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The Effects of Open Market Operations in a Model of Intermediation and Growth

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ABSTRACT

We examine an otherwise standard model of capital accumulation in which spatial separation and limited communication create a role for money and shocks to portfolio needs create a role for banks. In this context we examine the existence, multiplicity, and dynamical properties of monetary equilibria with positive nominal interest rates. Moderate levels of risk aversion can lead to the existence of multiple monetary steady states, all of which can be approached from a given set of initial conditions. In addition, even if there is a unique monetary steady state, monetary equilibria can be indeterminate, and oscillatory equilibrium paths can be observed. Thus financial market frictions are a potential source of both indeterminacies and endogenously arising economic volatility.

We also consider the consequences of monetary policy actions that rearrange the composition of government liabilities. Contractionary monetary policy activities can have complicated consequences, depending especially on the nature of the steady state equilibrium that obtains when there are multiple steady states. Under plausible conditions, however, a permanent contractionary change in monetary policy raises both the nominal rate of interest *and* the rate of inflation, and reduces long-run output levels. Thus liquidity provision by a central bank—just as by the banking system as a whole—can be growth promoting. Loose monetary policy also in conducive to avoiding development trap phenomena.

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Many mechanisms have now been established through which financial market conditions affect an economy's long-run level of real activity.[1] One that has received particular attention is the provision of liquidity: by providing liquidity banks affect the composition of savings, the availability of investment capital, and -- potentially -- the long-run capital stock.[2] However, often the largest bank in an economy is the central bank, which also typically is the financial institution that issues the economy's most liquid liabilities. Moreover, by conducting open market operations, central banks affect the liquidity of the entire financial system. How, then, do common monetary policy actions -- like changes in open market operations -- affect the long-run performance of an economy when they occur permanently? This is a question that has attracted surprisingly little formal attention, despite its obvious importance.

To attack this question, it is clearly necessary to analyze an economy with at least three kinds of assets: money, government bonds, and capital. In addition, in order for liquidity provision to "matter," it is natural to incorporate a formal liquidity -- providing sector -- such as a banking system -- into the analysis. Thus we need to consider an economy with a fairly rich set of asset markets and financial market institutions to analyze the long-run real consequences of open market activity.

Finally, there has been a good deal of interest in the potential existence of development traps (that is, cases in which intrinsically similar economies have much different long-run behavior), in the scope for indeterminacy and multiple equilibria, and in the potential for economies to experience endogenously arising volatility. Many, for instance Mints (1945), Simons (1948), and Friedman (1960), have argued that monetary policy substantially affects the scope for the indeterminacy of equilibria, especially in so far as the price level is concerned, and for equilibria that display "excessive" economic fluctuations. We therefore might wish to ask whether the open market stance of the central bank affects the potential for development trap phenomena to occur. If it does, this might significantly affect monetary policy prescriptions offered to developing economies. In addition, does "tight" monetary policy tend to reduce the scope for multiplicities to arise, or the scope for endogenously arising volatility? The latter

[1]For theoretical treatments of the linkages between financial markets and real activity see Greenwood and Jovanovic (1990), Bencivenga and Smith (1991), Levine (1991), Cooley and Smith (1992), Bencivenga, Smith and Starr (1993a, b, c), Khan (1993), Parente (1993), and Greenwood and Smith (1993). Empirical evidence appears in Patrick (1966), Cameron (1967), McKinnon (1973), and Shaw (1973). More formal empirical studies of the connections between growth and financial markets are presented by Goldsmith (1969), Antje and Jovanovic (1993), King and Levine (1993a, b), and Levine and Zervos (1995).

[2]Theoretical treatments of liquidity provision and real activity appear in Bencivenga and Smith (1991), Levine (1991), and Bencivenga, Smith, and Starr (1993a, b, c). Liquidity provision receives particular emphasis in the heuristic discussions of Cameron (1967), Hicks (1969), McKinnon (1973), and Shaw (1973).

questions have received little attention in the monetary growth literature.

In some sense this fact is not surprising. Most literature on the integration of financial market institutions into growth models considers purely real economies. In addition, most monetary growth models, at least in the absence of government budget deficits or externalities, deliver unique monetary steady states under any specification of monetary policy, and deliver unique dynamical equilibrium paths that monotonically approach them. Moreover, often either monetary policy must be neutral, in the long-run, or policy changes that are conducive to inflation raise the long-run level of real activity. Both results contradict a wealth of empirical evidence.[3]

This paper presents a monetary growth model with banks that operate to insure agents against random liquidity needs. In this context we examine the existence, multiplicity, and dynamical properties of competitive equilibria, and, in addition, we consider the consequences of permanent changes in monetary policy activity. More specifically, we analyze the effects of open market activity by a central bank, represented here by changes in the ratio of money to government bonds.

Under extreme choices of monetary policy, there may be no steady state competitive equilibria of our economy. We state conditions guaranteeing the existence of a nontrivial steady state equilibrium with valued fiat money and positive nominal interest rates. We also establish conditions under which there is a unique monetary steady state, or under which multiple steady states exists. A proliferation of steady state equilibria can occur whenever agents are sufficiently risk averse (specifically, when they are more risk averse than with logarithmic utility) and whenever monetary policy is sufficiently tight (that is, whenever the ratio of government bonds to money is sufficiently high). Interestingly, if monetary policy is sufficiently loose, there can exist at most one monetary steady state.

When multiple monetary steady state equilibria exist, it can easily transpire that at least one of them is a saddle, while one or more may be a sink. Since our economy has only one initial condition (the initial capital

[3]All of the theoretical literature described in footnote 1 considers purely real models. Monetary growth models like those of Diamond (1965), Mundell (1965), Tobin (1965), Sidrauski (1967a, b), Brock (1974, 1975), and Tirole (1985) yield the implications just described. Azariadis and Smith (1993), Boyd and Smith (1995), or Schreft and Smith (1994a) do present monetary growth models with a financial market friction in which all of these claims must be modified.

Finally, Calvo (1979) considers in some detail the scope for multiple steady state equilibria to arise, as well as their dynamical properties, in two classes of models. In a conventional monetary growth model where production occurs using only capital and labor, he derives the results stated in the text. When money enters the production function he finds that multiple monetary steady states can exist. We employ the former formulation.

Empirical evidence on the adverse effects of inflation for long-run real activity appears in Levine and Renelt (1992), de Gregorio (1992), Wynne (1993), and Bullard and Keating (1994).

stock), more than one steady state can potentially be approached. Indeed, we establish conditions under which a given economy, with a given initial capital stock, can either follow a saddle path approaching a steady state with a relatively low capital stock, or follow any of the continuum of paths that approach a locally stable, high-capital-stock steady state. Economies suffering the former fate will have permanently high nominal interest rates, which signal the existence of a significant distortion. Economies approaching the high-capital-stock steady state will have relatively low nominal and real rates of interest. In any event, two economies with similar, or even identical, initial capital stocks can end up with different long-run output levels.

The possibility that a locally stable, monetary steady state exists, even if the steady state is unique, also indicates that dynamical equilibria of our economy may be indeterminate, even when steady state equilibria are not. Moreover, it can easily transpire that paths approaching the steady state display damped oscillations as they do so. Indeed, we demonstrate that, when the bond-to-money ratio is low, there can exist equilibrium paths that approach no steady state, and display undamped or increasing oscillations. Finally, "tighter" monetary policy is shown to yield no obvious protection against local indeterminacy or endogenous volatility and it tends to be conducive to the occurrence of development traps. Thus the integration of more interesting financial market institutions into models of money and growth is a potential source of nonconvergence phenomena, an indeterminacy of equilibrium, and endogenously arising economic volatility.

The effects of permanent changes in open market activity depend, of course, on the number of steady state equilibria. When there is a unique monetary steady state, "contractionary" monetary policy actions (that is, increases in the ratio of bonds to money) raise the nominal rate of interest and lead to a reduction in steady state output. This occurs for an obvious reason: an increase in government bonds outstanding simply crowds out capital in private portfolios. Thus liquidity provision by the central bank is itself conducive to long-run capital formation.

When there are multiple monetary steady states, of course, a richer set of phenomena can be observed. However, the steady state that delivers the most conventional comparative static results has the feature that contractionary monetary policy actions raise the nominal interest rate, and reduce the steady state capital stock and output level.

Our vehicle for examining these issues is Diamond's (1965) model with money and government bonds, which we modify in one important respect. In particular, we introduce a form of spatial separation and limited communication into the environment in a manner following Townsend (1987) and, most explicitly, Champ, Smith, and Williamson (1992). We allow agents to be randomly relocated, and we assume that interlocation exchange requires currency. Thus agents who are relocated will wish to convert other assets into cash: this kind of randomly arising need for liquidity creates a role for banks that is familiar from Diamond and Dybvig (1983). In this context we analyze steady state and dynamical competitive equilibria and analyze the consequences of different levels of liquidity provision by the central bank.

The scheme of the paper is as follows. Section I describes the environment, while section II considers the nature of trade and the role for banks. Steady state equilibria are analyzed in section III, while section IV examines dynamical equilibria. Section V summarizes and concludes.

I. Environment

We consider an economy consisting of an infinite sequence of two-period-lived overlapping generations, plus an initial old generation. In addition, in each period agents are assigned to one of two locations; we assume that at each date the locations are completely symmetric and that, at the beginning of each period, each location contains a continuum of ex ante identical young agents with unit mass.

We let $t = 0,1,...$ index time. At each date t there is a single final commodity that is produced using a constant returns to scale technology with capital and labor as inputs. This technology is commonly available to all agents. Any agent using K_t units of capital and L_t units of labor can produce $F(K_t, L_t)$ units of the final good at t. We let $f(k_t) \equiv F(k_t, 1)$ denote the intensive production function, where $k_t \equiv K_t / L_t$ is the time t capital-to-labor ratio employed. We assume that f satisfies the following conditions: (a) $f(0) \geq 0$, (b) $f'(k) > 0 > f''(k)$, $\forall\, k \geq 0$, and (c) $f'(0) = \infty$.

At each date the final good can be either consumed or set aside as an investment to be converted into capital. One unit of the final good set aside at t becomes one unit of capital at t + 1 with probability one. Finally capital, once produced, is used in production of the final good and then depreciates completely.

Young agents at each date are endowed with a single unit of labor, which they supply inelastically, and old agents are retired. The initial old agents are each endowed with $k_0 > 0$ units of capital. No other agents have endowments of capital or of the final good at any date.

For simplicity we assume that agents value only second-period consumption, which we denote by c. Let

$u(c)$ denote the common utility function of all agents. We assume throughout that $u(c)$ has the CRRA form $u(c) = c^{1-\rho} / (1 - \rho)$ and, following Diamond and Dybvig (1983), we focus on the case $\rho > 1$.[4]

Three assets are available to agents in this economy. One is obviously the capital investment. We let i_t denote the per-capita quantity of the final good invested at t and k_t denote the per-capita capital stock. Clearly, in equilibrium, $k_{t+1} = i_t$.

The other assets are government liabilities. Let M_t denote the outstanding nominal money supply per capita in each location at t, and let B_t denote the nominal per-capita supply of bonds. Bonds mature in one period, and each bond issued at t is a sure claim to I_t units of currency at t + 1. Thus, I_t is the gross nominal interest rate on default-free government bonds. Money pays no interest, but it has a liquidity advantage over bonds that is described below. Let p_t be the time t price level, which is common across locations, and let $m_t \equiv M_t / p_t$ and $b_t \equiv B_t / p_t$ denote the per-capita values of the real quantity of money and bonds outstanding at date t.

The timing of events within a period is as follows. At the beginning of period t, young agents supply their single unit of labor and earn the real wage rate, w_t. Since agents care only about second-period consumption, all of their income is saved. Savings must be allocated among the three available assets. After these portfolio allocations are made, a fraction $\pi \in (0, 1)$ of all young agents in each location is randomly selected to be transferred to the other location. Although π is constant and known at the beginning of each period, only after savings decisions have been made do agents discover whether they must relocate.

We assume that neither capital investments, nor the consumption good, nor government bonds can be transported between locations. Thus money is the only asset that can be carried between locations, which is the source of its liquidity advantage.[5] In addition, we assume that spatial separation and limited communication preclude agents from exchanging privately issued claims originated in "the other" location; hence only currency is useful in interlocation exchange.[6]

For this reason, agents who discover that they are to be relocated will wish to convert their other asset holdings into currency. Thus random relocations play the same role here that "liquidity preference shocks" play in

[4]The case where $\rho \in [0, 1]$ is considered by Schreft and Smith (1994b).

[5]In effect relocated agents face a cash-in-advance constraint on consumption purchases, while agents who are not relocated do not. The inability to use bonds in interlocation exchange can be motivated by the realistic assumption that bonds are issued only in large denominations.

[6]This formulation follows Townsend (1987), Mitsui and Watanabe (1989), Hornstein and Krusell (1993), and Champ, Smith, and Williamson (1992). The last of these papers contains a detailed discussion of these assumptions and some defense of their realism for the United States and Canada around the turn of the century.

the Diamond-Dybvig (1983) model. As in that context, agents will wish to be insured against the event of premature asset liquidation. The efficient way for this insurance to be provided [see Greenwood and Smith (1993)] is through a bank that takes deposits, holds the primary assets in the model directly, and structures the returns paid to agents in a way that depends on whether they are relocated (in effect, on their date of withdrawal). As in Diamond-Dybvig, all savings will be intermediated through banks of this type. Thus agents who are to be relocated simply make withdrawals from their banks and are then transported to their new location. This timing of events is depicted in figure 1.

The final agent in the model is the government. We assume that the government has no direct expenditures and levies no direct taxes at any date. Thus the government need only manipulate the supply of its liabilities to guarantee that it can meet its interest obligations in each period. Let $R_t \equiv I_t p_t / p_{t+1}$ denote the gross real rate of interest on government bonds between t and t + 1. Then the government budget constraint is

(1) $$R_{t-1} b_{t-1} = (M_t - M_{t-1})/p_t + b_t ; \quad t \geq 0.$$

The initial conditions are $M_{-1} > 0$ and $B_{-1} = 0$. Finally, we assume that the government conducts monetary policy by choosing once and for all a ratio of bonds to money outstanding. That is,

(2) $$b_t / m_t \equiv \beta ; \quad t \geq 0,$$

where β is exogenously chosen by the government. Variations in β, which by assumption occur once and for all in the first period, can be construed as permanent open market operations. Finally, for reasons that will soon become apparent [see equation (15) below], β is assumed to satisfy the condition

(a.1) $$1 > \pi (1 + \beta).$$

II. Trade

In this section we describe the nature of exchange in asset and factor markets and discuss the equilibrium behavior of banks. Finally, the full set of general equilibrium conditions that our economy must satisfy is displayed.

A. Factor Markets

We assume that any agent can run the production process. Producers hire capital and labor in competitive factor markets that operate in each location. Hence capital and labor are each paid their marginal products, so the standard factor pricing relationships obtain:

(3) $$r_t = f'(k_t); \quad t \geq 0,$$

(4) $w_t = w(k_t) \equiv f(k_t) - k_t f'(k_t);\ t \geq 0,$

where r_t is the time t rental rate on capital and w_t is the real wage rate at t. Notice that $w' > 0$ holds. In addition, it will be convenient to make one additional technological assumption. Define $\Omega(k) \equiv k / w(k)$. Then assume that, for all k,

(a.2) $\Omega'(k) > 0.$

Assumption (a.2) is equivalent to

(a.2') $kw'(k) / w(k) \in (0, 1);\ k \geq 0.$

Assumption (a.2) [or (a.2')] holds if, for instance, f is any CES production function with an elasticity of substitution no less than one.[7]

B. Banks

Banks take deposits, hold the model's primary assets directly, and announce deposit return schedules that depend on the depositor's relocation status (or date of withdrawal). In addition, there is free entry into banking. Thus competition ensures that, in equilibrium, banks earn zero profits.

Let d_{mt} (d_{nt}) denote the gross real return on deposits offered by a typical bank to agents who are (are not) relocated at t. Banks announce these returns taking the returns offered by other banks as given. We seek a Nash equilibrium in financial markets--that is, a situation where, given the announcements of other banks, no bank has an incentive to alter its set of announced return schedules. Competition among banks for depositors implies that, in equilibrium, deposit return schedules are chosen to maximize the expected utility of a representative depositor, subject to a set of resource constraints, which we now describe. Given this behavior by banks, all savings will be intermediated. Thus each young agent at t deposits w_t with a bank.

Let m_t denote the quantity of real balances held by a representative bank at t, where all bank asset holdings are in per-depositor terms. Similarly, let b_t denote the bank's real bond holdings at t, and let i_t denote its level of time t capital investment. Then the bank faces the balance sheet constraint

(5) $m_t + b_t + i_t \leq w_t;\ t \geq 0.$

In addition, the bank promises to deliver the gross real return d_{mt} to all of its depositors who are relocated at t. By

[7] Lower elasticities of substitution are considered by Schreft and Smith (1994a). Allowing for lower elasticities of substitution strengthens the potential for multiple steady state equilibria to exist and enhances the potential for endogenously arising economic volatility. We comment further on this point below.

the law of large numbers, a fraction π of the bank's depositors are relocated; thus the bank promises a per depositor payment of $\pi\, d_{mt}\, w_t$ to agents who are relocated at t. Relocated agents must be given currency; therefore the bank's payments to "movers" are constrained by its holdings of real balances:

(6) $\pi\, d_{mt}\, w_t \leq m_t\,(p_t / p_{t+1});\ t \geq 0$

The gross real return on money, p_t / p_{t+1}, appears in (6) because agents who are relocated at t are given real balances at that date, which they then carry into t + 1. The real return on these money holdings is, of course p_t / p_{t+1}. The promised real return to depositors, d_{mt}, incorporates this consideration.

For the fraction $1 - \pi$ of its depositors who are not relocated at t, the bank has promised a gross return of d_{nt} per unit deposited. Thus the bank owes $(1 - \pi)\, d_{nt}\, w_t$ to nonmovers, which it pays at date t + 1. If $I_t > 1$, which we assume throughout, money is dominated in rate of return. Hence the bank will not carry real balances between periods; or, in other words, it holds money only as a reserve to pay agents who are relocated at t. Thus payments to nonmovers will be financed solely out of the bank's holdings of bonds and capital, so that

(7) $(1 - \pi)\, d_{nt}\, w_t \leq R_t\, b_t + r_{t+1}\, i_t;\ t \geq 0.$

must hold.

It will be convenient to represent the bank's choices in terms of the weights attached to different assets in its portfolio. To this end, let $\gamma_t \equiv m_t / w_t$ denote the bank's ratio of reserves to deposits, and let $\mu_t \equiv i_t / w_t$ denote the ratio of its capital investments to its deposits. Then equation (6) can be rewritten as

(6') $d_{mt} \leq \gamma_t\,(p_t / p_{t+1}) / \pi;\ t \geq 0,$

while (7) takes the form

(7') $d_{nt} \leq [r_{t+1}\, \mu_t + R_t\,(1 - \gamma_t - \mu_t)] / (1 - \pi);\ t \geq 0.$

In addition, $\gamma_t \geq 0$ and $\mu_t \geq 0$ must hold. However, we do *not* require that $\gamma_t + \mu_t \leq 1$ be satisfied because banks may in principle borrow from the government or from each other.

We have argued that, in any Nash equilibrium, banks must choose a return schedule (d_{mt}, d_{nt}) and a set of portfolio weights to maximize the expected utility of a representative depositor, subject to (6') and (7'). That is, banks choose d_{mt}, d_{nt}, γ_t, and μ_t to solve the problem

$\max\ \{\pi\,(d_{mt}\, w_t)^{1-\rho} + (1 - \pi)(d_{nt}\, w_t)^{1-\rho}\} / (1 - \rho)$

subject to (6'), (7'), $\gamma_t \geq 0$, and $\mu_t \geq 0$.

As is apparent from (7'), any equilibrium with positive capital investment must satisfy the no-arbitrage condition

(8) $R_t = r_{t+1}$; $t \geq 0$.

When (8) is satisfied, any bank is indifferent between investing in bonds and physical capital. In addition, when (8) is satisfied, the bank's equilibrium reserve-deposit ratio is given by

(9) $\gamma_t = \{1 + [(1-\pi)/\pi]\, I_t^{(1-\rho)/\rho}\}^{-1} \equiv \gamma(I_t)$.

Thus the function γ summarizes the equilibrium behavior of the bank; this function gives the reserve-deposit ratio as a function of the nominal interest rate alone. Notice that, with $\rho > 1$, $\gamma(I) \in [\pi, 1]\ \forall\ I \geq 1$ and that $\gamma(1) = \pi$.[8] In addition, it is straightforward to show that

(10) $I\gamma'(I)/\gamma(I) = [(\rho - 1)/\rho][1 - \gamma(I)] > 0$.

In particular, the representative bank has a reserve-deposit ratio that is increasing in I_t. This occurs for a simple reason: as I_t rises, the bank's optimal choice of reserve holdings is subject to standard income and substitution effects. When $\rho > 1$ holds, income effects dominate substitution effects, and increases in I_t induce banks to become more liquid. The nature of the analysis when $\gamma'(I) \leq 0$ holds is discussed in Schreft and Smith (1994b).

C. Equilibrium

In equilibrium the factor pricing relationships (3) and (4) must be satisfied, as must the no-arbitrage condition (8). In view of (3), (8) can be rewritten as

(11) $R_t = f'(k_{t+1})$; $t \geq 0$.

In addition, money supply must equal money demand at each date. Since all demand for money here derives from banks' demands for reserves, in equilibrium

(12) $m_t = \gamma(I_t)w(k_t)$; $t \geq 0$

is satisfied. Finally $k_{t+1} = i_t$ must hold. From (5), this condition is equivalent to

(13) $k_{t+1} = [w(k_t) - b_t - m_t]$; $t \geq 0$.

These conditions, plus the government budget constraint (1), constitute the full set of equilibrium conditions of the model. For future reference, we note that (1) can be rewritten as

(14) $R_{t-1} b_{t-1} = m_t - m_{t-1}(p_{t-1}/p_t) + b_t$; $t \geq 1$.

[8]It is easy to verify that the implied values d_{mt} and d_{nt} satisfy $d_{nt} > d_{mt}$ whenever $I_t > 1$ holds. Consequently non-movers never wish to withdraw prematurely.

III. Steady State Equilibrium

Equations (2) and (12) imply that, in equilibrium, $m_t + b_t = (1 + \beta)\,\gamma(I_t)\,w(k_t)$ must hold.

Substituting this condition into (13) yields the equilibrium law of motion for the capital stock:

(15) $\quad k_{t+1} = w(k_t)\,[1 - (1 + \beta)\,\gamma(I_t)\,];\ \ t \geq 0.$

In addition, using (2) in the government budget constraint and rearranging terms yields

(16) $\quad m_{t+1}(1 + \beta) = m_t\,(p_t / p_{t+1})\,[1 + \beta\,R_t\,(p_{t+1}/p_t)] \equiv m_t\,(p_t / p_{t+1})\,(1 + \beta\,I_t);\ \ t \geq 0.$

We now observe that, by definition,

(17) $\quad p_t / p_{t+1} = f'(k_{t+1})/\,I_t$

is satisfied. Then, equations (12), (16), and (17) imply that

(18) $\quad \gamma(I_{t+1}) = (1 + \beta\,I_t)\,\gamma(I_t)\,w(k_t)\,f'(k_{t+1})\,/\,(1 + \beta)\,I_t\,w(k_{t+1});\ \ t \geq 0.$

Equation (18) describes the evolution of the nominal interest rate over time.

We now turn our attention to the characterization of steady state equilibria. Dynamical equilibria are taken up in section IV.

A. Characterization

When $k_t = k_{t+1}$ and $I_t = I_{t+1}$ hold, equation (15) reduces to

(19) $\quad k / w(k) \equiv \Omega(k) = 1 - (1 + \beta)\,\gamma(I).$

At the same time, equation (18) becomes

(20) $\quad f'(k) = (1 + \beta)\,I\,/\,(1 + \beta\,I).$

Equations (19) and (20) express the relations between k and I that must obtain in any steady state equilibrium.

Clearly the bond-to-money ratio, β, is a central parameter of the model, which summarizes the "looseness" or "tightness" of monetary policy. Low values of β imply a large supply of money relative to bonds, and hence correspond to "loose" settings of monetary policy. When $\beta = 0$ monetary policy is as loose as possible. Technically matters are substantially different depending on whether $\beta > 0$ or $\beta = 0$ holds: we focus first on the former case. Having characterized steady state equilibria under each configuration of monetary policy, we will return to discuss why these two situations are so different.

1. The Case $\beta > 0$.

Equations (19) and (20) are depicted diagrammatically in figure 2 under the assumption that $\beta > 0$.

Assumption (a.2), along with $\gamma'(I) > 0$, implies that (19) describes a downward sloping locus in the figure: clearly [9]

(21) $dk / dI \big|_{(19)} = -(1 + \beta)\, \gamma'(I) / \Omega'(k) < 0.$

It is also easily verified that

(22) $dk / dI \big|_{(20)} = (1 + \beta) / (1 + \beta I)^2 f''(k) < 0.$

Thus (20) also defines a negatively sloped locus.

Since $\gamma(1) = \pi$ and $\Omega(0) = 0$ both hold, it is apparent that the locus defined by (19) passes through the points $(1, \Omega^{-1}[1 - \pi(1 + \beta)])$[10] and $(\gamma^{-1}(1 / (1 + \beta)), 0)$. Similarly, the locus defined by (20) contains the points $(1, (f')^{-1}(1))$ and $(\gamma^{-1}(1 / (1 + \beta)), (f')^{-1}(1 + \beta)\, \gamma^{-1}(1 / (1 + \beta)) / [1 + \beta\, \gamma^{-1}(1 / (1 + \beta))])$. Thus,when $\beta > 0$, there are two possible configurations of equations (19) and (20). These configurations are depicted in figures 2.a and 2.b. We now delineate these possibilities.

Case 1: $f'\{\Omega^{-1}[1 - \pi(1 + \beta)]\} < 1$. In this case the $I = 1$ intercept of equation (19) lies above that of equation (20). Since the $I = \gamma^{-1}(1 / (1 + \beta))$ intercept of (20) lies above that of (19), a steady state equilibrium, denoted (I^*, k^*), with $I^* > 1$ necessarily exists. This equilibrium may or may not be unique. Figure 2.a depicts the situation where a unique equilibrium exists.

Case 2: $f'\{\Omega^{-1}[1 - \pi(1 + \beta)]\} > 1$. In this case the $I = 1$ intercept of equation (19) lies below that of equation (20). As a consequence, if any steady state equilibria with $I > 1$ exist, there are necessarily at least two. We exhibit below several examples where two steady states with $I > 1$ exist, as shown in figure 2.b. If β is too large, however, equations (19) and (20) may have no intersections. In this event there are no steady state equilibria with $I > 1$, as the bond-money ratio is too large to allow the government to finance its interest obligations when $I > 1$.

As is apparent from figures 2.a and 2.b, if a steady state equilibrium with $I^* > 1$ exists, it must be the case that $f'(k^*) > 1$. In other words, capital overaccumulation, which is always possible in the standard Diamond (1965) model, does not occur here.

In order to characterize when multiple steady state equilibria with $I > 1$ exist, it will evidently be useful to know when equation (19) is steeper than equation (20) wherever the two intersect. This condition is stated in the following proposition.

[9]If assumption (a.2) fails to hold, then $\Omega(k)$ will not be monotonic, and neither will the locus defined by (19). It is then easy to see that there is far more scope for the existence of multiple steady state equilibria (and for endogenous volatility) when (a.2) is violated than when it is satisfied. This issue is considered in further detail by Schreft and Smith (1994a).

[10]Assumption (a.1) implies that $\Omega^{-1}[1 - \pi(1 + \beta)]$ is well-defined.

Proposition 1. Let (I^*, k^*) denote an intersection of equations (19) and (20). Then, at this intersection, $dk/dI\big|_{(19)} < dk/dI\big|_{(20)}$ holds iff

(23) $[k^* w'(k^*)/w(k^*)]\,[w(k^*)/k^* f'(k^*)]\,\{(1+\beta)\gamma(I^*)/[1-(1+\beta)\gamma(I^*)]\} > \{1 - [k^* w'(k^*)/w(k^*)]\}\,[\gamma(I^*)/I^* \gamma'(I^*)]\,(1+\beta I^*)^{-1}$

is satisfied.

Proposition 1 is proved in appendix A. It asserts that, whenever $\beta > 0$, and whenever (23) holds at some steady state with $I^* > 1$, there necessarily exists at least one additional steady state with $I > 1$. We now produce an example economy that possesses multiple monetary steady states.

Example 1. Let $f(k) = Ak^\alpha$, with $A = 0.55$ and $\alpha = 0.335$, and set $\pi = 0.25$ and $\rho = 2$. Table 1 reports the steady state equilibrium values of k and I -- at each of two steady state equilibria -- for three different values of $\beta > 0$. Evidently, when $\beta > 0$ holds, it is straightforward to produce multiple monetary steady states with $I > 1$.

2. The Case $\beta = 0$.

We now present the case where there are no bonds outstanding separately, for reasons that will soon be apparent. When $\beta = 0$ holds we have $\gamma^{-1}(1/(1+\beta)) = \infty$, a situation that clearly differs from figure 2. Also, for the purposes of this sub-section, we assume that the production function has the CES form

(24) $f(k) = (ak^\sigma + b)^{1/\sigma}$; $\sigma < 1$.

Assumption (a.2) requires that $\sigma \geq 0$ also hold.

We can now state the main result of this sub-section.

Proposition 2. Suppose that $\beta = 0$ and (24) hold. Then there is no steady state equilibrium where $I^* > 1$ holds and where (23) is satisfied.

The proof of proposition 2 appears in appendix B. The proposition asserts that, at any intersection of equations (19) and (20), equation (20) is necessarily more steeply sloped than equation (19). Hence there is at most one steady state equilibrium with $I > 1$, as shown in figure 3.

3. Discussion

As example 1 illustrates, no analog of proposition 2 will be generally available when $\beta > 0$ holds. Why, then, is the situation with $\beta = 0$ so different from the situation with $\beta > 0$? The answer has to do with the government's budget constraint. When $\beta = 0$, the government budget constraint implies that the steady state inflation rate must satisfy $p_{t+1} / p_t = I$. Note, in particular, that the steady state inflation rate is independent of I and k. In addition, when $\beta = 0$, (20) reduces to $f'(k) = I$.

When $\beta > 0$ holds, the government budget constraint (16) implies that the steady state inflation rate satisfies

(25) $\quad p_{t+1} / p_t = (1 + \beta I) / (1+ \beta).$

Thus the higher I, the higher is the steady state rate of inflation as the government is forced to finance a higher level of interest obligations on its debt.

When $\beta > 0$ and $\gamma'(I) > 0$ hold, multiple monetary steady states with $I > 1$ can exist. To see this, suppose that I^* is high. Then $\gamma(I^*)$ is also high, diverting savings away from capital formation. Hence $f'(k^*)$ is high, as is the steady state inflation rate. These facts validate the high nominal rate of interest. Similarly, when I^* is low so is $\gamma(I^*)$, $f'(k^*)$, and the steady state rate of inflation. This validates the low nominal interest rate. However, when $\beta = 0$ the steady state inflation rate is independent of I^*. This breaks the link between I^* and the steady state rate of inflation that is essential to the existence of multiple monetary steady state equilibria.

These observations allow us to state one immediate implication. As conventionally conceived, "tight" monetary policy corresponds to a high ratio of bonds-to-money, and conversely for "loose" monetary policy. The "loosest" possible monetary policy corresponds to $\beta = 0$: it is exactly this policy that is most conducive to the existence of a unique monetary state. "Tighter" monetary policies with higher values of β are exactly the policies that are most likely to lead to the existence of multiple monetary steady states and, as we will demonstrate, to development trap phenomena.

B. Comparative Statics

We now turn our attention to the comparative static consequences (that is, the consequences for steady state equilibria) of open market activity. In particular, an increase in β represents an increase in the bond-money ratio and hence corresponds to an open market sale. Conversely, a decrease in β constitutes an open market purchase. Thus higher levels of β correspond to "tighter" monetary policies, in conventional terminology.

The consequences of an increase in β depend on whether or not there is a unique monetary steady state and, in the case of multiple steady states, on which steady state obtains. We now consider the two possible cases.

Case 1. Suppose that there is a unique steady state equilibrium with $I^* > 1$, as in figure 2.a. It is readily verified that an increase in the bond-money ratio from β to β' shifts the locus defined by (19) downwards in figure 4.a. The same increase rotates the locus defined by (20) in a counter-clockwise manner. The result is depicted in figure 4.a. The steady state position of the economy shifts from point A to point B: clearly the nominal interest rate falls and the steady state capital stock rises as a result of the increase in β. Thus, when there is a unique steady state equilibrium, what are conventionally regarded as "contractionary" open market operations raise the long-run level of real activity, and ultimately reduce the nominal rate of interest. In particular, a "contractionary" change in monetary policy does not produce a long-run contraction.

Case 2. The situation with multiple steady state equilibria is depicted in figure 4.b. Here an increase in the bond-money ratio from β to β' shifts the loci defined by (19) and (20) in the manner just described. However, now there are two steady states with $I > 1$. If an increase in β causes the economy to move in the long-run from point C to point D (or point B), the effect of an increase in β is to reduce the nominal rate of interest and to raise the capital stock. This corresponds to the situation in case 1. However, if an increase in β shifts the economy from point A to point B (or point D), the result is to raise the nominal rate of interest in the long-run and to reduce the steady state capital stock.

Conventional wisdom is that "tighter" monetary policy (a higher value of β) raises nominal rates of interest, and potentially reduces output. If this conventional wisdom is correct (which it may not be), then the most interesting situation in this model is when the economy has multiple steady state equilibria (with $\beta > 0$) and asymptotically converges to the low-interest rate, high-capital-stock steady state. The issue of convergence is addressed in section IV.

IV. Dynamical Equilibria

In this section we characterize dynamical equilibria. We will be particularly concerned with three issues: (a) which, if any, of the steady state equilibria with $I > 1$ can be approached, (b) whether dynamical equilibria are unique, and (c) whether endogenously arising volatility will be displayed along dynamical equilibrium paths. As we will show, it can easily transpire that all steady state equilibria can be approached, given the initial capital stock k_o. Moreover, we cannot typically expect there to be a unique dynamical equilibrium when there are multiple steady state equilibria. It will also often be the case that there are multiple dynamical equilibria even if there is a unique monetary steady state with $I^* > 1$. Thus the operation of financial markets can easily be a source of indeterminacy of equilibrium. Finally, we will show that equilibria displaying endogenous volatility can easily be observed. We also describe conditions under which there exist cyclic equilibria where this volatility does not dampen. Thus the financial system can be a source not only of indeterminacy, but also of "excessive fluctuations".

A. The Case of Cobb-Douglas Production

In order to illustrate the possibilities that can arise, we begin with the case where $f(k) = A k^{\alpha}$, with $\alpha \in (0, 1)$. An analysis of more general production functions is undertaken below.

When production is of this Cobb-Douglas form, it is easily verified that $f'(k_{t+1}) / w(k_{t+1}) = [\alpha / (1 - \alpha)] / k_{t+1}$. Using this fact in equation (18), and using (15) to substitute out k_{t+1}, we obtain the following equilibrium law of motion for the nominal rate of interest:

(27) $\gamma(I_{t+1}) = \alpha(1 + \beta I_t)\gamma(I_t) / (1 - \alpha)(1 + \beta) I_t [1 - (1 + \beta)\gamma(I_t)];\ t \geq 0.$

As before, the cases $\beta > 0$ and $\beta = 0$ differ. We consider each case in turn.

1. $\beta > 0$.

When $\beta > 0$ holds, the fact that $\gamma(1) = \pi$ implies that (27) defines a locus passing through the point $(1, \gamma^{-1}\{\alpha\pi / (1 - \alpha)[1 - \pi(1+\beta)]\})$. If $\gamma^{-1}\{\alpha\pi / (1 - \alpha)[1 - \pi(1 + \beta)]\} > (<) 1$ holds,[11] then the $I_t = 1$ intercept of (27) is greater (less) than one. In addition, since $\gamma^{-1}(1) = \infty$, as $I_t \uparrow \gamma^{-1}(1 / (1 + \beta))$, $I_{t+1} \uparrow \infty$.

It remains to describe the slope of the locus defined by (27). We now state a result giving this slope at a steady state.

Proposition 3. Suppose that a steady state equilibrium exists with $I^* > 1$. Let $\lambda(\rho;\beta) \equiv dI_{t+1} / dI_t \big|_{I^*}$. Then

(28) $\lambda(\rho; \beta) = \{[(1 - \alpha) / \alpha](1 + \beta) I^* - [\rho / (\rho - 1)][1 - \gamma(I^*)]^{-1}\} / (1 + \beta I^*).$

Moreover, $\lambda(\rho; \beta) > 0$ holds if

(29) $[(\rho - 1) / \rho](1 + \beta I^*) \geq 1.$

Finally, if

(30) $\beta \geq 1 / (\rho - 1),$

then $\lambda(\rho; \beta) > 0$ holds at any steady state with $I^* > 1$.

Proposition 3 is proved in appendix C.

[11] $\gamma^{-1}\{\alpha\pi / (1 - \alpha)[1 - \pi(1 + \beta)]\} > (<) 1$ holds iff $\alpha / (1 - \alpha) > (<) 1 - \pi(1 + \beta)$. The latter condition, in turn, is equivalent to $\pi(1 + \beta) > (<) (1 - 2\alpha) / (1 - \alpha)$.

It is now apparent that there are three possible interesting configurations of the locus defined by (27). These are depicted in figures 5.a - 5.c. We describe each case in turn.

Case 1. $\gamma^{-1} \{\alpha \pi / (1 - \alpha) [1 - \pi (1 + \beta)]\} < 1$. In this case, which is depicted in figure 5.a, the $I_t = 1$ intercept of equation (27) is less than one. It follows that there exists at least one steady state with $I^* > 1$ and $\lambda(\rho; \beta) > 1$, and obviously it is possible that this steady state is unique. If there is a unique steady state then clearly it must be the case that $I_t = I^* \forall t$, so that the interest rate is unchanging over time.[12]

Case 2. $\gamma^{-1} \{\alpha \pi / (1 - \alpha) [1 - \pi (1 + \beta)]\} > 1$. When this inequality is satisfied there are two general possibilities. The first of them is depicted in figure 5.b. This configuration of (27) would necessarily obtain if, for instance, (30) was satisfied.

If $dI_{t+1} / dI_t > 0$ holds $\forall I_t$, then generically there are either no steady states with $I > 1$, or there are at least two of them. When multiple monetary steady states exist, the low-interest-rate steady state (I_A) can be approached from any initial $I_0 \in (1, I_B)$. The high-interest-rate steady state can only be approached by having $I_0 = I_B$. Since we have a free choice of I_0, clearly either steady state can be approached. Dynamical equilibrium paths approaching the low-interest-rate steady state will display locally monotone dynamics.

Table 1 reports values of dI_{t+1} / dI_t at the low-interest-rate steady state for the economy of example 1. The example corresponds to this case for the values of β reported.

Case 3. $\gamma^{-1} \{\alpha \pi / (1 - \alpha) [1 - \pi (1 + \beta)]\} > 1$. When this inequality holds, it is possible that $dI_{t+1} / dI_t < 0$ holds for some $I_t > 1$. When this occurs it is also possible that there exists a steady state equilibrium with $I^* > 1$ and $\lambda(\rho; \beta) < 0$. If such a steady state exists, there necessarily exists at least one additional steady state with a higher rate of interest, and with $dI_{t+1} / dI_t > 1$. As before we let I_A (I_B) denote the low (high)-interest-rate steady state.

Clearly both steady states can be approached by setting $I_0 = I_A$, or $I_0 = I_B$. Thus, as before, monetary equilibrium is indeterminate. It is also possible that the low-interest-rate steady state has $\lambda(\rho; \beta) > -1$, in which case it is asymptotically stable. Indeed this is the situation depicted in figure 5.c. Here not only is there an indeterminacy of monetary equilibrium, but paths approaching the low-interest-rate steady state display endogenous oscillations as they do so. Indeed, it is possible that there exist equilibria displaying cycles of two or more periods: one such equilibrium is displayed in figure 5.c. The possibility of equilibria displaying undamped oscillation is

[12]The $t = 0$ version of the government budget constraint implies that $(1 + \beta) m_0 = (1 + \beta) \gamma (I_0) w (k_0) = M_{-1} / p_0$. Thus clearly I_0 is a free initial condition; and in the case of a unique steady state we must set $I_0 = I^*$.

explored more formally in the next subsection.

2. $\beta = 0$.

When $\beta = 0$ holds we can provide a particularly sharp characterization of the dynamical behavior of the nominal rate of interest. In particular, we have already shown that, in this case, there is a unique steady state with I^* > 1. We now provide conditions under which that steady state is asymptotically stable, and hence under which the equilibrium is indeterminate. We also show that $\lambda(\rho; 0) < 0$ necessarily holds, so that any paths that do approach the steady state display endogenous oscillations as they do so. Finally, we state conditions under which there exists a value ρ_0 such that $\lambda(\rho_0; 0) = -1$. In this case, as ρ is varied, a flip bifurcation occurs at ρ_0, and equilibria displaying two-period cycles emerge. Hence not only are indeterminacy and endogenous volatility possible, but economic oscillations need not dampen over time.

When $\beta = 0$ holds, equation (27) reduces to

(27') $\gamma(I_{t+1}) = [\alpha / (1 - \alpha)]\, \gamma(I_t) / I_t\, [1 - \gamma(I_t)]; \; t \geq 0.$

The following lemma gives a useful transformation of this law of motion for I_t.

Lemma 1. (a) Define the function $h: [1, \infty) \rightarrow \Re_+$ by

(31) $h(I) \equiv \{[(1 - \alpha) / \alpha\,][(1 - \pi) / \pi\,]\, \gamma(I)\}^{-\rho}.$

Then equation (27) is equivalent to

(32) $I_t = h(I_{t+1}); \; t \geq 0.$

(b) $h'(I) < 0$ holds, $\forall I$. (c) If

(a.3) $\pi \geq 1 - \alpha,$

then $h: [1, \infty) \rightarrow [1, \infty)$.

The proof of lemma 1 appears in appendix D. We will maintain the assumption (a.3) for the remainder of this subsection.

The equilibrium law of motion (32) is depicted in figure 6. Evidently a steady state equilibrium with $I^* > 1$ exists iff $h^{-1}(1) > 1$: this condition is easily shown to be equivalent to

(33) $\pi > (1 - 2\alpha) / (1 - \alpha).$

It is also straightforward to verify that (33) is satisfied if (a.3) holds: thus (a.3) guarantees the existence of a steady state equilibrium with $I^* > 1$.

As is apparent from (27'), the steady state value I^* satisfies

(34) $\alpha / (1-\alpha) = I^* [1 - \gamma(I^*)] = [(1-\pi)/\pi] (I^*)^{1/\rho} / \{1 + [(1-\pi)/\pi] (I^*)^{(1-\rho)/\rho}\}$.

For fixed values of π and α obeying (a.3), (34) gives I^* as a differentiable function of ρ. We denote this relation by $I^* = g(\rho)$. Straightforward differentiation of (34) establishes that

(35) $\rho g'(\rho) / g(\rho) = \ln g(\rho) / \{1 + \rho [(1-\pi)/\pi] g(\rho) (1-\rho)/\rho\}$.

Thus, since $g(\rho) > 1$, $g'(\rho) > 0$ holds. In particular, as agents become more risk averse, the steady state nominal interest rate must increase.

We are now interested in establishing conditions under which the steady state equilibrium is and is not asymptotically stable. These conditions are stated in the next lemma.

Lemma 2. We have that

(36) $dI_{t+1} / dI_t \big|_{I^*} \equiv \lambda(\rho;0) = -[(1-\alpha)/\alpha] g(\rho)/(\rho-1)$.

Therefore the steady state is asymptotically stable (unstable) if

(37) $I^* = g(\rho) < (>) (\rho - 1) [\alpha / (1-\alpha)]$.

The proof of lemma 2 appears in appendix E. The lemma indicates that the steady state is asymptotically stable (unstable) if the steady state nominal interest rate is not too large (small). Evidently the steady state must be unstable if ρ is sufficiently close to one.

When the steady state is asymptotically stable, obviously there exists a continuum of dynamical equilibrium paths displaying damped oscillation, as depicted in figure 6. Hence indeterminacy and endogenously arising volatility must be observed. However, it may also be the case that two-period cycles exist, so that the interest rate oscillates in an undamped way. We now explore this possibility.

Define the function

(38) $Q(\rho) \equiv (\rho - 2) [(1-\pi)/\pi] \{[\alpha / (1-\alpha) (\rho - 1)\}^{(1-\rho)/\rho}$.

We then have the following result.

Proposition 4. Suppose that there exists a value $\hat{\rho}$ satisfying $Q(\hat{\rho}) > 1$. Then there also exists a value $\rho_0 > (2-\pi)/(1-\pi)$ such that $\lambda(\rho_0; 0) = -1$ and $\lambda_1(\rho_0; 0) \neq 0$. Moreover, there exist values of ρ near ρ_0 such that - at these values of ρ - equilibria with two period orbits exist.

Proposition 4 is proved in appendix F. The proposition states conditions guaranteeing that, for some values of ρ, there is an equilibrium where $I_t = I_0 > 1$, t odd, and $I_t = I_e > 1$, t even (with $I_e \neq I_0$.) Hence endogenous volatility exists and does not dampen asymptotically. These oscillatory equilibria can exist when the steady state is asymptotically stable; in this case there is obviously a large set of oscillatory equilibria.

For any specific parameter values, the existence of the desired value $\hat{\rho}$ is easily verified. We now illustrate this point with an example.

Example 2. Suppose that $\alpha = 1/3$ and that $\pi = 2/3$. It is easy to verify that $Q(8) = 1.0024$. Thus the desired value ρ_0 exists; here $\rho_0 < 8$ holds.

When the conditions of proposition 4 are satisfied, then, we can observe two-period cycles. One such cycle is depicted in figure 6.

When $\beta = 0$, monetary policy is "loose" in the sense we have previously defined. Loose monetary policies can be consistent with unique equilibria. Indeed equation (37) indicates that this must be the case if $1 \geq (\rho - 1)\,\alpha / (1 - \alpha)$ holds. However, when agents are fairly risk averse, setting $\beta = 0$ can easily imply the existence of an array of oscillatory equilibria. Hence, without a fairly precise knowledge of the level of risk aversion, it is not possible to know whether loose monetary policies yield unique equilibria or not.

3. The Evolution of the Capital Stock

Under the assumption of Cobb-Douglas production, the evolution of the sequence $\{I_t\}$ is governed by the scalar difference equation (27) and hence is independent of the evolution of $\{k_t\}$. However, the evolution of $\{k_t\}$ does depend on the equilibrium sequence $\{I_t\}$. We now briefly describe the dynamics of k_t as well as I_t.

Equation (15) implies that

(39) $$k_{t+1} - k_t = w(k_t)\,[1 - (1 + \beta)\,\gamma(I_t)] - k_t.$$

It follows that $k_{t+1} - k_t \geq 0$ obtains whenever

(40) $$\Omega(k_t) \leq 1 - (1 + \beta)\,\gamma(I_t).$$

Equation (40) at equality is depicted in figures 7.a - 7.c. Obviously the locus defined by (40) at equality is upward sloping, and points below (above) that locus have $k_{t+1} - k_t > (<)\ 0$.

There are now several possible configurations of the phase diagram. As is apparent from figures 5.a - 5.c, $I_{t+1} - I_t = 0$ holds iff $I_t = I^*$; that is, iff I_t corresponds to some steady state value. Using this observation, we now consider the four possibilities with respect to the nature of the phase diagram.

Case 1. Suppose that there is a unique steady state with $I^* > 1$, and that $\lambda(\rho; \beta) > 1$. Then if $I_t < (>) I^*$, $I_{t+1} - I_t < (>) 0$ holds. It follows that the phase diagram for the economy has the configuration depicted in figure 7.a. The unique steady state with $I^* > 1$ is a saddle, and paths approaching it do so monotonically, with $I_t = I^* \; \forall \; t$ and with k_t evolving according to $k_{t+1} = w(k_t)[1 - (1+\beta)\gamma(I^*)]$.

Case 2. Suppose that there is a unique steady state with $I^* > 1$, and that $\lambda(\rho; \beta) < 0$. This corresponds to the situation in figure 6; the phase diagram for this case is depicted in figure 7.b. Clearly if $I_t < (>) I^*$, $I_{t+1} - I_t > (<) 0$. The unique steady state can either be a sink [if $\lambda(\rho; \beta) > -1$] or a saddle [if $\lambda(\rho; \beta) < -1$]. In the former case fluctuations in both the nominal interest rate and the capital stock (real interest rate) can be observed along paths approaching the steady state. In the latter case paths approaching the steady state have $I_t = I^* \; \forall \; t$, and the capital stock monotonically approaches its steady state value, as in case 1.

Case 3. Now suppose that there are multiple monetary steady states with $I > 1$, and that each of them has $\lambda(\rho; \beta) > 0$ (as in figure 5.b). Let I_A (I_B) denote the low (high)-interest rate steady state. Then $I_{t+1} - I_t > 0$ holds if $I_t < I_A$ or $I_t > I_B$, and $I_{t+1} - I_t < 0$ holds if $I_t \in (I_A, I_B)$. The resulting phase diagram is depicted in figure 7.c. The low (high)-interest-rate steady state is a sink (saddle), and paths approaching each display locally monotone dynamics.

Case 4. Suppose that there are multiple monetary steady states with $I > 1$, and that the equilibrium law of motion for I_t has the configuration depicted in figure 5.c. The phase diagram corresponding to this situation is also represented by figure 7.c. Evidently the high-interest-rate steady state must be a saddle, while the low-interest-rate steady state may be either a sink or a saddle. In the latter case, paths approaching the low-interest-rate steady state display constant nominal rates of interest, and a monotonically evolving capital stock.

As the phase diagrams make apparent, in the case of multiple steady states either steady state can be approached from any nonnegative initial capital stock. The implications of this observation are discussed below.

1. The case of cycles.

Suppose that there is an equilibrium where the nominal interest rate has $I_t = I_o$, t odd, and $I_t = I_e$, t even, and to fix ideas let $I_o < I_e$ hold. We now describe the evolution of the capital stock in this case. To do so we define $x_t \equiv \ln k_t$, and $a_t \equiv \ln A[1 - (1+\beta)\gamma(I_t)]$; $t = e, o$.[13] Then it is easy to verify that $a_o > a_e$, and that x_t evolves according to

(41.a) $\quad x_{t+1} = a_o + (1-\alpha)a_e + (1-\alpha)^2 x_{t-1}$; t odd

[13]Of course $\beta = 0$ might hold.

(41.b) $x_{t+1} = a_e + (1 - \alpha)\, a_0 + (1 - \alpha)^2 x_t$; t even.

Evidently, the capital stock will itself asymptotically display a two-period cycle, with $x_t = x_e$, t even, and $x_t = x_0$, t odd. $x_0 < x_e$ will hold, so that the capital stock will be high exactly when the nominal rate of interest is high.

If k_0 satisfies $k_0 \in (\exp(x_0), [\exp(x_e - a_0)]^{1/\alpha})$, then both k_0 and k_1 lie between the asymptotic values for k_t corresponding to even and odd periods. As a consequence, $\{k_t\}$ will display increasing oscillation over time and will asymptotically converge to its own two period orbit.

B. More General Production Functions

Under Cobb-Douglas production, the evolution of the nominal interest rate is governed by a scalar difference equation. With more general production technologies this is no longer the case. We now consider more general production functions and limit our focus to the analysis of local dynamics.

The equilibrium time path of $\{k_t, I_t\}$ is governed by the dynamical system consisting of equations (15) and (18). To begin, it will be useful to transform (18) as follows. Define the function H(k) by

(42) $H(k) \equiv f'(k) / w(k)$.

It is straightforward to show that

(43) $k\, H'(k) / H(k) = -\, [k\, w'(k) / w(k)\,]\, [f(k) / kf'(k)]$.

Obviously H is a decreasing function.

Employing the definition of H in (18), and using (15) to substitute out k_{t+1}, we can rewrite the former equation as

(44) $\gamma\,(I_{t+1}) = (1 + \beta\, I_t)\, \gamma\,(I_t)\, w(k_t)\, H\{w(k_t)\, [1 - (1 + \beta)\gamma(I_t)]\} \,/\, (1 + \beta)\, I_t$.

We henceforth work with the dynamical system consisting of equations (15) and (44).

Suppose that we now consider a linear approximation to this system in a neighborhood of any steady state (k^*, I^*) with $I^* > 1$. Then we have

(45) $(k_{t+1} - k^*, I_{t+1} - I^*)' = J\, (k_t - k^*, I_t - I^*)'$,

where J is the Jacobian matrix

$$J = \begin{bmatrix} \partial k_{t+1} / \partial k_t & \partial k_{t+1} / \partial I_t \\ \partial I_{t+1} / \partial k_t & \partial I_{t+1} / \partial I_t \end{bmatrix}$$

It is straightforward to verify that, evaluated at the steady state,

(46) $\partial k_{t+1} / \partial k_t = k^* w'(k^*) / w(k^*)$

(47) $\partial k_{t+1} / \partial I_t = -(k^* / I^*) \{(1 + \beta) \gamma (I^*) / [1 - (1 + \beta) \gamma (I^*)]\} [I^* \gamma' (I^*) / \gamma (I^*)]$

(48) $\partial I_{t+1} / \partial k_t = (I^* / k^*) \{1 - [k^* w'(k^*) / w(k^*)] [f(k^*) / k^* f'(k^*)]\}[k^* w'(k^*) / w(k^*)] [\gamma (I^*) / I^* \gamma' (I^*)]$

(49) $\partial I_{t+1} / \partial I_t = 1 - [\gamma (I^*) / \gamma' (I^*)] / I^* (1 + \beta I^*) - [k^* H'(k^*) / H(k^*)] (1 + \beta) \gamma (I^*) / [1 - (1 + \beta) \gamma (I^*)]$.

Let $T(k^*, I^*)$ and $D(k^*, I^*)$ denote the trace and determinant of J, respectively. Then it is straightforward to calculate that

(50) $D(k^*, I^*) = [k^* w'(k^*) / w(k^*)] \{[1 - (1+ \beta) \gamma (I^*)]^{-1} - [\gamma (I^*) / I^* \gamma' (I^*)] / (1 + \beta I^*)\}$

(51) $T(k^*, I^*) = 1 + D(k^*, I^*) + x(k^*, I^*)$,

where

(52) $x(k^*, I^*) \equiv [k^* w'(k^*) / w(k^*)] [w(k^*) / k^* f'(k^*)] (1 + \beta) \gamma (I^*) [1 - (1 + \beta) \gamma (I^*)]^{-1} -$

$\{1 - [k^* w'(k^*) / w(k^*)]\} [\gamma (I^*) / I^* \gamma' (I^*)] (1 + \beta I^*)^{-1}$

The following result is then immediate.

Proposition 5. (a) $x(k^*, I^*) > (<) 0$ if (19) is steeper (flatter) than (20) where they intersect. (b) $T(k^*, I^*) >(<) 1 + D(k^*, I^*)$ if (19) is steeper (flatter) than (20) where they intersect. (c) $D(k^*, I^*) < 0$ holds iff

(53) $1 - (1 + \beta) \gamma (I^*) > (1 + \beta I^*) [1 - \gamma (I^*)] (\rho - 1) / \rho$.

Part (a) of the proposition is immediate from a comparison of equations (23) and (52), while part (b) follows directly from (51). Part (c) is implied by equations (10) and (50).

Proposition 5 allows us to say a great deal about local dynamics. As before, we consider $\beta = 0$ and $\beta > 0$ separately.

1. $\beta = 0$.

If $\beta = 0$ holds, then clearly (53) is satisfied. Moreover, in this situation there is a unique steady state equilibrium with $I^* > 1$; this steady state is depicted in figure 3. Since (19) is flatter than (20) where they intersect, $T(k^*, I^*) < 1 + D(k^*, I^*)$ holds. This observation implies that J has no eigenvalues greater than one, while $D(k^*, I^*) < 0$ implies that J has one positive eigenvalue. Thus the steady state equilibrium is either a saddle or a sink. Both situations can easily emerge, as we illustrate with the following examples.

Suppose that f(k) is of the CES form given in (24). Then it is readily verified that (19) and (20) reduce to

(54) $ak^{\sigma} / b = (1 + \beta) I [1 - (1 + \beta) \gamma (I)] / (1 + \beta I)$

and

(55) $a^{1/\sigma} [1 + (b/ak^{\sigma})]^{(1-\sigma)/\sigma} = (1 + \beta) I / (1 + \beta I)$.

In addition, $kw'(k) / w(k) = (1 - \sigma) (ak^{\sigma} / b) / [1 + (ak^{\sigma} / b)]$ and $kf'(k) / w(k) = ak^{\sigma} / b$ hold.

Example 3. Let $a = b = 1$, $\rho = 2$, $\sigma = 0.791746$, and $\pi = 0.154387$ hold. Then, with $\beta = 0$, it is easy to check that $k^* = 1$, $I^* = 1.2$, $\gamma(I^*) = 1/6$, $D(k^*, I^*) = -0.124953$, and $T(k^*, I^*) = -1.254223$ hold. Since $T(k^*, I^*) < -1 - D(k^*, I^*)$, J has an eigenvalue less than -1. It follows that the steady state with $I^* > 1$ is a saddle. Paths approaching it display locally monotone dynamics.

Example 4. This example is identical to example 3, except that $\rho = 3$ and $\pi = 0.150462$ hold. Then $k^* = 1$ and $I^* = 1.2$ satisfy (54) and (55), while $D(k^*, I^*) = -0.0624765$ and $T(k^*, I^*) = -0.437552$. The eigenvalues of J are 0.113398 and -0.55095. The steady state equilibrium is therefore a sink, and paths approaching it display damped oscillation.

In summary, the same situations that obtained under Cobb-Douglas production with $\beta = 0$ also can be observed in the presence of more general technologies.

2. $\beta > 0$.

When $\beta > 0$ holds, some phenomena can be observed that do not occur under Cobb-Douglas production. In particular, we can have two or more steady state equilibria with $I^* > 1$, and the low (high)-interest-rate steady state can be a sink (saddle). In addition, the eigenvalues of J, at the low-interest-rate steady state, can be complex conjugates. This creates an additional route through which oscillatory behavior can be observed. We now illustrate this point with an example.

Example 5. We assume that the production function is given by (24), with $a = b = 1$, and $\sigma = 11/13$. In addition, we set $\rho = 1.657462$ and $\pi = 0.071842$. Then it is straightforward to check that there is a steady state equilibrium with $k^* = 1$ and $I^* = 1.5$. At this steady state equation (23) is violated; therefore, with $\beta > 0$, an additional steady state equilibrium must exist with a higher interest rate and a lower capital stock.

For this example one readily calculates that $D(1, 1.5) = 1/130 = T(1, 1.5)$. Clearly, then, $T(1, 1.5)^2 < 4D(1, 1.5)$ holds, so that the eigenvalues of J are complex conjugates with modulus less than one. Paths approaching the low-interest-rate steady state must oscillate as they do so.

It is easy to show that, if $D(k^*, I^*) > 0$ holds at the low-interest-rate steady state, then $D(k^*, I^*) > 0$ holds at the high-interest-rate steady state as well. Therefore proposition 5 implies that the high-interest-rate steady state for this example is a saddle.

3. Summary

It is straightforward to construct examples where indeterminacy of equilibrium and endogenously arising volatility are observed. There are more methods by which this can be done when production technologies are not of the Cobb-Douglas form.

V. Conclusion

We have examined a model of capital accumulation in which spatial separation and limited communication create a role for money and random shocks to agents' portfolio needs create a role for banks. We have seen that in such a model there is considerable scope for the existence of multiple monetary steady states with positive nominal interest rates, for the indeterminacy of monetary equilibria, and for endogenously arising economic fluctuations.

These possibilities arise because the severity of the financial market frictions that agents face are--at least partly--endogenous in this environment. When nominal interest rates are low, agents perceive the costs of being forced to use currency in interlocation exchange as being correspondingly low, and hence they perceive financial markets as providing, relatively speaking, ample liquidity and as functioning "smoothly." When nominal interest rates are high, on the other hand, agents perceive high costs to being forced to use currency in interlocation exchange, there is a premium on liquidity, and the functioning of financial markets seems less smooth.

When the reserve-to-deposit ratio is increasing in the nominal interest rate ($\gamma' > 0$), low nominal interest rates (well-functioning financial markets) lead banks to invest heavily in capital. Hence the financial system operates with relative efficiency, holds low levels of government liabilities, and finances capital formation. High nominal interest rates, in contrast, are associated with banks holding comparatively large quantities of government liabilities, with the result that there is less financing of capital investment. The inefficiency of the financial system is detrimental to real activity. We have seen that a single economy can have either outcome as an equilibrium and that it is possible for either steady state equilibrium to be approached starting from a given initial capital stock. Thus intrinsically similar -- or even identical economies -- and even distinct economies with identical initial capital stocks can display highly disparate asymptotic behavior. In this sense the operation of the financial system can easily give rise to development trap phenomena.

It is also the case that the conduct of monetary policy can significantly influence the scope for development traps to exist. In particular, we have seen that when monetary policy is "loose enough" (that is, when $\beta = 0$), there is necessarily a unique monetary steady state. However, indeterminacies and endogenous fluctuations can be observed even when $\beta = 0$, and setting $\beta = 0$ does not imply that two identical economies will have identical long-run equilibria. Indeed, we have seen that some economies might experience undamped oscillation in interest rates, inflation, and real activity when $\beta = 0$, even though a unique steady state equilibrium exists.

"Tighter" monetary policy stances that raise β above zero raise the specter of development trap phenomena. In addition, once β exceeds zero and multiple steady state equilibria are observed, changes in monetary policy can have complicated effects depending on which steady state equilibrium obtains. Steady state equilibria delivering conventional comparative statics results will have the feature that contractionary monetary policies will raise the nominal rate of interest and result in lower output.

There are, of course, a number of issues that we have left unaddressed. One concerns the welfare consequences of variations in the monetary policy parameter β. Here the presence of multiple equilibria -- and potentially multiple steady state equilibria -- gives rise to a number of subtleties that are beyond the scope of the present paper. However, we can make a few casual observations. First, in the presence of multiple steady states with positive nominal interest rates, an increase in β raises steady state welfare in the high-interest-rate steady state if capital's share does not exceed one-half. Depending on parameter values, an increase in β might increase or reduce steady state welfare in the low-interest-rate steady state. Thus, in particular, there is no presumption that the unique steady state that arises with $\beta = 0$ cannot be improved upon by tightening monetary policy. Doing so, however, may increase the potential for development trap phenomenon.

A second issue that we have not discussed concerns the source of the non-neutrality of monetary policy in this model. In many overlapping generations models, "pure open market" operations are neutral, even with respect to the price level.[14] Neutrality obtains in those models only if open market activity is accompanied by a set of non-distorting taxes and transfers that offset any changes in the government's interest obligations on its liabilities. However, whether or not the necessary transfers can be constructed at all in economies with positive nominal interest rates is often an open question.[15] Here the movement of agents between locations, along with the spatial

[14]See, for instance, Wallace (1981), Chamley and Polemarchakis (1984), or Sargent and Smith (1987).
[15]For a detailed discussion of this issue in the context of a particular model, see Sargent and Smith (1987).

separation and limited communication that arises in the model, would make the existence of such transfers particularly problematic.

There are, of course, a number of dimensions along which the analysis could be extended. One would be the introduction of fiscal policy considerations, permitting an analysis of the consequences of varying a government's budget deficit. Another would be the introduction of bank regulation, which might well affect the scope for multiple equilibria. A third would be to allow for some additional informational asymmetries that could give rise to rationing in the market for investment capital. In such a context, open market operations might tend to alleviate or exacerbate the rationing of credit, as Williamson (1986, 1987) has demonstrated elsewhere. All of these would be interesting topics for future investigation.

Appendix

A. Proof of Proposition 1.

It is easy to check that

(A.1) $k\,\Omega'(k) / \Omega(k) = 1 - [kw'(k) / w(k)]$.

Equation (A.1) allows us to rewrite (21) as

(A.2) $dk/dI\,\big|_{(19)} = -(k/I)\,[I\gamma'(I)/\gamma(I)]\,(1+\beta)\,\gamma(I) / \{1 - [kw'(k)/w(k)]\}\,\Omega(k) =$

$-(k/I)\,[I\gamma'(I)/\gamma(I)]\,(1+\beta)\,\gamma(I) / \{1 - [kw'(k)/w(k)][1 - (1+\beta)\,\gamma(I)]$,

where the second equality follows from (19). We can also rewrite (22) as

(A.3) $dk/dI\,\big|_{(20)} = [(1+\beta)\,I/(1+\beta I)] / I\,(1+\beta I)\,f''(k) = (k/I)\,f'(k) / k\,f''(k)\,(1+\beta I) = (k/I)\,[k\,f'(k)/w(k)]\bullet$

$[w(k)/k\,f''(k)]\,[k\,(1+\beta I)]^{-1} = -(k/I)\,[k\,f'(k)/w(k)]\,[w(k)/kw'(k)]\,(1+\beta I)^{-1}$.

(23) follows immediately from (A.2) and (A.3). □

B. Proof of Proposition 2.

We must demonstrate that any potential steady state equilibrium with $I > 1$ violates (23). When $\beta = 0$, equation (23) reduces to

(A.4) $[k^*w'(k^*)/w(k^*)][w(k^*)/k^*f'(k^*)]\gamma(I^*) / [1 - \gamma(I^*)] > \{1 - [k^*w'(k^*)/w(k^*)]\}\,[\gamma(I^*)/I^*\gamma'(I^*)]$.

In addition, when (24) holds,

(A.5) $kw'(k)/w(k) = (1-\sigma)\,[ak^\sigma / (ak^\sigma + b)]$

and

(A.6) $kf'(k)/w(k) = ak^\sigma / b$.

Using equations (10), (A.5), and (A.6) in (A.4), we can reduce the latter condition to

(A.7) $\{(1-\sigma) / [1 + \sigma\,(ak^\sigma/b)]\}\gamma(I) > \rho/(\rho - 1)$.

But each term on the left-hand side of (A.7) is less than one, while $\rho/(\rho - 1) > 1$. Thus (A.7) and hence (23) cannot hold at any steady state, establishing the desired result. □

C. Proof of Proposition 3.

Differentiating equation (27), we obtain

(A.8) $[\gamma'(I_{t+1})/\gamma(I_{t+1})]\,dI_{t+1}/dI_t = \beta/(1+\beta I_t) + \gamma'(I_t)/\gamma(I_t) - 1/I_t + (1+\beta)\,\gamma'(I_t)/[1 - (1+\beta)\,\gamma(I_t)]$.

Imposing $I_t = I_{t+1} = I^*$ in (A.8) and rearranging terms yields

(A.9) $\lambda(\rho;\beta) \equiv dI_{t+1}/dI_t \mid_{I^*} = [1-(1+\beta)\gamma(I^*)]^{-1} - [\gamma(I^*)/I^*\gamma'(I^*)](1+\beta I^*)^{-1}$.

Moreover, imposing $I_t = I_{t+1} = I^*$ in (27) and rearranging terms, we note that any steady state interest rate with $I^* > 1$ satisfies

(A.10) $[1-(1+\beta)\gamma(I^*)]^{-1} = [(1-\alpha)/\alpha](1+\beta)I^*/(1+\beta I^*)$.

Then equations (10) and (A.10) imply that

(A.11) $\lambda(\rho;\beta) = \{[(1-\alpha)/\alpha](1+\beta)I^* - [\rho/(\rho-1)][1-\gamma(I^*)]^{-1}\}/(1+\beta I^*)$.

But this is exactly (28).

It follows from (A.11) that $\lambda(\rho;\beta) > 0$ holds iff

(A.12) $[(1-\alpha)/\alpha](1+\beta)I^*[(\rho-1)/\rho] > [1-\gamma(I^*)]^{-1}$.

However, $[1-\gamma(I^*)]^{-1} \le [1-(1+\beta)\gamma(I^*)]^{-1}$ holds. Using this fact, along with (A.10), we have that a sufficient condition for $\lambda(\rho;\beta) > 0$ is

(A.13) $[(\rho-1)/\rho](1+\beta I^*) \ge 1$.

But this is equation (29). Finally, (A.13) is obviously satisfied for any $I^* > 1$ if (30) holds. □

D. Proof of Lemma 1.

Equation (9) implies that

(A.14) $\gamma(I)/I[1-\gamma(I)] = [\pi/(1-\pi)]\, I^{-1/\rho}$.

Therefore (27') can be rewritten as

(A.15) $I_t = \{[(1-\alpha)/\alpha][(1-\pi)/\pi]\,\gamma(I_{t+1})\}^{-\rho}$.

This establishes part (a). Part (b) follows immediately from $\rho > 1$. For part (c) note that, since h is decreasing,

(A.16) $\lim_{I\to\infty} h(I) \le h(I')$

holds, $\forall I' \ge 1$. Clearly

(A.17) $\lim_{I\to\infty} h(I) = \{[(1-\alpha)/\alpha][(1-\pi)/\pi]\}^{-\rho} \ge 1$

holds iff $[(1-\alpha)/\alpha][(1-\pi)/\pi] \le 1$. But this is exactly (a.3). □

E. Proof of Lemma 2.

Differentiating (32) gives

(A.18) $dI_{t+1}/dI_t = h(I_{t+1})/I_t h'(I_{t+1})$.

Therefore

(A.19) $\lambda(\rho; 0) = h(I^*) / I^* h'(I^*)$.

Moreover, it is easy to verify that

(A.20) $Ih'(I) / h(I) = -\rho I\gamma'(I) / \gamma(I) = -(\rho - 1)[1 - \gamma(I)]$,

where the second equality follows from (10). Substituting (A.20) into (A.19), and using the first equality in (34), we obtain (36). (37) follows immediately. □

F. Proof of Proposition 4.

Equation (36) implies that, if ρ_0 exists, it satisfies

(A.21) $g(\rho_0) / (\rho_0 - 1) = \alpha / (1 - \alpha)$.

It is easily shown that (A.21) is equivalent to selecting ρ_0 to satisfy

(A.22) $1 = (\rho - 1)\gamma\{[\alpha / (1 - \alpha)](\rho - 1)\} / (\rho - 2) \equiv Z(\rho)$.

We therefore seek a value ρ_0 satisfying $Z(\rho_0) = 1$.

It will now be useful to state two preliminary results.

Lemma 3. $Z[(2 - \pi) / (1 - \pi)] > 1$ holds.

Proof. When $\rho = (2 - \pi) / (1 - \pi)$, $(\rho - 1) / (\rho - 2) = 1/\pi$ holds. Moreover, (a.3) implies that $[\alpha / (1 - \alpha)]\{[(2 - \pi) / (1 - \pi)] - 1\} > 1$. Therefore $Z[(2 - \pi) / (1 - \pi)] > \gamma(1) / \pi = 1$. □

Lemma 4. $Z(\rho) < 1$ holds iff

(A.23) $Q(\rho) > 1$.

Proof. Using the definitions of Z and γ, it follows that $Z(\rho) < 1$ holds iff

(A.24) $\rho - 1 < (\rho - 2)\{1 + [(1 - \pi) / \pi][(\alpha / (1 - \alpha))(\rho - 1)]^{(1-\rho)/\rho}\}$.

It is easy to verify that (A.24) is equivalent to (A.23). □

If there exists a value $\hat{\rho}$ such that $Q(\hat{\rho}) > 1$, it is easily checked that $\hat{\rho} > (2 - \pi) / (1 - \pi)$ must hold. Moreover, $Z(\hat{\rho}) < 1$. It then follows from the intermediate value theorem that there exists a value $\rho_0 \in ([(2 - \pi) / (1 - \pi)], \hat{\rho})$ such that $Z(\rho_0) = 1$.

We also wish to show that $\lambda_1(\rho_0; 0) \neq 0$. It is straightforward but tedious to demonstrate that

(A.25) $-\lambda_1(\rho;0) / \lambda(\rho;0)^2 = \left\{\ln\left[(\alpha / (1-\alpha))(\rho_0 - 1)\right] / \left[(1-\pi)/\pi\right]\left[(\alpha/(1-\alpha))(\rho_0 - 1)\right]^{(1-\rho_0)/\rho_0} + \rho_0^{-1}\right\} - (\rho_0 - 1)$.

Thus $\lambda_1(\rho_0; 0) \neq 0$ holds iff

(A.26) $\ln\{[\alpha/(1-\alpha)](\rho_o-1)\} \neq [\rho_o/(\rho_o-1)] + [\rho_o^2/(\rho_o-1)][(1-\pi)/\pi]\{[\alpha/(1-\alpha)](\rho_o-1)\}^{(1-\rho_o)/\rho_o}$.

We now observe that

(A.27) $[(1-\pi)/\pi]\{[\alpha/(1-\alpha)(\rho_o-1)]\}^{(1-\rho_o)/\rho_o} = \{1-\gamma\{[\alpha/(1-\alpha)](\rho_o-1)\}\}/\gamma[\alpha/(1-\alpha)(\rho_o-1)]$

holds. But then $Z(\rho_o) = 1$ implies that

(A.28) $[(1-\pi)/\pi]\{[\alpha/(1-\alpha)](\rho_o-1)\}^{(1-\rho_o)/\rho_o} = 1/(\rho_o-2)$.

Substituting this result into (A.26), it follows that $\lambda_1(\rho_0; 0) \neq 0$ holds iff

(A.29) $\exp[2\rho_0/(\rho_0-2)]/(\rho_0-1) \neq \alpha/(1-\alpha)$.

Straightforward differentiation of (38) establishes that $Q'(\rho) \geq 0$ holds iff

(A.30) $\exp[2\rho/(\rho-2)]/(\rho-1) \geq \alpha/(1-\alpha)$.

It is easily verified that (A.30) at equality has a unique solution, $\tilde{\rho}$ and that $Q'(\rho) > (<) 0\ \forall\ \rho < (>)\ \tilde{\rho}$.

Now $Q(\tilde{\rho}) \geq Q(\hat{\rho}) > 1$, so that $Z(\tilde{\rho}) < 1$ holds. It is also the case that $\lambda_1(\rho_0; 0) = 0$ holds iff $\rho_o = \tilde{\rho}$. But this is ruled out by $Q(\tilde{\rho}) > 1$.

It now remains to establish that there exist two-period orbits satisfying (32) for some values of ρ near ρ_0. To show this we work with the equation $I_t = h(I_{t+1})$. At the unique steady state

$dI_t/dI_{t+1}|_{I^*} = 1/\lambda(\rho_0; 0)$.

If there exist values $I_e > 1$ and $I_0 > 1$ (with $I_e \neq I_0$) such that $h(I_e) = I_0$ and $h(I_0) = I_e$, we have the desired two-cycle.

The existence of such values I_e and I_0, for some values of ρ in a neighborhood of ρ_0, is immediate from the following theorem on flip bifurcations. Its statement is taken from Azariadis (1993), p.97

Theorem. Consider the system $x_{t+1} = H(x_t, \rho)$ where $H: X \times \Omega \to X$, $H \in C^r$, and $r \geq 3$. Assume that (x^0, ρ_0) is a non-hyperbolic equilibrium for ρ_0, and that the eigenvalues of the Jacobian $D_x H(x^0, \rho_0)$ have moduli strictly less than unity with the exception of a single real eigenvalue $\lambda(\rho_0) = -1$. Then if $\lambda'(\rho_0) \neq 0$ as the parameter crosses the bifurcation point ρ_0 in some direction, the equilibrium loses stability. Moreover, in a sufficiently small neighborhood of (x^0, ρ_0) in $X \times \Omega$, this system has a periodic orbit of period 2 on one side of the bifurcation point ρ_o.

We have established that the map h satisfies the conditions of the theorem when $\alpha < 0.5$, and when there exists a value $\hat{\rho}$ with $Q(\hat{\rho}) > 1$. □

Table 1

	Low-interest-rate Steady State				High-interest-rate Steady State		
β	I	k	p_t/p_{t+1}	dI_{t+1}/dI_t	I	k	p_t/p_{t+1}
1.0	1.0718	.0747	.9653	.7554	1.5156	.0594	.7950
1.005	1.1094	.0728	.9480	.8101	1.4830	.0602	.8053
1.01	1.1703	.0702	.9212	.8929	1.4067	.0622	.8305

Other parameter values are: $A = 0.55$, $\alpha = 0.335$, $\pi = 0.25$, $\rho = 2.0$

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Figure 1

Timing of Events

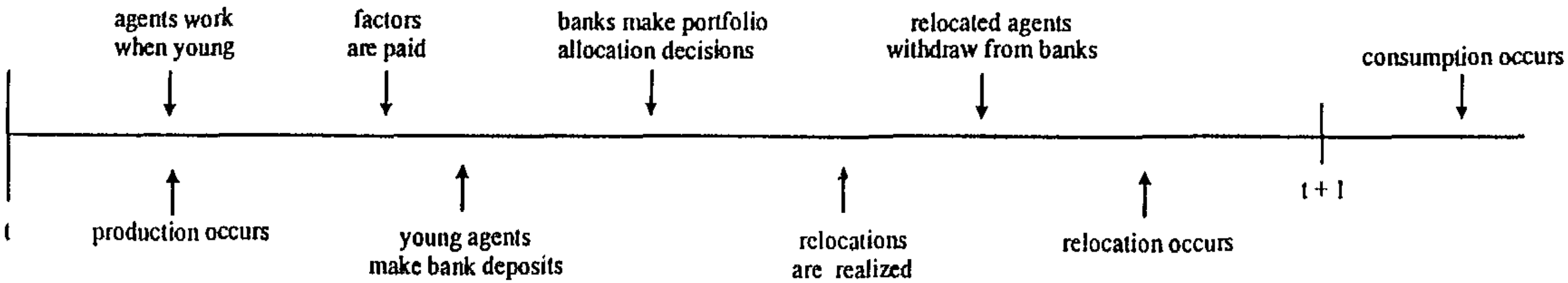

Figure 2.a

Case 1

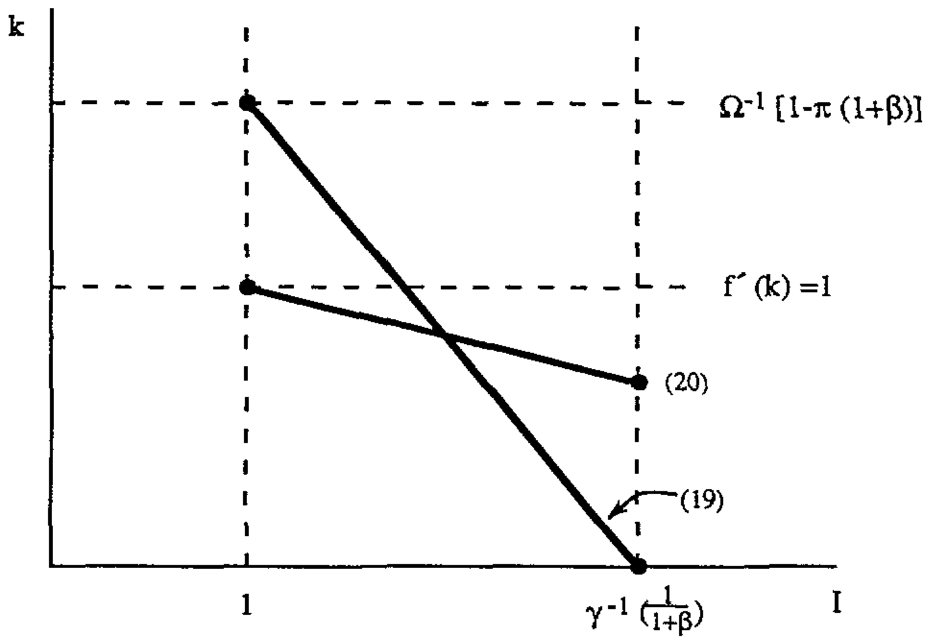

Figure 2.b

Case 2: Multiple Steady States

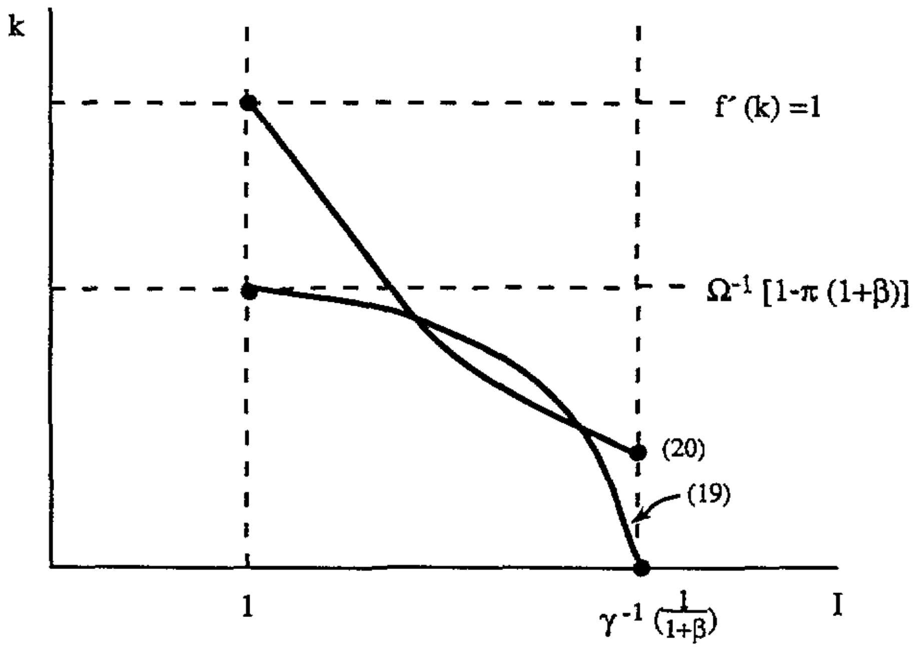

Figure 3

$\beta = 0$: A Unique Steady State

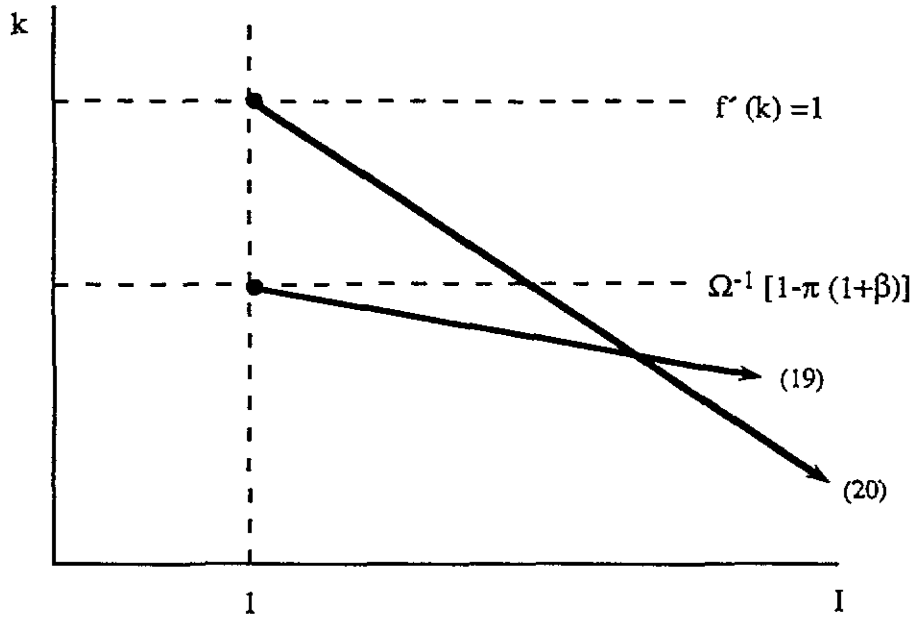

Figure 4.a

An Increase in β $[\beta' > \beta]$

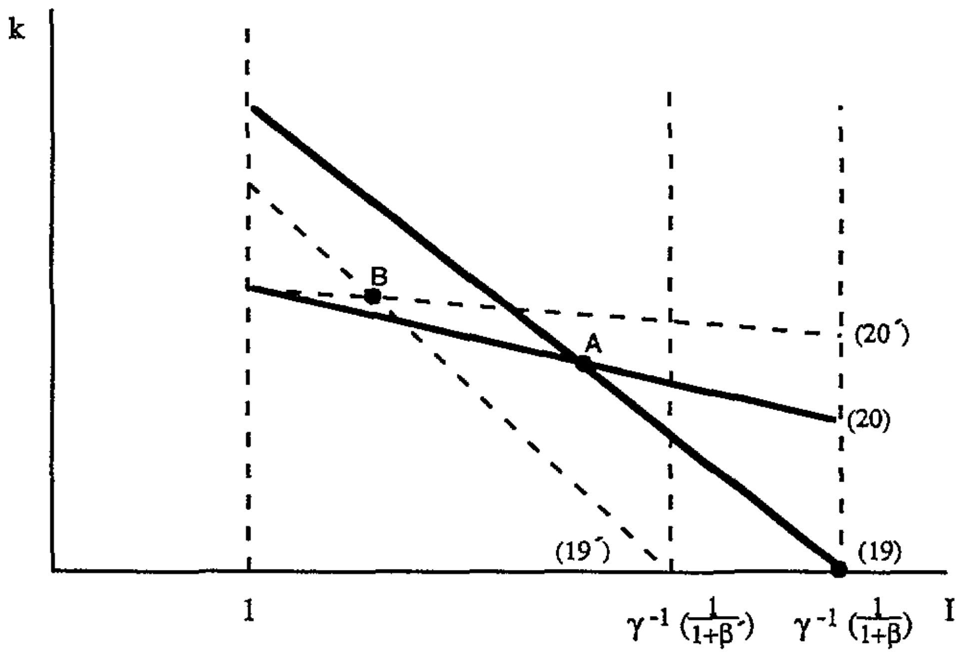

Figure 4.b

An Increase in β $[\beta' > \beta]$

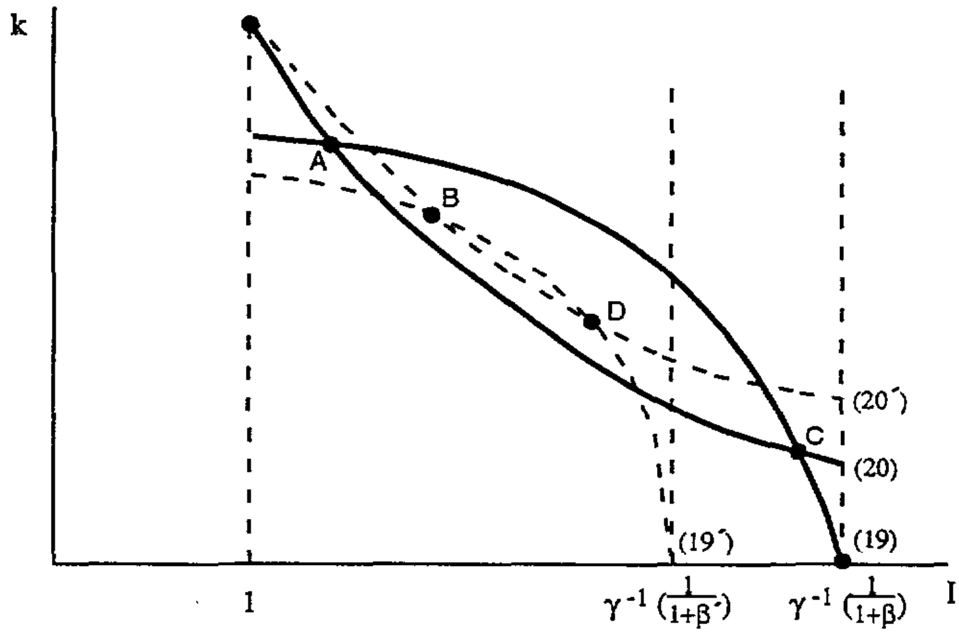

Figure 5.a

Case 1 $[\pi(1+\beta) < (1-2\alpha)/(1-\alpha)]$

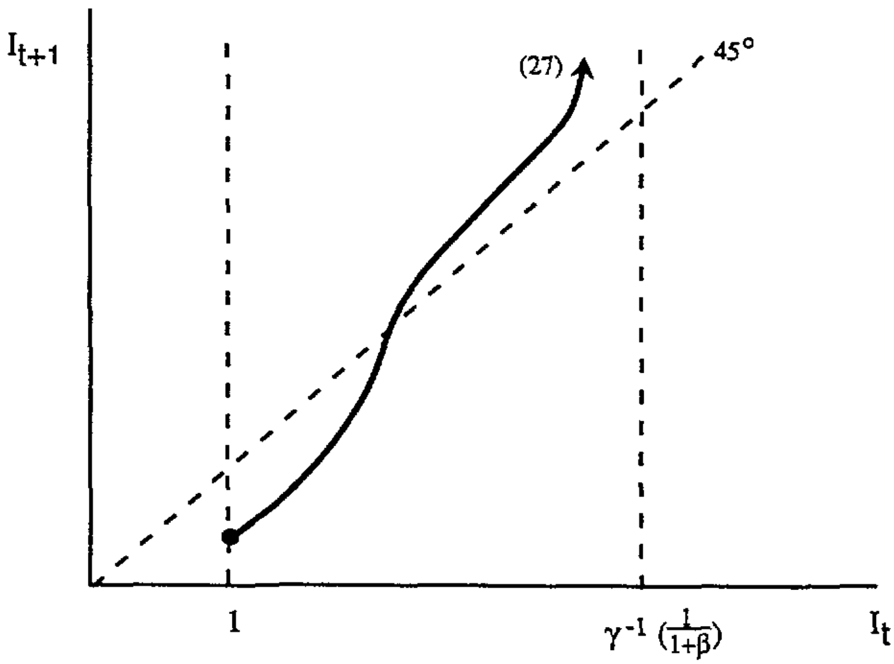

Figure 5.b

Case 2 [π (1+β) > (1-2α) / (1-α)]

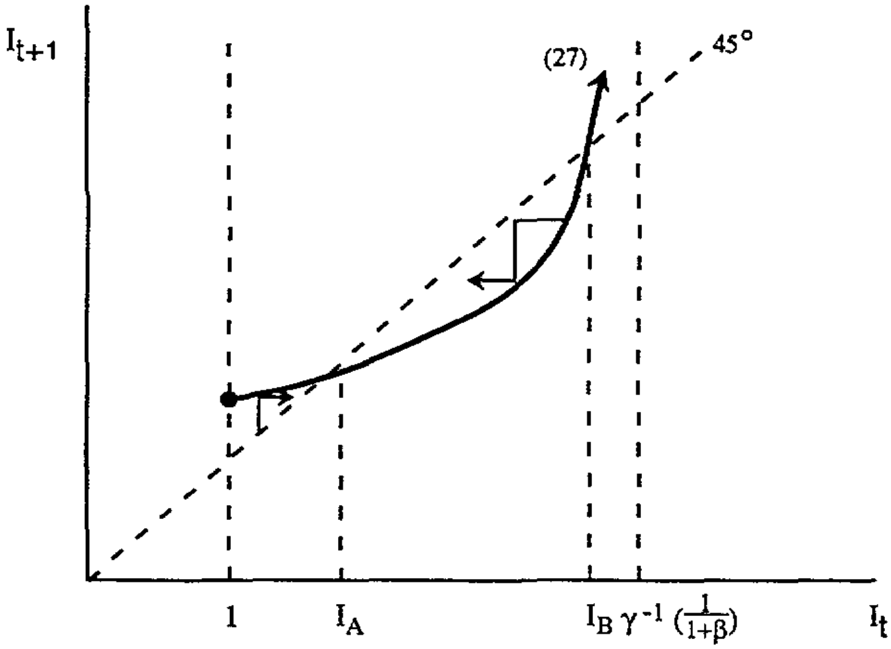

Figure 5.c

Case 3 [π (1+β) > (1-2α) / (1-α)]

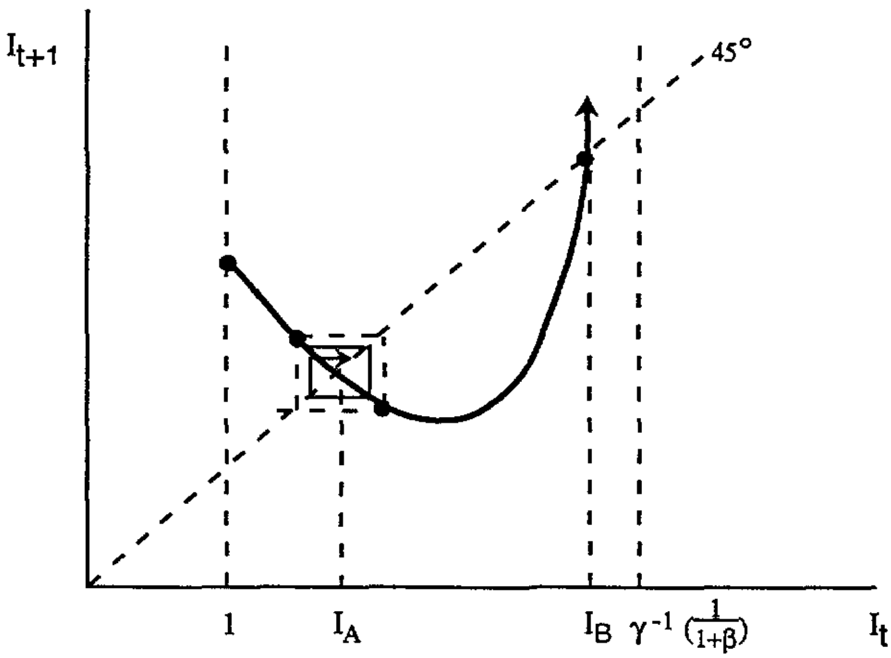

Figure 6

Cobb-Douglas Production [β =0]

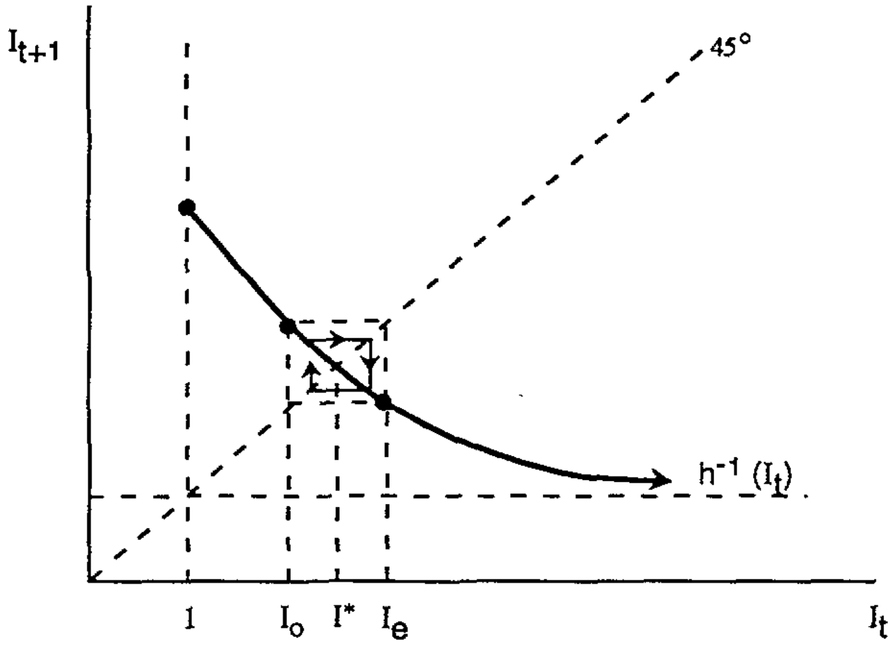

Figure 7.a

Phase Diagram: Case 1

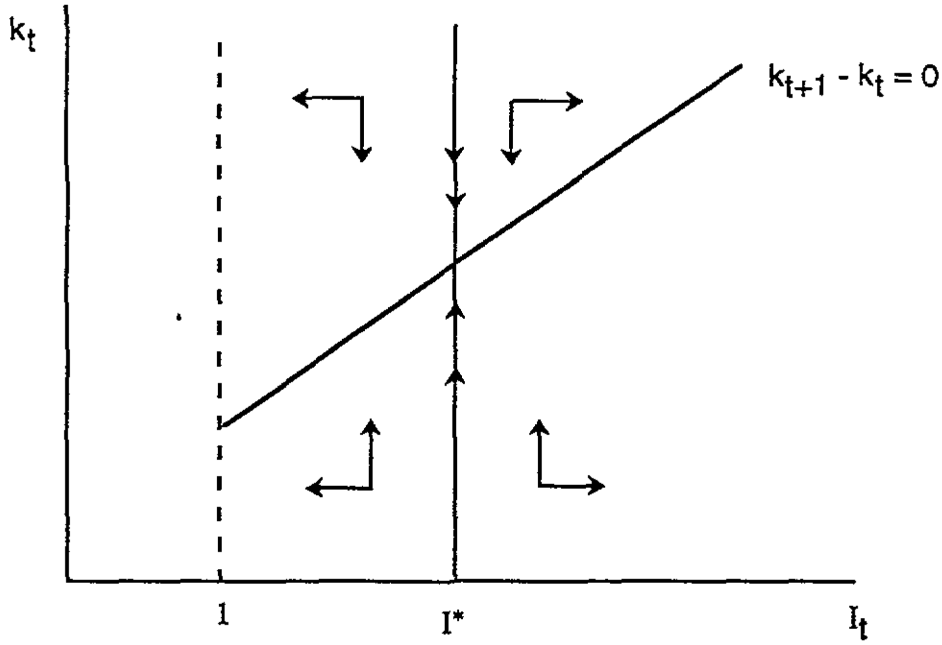

Figure 7.b

Phase Diagram: Case 2

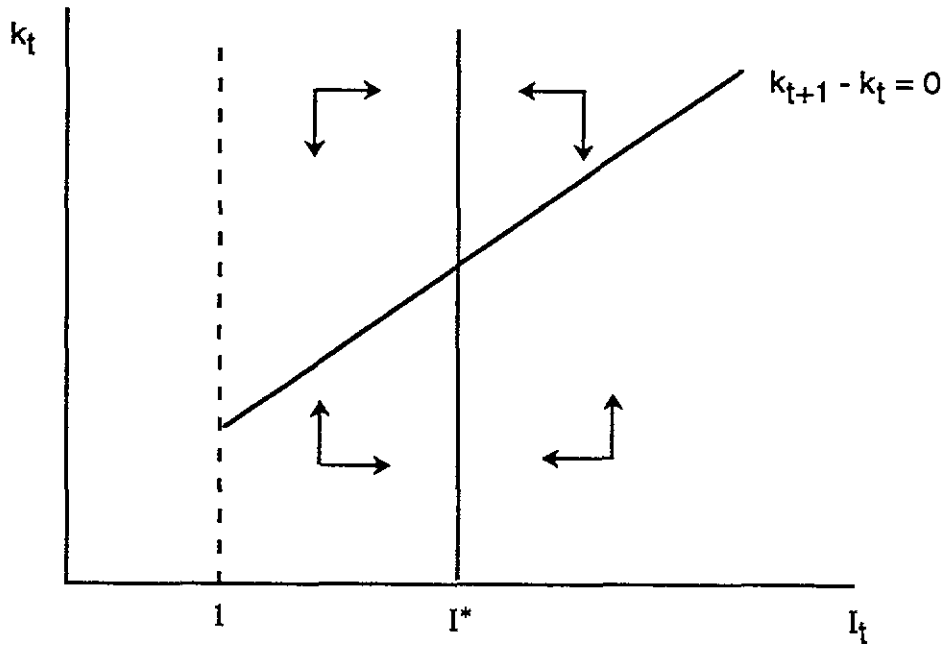

Figure 7.c

Phase Diagram: Cases 2 and 3