate.

Recall that the household's deposit response to a transitory money injection under full information is such that the shopper's and firm manager's real balances are unchanged. With planned transactions of the shopper and firm also unchanged, the nominal interest rate is unaffected by the shock. In contrast, with sluggish portfolio adjustment, the household is unable to respond to a current money shock by changing current deposits. Consequently, currency supplied by intermediaries to firms rises by more than in the full-information setting. In order to induce firms to disproportionately absorb extra liquidity, downward pressure is exerted on the nominal interest rate. This downward pressure, not present under full-information, is the liquidity effect on the nominal rate introduced by the sluggish-portfolio assumption.

The sluggish-portfolio assumption also influences responses of employment and output to money shocks relative to the full-information model. Recall that employment and output are unaffected by a positive, transitory shock to money growth in the full-information setting. Employment is unaffected since real transactions and the transaction-cost-adjusted real wages relevant for the worker's labor supply and firm's labor demand decisions are unaffected by the money shock. The neutrality of a transitory money shock disappears, however, in the sluggish-portfolio setting. With a positive, transitory shock to money growth and sluggish portfolio adjustment, household deposits do not change in the period of the shock. Consequently, the shopper transacts with its pre-shock planned currency balance and the firm ends up borrowing more currency than in the full-information setting. As the price level rises in response to the money injection, the shopper's real balance falls and, since $T_1^S(t)$ is decreasing in the shopper's real balance, the marginal transaction cost of increasing current consumption rises. This transaction-cost effect, not present in the full-information setting, by itself provides an incentive for the household to reduce current consumption and puts downward pressure on labor supply by reducing the transaction-cost-adjusted real wage, $\frac{W_t}{P_t(1+T_1^S(t))}$, facing the worker. In addition, as the firm's real balance rises given that the firm must disproportionately absorb currency injected into the economy, the marginal transaction cost of increasing current input acquisitions falls since $T_1^F(t)$ is decreasing in the firm's real loan balance. This transaction-cost effect, not present under full-information, by itself provides an incentive to increase current input acquisitions, including labor, since there is downward pressure on the transaction-cost-adjusted real wage, $\frac{W_t}{P_t}(1+T_1^F(t))$, facing the firm. If the positive effect on labor demand outweighs the negative effect on labor supply, a positive, transitory money shock can lead to an increase in equilibrium employment.

To summarize, a purely transitory monetary injection has no effect on employment, output, or the nominal interest rate in the full-information version of the model. In contrast, due to liquidity effects introduced by sluggish portfolio adjustment, the sluggish-portfolio version of the model allows for contemporaneous increases in employment and output and a decline in the nominal interest rate in response to a current, purely transitory money injection. When persistence is introduced in the money growth rate, a current money injection is nonneutral even in the full-information setting as anticipated inflation effects are introduced into the model. With persistence in money growth, the directions of contemporaneous responses of the nominal interest rate and real activities to a current money shock are in general ambiguous in both variants of the model. The directions of general equilibrium responses of variables to a money shock ultimately depend on values taken by the model's parameters, including parameters of the transaction-cost functions of the shopper and firm. The important point for our purposes is that by introducing the sluggish-portfolio assumption, liquidity effects not present in the full-information version of the model arise. The liquidity effects serve to exert downward pressure on the nominal interest rate following a monetary injection and upward pressure on labor demand.

The Full-Information, Shopping-Time Model

In the shopping-time economy, the shopper and firm manager sacrifice leisure in transactions. The household's problem is to maximize utility, $E_t\sum_{j=0}^{\infty}\beta^j\mu(C_{t+j},L_{t+j})$, subject to: $Y_t = f(K_t,Z_tH_t)$; $\mathcal{L}_t^S = N_t + X_t$; and, from (10), the household's nominal wealth evolution

$$M_{t+1} = M_t - N_t - P_tC_t + P_tY_t + \mathcal{L}_t^d - W_tH_t - P_t(K_{t+1} - (1-\delta)K_t) - (1+i_t^L)\mathcal{L}_t^d + W_t\tilde{H}_t + (1+i_t^L)(N_t+X_t) .$$

Total household leisure is now given by $L_t = 1-\tilde{H}_t - g^S\left[C_t,\frac{M_t-N_t}{P_t}\right] - g^F\left[\frac{W_t}{P_t}H_t + K_{t+1} - (1-\delta)K_t,\frac{\mathcal{L}_t^d}{P_t}\right]$, where leisure sacrificed by the shopper and firm in transactions are represented by the final two terms. Let $V(M_t,M_t^a,K_t,\kappa_t,s_t)$ now represent the value function corresponding to the household's problem in the shopping-time model, satifying

$$V(M_t,M_t^a,K_t,\kappa_t,s_t) = \max_{N_t,\mathcal{L}_t^d,K_{t+1},\tilde{H}_t,H_t,C_t} \Bigg\{ \mu\Bigg(C_t, 1-\tilde{H}_t - g^S\left[C_t, \frac{M_t-N_t}{P_t}\right]$$

$$- g^F\left[\frac{W_t}{P_t}H_t + K_{t+1} - (1-\delta)K_t, \frac{\mathcal{L}_t^d}{P_t}\right]\Bigg) + \beta\int V(M_{t+1},M_{t+1}^a,K_{t+1},\kappa_{t+1},s_{t+1})\Phi(s_{t+1}|s_t)\Bigg\} \quad (42)$$

where $N_t \in [0,M_t]$. An equilibrium consists of price, wage, and interest rate functions of the state, along with value function V satisyfing (42), and decision rules which solve the household's problem and satisfy the following market-clearing and aggregate-consistency conditions: $\mathcal{L}_t^d = \mathcal{L}_t^S$, $\tilde{H}_t = H_t$, $f(K_t,Z_tH_t) = C_t + K_{t+1} - (1-\delta)K_t$, $K_t = \kappa_t$, $M_t = M_t^a$.

Combining first-order and envelope conditions for the household's problem, decisions on N_t, $\mathcal{L}_t^d$, K_{t+1}, $\tilde{H}_t$, H_t , and C_t satisfy:

$$\mu_L(t)\frac{1}{P_t}g_2^S(t) + \beta\int\mu_M(t+1)\cdot i_t^L\Phi(s_{t+1}|s_t) = 0 \quad (43)$$

$$\mu_L(t)\frac{1}{P_t}g_2^F(t) + \beta\int\mu_M(t+1)\cdot i_t^L\Phi(s_{t+1}|s_t) = 0 \quad (44)$$

$$\mu_L(t)g_1^F(t) - [\mu_C(t) - \mu_L(t)g_1^S(t)]$$

$$+ \beta\int\Big\{\mu_L(t+1)g_1^F(t+1)\cdot(1-\delta) + [\mu_c(t+1) - \mu_L(t+1)g_1^S(t+1)]\cdot[f_K(t+1) + (1-\delta)]\Big\}\Phi(s_{t+1}|s_t) = 0 \quad (45)$$

$$-\mu_L(t) + [\mu_c(t) - \mu_L(t)g_1^S(t)]\frac{W_t}{P_t} = 0 \quad (46)$$

$$-\mu_L(t)g_1^F(t)\frac{W_t}{P_t} + [\mu_c(t) - \mu_L(t)g_1^S(t)]\left[f_H(t) - \frac{W_t}{P_t}\right] = 0 \quad (47)$$

$$\mu_M(t) - \beta\int\mu_M(t+1)\cdot(1+i_t^L)\Phi(s_{t+1}|s_t) = 0 \ , \quad (48)$$

where $g_j^i(t)$ is the partial derivative of shopping time function $g^i(t)$, i=S,F, with respect to its j^{th} argument, j=1,2, and $\mu_M(t) = \frac{1}{P_t}\{\mu_c(t) - \mu_L(t)[g_1^S(t) + g_2^S(t)]\}$ is the marginal utility value to the household of currency in period t which can be accounted for as follows. Consider the effect on the household of receiving an additional unit of currency in period t. The currency can purchase $\frac{1}{P_t}$ units of current consumption, the utility value of which is $\mu_c(t)\frac{1}{P_t}$. In addition, two shopping-time effects must be considered. First, the additional currency unit increases the shopper's real currency balance leading to a reduction in shopping time given by an amount $g_2^S(t) \cdot \frac{1}{P_t}$, where $g_2^S(t) < 0$. Second, by increasing consumption the shopper incurs an increase in shopping time given by $g_2^S(t) \cdot \frac{1}{P_t}$. The utility value of the effects on leisure of an extra unit of currency utilized by the shopper for consumption purchases, along with the utility value of increased consumption gives $\mu_M(t)$.

The term $[\mu_c(t) - \mu_L(t)g_1^S(t)]$, which appears in many of the conditions above, represents the marginal utility value of consumption after adjusting for the shopping-time effect on the household shopper due to a change in consumption. If, for example, the household increases current consumption by one unit, then the shopper sacrifices $g_1^S(t)$ units of leisure, the utility value of which is $\mu_L(t)g_1^S(t)$. The effective, or shopping-time adjusted, marginal utility value of an extra unit of consumption is therefore given by $[\mu_c(t) - \mu_L(t)g_1^S(t)]$.

The intuition behind the household optimality conditions above parallels the transaction-cost model with the exception that marginal transaction-cost effects arise now in terms of leisure rather than goods. Given our focus on employment, output, and interest rate responses to monetary impulses, note the following implications of the optimality conditions for interest rate and employment determination. First, note from (48) that the nominal interest rate in the full-information, shopping-time model, which we denote by $\hat{R}_t^{FI}$, can be expressed as[4]

$$\hat{R}_t^{FI} = (1 + i_t^L) = \frac{\mu_M(t)}{\beta E_t \mu_M(t+1)} .$$

In words, the nominal rate is equated to the appropriately discounted relative marginal utility values to the household of currency in periods t and t+1. In addition, according to conditions (43) and (44), currency is allocated within a period by the household so that the marginal utility value of the leisure benefit from liquidity to the shopper, $\mu_L(t)\frac{1}{P_t}g_2^S(t)$, equals the corresponding value to the firm, $\mu_L(t)\frac{1}{P_t}g_2^F(t)$.

Second, regarding employment determination, note from (46) that the household's worker balances the marginal utility cost of foregone leisure due to an increase in work effort, $-\mu_L(t)$, with the associated marginal utility benefit. The benefit arises from increased current household consumption afforded by increased wage receipts and is represented by the second term in (46). On the labor demand side, note that the first term in condition (47) represents the marginal utility value of leisure sacrificed by the firm in the process of hiring an additional unit of labor. According to (47), the firm will hire units of labor to where the marginal product, $f_H(t)$, exceeds real wage $\frac{W_t}{P_t}$. The real reward $f_H(t)$ from hiring an additional unit of labor must exceed the real wage sacrifice to compensate in terms of household utility for the utility value of leisure sacrificed by the firm manager in the process of hiring labor inputs.

Interest Rates and Effects of Money Shocks in the Shopping-Time Model

Effects of monetary shocks on interest rates and real activities implied by alternative variants of the shopping-time model qualitatively parallel those implied by the transaction-cost model. Contemporaneous responses of the nominal interest rate, employment, and output to a current positive shock to money growth when there is persistence in money growth can be summarized as follows. Responses in the full-information variant of the shopping-time model are governed by anticipated inflation effects on the model's variables. Adding the sluggish-portfolio assumption introduces additional, liquidity effects which exert downward pressure on the nominal interest rate and upward pressure on labor demand. In the sluggish-portfolio setting, since households are unable to respond in the period of a shock by altering deposits, firms must be induced to disproportionately absorb currency injected into the economy. This places downward pressure on the nominal rate given diminishing marginal benefits, in terms of leisure savings, to firms from additional liquidity. In addition, the marginal leisure cost $g_I^F(t)$ to a firm associated with acquiring additional inputs is decreasing in real balances borrowed from intermediaries. Therefore, as firms disproportionately absorb currency injected into the economy, it becomes less costly to acquire inputs, including labor. This puts upward pressure on labor demand which, if sufficiently strong, can lead to increased equilibrium employment in response to a positive money growth shock.

4. PARAMETER VALUES

With qualitative properties of the three monetary models in hand, we now turn to quantitatively evaluating the models. For each model, we must assign values to the parameters β, Ψ, γ, θ, α, δ, μ, ρ_θ, x, ρ_x, and the standard deviations of shocks to technology and money growth, $\sigma_{\varepsilon,\theta}$ and $\sigma_{\varepsilon,x}$, respectively. In addition, for the TC model, values must be assigned to parameters of the transaction-cost functions, d^S, d^F, A^S, A^F, and for the ST model values must be assigned to parameters of the shopping-time functions, q^S, q^F, Q^S, Q^F.

The value of β is set at .9926. Curvature of the utility function is determined by the parameter Ψ, which we set to -1 to allow for risk aversion. The preference parameter γ governs the share of period utility accounted for by leisure. We set γ = .76, a standard value in real business cycle models. We set θ, which is simply a scale variable for the productivity shock, equal to 1. The technology that we have specified has a Cobb-Douglas form of capital-labor substitution which conforms with the relatively constant share of output accruing to labor observed in the U.S. in spite of large secular real wage increases. Labor's share is governed by 1-α, which we set to .65, a standard value in real business cycle models. The .65 value for labor's share conforms to postwar data for the U.S. economy. The depreciation rate of capital, δ, and the average growth rate of technology, μ, are assigned values based on quarterly U.S. data for the period 1959:1 to 1989:4. These data are per capita investment, capital stock, and output series taken from CITIBASE. The depreciation rate of capital, δ, is set to .020, the average quarterly depreciation rate of capital for the U.S. for our sample period. The average growth rate of technology, μ, is set to .0041, the sample-average quarterly growth rate of per capita GNP.

The stochastic processes that we assume govern shocks to technology and money growth are specified in (14) and (15). Parameters of the productivity shock process are based on estimates by Burnside, Eichenbaum, and Rebelo (1990), who estimate a first-order autoregression for the linearly detrended logarithm of the technology shock, Z_t. Based on their estimates, we set ρ_θ = .9857 and $\sigma_{\varepsilon,\theta}$ = .01369. Parameters of the money-growth process are based on an estimated first-order autoregression for the sample period 1959:1-1989:4 using U.S. monetary base data adjusted for reserve requirement changes taken from CITIBASE.[5] From our estimate of (15), we set ρ_x = .7683 and $\sigma_{\varepsilon,x}$ = .0042. The average growth rate, x, is set to .0158, which is the full-sample average money growth rate. We also report results using ρ_x = .4953, $\sigma_{\varepsilon,x}$ = .0041, and x = .0187 corresponding to the period 1972:1 to 1989:4, given subsample instability in fitting the autoregression to the data. Using the lower value of ρ_x allows us to analyze the effects on the models' results of lower persistence in money growth.

With respect to the transaction-cost functions, we use the values $d^S = d^F = 1.89$ and $A^S = A^F = .003$ for the parameters which govern marginal transaction-cost effects. With these values, transaction costs account for 1.4 percent of GNP in nonstochastic steady state. Also, a one percent increase in real balances used by the shopper or firm implies a reduction of less than one percent in steady state total transaction costs, which amounts to approximately .01 percent of steady state GNP. We view these steady-state implications as an indication that our parameterization of the transaction-cost functions generates conservative transaction-cost effects in the model. For the shopping-time functions, we use the values $q^S = q^F = 1.89$, $Q^S = Q^F = .003$ for the parameters which govern marginal shopping-time effects. With these values, total household shopping time accounts for 1.1 percent of the household's total time endowment in nonstochastic steady state. If households can devote a maximum of 16 hours per day to market activity, the quarterly time endowment is 1460 hours. Given our parameterization of the shopping-time model, total household shopping time accounts for roughly sixteen hours per quarter. In addition, a one percent increase in real balances used by shoppers or firms implies a reduction in the household's steady-state shopping time of a little under 8.5 minutes per quarter. These implications indicate that our parameterization of the shopping-time functions generates conservative shopping-time effects in the model. Estimating parameters of the transaction cost and shopping time functions from data is difficult. We use parameter values which have steady state implications that are roughly consistent with those in Marshall (1987) for the TC model, and those in Den Haan (1990) and Kydland (1989) for the ST model. As there is uncertainty over values for the transaction cost and shopping time parameters, we report results using alternative values to check for sensitivity of the results to values taken by these parameters. Baseline values for all parameters of the models that are used in most of our simulations are summarized in Table 1.

5. QUANTITATIVE RESULTS

Using parameter values assigned above, along with decision rules obtained from the approximation procedure used to solve the models (details of which are in the Appendix), we simulate the model economies. To quantitatively evaluate the models, we begin by considering nonstochastic steady states. We then consider contemporaneous and longer-term responses of variables to monetary shocks, and compare volatilities of variables and cross correlations with output implied by the models with volatilities and cross correlations found in data from the U.S. economy.

Nonstochastic Steady State

For each model, the full-information and sluggish-portfolio variants share a common nonstochastic steady state. Table 2 displays the models' implications, given parameter values assigned above, for steady state ratios of capital-to-output, leisure-to-hours worked, and consumption-to-output. We also provide information on transaction costs for the TC model and shopping time for the ST model. Sample averages of U.S. data series for 1959:1-1989:4 are provided in the table for comparisons with implications of the models.[6]

The first three rows of Table 2 indicate that steady-state implications of the models when baseline parameter values are used are quite close to the U.S. data counterparts. The capital-to-output ratio for the TC and ST models is lower than the U.S. data counterpart, while the CIA and ST models imply leisure-to-hours worked ratios that are slightly higher than their data counterpart. Although slight alterations of values for share parameters α and γ bring these ratios closer to their data counterparts, we chose to work with $\alpha = .35$ and $\gamma = .76$ for comparability with Christiano's (1991) results for his CIA model.

For each model, given baseline parameterizations, the annualized steady state nominal interest rate is approximately 9.7 percent, which is considerably higher than the average annualized 30 day T-bill rate of 6.31 percent over the period 1959:1-1989:4. The relatively high nominal rate implied by the models is due largely to high average inflation which, in turn, arises from the average money growth rate, x, being set at a relatively high value. We set $x = .01582$ based on our estimated money growth model for the full sample period 1959:1-1989:4. When we lower the average growth rate of money to $x = .00737$, corresponding to the subsample period 1959:1-1971:4, the annualized steady state nominal interest rate for each model falls to around 6.2 percent.

To provide perspective on magnitudes of transaction-cost effects in the TC model, note in row 2 of Table 2 that in steady state, total transaction costs account for 1.4 percent of output. The marginal real transaction cost reduction due to a unit increase in real balances utilized either by the firm or shopper is -.0243. For comparison, Marshall (1987, Table 4) reports that his TC model implies a -.0306 marginal reduction. In our TC model, a one percent increase in real balances used either by the shopper or firm implies a reduction of less than one percent in steady state total transaction costs, which amounts to approximately .01 percent of steady state output. Information on sensitivity of steady state implications of the TC model to alternative values of parameters of the transaction-cost functions of the shopper and firm is provided in rows 4 and 5 of Table 2.

In the ST model, according to row 3 of Table 2, total shopping time accounts for 1.1 percent of the household's total time endowment in steady state. The marginal shopping-time reduction due to a unit increase in real balances utilized by the shopper or firm is -.0044. To gain perspective on the magnitude of the shopping-time effects, given the baseline parameterization of the

ST model, total household shopping time in steady state amounts to roughly sixteen hours per quarter if households can devote a maximum of 16 hours per day to market activity yielding a 1460 hour quarterly time endowment. In addition, a one percent increase in the shopper's or firm's real balance implies a reduction in the household's steady-state shopping time of a little under 8.5 minutes per quarter. Kydland (1989) finds in his ST model that a one percent increase in real balances around steady state implies a shopping time reduction of under 13 minutes per quarter. Rows 6 and 7 of Table 2 provide information on sensitivity of steady state implications of the ST model to alternative parameterizations of the shopping-time functions of the shopper and firm.

Contemporaneous Responses of Variables to a Money Shock

Table 3 reports contemporaneous responses of the nominal interest rate and employment to a one percentage point increase in the money growth rate for each model. Except for variations in the parameters ρ_x, $\sigma_{\epsilon,x}$, and x governing the money growth process, baseline parameter values were used in calculating the responses in the table. The percentage-point response of the nominal interest rate to a one-percentage-point increase in money growth is denoted by R_x in the table, while H_x denotes the percentage change in employment in response to the money shock.

Begin by noting from rows 1, 4, and 7 of Table 3 that $R_x = H_x = 0$ in the full-information variant of each model given a purely transitory ($\rho_x = 0$) shock to money growth. These results reflect neutrality of a transitory money shock in each model. When a temporary monetary injection is received by intermediaries, the fully informed households choose currency balances and deposits so that there are proportionate increases in nominal balances used by shoppers and firms equal to the proportionate increase in the price level. The result is equiproportionate increases in current and future wages and prices and no changes in current and future levels of investment, employment, output, or the nominal interest rate.

Now consider the CIA model, beginning with the results under full information when there is positive persistence in money growth shocks, $\rho_x > 0$. With positive persistence, anticipated inflation effects serve to increase the nominal interest rate and reduce employment in the period of a money shock, and the magnitudes of these responses increase with increased money-growth persistence. The nominal rate increases in the CIA model under full information along standard Fisherian lines due to increased anticipated inflation. Higher anticipated inflation also serves to reduce the expected reward to current work effort since in the CIA model current wage receipts of workers cannot be used in transactions in the period of the money shock. Thus, current labor supply and equilibrium employment are driven down.

Turning to the sluggish-portfolio variant of the CIA model, when there is positive money growth shock persistence, anticipated inflation effects compete with liquidity effects in determining the signs of responses of the nominal interest rate and employment to money shocks. Given a monetary injection, liquidity effects exert downward pressure on the nominal rate and upward pressure on labor demand, while increased anticipated inflation places upward pressure on the nominal rate and, acting as a tax on labor supply, exerts downward pressure on labor supply. Rows 2 and 3 of Table 3 show that with positive money growth persistence and sluggish portfolio adjustment, the nominal interest rate rises and employment falls in equilibrium in the period of a money shock. Thus, in the sluggish-portfolio CIA model evaluated using empirically reasonable parameter values, liquidity effects on the nominal interest rate and employment are dominated by anticipated inflation effects. In response to a monetary injection, the nominal rate rises and equilibrium employment falls.

Next, consider results for the TC and ST models. The results for full-information variants of these models are qualitatively the same as for the full-information CIA model. Rows 5 and 6 for the TC model, and rows 8 and 9 for the ST model reveal that with positive persistence in money growth shocks, the contemporaneous nominal interest rate response to a monetary injection is positive while employment responds negatively. The positive nominal interest rate responses reflect increased marginal values of liquidity to household shoppers and firms in equilibrium. Equilibrium employment responds negatively reflecting labor supply and demand responses given effects of the money shock on the transaction-cost or shopping-time adjusted real wages facing the household worker and firm.

The full-information versions of all three models (CIA, TC, ST) display the same qualitative results for contemporaneous nominal interest rate and employment responses to a money shock. In each model, with positive persistence in money growth shocks, the nominal interest rate rises and employment falls in the period of a monetary injection. Contingent on our parameterizations of the models, the full-information results indicate that the transaction-cost and shopping-time alternatives to the cash-in-advance model do not rationalize the view that a positive monetary innovation coincides with a reduction in the nominal interest rate and increased employment, at least in the short run. As we have seen, the sluggish-portfolio version of the CIA model also does not rationalize this view. Of interest, then, are the effects of introducing the Lucas-Fuerst sluggish-portfolio assumption into the TC and ST models. To see the effects, now consider the sluggish-portfolio results for the TC model in rows 5 and 6 of Table 3, and for the ST model in rows 8 and 9.

Results for the sluggish-portfolio TC and ST models with empirically reasonable parameterizations of the money growth process reveal that a monetary injection serves to increase the nominal interest rate and reduce equilibrium employment, and the magnitudes of these responses increase with increased money-growth persistence. The positive nominal interest rate response and negative employment response are also predicted by full-information versions of the TC and ST models when there is positive persistence in money growth shocks. Thus, adding the Lucas-Fuerst sluggish-portfolio assumption and resulting liquidity effects to the TC and ST models does not alter their implications for the signs of nominal interest rate and employment responses to monetary injections relative to the full-information setting.

To summarize, the results in Table 3 reveal that full-information versions of all three models fail to rationalize the conventional view that a positive money shock drives interest rates down and employment and output up, at least in the short run. In addition, contingent on our parameterizations of the models, introducing the Lucas-Fuerst sluggish-portfolio assumption does not overturn the models' predictions that nominal interest rates respond positively and employment negatively to monetary injections.

Contemporaneous and Longer-Term Responses of Variables to a Monetary Impulse

To provide a view of contemporaneous and longer-term effects of a monetary shock on variables, impulse responses are provided in Figure 1. The figure displays responses of variables for each model to a one standard-deviation money growth shock that occurs in period 10. The economies are in nonstochastic steady states prior to the shock, and the impulse responses are calculated using baseline parameter values. To facilitate comparisons across models, steady state values of variables for the TC and ST models have been normalized to equal steady state values in the CIA model.

There are four noteworthy features of the impulse responses. First, note that adding the Lucas-Fuerst portfolio rigidity to each model leads to only minor alterations in the implied nominal interest rate and employment responses to the money growth shock. Second, in full-information and sluggish-portfolio versions of each model, the money growth shock induces a contemporaneous negative consumption response and positive price level response. In the sluggish-portfolio CIA model, the negative consumption response becomes more pronounced relative to the full-information model. This reflects the CIA constraint on the household's shopper since the price level rises initially in response to the money shock while shopping balances cannot respond given the sluggish-portfolio assumption. The TC and ST models, by allowing greater flexibility to household shoppers than in the CIA model, generate substantially smaller consumption responses than in the CIA model. Even in the face of relatively larger positive price level effects, consumption responses to the money shock in the TC and ST models are quite small relative to the CIA model.

The third notable feature of the impulse responses in that the positive money growth shock leads to an increase, in the period of the shock, in transaction costs in the TC model and in total household shopping time in the ST model. These responses reflect that the initial positive price level effects of the money shock in the TC and ST models lead to reductions in real balances utilized by shoppers and firms. The reductions in real balances exert upward pressure on transaction costs and on household shopping time. Note, however, that the magnitudes of the transaction cost and shopping time responses to the money shock are small. The largest positive transaction cost and shopping time effects arise in the sluggish portfolio setting in the period of the money shock. The largest transaction cost effect is an increase in the transaction-cost-to-output ratio from .01321 to .01334. The largest shopping time effect is an increase in total household shopping time of roughly 9 minutes if a period is assumed to be a quarter and the time endowment is 1460 hours.

Next, note that in each model the nominal interest rate initially rises in response to the money shock and then declines to its steady state level over time. With $0 < \rho_x < 1$, money growth remains above its steady state level following the shock but declines over time. Consequently, inflation also declines over time to its steady state value. As a result, in the CIA model the nominal interest rate remains above its steady state value following the money shock and declines over time as inflation decreases. In the TC and ST models, an additional force is at work to keep the nominal rate above its steady state value in periods following the period of the money shock. The additional force providing persistence in the effects on the nominal interest rate of the money shock arises from persistent effects of the initial investment response on the firm's marginal transaction or leisure value of liquidity. In the TC model, for example, firms borrow to where the net nominal interest rate equals the marginal transaction value of additional real balances, $-T_2^F(t)$. Recall that the marginal transaction value of real balances to a firm in a period depends partly on investment in the period. Given a money shock, investment responds in the period of the shock which alters the marginal transaction value of liquidity to firms and, therefore, alters the nominal interest rate. Since the initial investment response also alters the subsequent path of capital accumulation, the marginal value of liquidity to firms and, consequently, the nominal interest rate in periods following even a transitory money shock are influenced. These nominal interest rate effects following the period of a money shock then subside over time as the initial investment response is eroded away by capital depreciation.

It is worth emphasizing that the TC and ST models have an avenue of persistence in the nominal interest rate effects of a money shock that is not present in the CIA model. In principle, if the TC or ST model under parameterizations other than what we use was to generate a dominant liquidity effect, leading to a decline in the nominal interest rate in the period of a positive money shock, the liquidity effect can persist through time. In contrast, as Christiano and Eichenbaum (1992) discuss, the CIA model analyzed here cannot generate persistent liquidity effects. Even if the liquidity effect of a positive money shock dominates the anticipated inflation effect in the period of the shock, the sluggish-portfolio CIA model generates only purely transitory liquidity effects. This is because households in periods following a monetary disturbance costlessly readjust their nominal portfolios.

Volatilities of Variables and Correlations with Output

In order to quantitatively evaluate the models along further dimensions, Table 4 provides measures of volatilities of variables and cross-correlations with output implied by the models. Corresponding moments for time series of variables drawn from U.S. data for the sample period 1959:1-1989:4 are also provided for comparison with the models' implications. For comparability, moments are calculated for actual and simulated time series that have been Hodrick-Prescott filtered. The first row of numbers for each variable listed in Table 4 gives moments for U.S. data. The moments calculated for the models are averages from 100 simulations using baseline parameter values. Actual standard deviations of variables are reported, except for consumption, employment, and investment for which we report standard deviations relative to the standard deviation of output.

Begin by considering volatilities of variables implied by the models and in actual U.S. data as measured by standard deviations. There are five noteworthy features of the volatilities reported in Table 4. First, note that in the CIA model, adding the Lucas-Fuerst sluggish-portfolio assumption substantially increases the relative (to output) consumption volatility compared to the full-information CIA model and U.S. data. This increase in consumption volatility arises in part from the binding cash constraint on the household shopper and an inability in the sluggish-portfolio setting of deposits and cash balances of the shopper to respond contemporaneously to a shock. When the price level changes in response to a shock, consumption changes equiproportionately and in the opposite direction exposing consumption to extra variability relative to the full-information setting.

In contrast, in the sluggish-portfolio variants of the TC and ST models, when the price level changes in response to a shock, consumption need not change by as much as in the CIA model. As real balances of shoppers fall given a price level increase, for example, shoppers in the TC (ST) model will respond by changing consumption, but also will allow for transaction cost (shopping time) adjustments. These latter responses attenuate the effect on consumption variability of adding the sluggish-portfolio assumption to the TC and ST models relative to the effect for the CIA model. In addition, in the TC and ST models shoppers have greater flexibility than in the CIA model in their contemporaneous consumption responses to shocks given that current wages in the TC and ST models can be used to finance current consumption expenditures. As Table 4 reveals, adding the sluggish-portfolio assumption to the TC and ST models increases consumption volatility relative to full-information variants of the models, but the increases are far less than for the CIA model. Consumption volatilities implied by the sluggish-portfolio TC and ST models are also much closer to the U.S. data counterpart than the volatility implied by the sluggish-portfolio CIA model.

The second feature of Table 4 to note is along the dimension of employment volatility. The CIA and TC models imply far less employment variability than found in U.S. data. An indivisible labor assumption used, for example, in Hansen (1985), or non-time-separable utility as considered in Kydland (1989) could presumably be used in these models to increase the implied employment volatility. The implications of the full-information and sluggish-portfolio ST models are rather striking. Employment volatility implied by either variant of the ST model is higher than employment volatility in the data.

Investment variability relative to the variability of output is low for each model relative to our measure for U.S. data. It is interesting to note, however, that Christiano (1991) reports a value of 2.590 as a measure of the standard deviation of investment relative to the standard deviation of output. The relative volatilities of investment implied by the models in this paper are fairly close to Christiano's measure. Christiano's investment data derive from per capita capital stock data given as the sum of the stock of consumer durables, producers structures and equipment, government nonresidential capital, and government and private residential capital. The capital stock data from which the relative investment volatility reported in Table 4 is derived exclude consumer durables and government capital and are therefore much less inclusive than Christiano's data. Evidently, the inclusion of consumer durables and government capital leads to a smoother investment series.

Overall, with the exception of employment variability in the ST model, the flexibility for shoppers and firms in the TC and ST models allowed by marginal transaction-cost and shopping-time adjustments to shocks leads to altenuated responses in real variables relative to the CIA model. Real variables are exposed to less variability in the TC and ST models than in the CIA model. The price level acts as much more of a shock absorber in the TC and ST models than in the CIA model exposing the price level in the former models to a great deal of variability. Price level volatility implied by all three monetary models is, in fact, higher than price volatility found in the data.

Now notice volatilities of nominal interest rates implied by the models. While the CIA models perform reasonably well in accounting for interest rate variability, the TC and ST models imply volatilities that are well below the U.S. data measure. The low interest rate volatilities implied by the TC and ST models are due to small marginal transaction-cost and shopping-time responses to shocks generated from our baseline parameterizations of the models.

In addition to volatilities of variables, Table 4 reports dynamic correlations of real output with select variables. The contemporaneous correlations of variables with real output in the models are generally above those found in the U.S. data. There are three notable features of the cross-correlations of variables with output reported in Table 4. First, the detrended price level is negatively correlated with output in the data and for all models. The negative price-output correlation in U.S. data is also reported in Kydland and Prescott (1990), and Christiano (1991). The negative price-output correlations in the models reflect that the importance of monetary shocks for the dynamics is small relative to the importance of technology shocks.[7]

Second, in the data and in all models except the sluggish-portfolio CIA model, the strongest cross-correlation of investment with output is the contemporaneous correlation. In the sluggish-portfolio CIA model, investment lags output by one period and the contemporaneous correlation between investment and output is well below that in the data. These implications for the CIA model stem from the Lucas-Fuerst portfolio rigidity which serves to attenuate financing of increased investment in the period of a technology shock. While the portfolio rigidity also attenuates nominal flows to firms in the TC and ST models, firms have the ability to increase investment even if currency flows are inhibited by incurring additional transaction or shopping-time costs. For this reason, the sluggish-portfolio versions of the TC and ST models do not share the sluggish-portfolio CIA model's low contemporaneous investment-output correlation or the prediction that investment lags output.

Third, note that detrended output in the data has a stronger correlation with future values of the nominal interest rate than with the contemporaneous nominal rate. While sluggish-portfolio versions of the CIA and TC models also predict that the nominal rate lags the cycle, these models imply that output is negatively correlated with the nominal interest rate contemporaneously and at a one period lead and lag. The signs of cross-correlations between output and nominal interest rates in all models are sensitive, however, to the random technology and money shock draws in the simulations. We have found both positive and negative cross-correlations between output and the nominal rate predicted by the models using alternative averages (over 100 simulations) of moments implied by the models.

Overall, the dynamic properties of the three monetary models indicate that, especially for the implied nominal interest rate and real variable responses to monetary shocks, the models have shortcomings when evaluated quantitatively using empirically reasonable parameter values. Confirming results in Christiano (1991), the CIA model with and without a Lucas-Fuerst nominal portfolio rigidity fails to rationalize the conventional view that nominal interest rates respond negatively and employment and output positively to positive money shocks. A sluggish-portfolio version of the CIA model also predicts high consumption volatility and low employment volatility relative to what is found in the data. Allowing shoppers and firms greater flexibility than in the CIA model in facilitating changes in real transactions using given real cash balances, as in the TC and ST models, leads to settings in which the implied consumption volatility is quite close to that in the data. Employment volatility in the TC model, however, is low relative to its data counterpart while the ST model implies relatively high employment volatility. In addition, the TC and ST alternatives to the CIA model do not, given our parameterizations of the models, rationalize the view that positive monetary innovations coincide with nominal interest rate reductions and increases in employment and output.

6. CONCLUSION

We have constructed three monetary, general equilibrium models. The approaches used to characterize monetary transactions services are: a cash-in-advance (CIA) approach, in which agents face cash constraints on goods purchases; a transaction-cost (TC) approach, in which agents sacrifice real resources to effect transactions; and a shopping-time (ST) approach, in which agents sacrifice leisure in transactions. The CIA approach is effectively a special case of the TC or ST model. In the CIA model, agents are required to accumulate real balances in advance of trading that are at least just sufficient for real transactions. If a cash-in-advance constraint is nonbinding, then there is no cost, in terms of goods or leisure, of increasing the volume of transactions with a given real money balance. If the constraint binds, however, there is an infinite marginal cost of increasing transactions. Relative to the CIA model, the TC and ST alternatives allow for curvature in the marginal transaction cost, either in terms of leisure or goods, of increasing the volume of real transactions to be executed with a given real money balance. The CIA model, consequently, can be viewed as an extreme parameterization of the TC and ST alternatives.

The three monetary models that we construct are used to examine their qualitative and quantitative implications for evolutions of nominal and real variables with a focus on implications for nominal interest rate and real-variable responses to monetary shocks. Contingent on our parameterizations of the models, neither of the three models, with or without the nominal-portfolio rigidity considered by Lucas (1990) and Fuerst (1992), imply interest rate and real-variable responses consistent with the conventional view that positive monetary innovations coincide with negative nominal interest rate innovations and positive employment and output responses.

Implications of the CIA model for interest rate and real responses to money shocks are likely to depend intimately on the form of the cash constraints used in the model. Implications of the TC and ST models similarly depend on the form of agents' transaction-cost or shopping-time functions. In light of recent results using a CIA approach by Christiano and Eichenbaum (1991, 1992), a promising avenue for future work would be to alter the cash constraints of the CIA model used here and the transaction-cost and shopping-time functions of our TC and ST models.

Christiano and Eichenbaum have modified the CIA model utilized here in directions which, under certain parameterizations, lead to negative interest rate and positive employment responses to monetary injections. Their modified model retains the Lucas-Fuerst portfolio rigidity analyzed here, but modifies firms' and shoppers' cash constraints. In particular, investment is eliminated from firms' cash-in-advance constraints thereby treating investment as a credit input and labor as a cash input, and current wage receipts are allowed to be used by shoppers in purchasing current consumption. Christiano and Eichenbaum report results where liquidity effects dominate anticipated inflation effects of money shocks so that the nominal interest rate falls in response to a money injection and employment rises. The nominal rate falls to induce firms to disproportionately absorb newly injected cash by borrowing from intermediaries. Employment rises as firms channel borrowed currency into increased labor demand and because current wages can be used to purchase current consumption thereby removing the negative anticipated inflation effect on current labor supply. The Christiano and Eichenbaum modifications of the CIA model used here involve altering the cash constraint on firms and shoppers, and can be incorporated also into the TC and ST models by modifying arguments of the firms' and shoppers' transaction-cost and shopping-time functions. It would be interesting to see whether liquidity effects dominate in equilibrium in versions of the TC and ST models modified along the lines of the Christiano and Eichenbaum CIA model modifications. It would also be useful to seriously address measurement issues surrounding parameterizations of transaction-cost and shopping-time functions used to characterize monetary transactions services.

REFERENCES

Burnside, C., Eichenbaum, M. and S. Rebelo (1990), "Labor Hoarding and the Business Cycle," Manuscript, Northwestern University.

Christiano, L.J. (1990), "Computational Algorithms for Solving Variants of Fuerst's Model," Research Department Working Paper 467, Federal Reserve Bank of Minneapolis.

Christiano, L.J. (1991), "Modelling the Liquidity Effect of a Money Shock," Quarterly Review, Federal Reserve Bank of Minneapolis, Winter 1991.

Christiano, L.J. and M. Eichenbaum (1991), "Liquidity Effects, Monetary Policy, and the Business Cycle," Manuscript, Northwestern University.

Christiano, L.J. and M. Eichenbaum (1992), "Liquidity Effects and the Monetary Transmission Mechanism," American Economic Review, 82, 346-353.

Den Hann, W.J. (1990), "The Optimal Inflation Path in a Sidrauski-Type Model with Uncertainty," Journal of Monetary Economics, 25, 389-409.

Fisher, I. (1930), The Theory of Interest, New York: Macmillan.

Fuerst, T.S. (1992), "Liquidity, Loanable Funds, and Real Activity," Journal of Monetary Economics, 29, 3-24.

Hansen, G.D. (1984), "Fluctuations in Total Hours Worked: A Study Using Efficiency Units," Manuscript, University of Minnesota.

Hansen, G.D. (1985), "Indivisible Labor and the Business Cycle," Journal of Monetary Economics, 16, 309-327.

Hansen, L.P. and K.J. Singleton (1983), "Stochastic Consumption, Risk Aversion, and the Temporal Behavior of Asset Returns," Journal of Political Economy, 91, 249-265.

Hodrick, R.J. and E.C. Prescott (1980), "Postwar Business Cycles: An Empirical Investigation," Manuscript, Carnegie-Mellon University.

Kydland, F.E. (1989), "The Role of Money in a Business Cycle Model," Discussion Paper 23, Institute for Empirical Macroeconomics, Federal Reserve Bank of Minneapolis of University of Minnesota.

Kydland, F.E. and E.C. Prescott (1990), "Business Cycles: Real Facts and a Monetary Myth," Quarterly Review, Federal Reserve Bank of Minneapolis, Spring 1990.

Lucas, R.E. (1990), "Liquidity and Interest Rates," Journal of Economic Theory, 50, 237-264.

Marshall, D. (1987), "Inflation and Asset Returns in a Monetary Economy with Transaction Costs," Manuscript, Northwestern University.

Sargent, T.J. (1973), "Interest Rates and Prices in the Long Run," Journal of Money, Credit, and Banking, 5, 385-449.

ENDNOTES

1. As long as the net loan rate is nonnegative in equilibrium, intermediaries will supply all their loanable cash to firms. This requirement will be satisfied throughout our analysis.

2. Typically such a constraint, along with the firm's loan-in-advance constraint which follows, is specified as a weak inequality allowing for the possibility that an agent may choose not to spend all cash. Since simulations are performed using parameter values for which agents drive their cash constraints to bind as equalities, we work with binding constraints in the model to ease exposition. We verify, in the simulations, that the cash constraints indeed bind.

3. There also exists an effective, or transaction-cost adjusted, expected nominal return in the model. To see this, note that the deposit return $(1+i_t^L)$ per unit deposit provides currency next period which the shopper can use. Accounting for transaction-cost effects on the shopper, a currency return of $(1+i_t^L)$ per unit of current deposits effectively increases the amount of currency that can be devoted to nominal consumption expenditure next period by an expected amount $(1+i_t^L)E_t(1-T_2^S(t+1))$. Next, note that a unit increase in current deposits implies that the shopper effectively has $1-T_2^S(t)$ fewer units of currency for current nominal expenditure. Expressing the transaction-cost adjusted expected return per unit deposit relative to the current nominal sacrifice of the shopper gives an effective nominal return which under full-information is

$$\tilde{R}_t^{FI,E} = \frac{(1+i_t^L)E_t(1-T_2^S(t+1))}{1-T_2^S(t)} = E_t(1-T_2^S(t+1)) \ .$$

The second equality follows from (35) which states $(1+i_t^L) = 1-T_2^S(t)$. This effective nominal return measures the expected future marginal transaction value of liquidity to the household. Notice from conditions (40) and (35) that we can alternatively express (40) as

$$E_t\left\{\frac{\mu_c(t)}{\beta\mu_c(t+1)}\frac{P_{t+1}(1+T_1^S(t+1))}{P_t(1+T_1^S(t))}\frac{1}{1-T_2^S(t+1)}\right\} = 1$$

which displays the relation in the full-information setting between the effective nominal interest rate and fundamentals. The fundamentals here consists of the marginal rate of substitution and anticipated gross inflation of the transaction-cost adjusted price of consumption.

4. There also exists an effective, or shopping-time adjusted, expected nominal return in the model which can be found following the development of the effective nominal return in the TC model in endnote 3.

5. We estimated the following money growth model: $x_t = (1-\rho_x)x + \rho_x x_{t-1} + \varepsilon_{x,t}$ using quarterly data on the monetary base adjusted for reserve requirement changes from Citicorp's <u>CITIBASE</u> data bank. Relative to the notation in the text, x_t is the growth rate of the per capita aggregate money stock, i.e. $x_t = \frac{X_t}{M_t^a}$, where $X_t = M_{t+1}^a - M_t^a$. The nonstochastic steady-state value of money growth is x. The results of our estimation are:

Period	Coefficients		St. Dev. of Shock
	$(1-\rho_x)x$	ρ_x	$\sigma_{\varepsilon,x}$
1959:1-1989:4	.0037	.7683	.0042
1959:1-1971:4	.0014	.8100	.0038
1972:1-1989:4	.0095	.4953	.0041

6. Data on per capita values of consumption, investment, the capital stock, and real output are from Citicorp's CITIBASE data bank. Per capita hours-worked data are based on Hansen's (1984) hours-worked series updated to 1989:4. The U.S. data presumably reflect some sort of allocation of goods and services, and perhaps time, expended in facilitating transactions to various components of the national product accounts. Thus, the U.S. data measures for the ratios of capital-to-output, leisure-to-hours worked, and consumption-to-output are likely to incorporate goods or time expended in transactions. The ratios for the models reported in Table 2 do not involve an allocation of transaction costs or shopping time of the shopper or firm to capital, consumption, or hours. Consequently, comparisons of ratios implied by the TC and ST models with sample averages of measures from U.S. data will be imprecise. However, the imprecision is likely to be very minor given that our parameterization of the TC (ST) model yields a small value for the steady state ratio of total transaction-costs-to-output (shopping-time-to-time endowment).

7. When the standard deviation of the unexpected portion of money growth, $\sigma_{\varepsilon,x}$, is set to zero, output volatilities implied by the models differ only trivially from those reported in Table 4. Monetary shocks have a relatively small influence on output volatilities in the models.

Appendix: The Solution Technique

Solving each variant of each model involves the following steps:

[1] Undertake stationary-inducing transformations of model variables. Variables in the model display growth in equilibrium stemming from sustained growth in the state of technology, Z_t, and in the money supply, X_t. Inducing stationarity is required for the solution method that we employ which involves approximations around a stationary equilibrium. To meet the stationarity requirement of the solution method, define the following transformed variables:

$$c_t = C_t\exp(-\mu t),\ k_{t+1} = K_{t+1}\exp(-\mu t),\ y_t = Y_t\exp(-\mu t),\ \bar{p}_t = P_t\exp(\mu t),\ \tilde{T}_t^S = T_t^S\exp(-\mu t),\ \tilde{T}_t^F = T_t^F\exp(-\mu t)\ ,$$

$$\bar{\kappa}_{t+1} = \kappa_{t+1}\exp(-\mu t)\ .$$

These transformations serve to remove the deterministic productivity trend. In addition, measure all nominal variables relative to the per capita aggregate money stock M_t^a :

$$m_t = \frac{M_t}{M_t^a},\ n_t = \frac{N_t}{M_t^a},\ p_t = \frac{\bar{p}_t}{M_t^a},\ x_t = \frac{X_t}{M_t^a},\ w_t = \frac{W_t}{M_t^a},\ \hat{\mathcal{L}}_t^d = \frac{\mathcal{L}_t^d}{M_t^a},\ \hat{\mathcal{L}}_t^s = \frac{\mathcal{L}_t^s}{M_t^a}\ .$$

These transformations serve to remove monetary growth.

Note that in the transaction-cost model, real transaction costs for the shopper and firm can be expressed in terms of transformed variables as, respectively, $\tilde{T}_t^S = T_t^S\exp(-\mu t) = T^S[c_t, \frac{m_t - n_t}{p_t}]$ and

$\tilde{T}_t^F = T_t^F\exp(-\mu t) = T^F\left[\frac{w_t}{p_t}H_t + k_{t+1} - (1-\delta^*)k_t, \frac{\hat{\mathcal{L}}_t^d}{p_t}\right]$ by virtue of the unit homogeneity of the functions T^S and T^F, where $(1-\delta^*) = (1-\delta)\exp(-\mu)$. In the shopping-time model, since g^S and g^F are each homogeneous of degree zero, total household leisure can be expressed in terms of transformed variables as $L_t = 1 - \tilde{H}_t - g^S\left[c_t, \frac{m_t - n_t}{p_t}\right]$

$$- g^F\left[\frac{w_t}{p_t}H_t + k_{t+1} - (1-\delta^*)k_t, \frac{\hat{\mathcal{L}}_t^d}{p_t}\right]\ .$$

[2] Formulate the discounted dynamic programming problem for the transformed (stationary) economy. In the full-information variant of the cash-in-advance model, for example, the problem can be expressed in terms of the value function as:

$$V(m_t, k_t, \bar{\kappa}_t, s_t) = \max_{k_{t+1}, \tilde{H}_t, \hat{\mathcal{L}}_t^d, n_t} \left\{ u(c_t, 1 - \tilde{H}_t) + \beta^* \int V(m_{t+1}, k_{t+1}, \bar{\kappa}_{t+1}, s_{t+1}) \Phi(s_{t+1} \mid s_t) \right\}$$

subject to the relevant constraints stated in terms of transformed variables, where $m_t^a = \frac{M_t^a}{M_t^a} = 1$ has been subsumed in the value function V and $\beta^* = \beta \exp[(1-\gamma)\Psi\mu]$.

[3] Combine first-order and envelope conditions for the household's problem stated in terms of stationary variables and impose equilibrium and aggregate consistency conditions for the stationary economy. This gives policy functions satisfying equilibrium, aggregate consistency, and household optimality conditions.

[4] Since closed-form solutions for endogenous model variables cannot be obtained, solve for the model's endogenous variables using the approximation technique of Christiano (1990) which involves the following:

- Linearize the optimality conditions, with equilibrium and aggregate consistency conditions imposed, by taking a first-order Taylor-series expansion about the model's nonstochastic steady-state.

- Conjecture decision rules for choice variables that are linear in the state variables (including shocks).

- Determine coefficient values for the conjectured linear decision rules using a method of undetermined coefficients.

For more details of the solution technique, see Christiano (1990, 1991).

TABLE 1

BASELINE PARAMETER VALUES

PARAMETER	VALUE	DESCRIPTION
β	.9926	subjective discount factor
Ψ	-1.0	utility function curvature
γ	.76	leisure share in utility
α	.35	capital share in production
δ	.02	depreciation
θ	1.0	productivity shock scale factor
μ	.0042	average growth rate of technology
x	.0158	average growth rate of money
ρ_θ	.9857	parameters of shock processes
$\sigma_{\varepsilon,\theta}$	.01369	
ρ_x	.7683	
$\sigma_{\varepsilon,x}$	.00416	
$d^S = d^F$	1.89	parameters of transaction-cost functions
$A^S = A^F$	.003	
$q^S = q^F$	1.89	parameters of shopping-time functions
$Q^S = Q^F$	.003	

TABLE 2

STEADY STATE IMPLICATIONS OF MODELS

ROW	MODEL	PARAMETERS		STEADY STATE VALUES (for FI and SP variants of each model)						
		Transaction Cost Parameters	Shopping Time Parameters	Capital / Output	Leisure[a] / Hours Worked	Consumption / Output	Annualized Nominal Interest Rate	Trans. Cost / Output	Shopping Time / Time Endowment	Marginal TC or ST Effects[b]
1	CIA	-------	-------	10.228	3.805	.742	9.736	-------	-------	-------
2	TC	$d^S = d^F = 1.89$ $A^S = A^F = .003$ (baseline)	-------	9.212	3.617	.719	9.736	.014	-------	$T_2^S = T_2^F = -.0243$
3	ST	-------	$q^S = q^F = 1.89$ $Q^S = Q^F = .003$ (baseline)	9.002	3.946	.739	9.736	-------	.011	$g_2^S = g_2^F = -.0044$
4	TC	$d^S = d^F = 1.89$ $A^S = A^F = .030$	-------	8.877	3.777	.698	9.736	.045	-------	$T_2^S = T_2^F = -.0243$
5	TC	$d^S = d^F = 1.39$ $A^S = A^F = .003$	-------	9.268	3.576	.720	9.736	.011	-------	$T_2^S = T_2^F = -.0243$
6	ST	-------	$q^S = q^F = 1.89$ $Q^S = Q^F = .030$	8.313	4.868	.759	9.736	-------	.031	$g_2^S = g_2^F = -.0050$
7	ST	-------	$q^S = q^F = 1.39$ $Q^S = Q^F = .003$	9.298	3.618	.731	9.736	-------	.013	$g_2^S = g_2^F = -.0042$
Sample Average for U.S. Data[c] 1959:1-1989:4				10.300	3.690	.621	6.31			

Rows (1)-(3) show results using baseline parameter values in Table 1.
Rows (4) and (5) show effects of altering the scale and power parameters in the transaction-cost functions of the shopper and firm in the TC model.
Rows (6) and (7) show effects of altering the scale and power parameters in the shopping-time functions of the shopper and firm in the ST model.

a. In steady state leisure is 1-H in the CIA and TC models, and is $1\text{-}H\text{-}g^S(\cdot\cdot)\text{-}g^F(\cdot\cdot)$ in the ST model.

b. For the TC (ST) model, in steady state real balances are allocated by the household to the shopper and firm so that the marginal value of real balances to the shopper equals the value to the firm, i.e. $T_2^F = T_2^S$ ($g_2^F = g_2^S$).

c. Data sources are provided in endnote 6.

TABLE 3

CONTEMPORANEOUS IMPACT OF A MONEY GROWTH SHOCK

Percentage-point change in the nominal interest rate, R_x, and percentage change in employment, H_x, in the period of a one-percentage point increase in money growth.[a]

ROW	MODEL	MONEY GROWTH PERSISTENCE[b]	MODEL VARIANT			
			Full-Information		Sluggish-Portfolio	
		ρ_x	R_x	H_x	R_x	H_x
1	CIA	0	.000	.000	-.087	-.155
2	CIA	.4953	.226	-.677	.125	-.815
3	CIA	.7683	.489	-1.455	.359	-1.543
4	TC	0	.000	.000	-.015	.018
5	TC	.4953	.041	-.029	.003	-.010
6	TC	.7683	.126	-.085	.039	-.066
7	ST	0	.000	.000	-.032	.042
8	ST	.4953	.038	-.089	.004	-.045
9	ST	.7683	.119	-.275	.081	-.225

a. $R_x = \frac{dR}{d\varepsilon_x}$ and $H_x = \frac{d\log H}{d\varepsilon_x}$ are evaluated in nonstochastic steady state.

b. $\rho_x = .4953$ along with $\sigma_{\varepsilon,x} = .0041$ and $x = .0187$ correspond to a U.S. data sample for 1972:1-1989:4; $\rho_x = .7683$ along with $\sigma_{\varepsilon,x} = .0042$ and $x = .0158$ correspond to a sample period 1959:1-1989:4. For data source, see endnote 5.

TABLE 4

MOMENTS FOR SELECT VARIABLES[a]

VARIABLE, v_t	MODEL[b]	STANDARD DEVIATION	CROSS CORRELATIONS BETWEEN OUTPUT AND OTHER VARIABLES: CORRELATION OF REAL OUTPUT WITH				
			v_{t-4}	v_{t-1}	v_t	v_{t+1}	v_{t+4}
Output (Y)	DATA	.0174	.182	.857	1.00	.857	.182
	CIA-FI	.0158	.081	.682	1.00	.682	.081
	CIA-SP	.0157	.080	.675	1.00	.682	.081
	TC-FI	.0150	.098	.703	1.00	.703	.098
	TC-SP	.0150	.098	.706	1.00	.706	.098
	ST-FI	.0139	.089	.691	1.00	.691	.089
	ST-SP	.0139	.090	.697	1.00	.697	.090
Consumption (C)	DATA	.505	.412	.730	.730	.609	.091
	CIA-FI	.434	-.033	.586	.926	.712	.232
	CIA-SP	.854	.068	.516	.756	.126	.006
	TC-FI	.535	.021	.661	.986	.742	.199
	TC-SP	.538	.023	.666	.986	.735	.196
	ST-FI	.544	.016	.651	.987	.728	.183
	ST-SP	.550	.022	.661	.989	.712	.177
Employment (H)	DATA	.943	.023	.564	.721	.704	.378
	CIA-FI	.699	.106	.579	.835	.517	-.019
	CIA-SP	.606	.100	.570	.836	.525	-.013
	TC-FI	.388	.175	.725	.984	.643	-.010
	TC-SP	.386	.174	.728	.984	.654	-.007
	ST-FI	1.536	.126	.705	.996	.664	.037
	ST-SP	1.537	.127	.710	.996	.670	.038

Investment (I)	DATA	3.215	.101	.798	.901	.817	.262
	CIA-FI	2.734	.127	.691	.985	.636	.012
	CIA-SP	2.558	.023	.395	.612	.875	.113
	TC-FI	2.329	.143	.718	.991	.669	.035
	TC-SP	2.321	.143	.720	.992	.678	.037
	ST-FI	2.343	.135	.706	.993	.655	.024
	ST-SP	2.321	.135	.712	.994	.677	.029
Price Level (P)	DATA	.0145	-.670	-.658	-.535	-.374	.153
	CIA-FI	.0210	.043	-.467	-.775	-.576	-.153
	CIA-SP	.0209	.044	-.456	-.771	-.570	-.152
	TC-FI	.0329	-.027	-.502	-.744	-.542	-.126
	TC-SP	.0329	-.029	-.505	-.743	-.540	-.124
	ST-FI	.0256	-.001	-.382	-.595	-.424	-.103
	ST-SP	.0257	-.003	-.387	-.591	-.542	-.126
Nominal Interest Rate (R)	DATA	.0032	-.057	.056	.302	.429	.453
	CIA-FI	.0042	.043	-.028	-.070	-.036	.013
	CIA-SP	.0039	.045	-.023	-.079	-.085	-.008
	TC-FI	.0007	.009	-.338	-.513	-.393	-.126
	TC-SP	.0007	.050	-.283	-.457	-.533	-.192
	ST-FI	.0010	.001	.054	.076	.051	.003
	ST-SP	.0010	.012	.059	.082	.045	-.001

a. Results in this table are based on data that have been logged and then Hodrick-Prescott filtered. Actual percent standard deviations are reported for all variables except C,I,H for which standard deviations have been divided by the standard deviation of real output. Nominal interest rates are expressed at a gross quarterly rate prior to logging and filtering. Data for variables, except employment, are from the CITIBASE tape defined as: Y = GNP82, C = CGS82 + GCN82, I = GIF82, P = PUNEW, R = FYGN3. Employment data are Hansen's (1984) series updated to 1989:4.

b. Model mnemonics are: CIA = cash-in-advance, TC = transaction cost, ST = shopping time, FI = full information, SP = sluggish portfolio.

FIGURE 1: RESPONSES TO A ONE-STANDARD-DEVIATION SHOCK TO MONEY GROWTH IN PERIOD 10.

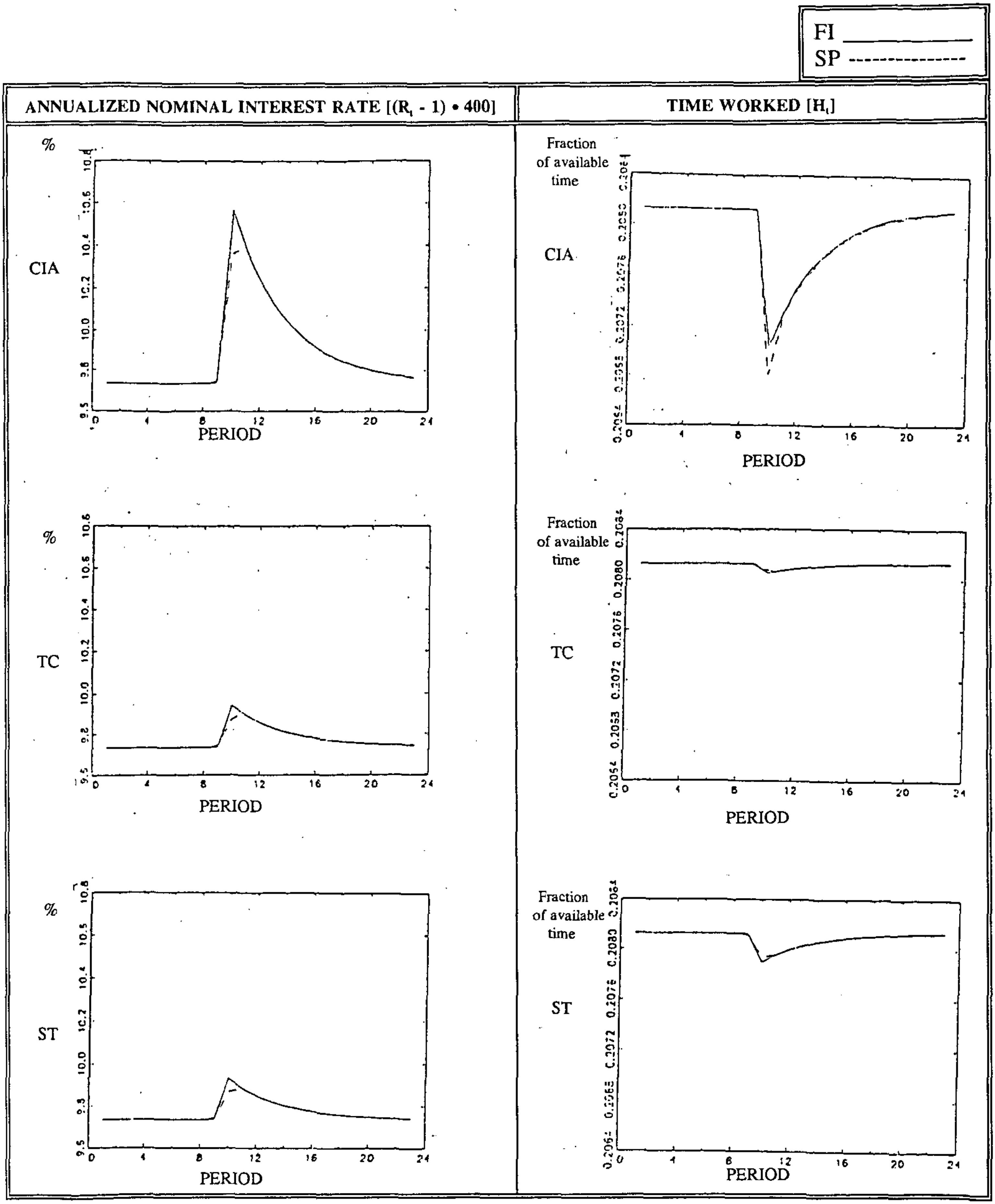
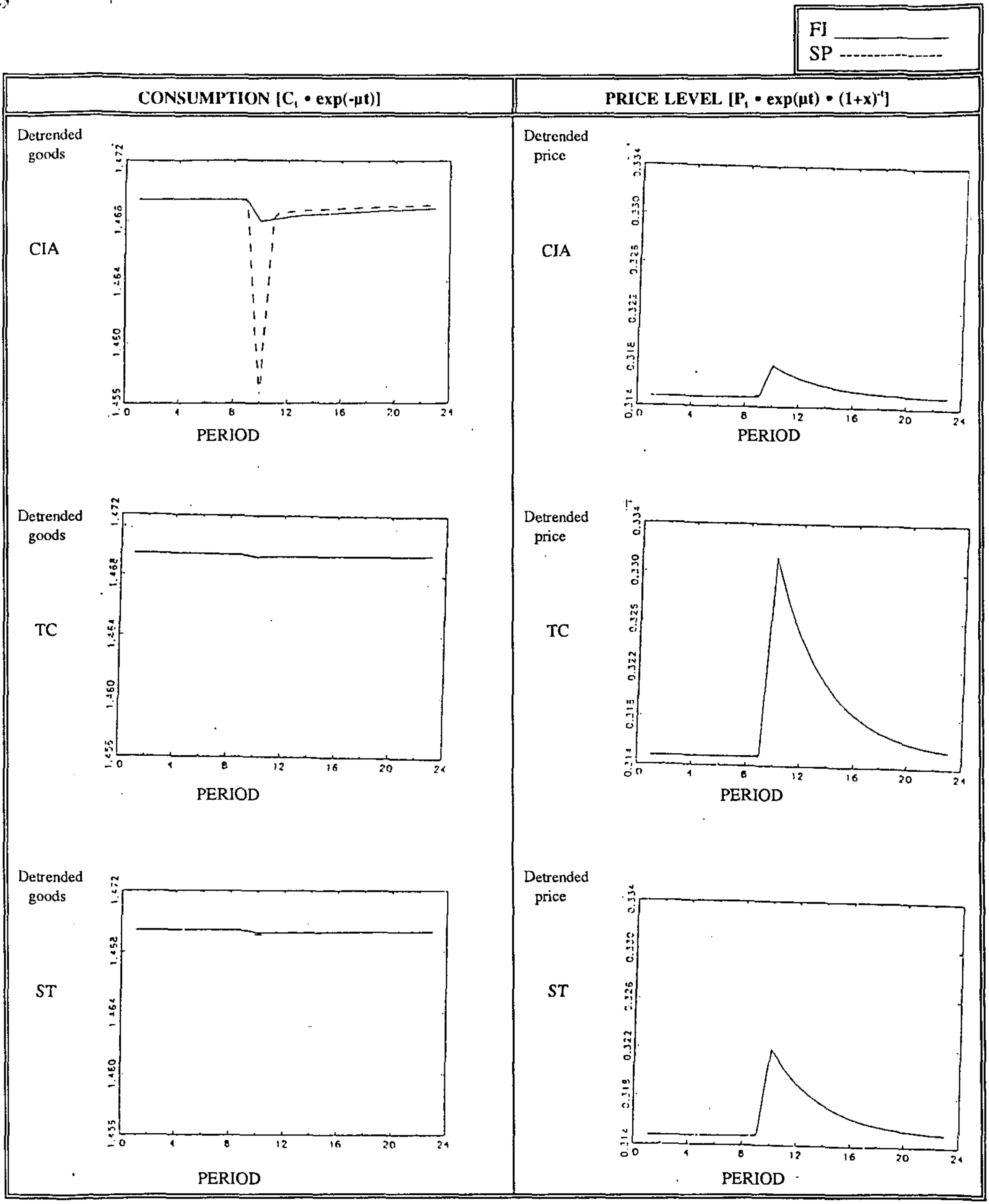
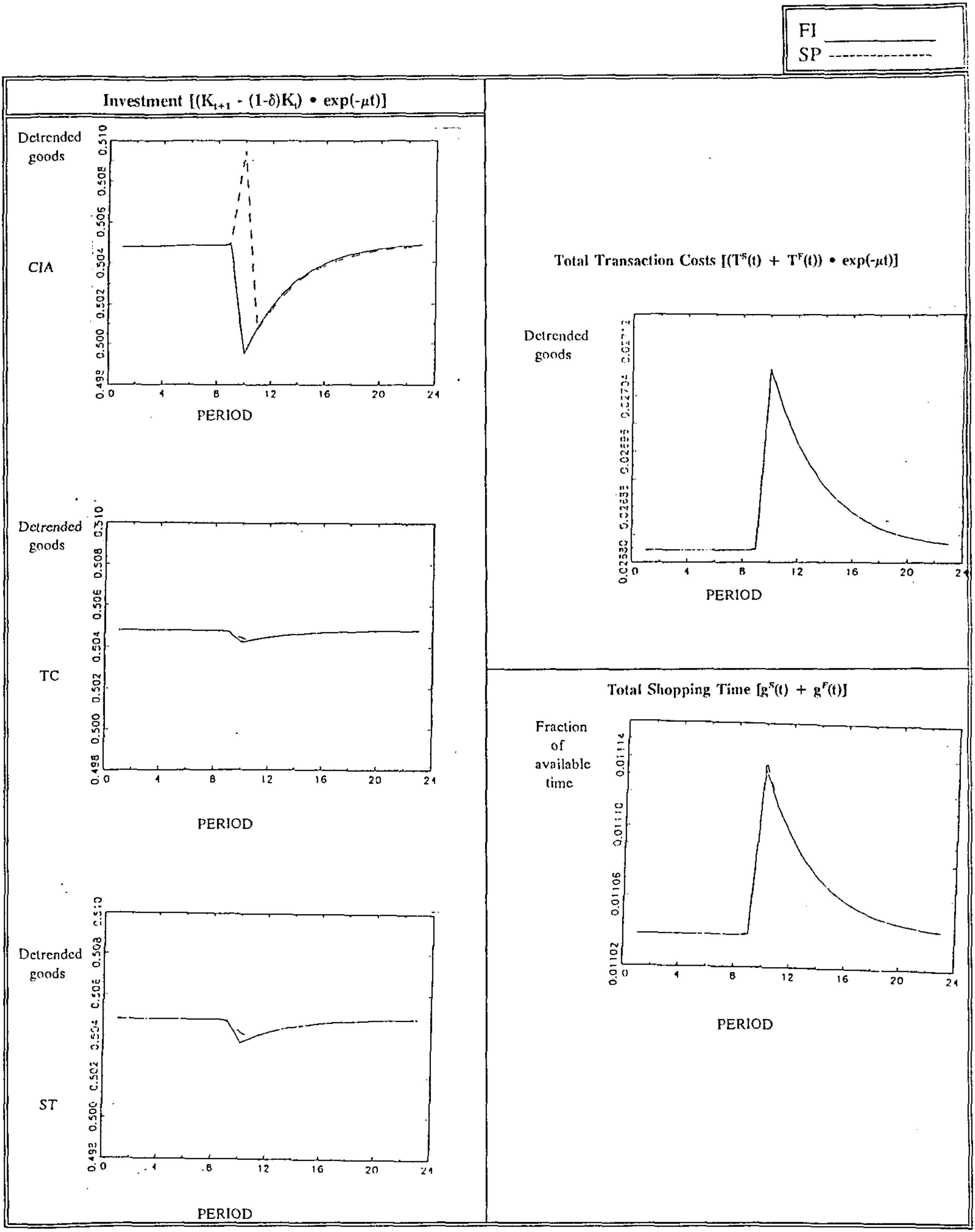


FIG. 1:



FI
SP
CONSUMPTION [$C_t \bullet \exp(-\mu t)$]
PRICE LEVEL [$P_t \bullet \exp(\mu t) \bullet (1+x)^{-t}$]
Detrended goods
CIA
TC
ST
Detrended price
PERIOD
0 4 8 12 16 20 24

FIG. 1:


FI
SP
Investment [(K_{t+1} - (1-δ)K_t) • exp(-μt)]
Detrended goods
CIA
0.498 0.500 0.502 0.504 0.506 0.508 0.510
0 4 8 12 16 20 24
PERIOD
Detrended goods
TC
0.498 0.500 0.502 0.504 0.506 0.508 0.510
0 4 8 12 16 20 24
PERIOD
Detrended goods
ST
0 4 8 12 16 20 24
PERIOD
Total Transaction Costs [(T^S(t) + T^F(t)) • exp(-μt)]
Detrended goods
0 4 8 12 16 20 24
PERIOD
Total Shopping Time [g^S(t) + g^F(t)]
Fraction of available time
0.01102 0.01106 0.01110 0.01114
0 4 8 12 16 20 24
PERIOD