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The Real Business Cycle: Intermediate Inputs and Sectoral Comovement

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ABSTRACT

We describe the postwar U.S. business cycle for the durable and nondurable goods producing sector. The business cycle is characterized by positive comovement of output, employment, and investment across the two sectors. We develop a two sector growth model to explain the observed pattern of comovements, and suggest that intermediate inputs produced by the nondurable goods sector for the durable goods sector play a crucial role.

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1 Introduction

A defining characteristic of the business cycle is the comovement of important macroeconomic time series. Dynamic general equilibrium models have been successful at explaining the comovement among different aggregate variables, but they have been less successful at explaining the comovement among variables across industrial sectors. We suggest that the comovement across sectors stems from the fact that the economy is strongly interrelated. That is, an industry's output is often used as an intermediate good in other industries and payments for intermediate goods are a substantial fraction of the total payments made for inputs.

In our study we focus on the comovement of employment, output, and investment between the durable and nondurable goods producing sector of the U.S. economy. We find that the properties of a two-sector stochastic growth model with intermediate goods are consistent with the observed comovement between the two sectors. In addition we find that this framework helps us to understand the observation that household investment is procyclical and is leading business investment over the cycle.

The properties of the U.S. business cycle have been documented as early as Mitchell (1913), Burns and Mitchell (1947), reviewed by Lucas (1977), and most recently by Murphy et al (1989) and Kydland and Prescott (1990). We expand on these facts by reporting some of the regularities found within and between the durable and nondurable goods sectors during the post war period. As it has been documented in other studies, we find that output as measured by real value-added, investment, and employment in the durables sector are more volatile than the same variables in the nondurables sector. In addition, we find that real value-added, investment, and employment in the two sectors are strongly procyclical and move together, and household investment leads business investment.

Current studies of the business cycle at a disaggregated level usually assume that a sector uses only primary inputs: capital, labor and land. In this case sectoral output is measured by real value-added. Input-output tables, however, show that in any sector payments for intermediate goods are a substantial fraction of total payments made to inputs. These intermediate goods are produced and used up in production during the accounting period. Unlike labor, intermediate goods are reproducible, and unlike capital, they depreciate completely within the accounting period. In Table 1, we have aggregated the 1982 Input-Output table of the U.S. into two sectors, the durable and the nondurable goods producing sectors. Nondurable goods which are used as intermediate inputs in the durable goods producing sector make up 26 percent of total payments to inputs in this sector. Conversely, payments for durable goods which are used as intermediate inputs in the nondurable goods producing sector are about four percent of total payments made to inputs in that sector.[1]

A complete study of the role that intermediate inputs play in the business cycle requires observations on gross output and all inputs, including intermediate inputs, for each sector. There is no such complete and consistent data set available. National Income Accounts at the sectoral level only contain information on value-added and primary factors of production, they do not contain information on gross output and all inputs. Jorgenson et al (1987), estimate sectoral output and total sectoral use of intermediate inputs for industries at the two digit SIC level of aggregation. For their data set a sector's output and use of intermediate inputs is in general procyclical, Hornstein and Praschnik (1994). At the current stage of our research we concentrate on the implications of intermediate inputs for the comovement of real value-added and primary factors of production in the durable and nondurable goods producing sector.

In our economy there are two sectors, a durable and a nondurable goods producing sector. Output from the nondurable goods sector is used as an intermediate input in the durable goods sector and as a consumption good. The durable goods sector produces capital goods only, which are used in both production sectors and by the household sector. Capital is accumulated with a time-to-build technology, Kydland and Prescott (1982), and installed capital is not mobile across sectors. Labor is mobile across the two sectors. The initial impact of a positive production shock to either sector is an increase of employment in the nondurable goods sector, a decrease of employment in the durable goods sector, and total employment increases because the nondurable goods sector is larger. Thus, as in other sectoral growth models, employment in the two sectors moves in opposite directions. In any case the production of intermediate inputs increases, which implies that in the

[1]The separation of the economy into durable and nondurable goods producing sectors is not perfect. Since durable goods are capital goods they should not appear as intermediate goods in the nondurable goods sector, also since nondurable goods are perishable they should not appear as final investment demand.

following period the marginal product of labor in the durable goods sector increases. This increases employment in the durable goods sector. Hence, after the initial period, employment in the two sectors moves together. We also demonstrate that the presence of intermediated inputs is crucial for the model's prediction that employment across sectors is positively correlated. If there are no intermediate inputs in the production of durable goods then the correlation of employment across sectors is negative.

The importance of intermediate inputs for the business cycle has been implicitly emphasized before by Long and Plosser (1983). They provide a disaggregated model of the U.S. economy where outputs of one sector are inputs to another sector. In their work Long and Plosser do not distinguish between capital and intermediate goods, and the equilibrium of their economy is characterized by constant employment in each sector over the business cycle. The observation that real business cycle models have a problem explaining the positive comovement of employment and output across sectors has been made before in the context of closed and open economies. For example, Benhabib et al (1992) reinterpret the standard growth model as a two sector economy, and observe that employment for investment purposes is procyclical and for consumption purposes it is countercyclical. They suggest that home production is what makes employment for both purposes procyclical. However, the distinction between the two types of employment in their economy is artificial. In the context of a two-country open economy with regional production of a homogeneous good Backus et al (1992) find that output and employment move in opposite directions in the two countries. Costello and Praschnik (1993) in a two-country open economy model with intermediate and final goods find that aggregate employment and output move together across countries, but within countries only sectoral output is positively correlated while sectoral employment is negatively correlated.

Murphy et al (1989) also emphasize the observed comovement of output and employment across industrial sectors. They argue that a multi-sector stochastic growth model with sectoral productivity shocks is consistent with this observation, if labor is sector specific and credit markets are imperfect. Furthermore, if such a model with intermediate inputs is to be consistent with their observation that the relative price of nondurables to durables is procyclical, then productivity shocks in the durable goods sector have to dominate productivity shocks in the nondurable goods sector. We show that measured total factor productivity in the durable goods sector is indeed more variable than in the nondurable goods sector. This implies that for our calibrated model of the U.S. economy the relative price of nondurable goods is procyclical.

The paper is organized as follows. In section two, we review the business cycle properties of the U.S. economy with particular emphasis on durable and nondurable goods production. In section three, we describe our artificial economy where nondurable goods serve as intermediate inputs in the production of durable goods. In section four, we calibrate the economy, that is we choose parameter values for the economy which are consistent with the long run properties of the U.S. economy. In section five we analyze the dynamic properties of the model economy. We summarize our results and offer conclusions in section six.

2 Durable and Nondurable Goods in the U.S. Business Cycle

We now describe the post-war U.S. business cycle with an emphasis on the behavior of the durable and nondurable goods producing sectors. We use annual data for the time period from 1948 to 1987. A variable's business cycle frequency is defined as the deviation from its long run trend as measured by the Hodrick-Prescott (1982) filter. A complete description of the data set is provided in the appendix. We find that value-added output and employment in the durable goods sector is more volatile than in the nondurable goods sector. With respect to the comovement of economic variables we find that value-added, employment and investment in both sectors are positively correlated with aggregate GNP and each other.

In the first column of Tables 2 and 3 we report the volatilities and contemporaneous correlations with aggregate GNP for the detrended time series. We will compare these statistics with the corresponding statistics generated by the economic model we describe in the next section. Note that we have excluded rental payments for owner and tenant occupied housing from aggregate GNP. These payments make up the major part of value-added originating in the real estate sector. We exclude these payments because we do not model the market provision of services from residential housing, rather all housing capital is in possession of the household.

Table 2 shows that the durable goods sector is much more volatile than the nondurable goods sector. The standard deviation of percentage deviations from trend for value-added and the capital stock in the durable goods sector is at least twice as big as the standard deviation for the corresponding variable in the nondurable goods sector, and employment in the durable goods sector is three times as volatile as in the nondurable goods sector. Value-added, capital stock, and employment volatility in the aggregate economy are a weighted average of their sectoral components. As was observed before, aggregate employment is almost as volatile as aggregate output, and the aggregate capital stock is less volatile than employment.

Our empirical and theoretical analysis abstracts from issues of growth. For the production side of the economy we therefore investigate the properties of detrended time series for inputs and outputs. For the components of final demand we investigate the behavior of the components' shares in aggregate GNP. In addition we also detrend these shares using the Hodrick-Prescott filter. This procedure is appealing since Hansen (1989) has shown that the business cycle properties of economies with deterministic growth are independent of the growth rate if the output-input ratios, factor income shares, and final demand shares are the same in these economies. We now describe the cyclical properties of final demand.

Total investment, that is business investment and household investment, is about twice as volatile as consumption. By household consumption we mean expenditures on nondurable goods and services, whereas expenditures for consumer durables are included in household investment, together with residential investment. For business investment we also observe that investment in the durable goods sector is more than twice as volatile as investment in the nondurable goods sector. Household investment is about one and a half times more volatile than aggregate investment, and about three times as volatile as household consumption of nondurables.

Most variables appear to be strongly procyclical, see Figures 1, 2 and 3. The contemporaneous correlation with aggregate GNP is at least 0.9 for value-added and employment in both sectors. The expenditure shares are not strongly correlated with aggregate GNP, with the exception of the consumption share which is strongly countercyclical. For the investment shares, household investment tends to lead and business investment tends to lag aggregate GNP.[2]

In Table 4, we report the comovement of employment, value-added, and investment across sectors. We see that employment, value-added, and investment in the durable goods sector is positively correlated with the same variable in the nondurable goods sector. Similar to contemporaneous correlations with aggregate GNP, employment and value-added are more strongly correlated across sectors, than are the investment expenditure shares. We observe that household investment is leading business investment in the cycle.

In the context of a disaggregated analysis of the business cycle it has been pointed out before, e.g. Greenwood et al (1992) or Murphy et al (1989), that the relative price of nondurable to durable goods is procyclical. Our data set does not include price series for durable and nondurable goods. As a substitute we investigate the behavior of the ratio of the nondurable value-added price index to the durable value-added price index. We find that this relative price is weakly countercyclical, see Figure 4.[3]

Real wages in the two sectors, that is nominal sectoral wages deflated by the aggregate GNP price index, are not perfectly correlated, but they move together, see Figure 5. This is of interest to us since we will model labor as perfectly mobile across the two sectors. Labor productivity in both sectors is procyclical and leading the cycle.[4] We now describe a model with intermediate inputs where sectoral productivity shocks can account for the observed volatilities and comovement that we have documented for the U.S. business cycle.

[2]In Table 3 we report only contemporaneous correlations with aggregate GNP. The correlation of aggregate GNP with lagged household investment is 0.33. The correlations of business investment, respectively investment in the durable and nondurable goods sectors with lagged aggregate GNP are 0.42, respectively 0.65 and 0.22.

[3]For a subsample of our data set, the period from 1964 to 1987, this relative price is indeed procyclical. In other work, Hornstein and Praschnik (1994) we investigate the behavior of sectoral intermediate input prices in the Jorgenson et al (1987) data set. We do not find a systematic pattern for the cyclicality of a sector's input price deflated by the sector's output price. In about half of all sectors this relative price is procyclical and in the other half it is countercyclical.

[4]The highest correlation between aggregate GNP and labor productivity, aggregate or sectoral, is the one for aggregate GNP with labor productivity lagged one year. These correlations are 0.78 for aggregate, 0.49 for durable goods, and 0.71 for nondurable goods sector labor productivity.

3 The Economy

Our economy is a two sector stochastic neoclassical growth model, where time is discrete and the horizon is infinite. There is a nondurable goods producing sector and a durable goods producing sector. The nondurable good can be used for consumption or as an intermediate input in the production of the durable good. As an intermediate input the nondurable good is put into inventory and becomes available for production in the next period. The durable good is capital and can be used for production in the two sectors or as a household durable good. Accumulation of capital takes time, which is characterized by a time-to-build investment technology. Total factor productivity in each sector follows a bivariate Markov process. There is also a government which imposes proportional taxes on capital and labor income. These taxes are rebated in a lump-sum fashion.

3.1 The Environment

There is an infinitely lived representative agent which maximizes expected life time utility

$$E\left[\sum_{t=0}^{\infty}\beta^{t}U\left(c_t,k_{ht},h_t\right)\right]. \tag{1}$$

where the period utility is a concave and increasing function U of the nondurable consumption good c, the services from the stock of the durable good k_h, and leisure h. The agent is endowed with T units of leisure.

There are two production sectors, the nondurable ($i = n$) and the durable goods ($i = d$) producing sectors. Both sectors use capital k_i and labor l_i as input. The durable goods sector also uses the nondurable good as an intermediate input m. Each sector's production technology is constant returns to scale in all inputs, and depends on some productivity parameter z_i,

$$\begin{aligned} q_{nt} &= F_n\left(k_{nt},l_{nt},z_{nt}\right), \\ q_{dt} &= F_d\left[G\left(k_{dt},l_{dt},z_{dt}\right),m_t\right]. \end{aligned} \tag{2}$$

We will discuss the separability assumption with respect to inputs in the durable goods production in the following section. A nondurable good produced in the current period can be consumed or used as an input to the production of durable goods in the following period. The resource constraint for nondurable goods is

$$c_t + \tilde{m}_t \leq q_{nt}. \tag{3}$$

where $m_{t+1} = \tilde{m}_t$.

Capital goods are accumulated using a technology with the time-to-build feature, cf. Kydland and Prescott (1982). An investment project which is used to produce a type $i = n, d, h$ capital good needs S_i periods to mature. Let s_{ijt} be the number of type i investment projects which are j periods from maturity in period t, that is these projects were started in period $t - S_i + j$ and will mature in period $t + j$. The different capital stocks then evolve according to the following laws of motion,

$$\begin{aligned} k_{i,t+1} &= (1-\delta_i)\, k_{it} + s_{i1t}, \\ s_{i,j-1,t+1} &= s_{i,j,t} \end{aligned} \tag{4}$$

where $j = 2, \ldots, S_i$ and $0 < \delta_i < 1$ is the depreciation rate of type i capital. An investment project of unit size which is j periods from maturity requires ϕ_{ij} units of the durable good to bring it into the next period. Investment is limited by the total production of durable goods,

$$\sum_{i=n,d,h} \sum_{j=1}^{S_i} \phi_{ij} s_{ijt} \leq q_{dt}. \tag{5}$$

Labor is perfectly mobile across the two production sectors, and its resource constraint is

$$l_{nt} + l_{dt} + h_t \leq T. \tag{6}$$

The natural log of productivity follows a bivariate AR(1) process with correlated innovations, $\ln \mathbf{z}_{t+1} = \Lambda \ln \mathbf{z}_t + \epsilon_{t+1}$,with $\mathbf{z} = (z_n, z_d)'$ and ϵ_t is iid normal with mean zero and covariance matrix Ω.

There is also a government which imposes proportional taxes on capital income net of depreciation originating in the business sector, and labor income. The tax rates are τ_k and τ_l. The tax revenue is rebated by a lump-sum payment Υ.

3.2 The Competitive Equilibrium

We now define a recursive competitive equilibrium along the lines of Stokey et al (1989). The representative household owns all inputs and sells them or rents their services to firms. Firms produce nondurable and durable goods and sell the goods to the household. All markets are competitive. For the equilibrium definition we will differentiate between aggregate and individual state and decision variables. Bold faced letters denote aggregate decision variables. Let the vectors of aggregate and individual state variables be

$$\begin{aligned} \mathbf{x} &= \left[(\mathbf{k}_i)_{i=n,d,h}, (\mathbf{s}_{ij})_{i=n,d,h;j=1,\dots,S_i-1}, \mathbf{m}\right] \text{ and } \mathbf{z} \\ x &= \left[(k_i)_{i=n,d,h}, (s_{ij})_{i=n,d,h;j=1,\dots,S_i-1}, m\right]. \end{aligned}$$

Let the vectors of aggregate and individual decision variables be

$$\begin{aligned} \mathbf{y} &= \left[(\mathbf{s}_{S_i i})_{i=n,d,h}, \tilde{\mathbf{m}}, (\mathbf{l}_i)_{i=n,d}, \mathbf{c}\right], \\ y &= \left[(s_{S_i i})_{i=n,d,h}, \tilde{m}, (l_i)_{i=n,d}, c\right], \end{aligned}$$

and the aggregate and individual state variables evolve according to the transition equations[5]

$$\begin{aligned} \left[\mathbf{x}_{t+1}^T, \mathbf{z}_{t+1}^T\right]^T &= \mathbf{H}(\mathbf{x}_t, \mathbf{z}_t, \mathbf{y}_t), \\ x_{t+1} &= H(x_t, y_t). \end{aligned}$$

In a stationary stochastic equilibrium prices and decision variables are functions of the aggregate and individual state of the world. Let $y = \mathbf{Y}(\mathbf{x}, \mathbf{z})$ be the aggregate and $y = Y(x, \mathbf{x}, \mathbf{z})$ be the individual decision rule. Normalize the price of the durable good at one. Let $p = [r_n, r_d, w, p_m, p_n]$ denote the vector of period prices, where r_i is the rental rate of capital in the two production sectors, w is the wage rate, p_m is the price of the intermediate input, and p_n the price of the nondurable good. Let $p = P(\mathbf{x}, \mathbf{z}) = [R_n, R_d, W, P_m, P_n](\mathbf{x}, \mathbf{z})$ denote prices as a vector-valued function of the aggregate state, and $\Upsilon(\mathbf{x}, \mathbf{z})$ the lump-sum tax rebate as function of the aggregate state of the economy.

The representative agent's dynamic optimization problem can now be defined as follows

$$\begin{aligned} V(x, \mathbf{x}, \mathbf{z}) \;=\; & \max \; U(c, k_d, T - l_n - l_d) + \beta E\left[V(x', \mathbf{x}', \mathbf{z}')\right] \\ & \text{s.t.} \; P_n(\cdot)\left[c + m'\right] + \textstyle\sum_{i=n,d,h} \sum_{j=1}^{S_i} \phi_{ij} s_{ij} \\ (PH) \quad & \le (1 - \tau_k)\left[(R_n(\cdot) - \delta_n) k_n + (R_d(\cdot) - \delta_d) k_d\right] \\ & + (1 - \tau_l) W(\cdot)\left[l_n + l_d\right] + P_m(\cdot) m + \Upsilon(\cdot), \\ & \text{and } \mathbf{H}, H, \mathbf{Y} \text{ given.} \end{aligned}$$

[5]Superscript T denotes the transpose of a matrix.

Firms in the nondurable and durable goods sector solve the static profit maximization problems

$$
\begin{aligned}
(PF_n) \quad \pi_n &= \max P_n(\cdot) F_n(k_n, l_n, z_n) - R_n(\cdot) k_n - W(\cdot) l_n \\
(PF_d) \quad \pi_d &= \max F_d [G(k_d, l_d, z_d), m] - R_d(\cdot) k_d - W(\cdot) l_d - P_m(\cdot) m
\end{aligned}
$$

At this point we comment on our convention of calling the nondurable good an intermediate input. In National Income Accounting (NIA) an intermediate good is a good which is produced and used up in production in the accounting period. In our analysis we interpret a period as a quarter but will concentrate on time aggregates representing a year. Thus although the nondurable good is produced to inventory in a quarter, it is produced and used in production in a year.

Define value-added in the durable goods sector as the sum of payments to primary factors of production, capital and labor, $va_d = wn_d + r_d k_d$. Value-added is the product of real value-added and the value-added price index, $va_d = p_{yd} y_d$.[6] Our assumption with respect to the separability of inputs in the production of durable goods implies that a value-added aggregate exists for this sector, that is real value-added is a function of the primary factors of production and productivity only, cf. Sato (1976) or Hulten (1978). Empirical work on production functions usually does not confirm the existence of a value-added aggregate, see for example Berndt and Wood (1975) or Jorgenson et al (1987). Existing sectoral models of the business cycle have proceeded on the assumption that such an aggregate exists. Our point is that even if such an aggregate exists, an explicit analysis of the environment including intermediate goods will improve our understanding of the business cycle.

We are now in a position to define a competitive equilibrium.

Definition 1 *A recursive stationary competitive equilibrium is a collection of functions $Y, \mathbf{Y}, V, P, \tau$ such that (1) (utility maximization) V satisfies the functional equation (PH) and Y is the associated optimal policy function, given $\mathbf{Y}$, P, and Υ, (2) (profit maximization) $\mathbf{k}_n, \mathbf{l}_n$ is the optimal policy for (PF_n), and $\mathbf{k}_d, \mathbf{l}_d, \mathbf{m}$ is the optimal policy for (PF_d), given P, and (3) (market clearing) $\mathbf{Y}(\mathbf{x}, \mathbf{z}) = Y(\mathbf{x}, \mathbf{x}, \mathbf{z})$.*

[6] In our numerical experiments we will approximate the real value-added aggregate with a Laspeyre quantity index, $y = q_d - \bar{p}_m m$, where the price weights are steady state prices.

4 Calibration

The model we analyze is similar to the environment of Kydland and Prescott (1982) except for the two-sector structure and the presence of intermediate inputs. As far as possible, our model economy will be parameterized along the lines of Kydland and Prescott (1982). We extend their procedure when we parameterize the production functions in both sectors and the degrees of substitutability between goods in production and consumption. We proceed by providing a parametric description of the environment.

The preferences of the representative agent are described as follows

$$U(c, k_d, l) = \left\{ \left([(\xi c)^\eta + ([1-\xi]\, k_d^\eta)]^{\gamma/\eta}\, h^{1-\gamma} \right)^{1-\sigma} - 1 \right\} / (1-\sigma), \qquad (7)$$

with $\eta \leq 1$, $0 < \xi < 1$, $\sigma > 0$ and $0 < \gamma < 1$. We assume that the consumption of nondurable goods and services from the stock of durable goods provides some composite consumption good. The elasticity of substitution between nondurable goods and durable goods is $1/(1-\eta)$, and the two goods become less substitutable as η declines. The elasticity of substitution between the composite consumption good and leisure is equal to one since preferences for these goods are Cobb-Douglas.

The production technology for nondurable and durable goods is described by the functions

$$\begin{aligned} q_{nt} &= z_{nt} k_{nt}^{\alpha_n} l_{nt}^{1-\alpha_n}, \\ q_{dt} &= \left[\left(\mu_y \left[z_{dt} k_{dt}^{\alpha_d} l_{dt}^{1-\alpha_d} \right] \right)^\rho + (\mu_m m_t)^\rho \right]^{1/\rho}, \end{aligned} \qquad (8)$$

with $0 < \alpha_n, \alpha_d < 1$, $\mu_y, \mu_m > 0$, and $\rho < 1$. We can define real value-added and the price index for real value-added in the durable goods sector as

$$\begin{aligned} y_{dt} &= z_{dt} k_{dt}^{\alpha_d} l_{dt}^{1-\alpha_d}, \\ p_{ydt} &= (r_{dt}/\alpha_d)^{\alpha_d} (w_t / [1-\alpha_d])^{1-\alpha_d} / z_{dt}. \end{aligned}$$

We choose parameter values for the economy by matching sample averages for the postwar U.S. economy with the corresponding values for the deterministic steady state, where productivity is fixed at one - its unconditional expected value. In a steady state, consumption, investment, employment, production, and the capital stocks are constant. The resource constraints for labor, the nondurable and the durable good are

$$\begin{aligned} l_n + l_d + h &= T, \\ c + m &= q_n = k_n^{\alpha_n} l_n^{1-\alpha_n}, \\ & \quad\; y_d = k_d^{\alpha_d} l_d^{1-\alpha_d}, \\ \delta_n k_n + \delta_d k_d + \delta_d k_d &= q_d = \left[(\mu_y y_d)^\rho + (\mu_m m)^\rho\right]^{1/\rho}. \end{aligned}$$

The real rate of return is $r = 1/\beta - 1$. The price of the durable good is normalized to one, the prices of all other goods are defined relative to the durable good. The price of installed capital is $Q_i = \sum_{j=1}^{S_i} \phi_{ij} (1+r)^{j-1}$, for $i = n, d$. This price reflects the present value of past resources used to build capital. The rental price of capital is $r_i = [(r+\delta_i)Q_i - \delta_i \tau_k] / (1-\tau_k)$, for $i = n, d$. First order conditions require that an input's marginal value product is equalized with its price

$$\begin{aligned} w &= (1-\alpha_n) p_n q_n / l_n, \\ w &= (1-\alpha_d) \left(\frac{\mu_y y_d}{q_d}\right)^\rho q_d / l_d, \\ r_n &= \alpha_d p_n q_n / k_n, \\ r_d &= \alpha_d \left(\frac{\mu_y y_d}{q_d}\right)^\rho q_d / k_d, \\ p_m &= \left(\frac{\mu_m m}{q_d}\right)^\rho q_d / m. \end{aligned}$$

The intertemporal optimality condition with respect to the intermediate input is

$$p_m = (1+r) p_n.$$

The first order conditions for the household are

$$\begin{aligned} (1-\tau_n) \frac{w}{p_n} &= \frac{1-\gamma}{\gamma} \frac{c}{h} \frac{(\xi c)^\eta + [(1-\xi) k_h]^\eta}{(\xi c)^\eta} \quad \text{and} \\ \frac{1-\xi}{\xi} &= \left(\frac{r+\delta_h}{p_n}\right)^{1/\eta} \left(\frac{c}{k_h}\right)^{1-1/\eta} \end{aligned}$$

where the first equation states that the marginal rate of substitution between nondurable consumption and leisure is equal to the after tax real wage, and the second equation is the intertemporal optimality condition with respect to household capital. This completes the description of the steady state. We now turn to the selection of parameter values for our baseline model.

We are choosing parameter values such that the models steady state corresponds to the average values of a collection of NIA measures for the postwar U.S. period. The economic environment of our model deviates from conventional NIA in an important way. In NIA the consumption of residential housing services, inclusive owner occupied housing, is part of GNP. In our model household capital is supposed to include all residential housing, thus residential housing services are not part of the model's GNP which is $va = p_n c + q_d$. In the following when we refer to GNP we mean GNP after deduction of value-added attributed to residential housing. The time period in our environment is supposed to represent a quarter.[7] We provide a summary of the steady state values of variables in Table 5 and of the parameter values in Table 6.

The average expenditure share on nondurable consumption goods and services, $p_n c/va$, is about 75 percent. This includes government spending. For our definition of durable versus nondurable goods we do not observe a distinct trend in the relative value-added prices, and we set the relative price of nondurable goods to one, $p_n = 1$.[8] Of total investment about 40 percent takes place in the nondurable goods sector, about 7 percent in the durable goods sector, and about 53 percent represents investment in household durables and residential housing. The capital stock in the nondurable (durable) goods sector is 3.4 (0.55) times that of quarterly GNP, household capital is five times that of quarterly GNP. The implied quarterly depreciation rates for capital in the nondurable, durable and household sector are respectively 2.94 percent, 3.18 percent, and 2.65 percent. For the time-to-build technology we follow Kydland and Prescott (1982) and assume that time to maturity for investment in the business sector is four periods, that is one year, and that $\phi_{ij} = 0.25$ for $i = n, d$ and $j = 1, ..., 4$. We assume that it takes less time for household investment to mature, half a year, and $\phi_{hj} = 0.5$ for $j = 1, 2$.

These observations are not sufficient to determine all parameter values. For our sample we observe that the average share of capital in the nondurable (durable) goods sector is about 29 (25) percent. While the two capital shares are not very different from each other, they are somewhat below the values used in related literature, cf. Christiano (1986) or Prescott (1986). We choose to treat value-added in both sectors symmetrically and select a capital share α of 0.3 for both sectors. We do not have reliable information on the elasticity

[7]The following statements about the average values of certain variables in the U.S. economy are based on our data set described in the appendix.

[8]By construction price indexes for GNP expenditure categories are equal in the base period.

of substitution between value-added and intermediate goods in the durable goods sector. Work by Berndt and Wood (1975) indicates that capital and energy are complementary inputs. Work by Jorgenson et al (1987) appears to show that for most two-digit SIC industries the share of value-added in the value of gross output is independent of the price of intermediate inputs. This would indicate an elasticity of one. For the baseline model we choose a value of 0.5 for the elasticity, but we also investigate the impact of other elasticity values for cross sectoral comovement. We do not use information on the actual share of intermediate inputs in the value of gross output, which is about 40 percent, cf. Table 1. The value implied by our calibration procedure is about 43 percent.

The preference parameters are determined as follows. We assume a coefficient of relative risk aversion of $\sigma = 2.0$. Information on the elasticity of substitution between nondurable and durable consumption goods is scarce. Eichenbaum and Hansen (1990) suggest that the two goods are perfect substitutes, Benhabib et al (1991) estimate an elasticity of 2.5. We choose an elasticity of 2. The average quarterly interest rate for the postwar period is about one percent, which implies a time preference parameter of $\beta \simeq .99$.

We now turn to government tax policy. Estimates of effective tax rates vary widely. Lucas (1990) suggests that the average tax on capital and labor income is 0.36. McGrattan (1991) reports average values for the capital tax rate between 0.4 and 0.5, and for the average labor tax rate values between 0.1 and 0.25. For the labor income tax we choose a proportional tax of $\tau_n = 0.3$. We do not fix the tax rate on capital, as an outcome of our calibration procedure its value is endogenously determined as $\tau_k = 0.77$. For the U.S. from 1953-79 Feldstein et al (1983) report values between 55 and 84 percent for the total effective tax rate on capital income. We are now in a position to determine all parameter values except the ones for the productivity process.

In order to get a rough idea on the magnitude of productivity fluctuations we first construct measures of total factor productivity for each sector. Total factor productivity is defined by

$$\ln z_{it} = \ln y_{it} - \alpha \ln k_{it} - (1-\alpha)\ln l_{it}, \text{ and } i = n, d$$

where y_i is real value-added, k_i is the capital stock, l_i is a labor input measure, and α is the selected value for the share of value-added paid to capital in the two sectors.[9] This procedure is justified in our theoretical framework since for each sector a value-added aggregate exists and changes in productivity affect only the value-added aggregate. Using ordinary least squares we estimate the process parameters as

$$\hat{\Lambda}^{a}_{U.S} = \begin{bmatrix} 0.952 & 0.013 \\ (0.023) & (0.031) \\ 0.004 & 0.908 \\ (0.006) & (0.095) \end{bmatrix} \text{ and } \hat{\Omega}^{a}_{U.S.} = \begin{bmatrix} 0.011^2 & 0.015^2 \\ 0.015^2 & 0.031^2 \end{bmatrix}$$

with standard deviations in parentheses. We see that total factor productivity in the durable goods sector is about three times as volatile as in the nondurable goods sector. The implied correlation coefficient between sectoral productivity innovations is 0.62.

Our estimates of the productivity process cannot be directly applied to our model since we have used annual data, and our model's basic time unit is a quarter. For the first order autocorrelation coefficients we follow the literature on the business cycle at the aggregate level. Most studies estimate and use first order autocorrelation coefficients around 0.96 with quarterly data, e.g. Hansen (1985) or Prescott (1988). The covariance matrix for productivity innovations is chosen as follows. First we assume that at the quarterly frequency, innovations to durable goods factor productivity are three times as volatile as innovations to nondurable goods factor productivity. Then we simulate our model and time aggregate data from quarterly to annual frequency. For each simulation we also estimate the stochastic process of each sector's total factor productivity at the model's annual frequency. We then choose Ω such that for the implied annual productivity process the standard deviations and correlations of productivity innovations match those we have observed for annual data in the U.S.. Following this procedure we select the following specification for the model's productivity process

$$\Lambda = \begin{bmatrix} 0.96 & 0.00 \\ 0.00 & 0.96 \end{bmatrix} \text{ and } \Omega = \begin{bmatrix} 0.0067^2 & 0.0104^2 \\ 0.0104^2 & 0.0233^2 \end{bmatrix}.$$

Using this specification in simulations of our model, we estimate the following

[9]The results with average capital share values from the sample are essentially the same.

average parameter values of a first order Markov process for annual data,

$$\hat{\Lambda}^a = \begin{bmatrix} 0.78 & -0.01 \\ (0.15) & (0.05) \\ 0.01 & 0.81 \\ (0.47) & (0.13) \end{bmatrix} \text{ and } \hat{\Omega}^a = \begin{bmatrix} 0.010^2 & 0.014^2 \\ (0.001) & \\ 0.014^2 & 0.031^2 \\ & (0.004) \end{bmatrix}$$

with an implied correlation coefficient between productivity innovations of 0.62 (0.11). The values are average values for correlation coefficients and standard deviations from the 50 samples generated, and the expressions in parentheses are their standard deviations across samples.

5 Findings

In this section we discuss the business cycle properties of our model economy. We solve for an approximation to the competitive equilibrium where decision rules are linear functions of the state variables using an algorithm similar to that described in Hansen and Prescott (1992).[10] We find that the business cycle in our model is comparable to the U.S. business cycle. The durable goods producing sector fluctuates more than the nondurable goods producing sector, and both sectors move together. In particular value-added, employment and investment in both sectors are strongly correlated. We also find that household investment leads business investment. We then argue that the cross-sectoral comovement of employment is not due to the opportunities for substitution in production, or the correlation of changes in productivity, but it follows from the presence of intermediate goods in the durable goods sector.

The business cycle of our model is characterized as follows. We first generate 50 random samples each consisting of 160 periods. Given that the time unit is one quarter each sample consists of 40 years. For each sample we define annual time series for all variables. For a flow variable like output, we define annual output as the sum of output over four consecutive quarters, and for a stock variable like capital, we take the annual capital stock to be the value in the fourth quarter. For prices we take the average over four consecutive quarters. In each sample on the production side of the economy

[10] A complete description of the algorithm is provided in the Appendix.

we detrend the log of a time series using the Hodrick-Prescott filter, and on the final demand side we detrend the share of each component in aggregate value-added. For the detrended time series we calculate their standard deviations and correlations with aggregate value-added. These results are reported in Tables 2, 3 and 4.

5.1 The Business Cycle at Annual Frequency

We now describe the business cycle properties of our model at an annual frequency. The second column of Table 2 shows volatilities, that is standard deviations of percentage deviations from trend, and correlations with GNP, that is aggregate value-added, for the model's variables at an annual frequency. In the model the volatility of GNP is about two percent, which is somewhat less than two thirds of U.S. GNP volatility. Of the inputs, the aggregate capital stock is almost as volatile as GNP and aggregate employment and labor productivity are about half as volatile as GNP. This is the opposite of what we observe for the U.S. economy.[11] For final demand, the expenditure share of aggregate investment is about three times as volatile as the share of consumption of nondurables. Compared with the U.S. economy, the consumption share fluctuates less and the investment share fluctuates more. This pattern is repeated at the sectoral level.

The durable goods producing sector fluctuates more and the nondurable goods producing sector fluctuates less than the aggregate economy, and this difference is more pronounced in the model than in the U.S. economy. Value-added in the durable goods producing sector is almost four times as volatile as GNP, and value-added in the nondurable goods producing sector is about three fourths as volatile as GNP. Compared with the U.S. economy the durable goods sector is more volatile relative to GNP, and the nondurable goods producing sector has about the same volatility relative to GNP. A similar observation applies to sectoral employment. Relative to GNP employment in the durable goods producing sector is somewhat more volatile in the model than in the data, and employment in the nondurable goods producing sector is substantially less volatile in the model than in the data.

[11] This is a feature our model has in common with many other real business cycle models. Presumably the model would do better along this dimension if an indivisibility constraint is introduced following Hansen (1985).

The model does not do very well with respect to the behavior of investment in the different sectors. Compared with the U.S. economy investment, especially in the durable goods producing sector, is excessively volatile. Essentially there are not enough frictions which prevent big changes of the investment volume in the durable goods sector.[12]

The model's business cycle is as persistent as the U.S. business cycle, the first order autocorrelation for GNP in the model and in the data is about the same. All variables move with GNP and the highest correlation coefficient is usually the contemporaneous one. The model does capture the lead and lag structure of investment, investment in the household sectors is leading the cycle and investment in the business sector is lagging the cycle. This is a feature where the model improves on other disaggregated real business cycle models, cf. Greenwood and Hercowitz (1991).

In the model the use of intermediate inputs in the durable goods producing sector is highly variable and strongly procyclical. Intermediate inputs are somewhat less volatile than value-added in the durable goods sector, and the intermediate inputs contemporaneous correlation with GNP is close to one. We do not have direct observations on the use of intermediate inputs in the U.S. economy with which we can compare this property, but the result is consistent with results we obtain based on the data set in Jorgenson et al (1987), Hornstein and Praschnik (1994). We also observe that the value-added deflator of nondurable goods relative to durable goods is procyclical, the sample standard deviation of the correlation coefficient, however, is large. The relative price of intermediate inputs, not reported here, is also procyclical. This result is consistent with the observation that the relative price of nondurables to durables is procyclical, cf. Murphy et al (1988) or Greenwood et al (1992). Simulations of the model economy with productivity shocks in the durable (nondurable) goods sector only, show a procyclical (countercycli-

[12]See the discussion of impulse response functions and Figure 10. An admittedly ad-hoc procedure to improve the performance of the model with respect to the behaviour of investment is to introduce adjustment costs to changes in the number of investment projects initiated in a period. Experiments show that for very small adjustment costs in the two business sectors and no adjustment costs in the household sector, sectoral investment behaves very much like in the U.S. economy with respect to volatility. By 'small' adjustment costs we mean that a one percentage point deviation from the steady state investment volume in a sector implies adjustment costs of 0.0005 percent of the steady state gross output in the durable goods sector.

cal) relative price of intermediate inputs. We now examine the cross sectoral comovement in more detail.[13]

5.2 Comovement Across Sectors

Employment in the durable goods producing sector and employment in the nondurable goods producing sector are strongly contemporaneously correlated with GNP and therefore also with each other. The same applies to sectoral value-added. In the model economy this pattern of comovement is due to the use of intermediate inputs in the production of durable goods. We illustrate this argument using impulse response functions and computational experiments.

In Table 4 we report cross-sectoral correlations for value-added, employment, and investment in the baseline model at annual frequency. With one exception we match the observed cross sectoral correlation pattern for the U.S. economy quite well. Sectoral employment and value-added are strongly contemporaneously correlated, and household investment leads business investment in the model. The model does not capture the positive contemporaneous correlation between investment in the durable and nondurable goods sector.

We want to argue that the strong cross-sectoral correlation of employment and value-added results from the presence of intermediate inputs, and it is not due to the assumed complementarity in production between intermediate inputs and the value-added aggregate, or the correlation of productivity innovations. For this purpose we perform three computational experiments. In the first experiment, we increase the elasticity of substitution between intermediate inputs and the value-added aggregate in the production of durable goods. In the second experiment we reduce the correlation between productivity innovations, and in the third experiment we assume that there are no intermediate inputs used in the production of durable goods.

In Figure 6A we graph the sample averages for cross-sectoral correlation coefficients of employment at various leads and lags. We do the same for

[13]At the quarterly frequency, the statistical properties of the model's aggregate variables are similar to that of a standard one sector real business cycle model. Estimated total factor productivity for the simulated aggregate economy is also similar to the process which is usually estimated for total factor productivity of the aggregate U.S. economy. We do not report results at the quarterly frequency to economize on space.

value-added in Figure 6B. In this experiment we increase the elasticity of substitution between value-added and intermediate inputs from 0.5 ($\rho = -1$) to 4 ($\rho = 0.75$). It is apparent from these graphs that only if the elasticity of substitution is substantially above unity, will sectoral employment and value-added move in opposite directions.

In the second experiment we reduce the correlation coefficient of sectoral productivity innovations, while keeping their standard deviations constant. In Figure 7A and 7B we graph the sample averages for cross-sectoral correlation coefficients of employment and value-added at various leads and lags. Figure 7A demonstrates that employment cross correlations are virtually independent of the correlation coefficient. Figure 7B shows that the positive comovement of sectoral value-added depends to a large extent on positively correlated productivity innovations. When innovations are independent, value-added in the durable goods sector moves independently of value-added in the nondurable goods sector. We now turn to the role intermediate inputs play.

We now investigate the behavior of our two sector economy when the durable goods producing sector does not use nondurable goods as an intermediate input. For this experiment we change the structure of our economy and assume that $\mu_m = 0$, and leave all other parameter values unchanged. We then characterize the business cycle of this economy as before for the economy with intermediate inputs. In the third column of Tables 2 and 3 we report standard deviations and contemporaneous correlations with aggregate value-added. The economy without intermediate inputs appears to be more volatile than the economy with intermediate inputs, especially with respect to capital stocks and expenditure shares. With the exception of the nondurable goods sector, the contemporaneous correlations with aggregate value-added are similar in the two economies. In the economy without intermediate inputs employment in the nondurable sector is countercyclical and value-added is only weakly procyclical. In Table 4 this is reflected in the fact that employment in the two sectors moves in opposite directions and sectoral value-added is almost uncorrelated.

We now interpret the positive comovement of sectoral employment, value-added, and investment in the baseline model using impulse response functions. In Figures 8 to 10 we illustrate the response of the model with intermediate inputs, starting from the steady state, to a positive one standard deviation productivity innovation in the nondurable goods sector. The unit of a period is a quarter. The qualitative features of the model economy's response to a productivity innovation in the durable goods sector is essentially the same.

Most sectoral real business cycle models have the feature that, following a change in relative productivities, labor is shifted out of the less productive application, here the durable goods sector, and into the more productive application, here the nondurable goods sector. This implies that employment in different production sectors moves in opposite direction. In the model economy this occurs only for the period when the innovation occurs. After this initial period employment in the durable goods sector follows employment in the nondurable goods sector with a one period lag, cf. Figure 8. For time aggregated annual data, employment in the two sectors moves together.

The behavior of employment in the durable goods sector is determined by the behavior of intermediate inputs, cf. Figure 8. In the standard one sector growth model employment increases following an increase in productivity, in order to produce more capital goods and smooth consumption over time. In the two sector economy with intermediate inputs, the same occurs following an increase in productivity in the nondurable goods producing sector. Production in the nondurable goods sector increases in order to provide more intermediate inputs for the durable goods sector, which will increase production of capital goods in the future. The increased use of intermediate inputs in the durable goods sector raises the marginal product of labor and employment in that sector increases.[14] In a two sector economy without intermediate inputs, this opportunity to increase production of capital goods is not available, and employment in the sector which is relatively more (less) productive stays above (below) its steady state level.

Real value-added and investment in the two production sectors also move together. The response of value-added in the two sectors to a productivity innovation is essentially the mirror image of the employment response in the two sectors, cf. Figure 9. Figure 10 illustrates an important shortcoming of our model: investment in the durable goods sector and the household sector is too volatile. Following a one standard deviation innovation to the nondurable (durable) goods sector's productivity, investment in the durable goods sector can deviate up to 15 (5) percent from its steady state value.

[14]The feature that employment in the durable goods sector is lagging is due to this time lag for intermediate inputs.

In Table 4 we have seen that business and household investment move together with household investment leading. From Figure 10 we see that this cannot be due to the response to productivity shocks in the nondurable goods sector, since household and business investment move in opposite directions. Nondurable goods become relatively less expensive and the household substitutes the consumption of nondurable goods for the consumption of services from the stock of durable goods. In Figure 11 we graph the response of investment to a one standard deviation productivity innovation in the durable goods sector. Because of the increased productivity capital is reallocated towards the durable goods sector and away from the nondurable goods sector. This time household investment also increases because the relative price of durable goods declines and the household substitutes the services of household capital for the consumption of nondurable goods. Since the productivity shocks in the durable goods sector are three times as volatile as in the nondurable goods sector and business investment fluctuations are dominated by investment in the durable goods sector, the net effect is that household investment is positively related to business investment in the cycle.

6 Conclusion

In this paper we have shown that a two sector stochastic growth model is consistent with positive comovement of employment and output in the durable and nondurable goods producing sector. A crucial element for this result is that nondurable goods serve as intermediate inputs in the production of durable goods. We also observe that measures of total factor productivity show higher productivity fluctuations in the durable goods sector than in the nondurable goods sector. Because of these differential productivity volatilities, the price of nondurables relative to durables is procyclical in the model economy. The differential productivity fluctuations also seem to imply that natural resource price shocks, like oil price changes, are not that important for the business cycle. The natural question to ask then is why are the shocks in the durable goods sector so big relative to the shocks in the nondurable goods sector.

Apparently we have also made some progress on the issue of investment comovement in multi-sector growth models. As is commonly observed household investment leads business investment over the business cycle. Our model can account for this observation. We believe that this speaks for the basic model structure since we did not intend that the model should address this issue. However, more work needs to be done on investment in multi sector-growth models, there is too much movement of capital between sectors, that is investment is much more volatile in the model than in the data. We will take up these issues in future work.

7 Appendix

7.1 The Data

The data are taken from the Citibase data set, unless otherwise stated. Real quantities are series in 1982 dollars. We use the series on population of the U.S. including armed forces overseas to express all variables in per capita terms. We define total value-added va as Gross National Product minus personal consumption expenditures on housing. The last series includes imputed rent for owner occupied housing and rent payments by tenants. Durables value-added va_d is defined as the value-added from the construction sector plus the manufacturing durables sectors. Nondurables value-added is $va_n = va - va_d$.

The total business capital stock k_b is the net stock of fixed private capital in the nonresidential sector. This includes the capital stocks from the agricultural, mining, construction, manufacturing, transportation, retail and wholesale, finance, insurance and real estate, and services sectors. The capital stock of the durables sector k_d is the net stock of fixed private capital in the construction and manufacturing durables sector. The capital stock of the nondurables sector is $k_n = k_b - k_d$. The household capital stock k_h is the net stock of durable goods owned by consumers plus residential real estate. The total capital stock is $k = k_b + k_h$. All capital stock series are constant cost series from the January 1986, August 1987, and August 1989 issues of the Survey of Current Business.

Total employment l is hours worked by full-time and part-time employees in all private industries and in the government sector. Employment in the durables sector l_d covers hours worked by full-time and part-time employees in the construction industry and manufacturing durables industries. Employment in the nondurables sector is $l_n = l - l_d$. These series are from the 1992 and 1993 National Income and Product Accounts supplement to the Survey of Current Business.

To calculate wages and labor shares in value-added we use series on compensation of employees in all industries W, compensation of employees in the manufacturing and construction sector W_d . Labor compensation in the nondurables sector is $W_n = W - W_d$. Real wages are labor compensation per labor input unit deflated with the GNP price index. From the OECD National Account Statistics, 1989, we take series on total net indirect taxes T, net indirect taxes in the durables sector T_d, and define net indirect taxes in the nondurables sector as $T_n = T - T_d$. Wage shares are calculated for value-added net of indirect taxes.

Consumption c covers personal consumption expenditures for nondurable goods and services minus personal consumption expenditures for housing. Household investment i_h covers personal consumption expenditures for durable goods plus investment in residential structures. Business investment i_b covers investment in nonresidential structures plus producers' durable equipment. Investment in the durable goods sector i_d covers investment in structures and equipment by the manufacturing durables and construction sectors. Historical cost values in current dollars for this series were provided by the U.S. Department of Commerce, Bureau of Economic Analysis. To obtain a constant dollar series it is deflated with the implicit price deflator for gross private domestic nonresidential investment. Investment in nondurables goods sector is $i_n = i_b - i_d$. Total investment is $i = i_b + i_h$.

7.2 The Algorithm

Consider the following representative agent problem

$$\begin{aligned} V(x, X, z) &= \max_y \{U(x, X, z, y, Y) + \beta E_z [V(x', X', z')]\} \\ s.t. \quad x' &= F(x, z, y) \\ X' &= F(X, z, Y) \\ z' &= G(z, \epsilon) \\ Y &= D(X, z) \end{aligned}$$

where x (X) respectively y (Y) is the individual (aggregate) state respectively control variable, and ϵ is iid normal with mean zero. Let $y = d(x, X, z)$ be the agent's optimal decision rule for this problem. In an equilibrium the perceived aggregate decision rule D must coincide with the individual decision rule $d(x, x, z) = D(x, z)$. Assume that the period return function U and the value function V are twice continuously differentiable, and that solutions to the maximization problem are interior. A necessary condition for optimality is

$$U_y(x,X,z,y,Y) + \beta E_z\left[F_y(x,z,y)^T V_x(x',X',z')\right] = 0,$$

and from the envelope theorem we have that

$$V_x(x,X,z) = U_x(x,X,z,y,Y) + \beta E_z\left[F_x(x,z,y)^T V_x(x',X',z')\right],$$

where the first derivative of a function is a column vector. Using the market equilibrium condition that $y = Y$ and $x = X$, we get the following system of equations

$$\begin{array}{lcccl} U_y(x,x,z,y,y) & & & + & \beta E_z\left[F_y(x,z,y)^T V_x(x',x',z')\right] & = & 0 \\ -U_x(x,x,z,y,y) & + & V_x(x,x,z) & + & \beta E_z\left[F_x(x,z,y)^T V_x(x',x',z')\right] & = & 0 \end{array}$$

We now choose linear approximations to the transition functions, and the first derivatives of the period return function and the value function and have

$$\begin{array}{lcccl} U_y^T\begin{bmatrix} x \\ y \\ z \end{bmatrix} & & & + & \beta F_y^T V_x \begin{bmatrix} F_x^T & F_y^T & F_z^T \\ 0 & 0 & G_z^T \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} & = & 0 \\ -U_x^T\begin{bmatrix} x \\ y \\ z \end{bmatrix} & + & V_x\begin{bmatrix} x \\ z \end{bmatrix} & + & \beta F_x^T V_x \begin{bmatrix} F_x^T & F_y^T & F_z^T \\ 0 & 0 & G_z^T \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} & = & 0 \end{array}$$

We are looking for a matrix V_x which is a solution to this system of equations. The procedure is similar to the approach described in Ceria and Rios-Rull (1993). We do not know if there exists a unique solution to this problem. In the numerical experiments the algorithm does converge. We have obtained solutions for the case where the proportional tax rates are zero. In this case the solution V_x is also a solution to the planning problem, and the matrix V_x is symmetric and negative definite. As we increase the tax rates to the values chosen in the calibration step, the implied impulse response functions for all variables change continuously with the tax rates.

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Table 1: Input-Output Table for the Durable and Nondurable Goods Producing Sector

Industries / Commodities	Nondurable Goods	Durable Goods	Final Demand	Total Output
Nondurable Goods	1,683,692 (40.5%)	351,414 (26%) [37.9%]	2,126,037	4,161,143
Durable Goods	184,698 (4.4%) [7.5%]	425,023 (31.4%)	743,293	1,353,014
Value Added	2,292,753	576,577	2,869,330	5,514,157

Note: The input-output table has been constructed from the 1982 Benchmark U.S. Input-Output Use Table, Survey of Current Business (1991), Table 2, pp.42–49. Values are millions of dollars at producers' prices. The durable goods sector consists of sectors 11 and 12, 20–23, and 35–64. The nondurable goods sector consists of all other sectors from 1 to 77. We have excluded sectors 78–85, which inludes among others government agencies and enterprises. Percentages in parentheses denote the input's share in payments to all inputs in the sector where the input is used. Square brackets denote the same input share when output is measured net of internal use of goods.

Table 2: Business Cycle Characteristics:
Standard Deviations of Percentage Deviations from Trend

Variable x	US Economy 1948-87	Intermediate Goods	No Intermediate Good
Aggregate Production			
GNP	3.29	1.89	2.15
va		(0.38)	(0.36)
Capital stock, total	1.41	1.61	2.18
k		(0.51)	(0.68)
	[0.43]	[0.84]	[1.01]
Employment, total	2.74	0.77	0.95
l		(0.19)	(0.29)
	[0.83]	[0.41]	[0.44]
Labor productivity,total	1.15	1.23	1.42
va/l		(0.23)	(0.28)
	[0.35]	[0.65]	[0.66]
Ratio of GDP deflators	3.04	2.94	2.70
		(0.60)	(0.60)
Production in Durable Goods Sector			
Value-added, durables	6.70	6.92	8.55
va_d		(1.84)	(2.24)
	[2.04]	[3.64]	[3.96]
Intermediate input		5.63	
m		(1.38)	
		[2.97]	
Capital stock, durables	3.50	3.83	5.03
k_d		(0.91)	(1.26)
	[1.06]	[2.03]	[2.33]
Employment, durables	6.03	4.02	5.84
l_d		(1.39)	(2.08)
	[1.83]	[2.09]	[2.69]
Labor productivity, durables	3.06	3.22	2.99
va_d/l_d		(0.69)	(0.60)
	[0.93]	[1.72]	[1.39]
Production in Nondurable Goods Sector			
Value-added, nondurables	2.62	1.30	1.28
va_n		(0.22)	(0.26)
	[0.80]	[0.70]	[0.61]
Capital stock, nondurables	1.43	1.68	2.90
k_n		(0.48)	(0.73)
	[0.43]	[0.89]	[1.35]
Employment, nondurables	1.98	0.31	0.59
l_n		(0.05)	(0.17)
	[0.60]	[0.17]	[0.27]
Labor productivity, nondurables	1.05	1.12	1.23
va_n/l_n		(0.21)	(0.26)
	[0.32]	[0.60]	[0.58]

Variable x	US Economy 1947-87	Intermediate Goods	No Intermediate Goods
Expenditure Shares in GNP			
Investment, total	4.49	4.69	6.64
i/va		(1.23)	(1.90)
Investment, business	4.79	5.35	7.49
i_b/va		(1.00)	(1.59)
Investment, durables	11.76	19.87	26.93
i_d/va		(3.24)	(5.21)
Investment, nondurables	4.32	7.15	15.92
i_n/va		(1.04)	(2.89)
Investment, households	7.41	7.42	11.57
i_h/va		(1.45)	(2.85)
Consumption	2.50	1.51	2.11
c/va		(0.34)	(0.49)

Note: The variables are defined in the Appendix. With the exception of expenditure shares the log of a variable is detrended using the Hodrick-Prescott filter with the smoothing parameter set at 400. For the model economies we provide for each variable the average value of its standard deviation from 50 samples with the respective sample standard deviation in parentheses. The entries in square brackets are the average values of variables' standard deviations relative to the standard deviation of aggregate GNP.

Table 3: Characteristics of the Business Cycle: Contemporaneous Correlations with Aggregate GNP

Variable x	US Economy 1948-87	Intermediate Goods	No Intermediate Goods
Production in Aggregate Economy			
GNP lagged.	0.63	0.59	0.62
$va(t-1)$		(0.13)	(0.11)
Capital stock	0.54	0.14	0.03
k		(0.17)	(0.21)
Employment	0.94	0.91	0.86
l		(0.04)	(0.06)
Labor Productivity	0.61	0.97	0.94
va/l		(0.02)	(0.04)
Ratio of GDP deflators	-0.16	0.55	0.67
		(0.20)	(0.15)
Production in Durable Goods Sector			
Value-Added	0.92	0.88	0.89
va_d		(0.06)	(0.05)
Intermediate Input		0.91	
m		(0.04)	
Capital stock	0.27	0.70	0.71
k		(0.12)	(0.09)
Employment	0.93	0.87	0.84
l_d		(0.05)	(0.07)
Labor Productivity	0.18	0.80	0.91
va_d/l_d		(0.11)	(0.05)
Production in Nondurable Goods Sector			
Value-Added	0.96	0.92	0.31
va_n		(0.04)	(0.27)
Capital stock	0.39	-0.10	-0.34
k_n		(0.18)	(0.20)
Employment	0.92	0.80	-0.76
l_n		(0.06)	(0.11)
Labor Productivity	0.66	0.84	0.70
va_n/l_n		(0.07)	(0.13)

Variable x	US Economy 1947-87	Intermediate Goods	No Intermediate Goods
Expenditure Shares in GNP			
Investment, total	0.26	0.80	0.81
i/va		(0.10)	(0.08)
Investment, business	0.34	0.48	0.32
i_b/va		(0.19)	(0.17)
Investment, durables	0.57	0.51	0.48
i_d/va		(0.04)	(0.07)
Investment, nondurables	0.17	0.18	-0.07
i_n/va		(0.20)	(0.15)
Investment, households	0.10	0.64	0.69
i_h/va		(0.11)	(0.11)
Consumption	-0.87	-0.83	-0.81
c/va		(0.09)	(0.08)

Note: See Table 2.

Table 4: Correlation of Employment, Value-Added and Investment Across Sectors.

	With Lags s =				
	-2	-1	0	+1	+2
US Economy, 1948-1987					
$l_n(t)$ and $l_d(t+s)$	0.00	0.53	0.91	0.51	-0.02
$va_n(t)$ and $va_d(t+s)$	0.18	0.51	0.76	0.40	-0.01
$i_n(t)/va(t)$ and $i_d(t+s)/va(t+s)$	-0.04	0.09	0.52	-0.13	-0.13
$i_b(t)/va(t)$ and $i_h(t+s)/va(t+s)$	0.16	0.34	-0.03	-0.11	-0.12
Economy with Intermediate Inputs					
$l_n(t)$ and $l_d(t+s)$	-0.14	0.12	0.71	0.78	0.34
	(0.15)	(0.16)	(0.07)	(0.07)	(0.18)
$va_n(t)$ and $va_d(t+s)$	0.16	0.29	0.62	0.45	0.06
	(0.25)	(0.24)	(0.15)	(0.14)	(0.17)
$i_n(t)/va(t)$ and $i_d(t+s)/va(t+s)$	0.42	0.42	-0.48	-0.17	-0.10
	(0.16)	(0.10)	(0.14)	(0.17)	(0.18)
$i_b(t)/va(t)$ and $i_h(t+s)/va(t+s)$	0.42	0.72	0.00	-0.04	-0.08
	(0.15)	(0.09)	(0.23)	(0.18)	(0.16)
Economy without Intermediate Inputs					
$l_n(t)$ and $l_d(t+s)$	-0.37	-0.81	-0.94	-0.52	-0.06
	(0.23)	(0.09)	(0.03)	(0.14)	(0.21)
$va_n(t)$ and $va_d(t+s)$	-0.05	-0.21	-0.12	-0.07	-0.16
	(0.25)	(0.25)	(0.26)	(0.26)	(0.23)
$i_n(t)$ and $i_d(t+s)$	0.47	0.16	-0.81	-0.13	-0.01
	(0.09)	(0.08)	(0.06)	(0.14)	(0.17)
$i_b(t)/va(t)$ and $i_h(t+s)/va(t+s)$	0.69	0.72	-0.13	-0.19	-0.16
	(0.11)	(0.05)	(0.18)	(0.16)	(0.16)

Note: See Table 2.

Table 5: Steady State Values for the Baseline Model.

value added, total,	va	4.00
value added, nondurable goods sector,	va_n	3.42
value added, durable goods sector,	va_d	0.57
output, total,	$q_n + q_d$	4.42
output, nondurable goods sector,	q_n	3.42
output, durable goods sector,	q_d	1.00
value share of intermediate inputs,	$p_m m/q_d$	0.43
capital, total,	$k_n + k_d$	35.80
capital, nondurable goods sector,	k_n	13.60
capital, durable goods sector,	k_d	2.20
capital, household durables,	k_h	20.00
employment, total,	$l_n + l_d$	2.21
employment share, nondurable goods sector,	l_n/l	0.86
employment share, durable goods sector,	l_d/l	0.14
intermediate input,	m	0.42
consumption share,	c/va	0.75
investment share, total,	$(i_n + i_d + i_h)/va$	0.25
investment share, nondurable goods sector,	i_n/va	0.10
investment share, durable goods sector,	i_d/va	0.02
investment share, household durables,	i_h/va	0.13
price of capital, nondurable goods sector,	Q_n	1.02
price of capital, durable goods sector,	Q_d	1.02
relative price of nondurables,	p_n	1.00

Table 6: Parameter Values for the Baseline Model.

1. Preferences		
time preference,	β	0.9901
elasticity of substitution between durables and nondurable,	$1/(1-\eta)$	2.0000
share of nondurable consumption,	ξ	0.9911
depreciation of household capital,	δ_h	0.0265
share of consumption index,	γ	0.4881
degree of relative risk aversion,	σ	2.0000
2. Technology		
depreciation of capital in nondurable goods sector,	δ_n	0.0294
depreciation of capital in durable goods sector,	δ_d	0.0318
capital share, nondurable good sector,	α_n	0.3000
capital share, durable goods sector,	α_d	0.3000
value-added coefficient,	μ_y	3.0811
intermediate inputs coefficient,	μ_m	5.5281
elasticity of substitution between intermediate goods and value-added index,	$1/(1-\rho)$	0.5000
3. Taxes		
labor income tax,	τ_n	0.3000
gross capital income tax,	τ_k	0.7702

Fig. 1: Value-Added Business Cycle for U.S. Economy: 1948-1987

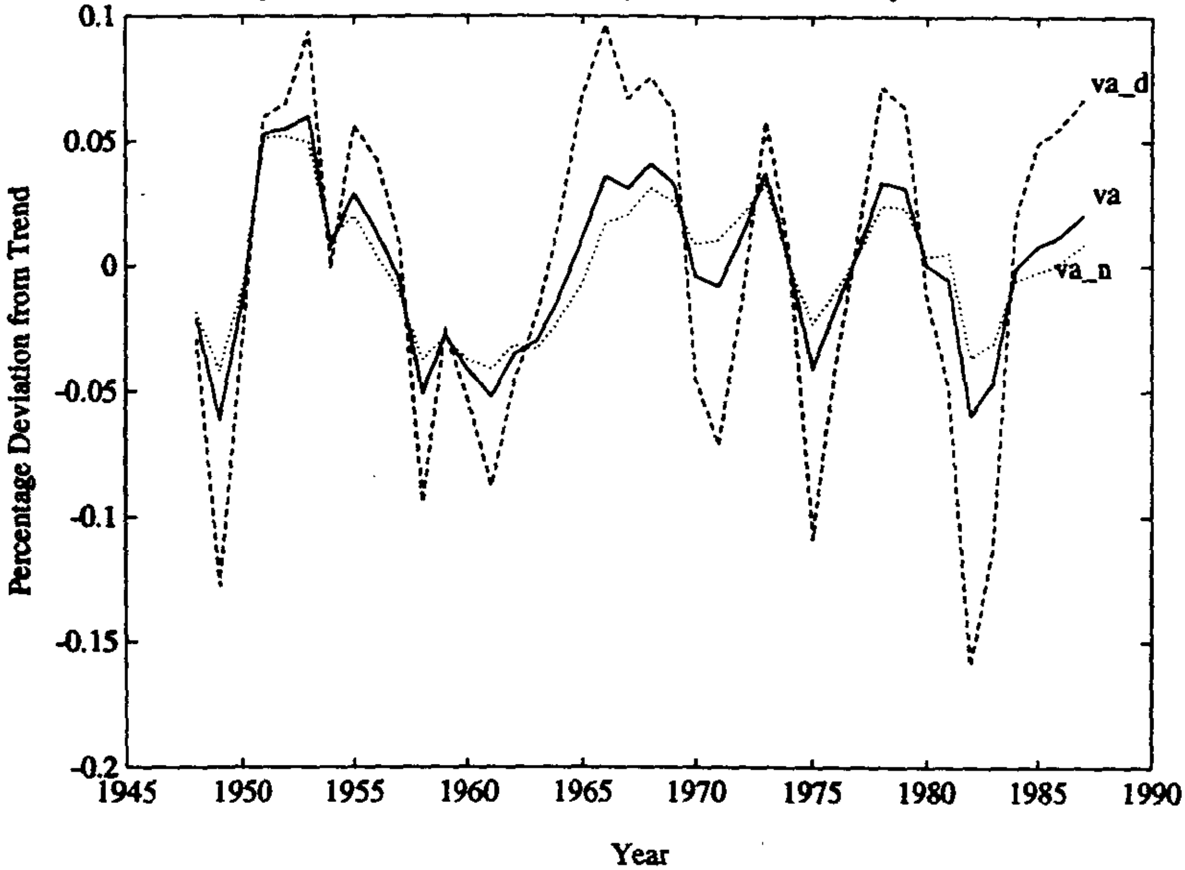

Fig. 2: Employment Business Cycle for U.S. Economy: 1948-1987

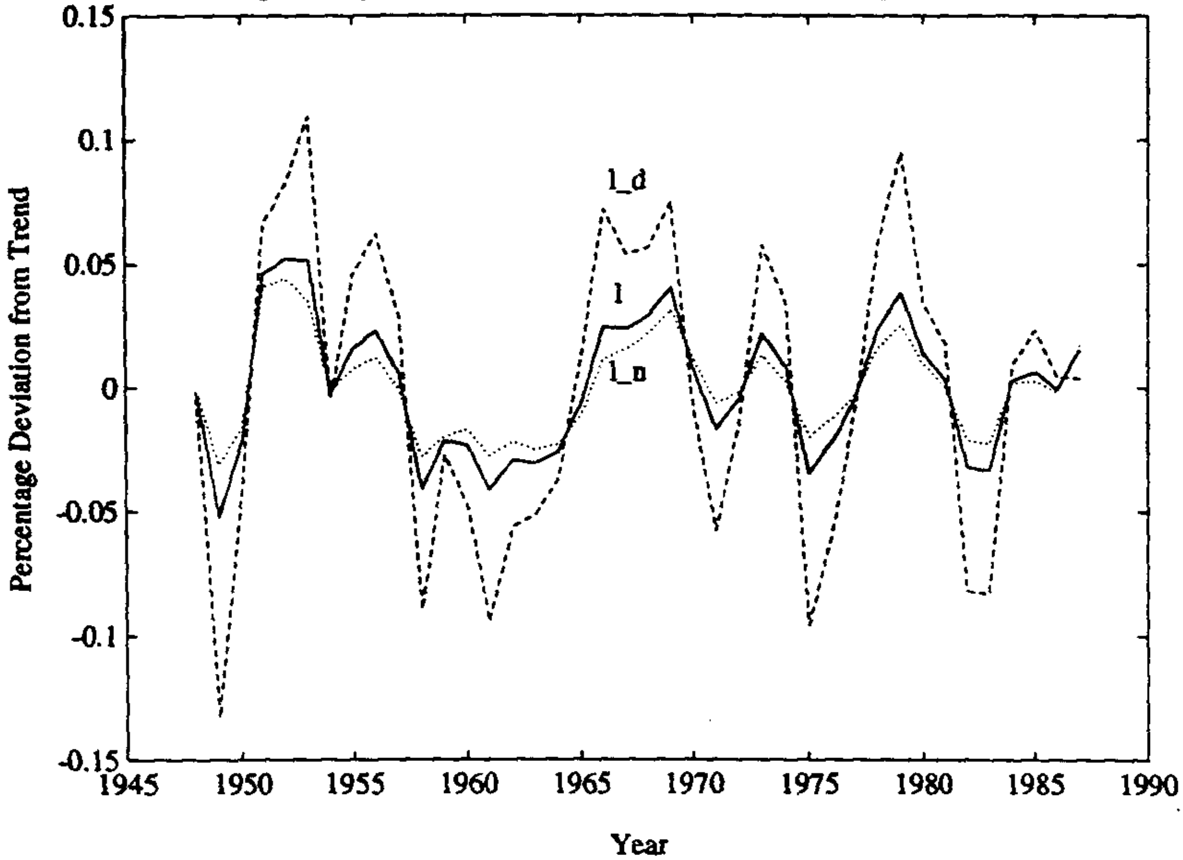

Fig. 3: Expenditure Share Business Cycle for U.S. Economy: 1948-1987

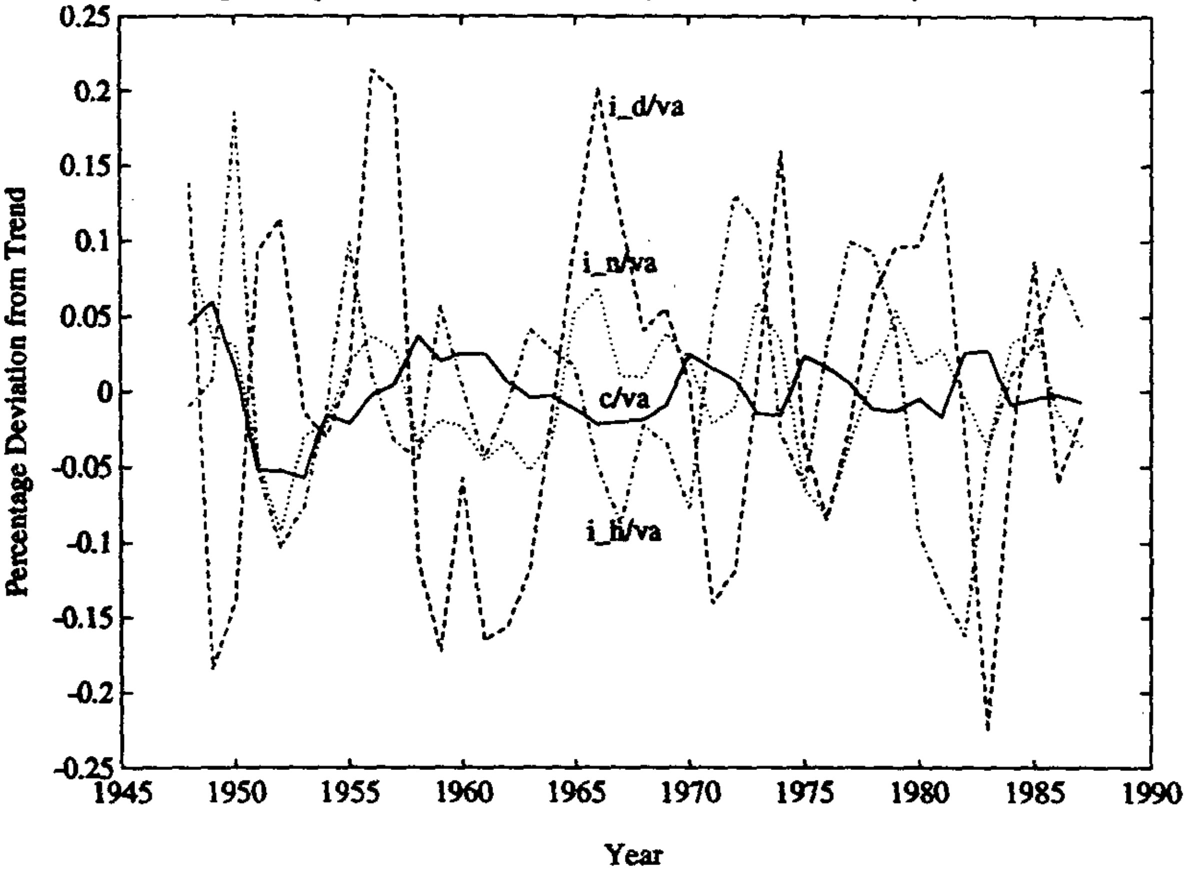

Fig. 4: GNP and the Relative Value-Added Deflator

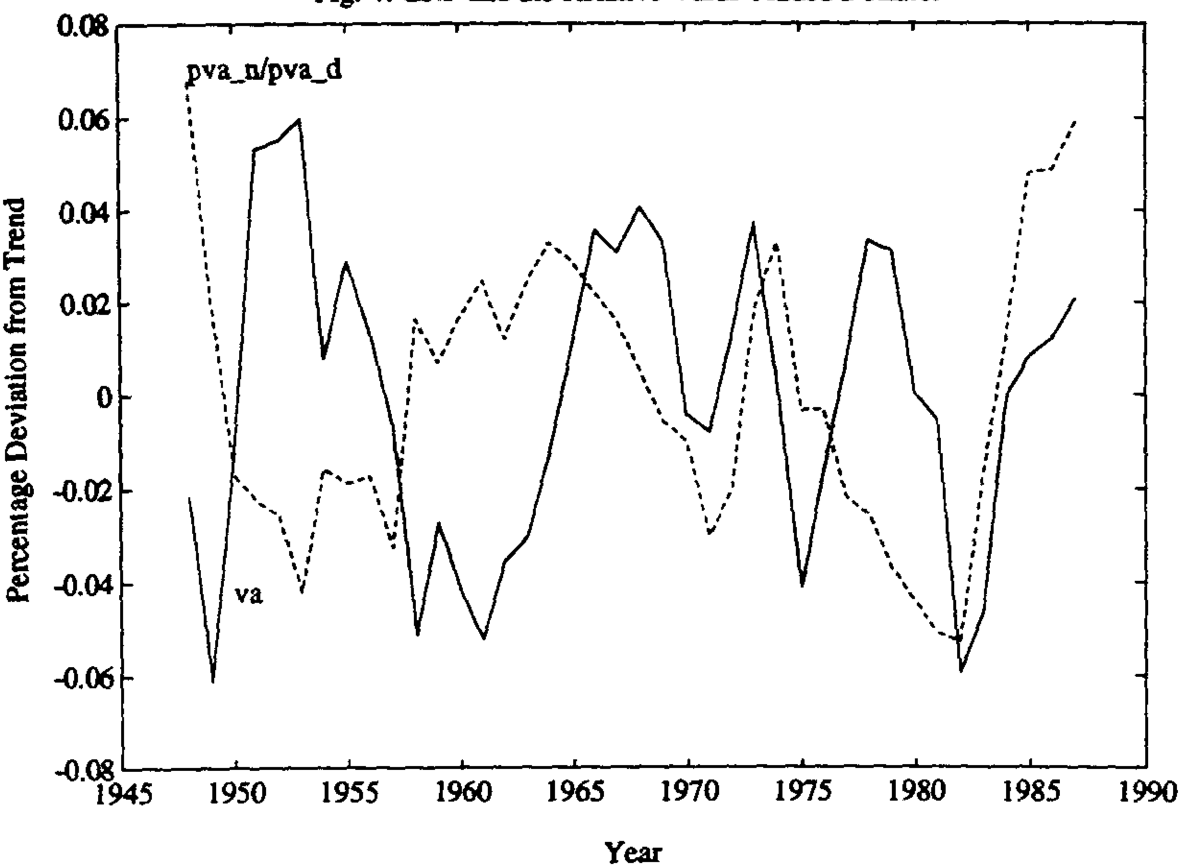

Fig. 5: Real Wages and GNP in the U.S. Economy: 1948-1987

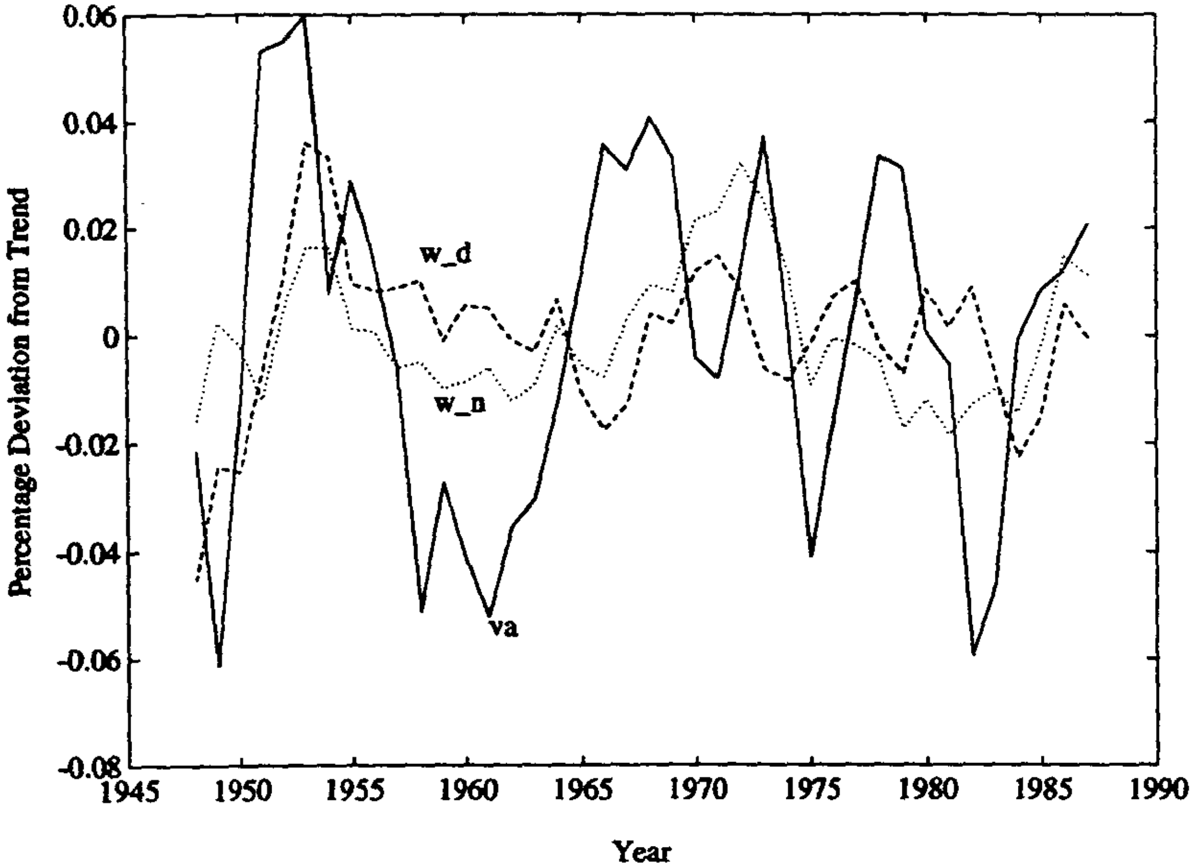

Fig. 6A: Employment Correlations and Substitution

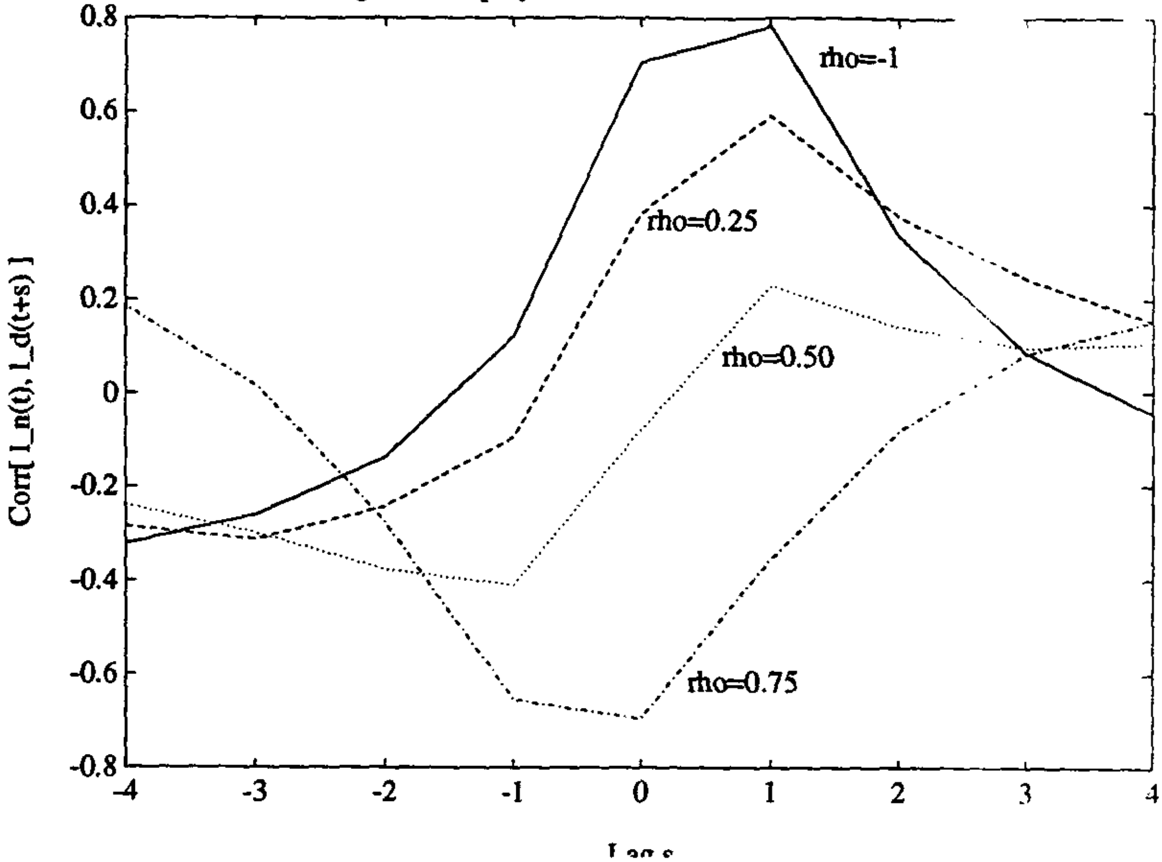

Fig. 6B: Value-Added Correlations and Substitution

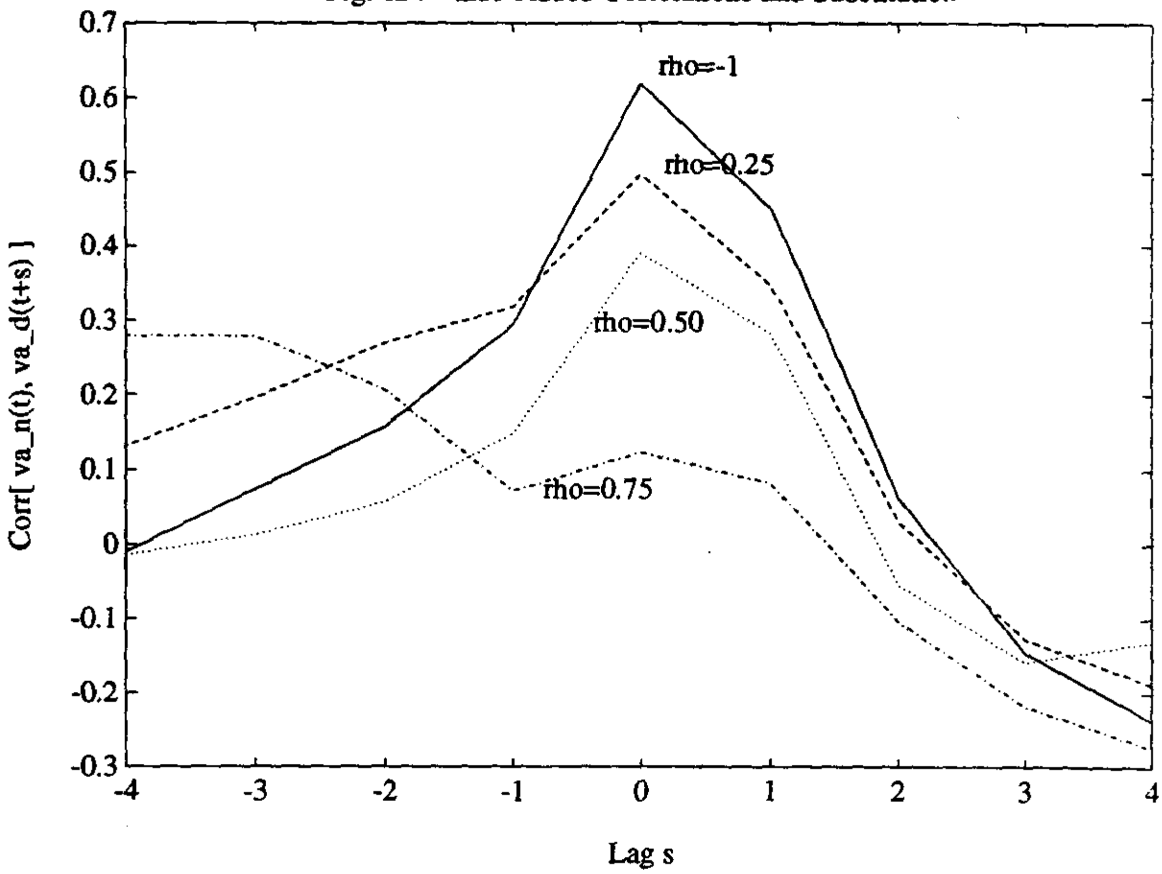

Fig. 7A: Employment Correlations and Productivity Correlations

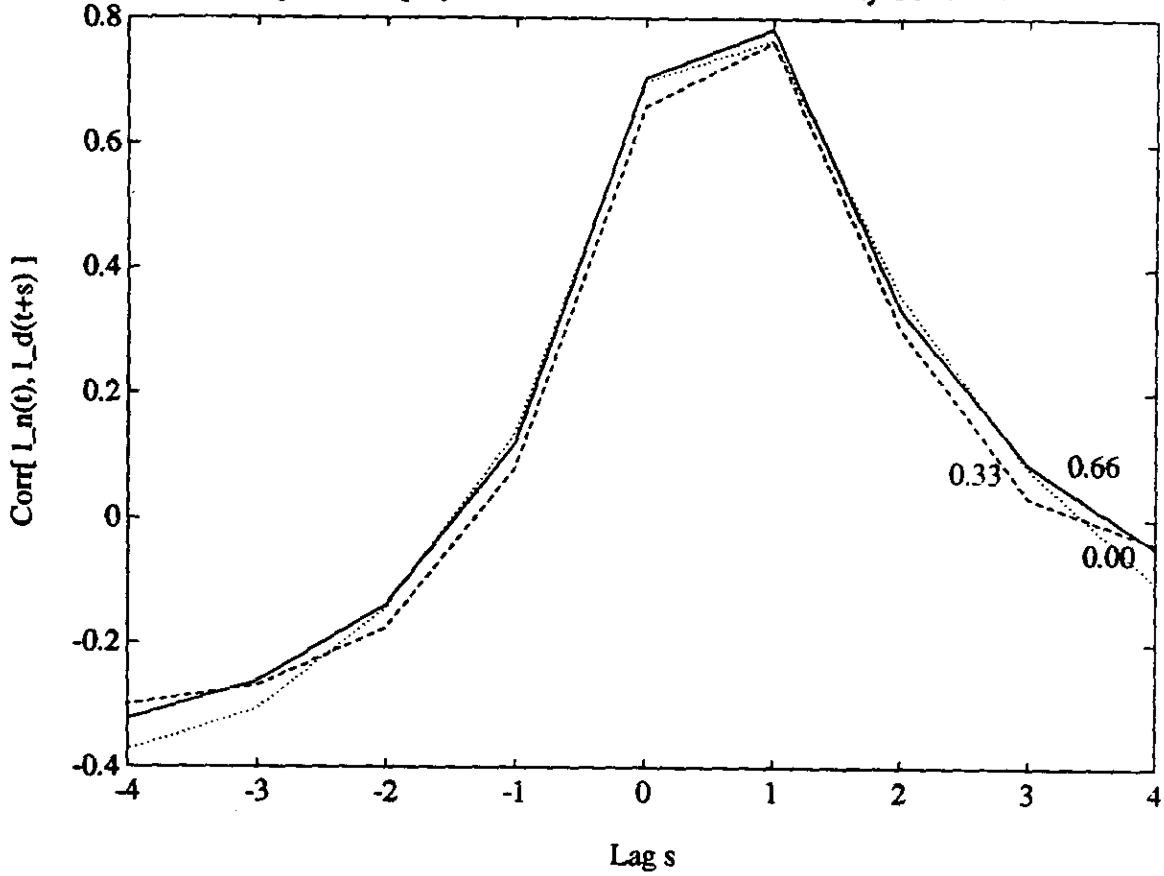

Fig. 7B: Value-Added Correlations and Productivity Correlations

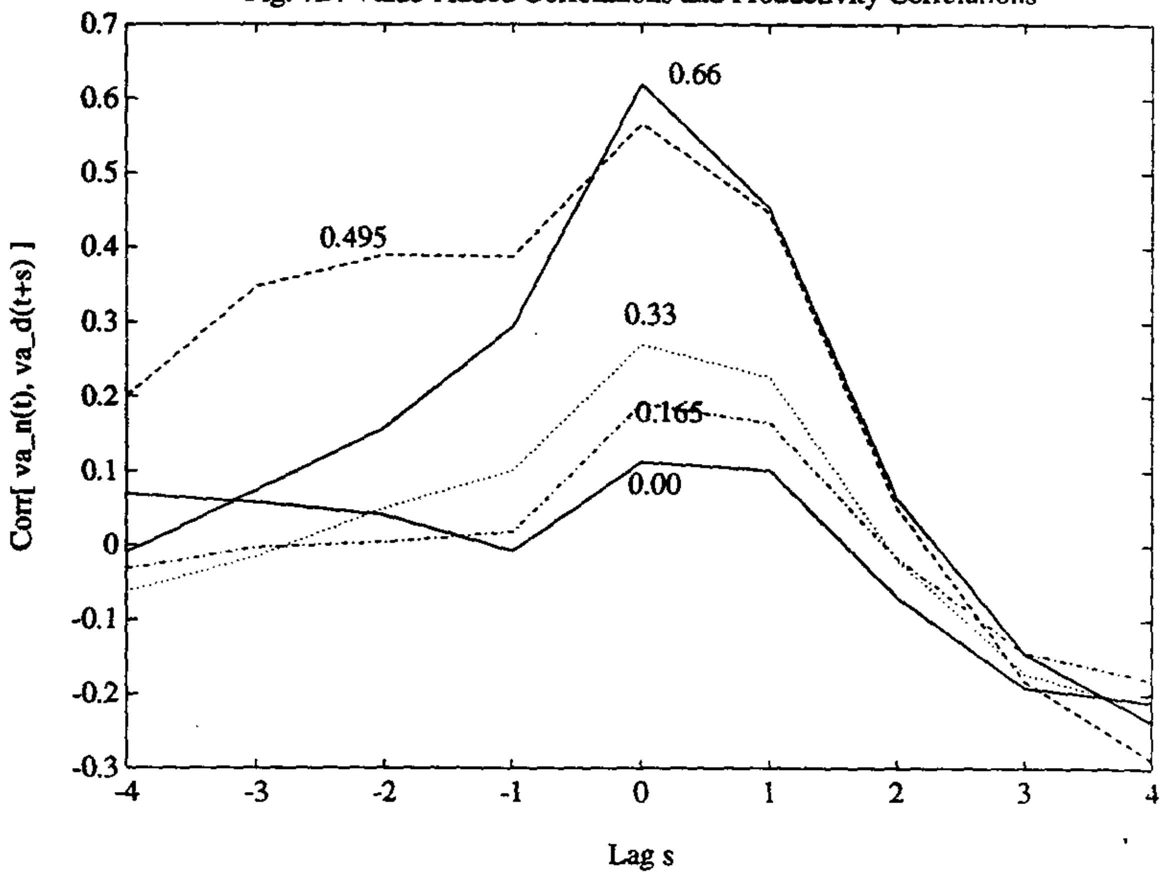

Fig.8: Employment Response to Nondurable Productivity Innovation

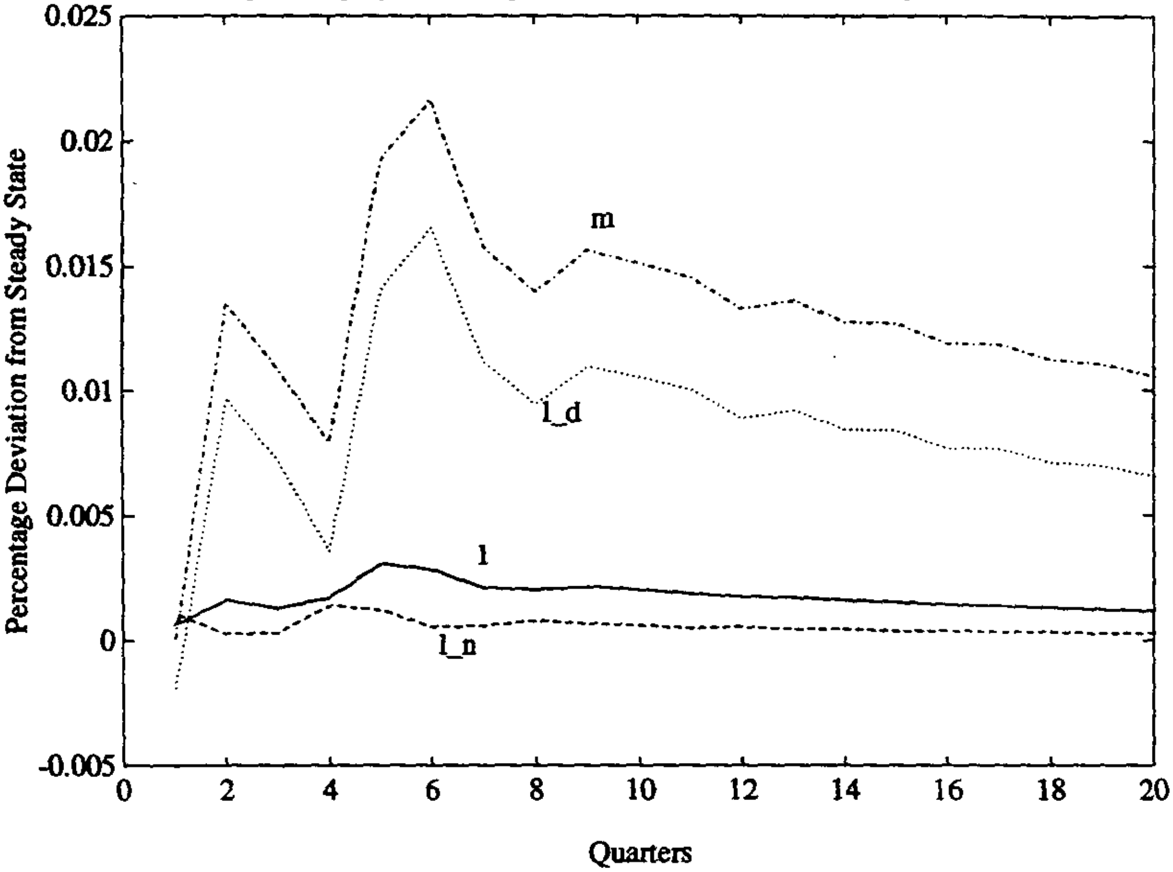

Fig.9: Output Response to Nondurable Productivity Innovation

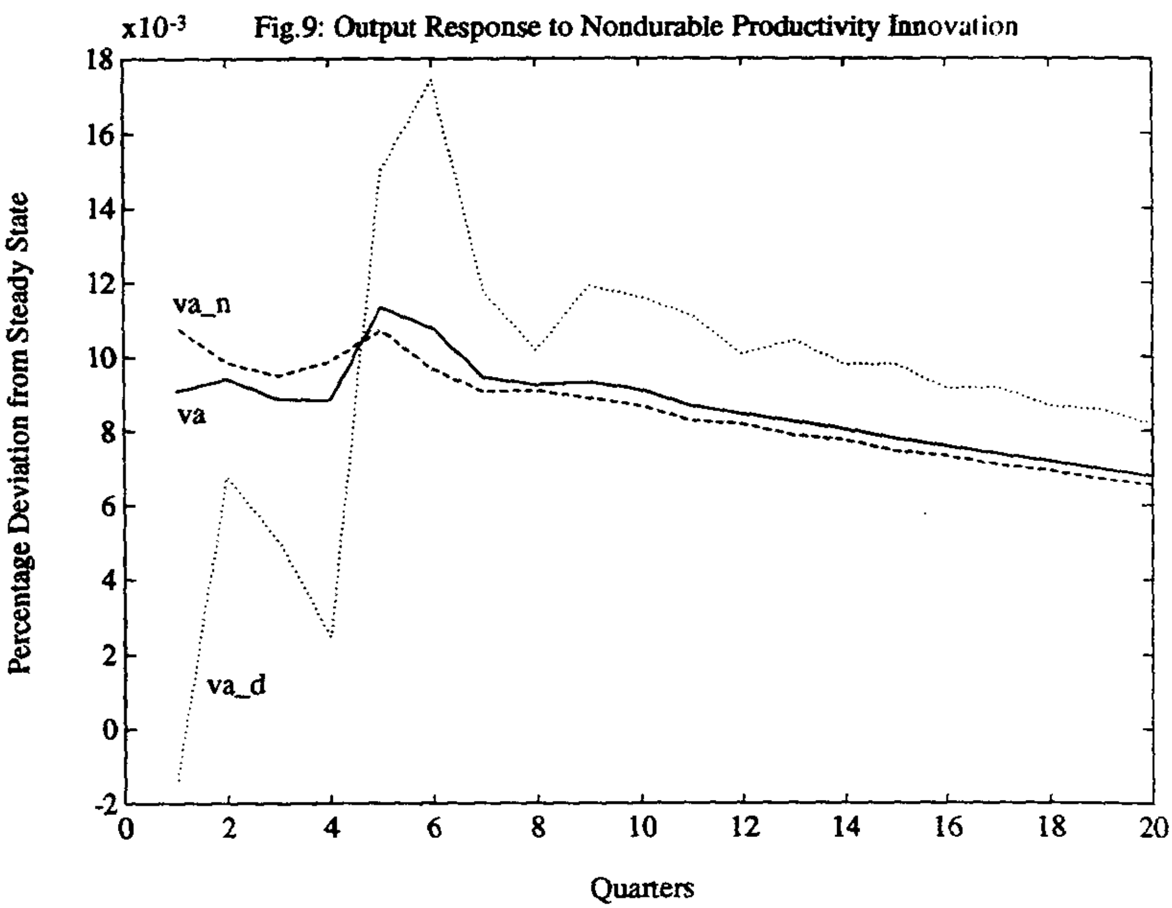

Fig.10: Investment Response to Nondurable Productivity Innovation

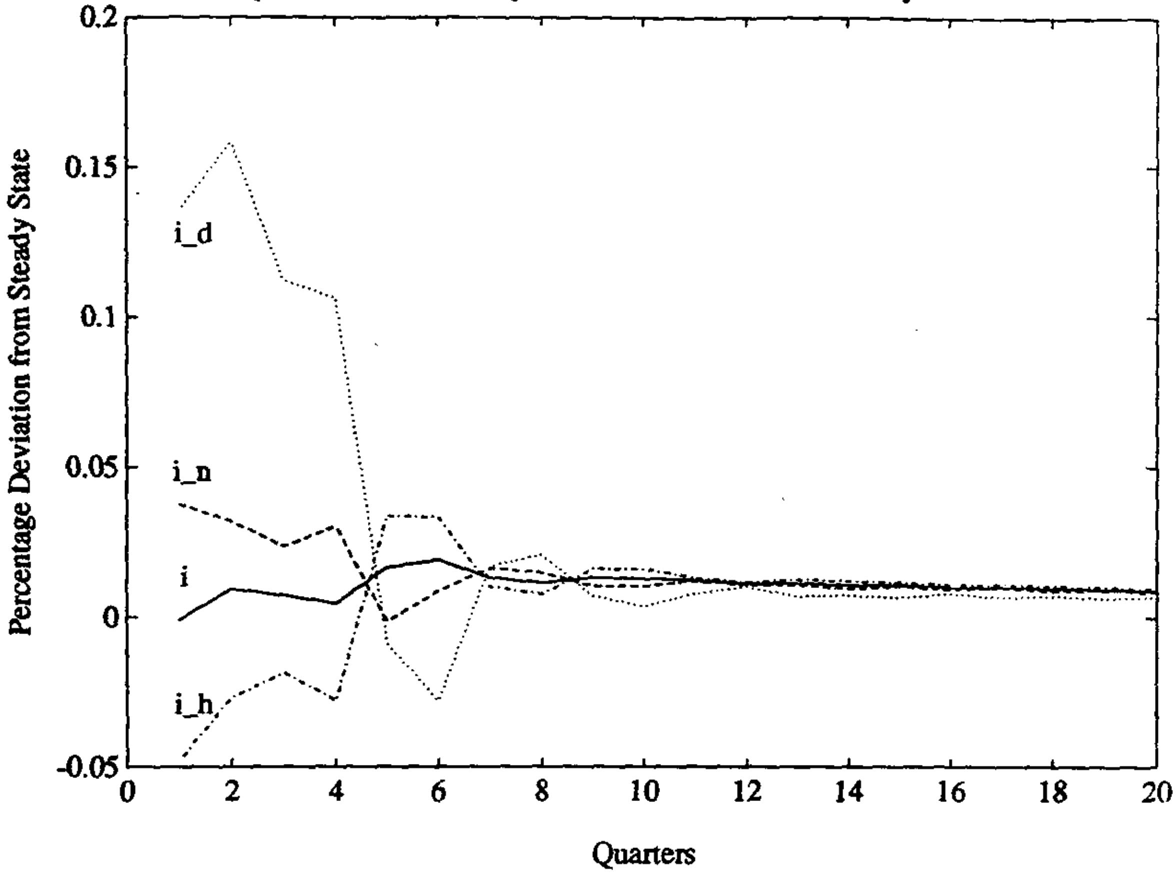

Fig.11: Investment Response to Durable Productivity Innovation