Bank of Portugal Lectures Summer 2006

Business Cycle Accounting

(An alternative to SVARs that uses more economics)

Based on work by Chari, Kehoe, and McGrattan

The Role of Quantitative Economic Models

- Allows systematic way of organizing data
- Introduces the power of abstraction
- Can be used to ask and answer questions about
  - changes in tax policy
  - changes in monetary policy
  - changes in regulatory policy
- History has not run every policy experiment of interest

Business Cycle Accounting

- Preliminary data analysis technique

- Goals:

  - Isolate promising classes of models/theories/stories

  - Guide development of theory

Idea of Approach

- Equivalence results:
  - Detailed models with frictions observationally equivalent to
  - Growth model with time-varying TFP and tax rates ("wedges")
- Accounting procedure:
  - Use theory plus data to measure wedges
  - Estimate stochastic process governing expectations
  - Feed wedges back one at a time and in combinations
  - How much of output, investment, labor accounted for by each?

Outline

- Business cycle accounting approach
  - Describe prototype growth model
  - Equivalence results
  - Applications to Great Depression and Postwar

Prototype Growth Model

- Consumption (c), labor (l), investment (x) solve

  $\max_{\{c_t, l_t, x_t\}} E \sum_{t=0}^{\infty} \beta^t U(c_t, l_t)$

  subject to

  $c_t + (1 + \tau_{xt}) x_t \leq (1 - \tau_{lt}) w_t l_t + r_t k_t + T_t$

  $k_{t+1} = (1 - \delta) k_t + x_t$

- Production: $y_t = A_t F(k_t, \gamma^t l_t)$

- Resource: $c_t + g_t + x_t = y_t$

Equations for Prototype Growth Model

- Efficiency wedge:

$$y_t = A_t F(k_t, \gamma^t l_t)$$

- Labor wedge:

$$-\frac{U_{lt}}{U_{ct}} = (1 - \tau_{lt})(1 - \alpha) y_t / l_t$$

- Investment wedge:

$$(1 + \tau_{xt}) U_{ct} = \beta E_t U_{ct+1} \left[\alpha y_{t+1} / k_{t+1} + (1 + \tau_{xt+1})(1 - \delta) \right]$$

- Government consumption wedge:

$$c_t + g_t + x_t = y_t$$

Business Cycle Accounting

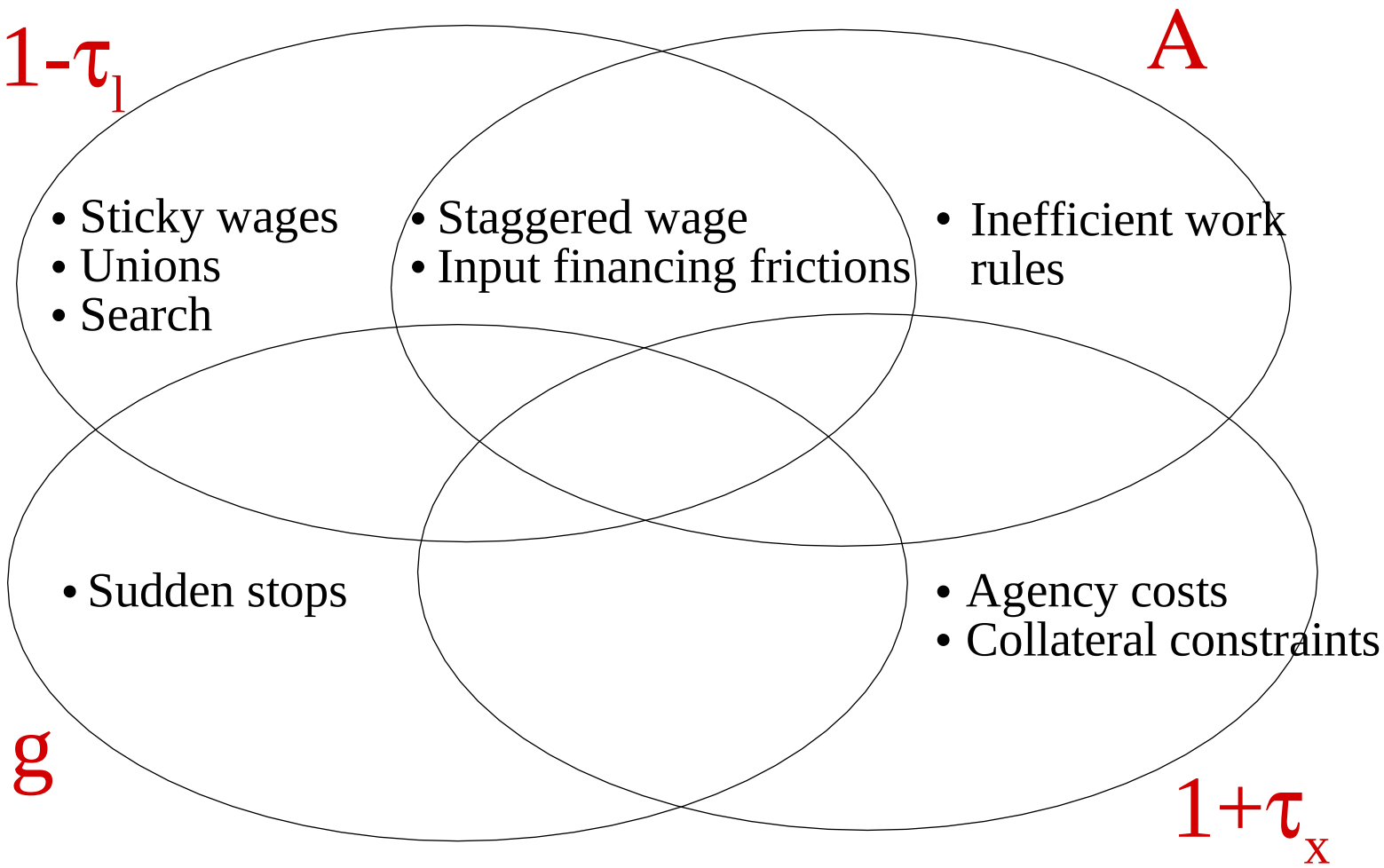
- Estimate stochastic processes for A, τ_l, τ_x, g

- Compute equilibria for prototype economy

- Generate realization by feeding in

  - A only
  - $1 - \tau_l$ only
  - $1 + \tau_x$ only
  - g only
  - combinations

Mapping Between Original and Prototype Models



Equivalence Results

Efficiency Wedges

$$y_t = A_t F(k_t, \gamma^t l_t)$$

- Changes in blueprints
- Misallocation of inputs across tasks/production units
  - within firms (work rules)
  - across firms (input financing frictions)

Example: Input Financing Frictions

- Final goods producers have technology $y = y_1^{1/2} y_2^{1/2}$

$$\max_{y_1, y_2} \; y - p_1 y_1 - p_2 y_2$$

- Intermediate goods producers have technology $y_i = l_i^{\alpha}$

$$\max_{l_i} \; p_i y_i - R(1 + \tau_i) w l_i$$

- Households supply labor $l = l_1 + l_2$ and solve

$$\max_{c,l} \; U(c, l) \text{ subject to } c = wl + \pi$$

Equivalent Prototype Economy

- Firm solves $\max_l \; Al^\alpha - wl$

- Household solves $\max_{c,l} \; U(c,l)$ subject to $c = (1-\tau)wl + \pi$

- Proposition: Allocations in two economies same if

$$A = \frac{(1+\tau_1)^{\frac{\alpha}{2}}(1+\tau_2)^{\frac{\alpha}{2}}}{[(1+\tau_1)+(1+\tau_2)]^\alpha}$$

$$1-\tau = \frac{1}{2}\left(\frac{1}{1+\tau_1}\right) + \frac{1}{2}\left(\frac{1}{1+\tau_2}\right)$$

Note: Increases in financing distortions makes TFP fall

Labor wedges

$$\frac{U_{lt}}{U_{ct}} = (1 - \tau_{lt}) F_{lt}$$

- Sticky wages
  - Labor wedges in prototype model
  - Staggering yields efficiency wedges
- Cartels/unions
  - Labor wedges in prototype model

Labor wedges

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- Sticky wages
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- Cartels/unions
  - Labor wedges in prototype model

Example: Sticky Wages

- Economy:
  - Event s_t, $s^t = (s_0, \ldots, s_t)$
  - Stochastic money growth $\mu(s^t)$
  - Utility $U(c, l, m) = u(c, l) + v(m)$
  - Production $F(k, l)$

- Define $\tau_l^*(s^t)$

$$\tau_l^*(s^t) = 1 - \frac{U_l^*(s^t)}{U_c^*(s^t)} \frac{1}{F_l^*(s^t)}$$

where '$*$' indicates equilibrium values

Equivalent Prototype Economy

- Real prototype economy with
  - Stochastic labor taxes $\tau_l(s^t)$
  - Utility $u(c, l)$
  - Production $F(k, l)$
- Proposition. Allocations in the two economies same if $\tau_l(s^t) = \tau_l^*(s^t)$.

Investment wedges

$$(1+\tau_{xt})U_{ct} = \beta E_t U_{ct+1}\left[\alpha y_{t+1}/k_{t+1} + (1+\tau_{xt+1})(1-\delta)\right]$$

- Models with financial frictions, e.g.,
  - Bernanke-Gertler
  - Carlstrom-Fuerst
  - Kiyotaki-Moore
- Map into prototype with investment wedges (See our appendix)

Application to the Great Depression

Measuring Wedges

- Stochastic Process for wedges $s_t = [\log A_t, \tau_{lt}, \tau_{xt}, \log g_t]$

$$s_{t+1} = P_0 + P s_t + Q \eta_{t+1}$$

- Preferences and technology
  - $U(c, l) = \log c + \psi \log(1 - l)$
  - $F(k, l) = A k^{\theta} l^{1-\theta}$

- With data for 1901-1940
  - fix parameters of technology and preferences
  - compute MLE estimates of P_0, P, Q

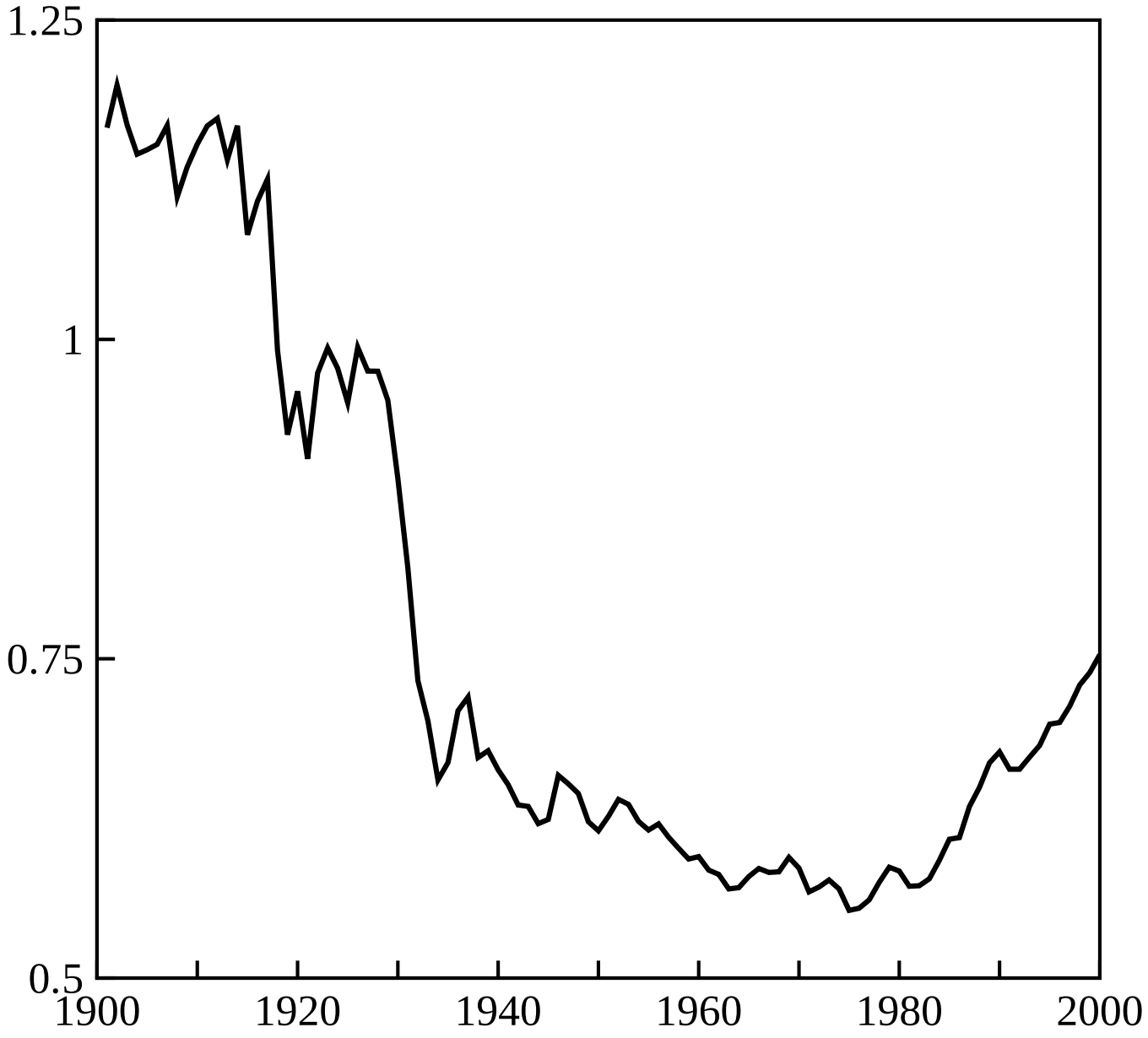
Recovering Wedges

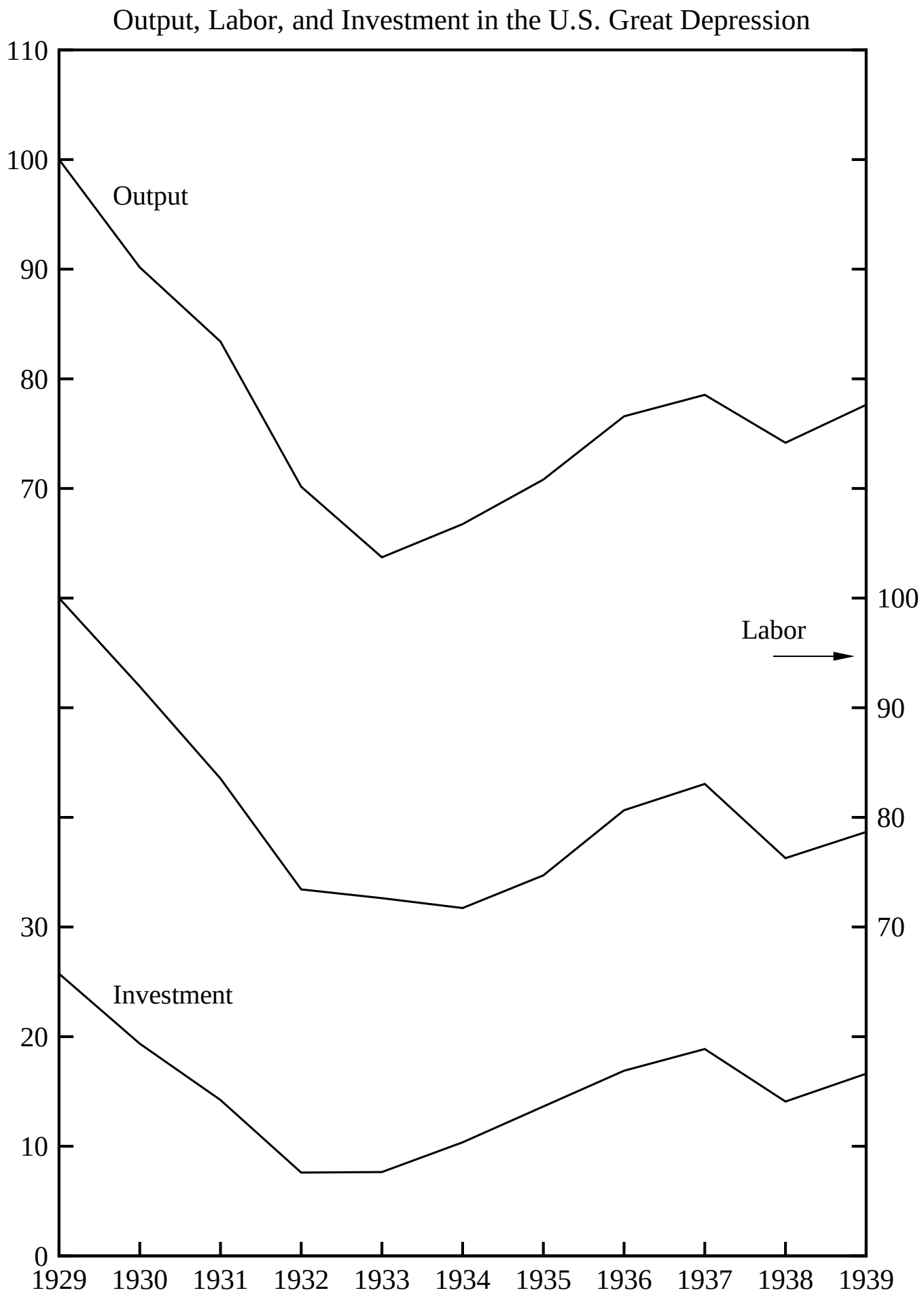
- Model decision rules are $c(s_t, k_t), x(s_t, k_t), l(s_t, k_t)$
- Set
  - $c(s_t, k_t) = c_t^{DATA}$
  - $x(s_t, k_t) = x_t^{DATA}$
  - $l(s_t, k_t) = l_t^{DATA}$

  with k_t defined recursively from accumulation equation
- Solve for values of $s_t = [\log A_t, \tau_{lt}, \tau_{xt}, \log g_t]$
- Inputting these values gives exactly same series as in data

Figure A1. Logarithm of U.S. Efficiency Wedge

Figure A2. U.S. Labor Wedge




Output, Labor, and Investment in the U.S. Great Depression
110
100
90
80
70
30
20
10
0
100
90
80
70
Output
Labor
Investment
1929
1930
1931
1932
1933
1934
1935
1936
1937
1938
1939

Figures 1–4
Examining the U.S. Great Depression
Annually, 1929–39; Normalized to Equal 100 in 1929

Figure 1
U.S. Output and Three Measured Wedges

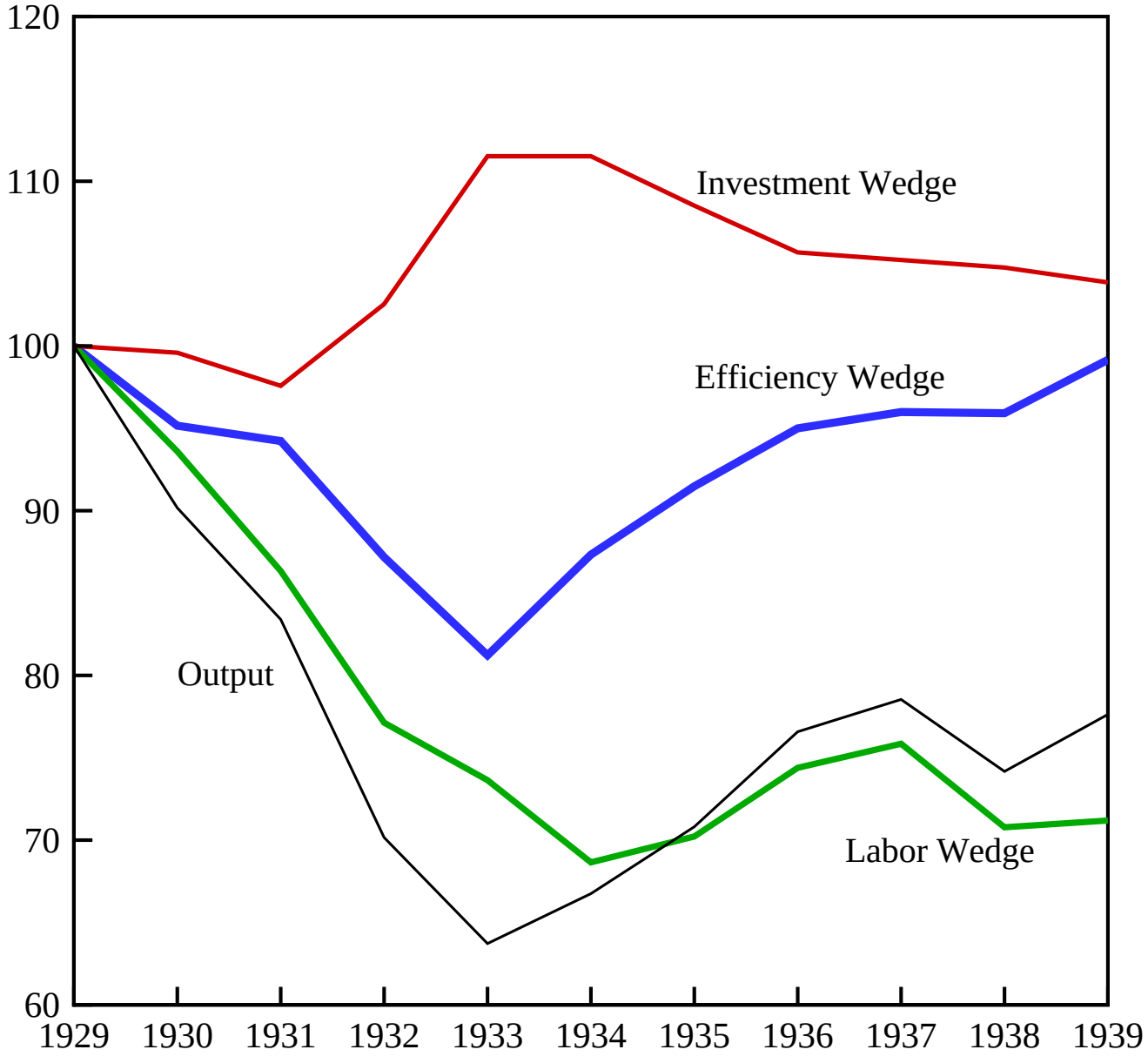
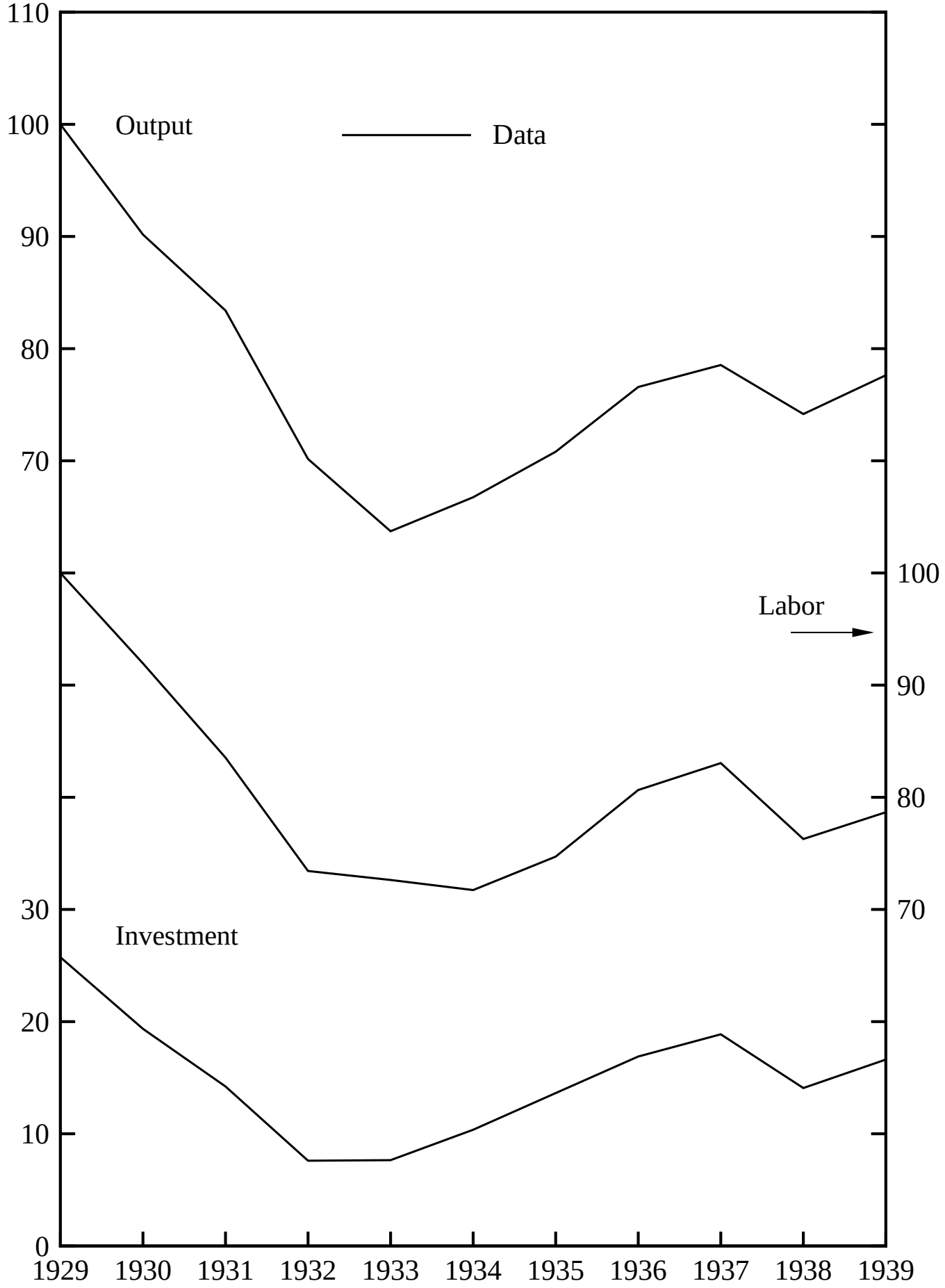


Figure 2
Data and Predictions of Models With Just One Wedge



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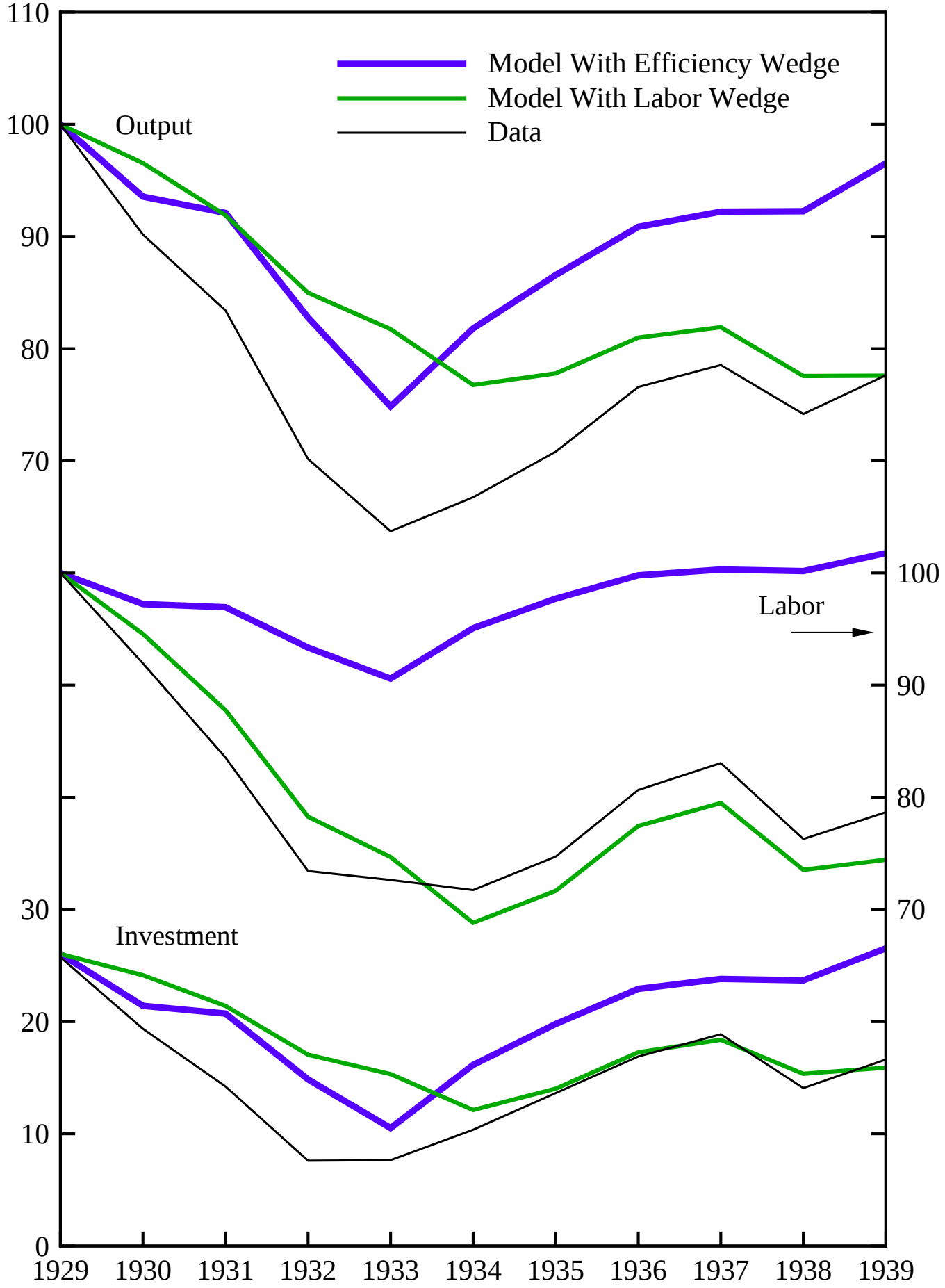


Figure 3
Data and Predictions of Model With Just the Investment Wedge

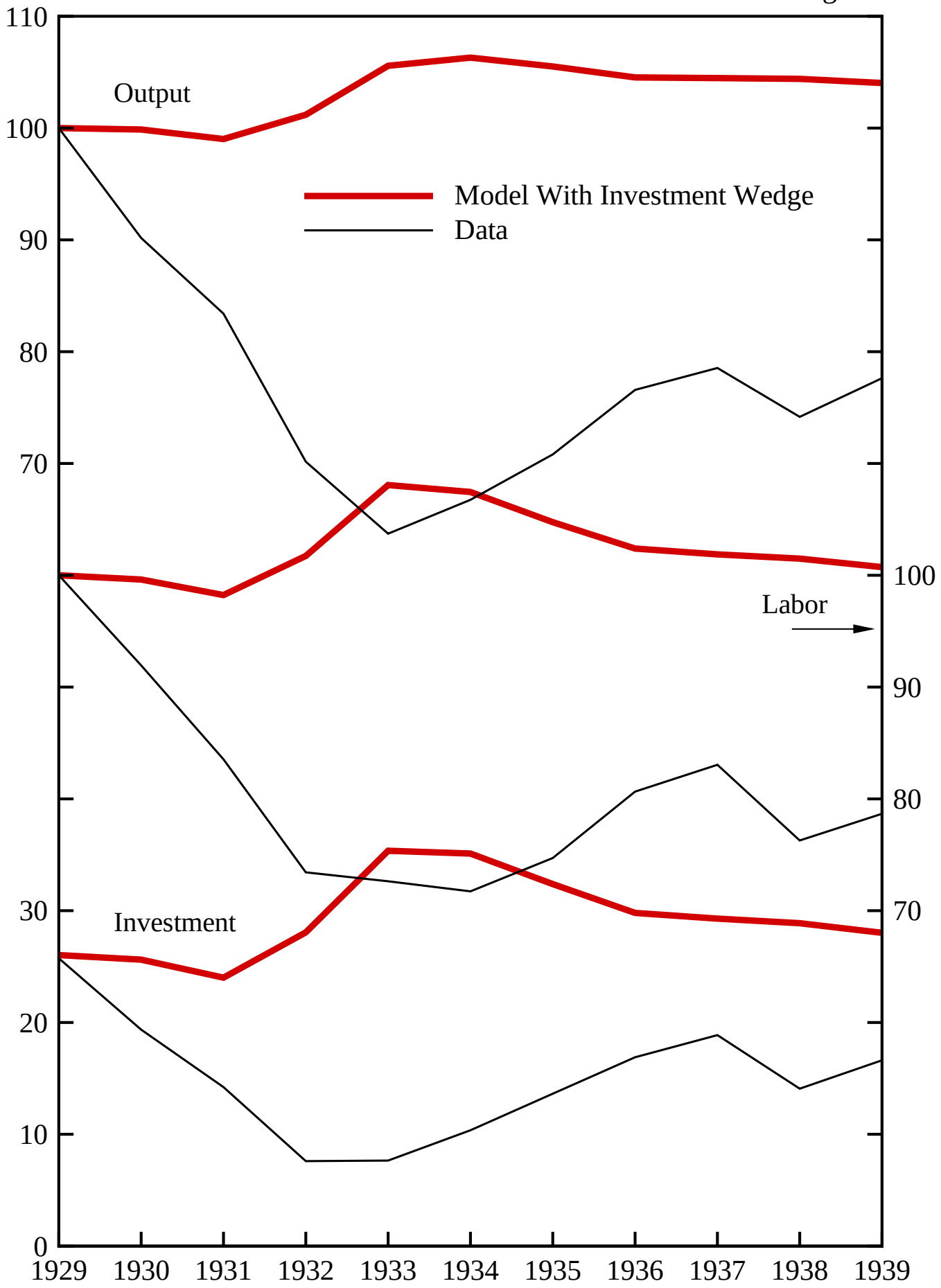


Figure 4
Data and Predictions of Models With All But One Wedge

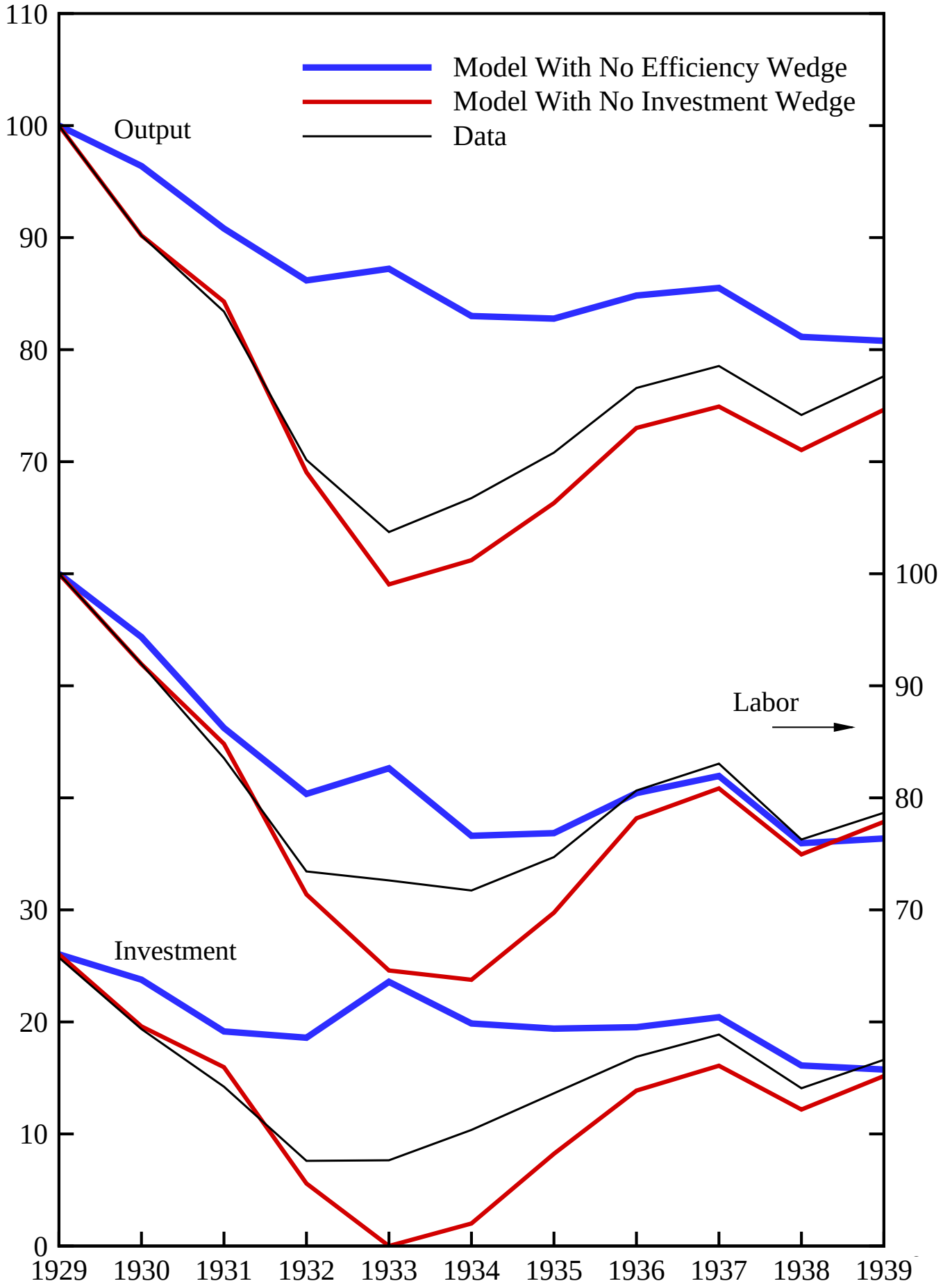


Figure A3
Data and Predictions of Model With Just the Government Consumption Wedge

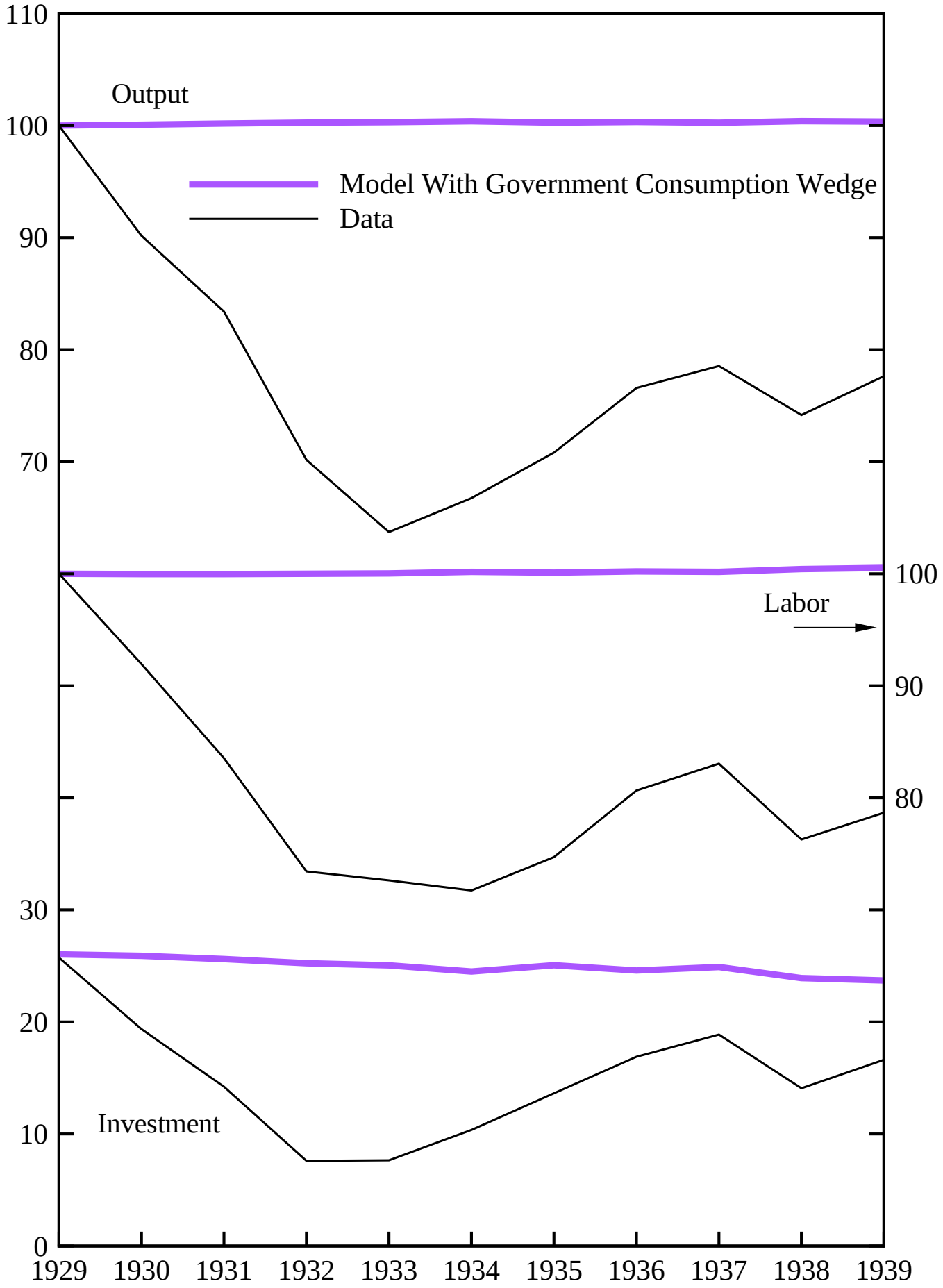
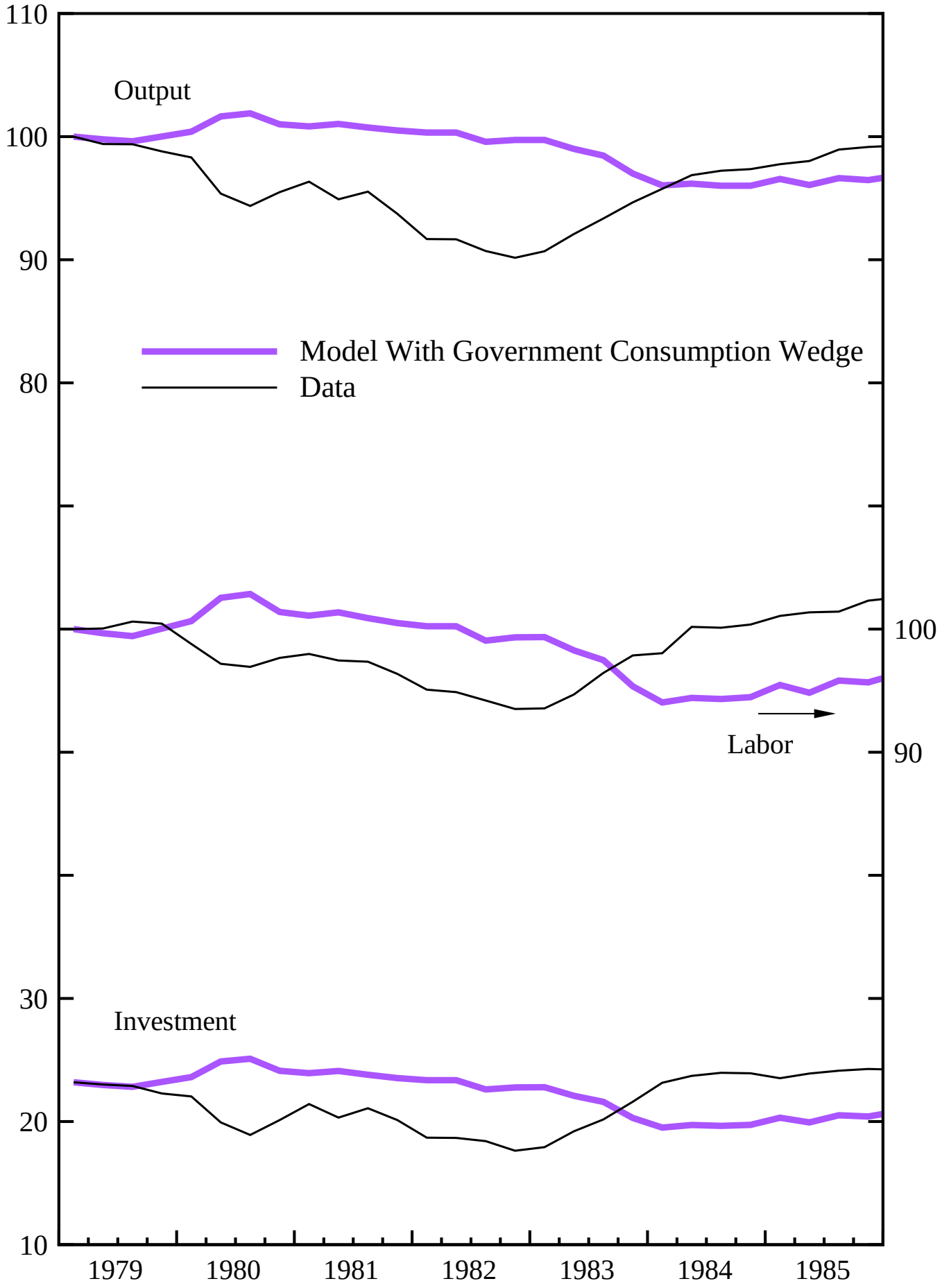


Figure A4
Data and Predictions of Model With Just the Government Consumption Wedge



Sensitivity to the Labor Elasticity

- Index the economy by i:

$$U(c,l) = \frac{c^{1-\sigma}}{1-\sigma} - \frac{l^{1+\zeta_i}}{1+\zeta_i}$$

$$\frac{U_l}{U_c} = (1-\tau_i)F_l \iff l^{\zeta_i} c^{\sigma}(1-\tau_i)(1-\theta)y/l$$

- Proposition: the ζ_2-economy has same equilibrium as the ζ_1-economy

  Proof: $1-\tau_2 = (1-\tau_1)l^{\zeta_2-\zeta_1}$

- Punchline: Different measured wedges—but same equilibrium

Sensitivity to the Capital Utilization

- Define technology as

$$y = A(kh)^{\theta}(nh)^{1-\theta}$$

where n=employed, h=hours, $l = nh$

- Fixed capital utilization: h constant, n varies

$$y = Ak^{\theta}l^{1-\theta}$$

- Variable capital utilization: h varies, n constant

$$y = Ak^{\theta}l^{1-\theta}$$

Sensitivity to the Capital Utilization

- Index the economy by i:

$$y = A_i k^{\theta} l^{\gamma_i} \quad \gamma_i = 1 - \theta \text{ or } 1$$

$$\frac{U_l}{U_c} = (1 - \tau_i)\gamma_i \frac{y}{l}$$

- Proposition: the γ_2-economy has same equilibrium as the γ_1-economy

  Proof: $A_2 = A_1 l^{\gamma_1 - \gamma_2}$ and $1 - \tau_2 = (1 - \tau_1)\gamma_1/\gamma_2$

- Punchline: Different measured wedges—but same equilibrium

Figures 9–12
Varying the Capital Utilization Specification
Great Depression Period, 1929–39

Figure 9
Measured Efficiency Wedges for Two Capital Utilization Specifications

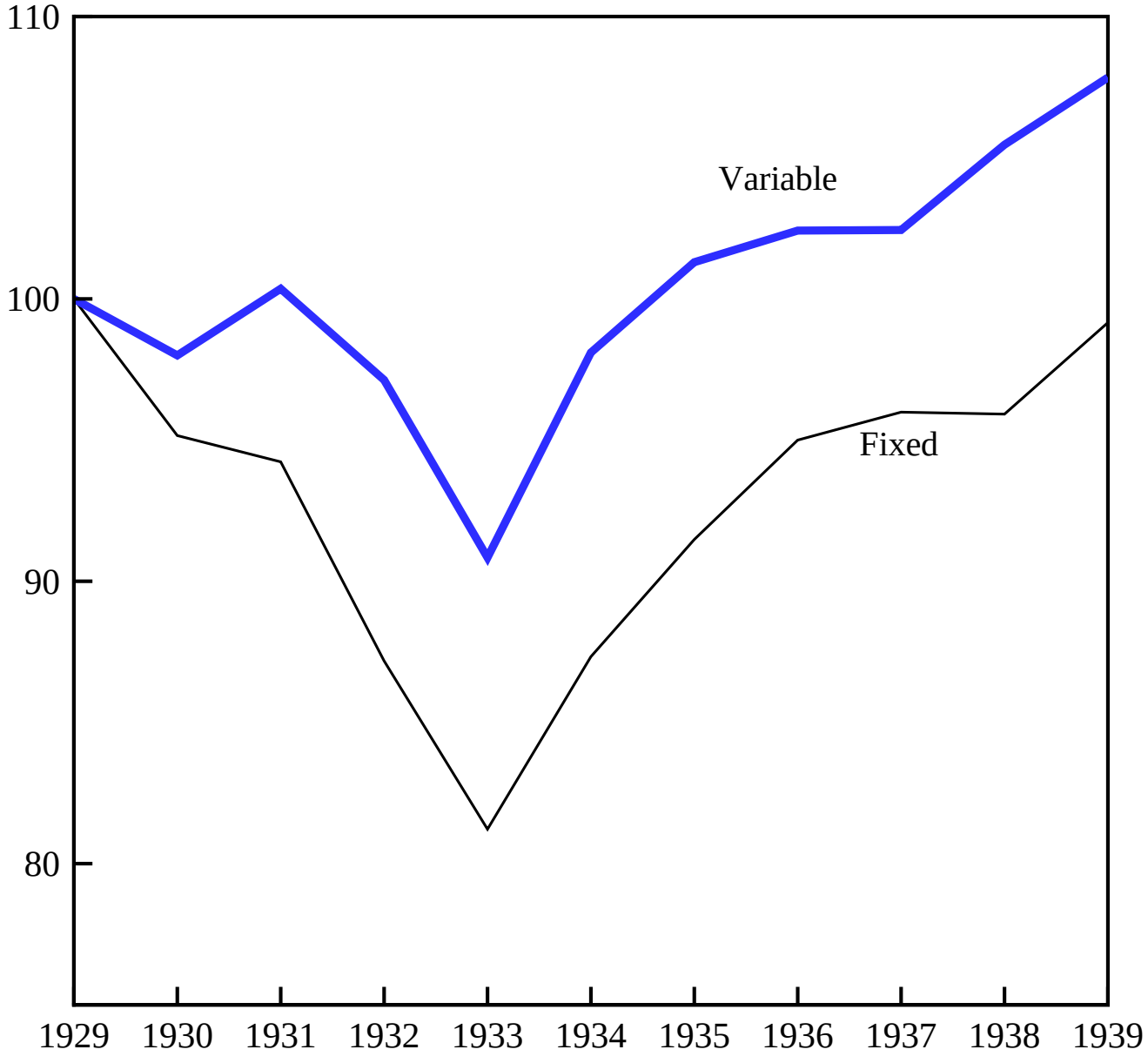
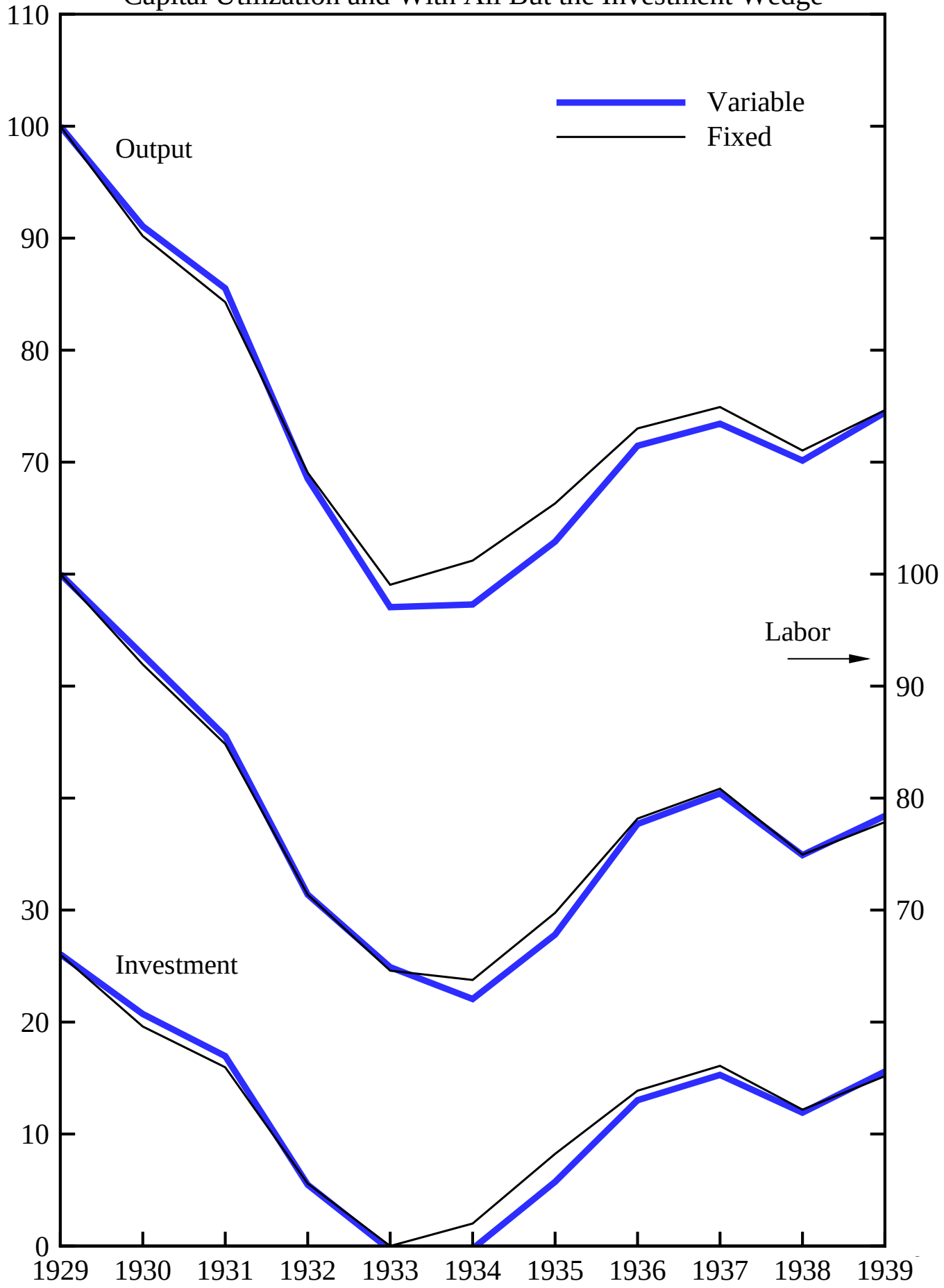


Figure 12
Predictions of Models with Fixed and Variable
Capital Utilization and With All But the Investment Wedge



Recap

- Our application to Great Depression shows
  - Labor and efficiency wedges generate almost all variation
  - While investment wedges are the focus of the literature
  - Size of wedges not relevant—equilibrium responses are.

Application to the Postwar

Figures 5–8
Examining the 1982 U.S. Recession
Quarterly, 1979:1–1985:4; Normalized to Equal 100 in 1979:1

Figure 5
U.S. Output and Three Measured Wedges

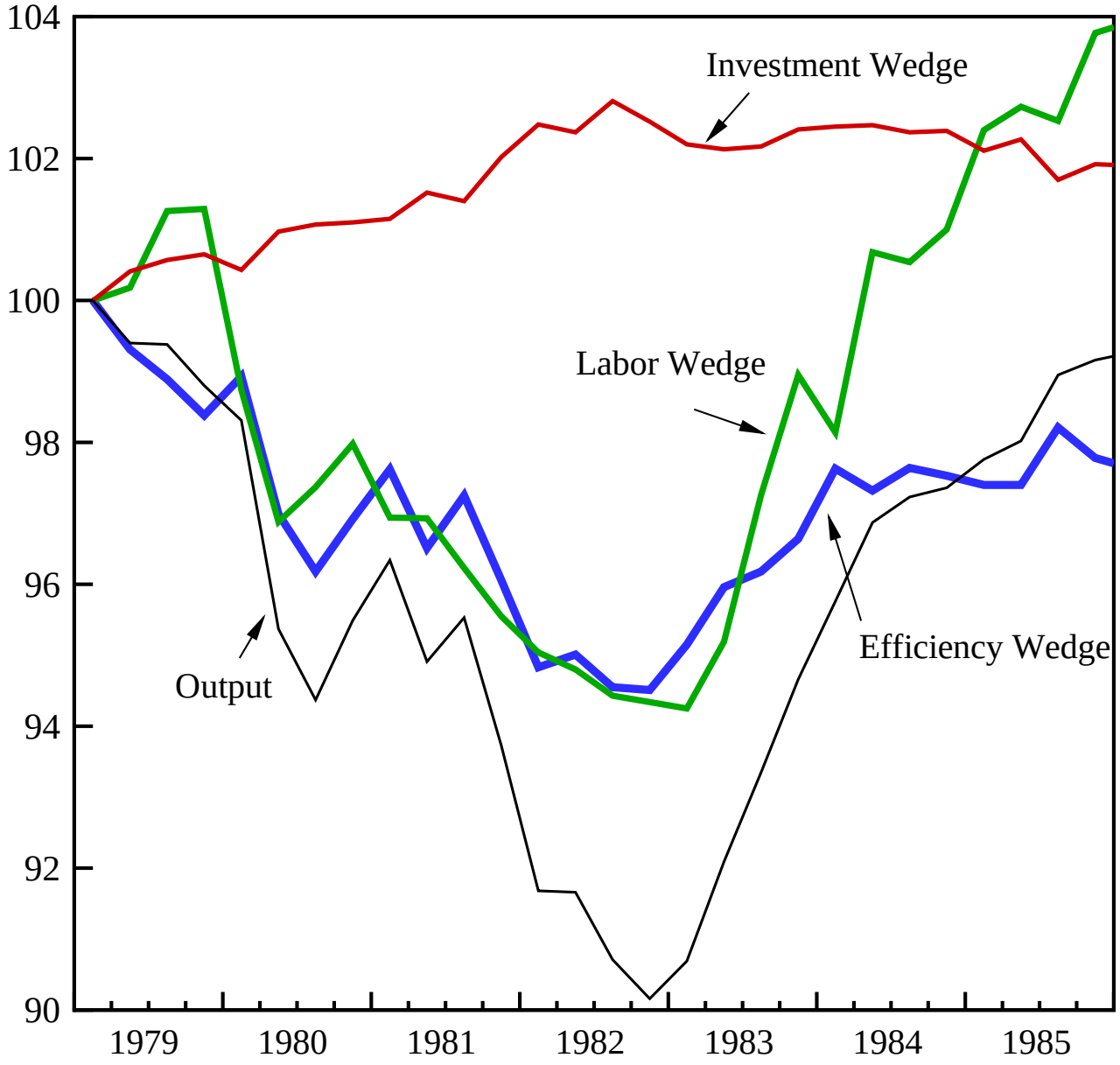


Figure 6
Data and Predictions of Models with Just One Wedge

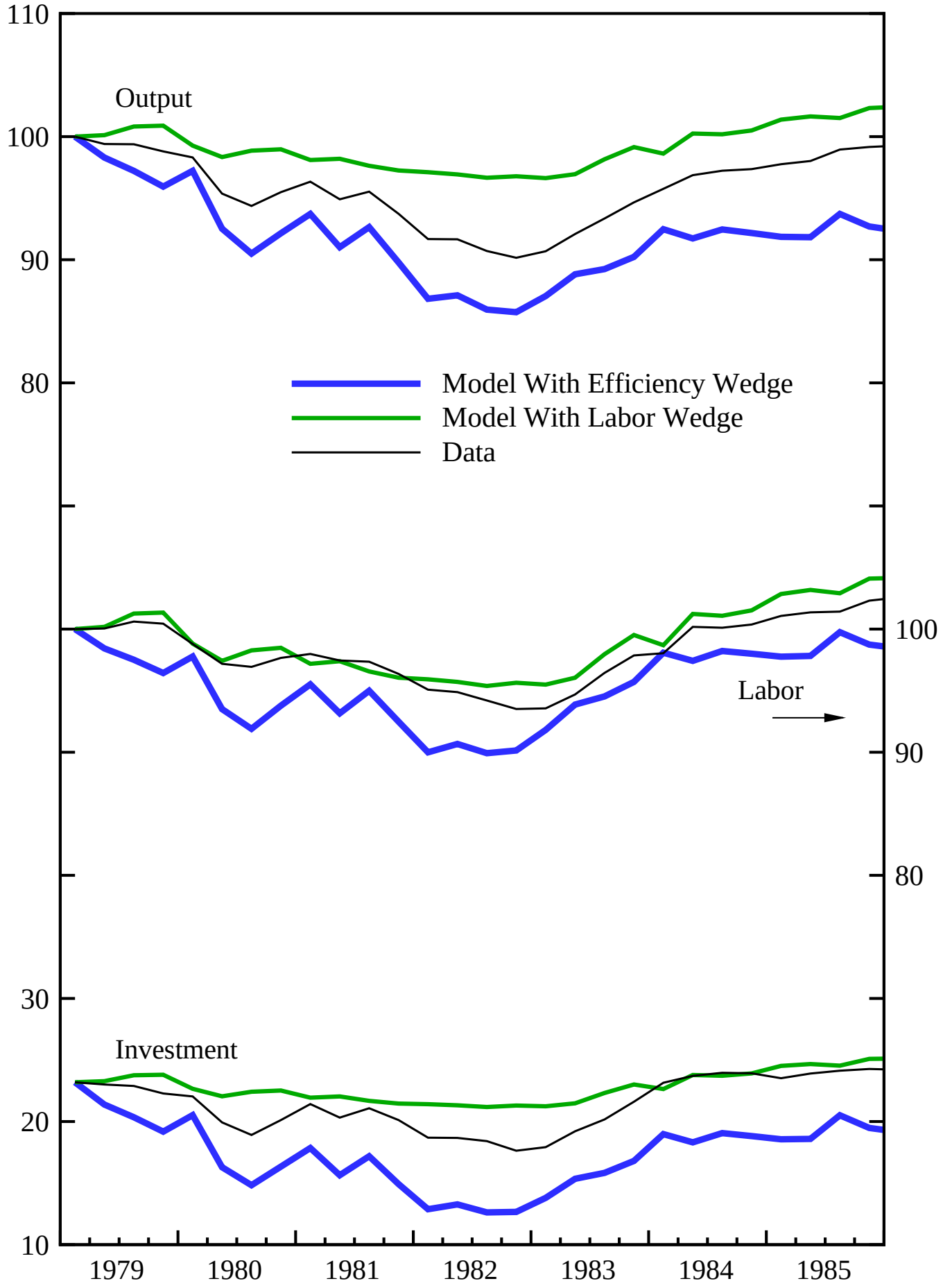


Figure 7
Data and Predictions of Model With Just the Investment Wedge

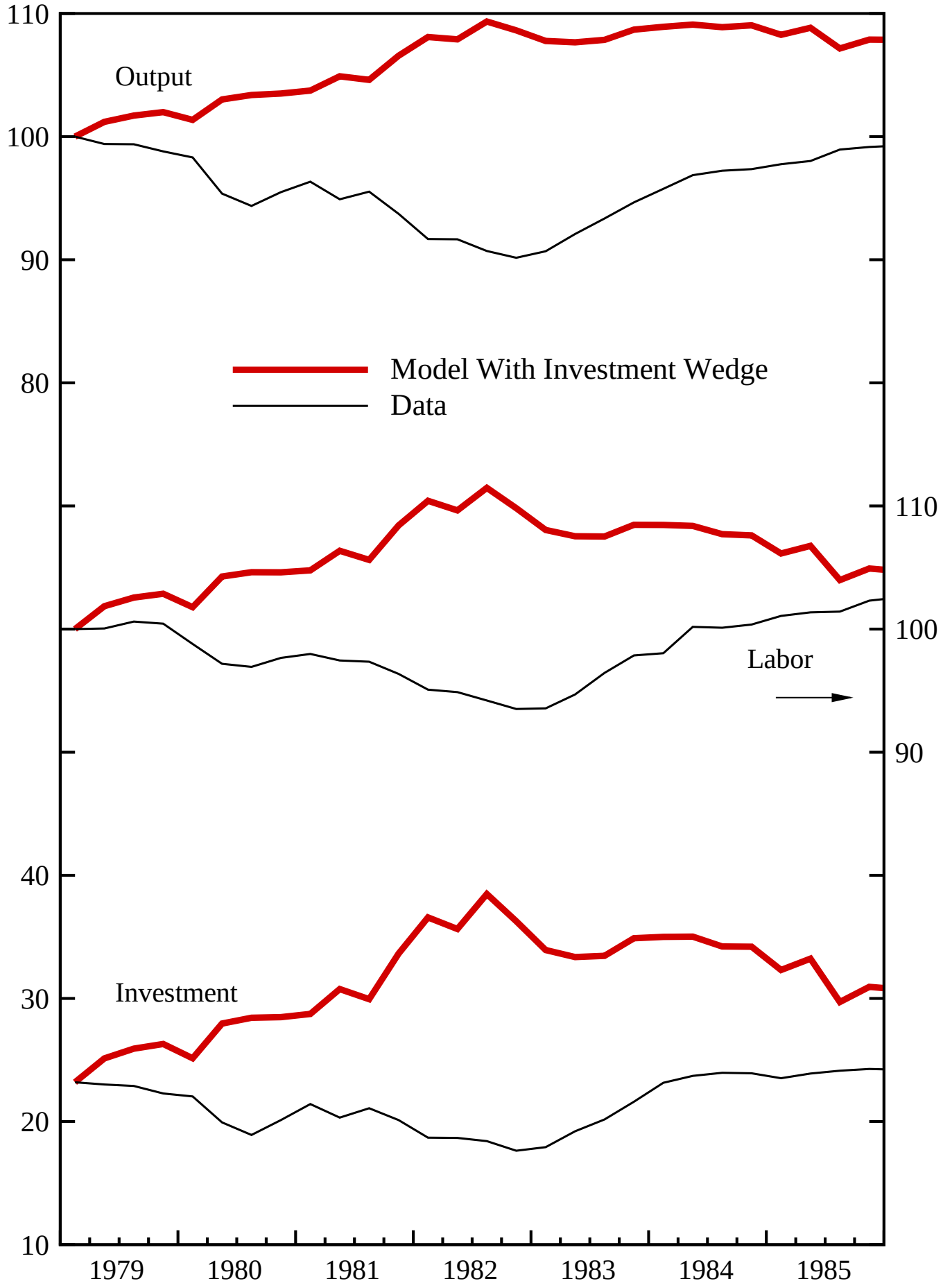
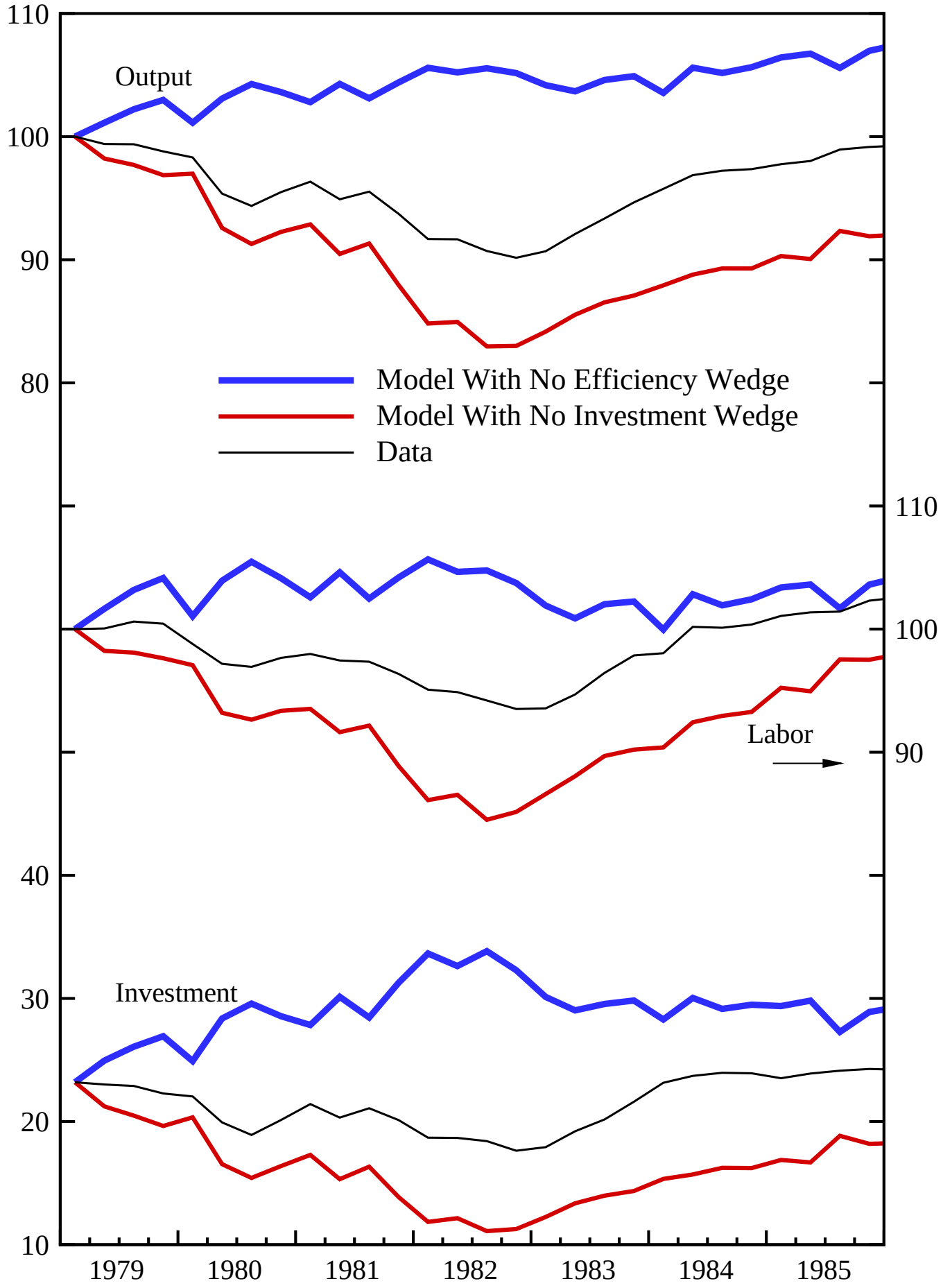


Figure 8
Data and Predictions of Models With All But One Wedge



Properties of the Wedges, 1959:1–2004:3

A. Summary Statistics		
Wedges	Standard Deviation Relative to Output	Correlation With Output
Efficiency	.62	.85
Labor	.92	.71
Investment	.26	−.70
Government Consumption	1.51	−.33
B. Cross Correlations		
Efficiency, Labor		.30
Efficiency, Investment		−.85
Efficiency, Government Consumption		−.34
Labor, Investment		−.15
Labor, Government Consumption		−.38
Investment, Government Consumption		−.15

Properties of the Output Components, 1959:1–2004:3

A. Summary Statistics		
Components	Standard Deviation Relative to Output	Correlation With Output
Efficiency	1.59	.86
Labor	.56	.65
Investment	.78	−.70
Government Consumption	.49	−.37
B. Cross Correlations		
Efficiency, Labor		.23
Efficiency, Investment		−.86
Efficiency, Government Consumption		−.36
Labor, Investment		−.13
Labor, Government Consumption		−.36
Investment, Government Consumption		−.12

Recap

- Approach useful for isolating relevant class of theories

- Should focus on
  - Labor and efficiency wedge
  - Models with multivariate and correlated shock processes

- Should **not** focus on
  - Investment wedges
  - Models with single or uncorrelated shock processes