Bank of Portugal Lectures Summer 2006

International Business Cycles

With and Without Frictions

Based on work by Kehoe and Perri

Anomalies in IRBC complete markets models

1) *The consumption anomaly*

Data	Model
Corr(Y^{us} Y^{eu}) > Corr(C^{us} C^{eu})	Opposite

2) *The inputs anomaly*

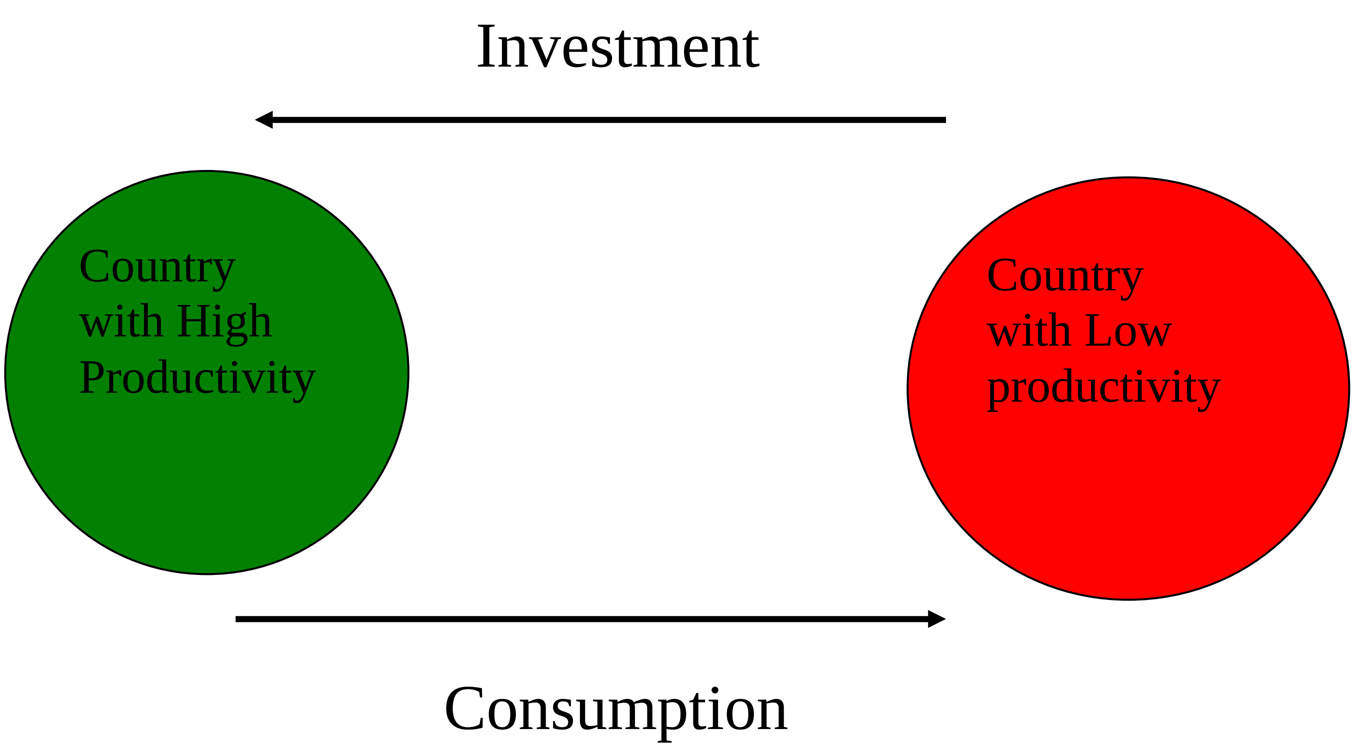
Data	Model
Corr(Inv^{us} Inv^{eu}) > 0 Corr(Emp^{us} Emp^{eu}) > 0	Opposite

- **Objective of this paper:**
  Develop a model that explains (1) and (2)

<u>What causes the anomalies ?</u>

- Key reason:
  Assumption of complete Internantional financial markets

<u>Risk Sharing and Productive efficiency</u>



<u>Solution</u>
Frictions in international capital markets

<u>Exogenous or Endogenous</u> ?

- Exogenous incomplete markets
  Restricts the menu of tradable assets
  Can solve (1) but not (2)

- Endogenous incomplete markets
  Asset Markets unrestricted but
  debt constrained
  - Contracts can be enforced only through exclusion from credit market
  - Every allocation has to make every country in every state at least as happy as in autarky

- Results
  Endogenous incomplete can solve
  (1) and (2)

<u>OUTLINE</u>

- Model Economies

  -Complete Markets
  -Enforcement
  -Bond

- Solution

- Parameter Values

- Findings

- Decentralization

The economy

- Two countries: $i = 1, 2$. One Good
- Finitely many events s_t
- $s^t = (s_0, .., s_t)$ is an history
- $\pi(s^t) = t_0$ probability of s^t
- Immobile labor, mobile capital
- Preferences:

$$\sum_{t=0}^{\infty} \sum_{s^t} \beta^t \pi(s^t) U(c_i(s^t), l_i(s^t))$$

- Technology: CRS

$$F(k_i(s^{t-1}), A_i(s^t)l_i(s^t))$$

- $A_i(s^t)$ is a country specific technology shock

- Capital Accumulation Equation

$$k_i(s^t) = (1 - \delta)k_i(s^{t-1}) + x_i(s^t)$$

- World Resource Constraint

$$\sum_i c_i(s^t) + x_i(s^t) \leq \sum_i F(k_i(s^{t-1}), A_i(s^t)l_i(s^t))$$

Note: $k_i(s^{t-1})$ is capital in place in i

at the beginning of t

Complete Markets allocation

Max weighted sum of utilities

s.t.

World resource constraint

Enforcement constraints

$$\sum_{r=t}^{\infty} \sum_{s^r} \beta^{r-t} \pi(s^r|s^t) U(c_i(s^r), l_i(s^r)) \geq V^i(k_i(s^{t-1}), s^t)$$

$$i = 1, 2 \quad \forall s^t$$

where the value of Autarky is

$$V^i(k_i(s^{t-1}), s^t) = \max \sum_{t=r}^{\infty} \sum_{s^r} \beta^t \pi(s^r|s^t) U(c_i(s^r), l_i(s^r))$$

s.t.closed economy BC

$$c_i(s^r) + k_i(s^r) \leq F(k_i(s^{r-1}), A_i(s^r) l_i(s^r)) + (1-\delta) k_i(s^{r-1})$$

- Idea behind enforcement constraints

Efficient Allocations with enforcement constraints

Allocations $\{c_i(s^t), l_i(s^t), k_i(s^t)\}$, that solve

$$\max \lambda_1 \sum_{t=0}^{\infty} \sum_{s^t} \beta^t \pi(s^t) U(c_1(s^t), l_1(s^t)) + \\ \lambda_2 \sum_{t=0}^{\infty} \sum_{s^t} \beta^t \pi(s^t) U(c_2(s^t), l_2(s^t))$$

Subject to:

- Resource Constraint $\quad \forall\ s^t$
- Enforcement Constraints $\forall\ s^t \qquad i = 1, 2$

Non standard D.P. problem

A little bit of algebra

$$\sum_{t=0}^{\infty} \beta^t \lambda U(c_t, l_t) + \sum_{t=0}^{\infty} \beta^t \mu_t \left[\sum_{r=t}^{\infty} \beta^{r-t} U(c_t, l_t) - V(k_t) \right] \tag{L1}$$

Where $\beta^t \mu_t$ is the multiplier of date t enforcement constraint

$$\sum_{t=0}^{\infty} \beta^t \lambda U(c_t, l_t) + \sum_{t=0}^{\infty} \mu_t \sum_{r=t}^{\infty} \beta^r U(c_t, l_t) - \sum_{t=0}^{\infty} \mu_t \beta^t V(k_t) \tag{L2}$$

$$\lambda U(c_0,l_0)+\lambda\beta U(c_1,l_1)+\lambda\beta^2 U(c_2,l_2)\cdots+\lambda\beta^t U(c_t,l_t)+\cdots$$

$$\mu_0 U(c_0,l_0)+\mu_0\beta U(c_1,l_1)+\mu_0\beta^2 U(c_2,l_2)+\cdots+\mu_0\beta^t U(c_t,l_t)+\cdots$$

$$\mu_1\beta U(c_1,l_1)+\mu_1\beta^2 U(c_2,l_2)+\cdots+\mu_1\beta^t U(c_t,l_t)+\cdots$$

$$\mu_2\beta^2 U(c_2,l_2)+\cdots+\mu_2\beta^t U(c_t,l_t)+\cdots$$

$$\vdots \qquad \vdots \qquad \vdots \qquad \vdots$$

$$\mu_t\beta^t U(c_t,l_t)+\cdots$$

$$-\mu_0 V(k_0)-\mu_1\beta V(k_1)-\mu_2\beta^2 V(k_2)-\cdots-\mu_t\beta^t V(k_t)-\cdots$$

Let $M_{t-1}=\lambda+\mu_0+..+\mu_{t-1} \qquad M_{-1}=\lambda$

$$\sum_{t=0}^{\infty}\beta^t\left\{M_{t-1}U(c_t,l_t)+\mu_t\left[U(c_t,l_t)-V(k_t)\right]\right\} \text{ (L3)}$$

Lagrangean:

$$\sum_{t=0}^{\infty} \sum_{s^t} \sum_{i} \beta^t \pi(s^t)[M_i(s^{t-1})U(c_i(s^t), l_i(s^t)) +$$
$$\mu_i(s^t)[U(c_i(s^t), l_i(s^t)) - V_i(k_i(s^{t-1}), s^t)]]$$

plus resource constraint terms.

Weights

$$M_i(s^t) = M_i(s^{t-1}) + \mu_i(s^t)$$

$$M_i(s^{-1}) = \lambda_i.$$

F.O.C.

$$\frac{U_c^1(s^t)}{U_c^2(s^t)} = \frac{M_2(s^t)}{M_1(s^t)}, \qquad (c_i(s^t))$$

$$-\frac{U_l^i(s^t)}{U_c^i(s^t)} = F^i(s^t), \qquad (l^i(s^t))$$

$$U_c^i(s^t) = \beta \sum_{s^{t+1}} \pi(s^{t+1}|s^t) \qquad (k^i(s^t))$$

$$\left[\frac{M_i(s^{t+1})}{M_i(s^t)} U_c^i(s^{t+1})[F_k^i(s^{t+1}) + 1 - \delta] - \frac{\mu_i(s^{t+1})}{M_i(s^t)} V_1^i(s^{t+1})\right]$$

$$\sum_{r=t}^{\infty} \sum_{s^r} \pi(s^r|s^t)\beta^{r-t} U^i(s^r) \geq V^i(s^t) \qquad (\mu^i(s^t))$$

plus complementary slackness conditions.

Multipliers' normalization

$$v_i(s^t) = \frac{\mu_i(s^t)}{M_i(s^t)}$$

$$M_i(s^t) = \frac{M_i(s^{t-1})}{1 - v_i(s^t)}$$

$$z(s^t) = \frac{M_2(s^t)}{M_1(s^t)} \qquad \text{Relative weight of 2}$$

$$z(s^t) = \frac{(1 - v_1(s^t))}{(1 - v_2(s^t))} z(s^{t-1}).$$

- <u>F.O.C.</u>

$$\frac{U_c^1(s^t)}{U_c^2(s^t)} = z(s^t), \qquad (c_i(s^t))$$

- Reduce risk sharing

$$-\frac{U_l^i(s^t)}{U_c^i(s^t)} = F_2^i(s^t), \qquad (l_i(s^t))$$

$$U_1^i(s^t) = \beta \sum \pi(s^{t+1}|s^t) \qquad (k_i(s^t))$$

$$\left[\frac{U_1^i(s^{t+1})}{1 - v_i(s^{t+1})}[F_1^i(s^{t+1}) + 1 - \delta] - \frac{v_i(s^{t+1})}{1 - v_i(s^{t+1})} V_1^i(s^{t+1})\right]$$

- Reduce Productive Efficiency

Plus enforcement constraints and complementary slackness

- F.O.C. Interpretation

Model Solution

State : $S_t = (z(s^{t-1}), k_1(s^{t-1}), k_2(s^{t-1}), A_1(s_t), A_2(s_t))$

Find stationary decision rules

$c_i(S_t)$, $l_i(S_t)$, $k_i(S_t)$, $v_i(S_t)$

that satisfy:

F.O.C., Resource Constraint, Enforcement Constraints,

Complementary Slackness

Solution Method

- Grid over the state space and piecewise bi-linear approximation of value functions and policy functions

- Backward Recursion from t=T

  t$\geq T$: No enforcement constraints

A bond economy

- Agents' problem

$$\max \sum_{t=0}^{\infty} \sum_{s^t} \beta^t \pi(s^t) U(c_i(s^t), l_i(s^t)) \qquad i = 1, 2$$

$s.t.$

$$\begin{aligned} & c_i(s^t) + x_i(s^t) + q(s^t) b_i(s^t) \\ \leq \; & w(s^t) l_i(s^t) + r_i(s^t) k_i(s^{t-1}) + b_i(s^{t-1}) \end{aligned}$$

$$k_i(s^t) = (1 - \delta) k_i(s^{t-1}) + x_i(s^t)$$

$$-\bar{b} \leq b_i(s^t) \leq \bar{b}$$

$b_i(s^t) =$uncontingent bond maturing at time $t + 1$

$q(s^t) =$ price of bond

- Market clearing

$$b_1(s^t) + b_2(s^t) = 0.$$

Parameter values

- Preferences

$$U(c,l) = \frac{(c^{\gamma}(1-l)^{1-\gamma})}{(1-\sigma)}^{1-\sigma}$$

$\beta = 0.99$ $\gamma = 0.34$ $\sigma = 2.0$

- Technology

$$F(k, Al) = k^{\alpha}(Al)^{1-\alpha}.$$

$\alpha = 0.36$

$$k' = (1-\delta)k + x$$

$\delta = 0.025$

- Process for Shocks

- Backus, Kehoe, Kydland (without capital):

$$\begin{vmatrix} \log A_{1t+1} \\ \log A_{2t+1} \end{vmatrix} = \begin{vmatrix} a_1 & a_2 \\ a_3 & a_4 \end{vmatrix} \begin{vmatrix} \log A_{1t} \\ \log A_{2t} \end{vmatrix} + \begin{vmatrix} \varepsilon_{1t+1} \\ \varepsilon_{2t+1} \end{vmatrix}$$

with

$a_2 = a_3 = 0.088$

$a_1 = a_4 = 0.906$

- Kolmann (with capital)

$a_2 = a_3 = 0$

$a_1 = a_4 = .95$

- VAR(1) $\longrightarrow$ 9 States MARKOV CHAIN

Decentralization of the efficient allocation

with enforcement constraints

Country 1 private agents

- Budget constraint

$$\begin{aligned} & c_1(s^t) + k_1(s^t) + \sum_{s^{t+1}} q(s^t, s_{t+1}) b_1(s^t, s_{t+1}) + T_1(s^t \\ = \ & (r_1(s^t) + 1 - \delta)(1 - \tau_1^k(s^t)) k_1(s^{t-1}) \\ & + w_1(s^t) l_1(s^t) + b_1(s^t) \chi(b_1(s^t), s^t, s_{t+1}) \end{aligned}$$

Country 1 Goverment

- Tax on payments to foreigners

$$\chi(b_1(s^t), s^t, s_{t+1}) = \begin{cases} (1 - \tau_1^b(s^t, s_{t+1})) & \text{if } b_2(s^t) > 0 \\ 0 & \text{if } b_2(s^t) \leq 0 \end{cases}$$

- Capital tax & Lump sum transfers

$$\tau_1^k(s^t) \qquad T_1(s^t)$$

Sustainable equilibrium

Agents are competitive

Governments are not competitive

- Sequential optimality of goverments
- Sequential optimality of private agents
- Market equilibrium

Proposition:

Autarky: Worst sustainable equilibrium.

Proposition:

Any allocation that satisfies:

- *Enforcement constraints*

- *Resource constraint*

is sustainable.

We focus on equal weights, efficient outcomes.

Bond taxes and multipliers

Planner

$\forall\ s^{t+1}$ in which C1 constraint is binding:

$$\frac{U_c^1(s^{t+1})}{U_c^2(s^{t+1})} = \left[1 - \nu_1(s^{t+1})\right] \frac{U_c^1(s^t)}{U_c^2(s^t)}$$

Equilibrium

$\forall\ s^{t+1} : b_1(s^t, s^{t+1}) < 0$

$$\frac{U_c^1(s^{t+1})}{U_c^2(s^{t+1})} = \left[1 - \tau_1(s^{t+1}, s^t)\right] \frac{U_c^1(s^t)}{U_c^2(s^t)}$$

If C1 constraint is binding in s^{t+1}:

C1 is a debtor in s^{t+1} and

in s^t C1 tax payments to foreigners in s^{t+1}

Set

$$\tau_1(s^{t+1}, s^t) = \nu_1(s^{t+1})$$

Intuition

Capital tax and multipliers

Planner

$$U_c^i(s^t) = \beta \sum \pi(s^{t+1}|s^t)$$

$$\left[\frac{U_c^i(s^{t+1})}{1 - v_i(s^{t+1})}[F_k^i(s^{t+1}) + 1 - \delta] - \frac{v_i(s^{t+1})}{1 - v_i(s^{t+1})} V_k^i(s^{t+1})\right]$$

Equilibrium

$$U_c^i(s^t) = \beta \sum \pi(s^{t+1}|s^t)$$

$$U_c^i(s^{t+1})[F_k^i(s^{t+1}) + 1 - \delta](1 - \tau_k^1(s^t))$$

If C1 constraint is binding for some s^{t+1}:

C1 tax or subsidize capital income s^{t+1}

Intuition

TABLE I

PARAMETER VALUES

Experiments		Parameters
Baseline Experiments		
	Preferences	$\beta = .99,\ \sigma = 2,\ \gamma = .36$
	Technology	$\alpha = .36,\ \delta = .025$
	Productivity shocks	$a_1 = .95,\ a_2 = 0$
		$\mathrm{var}(\varepsilon_1) = \mathrm{var}(\varepsilon_2) = .007^2,\ \mathrm{corr}(\varepsilon_1, \varepsilon_2) = .25$
Sensitivity Experiments		
	Adjustment costs[a]	$\phi = .6$
	High persistence	$a_1 = .99,\ a_2 = 0$
	High spillover	$a_1 = .85,\ a_2 = .15$
	BKK	$a_1 = .906,\ a_2 = .088$

[a] In the other sensitivity analysis experiments, the adjustment cost parameter in the bond economy is set to match the relative volatility of investment in the data.

Business Cycle Statistics: Baseline Parameters

Statistic	Data	Economy with: No Adjustment Costs: Complete Markets	No Adjustment Costs: Bond	No Adjustment Costs: Enforcement	Adjustment Costs: Complete Markets	Adjustment Costs: Bond
Volatility						
% Standard deviations						
GDP	1.72 (.20)	2.01	1.94	1.33	1.37	1.34
Net Exports/GDP	0.15 (.01)	13.04	12.42	0.06	0.36	0.33
% Standard deviations relative to GDP						
Consumption	0.79 (.05)	0.19	0.21	0.28	0.27	0.29
Investment	3.24 (.17)	25.23	25.06	3.04	3.42	3.24
Employment	0.63 (.04)	0.56	0.54	0.50	0.52	0.49
Domestic Comovement						
Correlations with GDP						
Consumption	0.87 (.03)	0.90	0.93	0.93	0.90	0.94
Investment	0.93 (.02)	0.07	0.08	0.99	0.95	0.95
Employment	0.86 (.03)	0.99	0.99	0.99	0.99	0.99
Net Exports/GDP	−0.36 (.09)	0.06	0.06	0.27	−0.02	−0.05
International Correlations						
Home and Foreign GDP	0.51 (.13)	−0.46	−0.43	0.25	0.09	0.12
Home and Foreign Consumption	0.32 (.17)	0.28	0.13	0.29	0.77	0.62
Home and Foreign Investment	0.29 (.17)	−0.99	−0.99	0.33	−0.17	−0.09
Home and Foreign Employment	0.43 (.11)	−0.58	−0.53	0.23	−0.15	−0.04

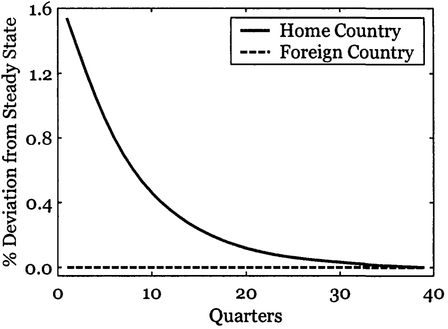


FIGURE 1.—Productivity shocks.

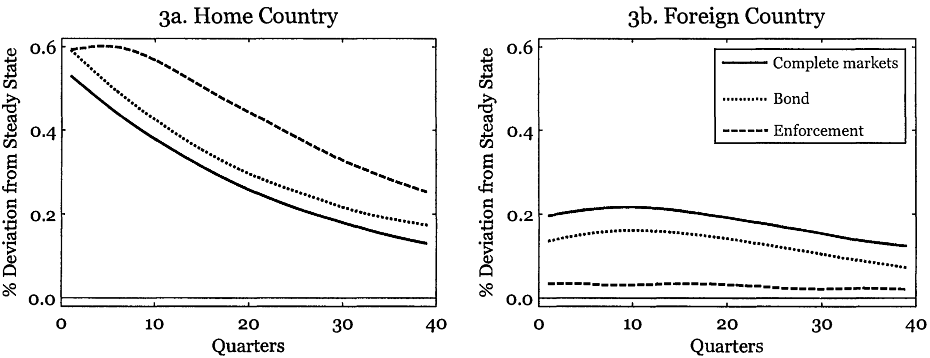


FIGURE 3.—Impulse responses to a home productivity shock—consumption.

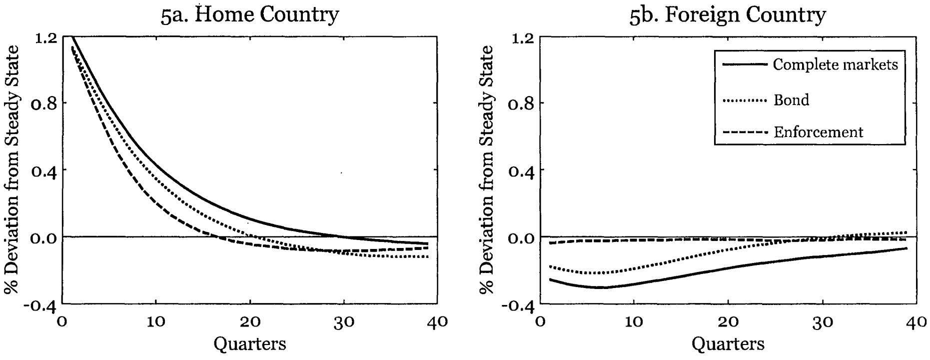


FIGURE 5.—Impulse responses to a home productivity shock—employment.

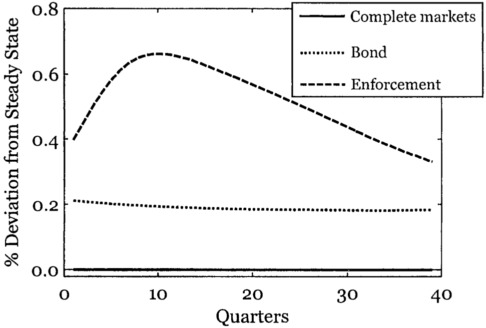


FIGURE 7.—Impulse responses to a home productivity shock—foreign/home ratio of marginal utilities.