

Federal Reserve Bank of Minneapolis  
Staff Report 147

November 1991

## OPTIMAL FISCAL AND MONETARY POLICY: SOME RECENT RESULTS

V. V. Chari

Federal Reserve Bank of Minneapolis

Lawrence J. Christiano

Federal Reserve Bank of Minneapolis

Patrick J. Kehoe

Federal Reserve Bank of Minneapolis and  
University of Minnesota

### ABSTRACT

---

This paper studies the quantitative properties of fiscal and monetary policy in business cycle models. In terms of fiscal policy, optimal labor tax rates are virtually constant and optimal capital income tax rates are close to zero on average. In terms of monetary policy, the Friedman rule is optimal—nominal interest rates are zero—and optimal monetary policy is activist in the sense that it responds to shocks to the economy.

---

The views expressed herein are those of the author and not necessarily those of the Federal Reserve Bank of Minneapolis or the Federal Reserve System.

A fundamental question in macroeconomics is, How should fiscal and monetary policy be set over the business cycle? In three recent papers (Chari, Christiano, and Kehoe 1990a,b,c), we have analyzed various aspects of this question. In this paper, we summarize our findings. In our models, optimal fiscal and monetary policy have four properties:

- Tax rates on labor are roughly constant over the business cycle.
- Capital income taxes are close to zero on average.
- The *Friedman rule* is optimal: Nominal interest rates are zero.
- Monetary policy responds to shocks: Money is countercyclical with respect to technology shocks and procyclical with respect to government consumption.

Our framework combines features of two distinguished traditions in economics: a public finance tradition and a more recent tradition of business cycle theory. The public finance tradition we follow stems from Ramsey (1927), who considers the problem of choosing an optimal tax structure when only distorting taxes are available. The business cycle tradition we follow stems from Kydland and Prescott (1982) and Long and Plosser (1983), who, along with others in this tradition, analyze the quantitative role of shocks to technology and government consumption in generating fluctuations in output and employment. We extend this framework to a monetary business cycle model using the cash-credit good construct of Lucas and Stokey (1983).

Merging these traditions allows us to develop a quantitative framework to analyze fiscal and monetary policy. We model policy choice by assuming that a technology exists through which the government can commit to a sequence of state-contingent policies. An *optimal* policy is one such sequence which maximizes the welfare of the representative agent subject to the constraint that the resulting outcomes constitute a competitive equilibrium.

We analyze fiscal and monetary policy in several closely related models. We specify the parameters for preferences and technology to be similar to those used in the public finance and business cycle literature. The stochastic processes for technology shocks and government consumption are chosen to mimic those in the postwar U.S. economy. With these specifications, we show that the optimal policies for our model economies have the four properties listed above.

In terms of the properties of fiscal policy, optimal tax policies should smooth distortions over time and states of nature. This involves running a surplus in “good times” and a deficit in “bad times.” In our models, good times are associated with above-average technology shocks and below-average government consumption; bad times, with the converse. For reasonable parameter values, smoothing tax distortions turns out to imply that the tax rates on labor (or consumption) should be essentially constant. Smoothing tax distortions also implies that capital tax rates should be close to zero on average, a result reminiscent of one in the deterministic literature (Judd 1985 and Chamley 1986).<sup>2</sup>

In terms of the properties of monetary policy, if the models had lump-sum taxes, then following the Friedman rule would be optimal. Phelps (1973) argues that in models with distorting taxes, it is optimal to tax all goods, including the liquidity services derived from holding money. Hence, Phelps argues that in such models the Friedman rule is not optimal. In our monetary model, however, even though the government has distorting taxes, the Friedman rule turns out to be optimal. In our model, deviating from the Friedman rule amounts to taxing a subset of consumption goods, called *cash goods*, at a higher rate than other consumption goods. Optimality requires that all types of consumption goods be taxed at the same rate; thus, optimality requires following the Friedman rule.

The cyclical properties of optimal monetary policy amount to requiring that the government inflate relatively in bad times and deflate relatively in good times. In effect, then, such a policy

allows the government to use nominal government debt as a shock absorber. In the model, the government would like to issue real state-contingent debt in order to insure itself from having to sharply raise and lower tax rates when the economy is hit with shocks. The government achieves this outcome by issuing nominal noncontingent debt and then inflating or deflating to provide the appropriate ex post real payments. In bad times, inflating is optimal, so the real debt payments are relatively small. In good times, deflating is optimal, so the real debt payments are relatively large.

The plan of this paper is as follows. Section 1 outlines a simple version of Lucas and Stokey's (1983) model without capital or money and describes the basic theoretical framework underlying the analysis. Section 2 develops a model with capital and derives its implications for fiscal policy. Section 3 develops a monetary model without capital and derives its implications for monetary policy. Section 4 discusses the scope and applicability of the analysis.

## 1. A REAL ECONOMY

Consider a simple production economy populated by a large number of identical infinitely lived consumers. In each period  $t = 0, 1, \dots$ , the economy experiences one of finitely many events  $s_t$ . We denote by  $s^t = (s_0, \dots, s_t)$  the history of events up through and including period  $t$ . The probability, as of period zero, of any particular history  $s^t$  is  $\mu(s^t)$ . The initial realization  $s_0$  is given. This suggests a natural commodity space in which goods are differentiated by histories.

In each period  $t$ , there are two goods: labor and a consumption good. A constant returns-to-scale technology is available to transform one unit of labor  $\ell(s^t)$  into one unit of output. The output can be used for private consumption  $c(s^t)$  or government consumption  $g(s^t)$ . Throughout, we will take government consumption to be exogenously specified. Feasibility requires that

$$c(s^t) + g(s^t) = \ell(s^t). \tag{1}$$

The preferences of each consumer are given by

$$\sum_{t, s^t} \beta^t \mu(s^t) U(c(s^t), \ell(s^t)) \quad (2)$$

where the discount factor  $0 < \beta < 1$  and  $U$  is increasing in consumption, decreasing in labor, strictly concave, and bounded.

Government consumption is financed by proportional taxes on the income from labor  $\tau(s^t)$  and by debt. Government debt has a one-period maturity and a state-contingent return. Let  $b(s^t)$  denote the number of units of debt issued at state  $s^t$  and  $R_b(s^{t+1})b(s^t)$  denote the payoff at any state  $s^{t+1} = (s^t, s_{t+1})$ . The consumer's budget constraint is

$$c(s^t) + b(s^t) \leq (1 - \tau(s^t))\ell(s^t) + R_b(s^t)b(s^{t-1}). \quad (3)$$

Let  $b_{-1}$  denote the initial stock of debt. Consumer purchases of government debt are bounded above and below by some arbitrarily large constants. Let  $x(s^t) = (c(s^t), \ell(s^t), b(s^t))$  denote an allocation for consumers at  $s^t$ , and let  $x = (x(s^t))$  denote an allocation for all  $s^t$ .

The government sets tax rates on labor income and returns for government debt to finance the exogenous sequence of government consumption. The government's budget constraint is

$$b(s^t) = R_b(s^t)b(s^{t-1}) + g(s^t) - \tau(s^t)\ell(s^t). \quad (4)$$

Let  $\pi(s^t) = (\tau(s^t), R_b(s^t))$  denote the government policy at  $s^t$ , and let  $\pi = (\pi(s^t))$  denote the policy for all  $s^t$ .

Note that for notational simplicity we have not explicitly included markets in private claims. Since all consumers are identical, such claims will not be traded in equilibrium; hence, their absence

will not affect the equilibrium. Thus, we can always interpret this model as having complete contingent private claims markets.

Consider now the policy problem faced by the government. Suppose an institution or a commitment technology exists through which the government can bind itself to a particular sequence of policies once and for all at period zero. We model this by having the government choose a policy  $\pi = (\pi(s^t))$  at the beginning of time and then having consumers choose their allocations. Since the government needs to predict how consumer allocations and prices will respond to its policies, consumer allocations and prices are described by rules that associate allocations with government policies. Formally, allocation rules are sequences of functions  $x(\pi) = (x(s^t | \pi))$  that map policies  $\pi$  into allocations  $x$ . We then have this definition:

A *Ramsey equilibrium* is a policy  $\pi$  and an allocation rule  $x(\cdot)$  that satisfy

- *Government maximization:* The policy  $\pi$  maximizes

$$\sum_{t, s^t} \beta^t \mu(s^t) U(c(s^t | \pi), \ell(s^t | \pi))$$

subject to (4) with allocations given by  $x(\pi)$ .

- *Consumer maximization:* For every  $\pi'$ , the allocation  $x(\pi')$  maximizes (2) subject to the bounds on debt purchases and to (3) evaluated at the policy  $\pi'$ .

The allocations in a Ramsey equilibrium solve a simple programming problem called the *Ramsey allocation problem*. For convenience, let  $U_c(s^t)$  and  $U_\ell(s^t)$  denote the marginal utilities of consumption and labor at state  $s^t$ . We have, then,

*Proposition 1* (The Ramsey Allocations). The consumption and labor allocations in the Ramsey equilibrium solve the Ramsey allocation problem

$$\sum_{t,s^t} \beta^t \mu(s^t) U(c(s^t), \ell(s^t)) \quad (5)$$

subject to

$$c(s^t) + g(s^t) = \ell(s^t) \quad (6)$$

$$\sum_{t,s^t} \beta^t \mu(s^t) [U_c(s^t) c(s^t) + U_\ell(s^t) \ell(s^t)] = U_c(s_0) [R_b(s_0) b_{-1}]. \quad (7)$$

*Proof.* In the Ramsey equilibrium, the government must satisfy its budget constraint taking as given the allocation rule  $x(\pi)$ . These requirements impose restrictions on the set of allocations the government can achieve by varying its policies. We claim that these restrictions are summarized by constraints (6) and (7). We first show that these restrictions imply (6) and (7). To see that the restrictions imply (6), note that (3) holds with equality under the allocation rule  $x(\cdot)$ . We can add (3) and (4) to get (6); thus, these requirements imply that feasibility is satisfied. We next show that these requirements imply (7). Consider the allocation rule  $x(\pi)$ . For any policy  $\pi$ , we describe the necessary and sufficient conditions for  $c$ ,  $\ell$ , and  $b$  to solve the consumer's problem. Let  $p(s^t)$  denote the Lagrange multiplier on constraint (3). Then, by Weitzman's (1973) theorem, these conditions are constraint (3) together with first order conditions for consumption and labor:

$$\beta^t \mu(s^t) U_c(s^t) \leq p(s^t), \quad \text{with equality if } c(s^t) > 0 \quad (8)$$

$$\beta^t \mu(s^t) U_\ell(s^t) \leq -p(s^t)(1 - \tau(s^t)), \quad \text{with equality if } \ell(s^t) > 0; \quad (9)$$

first order conditions for bonds:

$$\left[ p(s^t) - \sum_{s^{t+1}} p(s^{t+1}) R_b(s^{t+1}) \right] b(s^t) = 0; \quad (10)$$

and the transversality condition. This condition specifies that, for any infinite history  $s^\infty$ ,

$$\lim p(s^t)b(s^t) = 0 \quad (11)$$

where the limits are taken over sequences of histories  $s^t$  contained in the infinite history  $s^\infty$ .

Multiplying (3) by  $p(s^t)$ , summing over  $t$  and  $s^t$ , and using (10) and (11) gives

$$\sum_{t, s^t} p(s^t)[c(s^t) - (1 - \tau(s^t))\ell(s^t)] = p(s_0)R_b(s_0)b_{-1}. \quad (12)$$

Using (8) and (9), we can rewrite (12) as

$$\sum_{t, s^t} \beta^t \mu(s^t) [U_c(s^t)c(s^t) + U_\ell(s^t)\ell(s^t)] = U_c(s_0)[R_b(s_0)b_{-1}]. \quad (13)$$

Thus, (6) and (7) are implied by the requirements that the government must satisfy its budget constraint and that allocations are consistent with the allocation rule  $x(\cdot)$ .

Next, given any set of allocations  $c$  and  $\ell$  that satisfy (6) and (7), we can construct sequences of bond holdings, returns on debt, and sequences of tax rates on labor income such that these allocations are consistent with the allocation rule  $x(\cdot)$  and the government's budget constraint. Construct the bond allocation  $b(s^t)$  as follows. In equilibrium, (3) holds with equality. Multiply this equation by  $p(s^t)$  and sum over all dates and states following  $s^t$ ; then use (8)–(11) to obtain

$$b(s^t) = \sum_{t=r+1}^{\infty} \sum_{s^t} \beta^{t-r} \mu(s^t | s^r) [U_c(s^t)c(s^t) + U_\ell(s^t)\ell(s^t)] / U_c(s^r). \quad (14)$$

Construct the tax rates on labor income by noting that the consumer's first order conditions imply that

$$1 - \tau(s^t) = - \frac{U_\ell(c(s^t), \ell(s^t))}{U_c(c(s^t), \ell(s^t))}. \quad (15)$$

The returns on debt can be found by substituting (14) and (15) into (3). Therefore, (6) and (7) completely characterize the restrictions imposed on allocations by the requirements that when choosing a policy the government must satisfy its budget constraint and the resulting allocations are determined by the allocation rule  $x(\cdot)$ . Since in the Ramsey equilibrium the government chooses a policy that maximizes the welfare of consumers, it follows that the allocations in the Ramsey equilibrium solve (5).  $\square$

For convenience later, write the Ramsey allocation problem as

$$\max \sum_{t, s^t} \beta^t \mu(s^t) W(c(s^t), \ell(s^t), \lambda) \quad (16)$$

subject to (6). Here  $\lambda$  is the Lagrange multiplier on constraint (7), which is called the *implementability constraint*. Note that the function  $W$  simply incorporates the implementability constraint into the maximand. For  $t \geq 1$ ,

$$W(c(s^t), \ell(s^t), \lambda) = U(c(s^t), \ell(s^t)) + \lambda [U_c(s^t)c(s^t) + U_\ell(s^t)\ell(s^t)] \quad (17)$$

and for  $t = 0$ ,  $W$  equals the right side of (17), evaluated at  $s_0$ , minus  $\lambda U_c(s_0)[R_b(s_0)b_{-1}]$ . The first order conditions for this problem imply that

$$- \frac{W_\ell(c(s^t), \ell(s^t), \lambda)}{W_c(c(s^t), \ell(s^t), \lambda)} = 1. \quad (18)$$

Notice that (18) together with (6) implies that the allocations for consumption and labor depend only on the current realization of government consumption, not separately on the entire history of realizations. Thus, in a Ramsey equilibrium,  $c(s^t) = \bar{c}(g_t)$  and  $\ell(s^t) = \bar{\ell}(g_t)$ , where  $g_t = g(s^t)$ . We are interested in the implications of this feature of the Ramsey allocations for tax rates. To develop these, note that the consumer's first order conditions imply that (15) holds in a Ramsey

equilibrium. Since the allocations depend only on the current realization of government consumption, so do the tax rates. Hence, Ramsey tax rates satisfy  $\tau(s^t) = \bar{\tau}(g_t)$ . Since the Ramsey tax rates depend only on the current level of government consumption, these rates inherit the persistence properties of the process on government consumption. For example, if the process on government consumption is i.i.d., then so are the tax rates; if this process is highly persistent, then so are the tax rates.

From a quantitative standpoint, an important question is, How responsive should tax rates be to shocks? For a plausibly parameterized version of the model considered here, it turns out that optimal tax rates on labor are essentially constant. (See also the model in the next section.) In particular, the revenues from labor taxation are much smoother than government consumption. The government keeps its revenues smoother than its consumption by using debt policy as a shock absorber. In periods of high consumption, for example, the government accomplishes this by both selling more debt and lowering the return on inherited debt. Such a debt policy lets the government smooth tax distortions while satisfying its budget constraint.

The idea that governments should sell more debt in periods of higher-than-average consumption is common to many models. The rather novel idea that in such periods the government should also lower the rate of return on inherited debt is due to Lucas and Stokey (1983). To see how this idea works, suppose that government shocks follow a two-state Markov process, and interpret the states as wartime and peacetime. Suppose the economy starts in wartime with no inherited debt. It is easy to show that since the shocks are Markov, the value of the inherited debt  $R_b(s^t)b(s^{t-1})$  depends only on the current state  $s_t$ . Thus, the value of the inherited debt is the same in any period of war as it was at the initial date, namely, zero. Since it can be shown that the debt issued into a state of war is not zero, the return on debt  $R_b$  in wartime is zero.

The intuition for this result is as follows. In the initial period, there is a war, and to smooth tax distortions, the government issues debt. If the war continues, the government cancels the debt. If peace breaks out, the government pays a relatively high rate of return on the debt to compensate debt holders for the losses they suffer in wartime. Such a state-contingent return policy for the debt lets the government run a deficit in wartime and a surplus in peacetime and yet still maintain a stationary pattern for the debt.

In this model, the government implements the state-contingent return policy by directly changing the ex post return on the debt. Another way for the government to implement a state-contingent return policy is to issue nominal debt and use inflation to alter the real rates of return appropriately. We explore this way of implementing the shock absorber role for debt in section 3.

Here we have shown that optimal labor tax rates inherit the persistence properties of the underlying shocks and that debt acts as a shock absorber. These results are quite different from received wisdom. Following Barro (1979), many macroeconomists—including Mankiw (1987) and Judd (1989)—have argued that tax rates should follow a random walk regardless of the persistence properties of the underlying shocks. These arguments have been based on partial equilibrium models that assume a constant rate of return on debt and a loss function for the government which depends directly on the tax rates rather than on the allocations. One conjecture is that if we restrict the government to issue only real state-noncontingent debt, then our general equilibrium model will also produce tax rates close to a random walk. For an analysis of this conjecture and a general discussion of the random walk theory of taxation, see Chari, Christiano, and Kehoe 1990a.

## 2. A REAL ECONOMY WITH CAPITAL

Now consider modifying the economy in section 1 to incorporate a constant returns-to-scale technology which transforms labor  $\ell(s^t)$  and capital  $k(s^{t-1})$  into output by a production function

$F(k(s^{t-1}), \ell(s^t), s_t)$ . Notice that the production function incorporates a stochastic shock. The output can be used for private consumption  $c(s^t)$ , government consumption  $g(s^t)$ , and new capital  $k(s^t)$ .

Feasibility requires that

$$c(s^t) + g(s^t) + k(s^t) = F(k(s^{t-1}), \ell(s^t), s_t) + (1 - \delta)k(s^{t-1}) \quad (19)$$

where  $\delta$  is the depreciation rate on capital. The preferences of each consumer are as before.

Government consumption is financed by proportional taxes on the income from labor and capital and by debt. Let  $\tau(s^t)$  and  $\theta(s^t)$  denote the tax rates on the income from labor and capital. Government debt has a one-period maturity and a state-contingent return. Let  $b(s^t)$  denote the number of units of debt issued at state  $s^t$  and  $R_b(s^{t+1})b(s^t)$  denote the payoff at any state  $s^{t+1} = (s^t, s_{t+1})$ . The consumer's budget constraint is

$$c(s^t) + k(s^t) + b(s^t) \leq (1 - \tau(s^t))w(s^t)\ell(s^t) + R_b(s^t)b(s^{t-1}) + R_k(s^t)k(s^{t-1}) \quad (20)$$

where  $R_k(s^t) = 1 + [1 - \theta(s^t)](r(s^t) - \delta)$  is the gross return on capital after taxes and depreciation and  $r(s^t)$  and  $w(s^t)$  are the net before-tax returns on capital and labor. Competitive pricing ensures that these returns equal their marginal products, namely, that  $r(s^t) = F_k(s^t)$  and  $w(s^t) = F_\ell(s^t)$ , where  $F_k(s^t)$  and  $F_\ell(s^t)$  denote the marginal products of capital and labor at state  $s^t$ . Consumer purchases of capital are constrained to be nonnegative, and the purchases of government debt are bounded above and below by some arbitrarily large constants. Let  $x(s^t) = (c(s^t), \ell(s^t), k(s^t), b(s^t))$  denote an allocation for consumers at  $s^t$ , and let  $x = (x(s^t))$  denote an allocation for all  $s^t$ .

In this economy, the government sets tax rates on labor and capital income and returns for government debt to finance the exogenous sequence of its consumption. The government's budget constraint is

$$b(s^t) = R_b(s^t)b(s^{t-1}) + g(s^t) - \tau(s^t)w(s^t)\ell(s^t) - \theta(s^t)(r(s^t) - \delta)k(s^{t-1}). \quad (21)$$

Let  $\pi(s^t) = (\tau(s^t), \theta(s^t), R_b(s^t))$  denote the government policy at  $s^t$ , and let  $\pi = (\pi(s^t))$  denote the policy for all  $s^t$ .

A Ramsey equilibrium for this economy is defined analogously to that in section 1. As is well-known, in the Ramsey equilibrium the government has an incentive to set the initial tax rate on capital income to be as large as possible. To make the problem interesting, we adopt the convention that the initial tax rate  $\theta(s_0)$  is fixed at some rate, say, zero. Then the consumption, labor, and capital allocations in the Ramsey equilibrium solve this Ramsey allocation problem:

$$\sum_{t, s^t} \beta^t \mu(s^t) U(c(s^t), \ell(s^t))$$

subject to (19) and

$$\sum_{t, s^t} \beta^t \mu(s^t) [U_c(s^t)c(s^t) + U_\ell(s^t)\ell(s^t)] = U_c(s_0)[R_k(s_0)k_{-1} + R_b(s_0)b_{-1}]. \quad (22)$$

(For details, see Chari, Christiano, and Kehoe 1990b.)

For convenience later, write the Ramsey allocation problem as

$$\max \sum_{t, s^t} \beta^t \mu(s^t) W(c(s^t), \ell(s^t), \lambda) \quad (23)$$

subject to (19). Here  $\lambda$  is the Lagrange multiplier on the implementability constraint (22). For  $t \geq 1$ ,

$$W(c(s^t), \ell(s^t), \lambda) = U(c(s^t), \ell(s^t)) + \lambda [U_c(s^t)c(s^t) + U_\ell(s^t)\ell(s^t)] \quad (24)$$

and for  $t = 0$ ,  $W$  equals the right side of (22), evaluated at  $s_0$ , minus  $\lambda U_c(s_0)[R_k(s_0)k_{-1} + R_b(s_0)b_{-1}]$ . The first order conditions for this problem imply that

$$-W_\ell(s^t)/W_c(s^t) = F_\ell(s^t) \quad (25)$$

and

$$1 = \sum_{s^{t+1}} \beta \mu(s^{t+1}|s^t) (W_c(s^{t+1})/W_c(s^t)) [1 - \delta + F_k(s^{t+1})]. \quad (26)$$

We begin our analysis of optimal fiscal policy for this model by considering a nonstochastic version of the model in which the stochastic shock in the production function is constant. Government consumption is also constant, so  $g(s^t) = g$ . Suppose that under the Ramsey plan the allocations converge to a steady state. In such a steady state,  $W_c$  is constant. Thus, from (26),

$$1 = \beta[1 + F_k - \delta]. \quad (27)$$

The consumer's intertemporal first order condition is

$$U_{ct} = \beta U_{ct+1} [1 + (1 - \theta_{t+1})(F_{kt+1} - \delta)]. \quad (28)$$

In a steady state,  $U_c$  is a constant, so (28) reduces to

$$1 = \beta[1 + (1 - \theta)(F_k - \delta)]. \quad (29)$$

Comparing (27) and (29), we can see that in a steady state the optimal tax rate on capital income,  $\theta$ , is zero. This result is due to Chamley (1986).

In Chari, Christiano, and Kehoe 1990b, we show that an analogous result holds in stochastic economies; namely, the value of tax revenue across states of nature is approximately zero in a stationary equilibrium. However, the state-by-state capital taxes are not uniquely determined and can be quite different from zero. A review of some features of the model in section 1 will help explain why. In that model, state-contingent government debt plays a key role in smoothing tax distortions over time and over states of nature. One way to implement the required state-contingency of debt

payments is to use state-contingent taxes on private assets, which in that model are the same as government debt. In this model, private assets include capital as well as government debt. In it both state-contingent taxes on capital and state-contingent debt play analogous roles in smoothing tax distortions over time and over states of nature. Arbitrage conditions require that the ex ante rates of return on both types of assets be equalized. However, the pattern of ex post tax rates on capital and the rates of returns on bonds can be structured in many ways and still meet the ex ante arbitrage conditions and raise the same revenue in each state of nature. We will focus on just one of these ways. We suppose that the government is restricted to making capital tax rates not contingent on the current state. Under this assumption, the policy is uniquely determined and government debt is the only shock absorber.

In Chari, Christiano, and Kehoe 1990b, we explore the quantitative properties of optimal policy in a parameterized version of the model. We consider preferences of the form

$$U(c, \ell) = [c^{1-\gamma}(L-\ell)^\gamma]^\psi / \psi \quad (30)$$

where  $L$  is the endowment of labor. This class of preferences has been widely used in the literature (Kydland and Prescott 1982; Christiano and Eichenbaum 1990; Backus, Kehoe, and Kydland 1991).

The production technology is given by

$$F(k, \ell, z, t) = k^\alpha [e^{\rho t + z \ell}]^{(1-\alpha)}. \quad (31)$$

Notice that the production technology has two kinds of labor augmenting technological change. The variable  $\rho$  captures deterministic growth in this change. The variable  $z$  is a technology shock that follows a symmetric two-state Markov chain with states  $z_\ell$  and  $z_h$  and transition probabilities  $\text{Prob}(z_{t+1} = z_i | z_t = z_j) = \pi$ ,  $i = \ell, h$ . Government consumption is given by  $g_t = g e^{\rho t}$ , where  $\rho$  is the deterministic growth rate and  $g$  follows a symmetric two-state Markov chain with states  $g_\ell$  and

$g_h$  and transition probabilities  $\text{Prob}(g_{t+1} = g_i | g_t = g_i) = \phi$ ,  $i = \ell, h$ . Notice that without technology or government consumption shocks, the economy has a balanced growth path along which private consumption, capital, and government consumption grow at rate  $\rho$  and labor is constant.

We consider two parameterizations of this model. (See Table 1.) Our *baseline* model has  $\psi = 0$  and thus has logarithmic preferences. Our *high risk aversion* model has  $\psi = -8$ . The rest of the parameters for preferences and the parameters for technology are the annualized versions of those used by Christiano and Eichenbaum (1990). We choose the three parameters of the Markov chain for government consumption to match three statistics of the postwar U.S. data: the average value of the ratio of government consumption to output, the variance of the detrended log of government consumption, and the serial autocorrelation of the detrended log of government consumption. We construct the Markov chain for the technology parameters by setting the mean of the technology shock equal to zero and use Prescott's (1986) statistics on the variance and serial correlation of the technology shock to determine the other two parameters.

For each setting of the parameter values, we simulate our economy starting from the steady state of the deterministic versions of our models. In Table 2 we report some properties of the fiscal variables for our baseline model. The table shows that tax on labor income fluctuates very little. For example, if the labor tax rate were approximately normally distributed, then 95 percent of the time the tax rate would fluctuate between 27.89 percent and 28.25 percent. The tax on capital income is zero. This is to be expected from the analytic results in Chari, Christiano, and Kehoe 1990b since with  $\psi = 0$  the utility function is separable between consumption and leisure and homothetic in consumption. For such preferences, this paper shows that the tax on capital is zero in all periods but the first.<sup>3</sup> In the baseline model, the tax on private assets has a large standard deviation.

In Table 2 we also report some properties of the fiscal policy variables for the high risk aversion model. Here, too, the tax rate on labor fluctuates very little. The tax rate on capital income has a mean of  $-0.13$  percent, which is close to zero. We find this feature interesting because it suggests that our analytical result approximately holds for the class of utility functions commonly used in the literature. This feature also suggests that Chamley's (1986) result on the undesirability of the taxation of capital income in a deterministic steady state approximately holds in stochastic steady states of stochastic models. As in the baseline model, we find here that the standard deviation of the tax on private assets is large.

To gain an appreciation of the magnitudes of some of the numbers for our model economies, we compute analogous numbers for the U.S. economy. In Table 2, we report these as well. For the labor tax rate, we use Barro and Sahasakul's (1983) estimate of the average marginal labor tax rate. The standard deviation of this rate is 2.39 percent, which is approximately 25 times the standard deviation in our baseline model. For the tax rate on capital income, we use Jorgenson and Sullivan's (1981) estimate of the effective corporate tax rate. This number probably underestimates the ex ante rate since it ignores the taxation of dividends and capital gains received by individuals. The mean effective rate in the data is 28.28 percent while our baseline model has an ex ante tax rate of zero. Finally, the standard deviation of the innovation in the tax on private assets in the baseline model is about six times that in the data.<sup>4</sup>

### 3. A MONETARY ECONOMY

Now we study the properties of the optimal inflation tax using a version of Lucas and Stokey's (1983) cash-credit goods model. We study both the mean inflation rate and its cyclical properties. Friedman (1969) has argued that monetary policy should follow a rule: set nominal interest rates to zero. For a deterministic version of our economy, this would imply deflating at the

rate of time preference. Phelps (1973) argues that Friedman's rule is unlikely to be optimal in an economy with no lump-sum taxes. His argument is that optimal taxation generally requires using all available taxes, including the inflation tax. Thus, Phelps argues that the optimal inflation rate is higher than the Friedman rule implies.

In sections 1 and 2, we have shown how real state-contingent debt can serve a useful role as a shock absorber. Here we allow the government to issue only nominal state-noncontingent debt. We examine how the government should optimally use monetary policy to make this debt yield the appropriate real state-contingent returns.

Consider, then, a simple production economy with three goods. The goods are labor  $\ell$  and two consumption goods: a cash good  $c_1$  and a credit good  $c_2$ . A stochastic constant returns-to-scale technology transforms labor into output according to

$$c_1(s^t) + c_2(s^t) + g(s^t) = z(s^t)\ell(s^t) \quad (32)$$

where  $z(s^t)$  is a technology shock and, again,  $g(s^t)$  is government consumption. The preferences of each consumer are given by

$$\sum_t \sum_{s^t} \beta^t \mu(s^t) U(c_1(s^t), c_2(s^t), \ell(s^t)) \quad (33)$$

where  $U$  has the usual properties.

In period  $t$ , consumers trade money, assets, and goods in particular ways. At the start of period  $t$ , after observing the current state  $s_t$ , consumers trade money and assets in a centralized securities market. The assets are one-period state-noncontingent nominal claims. Let  $M(s^t)$  and  $B(s^t)$  denote the money and nominal bonds held at the end of the securities market trading. Let  $R(s^t)$  denote the gross nominal return on these bonds payable in period  $t + 1$  in all states  $s^{t+1}$ . After this trading, each consumer splits into a worker and a shopper. The shopper must use the money to

purchase cash goods. To purchase credit goods, the shopper issues nominal claims which are settled in the securities market in the next period. The worker is paid in cash at the end of each period.

This environment leads to this constraint for the securities market:

$$\begin{aligned} M(s^t) + B(s^t) = & R(s^{t-1})B(s^{t-1}) + M(s^{t-1}) - p(s^{t-1})c_1(s^{t-1}) \\ & - p(s^{t-1})c_2(s^{t-1}) + p(s^{t-1})(1 - \tau(s^{t-1}))z(s^{t-1})\ell(s^{t-1}). \end{aligned} \quad (34)$$

The left side of (34) is the nominal value of assets held at the end of securities market trading. The first term on the right side is the value of nominal debt bought in the preceding period. The next two terms are the shopper's unspent cash. The next is the payments for credit goods, and the last is the after-tax receipts from labor services. Besides this constraint, we will assume that the real holdings of debt,  $B(s^t)/p(s^t)$ , are bounded below by some arbitrarily large constant. Purchases of cash goods must satisfy a cash-in-advance constraint:

$$p(s^t)c_1(s^t) \leq M(s^t). \quad (35)$$

Money is introduced into and withdrawn from the economy through open market operations in the securities market. The constraint facing the government in this market is

$$\begin{aligned} M(s^t) - M(s^{t-1}) + B(s^t) = & R(s^{t-1})B(s^{t-1}) + p(s^{t-1})g(s^{t-1}) \\ & - p(s^{t-1})\tau(s^{t-1})z(s^{t-1})\ell(s^{t-1}). \end{aligned} \quad (36)$$

The terms on the left side of this equation are the assets sold by the government. The first term on the right is the payments on debt incurred in the preceding period, the second is the payment for government consumption, and the third is tax receipts. Notice that government consumption is bought on credit.

The consumer's problem is to maximize (33) subject to (34) and (35) and the bound on debt. Money earns a gross nominal return of 1. If bonds earn a gross nominal return of less than 1, then the consumer can make infinite profits by buying money and selling bonds. Thus, in any equilibrium,  $R(s^t) \geq 1$ . The consumer's first order conditions imply that  $U_1(s^t)/U_2(s^t) = R(s^t)$ ; thus, in any equilibrium, this constraint must hold:

$$U_1(s^t) \geq U_2(s^t). \quad (37)$$

This feature of the competitive equilibrium constrains the set of Ramsey allocations.

A Ramsey equilibrium for this economy is defined in the obvious way. As is well-known, if the initial stock of nominal assets held by consumers is positive, then welfare is maximized by increasing the initial price level to infinity. If the initial stock is negative, then welfare is maximized by setting the initial price level so low that the government raises all the revenue it needs without levying any distorting taxes. To make the problem interesting, we set the initial nominal assets of consumers to zero. Let  $a(s_0)$  denote initial real claims that the government holds against private agents. As we show in Chari, Christiano, and Kehoe 1990c, the Ramsey allocation problem is

$$\max \sum_t \sum_{s^t} \beta^t \mu(s^t) U(c_1(s^t), c_2(s^t), \ell(s^t))$$

subject to (32), (37), and

$$\sum_t \sum_{s^t} \beta^t \mu(s^t) [U_1(s^t)c_1(s^t) + U_2(s^t)c_2(s^t) + U_3(s^t)\ell(s^t)] = U_2(s_0)a(s_0). \quad (38)$$

For convenience in studying the properties of the Ramsey allocation problem, let

$$W(c_1, c_2, \ell, \lambda) = U(c_1, c_2, \ell) + \lambda [U_1 c_1 + U_2 c_2 + U_3 \ell] \quad (39)$$

where  $\lambda$  is the Lagrange multiplier on the implementability constraint (38). The Ramsey allocation problem is, then, to maximize

$$\sum_t \sum_{s^t} \beta^t \mu(s^t) W(c_1(s^t), c_2(s^t), \ell(s^t), \lambda)$$

subject to (32) and (37). Consider utility functions of the form

$$U(c_1, c_2, \ell) = h(c_1, c_2)v(\ell) \tag{40}$$

where  $h$  is homogenous of degree  $k$  and the utility function has the standard properties. We then have

*Proposition 2* (The Optimality of the Friedman Rule). For utility functions of the form (40), the Ramsey equilibrium has  $R(s^t) = 1$  for all  $s^t$ .

*Proof.* Consider for a moment the Ramsey problem with constraint (37) dropped. A first order condition for this problem is

$$W_1(s^t)/W_2(s^t) = 1. \tag{41}$$

For utility functions of the form (40),

$$W = hv + \lambda[c_1 h_1 v + c_2 h_2 v + \ell h v'].$$

Since  $h$  is homogenous of degree  $k$ ,  $c_1 h_1 + c_2 h_2 = kh$ . Thus,  $W = h(c_1, c_2)Q(\ell, \lambda)$  for some function  $Q$ . Combining this feature with (41) gives

$$1 = W_1/W_2 = U_1/U_2. \tag{42}$$

Since the solution to this less-constrained problem satisfies (37), it is also a solution to the Ramsey problem. Then the consumer's first order condition  $U_1(s^t)/U_2(s^t) = R(s^t)$  implies that  $R(s^t) = 1$ .  $\square$

In Chari, Christiano, and Kehoe 1990c, we show that the Friedman rule is optimal for more general utility functions of the form

$$U(c_1, c_2, \ell) = V(h(c_1, c_2), \ell)$$

where  $h$  is homothetic. We also show that the Friedman rule is optimal for money-in-the-utility-function economies and transaction cost economies which satisfy a similar homotheticity condition.

The intuition for this result is as follows. In this economy, the tax on labor income implicitly taxes consumption of both goods at the same rate. A standard result in public finance is that if the utility function is separable in leisure and the subutility function over consumption goods is homothetic, then the optimal policy is to tax all consumption goods at the same rate (Atkinson and Stiglitz 1972). If  $R(s^t) > 1$ , the cash good is effectively taxed at a higher rate than the credit good since cash goods must be paid for immediately but credit goods are paid for with a one-period lag. Thus, with such preferences, efficiency requires that  $R(s^t) = 1$  and, therefore, that monetary policy follow the Friedman rule.

This intuition is not complete, however. As we mentioned earlier, the Friedman rule turns out to be optimal even in many models with money in the utility function or with money facilitating transactions. In such models, money and consumption goods are taxed at different rates. Specifically, money is not taxed at all while consumption goods are. Thus, the Phelps (1973) argument turns out to be more tenuous than it first appears. (For analyses of optimality of the Friedman rule in various deterministic models of money with distorting taxes, see Kimbrough 1986, Faig 1988, and Woodford 1990.)

We turn now to some numerical exercises which examine the cyclical properties of monetary policy in our model. In these exercises, we consider preferences of the form

$$U(c, \ell) = [c^{1-\gamma}(L-\ell)^\gamma]^\psi / \psi$$

where  $L$  is the endowment of labor and

$$c = \left[ (1-\sigma)c_1^\nu + \sigma c_2^\nu \right]^{1/\nu}.$$

The technology shock  $z$  and government consumption both follow the same symmetric two-state Markov chains as in the model in section 2.

For preferences, we set the discount factor  $\beta = 0.97$ , we set  $\psi = 0$ , which implies logarithmic preferences between the composite consumption good and leisure, and we set  $\gamma = 0.8$ . These values are the same as those in Christiano and Eichenbaum 1990. The parameters  $\sigma$  and  $\nu$  are not available in the literature, so we estimate them using the consumer's first order conditions. These conditions imply that  $U_{1t}/U_{2t} = R_t$ . For our specification of preferences, this condition can be manipulated to be

$$\frac{c_{2t}}{c_{1t}} = \left[ \frac{\sigma}{1-\sigma} \right]^{1/1-\nu} R_t^{1/1-\nu}. \quad (43)$$

With a binding cash-in-advance constraint,  $c_1$  is real money balances and  $c_2$  is aggregate consumption minus real money balances. We measure real money balances by the monetary base,  $R_t$  by the return on three-month Treasury bills, and consumption by consumption expenditures. Taking logs in (43) and running a regression using quarterly data for the period 1959–89 gives  $\sigma = 0.57$  and  $\nu = 0.83$ .

Our regression turns out to be similar to those used in the money demand literature. To see this, note that (43) implies that

$$\frac{c_{1t}}{c_{1t} + c_{2t}} = \left[ 1 + \left( \frac{\sigma}{1-\sigma} \right)^{1/1-\nu} R_t^{1/1-\nu} \right]^{-1}. \quad (44)$$

Taking logs in (44) and then taking a Taylor's expansion yields a money demand equation with consumption in the place of output and with the restriction that the coefficient of consumption is 1. Our estimates imply that the interest elasticity of money demand is 4.94. This estimate is somewhat smaller than estimates obtained when money balances are measured by M1 instead of the base.

Finally, we set the initial real claims on the government so that, in the resulting stationary equilibrium, the ratio of debt to output is 44 percent. This is approximately the ratio of U.S. federal government debt to GNP in 1989. For the second parameterization, we set  $\psi = -8$ , which implies a relatively high degree of risk aversion. For the third, we make both technology shocks and government consumption i.i.d.

In Table 3 we report the properties of the labor tax rate, the inflation rate, and the money growth rate for our monetary models. In all three, the labor tax rate has the same properties it did in the real economy with capital: it fluctuates very little, and it inherits the persistence properties of the underlying shocks.

Consider next the inflation rate and the money growth rate. Recall that for these monetary models the nominal interest rate is identically zero. If government consumption and the technology shock were constant, then the price level and the money stock would fall at the rate of time preference, which is 3 percent. In a stochastic economy the inflation rate and the money growth rate vary with consumption. Therefore, the mean inflation rate depends not only on the rate of time preference, but also on the covariance of the inflation rate and the intertemporal marginal rate of substitution. This effect causes the inflation rate and the money growth rate to rise with an increase in the coefficient of risk aversion.

In the monetary models, the autocorrelations of the inflation rate are small or negative. Thus, they are far from a random walk. The correlations of inflation with government consumption and with the technology shock have the expected signs. Notice that these correlations have opposite signs, and in the baseline and high risk aversion models, this leads to inflation having essentially no correlation with output. The most striking feature of the inflation rates is their volatility. In the baseline model, for example, if the inflation rate were normally distributed, it would be higher than 20 percent or lower than  $-20$  percent approximately a third of the time. The inflation rates for the high risk aversion model are even more volatile. The money growth rate has essentially the same properties as the inflation rate.

Note that our results are quite different from those of Mankiw (1987). Using a partial equilibrium model, he argues that optimal policy implies that inflation should follow a random walk. It might be worth investigating whether there are any general equilibrium settings which rationalize Mankiw's argument.

#### 4. CONCLUSIONS

In this paper, we have summarized four properties of optimal fiscal and monetary policy in a particular class of models. We have obtained sharp quantitative properties of such policies using reasonable parameter values of standard models of macroeconomics and public finance. These models abstract from a host of issues, including income distribution, heterogeneity, and externalities. The monetary models also abstract from intermediation and nominal rigidities. Thus, these models focus attention on intertemporal efficiency. We think that the forces driving our results will be present in dynamic models generally.

We have focused on calculating the optimal policies in quantitative models. But, as Lucas and Stokey (1983, p. 87) point out, "a policy or policy rule that is optimal in a theoretical model that

is an approximation to reality, can only be approximately optimal applied in reality.” Furthermore, simple policy rules are preferable to complicated state-contingent policies. These considerations suggest that, in practice, we should look for policy rules that are simple approximations to the complicated optimal policies and that continue to perform well for minor perturbations of the model. We think the quantitative framework summarized here will be useful in comparing the performance of simple policy rules. Clearly, a lot more work needs to be done with serious quantitative models before the models can be used to make practical policy proposals. However, we think that the research summarized here represents a step toward a quantitative analysis of optimal policy design and that the general methodology will be useful in more elaborate studies.

## Notes

<sup>1</sup>This is an edited version of a paper that was prepared for a November 1990 conference sponsored by the Federal Reserve Bank of Cleveland and published in a special issue of the *Journal of Money, Credit, and Banking* (August 1991, part 2, vol. 23, no. 6, pp. 519–39). The paper appears here with the permission of the Federal Reserve Bank of Cleveland. © All rights reserved. 0022-2879/91 \$1.00 + 0. The authors thank Nick Bull and George Hall for excellent research assistance. The views expressed here are those of the authors and not necessarily those of the Federal Reserve Bank of Minneapolis or the Federal Reserve System. Kehoe's research was partially supported by the National Science Foundation.

<sup>2</sup>The public finance literature on various aspects of optimal capital income taxes is voluminous. It includes Atkinson 1971, Diamond 1973, Pestieau 1974, and Atkinson and Sandmo 1980. (See also the Auerbach 1985 and Stiglitz 1987 surveys.) These analyses primarily deal with overlapping generations models while we use a model with infinitely lived agents. For other analyses of optimal taxation in business cycle models, see King 1990 and Zhu 1990. For analyses of optimal taxation with human and physical capital in an infinite-lived agent model, see Bull 1990 and Jones, Manuelli, and Rossi 1990.

<sup>3</sup>Separability between consumption and leisure and homotheticity in consumption are the well-known conditions under which the optimal policy is uniform consumption taxes in all periods except the first. (See Atkinson and Stiglitz 1972 for an analysis in a partial equilibrium setting.) In our model, uniform consumption taxes are equivalent to zero capital income taxes; thus, with  $\psi = 0$ , the result that capital income taxes are zero in a stochastic steady state is not surprising. More interesting is the result that, even for the high risk aversion model, which is not separable between

consumption and leisure, the mean of the capital income tax is close to zero in a stochastic steady state.

<sup>4</sup>We compute the tax on private assets by first constructing a value for total debt. Following Jorgenson and Sullivan (1981), we note that the present value of depreciation allowances is a claim on the government similar to conventional debt. We thus define total debt to be the sum of the market value of federal debt and the value of depreciation allowances. We compute an innovation in this sum by regressing it on two lags of these variables: federal government expenditures net of interest payments, Hansen's (1984) Solow residual series, and the sum itself. For further details, see Chari, Christiano, and Kehoe 1990b.

## LITERATURE CITED

- Atkinson, A. B. 1971. "Capital Taxes, the Redistribution of Wealth and Individual Savings." *Review of Economic Studies* 38 (April): 209–27.
- Atkinson, A. B., and A. Sandmo. 1980. "Welfare Implications of the Taxation of Savings." *Economic Journal* 90 (September): 529–49.
- Atkinson, A. B., and J. E. Stiglitz. 1972. "The Structure of Indirect Taxation and Economic Efficiency." *Journal of Public Economics* 1 (April): 97–119.
- Auerbach, Alan J. 1985. "The Theory of Excess Burden and Optimal Taxation." In *Handbook of Public Economics*, vol. I, edited by Alan J. Auerbach and Martin Feldstein, pp. 61–127. Amsterdam: North-Holland.
- Backus, David K., Patrick J. Kehoe, and Finn E. Kydland. 1991. "International Real Business Cycles." Research Department Working Paper 426R, Federal Reserve Bank of Minneapolis.
- Barro, Robert J. 1979. "On the Determination of the Public Debt." *Journal of Political Economy* 87 (October): 940–71.
- Barro, Robert J., and Chaipat Sahasakul. 1983. "Measuring the Average Marginal Tax Rate From the Individual Income Tax." *Journal of Business* 56 (October): 419–52.
- Bull, Nicholas. 1990. "Optimal Taxation in an Endogenous Growth Model With Human Capital." Manuscript, Federal Reserve Bank of Minneapolis.
- Chamley, Christophe. 1986. "Optimal Taxation of Capital Income in General Equilibrium With Infinite Lives." *Econometrica* 54 (May): 607–22.
- Chari, V. V., Lawrence J. Christiano, and Patrick J. Kehoe. 1990a. "On the Random Walk Theories of Optimal Taxation." Manuscript, Federal Reserve Bank of Minneapolis.

- \_\_\_\_\_. 1990b. "Optimal Fiscal Policy in a Business Cycle Model." Manuscript, Federal Reserve Bank of Minneapolis.
- \_\_\_\_\_. 1990c. "The Friedman Rule in Economies With Tax Distortions." Manuscript, Federal Reserve Bank of Minneapolis.
- Christiano, Lawrence J., and Martin Eichenbaum. 1990. "Current Real Business Cycle Theories and Aggregate Labor Market Fluctuations." Institute for Empirical Macroeconomics Discussion Paper 24, Federal Reserve Bank of Minneapolis.
- Diamond, Peter A. 1973. "Taxation and Public Production in a Growth Setting." In *Models of Economic Growth*, edited by James A. Mirrlees and N. H. Stern, pp. 215–35. New York: Wiley.
- Faig, Miquel. 1988. "Characterization of the Optimal Tax on Money When It Functions as a Medium of Exchange." *Journal of Monetary Economics* 22 (July): 137–48.
- Friedman, Milton. 1969. "The Optimum Quantity of Money." In *The Optimum Quantity of Money and Other Essays*, pp. 1–50. Chicago: Aldine.
- Hansen, Gary D. 1984. "Fluctuations in Total Hours Worked: A Study Using Efficiency Units." Working Paper, University of Minnesota.
- Jones, Larry E., Rodolfo E. Manuelli, and Peter E. Rossi. 1990. "Optimal Taxation in Convex Models of Equilibrium Growth." Manuscript, Northwestern University.
- Jorgenson, Dale W., and Martin A. Sullivan. 1981. "Inflation and Corporate Capital Recovery." In *Depreciation, Inflation and the Taxation of Income From Capital*, edited by C. R. Hulten. Washington, D.C.: Urban Institute Press.
- Judd, Kenneth L. 1985. "Redistributive Taxation in a Simple Perfect Foresight Model." *Journal of Public Economics* 28 (October): 59–83.

- \_\_\_\_\_. 1989. "Optimal Taxation in Dynamic Stochastic Economies: Theory and Evidence."  
Working Paper, Hoover Institution.
- Kimbrough, Kent P. 1986. "The Optimum Quantity of Money Rule in the Theory of Public Finance." *Journal of Monetary Economics* 18 (November): 277-84.
- King, Robert G. 1990. "Observable Implications of Dynamically Optimal Taxation." Manuscript, University of Rochester.
- Kydland, Finn E., and Edward C. Prescott. 1982. "Time to Build and Aggregate Fluctuations." *Econometrica* 50 (November): 1345-70.
- Long, John B., Jr., and Charles I. Plosser. 1983. "Real Business Cycles." *Journal of Political Economy* 91 (February): 39-69.
- Lucas, Robert E., Jr., and Nancy L. Stokey. 1983. "Optimal Fiscal and Monetary Policy in an Economy Without Capital." *Journal of Monetary Economics* 12 (July): 55-93.
- Mankiw, N. Gregory. 1987. "The Optimal Collection of Seigniorage: Theory and Evidence." *Journal of Monetary Economics* 20 (September): 327-41.
- Pestieau, Pierre M. 1974. "Optimal Taxation and Discount Rate for Public Investment in a Growth Setting." *Journal of Public Economics* 3 (August): 217-35.
- Phelps, E. S. 1973. "Inflation in the Theory of Public Finance." *Swedish Journal of Economics* 75 (March): 67-82.
- Prescott, Edward C. 1986. "Theory Ahead of Business Cycle Measurement." *Federal Reserve Bank of Minneapolis Quarterly Review* 10 (Fall): 9-22.
- Ramsey, Frank P. 1927. "A Contribution to the Theory of Taxation." *Economic Journal* 37 (March): 47-61.

- Stiglitz, Joseph E. 1987. "Pareto Efficient and Optimal Taxation and the New New Welfare Economics." In *Handbook of Public Economics*, vol. II, edited by Alan J. Auerbach and Martin Feldstein, pp. 991–1042. Amsterdam: North-Holland.
- Weitzman, Martin L. 1973. "Duality Theory for Infinite Horizon Convex Models." *Management Science* 19 (March): 783–89.
- Woodford, Michael. 1990. "The Optimum Quantity of Money." In *Handbook of Monetary Economics*, vol. II, edited by Benjamin M. Friedman and Frank H. Hahn, pp. 1067–1152. Amsterdam: North-Holland.
- Zhu, Xiaodong. 1990. "Optimal Fiscal Policy in a Stochastic Growth Model." Manuscript, University of Chicago.

Table 1

## Parameter Values for the Real Models

Models	Parameters and Values			
<i>Baseline Model</i>				
Preferences	$\gamma = 0.80$	$\psi = 0$	$\beta = 0.97$	$L = 5,475$
Technology	$\alpha = 0.34$	$\delta = 0.08$	$\rho = 0.016$	
Markov Chains for				
Government Consumption	$g_\ell = 350$	$g_h = 402$	$\phi = 0.95$	
Technology Shock	$z_\ell = -0.04$	$z_h = 0.04$	$\pi = 0.91$	
<i>High Risk Aversion Model</i>				
Preferences	$\psi = -8$			

Source: Chari, Christiano, and Kehoe 1990b

Table 2

## Properties of the Real Models and the U.S. Economy

Tax Rates	Models		U.S. Economy
	Baseline	High Risk Aversion	
<i>Labor</i>			
Mean	28.07	33.67	24.76
Standard Deviation	.09	.20	2.39
Autocorrelation	.83	.91	.77
<i>Capital</i>			
Mean	.00	-.13	28.28
Standard Deviation	.00	3.82	8.75
Autocorrelation	—	.85	.74
<i>Private Assets</i>			
Mean	.15	.22	.00
Standard Deviation	4.12	4.70	.73
Autocorrelation	.02	.04	-.32

**Notes:** All statistics are based on 400 simulated observations. The means and standard deviations are in percentage terms. For the U.S. economy, the labor tax rate is measured by the average marginal tax rate of Barro and Sahasakul (1983), the capital tax rate is measured by the effective corporate tax rate of Jorgenson and Sullivan (1981), and the tax on private assets is constructed as described by Chari, Christiano, and Kehoe (1990b). For the baseline model, the capital tax rate is zero; thus, its autocorrelation is not defined.

Table 3

## Properties of the Monetary Models

	Models		
	Baseline	High Risk Aversion	I.I.D.
<i>Labor Tax</i>			
Mean	20.05	20.18	20.05
Standard Deviation	.11	.06	.11
Autocorrelation	.89	.89	.00
Correlation With Government Consumption	.93	-.93	.93
Technology Shock	-.36	.35	-.36
Output	.03	-.06	.02
<i>Inflation</i>			
Mean	-.44	4.78	-2.39
Standard Deviation	19.93	60.37	9.83
Autocorrelation	.02	.06	-.41
Correlation With Government Consumption	.37	.26	.43
Technology Shock	-.21	-.21	-.70
Output	-.05	-.08	-.48
<i>Money Growth</i>			
Mean	-.70	4.03	-2.78
Standard Deviation	18.00	54.43	3.74
Autocorrelation	.04	.07	.00
Correlation With Government Consumption	.40	.28	.92
Technology Shock	-.17	-.20	-.36
Output	.00	-.07	.02