

Sequential Markets and the Suboptimality of the Friedman Rule

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Abstract

A cash-in-advance model with sequential markets is constructed, where unanticipated monetary injections are nonneutral and can potentially produce large liquidity effects. However, if the monetary authority adheres to an optimal money rule, money should not respond to unanticipated shocks, so that a Friedman rule is suboptimal and the monetary authority does not exploit the liquidity effect. Quantitatively, the model can generate variability in money

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and nominal interest rates close to what is observed, and can produce data with no obvious evidence of the existence of liquidity effects.

1. INTRODUCTION

This paper attempts to shed some light on two puzzles in monetary economics. The first of these puzzles is that central banks do not conduct monetary policy as standard monetary theory predicts they should. In most monetary models (with some notable exceptions, see Woodford 1990a) optimal monetary policy is prescribed by a Friedman rule (Friedman 1969). That is, the stochastic process for the money supply should be such that the nominal interest rate is zero in all states of the world. The Friedman rule is optimal in models with a cash-in-advance constraint, with money-in-the-utility-function, or with a role for money in economizing on transactions costs. Also, Chari, Christiano, and Kehoe (1993) find conditions under which the Friedman rule is optimal in a variety of environments with distorting taxes, where a priori reasoning would suggest that a moderate inflation tax might be desirable. The Friedman prescription is striking, in that it differs so markedly from practice; nominal interest rates on default-free government securities are typically positive and variable (and sometimes highly variable).

The second puzzle relates to the elusive "liquidity effect." The liquidity effect is defined to be a negative response of the nominal interest rate to a positive money injection. It seems clear that a belief in the liquidity effect is strong among many macroeconomists. However, the existence of a liquidity effect is at best weakly supported by empirical evidence (see Leeper and Gordon 1992 and Christiano and Eichenbaum 1992).

In this paper, a model is constructed in which money is nonneutral and where there potentially exist strong liquidity effects. An unanticipated monetary injection causes a less-than-proportionate short-run increase in the price level, the nominal interest rate falls, and output increases. However, if the monetary authority sets the money supply optimally, then it will not exploit the liquidity effect. That is, it should

eliminate all variability in unanticipated money, and permit the nominal interest rate to vary. As the liquidity effect is never exploited by the monetary authority at the optimum, the data generated by the model will exhibit no obvious evidence that a liquidity effect exists. Though this implication of the model appears to fit the qualitative facts, we want to explore further, in asking whether the operating characteristics of our model under an optimal money rule provide a quantitative match with U.S. experience for the period 1954-1992.

In spite of the weak empirical evidence supporting the existence of liquidity effects, there has been a significant quantity of recent theoretical and quantitative work which seeks to "explain" these effects. Lucas (1990) and Fuerst (1992) study a class of cash-in-advance models with distributional effects of monetary injections, building on earlier work by Grossman and Weiss (1983) and Rotemberg (1984). In Lucas's model, the goods market and financial market are effectively segmented within a period. When an open market purchase occurs, the financial market is forced to absorb its effects, and the nominal interest rate falls. Christiano (1990) and Christiano and Eichenbaum (1992) study quantitative versions of Lucas-Fuerst type models.

Putting aside the question of whether distribution effect models can explain empirical observations, it can be argued that they lack plausibility. In these models, the liquidity effect lasts for the length of time financial markets and goods markets are effectively segmented. The market segmentation interval is not likely to last long in practice; in the Grossman-Weiss and Rotemberg frameworks, this interval is the length of time between visits to the bank. Thus, it seems a stretch to argue that such models could be used to explain phenomena which occur at a monthly or quarterly frequency.

An alternative class of models to the distribution effect models, which share a similar nonneutrality of unanticipated money, are the environments with sequential

markets studied by Lucas (1989), Eden (1994), Woodford (1990b), and Lucas and Woodford (1992). Lucas (1989) interpreted his model as one where prices are set in advance in a sequence-of-markets economy, while Lucas and Woodford study a model with essentially the same implications, where transactions proceed sequentially subject to imperfect information about the current monetary shock. In these models, the full extent of nominal demand for goods is not revealed until the last market in the sequence clears, and unanticipated monetary injections are therefore nonneutral. When the only source of aggregate uncertainty is a money supply disturbance, it is optimal to reduce the variability in unanticipated money to zero, just as in older money-surprise models (Lucas 1972).

The model here builds on the work of Lucas (1989), Eden (1994), Woodford (1990b), and Lucas and Woodford (1992). We retain the essential sequential markets characteristic of transactions in goods markets, but adapt the model so that it can handle asset pricing and quantitative analysis. The model involves a cash-in-advance constraint, with the preferences of the representative household defined over cash goods and credit goods (Lucas and Stokey 1987). A household consists of a continuum of buyers and a seller, who are together at the beginning of the period. While the buyers take cash to the market for cash goods, the seller sells cash goods across a set of markets which are spatially separated. At the beginning of the period, the household has a given amount of productive capacity which must be allocated at that time across cash goods markets and in the market for credit goods. After productive capacity is put in place, the household learns the current money shock and preference shock, and the asset market opens. A positive preference shock acts to increase the marginal utility of cash goods, and can be interpreted as a positive money demand shock.

Cash goods markets open sequentially and, effectively, the number of cash goods

markets which open in any period depends on the aggregate quantity of money in circulation. The household will then (in equilibrium) allocate a positive quantity of capacity to any cash goods market with a positive conditional probability of opening in the current period. When an unanticipated money injection occurs, the average price of the cash good and output rise. In general, there will be a liquidity effect on the nominal interest rate, which works somewhat similarly to the liquidity effect in traditional fixed-price Keynesian models, though prices are not fixed in advance here. With a positive money injection, equilibrium asset prices must move in order to induce the representative household to take the higher stock of real balances to cash goods markets, and to hold zero nominal bonds. With an intertemporal elasticity of substitution less than one, this requires that the nominal interest rate must fall to make cash balances more attractive.

In the absence of preference shocks, the optimal money rule is identical to that in Lucas(1989) and Lucas and Woodford (1992). That is, monetary uncertainty is suboptimal, as it reduces average consumption and increases consumption variability. Thus, a Friedman rule is optimal; the nominal interest rate should be zero in all states of the world, with the money supply responding to real shocks which are known when capacity is put in place. When there are unanticipated preference shocks, the optimal money rule cannot be characterized analytically. There are two effects at work here. First, if there is a positive preference shock, then the representative household has a greater preference for cash goods relative to credit goods, but more cash goods will not be forthcoming unless there is a cash injection. If monetary policy responds to unanticipated preference shocks, which are revealed by unanticipated movements in the nominal interest rate, then this can act to increase expected utility. However, the second effect comes into play, in that variability in unanticipated money will imply that production is in general below capacity, and this will act to reduce welfare. Here,

the optimal money rule must be determined quantitatively.

The first set of quantitative exercises we consider serve to illustrate some of the operating characteristics of the model and the size of the potential welfare losses from suboptimal money rules. We consider a version of the model where the money supply is exogenous, all aggregate uncertainty is associated with variability in the money supply, and a Friedman rule is optimal. We compute equilibria for various parameter values given a money supply process which replicates the properties of the stochastic process followed by the monetary base over the period 1954-1992. Monetary variability produces variability in the nominal interest rate equal to or larger than observed variability with moderate levels of the intertemporal elasticity of substitution. The correlation between detrended money and the nominal interest rate generated by the model is negative and large in absolute value, while the corresponding correlation in the data is close to zero. Therefore, if exploitation of the liquidity effect had been an important part of Federal Reserve operating procedure over the period 1954-1992, then our model would not be able to explain the data.

With an exogenous money supply and no shocks to preferences or technology, we calculate the welfare gain associated with changing the money supply process, from one that replicates the data to a Friedman rule. This welfare calculation is done in two steps. We first reduce the variance about trend in money to zero, and then reduce the trend money growth rate to minus the discount rate. The second step corresponds to the experiment conducted in Cooley and Hansen (1989). In a standard cash-in-advance model, the cost (in terms of average consumption) of variability in money is negligible, while in our model the cost of variability is from 2.6 to 30 times the cost of suboptimal trend money growth (for the parameter values considered). Therefore, in this model, variability in money and inflation is a big deal (or at least a relatively big deal).

The second set of quantitative exercises involves introducing preference shocks and shocks to productive capacity, with the stochastic processes for these shocks calibrated to replicate the sample behavior of the velocity of money and consumption. The money supply is determined endogenously so as to maximize the expected utility of the representative household in equilibrium. For the parameter values considered, it is always optimal to reduce the variability in unanticipated money to zero. As preference shocks cause unanticipated movements in the nominal interest rate which do not induce any response by the monetary authority at the optimum, the nominal interest rate will be positive, on average, and variable. We show that, for reasonable parameter values, the variability in the nominal interest rate and money about trend are on the order of what is observed in the data. Also, the correlation between detrended money and the nominal interest rate is close to zero, as in the data. Therefore, though there exists a strong liquidity effect in the model, it is suboptimal for the monetary authority to exploit it. If the monetary authority adheres to an optimal money rule, the data will provide no obvious indication that liquidity effects exist.

The results we get here are quite different from those of Woodford (1990b), who suggests that, when there are unanticipated real shocks, the monetary authority should respond to them and smooth nominal interest rates. Woodford's model differs from ours in that, in his model, money is injected through the loan market, which clears sequentially. It seems more natural to assume, as we do, that there is a dichotomy between financial markets and goods markets; financial markets are relatively frictionless, while goods markets are more poorly integrated and subject to trading frictions. Woodford also does not examine asset pricing or quantitative issues, as we do here.

The remainder of the paper proceeds as follows. In Section 2 the model is constructed, and a competitive equilibrium is defined in Section 3. The properties of the competitive equilibrium are studied in Section 4, and Section 5 contains an analysis

of optimal monetary policy. In Section 6, the results of some quantitative exercises are reported. Section 7 contains a summary and conclusion.

2. THE MODEL

This is a cash-in-advance model with cash goods and credit goods, where the cash-in-advance constraint and the transactions mechanism can be motivated by the following locational structure. There are N households (N large), indexed by $i = 1, 2, \dots, N$, each consisting of a seller and a continuum of buyers indexed by $s \geq 0$. Given symmetry, we will economize on notation by dropping i subscripts wherever possible. Household i begins each period $t = 0, 1, 2, \dots$, with M_t units of currency and B_t units of one-period nominal bonds (each of which is a promise to pay one unit of currency on the asset market in period t), and with the buyers and the seller together at location i . The household has preferences given by

$$E_0 \sum_{t=0}^{\infty} \beta^t u(c_t^1, c_t^2, \theta_{t+1}),$$

where $0 < \beta < 1$, c_t^1 is consumption of the good produced at location $(i+1) \pmod{N}$ (the cash good), c_t^2 is consumption of the good produced at location i (the credit good), and θ_{t+1} is a shock to preferences which is unknown at the beginning of period t . Assume that the utility function is twice differentiable, increasing, and strictly concave in its first two arguments, that $\frac{u_1(0, c_2, \theta)}{u_2(0, c_2, \theta)} = \infty$ for $c_2 > 0$, that $\frac{u_1(c_1, 0, \theta)}{u_2(c_1, 0, \theta)} = 0$ for $c_1 > 0$, that $u_{13}(c_1, c_2, \theta) > 0$, and that $u_{23}(c_1, c_2, \theta) < 0$. That is, higher θ implies a greater preference for cash goods relative to credit goods. As purchases of cash goods require currency, the shock to preferences can be interpreted as a money demand shock. This approach to modeling random liquidity needs by way of preference shocks is similar to what is done in some types of banking models (e.g. Diamond-Dybvig 1983, Champ, Smith, and Williamson 1992).

At the beginning of each period, each household has y_t units of productive capacity available, where y_t is a random variable. This productive capacity can be used to produce credit goods at location i and to put productive capacity in place for producing cash goods at location i . One unit of productive capacity is capable of producing one unit of the cash good or one unit of the consumption good. Cash goods are sold at location i on a continuum of spatially-separated markets indexed by $s \geq 0$. Letting $x_t(s)$ denote the quantity of productive capacity allocated to market s , the quantity of credit goods produced is $y_t - \int_0^\infty x_t(s) ds$.

Once productive capacity is installed, the household receives a cash transfer of Υ_{t+1} units of currency from the government. The government's budget constraint, in per capita terms, is

$$\bar{M}_{t+1} - \bar{M}_t = \Upsilon_{t+1}, \quad (1)$$

where \bar{M}_t is the per-capita money stock at the beginning of period t . The government sets money transfers in such a way that the money supply follows a trend-stationary process

$$\bar{M}_{t+1} = \bar{M}_0 z^t \phi_{t+1}, \quad (2)$$

where $\bar{M}_0 > 0$, $z > 0$, and ϕ_{t+1} is a random variable which is unknown at the beginning of period t . The money supply process in (2) is somewhat unconventional for this type of model; usually the (log of the) money supply is assumed to be stationary in first differences. We adopt this specification to be consistent with the way we treat the data, and because this will make our points more forcefully, as we will see later.

At the beginning of the period, the household knows θ_t , ϕ_t , and y_t . The preference shock θ_{t+1} is revealed after productive capacity is installed, at the same time as the household learns ϕ_{t+1} . Assume that (θ_t, ϕ_t, y_t) follows a first-order Markov process.

After θ_{t+1} and ϕ_{t+1} are learned, and the household receives the cash transfer, cash holdings can be adjusted on the asset market, following which buyers go to location

$i + 1$ to buy cash goods with currency, while the seller remains at location i to sell cash goods. The transactions process for the exchange of cash goods and currency works in the following manner. The household chooses a subset of the continuum of buyers to go to the cash goods markets. Without loss of generality, let those buyers with $s \leq s_t^*$ be those who are chosen to participate, where $s_t^* \geq 0$. Buyer s is then allocated a quantity of cash $M_t(s)$, $s \leq s_t^*$. Each buyer receives a place in line at random, and the cash goods markets open in sequence, i.e. buyer s receiving place in line r goes to market r when it opens with quantity of cash $M_t(s)$. Buyers cannot communicate with each other after they learn their place in line. There is a constraint that no more than $M_0 z^t$ units of cash can be spent per unit interval on the continuum of cash goods markets. Also, we assume that buyers are not permitted to leave a cash goods market with a positive quantity of currency,¹ and that the monetary authority will tax away any cash balances taken by a buyer to a market with $s \geq \phi_{t+1}$. In what follows, markets with $s \leq \phi_{t+1}$ will be denoted "open markets," and those with $s > \phi_{t+1}$ will be "closed."

Let $P_t^1(s)$ denote the price of a cash good sold in market s , and note that there are no opportunities for arbitrage across cash goods markets, ex post. That is, once capacity is put in place, production cannot be moved across markets, and buyers cannot arbitrage price differences across markets. We will conjecture an equilibrium where $P_t^1(s)$ is monotonic increasing in s , for $s \leq \phi_{t+1}$, with $P_t^1(s) = 0$ for $s > \phi_{t+1}$. We will later verify that the equilibrium price schedule in fact has these properties. Facing this price schedule, and given the behavior of the monetary authority and the constraints on money holdings, it is optimal for the household to set $s_t^* = \min(\frac{M_t + \Upsilon_{t+1}}{M_0 z^t}, \phi_{t+1})$ and $M_t(s) = M_0 z^t$ for all $s \leq s_t^*$ [supposing that the quantity of cash taken to cash goods

¹This assumption rules out the possibility that the buyer in a market with a high price would choose to forego purchasing consumption goods, and hold cash over to the following period.

markets is $\min(M_t + Y_{t+1}, \phi_{t+1} M_0 z^t)$, i.e. the cash-in-advance constraint is binding]. That is, the household wishes to, as much as possible, confine its purchases to markets with low s . Now, when households go to the asset market, the relevant price for their decision-making is the implicit price P_t^1 for all cash goods across markets $s \leq s_t^*$. The cash-in-advance constraint is

$$Q_t B_{t+1} + P_t^1 c_t^1 \leq M_t + B_t + Y_{t+1}, \quad (3)$$

where Q_t is the price in period t of a promise to pay one unit of currency in period $t + 1$.

While this model is similar in spirit to that of Lucas and Woodford (1992), there are some important differences, which are intended to simplify the analysis and to permit the determination of asset prices. In particular, Lucas and Woodford study a transactions process in the market for goods in which transactions proceed sequentially, and where sellers are uninformed about the current money shock at the time the goods market opens. Here, since the asset market opens after the money shock is known and before the goods market opens, there is a potential for asset prices to provide information about the current money supply. This information would be useful for producers in allocating capacity across markets. However, in our model production decisions are locked in at the time the money shock is revealed. In spite of the simplifications we have made here relative to Lucas and Woodford (1992), our model has much the same implications with regard to the role of money, as we will show.

While the transactions process here may seem somewhat contrived, what is important is that the actions of the monetary authority can affect the distribution of cash holdings, and the distribution of nominal demand, across cash goods markets after production decisions are locked in place. In fact, this model can be interpreted as capturing a distributional effect of monetary injections in goods markets, while

the models of Lucas (1990) and Fuerst (1992) focus on the distribution effects of monetary policy in financial markets vis-a-vis goods markets.

The household's budget constraint is

$$\begin{aligned} M_{t+1} + Q_t B_{t+1} + P_t^1 c_t^1 + P_t^2 c_t^2 + P_t^2 \int_0^\infty x_t(s) ds \\ \leq M_t + B_t + Y_{t+1} + P_t^2 y_t + \int_0^{\phi_{t+1}} P_t^1(s) x_t(s) ds, \end{aligned} \quad (4)$$

where P_t^2 is the money price of the credit good. Note here, that in contrast to Lucas and Stokey's (1987) model, the prices of cash and credit goods may differ here, as there are no opportunities for goods arbitrage after productive capacity is put in place.

3. COMPETITIVE EQUILIBRIUM

For convenience, we scale all nominal variables by dividing by trend money, $\bar{M}_0 z^t$, and use lower case letters to denote these normalized variables. For example, $p_t^1 \equiv \frac{P_t^1}{\bar{M}_0 z^t}$. We can then use (2) to rewrite the constraints (3) and (4), respectively, as

$$Qb'z + p_1 c_1 \leq m + b + \tau', \quad (5)$$

$$\begin{aligned} m'z + Qb'z + p_1 c_1 + p_2 c_2 + p_2 \int_0^\infty x_t(s) ds \\ \leq m + b + \tau' + p_2 y + \int_0^{\phi'} p_1(s) x(s) ds, \end{aligned} \quad (6)$$

where for convenience we drop t subscripts, use primes to denote variables dated $t+1$, and use subscripts instead of superscripts on prices.

Now, defining $v(m, b, \theta, \phi, y)$ as the value function for the household's optimization problem, the Bellman equation for this problem is

$$\begin{aligned} v(m, b, \theta, \phi, y) = \\ \max E [\max \{u(c_1, c_2, \theta') + \beta E[v(m', b', \theta', \phi', y') \mid \theta', \phi', y']\} \mid \theta, \phi, y] \end{aligned} \quad (7)$$

subject to (5) and (6). Here, the outer maximization is with respect to $x(s)$, and the inner maximization is with respect to c_1, c_2, m' , and b' . Let λ denote the multiplier

associated with the cash-in-advance constraint (5), and μ the multiplier associated with the household's budget constraint, and assume that the value function is differentiable and well-behaved. We will assume throughout that it is optimal for the household to spend all its cash holdings in the markets for cash goods. In equilibrium, we will then have $s^* = \phi'$, so that the demand for cash goods is positive at any open production outlet, i.e. for any s such that $s \leq \phi'$. Therefore, in a competitive equilibrium we must have $x(s) > 0$ for all s such that $\Pr[s \leq \phi' \mid \theta, \phi, y] > 0$. Then, given the first-order condition for $x(s)$ from the household's optimization problem, the following arbitrage condition must hold for each s such that $\Pr[s \leq \phi' \mid \theta, \phi, y] > 0$.

$$E\{\mu[p_1(s) - p_2] \mid \theta, \phi, y\} = 0. \quad (8)$$

Note in (8) that, if $\mu, p_1(s)$, and p_2 are known at the beginning of the period (which, in general, they are not), then all goods prices are equalized, as in the model of Lucas and Stokey (1987).

The first-order conditions from the inner maximization problem in (7) give (maximizing with respect to c_1, c_2, m' , and b' , respectively):

$$u_1(c_1, c_2, \theta') - (\lambda + \mu)p_1 = 0, \quad (9)$$

$$u_2(c_1, c_2, \theta') - \mu p_2 = 0, \quad (10)$$

$$\beta E[v_1(m', b', \theta', \phi', y') \mid \theta', \phi', y] - \mu z = 0, \quad (11)$$

$$\beta E[v_2(m', b', \theta', \phi', y') \mid \theta', \phi', y] - (\lambda + \mu)Qz = 0. \quad (12)$$

We also have the following envelope conditions.

$$v_1(m, b, \theta, \phi, y) = \lambda + \mu, \quad (13)$$

$$v_2(m, b, \theta, \phi, y) = \lambda + \mu. \quad (14)$$

Using (13) and (14) to substitute in (11) and (12), we obtain

$$\beta E[\lambda' + \mu' \mid \theta', \phi', y] - \mu z = 0, \quad (15)$$

$$\beta E[\lambda' + \mu' \mid \theta', \phi', y] - (\lambda + \mu)Qz = 0. \quad (16)$$

Now, the equilibrium conditions for the money and bond markets, respectively, are

$$m = \frac{\phi}{z}, \quad (17)$$

$$b = 0, \quad (18)$$

and equilibrium in the market for cash goods, a binding cash-in-advance constraint, and $s^* = \phi'$ imply that

$$\begin{aligned} p_1(s)x(s) &= 1, s \leq \phi', \\ p_1(s) &= 0, s > \phi', \end{aligned} \quad (19)$$

and

$$p_1 c_1 = \phi', \quad (20)$$

where

$$c_1 = \int_0^{\phi'} x(s) ds. \quad (21)$$

The economy's resource constraint gives

$$c_2 = y - \int_0^{\infty} x(s) ds. \quad (22)$$

Given that (θ, ϕ, y) follows a first-order Markov process, an equilibrium is a set of quantity schedules $\{x(s, \theta, \phi, y), c_1(\theta, \phi, y, \theta', \phi'), c_2(\theta, \phi, y)\}$, and a set of price schedules $\{p_1(s, \theta, \phi, y), p_1(\theta, \phi, y, \theta', \phi'), p_2(\theta, \phi, y, \theta', \phi')\}$ satisfying (8)-(10) and (15)-(22). Note that $x(s)$ is chosen before θ' and ϕ' are known, so that it depends only on (θ, ϕ, y) . Therefore, from (19), $p_1(s)$ does not depend on θ' and ϕ' , for $s \leq \phi'$. Also, given that $x(s)$ depends only on (θ, ϕ, y) , the same is true for c_2 , from (22).

4. PROPERTIES OF THE COMPETITIVE EQUILIBRIUM

To simplify, we can use (9) and (10) to substitute for the multipliers λ and μ in (8), (15), and (16). Then, use (19) and (20) to substitute for $p_1(s)$ and p_1 , respectively, to get

$$\begin{aligned} [x(s)]^{-1} E[u_2(c_1, c_2, \theta') \frac{I(s, \phi')}{p_2} \mid \theta, \phi, y] \\ - E[u_2(c_1, c_2, \theta') \mid \theta, \phi, y] = 0, \end{aligned} \quad (23)$$

$$\beta E\left[\frac{u_1(c'_1, c'_2, \theta'') c'_1}{\phi''} \mid \theta', \phi', y\right] - \frac{u_2(c_1, c_2, \theta')}{p_2} z = 0, \quad (24)$$

$$\beta E\left[\frac{u_1(c'_1, c'_2, \theta'') c'_1}{\phi''} \mid \theta', \phi', y\right] - \frac{Qz u_1(c_1, c_2, \theta') c_1}{\phi} = 0. \quad (25)$$

In equation (23), $I(s, \phi') = 1$ for $s \leq \phi'$, and $I(s, \phi') = 0$ otherwise. From (23), note that $E[u_2(c_1, c_2, \theta') \frac{I(s, \phi')}{p_2} \mid \theta, \phi, y]$ is monotone decreasing in s , which implies that $x(s)$ is monotone decreasing in s . Therefore, from (19) $p_1(s)$ is monotone increasing in s . This implies that p_1 is monotone increasing in ϕ' , since $p_1(s)$ does not depend on ϕ' . That is, unanticipated money shocks cause an increase in the price of cash goods. However, since the quantity of cash goods increases with ϕ' from (21), then given (20) the price of cash goods rises less than in proportion to the unanticipated increase in money.

Since $x(s)$ depends only on (θ, ϕ, y) , from (22), c_2 depends only on (θ, ϕ, y) . Then, from (21), c_1 depends only on (θ, ϕ, y, ϕ') . Therefore, the innovation in the current consumption allocation is independent of the innovation in the preference shock θ , unless innovations in money are correlated with innovations in the preference shock. This also implies that, from (20), the innovation in p_1 is independent of the innovation in the preference disturbance unless money innovations are correlated with innovations in the preference shock. However, from (23) and (24) it is apparent that innovations in the preference shock will in general affect the price of the credit good, p_2 , and the price of the nominal bond, Q .

Now, suppose that (θ, ϕ, y) is i.i.d., and that θ , ϕ , and y are mutually independent. Then, the following conditions hold:

$$E\left[\frac{u_1(c_1, c_2, \theta'')c_1}{\phi''} \mid \theta', \phi', y\right] = k, \quad (26)$$

$$E[u_2(c_1, c_2, \theta') \mid \theta, \phi, y] = f(y) \quad (27)$$

where k is a positive constant. Then, substituting in (23) using (24), (26), and (27), we get

$$x(s) = \frac{\beta k \Pr[s \leq \phi']}{f(y)z} \quad (28)$$

Therefore, if the support of the distribution of ϕ' is the interval $[\phi_\ell, \phi_h]$, with $0 < \phi_\ell < \phi_h$, and the probability distribution function is continuous and increasing on this interval then, given y , $x(s)$ is continuous and increasing on the interval $[0, \phi_h]$, with $x(s)$ a positive constant on $[0, \phi_\ell]$, $x(s)$ strictly decreasing on $[\phi_\ell, \phi_h]$, and $x(s) = 0$ for $s > \phi_h$.

Using (26) to substitute in (25) and rearranging, we obtain

$$Q = \frac{\beta k \phi'}{z u_1(c_1, c_2, \theta') c_1}. \quad (29)$$

In (29), ϕ' has three effects on the price of the nominal bond, Q . First, there is an anticipated inflation effect, which works directly through ϕ' in the numerator. A high current realization of the money shock implies that money is high relative to trend this period, which implies that money is expected to be lower relative to trend next period. That is, a high ϕ' signals low money growth next period and low inflation, so that the nominal interest rate will tend to fall. This anticipated inflation effect here has the opposite sign from what we would get if the money supply process were stationary in first differences with positive first-order autocorrelation in the first differences. As we remarked earlier, the money supply process is specified as in (2) to be consistent with how we treat the data. Also, as will be explained later, this specification actually works against our ability to explain the data.

The second effect of ϕ' on Q in (29) is an anticipated inflation effect working through c_1 in the denominator. If ϕ' is high, then purchases of cash goods is high, which tends to lower the current price of cash goods and increase anticipated inflation. This second effect causes Q to fall and the nominal interest rate to rise. Since $\frac{\phi'}{c_1}$ increases when ϕ' rises, the net effect of the two anticipated inflation effects is a negative anticipated inflation effect on the nominal interest rate.

The third effect works through $u_1(c_1, c_2, \theta')$ in the denominator in equation (29). That is, holding constant c_2 and θ' , the marginal utility of cash goods falls with an increase in ϕ' , since c_1 increases. This is simply a real interest rate effect. That is, the representative agent attempts to smooth consumption by buying bonds, forcing down the real interest rate. This effect acts to increase Q and reduce the nominal interest rate and we will label it the "liquidity effect." Indeed, I believe that this corresponds closely to most central bankers' notions of what constitutes a liquidity effect. A widely-held belief among central bankers is that unanticipated money injections by the central bank will reduce *real* interest rates in the short run.

The liquidity effect in this model works in a similar fashion to the traditional liquidity effect in fixed price Keynesian models, in that asset prices must adjust so that agents are willing to hold a larger stock of real cash balances. However, in this explicit dynamic optimizing framework, intertemporal substitution is the key ingredient of the liquidity effect.

Now, from (28), (22), and (21), there is no variation in c_1 or c_2 caused by the preference shock θ' . However, from (29), a positive preference shock (an increased preference for cash goods) causes $u_1(c_1, c_2, \theta')$ to increase, so that Q falls and the nominal interest rate rises. In an attempt to obtain more currency to purchase cash goods, households attempt to sell nominal bonds, which are in zero net supply in equilibrium, and their price falls.

From (22), (23), (24), and (26), higher y tends to cause $E[u_2(c_1, c_2, \theta') | \theta, \phi, y]$ to decrease, resulting in an increase in $x(s)$ for each s . From (21) and (20), the price of cash goods, p_1 , will decrease, but the effects on p_2 and Q are ambiguous.

5. OPTIMAL MONETARY POLICY

5.1 No Preference Shocks

In the case where there are no preference shocks i.e. $\theta_t = \theta$ for all t , optimal monetary policy is determined in a manner similar to Lucas and Woodford (1992) or Lucas (1989). In this case, any shocks to the money supply which occur after productive capacity is locked in at the beginning of the period can only reduce welfare. Variability in unanticipated money implies that, in general, $c_t^1 + c_t^2 < y_t$. If the current money supply is known at the beginning of the period, then productive capacity is always fully utilized. An optimal money rule will thus allow the current money innovation to depend only on y . We wish to find a function $\phi(y)$ and a z^* such that, in a competitive equilibrium, the expected utility of the representative household is maximized if $\phi' = \phi(y)$ and the trend money growth factor $z = z^*$. A social planner who could allocate cash goods and credit goods to the representative household to maximize welfare faces a static problem of maximizing period utility subject to the constraint that $c_1 + c_2 = y$. Letting $c_1(y)$ and $c_2(y)$ denote the planner's optimal choices, the optimal allocation satisfies

$$u_1[c_1(y), y - c_1(y)] = u_2[c_1(y), y - c_1(y)] \quad (30)$$

and

$$c_1(y) + c_2(y) = y \quad (31)$$

Now, we wish to characterize the monetary rule $\phi(y)$ which supports the allocation given by (30) and (31) as a competitive equilibrium. Note first that prices are known

at the beginning of the period, so that (8) implies that $p_1(s) = p_2 = p$ for all s . Then, we have $x(s) = p^{-1}$ from (19) for all $s \leq \phi(y)$, with $x(s) = 0$ otherwise. Now, substituting in (24) using (20), we get

$$\beta E\left[\frac{u_1(c_1', c_2', \theta)c_1'}{\phi'} \mid y\right] - \frac{u_2(c_1, c_2, \theta)}{\phi} c_1 z = 0,$$

Given (30), the optimal monetary rule must then satisfy

$$\beta E\left[\frac{u_1[c_1(y'), y' - c_1(y'), \theta]c_1(y')}{\phi(y')} \mid y\right] - \frac{u_1[c_1(y), y - c_1(y), \theta]c_1(y)}{\phi(y)} z^* = 0. \quad (32)$$

Taking unconditional expectations through equation (32) and applying the law of iterated expectations implies that $z^* = \beta$. Then, (32) and (25) imply that $Q = 1$ i.e. the cash-in-advance constraint does not bind and the nominal interest rate is zero. Thus, a standard Friedman rule is optimal. Setting $z^* = \beta$ in (32), $\phi(y)$ can be determined only up to a factor of proportionality, which will not matter for the behavior of money relative to trend. If we normalize by letting $\phi(y^*) = 1$ for some y^* , then from (32) we get

$$\phi(y) = \frac{u_1[c_1(y), y - c_1(y), \theta]c_1(y)}{u_1[c_1(y^*), y^* - c_1(y^*), \theta]c_1(y^*)}$$

Therefore, if (??) holds, if both goods are normal, and if $\frac{\partial^2 u}{\partial c_1 \partial c_2} \leq 0$, then $\phi(y)$ is decreasing in y . Thus, if aggregate output is high, then money should be low relative to trend.

With no preference shocks, the variance in unanticipated money should be reduced to zero, as in the models of Lucas (1972), Lucas (1989), and Lucas and Woodford (1992). Variability in unanticipated money reduces welfare in part because it induces more variability in aggregate consumption, but the main source of the welfare loss is the reduction in average consumption. That is, money variability implies that, on average, there exists unused production capacity, and capacity is always fully utilized

if the variability in unanticipated money is eliminated. An optimal money rule also serves to set anticipated money so as to eliminate the distortion due to inflation that is reflected in a positive nominal interest rate.

5.2 Preference Shocks

With unanticipated preference shocks, we create the potential for monetary policy to react within the period. Here, once productive capacity is locked in, preference shocks will not alter consumption allocations unless the current innovation in money depends on the preference shock. In these circumstances, it may be beneficial for the monetary authority to create a positive money surprise on observing a positive preference shock. When there is a positive preference shock, this implies a greater demand for cash goods relative to credit goods, which cannot be realized unless there is an unanticipated cash injection before the market for cash goods opens. Note that the monetary authority need not directly observe the preference shock in order to accommodate a fluctuating demand for cash goods, as the preference shock can be inferred from movements in the nominal interest rate. High demand for cash goods is reflected in a high nominal interest rate, so that the appropriate cash injections may act to smooth nominal interest rates (through the anticipated inflation and liquidity effects).

There is a cost, however, from allowing unanticipated money to accommodate preference shocks. With variability in the money supply, there will in general be unused capacity, and total consumption will be lower on average than it would otherwise be. Therefore, the monetary authority faces a tradeoff, in that variable money might potentially allocate resources more efficiently while reducing the size of the pie. Also, it is not clear in this context what the optimal trend money growth rate should be.

Because the model's transactions mechanism will matter here for the determination

of the optimal money rule, a simple analytical characterization is not possible, in contrast to the previous section. In the next section, we will determine optimal money rules numerically.

6. QUANTITATIVE EXERCISES

In all the numerical exercises, we use the utility function

$$u(c_1, c_2) = \frac{\left\{ \left[\theta c_1^{-\rho} + (1 - \theta) c_2^{-\rho} \right]^{-\frac{1}{\rho}} \right\}^{1-\gamma} - 1}{1 - \gamma},$$

where θ is the preference shock, $\frac{1}{\gamma}$ is the intertemporal elasticity of substitution, and the intratemporal elasticity of substitution is $\frac{1}{1+\rho}$. We interpret a period as one quarter and, consistent with most numerical work with real business cycle models (e.g. Hansen 1985), we set the discount factor $\beta = .99$.

6.1 Monetary Shocks Only

In this section, we will treat the money supply as exogenous, and eliminate preference shocks and shocks to productive capacity. Here, our interest is in studying the operating characteristics of the model in response to an exogenous money supply process which mimics the behavior of the money stock in the United States over the period 1954-1992. Our purpose here is not to fit the data, but to derive some understanding of the quantitative response of the model to monetary shocks, as well as to determine the potential welfare losses from suboptimal money rules. Our results here will motivate our approach in the next section, where we are more interested in the match between the predictions of the model and the U.S. data for the period of interest.

Tables 1 and 2 show some summary statistics for U.S. quarterly time series data over the period 1954:1-1992:4. The consumption measure includes nondurables and

services, and the price level measure is the corresponding price deflator. The nominal interest rate is the 90-day Treasury bill rate. The measure of money used is the monetary base (adjusted for reserve requirements by the St. Louis Fed). The arguments are strong for using the monetary base rather than M1 or M2 (or nonborrowed reserves, for that matter) for this type of model, as broader monetary aggregates include inside assets and thus do not correspond to the notion of money in a cash-in-advance model. Velocity is the nominal value of consumption divided by the monetary base. The data were transformed by first taking logs (except for the interest rate), and then using a Hodrick-Prescott filter (see Prescott 1986) with a smoothing parameter of 1600 to detrend. Of particular note in Table 2 is that the correlation between money and the nominal interest rate is close to zero.

TABLE 1: STANDARD DEVIATIONS OF % DEVIATIONS FROM TREND; U.S. QUARTERLY DATA, 1954:1-1992:4

Cons.	Mon. Base	Velocity	Nom. Rate	Price Level
0.813	0.893	1.150	1.229	0.942

TABLE 2: CORRELATION MATRIX FOR % DEVIATIONS FROM TREND; U.S. QUARTERLY DATA, 1954:1-1992:4

	Cons.	M. Base	Velocity	Nom. Rate	Price Level
Cons.	1.000	0.311	-0.092	0.168	-0.670
M. Base		1.000	-0.780	-0.023	-0.277
Velocity			1.000	0.439	0.563
Nom. Rate				1.000	0.369
Price Level					1.000

The parameters for this set of simulation exercises are set as follows. First, we

normalize by setting $y_t = 1$ for all t . For the stochastic process for the money supply, we set $z = 1.011$ so that the model (in the absence of variability about trend) generates an inflation rate equal to the average inflation rate over the sample. The deviation from trend in the money supply, ϕ_t , was modeled as a two-state first-order Markov process, with $\Pr[\phi_{t+1} = \phi_i | \phi_t = \phi_i] = \pi_\phi$, $i = 1, 2$. We set $\phi_1 = .9911$, $\phi_2 = 1.0089$, and $\pi_\phi = .894$, so that the model replicates the unconditional variance and first-order serial correlation in detrended money over the sample period. We set $\theta = .48$ so as to match the average velocity of money over the sample period.

Tables 3 and 4 show some selected statistics from simulations with different parameter values for ρ and γ (the only remaining free parameters). This illustrates some of the operating characteristics of this economy in response to exogenous money shocks. Note, in Table 3, that variability in the nominal interest rate tends to rise as γ rises and as ρ rises. That is, variability increases as the intertemporal and intratemporal elasticities of substitution decrease. The liquidity effect on nominal interest rates increases with γ ; note that variability in the nominal interest rate increases sharply as γ increases. For moderate levels of γ , the liquidity effect accounts for much of the variability in the nominal interest rate. The liquidity effect and anticipated inflation effect produce a large amount of variability in the nominal interest rate in the sense that, for moderate levels of intertemporal substitution ($\gamma < 2$), the model is capable of generating variability in the nominal interest rate equal to or larger than the variability observed in the data (Table 1).

**TABLE 3: SIMULATIONS WITH EXOGENOUS MONEY SUPPLY;
STANDARD DEVIATION OF THE NOMINAL INTEREST RATE**

	$\gamma = .5$	$\gamma = 1$	$\gamma = 2$	$\gamma = 5$
$\rho = -.8$.8451	.8467	1.1082	2.6789
$\rho = -.5$.7596	.7436	1.1103	3.1207
$\rho = 0$.7269	.8301	1.3598	3.5813
$\rho = .5$.8161	1.0411	1.6799	4.0142
$\rho = 1$	1.0269	1.3264	2.0305	4.4455

**TABLE 4: SIMULATIONS WITH EXOGENOUS MONEY SUPPLY;
CORRELATION BETWEEN THE NOMINAL INTEREST RATE AND
DETRENDED MONEY**

	$\gamma = .5$	$\gamma = 1$	$\gamma = 2$	$\gamma = 5$
$\rho = -.8$	-.6458	-.8074	-.8710	-.6827
$\rho = -.5$	-.6933	-.9335	-.9299	-.6649
$\rho = 0$	-.8967	-1.000	-.8754	-.6430
$\rho = .5$	-.9972	-.9574	-.8135	-.6251
$\rho = 1$	-.9608	-.8842	-.7629	-.6101

In Table 4, note that there is strong negative correlation between the nominal interest rate and money for all parameter values considered. Therefore, given that the observed interest rate/money correlation in the data is close to zero, it will be difficult for this model to replicate the data if there is much variability in unanticipated money.

The next exercise is to quantify the welfare losses from a suboptimal money rule, again taking the money supply to be exogenous. We know from the previous section that, in the absence of preference shocks and given that $y_t = 1$ for all t , the optimal money rule is $\phi_t = 1$ for all t , and $z = \beta = .99$, a Friedman rule. In Tables 5-7, we

compute the welfare gain in moving from the observed money supply process to the optimal rule. We perform this welfare calculation in two stages. First, the variability in money is reduced to zero, holding constant the trend money growth rate. We then calculate the welfare gain from reducing z from 1.011 to .99. As a benchmark, we compare the welfare costs in our model (the “transactions model”) to those in a Lucas-Stokey cash/credit goods model (the “cash-in-advance” model) which is identical to ours except that production capacity does not have to be allocated in advance of learning the current shock. The welfare gains are computed as the percentage increase in consumption of all goods in all states of the world that would be required to make a consumer indifferent between the suboptimal money rule and the optimal one.

**TABLE 5: SIMULATIONS WITH EXOGENOUS MONEY SUPPLY;
WELFARE GAIN FROM ELIMINATING MONEY VARIABILITY;
TRANSACTIONS MODEL (% OF CONSUMPTION)**

	$\gamma = .5$	$\gamma = 1$	$\gamma = 2$	$\gamma = 5$
$\rho = -.8$.0681	.0682	.0683	.0685
$\rho = -.5$.0782	.0783	.0784	.0788
$\rho = 0$.0817	.0817	.0819	.0823
$\rho = .5$.0828	.0829	.0831	.0835
$\rho = 1$.0835	.0836	.0837	.0842

**TABLE 6: SIMULATIONS WITH EXOGENOUS MONEY SUPPLY;
WELFARE GAIN FROM ELIMINATING MONEY VARIABILITY;
CASH-IN-ADVANCE MODEL (% OF CONSUMPTION)**

	$\gamma = .5$	$\gamma = 1$	$\gamma = 2$	$\gamma = 5$
$\rho = -.8$.00007	.00007	.00007	.00008
$\rho = -.5$.00006	.00006	.00006	.00006
$\rho = 0$.00004	.00004	.00004	.00004
$\rho = .5$.00003	.00003	.00003	.00003
$\rho = 1$.00002	.00002	.00002	.00002

**TABLE 7: SIMULATIONS WITH EXOGENOUS MONEY SUPPLY;
WELFARE GAIN FROM REDUCING MONEY GROWTH TO THE
OPTIMUM; TRANSACTIONS MODEL AND CASH-IN-ADVANCE
MODEL (% OF CONSUMPTION)**

	$\gamma = .5$	$\gamma = 1$	$\gamma = 2$	$\gamma = 5$
$\rho = -.8$.0261	.0261	.0261	.0261
$\rho = -.5$.0109	.0109	.0109	.0109
$\rho = 0$.0055	.0055	.0055	.0055
$\rho = .5$.0037	.0037	.0037	.0037
$\rho = 1$.0028	.0028	.0028	.0028

Note first, in Table 5, that the welfare cost of monetary variability in the transactions model is not affected much by ρ and γ . The cost of variable money is large, both in comparison to the cost of variability in the cash-in-advance model (which is negligible, see Table 6), and in comparison to the cost due to the deviation of trend money from the optimum (Table 7). In Table 7, the costs of suboptimal trend money growth are identical in the transactions model and the cash-in-advance model. Note,

from Tables 5 and 7, that the cost of money variability in the transactions model is at the least 2.6 times, and at most 30 times, the cost of suboptimal trend money growth. That is, in this model variability in money is a big deal, in welfare terms, relative to suboptimal trend money growth.

The welfare costs we compute here, even those arising from variability in money, are small relative to the welfare costs of inflation computed by Cooley and Hansen (1989). Cooley and Hansen find a welfare cost of 10% annual inflation on the order of .5% of consumption. We conjecture that the differences between Cooley and Hansen's results and ours are due to the effects of inflation on the capital stock in Cooley and Hansen's model. If this conjecture is correct, this should imply that if we allowed for capital accumulation in our model, then the welfare costs of suboptimal money rules would increase substantially.

6.2 Optimal Monetary Policy With Shocks to Preferences and Productive Capacity

In this section, we compute solutions to our model when θ_t and y_t are stochastic. Here, the money supply is determined endogenously, in that the monetary authority is assumed to set a money supply rule which maximizes the unconditional expected utility of the representative agent. As we observed in Section 5, we cannot characterize the solution analytically in this case.

Here, we assume that θ_t and y_t are independent, and that each follows a two-state first-order Markov process. That is, $\Pr[\theta_{t+1} = \theta_i | \theta_t = \theta_i] = \pi_\theta, i = 1, 2$, and $\Pr[y_{t+1} = y_i | y_t = y_i] = \pi_y, i = 1, 2$. We set θ_1, θ_2 , and π_θ so that the model replicates the unconditional mean, unconditional variance, and first-order serial correlation in the velocity of money observed over the sample. This is for the case where money growth is set to a constant which generates an inflation rate equal to the mean

inflation rate over the sample, and where $\rho = 0$. The parameters y_1 , y_2 , and π_y are set so that the model will replicate the unconditional mean, the unconditional variance, and the first-order autocorrelation in consumption in the case where capacity is always fully utilized. That is, we set $\theta_1 = .4861$, $\theta_2 = .4739$, $\pi_\theta = .90$, $y_1 = 1.00813$, $y_2 = .99187$, and $\pi_y = .92$.

The optimal money supply rule consists of a trend money growth factor, z^* , and a rule as to how ϕ_{t+1} should respond to y_t , θ_t , and θ_{t+1} . Here, we want to determine ϕ_{ijk}^* , $i, j, k = 1, 2$, where $\phi_{t+1} = \phi_{ijk}^*$ when $y_t = y_i$, $\theta_t = \theta_j$, and $\theta_{t+1} = \theta_k$. Note that ϕ_{ijk}^* is determinate only up to a factor of proportionality. Therefore, we can normalize by setting $\phi_{111}^* = 1$, so that the optimal money supply rule is determined by 8 parameters.

The algorithm for computing solutions works as follows. Given an initial money rule, we compute an equilibrium through an iterative procedure using equations (23)-(25). At each stage of the iteration, we check whether the cash-in-advance constraint is binding, and adjust the money supply rule accordingly if it is not. Once convergence to an equilibrium is achieved, we compute unconditional expected utility, and then use an optimization routine to determine how to adjust the money supply rule to increase welfare. A new money supply rule is determined and another equilibrium is computed, etc., until the optimization routine determines that an optimum has been achieved.

Solutions were computed for $\rho = 0$ and $\gamma = .5, 1, 2, 5$. In all cases, it is optimal to completely eliminate variance in unanticipated money, i.e. $\phi_{ij1}^* = \phi_{ij2}^*$ for all $i, j = 1, 2$. Potentially, unanticipated money might play a role here in generating more consumption of cash goods when the demand for these goods is unexpectedly high. However, the welfare loss from such a policy, which results from lower average consumption, is so high that zero variability in unanticipated money is optimal.

Therefore, in spite of the fact that there are potentially large negative effects on the nominal interest rate from an unanticipated increase in the money supply in this model, the monetary authority does not exploit this feature at the optimum.

If the monetary authority were following a Friedman rule, maintaining a zero nominal interest rate would require positive variability in unanticipated money. Thus, the nominal interest rate will be positive, on average, at the optimum, and it will be variable. Table 8 shows the variability in the nominal interest rate and in money, about trend, for the different parameter values. Note that variability in both variables tends to increase with γ for $\gamma \geq 1$. For $\gamma = 2$, we obtain variability in money and nominal interest rates that is close to what is observed in the data (Table 1). As a benchmark, compare the statistics in Table 8 to those in Table 9 for the cash-in-advance model, where a Friedman rule is optimal. As the nominal interest rate is zero in all states of the world, its variance is zero. Note in Table 9 that, for all parameter values, the unconditional variance in money about trend is much greater than it is in the data.

TABLE 8: STANDARD DEVIATIONS OF % DEVIATIONS FROM TREND; TRANSACTIONS MODEL WITH AN OPTIMAL MONEY RULE; $\rho = 0$

	$\gamma = .5$	$\gamma = 1$	$\gamma = 2$	$\gamma = 5$
Nom. Rate	1.002	1.027	1.078	1.237
Money	0.410	0.000	0.810	3.250

TABLE 9: STANDARD DEVIATIONS OF % DEVIATIONS FROM TREND; CASH-IN-ADVANCE MODEL WITH AN OPTIMAL MONEY RULE; $\rho = 0$

	$\gamma = .5$	$\gamma = 1$	$\gamma = 2$	$\gamma = 5$
Nom. Rate	0.000	0.000	0.000	0.000
Money	1.310	1.270	1.550	3.570

Though the model is capable of predicting second moment properties of money and nominal interest rates on the order of what is observed in the data, it does less well with respect to first moments. For example, as with most representative agent models, the real rate of interest the model predicts is higher than is observed in the data. Also, though the optimal money growth factor, z^* , is greater than the discount factor, $\beta = .99$, it is not very much larger (see Table 10). Though money growth is higher than it is with a Friedman rule, trend deflation is optimal.

TABLE 10: OPTIMAL TREND MONEY GROWTH FACTORS; TRANSACTIONS MODEL; $\rho = 0$

$\gamma = .5$	$\gamma = 1$	$\gamma = 2$	$\gamma = 5$
.9925	.9925	.9927	.9930

Table 11 shows the standard deviations of deviations from trend for all variables with $\rho = 0$ and $\gamma = 2$. Here, note that the stochastic processes for θ_t and y_t were calibrated so that the variability of velocity and consumption would be close to what they are in the data (see Table 1).² Of note here is that the variability in the price level is more than twice what it is in the data (see Table 11 and Table 1).

²The variability in velocity is not exactly what it is in the data, as variability depends on parameter values, and the calibration was done for gamma = 1.

Finally, we report the correlation matrix for the case $\rho = 0$ and $\gamma = 2$ in Table 12. Here, note particularly that the correlation between the nominal interest rate and the money supply is close to zero, as in the data (Table 1). In this model there exist strong liquidity effects, and strong tendencies for variability in unanticipated money to produce a negative correlation between money and nominal interest rates. However, in equilibrium there is no evidence of a liquidity effect in unconditional correlations. It is hard to see how the model would produce *any* statistical evidence of a liquidity effect, as the monetary authority never exploits it in equilibrium.

TABLE 11: STANDARD DEVIATIONS OF % DEVIATIONS FROM TREND; $\rho = 0, \gamma = 2$

Cons.	Mon. Base	Velocity	Nom. Rate	Price Level
0.813	0.810	1.130	1.078	1.980

TABLE 12: SIMULATION WITH AN OPTIMAL MONEY RULE; TRANSACTIONS MODEL; CORRELATION MATRIX FOR $\gamma = 2, \rho = 0$

	Cons.	M. Base	Velocity	Nom. Rate	Price Level
Cons.	1.000	-1.000	0.000	0.000	-0.822
M. Base		1.000	-0.009	0.009	0.816
Velocity			1.000	-0.999	0.569
Nom. Rate				1.000	-0.569
Price Level					1.000

Some of the correlations in Table 12 are quite at variance with those from the sample data in Table 2. Most of the problems can be traced to the behavior of velocity. In particular, note that the correlation between velocity and the nominal interest rate is close to -1, while it is positive in the data. This correlation arises because of the fact

that, from equation (24), the marginal utility of the credit good and its price falls, when there is an unanticipated positive preference shock which drives up the nominal interest rate. Therefore, as the price of the cash good, aggregate consumption, and the money supply are invariant to the preference shock at the optimum, the price level falls and velocity decreases. It is this effect which also explains the excess volatility in the price level (see Tables 11 and 1). This one aspect of the behavior of velocity in the model could be corrected by altering the specification of the preference shock so that a positive preference shock increases the marginal utility of both goods. However, velocity in the model would then not vary enough relative to velocity in the data. Thus, the puzzle associated with the behavior of the velocity of money in cash-in-advance models, studied by Hodrick, Kocherlakota, and Lucas (1991), is also a puzzle for this model.

7. SUMMARY AND CONCLUSION

The sequential markets model constructed here exhibits a nonneutrality of unanticipated money. A positive money surprise causes a less-than-proportionate increase in the price of cash goods, and production of cash goods increases. The model is also capable of generating a large negative response of the nominal interest rate to an unanticipated monetary injection, i.e. liquidity effects can be large. In spite of the existence of a potentially important liquidity effect, quantitative exercises show that the monetary authority should not exploit this effect, even in the presence of unanticipated preference shocks. With an optimal money supply rule, the nominal interest rate is in general positive and variable, so that a Friedman rule is suboptimal. In this model it is optimal to eliminate variability in unanticipated money, as the welfare cost of monetary uncertainty is generally quite large relative to the welfare cost of the inefficiency reflected in a positive nominal interest rate. Though the optimal

money rule is not a "Friedman" rule, it has a monetarist flavor consistent with the thrust of Friedman's Presidential address (Friedman 1968).

A calibrated version of the model with shocks to preferences and capacity, where the money supply rule is chosen endogenously to maximize welfare, is capable of yielding variability in money and the nominal interest rate (about trend) close to what was observed in the United States over the period 1954-1992. Also, the model can produce a correlation between detrended money and the nominal interest rate close to zero, as observed in the data. In spite of the existence of a strong liquidity effect, the data may show no obvious evidence of such an effect, as the monetary authority does not exploit the liquidity effect at the optimum.

There are some features of the data that the model does not replicate, and these seem to be tied mainly to the anomalous behavior of velocity. This behavior would appear to be difficult to correct. That is, cash-in-advance models, including this one, seem incapable of correctly predicting the observed behavior of the velocity of money, as pointed out by Hodrick, Kocherlakota, and Lucas (1991).

The most straightforward extension of the model would be to include capital and investment. A model with these features would be capable of evaluating the relative importance of money and real disturbances for aggregate fluctuations. Also, an attractive feature of such a model is that it could be used to explain inventory behavior. That is, we could allow productive capacity to be used for production in all markets (including those markets where there are no buyers), and have unsold goods go into inventory. Here, some source of randomness in the transactions mechanism might be required, in addition to monetary uncertainty, to capture the observed behavior of inventories.

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