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Aggregate Implications of Innovation Policy*

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ABSTRACT

We present a tractable model of innovating firms and the aggregate economy that we use to assess the link between the responses of firms to changes in innovation policy and the impact of those policy changes on aggregate output and welfare. We argue that the key theoretical determinant of the relative long-run aggregate impact of alternative policies is their impact on the expected profitability of entering firms. We show that, to a first-order approximation, a wide range of policy changes have a long-run aggregate impact in direct proportion to the fiscal expenditures on those policies, and that to evaluate the aggregate impact of such policy changes, there is no need to calculate changes in firms' decisions in response to these policy changes.

We use these results to compare the relative magnitudes of the impact on aggregates in the long run of three innovation policies in the United States: the Research and Experimentation Tax Credit, federal expenditure on R&D, and the corporate profits tax. We argue that the corporate profits tax is a relatively important policy through its negative effects on innovation and physical capital accumulation that may well undo the benefits of federal support for R&D. We also use a calibrated version of our model to examine the absolute magnitude of the impact of these policies on aggregates. We show that, depending on the magnitude of spillovers, it is possible for changes in innovation policies to have a very large impact on aggregates in the long run. However, over a 15-year horizon, the impact of changes in innovation policies on aggregate output is not very sensitive to the magnitude of spillovers.

On the basis of these results we conclude that, while it is possible to make comparisons about the relative importance of different policies and sharp predictions about their aggregate impact in the medium term, it is very difficult to shed much light on the implications of innovation policies for long-run aggregate outcomes and welfare without accurate estimates as to the magnitude of innovation spillovers.

I. Introduction

How do changes in economic policies that affect the costs and benefits to firms of innovative activity impact aggregate investments in innovation, output, productivity, and welfare? In this paper, we address this question using a model that focuses on the role of innovative activities by heterogeneous firms in contributing to aggregate productivity improvements in the economy. Our model is rich enough to capture the dynamic decisions of heterogeneous firms to both improve existing products (process innovation) and create new products (product innovation), and yet tractable enough to aggregate up from firm-level decisions to obtain a deeper understanding of how aggregate innovative activities, output, productivity, and welfare should be expected to respond in general equilibrium to changes in innovation policies.

There is a very large empirical literature, following from the work of Zvi Griliches and many others, that uses detailed firm- and industry-level data to assess the impact of changes in innovation policies on firms' decisions to engage in innovative activities.¹ At the same time, there is also a very large macroeconomic literature that aims to assess the aggregate implications of changes in innovation policy.² In this paper, we develop a model that integrates these two perspectives on the implications of innovation policy. As in Atkeson and Burstein (2010), our model extends Hopenhayn's (1992), Atkeson and Kehoe's (2005), and Luttmer's (2007) model of firm dynamics with entry of new firms or products to include a process innovation decision by incumbent firms following Griliches' (1979) model of knowledge capital. Our model features both physical capital and intangible capital accumulated by firms, and spillovers from firms' innovative activity.³ We view our model as a tractable benchmark for examining the link between the responses of innovative firms to changes in innovation policy (modeled as taxes and subsidies on innovative activity) and the impact of those policy changes on aggregate investments in innovation, output, productivity, and welfare.⁴

We first show that one can use a simple two-step algorithm to assess the impact of

¹See, e.g., the surveys in Congressional Budget Office (2005) and Hall, Mairesse, and Mohnen (2010).

²See, e.g., Jones and Manuelli (2005) for a review of the impact of policies on growth in neoclassical endogenous growth models. See, e.g., Segerstrom (2000), Howitt (2004), and Aghion and Howitt (2006) for a review of the impact of competition and innovation policy on growth. See, e.g., Kortum (1997) and Jones and Williams (1998) for work integrating observations on innovation at the firm or industry level into a macroeconomic framework. Also see, e.g., Lee and Gordon (2005); OECD (2003, 2005), and Congressional Budget Office (2005) for reviews of the empirical work on the impact of innovation policy on aggregate outcomes.

³Our model extends McGrattan and Prescott (2005a,b) by modeling the accumulation of firm-specific intangible capital. See Klenow and Rodríguez-Clare (2005) for a review of spillovers in models of economic growth.

⁴In this paper we abstract from policies that change intellectual property rights, such as patents. We will consider these alternative innovation policies in future work.

changes in innovation policies on aggregate innovative activities, output, and productivity in the long run. In the first step, one uses a straightforward procedure to measure the impact of changes in innovation policies on firm profitability. In this step, the analyst only needs to compute the impact at the margin of a change in innovation policy on the profitability of a typical firm. There is no need in this first step for the analyst to take into account, at least locally, the dynamic response in the innovative activity of the typical firm to the change in innovation policy. For example, to compute the impact on aggregates in the long-run of a change in the Tax Credit for Research and Experimentation (R&E), in this first step, the analyst would simply need to compute the expected impact on the after-tax profitability of the typical firm of this change in the tax credit, holding fixed current levels of innovation expenditure and other decisions by firms.⁵ In the second step, the analyst uses the macroeconomic structure of the model to infer the long-run changes in aggregate innovative activities, output, and productivity that must accompany, in general equilibrium, the change in firm profitability computed in the first step.

We use this algorithm to establish several analytical results on the long-run impact of changes in innovation policies. We show first that, globally, a uniform subsidy to innovative activity has an equivalent impact on aggregates as a direct subsidy to firm profitability because subsidizing the cost of innovation is equivalent to subsidizing the returns to innovation. We also provide conditions under which these two policies have the same fiscal cost. Second, we show that, locally, changes in the subsidy to any particular type of innovative activity all have an equivalent impact on aggregates as long as these policy changes have the same impact on firm profitability. In establishing this second result, we show that in the long run, the impact of a change in innovation policy on firms' process innovation decisions must be offset, in equilibrium, by a change in firms' product innovation decisions so as to result in the same aggregate response of output as would be achieved by a uniform subsidy to all innovative activities. Together, these two results imply that in our model the details of heterogeneous firms' responses to a particular change in innovation policy are not of first-order importance for aggregate outcomes, and that there is no special role for innovation policies distinct from a policy of subsidizing the profits of firms directly, even in the presence of spillovers from innovative activities. Third, we provide conditions under which, to a first-order approximation, the relative impact of a policy change on firm profitability and on aggregates is proportional to

⁵In this respect, our procedure differs from the current standard in the literature that focuses on measuring the impact of changes in innovative policies on the user cost of R&D and uses firm- or industry-level data to measure the elasticity of R&D expenditure to changes in this user cost (see, e.g., Mansfield 1986; Hall and Van Reenen 2000; Griffith, Redding, and Van Reenen 2001; and Bloom, Griffith, and Van Reenen 2002).

the impact of that policy change on government's fiscal expenditures. These results simplify policy evaluation because there is no need to calculate changes in effective marginal tax or subsidy rates nor changes in firms' decisions in response to these policy changes to evaluate their aggregate impact.

We next use our model in two quantitative applications. First, we compare the *relative* magnitudes of the impact on aggregates in the long run of three policies affecting innovative activity by firms in the United States: the R&E Tax Credit, federal expenditure on R&D, and the corporate profits tax.⁶ We use data on fiscal expenditures on these policies in the United States to first argue that federal expenditure on R&D is much more important than the R&E Tax Credit in terms of its aggregate impact on the long-run accumulation of both tangible and intangible capital. We also argue that the corporate profits tax (which is a combination of a tax on variable profits, a subsidy on innovation, and a tax on physical capital) is a relatively important policy, in comparison to the R&E Tax Credit and federal expenditure on R&D, in terms of its negative long-run impact on aggregates. To derive this result, we develop analytical results regarding differences in the aggregate impact of taxes on the use of physical capital by firms and of subsidies to innovation.

In our second quantitative application, we use a calibrated version of our model to examine the *absolute* magnitude of the impact on aggregates of three policies: a uniform subsidy to all innovative activities, a subsidy to process innovation, and a subsidy to the use of physical capital by firms. We choose the size of these policies so that they all lead to fiscal expenditures of 3% of GDP. Our model implies that while the response of aggregate investments in innovation to these policies changes can be pinned down relatively precisely, there is a great deal of uncertainty regarding the impact of these policy changes on aggregate output in the long run and on welfare, depending on the magnitude of the spillovers from innovative activity. For example, in terms of welfare, the equivalent variation in consumption ranges from between 0 with no spillovers to roughly 50% with strong spillovers. In contrast, we find that if we consider the transition over a 15-year horizon, these policy changes have a similar impact on aggregate investments on innovation and on aggregate output regardless of the magnitude of spillovers. The large degree of uncertainty about the long-run impact of policy changes is not apparent at a 15-year horizon because the model's transition dynamics become extremely slow as the spillovers from innovative activities become large. On the basis of these results, we conclude that it should be very difficult to use data on the response of aggregate

⁶See Congressional Budget Office (2007) for a description of the range of innovation policies in the United States.

outcomes to changes in innovation policies over a medium-term horizon to shed much light on the magnitude of spillovers and hence on the implications of changes in innovation policies for long-run aggregate output and productivity, and for welfare.

Our model's implications for the impact of changes in innovation policy on long-run outcomes and welfare do depend on its macroeconomic structure. Our model nests both semi-endogenous growth and endogenous growth through expanding varieties, as in Romer (1990) and Acemoglu (2009), chapter 13. In deriving our main results, we assume that the parameters of our model lie in a region such that an increase in the productivity of incumbent firms in response to a change in policy crowds out the profits of entering firms. With this assumption, our model is a semi-endogenous growth model in which innovation policies affect the level of aggregate activity but not its growth rate, which is determined by the exogenous growth rate of general scientific knowledge and the population. There is a knife-edged set of parameter values for which the profits of entering firms are independent of other firms' productivities. With parameters set in this knife-edge case, our model is an endogenous growth model in which innovative activity by firms is the engine of growth. Note that the standard endogenous growth models such as the model of expanding varieties of Romer (1990), the quality ladders model of Grossman and Helpman (1991) and Klette and Kortum (2004), and the Schumpeterian models of Aghion and Howitt (1992), as well as many of the variants of these models discussed in Acemoglu (2009), chapters 13–14, satisfy this knife-edged property. We find that the quantitative implications of our model for output at a finite horizon and for welfare are continuous in the extent to which an increase in the productivity of incumbent firms in response to a change in policy crowds out the profits of entering firms. Hence, we argue that our framework nests fully endogenous growth models for all practical purposes.

Our baseline model of firm dynamics does not nest quality ladders or Schumpeterian models of firm dynamics in which entering firms directly displace incumbent firms in the production of existing products. Our line of argument, however, can be extended to cover models of this type as well. In particular, in an online appendix, we show that our main analytic results go through in a Klette and Kortum (2004) type quality ladders version of the model.

The paper is organized as follows. Section II presents the model. Section III characterizes the balanced growth path. Section IV describes optimal innovation policies. Section V presents analytic results on global policy equivalence and Section VI on the aggregate impact, in the long run, of policy changes. Section VII examines the relative magnitudes of the

impact of the three policies that affect innovative activity in the United States. Section VIII presents the results from our calibrated model. Section IX discusses the use of observations on the aggregate implications of changes in the research intensity of the economy to infer the degree of spillovers.

II. The Model

In this section, we describe the physical environment, innovation policies, the equilibrium, and the implications of our model for firm dynamics.

A. Physical Environment

Time is discrete and labeled $t = 0, 1, 2, \dots$. There are two final goods, the first of which we call the *consumption good*. The representative household has preferences over this consumption good, C_t , given by $\sum_{t=0}^{\infty} \beta^t L_t \log(C_t/L_t)$, where L_t is the population at date t , which grows at a steady rate of g_L so $L_{t+1} = \exp(g_L)L_t$. We assume that $\beta \exp(g_L) < 1$.

Output of the consumption good, Y_t , is used for three purposes. First, as consumption by the representative household. Second, as investment in physical (tangible) capital, $K_{t+1} - (1 - \delta_k) K_t$, where K_t denotes the aggregate capital stock and δ_k denotes the depreciation rate of physical capital. Third, as an input into innovative activity, X_t . The resource constraint of the final consumption good is

$$C_t + K_{t+1} - (1 - \delta_k) K_t + X_t = Y_t. \quad (1)$$

At each date t , there is a continuum of size N_t of incumbent firms, each producing distinct intermediate goods. These intermediate goods producing firms are distinguished by an index z of their current productivity, and we let $J_t(z)$ denote the distribution of z across incumbent firms at date t (so $\int_z dJ_t(z) = N_t$). The consumption good is produced as a constant elasticity of substitution (CES) aggregate of the output of this continuum of intermediate goods, according to

$$Y_t = \left(\int_z y_t(z)^{(\rho-1)/\rho} dJ_t(z) \right)^{\rho/(\rho-1)}, \quad (2)$$

where $y_t(z)$ is the output of an intermediate good producing firm with productivity index z at date t . We assume that $\rho > 1$.

An incumbent intermediate good producing firm with productivity index z produces

its differentiated output with physical capital, k , and labor, l , according to

$$y_t(z) = \exp(z_t)^{1/(\rho-1)} k_t(z)^\alpha l_t(z)^{1-\alpha}, \quad (3)$$

where $0 < \alpha < 1$. Here, the productivity of the individual firm is given by $\exp(z_t)^{1/(\rho-1)}$.⁷ As we show below, this normalization of productivity is convenient as the equilibrium size of the firm, measured in terms of labor $l_t(z)$, capital $k_t(z)$, revenues, or variable profits, is directly proportional to $\exp(z)$.

To innovate, firms must use a second final good, which we refer to as the *research good*, as an input. Intermediate goods producing firms invest in two types of innovation in this economy: process and product innovation. Incumbent intermediate goods producing firms engage in *process innovation* to increase their productivity index z from one period to the next. For an incumbent firm to increase its productivity index from z in period t to $z' = z + g_z$ in period $t + 1$, it requires an expenditure of $c(\exp(g_z)) \exp(z)$ units of the research good in period t , where $c(x)$ is an increasing and convex function.⁸ We denote the curvature of the function $c(x)$ by $\eta_c = \frac{c''(x)x}{c'(x)}$.

Firms invest in *product innovation* as follows. A new firm producing a new variety can be created in period $t + 1$ by an expenditure of n_e units of the research good in period t . For simplicity, we assume that all newly created firms start with productivity index \bar{z} , which we normalize to 0.⁹

Letting $M_t \geq 0$ denote the mass of investment in new products in period t , then the resource constraint for the research good is given by

$$n_e M_t + \int_z c(\exp(g_{zt}(z))) \exp(z) dJ_t(z) = Y_{rt}, \quad (4)$$

where $g_{zt}(z)$ is the growth rate of the productivity index z chosen by intermediate goods producing firms with current index z in period t and Y_{rt} is the output of the research good in period t .

We assume that a fraction δ_f of all incumbent firms and newly created firms exit

⁷Extending the model to allow for decreasing returns to scale in the production of intermediate goods and a richer input-output structure (i.e., final goods and intermediate goods are produced using the consumption good, capital and labor) leaves our main results unchanged.

⁸With this specification of process innovation costs, the cost for a firm of growing by a given percentage scales in direct proportion to the current size of the firm, which as we show below is proportional to $\exp(z)$.

⁹It is straightforward to allow for exogenous growth in \bar{z} so that new goods improve over time. This is the approach taken in Atkeson and Kehoe (2005).

exogenously at the beginning of each period. Thus, the evolution of the number of firms is given by

$$N_{t+1} = (1 - \delta_f)(M_t + N_t). \quad (5)$$

The research good is produced as a Cobb-Douglas combination of the consumption good and labor according to

$$Y_{rt} = A_t H_t^\gamma L_{rt}^\lambda X_t^{1-\lambda}, \quad \text{where} \quad (6)$$

$$H_{t+1} = (1 - \delta_r)H_t + Y_{rt}. \quad (7)$$

Here, L_{rt} and X_t denote the labor and consumption good, respectively, used in the production of the research good, and A_t represents the stock of basic scientific knowledge that is assumed to evolve exogenously, growing at a steady rate of g_A so $A_{t+1} = \exp(g_A)A_t$. Increases in this stock of knowledge improve the productivity of resources devoted to innovative activity. We interpret A as a worldwide stock of scientific knowledge that is freely available for firms to use in innovative activities. The determination of A is outside the scope of our analysis.

The variable H_t is the spillover from cumulative innovative activity by firms, here modeled as an external learning effect — in innovating, researchers gain knowledge and experience useful for further innovation that is not captured as part of the private return of either the firm or the workers engaged in innovation. The parameter δ_r is the rate at which this external learning effect depreciates. In what follows, we assume that $0 \leq \gamma < 1$.

The amount of labor used in current production, L_{pt} , is given by $L_{pt} = \int_z l_t(z) dJ_t(z)$. The resource constraint for labor requires that labor used in current production plus labor used in the production of the research good must sum to a fixed total population L_t , that is, $L_{pt} + L_{rt} = L_t$. Labor is mobile between intermediate goods production and innovation. The amount of physical capital used in current production must satisfy the constraint $K_t = \int_z k_t(z) dJ_t(z)$.

Finally, the law of motion of the distribution of z across incumbent firms, $J_t(z)$, is determined by the exogenous exit rate, δ_f , the process innovation decisions of incumbent firms, $g_{zt}(z)$, and the mass of investment in new products, M_t , in a standard manner.

B. Policies and Equilibrium

In this subsection, we describe the decentralization of this economy and define equilibrium with a collection of policies. In this decentralization, we assume that the representative household owns the physical capital stock and rents it to the intermediate good producing

firms at rental rate R_{kt} . Each period, the household faces a budget constraint given by

$$C_t + K_{t+1} = [R_{kt} + (1 - \delta_k)] K_t + W_t L_t + D_t - E_t,$$

where W_t , D_t , and E_t denote the economy-wide wage, aggregate dividends paid by intermediate good firms, and aggregate fiscal expenditures on policies (which are financed by lump-sum taxes collected from the representative household), respectively. We also define an interest rate for bonds denominated in the final consumption good, \bar{r}_t , which with log preferences is given by $1 + \bar{r}_t = \beta^{-1} (C_{t+1}/L_{t+1}) / (C_t/L_t)$. We find it useful to denote the interest rate denominated in terms of the research good as r_t . This interest rate is defined by $(1+r_t) = (1+\bar{r}_t) \frac{P_{rt}}{P_{rt+1}}$. The Euler equation for physical capital is given by $1+\bar{r}_t = R_{kt+1} + 1 - \delta_k$.

Intermediate good producing firms are offered three types of subsidies and are subject to one tax. These are abstract policies that are useful in deriving analytical results. In our quantitative work below we describe a mapping between actual policies and these abstract policies. The policies are as follows. First, firms receive a subsidy to variable profits from production (defined below), which we denote by τ_p . Second, firms receive a subsidy to process innovation, which we denote by τ_g . Third, firms receive a subsidy to product innovation, which we denote by τ_e . We refer to the subsidies τ_g and τ_e as *innovation policies*. We show below that the subsidy to variable profits, τ_p , has an equivalent impact to a uniform change in these innovation policies and hence can also be considered as an innovation policy. Finally, firms are subject to a tax on their use of physical capital, which we denote by τ_k .

As we discuss below, the equilibrium of our model has the standard inefficiency arising from a monopoly markup in the production of intermediate goods. The policies that we consider above are not sufficient to undo the distortions from this markup. To have enough policies to implement the socially optimal allocation, we allow for a per-unit subsidy on production of the consumption good, τ_s , that can be set to undo the distortions from the markup (note that subsidizing the production of the consumption good is equivalent to subsidizing the sales of intermediate good producers).

Competitive firms producing the final consumption good choose inputs and output to maximize profits each period subject to (2). By standard arguments, in equilibrium prices must satisfy $(1 + \tau_s) P_t = [\int_z (p_t(z))^{1-\rho} dJ_t(z)]^{\frac{1}{1-\rho}}$, where $p_t(z)$ is the price set by firms with productivity index z , and P_t is the price of the final good paid by the representative household. We normalize P_t to 1. This profit maximization problem also gives input demands $y_t(z) = (1 + \tau_s)^\rho p_t(z)^{-\rho} Y_t$.

The *variable profits* from production of a firm with productivity index z in period t are given by $(1 + \tau_p) [p_t(z)y_t(z) - (1 + \tau_k) R_{kt}k_t(z) - W_t l_t(z)]$. This firm chooses price and quantity, $p_t(z)$ and $y_t(z)$, to maximize these variable profits subject to the demand above and the production function (3). Profit maximization implies that a firm with productivity index z in period t sets its price at

$$p_t(z) = \frac{\rho}{\rho - 1} \frac{1}{\alpha^\alpha (1 - \alpha)^{1-\alpha}} \frac{[(1 + \tau_k) R_{kt}]^\alpha W_t^{1-\alpha}}{\exp(z)^{1/(\rho-1)}} \quad (8)$$

and its variable profits from production can be written as $(1 + \tau_p) \Pi_t \exp(z)$, with the *constant in variable profits* Π_t defined by

$$\Pi_t = \kappa_0 (1 + \tau_s)^\rho (1 + \tau_k)^{\alpha(1-\rho)} (R_{kt}^\alpha W_t^{1-\alpha})^{1-\rho} Y_t, \quad (9)$$

where κ_0 is a constant.¹⁰ As these equations make clear, the variable profits earned by an incumbent firm scale with its productivity index z in direct proportion to $\exp(z)$.

Firms' investments in process innovation (indexed by g_z) are governed by the Bellman equation

$$\frac{V_t(z)}{P_{rt}} = \max_{g_{zt}} \left[(1 + \tau_p) \frac{\Pi_t}{P_{rt}} - (1 - \tau_g) c(\exp(g_{zt})) \right] \exp(z) + \frac{(1 - \delta_f) V_{t+1}(z + g_z)}{P_{rt+1}}. \quad (10)$$

The value function $V_t(z)$ corresponds to the expected discounted present value of dividends paid by an incumbent firm with current productivity index z . The first term in the right-hand side of (10) is the current dividend denominated in terms of the research good. It comprises variable profits less expenditures on process innovation. Observe that the value function in this Bellman equation takes the form $\frac{V_t(z)}{P_{rt}} = \bar{V}_t \exp(z)$, so that the value of an incumbent firm is directly proportional to the index of the firm's productivity $\exp(z)$. We refer to the factor of proportionality \bar{V}_t as the *profitability* of firms in period t .

The zero-profit condition governing product innovation is given by

$$(1 - \tau_e) n_e = \frac{(1 - \delta_f)}{1 + r_t} \bar{V}_{t+1} \quad (11)$$

as long as there is positive investment in product innovation, and an inequality with the cost

¹⁰In particular, $\kappa_0 = \rho^{-\rho} (\rho - 1)^{\rho-1} [\alpha^\alpha (1 - \alpha)^{1-\alpha}]^{\rho-1}$.

of product innovation exceeding the profits otherwise. The dividend paid by an entering firm, denominated in terms of the research good, is $-(1 - \tau_e) n_e$.

To aggregate the decisions of intermediate good producers, we define an index of *total productivity* at time t , $N_t Z_t$, where Z_t is an index of *average productivity* at time t given by

$$Z_t = \frac{1}{N_t} \int_z \exp(z) dJ_t(z).$$

It is straightforward to show that in equilibrium, aggregate output of the consumption good is

$$Y_t = (N_t Z_t)^{1/(\rho-1)} K_t^\alpha L_{pt}^{1-\alpha}. \quad (12)$$

In parallel to physical capital, we refer to the index of total productivity $N_t Z_t$ as the aggregate stock of intangible capital in firms. In our applications, we use the sum of expenditures on consumption, C_t , and investment in physical capital, $K_{t+1} - (1 - \delta_k) K_t$, as our measure of aggregate output (GDP) because in the National Income and Product Accounts, expenditures on innovative activities, $P_{rt} Y_{rt}$, are typically expensed rather than counted as a part of final output. Using (1), we have $GDP_t = Y_t - X_t$.¹¹

C. Factor Shares, Research Intensity, and the Allocation of Labor

With CES aggregators and Cobb-Douglas production functions, aggregate revenues of intermediate goods firms, $(1 + \tau_s) Y_t$, are split into three components. A share $1/\rho$ accrues to variable profits from production (exclusive of the subsidy on variable profits), $\Pi_t N_t Z_t = \frac{1}{\rho} (1 + \tau_s) Y_t$, a share $\alpha(\rho - 1)/\rho$ is paid to physical capital (inclusive of taxes on physical capital), $(1 + \tau_k) R_{kt} K_t = \frac{\alpha(\rho-1)}{\rho} (1 + \tau_s) Y_t$, and a share $(1 - \alpha)(\rho - 1)/\rho$ is paid as wages to production labor, $W_t L_{pt} = \frac{(1-\alpha)(\rho-1)}{\rho} (1 + \tau_s) Y_t$.

Production of the research good is undertaken by competitive firms that take the spillover from innovation H_t as given. Cost minimization in the production of the research good implies that the price of the research good, P_{rt} , is equal to

$$P_{rt} = \frac{1}{H_t^\gamma A_t} \frac{1}{\lambda^\lambda (1 - \lambda)^{(1-\lambda)}} W_t^\lambda. \quad (13)$$

¹¹In using this measure of GDP, we are abstracting from measurement problems arising from the introduction of new products. Under the assumption that Y from (12) is a physical good that is traded in the market, these measurement problems for GDP are less likely to arise.

It also implies

$$W_t L_{rt} = \lambda P_{rt} Y_{rt}, \text{ and } X_t = (1 - \lambda) P_{rt} Y_{rt}. \quad (14)$$

Using the factor shares above and (14), the allocation of labor between production and research is

$$\frac{L_{pt}}{L_{rt}} = \frac{(1 - \alpha)(\rho - 1)\Pi_t N_t Z_t}{\lambda P_{rt} Y_{rt}}. \quad (15)$$

We define the research intensity of the economy, s_r , as the ratio of spending on innovative activities to GDP, $s_{rt} = P_{rt} Y_{rt} / (Y_t - X_t)$. Using (i) $X_t = (1 - \lambda) P_{rt} Y_{rt}$, (ii) $Y_t / (\Pi_t N_t Z_t) = \rho / (1 + \tau_s)$ together with equation (15), we can express the research intensity of the economy as

$$s_{rt} = \left[\frac{\rho \lambda}{(1 - \alpha)(\rho - 1)(1 + \tau_s)} \frac{L_{pt}}{L_{rt}} - (1 - \lambda) \right]^{-1}. \quad (16)$$

An equilibrium in this economy is a collection of sequences of aggregate prices $\{\bar{r}_t, P_{rt}, R_{kt}, W_t\}$, prices for intermediate goods $\{p_t(z)\}$, sequences of aggregate quantities $\{Y_t, K_t, X_t, C_t, L_{pt}, L_{rt}, H_t\}$, quantities of the intermediate goods and allocations of physical capital and labor $\{y_t(z), k_t(z), l_t(z)\}$, sequences of $\{\Pi_t\}$, and sequences of firm value functions and process innovation decisions $\{V_t(z), g_{zt}(z)\}$ together with distributions of firms, mass of incumbent firms, measures of product innovation, and aggregate productivities $\{J_t(z), N_t, M_t, Z_t\}$ such that, given a set of policies $\{\tau_p, \tau_e, \tau_g, \tau_k, \tau_s\}$, initial stocks $\{A_0, L_0, H_0, K_0\}$, and an initial distribution of firms $J_0(z)$, households maximize their utility subject to their budget constraint, intermediate good firms maximize profits, all of the feasibility constraints are satisfied, and the distribution of firms evolves as described above.

D. Firm Profitability and Firm Dynamics

Our model has two main implications for firm dynamics that we use frequently in our results below. First, the growth rate of incumbent firms is independent of firm size and is determined by what we term the level of firm profitability. Second, in any period with positive product innovation, the level of firm profitability is determined by the cost of product innovation. Putting these two implications together, we get a simple formula for determining how changes in innovation policy impact process innovation by incumbent firms.

To derive the two implications regarding firm dynamics, observe from (10) that all

incumbent firms in period t choose the same growth rate g_{zt} , given by the solution to

$$(1 - \tau_g) c'(\exp(g_{zt})) = \frac{(1 - \delta_f)}{1 + r_t} \bar{V}_{t+1}. \quad (17)$$

This result implies that the growth rate of incumbent firms in our model, g_{zt} , is always independent of firm size. With this result, we can write the law of motion for the index of total productivity as

$$N_{t+1} Z_{t+1} = (1 - \delta_f) \exp(g_{zt}) N_t Z_t + (1 - \delta_f) M_t \exp(\bar{z}). \quad (18)$$

Combining the two expressions governing process and product innovation, (11) and (17), we get that in any period in which there is positive product innovation, the growth rate g_z of incumbent firms (the level of process innovation) is simply determined by

$$(1 - \tau_g) c'(\exp(g_{zt})) = (1 - \tau_e) n_e. \quad (19)$$

To ensure the existence of equilibrium, we need parameter restrictions to ensure that profitability for incumbent firms \bar{V} is well defined (finite) and that the index of average productivity, Z , is also well defined. Both of these conditions are restrictions on the growth rate of incumbent firms, g_{zt} . We present these restrictions in the next section for the case in which the economy is on a balanced growth path with positive product innovation every period.

III. Balanced Growth Path with Product Innovation

In our results on the aggregate implications of policy changes in the long run, we analyze the change in our economy's balanced growth path corresponding to a given change in policy. We now characterize a balanced growth path (BGP) with product innovation in our model economy. On a BGP with product innovation, a subset of the variables is constant over time and other variables grow at a constant rate. In particular, on a BGP, the allocation of labor between production and innovation remains constant as does the profitability of firms in terms of the research good, the interest rate in terms of both the final good and the research good, the rental rate of capital, and the index of average productivity of incumbent firms. Output of the consumption and research goods, consumption, the stock of physical capital, total productivity, wages, and cumulated experience with innovation all grow at constant rates.

Our model has two types of BGPs with product innovation, one with semi-endogenous growth and one with endogenous growth, depending on parameter values. Specifically, we define a parameter θ as a function of the parameters γ, ρ, α , and λ :

$$\theta = 1 - \frac{(1 - \lambda)}{(\rho - 1)(1 - \gamma)(1 - \alpha)}. \quad (20)$$

Our model is a semi-endogenous growth model with the growth rate along the BGP determined by the exogenous growth rates of scientific knowledge g_A and of the population g_L if $\theta > 0$. In this case, it is not possible to have fully endogenous growth because an increase in the index of total productivity NZ results in general equilibrium in a decline in the profitability of firms, and thus a decline of firms' investments in innovative activity. Growth can occur only to the extent that scientific progress reduces the cost of innovation and/or growth in population increases market size. We focus on equilibria that satisfy this parameter restriction in deriving our main results.

Our model is an endogenous growth model with the growth rate along the BGP determined by firms' investments in innovative activity only in the knife-edged case in which $\theta = 0$. In this case, it is possible to have endogenous growth because an increase in the index of total productivity NZ has no impact on the profitability of firms and thus firms find it optimal to continue to invest in innovation even as the productivity of their competitors grows. To the best of our knowledge, all endogenous growth models with innovation by firms satisfy some version of this knife-edge parameter restriction.¹²

We now characterize a BGP with product innovation and semi-endogenous growth. We provide additional details of this characterization in the appendix.

A BGP with Product Innovation and Semi-endogenous Growth. When $\theta > 0$, the conditions characterizing a BGP have a block recursive structure that lies at the heart of our analytic results. We first characterize aggregate growth rates, the interest rate, and the rental rate of capital along a BGP with product innovation. Along such a BGP, these variables are independent of policies. We then characterize the growth rate of incumbent firms, the average productivity of firms, and the equilibrium level of firm profitability from the zero-profit condition for product innovation. Finally, we solve for the levels along the BGP

¹²If $\theta < 0$, then our model does not have a BGP, as in this case the profitability of firms and their investments in innovation accelerate as the index of total productivity NZ grows. Our knife-edge condition is different from the one emphasized in Jones (1999) on the technology governing innovation. Our condition encompasses both the technology governing innovation and spillovers, as well as the impact of product market competition on the profitability of creating new products.

of the other aggregate variables consistent with the equilibrium level of firm profitability.

The growth rates and the interest rate are determined from equations (1), (4), (6), (7), and (12). In particular, the growth rate of the measure of incumbent firms g_N is given by

$$g_N = \frac{1}{\theta(1-\gamma)}(g_A + g_L), \quad (21)$$

and the growth rate of total productivity (NZ) is equal to the growth of the measure of incumbent firms, g_N , with Z constant. The growth rate of output of the consumption good (and hence consumption, physical capital, and the input into innovative activity X) is given by $g_Y = g_N / [(\rho - 1)(1 - \alpha)] + g_L$, the growth rate of output of the research good is given by $g_{Y_r} = g_N$, the growth rate of the wage is $g_Y - g_L$, the rental rate of capital is constant and given by $R_k = \beta^{-1} \exp(g_Y - g_L) - 1 + \delta_k$, and the interest rate in terms of the research good is given by $r = \beta^{-1} \exp(g_N - g_L) - 1$.

The growth rate of incumbent firms, g_z , is constant and, with positive product innovation, is determined as the solution to (19). Average productivity is given by

$$Z = \frac{\exp(g_N) - (1 - \delta_f)}{\exp(g_N) - (1 - \delta_f) \exp(g_z)}. \quad (22)$$

The equilibrium level of firm profitability \bar{V} is solved from the zero profit condition for product innovation, (11). The constant in firm profits, Π/P_r , is determined from

$$\bar{V} = \frac{1+r}{1+r - (1-\delta_f)\exp(g_z)} \left[(1+\tau_p) \frac{\Pi}{P_r} - (1-\tau_g)c(\exp(g_z)) \right]. \quad (23)$$

We solve for aggregates along the BGP as follows. The allocation of labor between production and research is determined from

$$\frac{L_p}{L_r} = \frac{(1-\alpha)(\rho-1)}{\lambda} \frac{\Pi/P_r}{n_e \frac{\exp(g_N) - (1-\delta_f)\exp(g_z)}{1-\delta_f} + c(\exp(g_z))} \quad (24)$$

and labor market clearing, $L_{pt} + L_{rt} = L_t$. For a given level of the stock of basic scientific knowledge A_t , we solve for the mass of firms N_t from

$$\frac{\Pi}{P_r} = \kappa_1 (1 + \tau_s)^{\frac{1-\lambda}{(1-\gamma)(1-\alpha)}} (1 + \tau_k)^{\alpha(\rho-1)(\theta-1)} (A_t)^{\frac{1}{1-\gamma}} (N_t Z)^{-\theta} (L_{rt})^{\frac{\gamma}{1-\gamma}} L_{pt}, \quad (25)$$

where κ_1 is a constant. Finally, we solve for aggregate output, Y_t , using (12), the stock

of physical capital, K_t , using the factor shares of physical capital and production-labor, the consumption good used in the production of the research good, X_t , using (14), and consumption, C_t , using (1).

There are two ways to see why the restriction that $\theta > 0$ ensures that the BGP of our model features semi-endogenous growth. The first is mechanically from equation (21): when $\theta > 0$ (and $\gamma < 1$), the growth rates are necessarily pinned down by resource constraints and production functions. The second is to see from equation (25) that $\theta > 0$ is required to obtain a unique solution for the total productivity of firms $N_t Z_t$, for given levels of L_t and A_t , once Π/P_r is determined by (23). The parameter θ determines to what extent an increase in total productivity NZ crowds out firm profitability, taking into account all the general equilibrium effects on aggregate prices and quantities. In contrast, when $\theta = 0$, the general equilibrium effects exactly cancel and the crowding-out effect is not operative. In this case, it is possible to have a BGP with fully endogenous growth driven by firms' innovative activity.

IV. Optimal Innovation Policies

We now characterize optimal policy on the BGP with product innovation in the semi-endogenous growth case of our model. The equilibrium of our model has two inefficiencies. The first is the standard inefficiency arising from a monopoly markup in the production of intermediate goods that distorts the mix of labor and consumption goods used in the production of the research good (when $\lambda < 1$) and the ratio of physical capital to labor used in the production of intermediate goods (when $\alpha > 0$). The second inefficiency arises because agents do not internalize the spillover from experience in innovative activities (when $\gamma > 0$). Hence, the equilibrium level of innovative activity is too low. A planner can undo these distortions with a subsidy to production of the consumption good and a uniform subsidy to process and product innovation ($\tau_g = \tau_e$).

Proposition 1: If the social optimum allocation has a BGP with product innovation and semi-endogenous growth, then that optimal BGP corresponds to the equilibrium BGP with a subsidy to the production of the consumption good given by

$$\tau_s^* = \frac{1}{\rho - 1}$$

and a uniform subsidy to process and product innovation given by

$$\tau_g^* = \tau_e^* = 1 - \gamma \frac{\beta \exp(g_L) [\exp(g_N) - (1 - \delta_r)]}{[\exp(g_N) - \beta \exp(g_L) (1 - \delta_r)]}, \quad (26)$$

where g_N is given by (21). The other policies are set to zero.

Proof: See Appendix.

Clearly, the extent of the optimal innovation subsidy depends on the spillover parameter γ . To verify that the social optimum has product innovation, one simply must check the conditions for a BGP with product innovation described in the Appendix. With endogenous growth, Proposition 1 applies, except that the socially optimal growth rate of production of the research good g_N must be solved for endogenously, as described in the Appendix.

V. Global Policy Equivalence

In this section we show that a uniform subsidy to all innovative activity, $\tau_g = \tau_e$, has an equivalent impact on equilibrium allocations as a direct subsidy to firms' variable profits in the sense that the equilibrium allocations remain constant as long as the ratio $(1 + \tau_p) / (1 - \tau_g)$ remains constant. We also provide conditions under which these two policies have the same fiscal impact. Through these results, we argue that in our model there is no special role for innovation policies distinct from a policy of subsidizing the variable profits of firms, even in the presence of spillovers from innovative activities.

Proposition 2: Let $\{\tau_p, \tau_g, \tau_e, \tau_k, \tau_s\}$ be a set of policies that are constant over time with $\tau_g = \tau_e$, and let there be an equilibrium allocation with product innovation every period corresponding to these policies. Let $\{\tilde{\tau}_p, \tilde{\tau}_g, \tilde{\tau}_e, \tilde{\tau}_k, \tilde{\tau}_s\}$ be a set of alternative policies with $\tau_k = \tilde{\tau}_k$, $\tau_s = \tilde{\tau}_s$, $\tilde{\tau}_g = \tilde{\tau}_e$, and $\frac{1+\tilde{\tau}_p}{1-\tilde{\tau}_g} = \frac{1+\tau_p}{1-\tau_g}$. Then the equilibrium allocations corresponding to these two sets of policies are equal.

Proof: See Appendix.

The intuition for this result is straightforward from inspection of three equilibrium conditions: (10), (11), and (19). A uniform subsidy to process and product innovation is equivalent to changing the equilibrium price of the research good. Since variable profits are the returns to innovative activities, a subsidy to these profits is a subsidy to the returns to innovation. One can achieve the same aggregate results by subsidizing the returns to or the costs of innovative activity.¹³

We now compare the fiscal cost of these alternative policies on a balanced growth path.

¹³Note that the assumption that the spillovers from innovative activity impact equally the cost of both process and product innovation (without favoring either type of innovative activity) is important in deriving this result on the equivalence between innovation subsidies and subsidies to firm profits, as well as the result on the local equivalence of changes in the subsidy to any particular type of innovative activity that we present in the following section.

Aggregate fiscal expenditures on policies at any point in time are given by

$$E = \tau_p \Pi N Z + \tau_g P_r c(\exp(g_z)) N Z + \tau_e P_r n_e M - \tau_k R_k K + \tau_s Y. \quad (27)$$

To compare the fiscal cost of the two policies, note that on a BGP, the free-entry condition (11) can be rewritten using (23), and $M/(NZ) = [\exp(g_N) - (1 - \delta_f) \exp(g_z)] / (1 - \delta_f)$ as

$$(1 - \tau_e) P_r n_e M \xi = (1 + \tau_p) \Pi N Z - (1 - \tau_g) P_r c(\exp(g_z)) N Z, \quad (28)$$

where

$$\xi = \frac{1}{\beta \exp(g_L)} \frac{1 - \beta \exp(g_L) (1 - \delta_f) \exp(g_z - g_N)}{1 - (1 - \delta_f) \exp(g_z - g_N)} \geq 1. \quad (29)$$

Since the result in Proposition 2 implies that the two policy alternatives implement the same allocations on a BGP, equation (28) implies that

$$E - \tilde{E} = (\tilde{\tau}_e - \tau_e) P_r n_e M (\xi - 1), \quad (30)$$

where \tilde{E} denotes fiscal expenditures under the alternative policies.

To understand expression (30), note that equation (28) and Proposition 2 imply that the discounted expected value of subsidies offered to an entering firm are the same under the two policy alternatives. The fiscal impact of the two policy alternatives differs to the extent that the timing of the subsidies offered to an entering firm differs. In particular, subsidies to product innovation are paid up front, whereas subsidies to variable profits and process innovation are paid at later dates. The variable ξ reflects the impact of discounting on the calculation of fiscal impact of these subsidies.

The parameter ξ defined in expression (29) is related to observables on the BGP as follows. Note first that if $\beta \exp(g_L) = 1$, then $\xi = 1$. In our model, $\beta \exp(g_L)$ is equal to the ratio of the growth of aggregate GDP to the consumption interest rate, $\beta \exp(g_L) = \exp(g_Y) / (1 + \bar{r})$. Thus, $\xi = 1$ if the growth rate of aggregate GDP equals the consumption interest rate. Since we require that the consumption interest rate be at least as large as the growth of aggregate GDP, we have $\xi \geq 1$. Second, from equation (18) in the BGP, the term $1 - (1 - \delta_f) \exp(g_z - g_N)$ in expression (29) is equal to the share of production employment accounted for by new products. We use these two observations in our calibration below to argue that $\xi = 1.16$ in the United States.

Thus, to summarize, we have that the fiscal cost of a uniform subsidy to innovative

activities is slightly lower than the fiscal cost of a subsidy to variable profits that implements the same allocation when the interest rate exceeds the growth rate of the economy because the portion of the innovation subsidy that is paid as a subsidy to product innovation is paid up front to entering firms.

Note that in Proposition 2, we impose that policies are constant over time. Thus, we do not consider subsidies that are offered to incumbent firms but are not offered in the future to entering firms. That is, for the set of policies that we consider here, entering firms anticipate that they will receive the same subsidies in the future as the current incumbents. As we discuss in greater detail in the next section, it is the impact of policy on the incentives of firms to enter that is critical for their aggregate effects.

VI. Calculating Aggregate Effects of Changes in Policies

In this section we present a two-step algorithm for computing the implications for the change in aggregates from one BGP to another resulting from a change in policies in the case that our model has a BGP with semi-endogenous growth and positive product innovation. We first use this algorithm to derive analytical results on the effects of changes in innovation policies. We derive conditions under which the long-run impact of changes in innovation policies is, to a first-order approximation, directly proportional to the fiscal impact of those policy changes. Hence, under these conditions, the relative magnitudes of the impact of different innovation policy changes can be measured simply from the relative magnitudes of the fiscal impact of these policy changes. We then analyze the impact of changes in the tax on physical capital and compare these results with those that we have obtained for innovation policies. In the Appendix, we briefly discuss how to extend our algorithm in the case where our model has a BGP with endogenous growth.

Our algorithm takes advantage of the block recursive structure of the equations characterizing a BGP in our model with semi-endogenous growth and positive product innovation. When our model has semi-endogenous growth, aggregate growth rates and the interest rate in the BGP are invariant to policy changes and are as described in Section III above.

In the first step of our algorithm, we solve for the equilibrium level of firm profitability, \bar{V} , the growth rate of incumbent firms, g_z , the level of average productivity, Z , and the constant in firm profits, Π/P_r , from (11), (19), (22), and (23). In the second step, we solve for the allocation of labor between production and research from (24) and labor market clearing, and for the mass of firms N_t from (25). All other aggregates in a BGP are pinned down by these variables. We now use this algorithm to establish two propositions on the aggregate impact of a change in innovation policies on BGP allocations.

A. Changes in Subsidy to Variable Profits

In the following proposition, we consider the aggregate impact of changes in the subsidy to variable profits.

Proposition 3: If our model has a BGP with semi-endogenous growth and positive product innovation, then a change in the subsidy to variable profits of size $\Delta \log(1 + \tau_p)$ results in the following changes to the aggregates along the BGP:

1. The equilibrium level of firm profitability, \bar{V} , the growth rate of incumbent firms, g_z , and average productivity, Z , are unchanged;
2. The constant in firm profits, Π/P_r , changes by $\Delta \log(\Pi/P_r) = -\Delta \log(1 + \tau_p)$;
3. The aggregate allocation of labor between production and research changes by $\Delta \log(L_p/L_r) = -\Delta \log(1 + \tau_p)$;
4. To a first-order approximation, the change in the index of total productivity is $\Delta \log(NZ) = \frac{1}{(1-\gamma)\theta} \frac{L_p}{L} \Delta \log(1 + \tau_p)$;
5. To a first-order approximation, the change in aggregate output, GDP , is

$$\Delta \log GDP = \left[\left(1 + \frac{1}{(1-\gamma)(\rho-1)(1-\alpha)\theta} \right) \frac{L_p}{L} - \frac{Y}{GDP} \right] \Delta \log(1 + \tau_p) ;$$

6. To a first-order approximation, the change in the research intensity of the economy, s_r , is $\Delta \log s_r = \frac{Y}{GDP} \Delta \log(1 + \tau_p)$.

Proof: See Appendix.

Proposition 3 is the simplest application of our two-step algorithm for computing the aggregate implications of changes in innovation policies, and it illustrates the logic of the algorithm in a straightforward manner. We start in result 1 with the observation that the equilibrium level of firm profitability \bar{V} must remain constant in response to a change in the subsidy to variable profits to satisfy the equilibrium condition that there be zero profits associated with product innovation. In result 2, we see that to keep the level of firm profitability constant, the constant in variable profits, Π/P_r , must change to offset the direct impact of the subsidy to variable profits on firm profitability. This point follows immediately because a subsidy to variable profits has no impact on the level of process innovation, g_z , or on the average productivity of firms, Z . In terms of firm dynamics, the only impact of a subsidy to variable profits is to encourage product innovation. Because the subsidy to variable profits has no impact on the life cycle of a typical firm, the use of the research good for innovative activities per firm is unchanged. This observation is the key to our calculation of the aggregate reallocation of labor between production and research in result 3.

The resulting response of total productivity NZ then follows from the downward-sloping relationship between total productivity and the constant in variable profits that follows from our assumption that this is a semi-endogenous growth model. The response of total productivity to changes in innovation policy in this case is self-limiting because, as we have discussed above, an increase in the total productivity of incumbent firms crowds out the profitability of new firms and hence crowds out product innovation. As is evident in the expression in result 4, the slope of this relationship depends inversely on θ , which measures the extent to which an increase in the productivity of all firms crowds out the profitability of a given firm. If θ is large, so that the crowding out is strong, then the response of total productivity associated with a given change in Π/P_r is small. In this case, the response of innovative activity to a change in policy is crowded out before a large response of total productivity can occur. In contrast, as θ approaches zero, this crowding-out effect is weak and the corresponding response of total productivity becomes large. The remaining aggregates follow immediately from the responses of the aggregate allocation of labor and total productivity.

B. Changes in Subsidies to Process or Product Innovation

In the next proposition, we consider the aggregate impact of changes in subsidies to process and/or product innovation individually. In this case, changes in policies do affect the level of process innovation and the average productivity of firms, as well as the level of product innovation.

Proposition 4: If our model has a BGP with semi-endogenous growth and positive product innovation, and policies are initially set so that $\tau_g = \tau_e$, then a change in the subsidy to process innovation of size $\Delta \log(1 - \tau_g)$ and/or a change in the subsidy to product innovation of size $\Delta \log(1 - \tau_e)$ results in the following changes to the aggregates along the BGP:

1. The equilibrium level of firm profitability, \bar{V} , changes by $\Delta \log \bar{V} = \Delta \log(1 - \tau_e)$;
2. To a first-order approximation, the growth rate of incumbent firms, g_z , and average productivity, Z , change by

$$\Delta g_z = \frac{1}{\eta_c} [\Delta \log(1 - \tau_e) - \Delta \log(1 - \tau_g)]$$

$$\Delta \log Z = \frac{(1 - \delta_f) \exp(g_z)}{\exp(g_N) - (1 - \delta_f) \exp(g_z)} \frac{1}{\eta_c} [\Delta \log(1 - \tau_e) - \Delta \log(1 - \tau_g)];$$

3. To a first-order approximation, the constant in firm profits, Π/P_r , the aggregate allocation of labor between production and research, L_p/L_r , the index of total productivity, NZ , aggregate output, GDP , and the research intensity of the economy, s_r , change as in Proposition 3 with $-[s_g \Delta \log(1 - \tau_g) + (1 - s_g) \Delta \log(1 - \tau_e)]$ replacing $\Delta \log(1 + \tau_p)$, where s_g is the ratio of process innovation costs to variable profits, $s_g = \frac{(1-\tau_g)c(\exp(g_z))}{(1+\tau_p)\Pi/P_r}$.

Proof: See Appendix.

The application of our algorithm here in Proposition 4 to the case of changes to the individual subsidies to process and product innovation one at a time or in combination applies only to a first approximation because now firm dynamics, characterized by firms' investments in process innovation g_z and average productivity Z , do change in response to the change in subsidies. In fact, in response to a change in innovation policy (in terms of a change in the subsidy to either process or product innovation), incumbent firms in our model can exhibit a wide range of different responses of their investments in process innovation depending on the curvature of the process innovation cost function given by η_c .

Two key insights are needed to establish that, to a first approximation, this response in incumbent firms' investments in process innovation does not matter for the aggregates. The first insight is that, to a first-order approximation, the change in the constant in firm profits Π/P_r required to maintain zero profits to product innovation does not depend on the induced response of firms' process innovation g_z . This first insight follows directly from the envelope condition in firms' Bellman equation defining firm profitability \bar{V} : since firms are choosing process innovation optimally in the original allocation, to a first-order approximation the change in firm profitability induced by a change in process innovation is zero. The second insight is that to a first-order approximation, the aggregate reallocation of labor between production and research also does not depend on the response of firms' investments in process innovation. This second insight is more subtle and depends both on the assumption that in the original allocation the subsidies to both types of innovation are equal ($\tau_e = \tau_g$) and on the result that all firms choose the same process innovation rate g_z . With these additional equalities, we have that, to a first-order approximation, the use of the research good for innovative activities per firm is again unchanged, and hence the aggregate reallocation of labor is the same as in Proposition 3.

Finally, consider the response of the index of total productivity NZ to the change in policy. Here, with a change in the subsidy to process or product innovation separately, as noted in point 2, firms' investments in process innovation g_z change and the resulting level

of the average productivity of firms Z also changes. The size of these responses depends on the curvature of firms' process innovation cost function η_c . If this cost function has low curvature, the response of process innovation and average productivity can be quite large. In equilibrium, however, it is the crowding-out effect of an increase in total, not average, productivity on firm profits that determines the aggregate response of the economy. It is irrelevant for aggregates whether this response of total productivity is achieved through a large increase in average productivity due to a large increase in process innovation and a small change in product innovation or through a small change in average productivity and a large change in product innovation. If the average productivity of incumbent firms rises substantially in response to a change in policy, then the crowding-out effect implies that there is less room for the creation of new products.

In the Appendix we present a corollary of Proposition 4 which establishes that, to a first-order approximation, a change in the subsidy to process or product innovation individually has the same aggregate impact on the index of total productivity and output as a change in the subsidy to variable profits as long as these policy changes have the same impact on firm profitability holding fixed firms' process innovation decisions. This corollary reinforces the point that, to a first-order approximation, information on the response of firms' investments in process innovation to a change in innovation policy is not informative for the aggregate implications of that policy change: what is needed to evaluate policy, at least locally, is information on the impact of that policy change on the constant in firm variable profits holding fixed firms' decisions to invest in process innovation. As we have seen in Proposition 4, changes in the individual subsidies to process or product innovation do affect firms' process innovation decisions. In contrast, as we have seen in Proposition 3, changes in the subsidy to firms' variable profits have no effect on firms' process innovation decisions and hence no impact on average productivity. As this corollary establishes, despite this contrast, to a first-order approximation, these policy changes have the same impact on aggregates as long as they have the same impact on firm profitability holding fixed firms' process innovation decisions.

Note that the long-run change in aggregate consumption, C , corresponding to a change in these policies that leads to a given change in Π/P_r is the same in Proposition 3 and Proposition 4 and its corollary. This result follows from the fact that the change in the ratio of investment in physical capital to GDP is the same across all of these policies.

C. Using Fiscal Impact to Compare Aggregate Effects of Innovation Policy Changes

In Propositions 3 and 4 we computed the aggregate impact of a given change in the loga-

rithm of innovation subsidies. To use these results in applications, one would have to measure changes in effective marginal subsidy rates. In applying our algorithm to measure the aggregate impact of policies in the data, we find it more convenient to compute the aggregate impact of a change in subsidies measured in terms of the change in aggregate fiscal expenditures on these policies. We do this in the next proposition.

Proposition 5: Let our model have a BGP with semi-endogenous growth and positive product innovation, and suppose policies are initially set so that $\tau_p = \tau_g = \tau_e = 0$. Suppose that these policies change by $\Delta\tau_p$, $\Delta\tau_g$, $\Delta\tau_e$, and $\Delta\tau_s = 0$. Then, to a first-order approximation, the log change in the constant in firm profits, Π/P_r , is given by

$$\Delta \log (\Pi/P_r) = - [\Delta\tau_p + s_g\Delta\tau_g + (1 - s_g) \Delta\tau_e] , \quad (31)$$

and the change in aggregate expenditures on subsidies E is given by

$$\Delta E = \Pi NZ \left[\Delta\tau_p + s_g\Delta\tau_g + \frac{(1 - s_g)}{\xi} \Delta\tau_e \right] , \quad (32)$$

where ξ is defined in expression (29).

Proof: See Appendix.

Recall from Propositions 3 and 4 that, to a first-order approximation, changes in the aggregate allocation of labor between production and research, L_p/L_r , aggregate output, GDP , and the research intensity of the economy, s_r , are all proportional to the negative of the log change in the constant in variable profits, Π/P_r . Hence, what Proposition 5 implies is that if we start from a situation with no policies (but for τ_s and τ_k), a change in the subsidy to variable profits and a change in the subsidy to process innovation all have aggregate impacts that are directly proportional to their aggregate fiscal impact. In contrast, a change in the subsidy to product innovation has an aggregate impact that is directly proportional to ξ times its impact on expenditures in this subsidy. The intuition for this adjustment by ξ is the same as that discussed in Section V. When the interest rate is equal to the growth rate of the economy, $\xi = 1$, the aggregate impact of all of these policy changes is directly proportional to the fiscal impact of these changes.

Summarizing these results, we have that for changes in τ_p and τ_g , the log change in GDP per dollar change in fiscal expenditures is, to a first-order approximation,

$$\frac{\Delta \log GDP}{\Delta E} = \frac{1}{\Pi NZ} \left[\left(1 + \frac{1}{(1 - \gamma)(\rho - 1)(1 - \alpha)\theta} \right) \frac{L_p}{L} - \frac{Y}{GDP} \right] , \quad (33)$$

and for changes in τ_e , the log change in GDP per dollar change in fiscal expenditures is ξ times the ratio in the equation above.

D. Changes in Tax on Use of Physical Capital

In the next proposition we characterize the aggregate impact of a change in the tax τ_k on the firms' use of physical capital. A change in this tax has new effects that arise from the impact of this tax on intermediate goods firms' physical capital to output ratio and also because changes in this tax do not alter the allocation of labor between production and research.

Proposition 6: If our model has a BGP with semi-endogenous growth and positive product innovation, then a change in the tax to the use of physical capital of size $\Delta \log(1 + \tau_k)$ results in the following changes to the aggregates along the BGP:

1. The equilibrium level of firm profitability, \bar{V} , the constant in firm profits, Π/P_r , the growth rate of incumbent firms, g_z , and average productivity, Z , are unchanged;
2. The aggregate allocation of labor between production and research, L_p/L_r , and the research intensity of the economy, s_r , are unchanged;
3. To a first-order approximation, aggregate output, GDP , changes by

$$\Delta \log GDP = -\frac{\alpha}{(1-\alpha)\theta} \Delta \log(1 + \tau_k).$$

Proof: See Appendix.

To understand the different aggregate implications of the policies considered in Propositions 3, 4, and 6, it is helpful to compare the impact of these policies on the innovation intensity of the economy s_r (the ratio of intangible investment to output) and on the ratio of physical capital investment to output of the final consumption good (which is proportional to K/Y). The innovation policies that we considered in Propositions 3 and 4 affect s_r but do not affect K/Y . In contrast, the policy that we considered in Proposition 6, i.e., a change in the tax on the use of physical capital, affects K/Y but does not affect s_r . Note that a change in the tax on the use of physical capital has an additional direct effect of changing the allocation of aggregate output between consumption and investment in physical capital. This effect is a standard implication of taxing physical capital.

We can use Proposition 6 to derive a relationship between the change in GDP and the change in fiscal expenditures from changes in the tax on the use of physical capital. Under the assumption that initially $\tau_k = 0$, this relationship, to a first-order approximation, is given

by

$$\frac{\Delta \log GDP}{\Delta E} = \frac{1}{\Pi NZ} \frac{1}{(\rho - 1)(1 - \alpha)\theta}. \quad (34)$$

Comparing (33) and (34), we see that the change in GDP relative to the change in fiscal expenditure from changes in the tax on physical capital can, in general, be higher or lower than that from changes in innovation policies. Hence, one cannot use the relative magnitudes of aggregate impact of these policy changes without specifying a full set of model parameters.

VII. Applying Our Algorithm

To this point, we have considered abstract policies. In this section we apply the results in Section VI to assess the relative magnitudes of the impact on aggregates of three current policies affecting innovation activity by firms in the United States: the Research and Experimentation (R&E) Tax Credit, federal spending on research and development (R&D), and the corporate profits tax. As we discuss in greater detail below, we model changes in all three of these policies as combinations of changes in our abstract policies.

We consider the aggregate impact of eliminating each of these three policies in turn. To measure the relative impact of these policies on aggregates, we use the logic of our results in Proposition 5 that, to a first-order approximation, the relative magnitudes of the aggregate effects of two different changes in innovations policy are given by the relative magnitudes of the impact of those policy changes on fiscal expenditures, holding all firms' decisions fixed.

In the statement of Proposition 5, we assumed that policies were set initially equal to zero for calculating the impact of a policy change on fiscal expenditure. In our application, we use a related first-order approximation in which we consider the aggregate impact of eliminating a subsidy that is initially set at a value different from zero. According to this approximation, the aggregate impact of eliminating a policy is also given by (33), where ΔE now denotes current expenditure on the policy being eliminated. The details of this approximation are given in the Appendix.

To compare the relative size of the impact of these three policies on aggregates in the United States, we use data from 2007 to measure fiscal expenditures on these three policies. According to the Office of Management and Budget (2009), the fiscal expenditure on the Research and Experimentation Tax Credit in 2007 was \$10 billion. The same data source lists federal spending on R&D of \$139 billion.¹⁴ Finally, the National Income and Product

¹⁴Federal spending on R&D is often grouped into five categories: basic research, applied research, development, R&D equipment, and R&D facilities (see, e.g., Office of Management and Budget 2009). Over 50% of federal spending on basic research is in Health and Human Services. The three leading agencies in applied

Accounts (Table 6.18) shows revenue of \$445 billion from corporate profits taxes.

We now describe in greater detail how we map the R&E Tax Credit, federal spending on R&D, and the corporate profits tax into our framework. We model the R&E Tax Credit as a combination of subsidies to process and product innovation of sizes τ_g^{re} and τ_e^{re} , where the precise size of these subsidies depends on the details of the rules on eligibility for the tax credit.¹⁵ We abstract from these complications of the policy because, as we have shown in our model, all that we need to calculate the relative magnitude of these policies on aggregate in the BGP is the aggregate fiscal impact of the R&E Tax Credit. According to our model, in the end the R&E Tax Credit is simply a complicated and perhaps administratively expensive way of subsidizing firms' variable profits.

We model federal spending on R&D as a direct subsidy to the production of the research good, which is equivalent to a uniform subsidy to process and product innovation of size τ_g^{rd} and τ_e^{rd} .

Finally, we model the corporate profits tax as a combination of taxes on variable profits and physical capital, and subsidies to innovative activities. The corporate profits tax includes a tax on variable profits simply because variable profits are a part of corporate profits. As is standard, the corporate profits tax includes a tax on the use of physical capital (relative to the use of other inputs such as labor) depending on the extent to which this capital is equity financed (as opposed to debt financed or leased), and the extent of the deductibility of depreciation of physical capital (both of these factors affect the extent to which spending on physical capital by firms is deductible from corporate profits). The corporate profits tax includes a subsidy to process innovation by incumbent firms if these firms partly expense this innovative activity so that they are deducted from corporate profits. To the extent that expenditures on product innovation can also be partly expensed, the corporate profits tax can also include a subsidy to product innovation.¹⁶

To the extent that elimination of any of these three policies amounts to a change in

research are Health and Human Services, Defense, and Energy. The Defense Department and NASA account for the overwhelming majority of spending on development. Expenditures on equipment and facilities are spread across a variety of agencies. Spending in 2007 was allocated among \$28 billion on basic research, \$27 billion on applied research, \$80 billion on development, and \$4 billion on equipment and facilities. For comparison, business spending on R&D in 2007 was measured at \$260 billion.

¹⁵In practice, the R&E Tax Credit in the United States is a complicated policy that defines *qualified* research expenses and offers a credit only for those expenses that are incremental over a baseline amount that is also defined in the regulations (see, e.g., Hall 2001).

¹⁶Gentry and Hubbard (2000), Cullen and Gordon (2007), and McGrattan and Prescott (2005a) discuss incomplete offsets of investment in intangible capital against corporate profits. Empirical work by Gentry and Hubbard (2000), Lee and Gordon (2005), Cullen and Gordon (2007), and Djankov et al. (2010) find that corporate taxes reduce entry of new firms.

the overall subsidy to variable profits, τ_p , or to a change in the overall subsidy to process innovation, τ_g , then from Proposition 5, the relative aggregate impact, to a first-order approximation, of eliminating these policies can be measured directly from their relative fiscal expenditures (as currently measured) — no parameters of the model enter into this calculation.

To the extent that elimination of any of these policies amounts to a change in the overall subsidy to product innovation, τ_e , the relative aggregate impact, to a first-order approximation, of eliminating these policies can be measured from the fiscal expenditure of the component of the policy affecting the subsidy to product innovation, scaled up by the parameter ξ reflecting the difference in the discounted present value and cross section of firm dividends. For example, in the case of the R&E Tax Credit, one needs to scale up fiscal expenditure on that portion of the tax credit that is paid for the development of new products or firms. In the case of the corporate profits tax, one needs to scale up fiscal expenditure arising from the expensing of product innovation costs.¹⁷

We now discuss how we calibrate the parameter ξ defined in expression (29). We set the ratio of the growth of aggregate GDP to the consumption interest rate, $\exp(g_Y) / (1 + \bar{r}) = \beta \exp(g_L)$ to 0.99. We set the share of production employment accounted for by new products, the term $1 - (1 - \delta_f) \exp(g_z - g_N)$, to 0.063, which is the share of employment in new establishments in the United States in 2007.¹⁸ With these numbers, $\xi = 1.16$. As a robustness check, consider data from Broda and Weinstein (2010) on the sales share of newly created consumer products obtained from ACNielsen’s Homescan database. They report a sales share for newly created products of 9%, which leads to a value of $\xi = 1.1$. Hence, this scaling of fiscal expenditures on policies affecting the subsidy to product innovation does not substantially alter the relative ranking of the aggregate implications of the three policies that we consider from the ranking one would obtain from directly comparing fiscal expenditures.

On the basis of the data on fiscal expenditures and our value of the parameter ξ , we conclude that, to a first-order approximation, the long-term impact on aggregate output of federal spending on R&D is substantially larger than the aggregate impact of the R&E Tax Credit.

¹⁷The Office of Management and Budget (2009) provides an estimate for 2007 of the tax expenditures arising from the fact that corporate R&D expenditures are expensed rather than counted as investment, at \$5 billion. This is a small portion of the revenue collected from the corporate profits tax. The portion of these tax expenditures corresponding to product innovation must be even smaller.

¹⁸The source of this figure is the 2007 Business Dynamics Statistics from the U.S. Small Business Administration. Employment in entering establishments is the sum of employment in new firms (3.3%), as reported in Haltiwanger, Jarmin, and Miranda (2010), and employment in new establishments by existing firms (3%).

To use our methodology to compare the aggregate impact of the R&E Tax Credit and federal spending on R&D with the aggregate impact of the corporate profits tax, we must divide the revenue currently collected from the corporate profits tax into a component that corresponds to innovation policies τ_p , τ_g , and τ_e , and a component corresponding to a tax on physical capital τ_k . As we have seen in Proposition 6, the impact of a tax on physical capital on aggregate output per dollar of fiscal revenue collected from that tax differs from the impact of innovation policies on aggregate output per dollar of fiscal spending on these policies, depending on the model's parameter values. Hence, we cannot make a direct comparison of the impact of the corporate profits tax with the impact of the other innovation policies until we choose parameter values for the model in the next section.

We find that, once we parameterize the model in the next section, the long-run impact of the tax on the use of physical capital on aggregate output per dollar of fiscal revenue (given by expression 34) exceeds that of innovation policies (given by expression 33), unless the spillover parameter γ is quite high (justifying an optimal uniform innovation subsidy of roughly 30%). Therefore, given the large revenues collected from the corporate profits tax in comparison with spending on the R&E Tax Credit and federal spending on R&D, in our calibrated model the corporate profits tax has a significantly larger negative impact on aggregate output in comparison with both the federal R&E Tax Credit and spending on R&D's positive impact on aggregate output, unless γ is very high. This result holds even if the corporate profits tax is largely a tax on the use of physical capital. We thus conclude in our calibrated model that the corporate profits tax is a relatively important policy (in comparison with the R&E Tax Credit and federal expenditure on R&D) in terms of its aggregate effects on the long-run accumulation of both tangible and intangible capital.

VIII. Quantitative Analysis on Aggregate Effects of Changes in Policies

We now perform a calculation of the absolute (as opposed to relative) magnitude of the aggregate impact of a change in innovation policy in a calibrated version of our model. We consider, for simplicity, an economy that is on a BGP with subsidies and taxes set equal to zero, and conduct several policy experiments. First, we ask what is the aggregate impact of a uniform subsidy to innovative activities ($\tau_g = \tau_e$) that on the new BGP has fiscal expenditures of 3% of GDP. For the U.S. economy, this figure would have been approximately \$420 bn in 2007 (i.e., similar in magnitude to revenues collected from the corporate profits tax that year). In our second and third experiments, we calculate the aggregate impact of a subsidy to process innovation τ_g and a subsidy to the use of physical capital $-\tau_k$, respectively, that on the new BGP have a fiscal expenditure of 3% of GDP. We consider both the long-run

response and the transition dynamics from one BGP to another.

A. Calibration

To conduct this exercise, we must choose the parameters of our model. Table 1 lists the target moments and parameter values. A time period is defined to be a year. We normalize the level of population, L , at time zero to 1 and have it grow at rate $g_L = 0.01$. We normalize the level of scientific knowledge, A_r , at time zero to 1 and set its growth rate, g_A , so that, given the other parameter choices, the growth rate of output per capita is 2%. We normalize the cost of product innovation, n_e , to 1. We parameterize the cost function for process innovation, $c(\exp(g_z)) = c_0 [c_1 + \exp(g_z)^{1+\eta_c}]$. Recall from expression (19) that the parameter η_c determines the response of process innovation by incumbent firms to changes in innovation policies. In our first and third experiments, the optimal choice of g_z is not altered by the change in policy, both on the BGP and on the transition path. Therefore, in these two experiments, the choice of parameter η_c does not affect our results. From Proposition 4, we have that to a first-order approximation, the parameter η_c does not affect the aggregate effects in the BGP of a change in the subsidy to process innovation. To check the accuracy of this result when the policy changes are large, we set $\eta_c = 10$ so that the growth rate of incumbent firms increases by 1 percentage point for every 10 percentage point change in the subsidy rate to process innovation.

We choose the parameters β , δ_k , δ_f , and λ as follows. The discount factor β corresponds to the ratio of the growth rate of per capita GDP to the interest rate (in terms of the consumption good). We use a per capita growth rate of GDP of 2% and an interest rate of 4%, which implies $\beta = 0.98$. We set the depreciation rate on physical capital to $\delta_k = 0.10$. We choose the exogenous exit rate of incumbent firms to $\delta_f = 0.053$, which corresponds to the employment-weighted exit rate of U.S. establishments in 2007.¹⁹ We set the share of labor in the production function of the research good to $\lambda = 0.01$. This choice is guided by the data from the NIPA satellite accounts on the price of inputs into R&D. In our model, the ratio of the relative price of research inputs to the price of final output is directly proportional to W^λ . Given that in the data, the relative price of research inputs to the GDP deflator shows no trend, we set λ close to zero.

To calibrate the remaining parameters, we use as targets a standard share of labor in GDP of 66% and a share of dividends to owners of firms (payments to intangible capital)

¹⁹The source of this figure is the 2007 Business Dynamics Statistics from the U.S. Small Business Administration. We include exit of all establishments, whether or not the firm that owns the establishment actually exits.

in GDP of 1% (obtained from McGrattan and Prescott 2005b). We denote this share of dividends by $\pi = (\Pi NZ - P_r Y_r) / GDP$. The share of rental payments to physical capital in GDP is calculated as a residual equal to 33%. The expressions for the factor shares presented above together with $\pi = 0.01$ imply $\alpha = 1/3$. We use as a target an innovation intensity of the economy (share of intangible investment in GDP) of $s_r = 0.15$ (this estimate is taken from Corrado, Hulten, and Sichel 2009).

Note that the parameter ξ is related to s_r , π , and s_g , by²⁰ $\xi = (1 - s_g) \left(\frac{s_r}{s_r + \pi} - s_g \right)^{-1}$. Our calibration of $\xi = 1.16$ above implies $s_g = 0.54$. We choose the parameters ρ , c_0 , c_1 , g_z , and g_N to hit our target values of s_r , s_g , π , $g_{Y/L}$, and the employment share of new products, using the equations described above. This procedure results in values of parameters $\rho = 7.15$, $g_z = 0.07$, and $g_N = 0.08$.²¹ Note that, with λ close to zero, L_p/L is essentially one and $GDP/Y = 1 / (1 + s_r) = 0.86$.

This calibration procedure does not pin down the choice of parameters γ and δ_r governing the spillover of the research good (that is, given our calibration procedure, the values of all target and parameters other than g_A of our calibration are independent of γ and δ_r). Hence, the value of θ is not pinned down either. We report results below for a wide variety of values of γ ranging from 0 to 0.74 (so that θ varies from $\theta = 0.75$ when $\gamma = 0$, to $\theta = 0.07$ when $\gamma = 0.74$), and set $\delta_r = 0.10$. In our calibration, the optimal level of the subsidy to innovative activity as a function of γ , from equation (26), is $\tau_g^* = \tau_e^* = 0.944 * \gamma$. Hence, the optimal innovation subsidy ranges from 0 to 0.7. We briefly discuss the sensitivity of our results to changes in ρ , λ , and δ_r .

B. Long-Run Impact

We now use our calibrated model to assess the long-run impact of our three policy experiments: a uniform subsidy to innovative activities ($\tau_g = \tau_e = 0.162$), a subsidy to process innovation ($\tau_g = 0.221$), and a subsidy on the use of physical capital ($\tau_k = -0.084$). In each of these policy experiments, we start from a baseline of zero policies and assume that the changes in subsidy rates are unanticipated and permanent, and result in fiscal expenditures of 3% of GDP in the BGP.

Figure 1 displays the log change in GDP from the initial BGP to the new BGP under our three experiments. Recall that in our model, since the long-run growth rate is independent

²⁰We obtain this expression from equation (28), the definition of the research intensity of the economy as s_r together with the resource constraint for the research good (4), and the definition of π .

²¹We do not report the values of c_0 and c_1 since they do not have any particular interpretation other than resulting in the target g_z and s_g . We also note our calibration procedure results in a unique choice for all of our parameters.

of policy, GDP across BGPs differs by a fixed log difference at each date. This log difference also corresponds to the long-run change in GDP when GDP is detrended by the exogenous growth rate.

When $\gamma = 0$, the log change in GDP across BGPs is relatively small: 0.027 in Experiment 1, 0.019 in Experiment 2, and 0.058 in Experiment 3. As γ gets large, the change in GDP clearly becomes much larger. For example, when $\gamma = 0.74$ the log change in GDP is 2.29 in Experiment 1 (so that GDP increases by a factor of $\exp(2.29) = 9.88$), 2.11 in Experiment 2, and 0.62 in Experiment 3. If γ is increased slightly further, so that θ is driven closer to 0, our model becomes an endogenous growth model and this change in GDP across BGPs rises toward infinity. Clearly, our model's implications for the long-run impact of a given change in policies vary tremendously depending on the assumed spillover parameter.²² The changes in aggregate consumption in each of these experiments are of similar magnitudes to the changes in GDP shown in Figure 1.

Note that the changes in GDP corresponding to Experiments 1 and 2 are quite similar, even though these two different changes in innovation policies have very different implications for observed changes in firm dynamics. In Experiment 1, there is no change in the process innovation decisions of incumbent firms, and hence the average productivity of incumbent firms Z remains unchanged. The increases in GDP that occur arise due to increases in product innovation and physical capital. On the other hand, in Experiment 2, investments in process innovation by incumbent firms grow considerably, raising the average growth rate of incumbent firms by roughly 1.5 percentage points. As a result, the average productivity of incumbent firms rises by a factor of 1.6. Of course, this change in the behavior of incumbent firms does not have a significant impact on aggregate output because of the offsetting response of product innovation.

Note also that the change in GDP corresponding to a change in a subsidy to the use of physical capital in Experiment 3 is larger (smaller) than the changes in GDP corresponding to a change in innovation subsidies in Experiments 1 and 2 when γ is below (above) roughly 0.3 (which is associated with an optimal innovation subsidy of $\tau_g^* = \tau_e^* = 0.29$). That is, as we discussed above, in our calibration the impact of a change in the physical capital tax

²²The first-order approximations of the change in GDP corresponding to a change in policies derived in Section VI are quite accurate even for these relatively large changes in policies. In Experiment 1, the approximation is within 10% of the exact answer for all values of γ . The main error is coming from the log-linear approximation of the equation $GDP = Y - X$. In Experiment 2, the first-order approximation is within 20% of the exact answer for all values of γ . The approximation error is the result of higher order terms from changes in the growth rate of incumbent firms g_z . In Experiment 3, the first-order approximation is extremely accurate.

per dollar of fiscal expenditure is larger than the impact of innovation subsidies per dollar of fiscal expenditure when γ is low.

The research intensity of the economy increases from $s_r = 0.15$ on the initial BGP to $s_r = 0.185$ on the new BGP in Experiment 1 and to $s_r = 0.183$ in Experiment 2, whereas it remains unchanged in Experiment 3. As discussed in Section VI, these changes in the research intensity of the economy across BGPs are independent of the spillover parameter γ . None of these results depend on the choice of δ_r .

We now ask what our model’s implications are for the behavior of aggregates during the transition from the initial to the new BGP.

C. Transition Dynamics

To compute the transition dynamics from one BGP to another, we solve the model numerically. We report on the behavior of aggregates over the first 15 years of the transition path. We focus on this 15-year horizon to show our model’s implications for aggregates over a horizon that is relevant for applied work on the consequences of actual policy changes. In each of these experiments, we are using the same change in subsidy rates as we described above. We find that, in all of our experiments, in the 15th year of the transition, the ratio of fiscal expenditure on subsidies to GDP is always close to 3%.

Figure 2 displays the log change in detrended GDP 15 years after the policy change for each of our three experiments, for the same range of values of γ used in Figure 1. In this figure, we see that the range of responses of GDP over a 15-year horizon is much smaller than the range of long-run responses shown in Figure 1. In particular, in contrast to Figure 1, in all cases, the log change in detrended GDP is less than 0.05. These changes in detrended GDP amount to changes in the average annual growth rate of GDP during the first 15 years of transition of less than 1/3 of 1 percentage point. Hence, our model’s implications for the impact of a given change in policies over a 15-year horizon do not vary so much with the assumed spillover parameter γ .²³

In Figure 3, we plot the change (in percentage points) of the research intensity of the economy, s_r , 15 years after the policy change in each of our experiments. This change in the research intensity is roughly the same as the long-run change across all the values of γ in our three policy experiments.

Note that in the case of low spillovers, the change in GDP over the first 15 years amounts to a large fraction of the long-run change in GDP across BGPs (e.g., 0.49 in Exper-

²³The impact of a change in the subsidy to the use of physical capital over a 15-year horizon is larger than the impact of a change in innovation subsidies for a wide range of spillover parameters γ .

iment 1 whereas $\gamma = 0$), whereas in the case of high spillovers, the change in GDP over the first 15 years is a small fraction of the long-run change in GDP (e.g., 0.02 in Experiment 1 when $\gamma = 0.74$). This difference in responses over different horizons follows from the result that the transition dynamics of our model get significantly slower as the spillover parameter increases. For example, in Experiment 1, when $\gamma = 0.74$ it takes more than 250 years for the cumulative change in GDP to reach half the long-run change (in contrast, when $\gamma = 0$, the half-life is 16 years). If γ is increased slightly further, so that if θ is driven closer to 0, our model becomes an endogenous growth model and the half-life approaches infinity.²⁴

Consider now the impact of our policy changes on welfare when transition dynamics are taken into account. In Figure 4 we show the overall welfare gains (defined as the logarithm of the equivalent variation in consumption), taking into account the whole transition for the range of spillovers considered in Figures 1–2 in each of our three policy experiments. As this figure shows, our model implies that the welfare gains from our policy experiments are very small when there are no spillovers,²⁵ and they are very large when the spillover is large (e.g., in Experiment 1, when $\gamma = 0.74$, consumption in the initial BGP would have to be permanently multiplied by $\exp(0.47) = 1.6$ in order to provide the same level of utility as that obtained in the transition after the policy change).

Thus, although the implications of our model for aggregates over a 15-year horizon are not very sensitive to the assumed level of spillovers, the welfare implications of our model are very sensitive to the choice of the spillover parameter. In Appendix B, we perform a sensitivity analysis of our results with respect to changes in parameters ρ , λ , and δ_r .

IX. What Do We Know about the Long-Run Impact of Innovation Policy?

With our calibrated model, we can answer the question of how permanent changes in innovation policy affect the research intensity of the economy and the levels of GDP and other aggregates over a 15-year horizon even if we do not have precise information about the strength of spillovers from innovative activity. In particular, we found that a permanent subsidy to innovative activities with a fiscal impact of 3% of GDP per year raises the research intensity of the economy by roughly 3 to 4 percentage points both in the long run and at a 15-year

²⁴This slowdown of the transition dynamics as the spillover parameter increases is not likely to be an artifact of our specific calibration. For example, in a standard growth model extended to include spillovers to the accumulation of physical capital, it is straightforward to show that there is a trade-off between the size of the steady-state effects and the speed of transition from a policy change as the share of physical capital or the strength of spillovers varies.

²⁵If the subsidy on the production of the final good, τ_s , were optimally chosen to eliminate effective markups (as opposed to being set at $\tau_s = 0$) and $\gamma = 0$, then our policy changes would reduce welfare.

horizon. This increase in research intensity is associated with an increase in GDP at a 15-year horizon ranging between 1 and 5 percentage points for a wide range of possible values of the spillover parameter γ .

What we cannot answer is the question of how aggregate output and welfare respond in the long run to a permanent policy change. We cannot answer this question because we do not know how aggregate output responds in the long run to a policy-induced change in the research intensity of the economy. From Propositions 3 and 4, we have that changes in the aggregate research intensity driven by changes in three of our policies are related to aggregate output in the BGP, to a first-order-approximation, by

$$\Delta \log GDP = \left[\left(1 + \frac{1}{(1-\gamma)(\rho-1)\theta} \right) \frac{L_p}{L} \frac{1}{1+(1-\lambda)s_r} - 1 \right] \Delta \log s_r.$$

What we have shown is that the term in square brackets is highly sensitive to the strength of spillovers, γ , and the implied value of θ .

Moreover, as we have seen above, our model implies very different relationships between research intensity and aggregate output depending on the time horizon. This problem is particularly severe when the spillovers are high. Given the slowdown in the transition dynamics as the spillover parameter increases, we suspect it would be difficult in practice to use data on the response of GDP to a policy change over a 15-year horizon to infer the degree of spillovers. This is because it would be difficult in practice to tell the difference between a 15-year cumulative, detrended, GDP change in the range of those shown in Figure 2. Using cumulative changes in GDP for shorter time horizons, these differences would be even smaller. Hence, we are skeptical that one could uncover useful measures of the aggregate long-run implications of a policy-induced change in the research intensity of the economy with no more than one or two decades of data.

To conclude, our model establishes a benchmark for evaluating the theoretically predicted impacts of policy changes on firm responses observed in micro data and the relationship between those firm-level responses and the responses of macroeconomic aggregates. To do so, we have gained a great deal of tractability with a number of stark assumptions, such as a common growth rate for all continuing firms (Gibrat's law), constant markups from CES demand, constant factor shares from Cobb-Douglas production functions, symmetric spillovers from both process and product innovation, and perhaps most importantly, completely elastic product innovation. Further research is needed to assess the sensitivity of our results to empirically plausible deviations from these strong assumptions.

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Table 1: Model Calibration

Calibrated parameters		Targets
Growth rate of population, g_L	0.01	Growth rate of population = 0.01
Depreciation rate of physical capital, δ_k	0.1	Depreciation rate of physical capital = 0.10
Curvature in the innovation cost function, η_c	10	Growth rate of incumbents rises 1% for every 10% increase in τ_g
Growth rate of number of varieties, g_N	0.082	Growth rate of per capita GDP = 0.02
Discount factor, β	0.98	Difference between interest rate and growth rate = 0.01
Exit rate of incumbent firms, δ_f	0.053	Employment-weighted exit rate of U.S. establishments in 2007
Share of labor in production function of research good, λ	0.01	Trend difference, U.S. price of inputs into R&D and GDP deflator
Share of physical capital, α	1/3	Share of labor in GDP = 0.66
Elasticity of substitution across products, ρ	7.15	Research intensity (share of intangible investment in GDP), $s_r = 0.15$
Parameters in innovation cost function, c_0 and c_1	0.0147 and 0.0445	Share of dividends (payments to intangible capital) in GDP = 0.01
		Share of production employment by new establishments = 0.063
<p>Imply ratio of process-innovation costs to variable profits, $s_g = 0.54$, growth rate of z, $g_z = 0.07$, and $\xi = 1.16$</p>		
Parameters governing spillover from cumulative innovative activity		
Share, γ	0 to 0.74	Implies θ ranging between 0.76 (when $\gamma = 0$) and 0.07 (when $\gamma = 0.74$)
Depreciation rate, δ_r	0.1	Sensitivity, $\delta_r = 0.05$ and $\delta_f = 0.15$
Other parameters that do not affect results		
Population at time zero, L	1	
Level of scientific knowledge at time 0, A_r	1	
Cost of product innovation, n_e	1	
Policies		
Taxes and subsidies on initial balanced growth path (BGP): $\tau_j = 0$, $j = p,s,g,e,k$		
Taxes and subsidies on new BGP:		
Experiment 1: $\tau_g = \tau_e = 0.162$ and $\tau_j = 0$, $j = p,k,s$		Fiscal expenditures = 3% of GDP on new BGP
Experiment 2: $\tau_g = 0.221$ and $\tau_j = 0$, $j = p,e,k,s$		
Experiment 3: $\tau_k = -0.084$ and $\tau_j = 0$, $j = p,g,e,s$		

Figure 1: GDP, log change across balanced growth paths

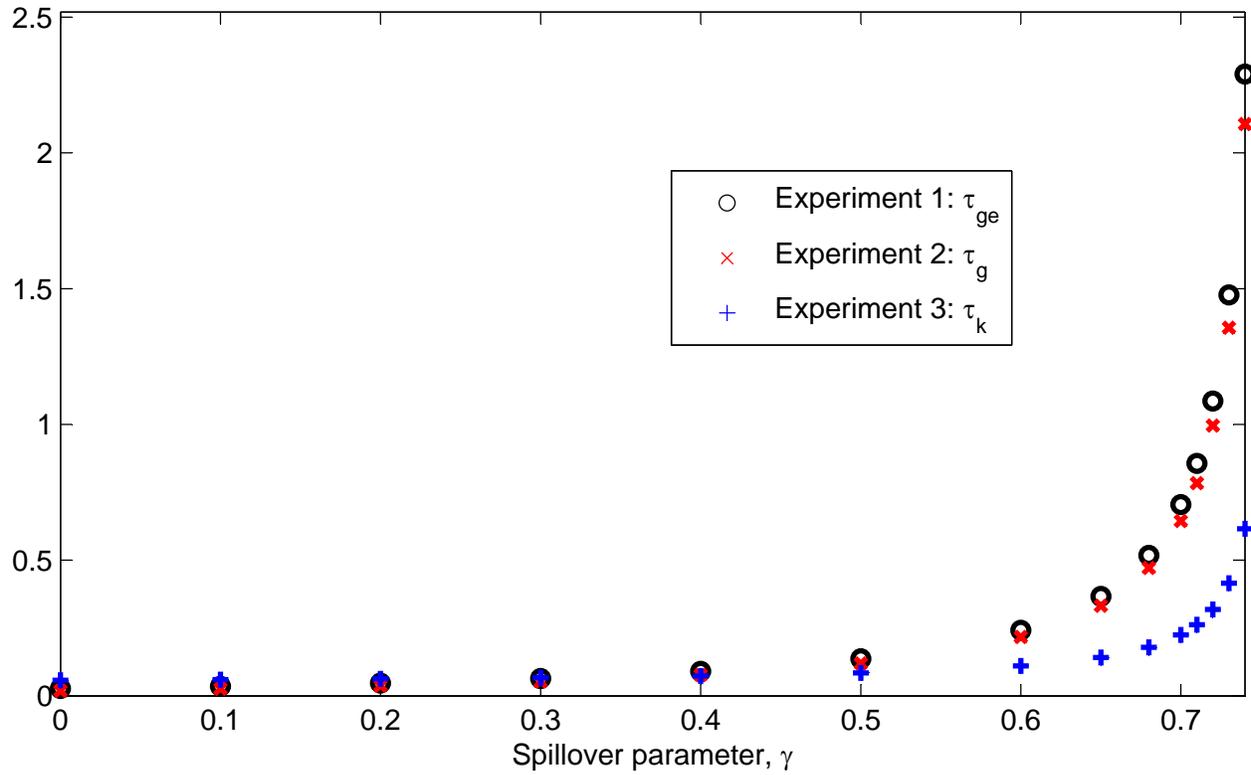


Figure 2: GDP, log change 15 years after policy change

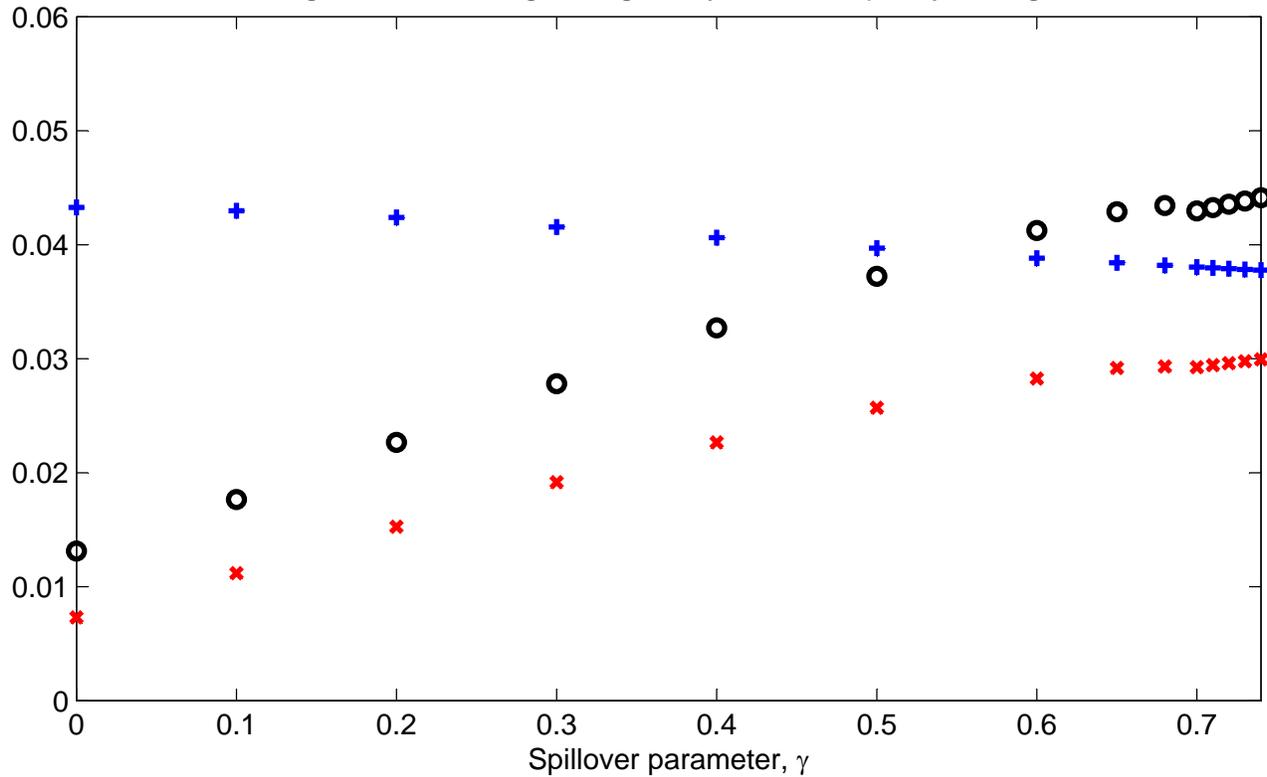


Figure 3: Research intensity of the economy, change 15 years after policy change

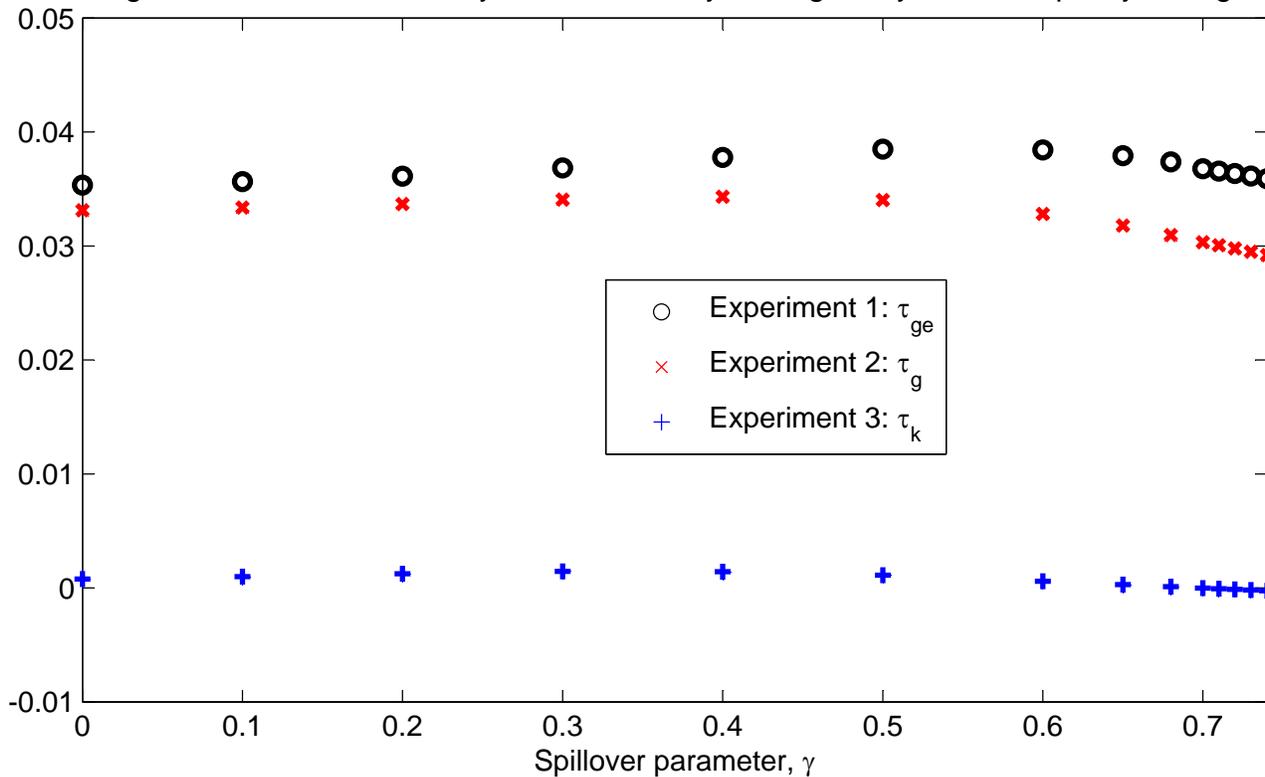
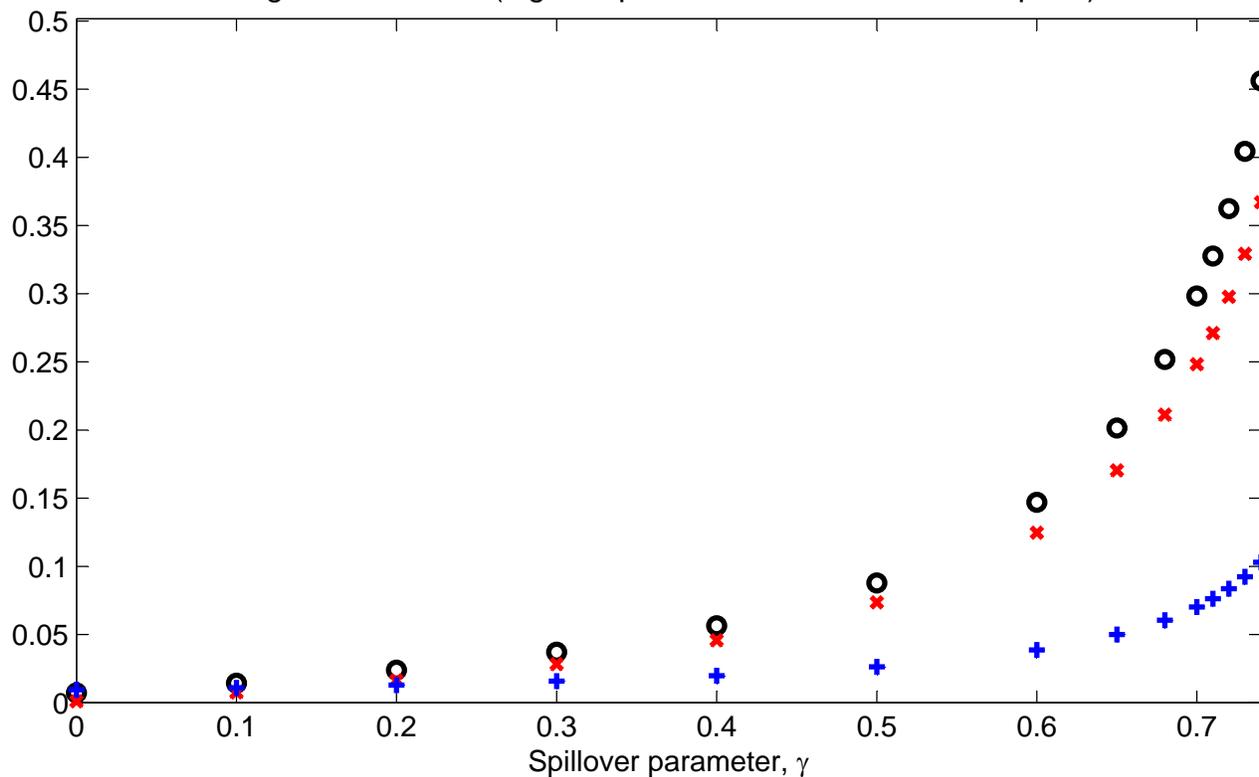


Figure 4: Welfare (log of equivalent variation in consumption)



Appendix A: Proofs

Derivation of BGP. We first provide additional details of the derivation of the BGP with semi-endogenous growth and with product innovation. We first introduce two equations that we use, which are implied from the factor shares presented above:

$$W_t L_{pt} = (1 - \alpha)(\rho - 1) \Pi_t N_t Z_t, \text{ and} \quad (\text{A1})$$

$$\frac{K_t}{L_{pt}} = \frac{\alpha}{1 - \alpha} \frac{W_t}{R_{kt}(1 + \tau_k)}. \quad (\text{A2})$$

The growth rate of the measure of incumbent firms g_N is derived from equations (1), (4), (6), (7), and (12). The growth rate of output of the consumption good (and hence consumption, physical capital, and the input into innovative activity X) is given by $g_Y = g_N / [(\rho - 1)(1 - \alpha)] + g_L$ (from equations (1) and (12)), the growth rate of output of the research good is given by $g_{Yr} = g_N$ (from equation (4)), the growth rate of the wage is $g_Y - g_L$ (from equation (A1)), and the rental rate of capital is constant and given by $R_k = \beta^{-1} \exp(g_Y - g_L) - 1 + \delta_k$ from equations $1 + \bar{r}_t = R_{kt+1} + 1 - \delta_k$ and

$$1 + \bar{r}_t = \frac{1}{\beta} \frac{C_{t+1}/L_{t+1}}{C_t/L_t}. \quad (\text{A3})$$

Finally, using (A3), (13), and $(1 + r_t) = (1 + \bar{r}_t) \frac{P_{rt}}{P_{rt+1}}$, the interest rate in terms of the research good is given by $r = \beta^{-1} \exp(g_N - g_L) - 1$.

To ensure that average productivity Z is finite in a BGP, we must have that the solution for the growth rate of incumbent firms g_z not be too large. Specifically, we require the restriction that

$$(1 - \delta_f) \exp(g_z) < \exp(g_N). \quad (\text{A4})$$

If condition (A4) is violated, the BGP does not have positive product innovation.

To ensure that a solution for Π/P_r exists, we must have $(1 - \delta_f) \exp(g_z) < 1 + r$, which is satisfied due to (A4) and our assumption that $\beta \exp(g_L) < 1$.

Equation (24) is obtained from (4), (15), and (22). Equation (25) is obtained from (6), (7), (9), (12), (13), (14), (A1), and (A2), imposing that the spillover from cumulative innovative activity, H , is equal to its level along a BGP. The constant κ_1 is given by $\kappa_1 = [(1 - \alpha) / \lambda]^{\frac{\gamma - \lambda}{1 - \gamma}} \left[\alpha^{\frac{\alpha}{1 - \alpha}} (1 - \lambda) \right]^{\frac{1 - \lambda}{1 - \gamma}} \rho^{-1} \left[\frac{\rho - 1}{\rho} \right]^{\frac{\alpha}{1 - \alpha} + \frac{\gamma - \lambda}{(1 - \gamma)(1 - \alpha)}} [\exp(g_N) + \delta_r - 1]^{\frac{-\gamma}{1 - \gamma}} (R_k)^{\alpha(\theta - 1)(\rho - 1)}$.

In equation (25), the parameter θ determines to what extent an increase in total productivity NZ crowds out firm profitability, taking into account all the general equilibrium

effects on aggregate prices and quantities. These general equilibrium effects are as follows:

1. An increase in total productivity NZ of 1% increases output of the consumption good Y by $1/[(\rho - 1)(1 - \alpha)]\%$ (taking into account the direct effect on productivity and the indirect effect through an increase in the capital stock), increasing the demand and hence the profitability of any individual firm by the same percentage.
2. An increase in the total productivity NZ of 1% increases the wage by $1/[(\rho - 1)(1 - \alpha)]\%$ and hence the marginal cost of production by $1/(\rho - 1)\%$, reducing the profitability of any individual firm measured in units of the final consumption good by 1%.
3. An increase in the total productivity NZ of 1% increases the wage by $1/[(\rho - 1)(1 - \alpha)]\%$ and increases the spillover from cumulative innovative activity on the BGP by $(1 - \lambda) / [(\rho - 1)(1 - \gamma)(1 - \alpha)]\%$. These two effects change the price of the research good by $(\lambda - \gamma) / [(\rho - 1)(1 - \gamma)(1 - \alpha)]\%$.

Adding these three effects gives a combined effect of negative $\theta\%$. When $\theta > 0$, the cumulative effect of these three general equilibrium effects is negative, meaning that an increase in the total productivity of incumbent firms crowds out the profitability of firms and hence crowds out the incentive for these firms to continue to innovate. Without an exogenous fall in the price of the research good brought about by exogenous scientific progress and/or an exogenous fall in wages brought about by an exogenous increase in population, productivity growth due to innovation by firms must cease in the long run.

BGP without Product Innovation. On a BGP without product innovation, $\exp(g_N) = 1 - \delta_f$ and $g_N + g_z$ is equal to the right-hand side of expression (21) — at these growth rates, condition (A4) is violated. The free-entry condition (11) does not apply. Instead, firm profitability adjusts so that expression (17) holds at the specified level of g_z . Once this level of profitability is obtained, aggregates are solved for by setting $M = 0$, taking N_t as determined by initial conditions, and solving for the level of Z_t and L_{pt}/L_{rt} using equations (24) and (25), for given levels of A_t and L_t . This model can also have fully endogenous growth of the form described below, if $g_A = g_L = \theta = 0$.

BGP with Endogenous Growth. We now briefly describe a BGP with fully endogenous growth. To have a BGP with endogenous growth, we need $\theta = 0$ and $g_A + g_L = 0$. In this case, we lose the recursive structure that we use in the semi-endogenous case because we lose equation (21). To solve for the BGP at a point in time, we must choose a value of N_t and solve simultaneously for the rest of the variables using the remaining equations of the

semi-endogenous growth case. Note that the level of N_t is not pinned down by the conditions for a BGP. Instead, the equilibrium level of N_t must be determined along the transition path from a given initial condition N_0 (analogously to the level of capital in an AK model).

An alternative assumption for generating endogenous growth is to have $\lambda = \gamma = 1$ and $g_A = g_L = 0$. This is the case discussed in Acemoglu (2009), chapter 13, to distinguish between semi-endogenous ($\gamma < 1$) and endogenous ($\gamma = 1$) growth. While we have assumed $\gamma < 1$, the case with $\lambda = \gamma = 1$ corresponds to the limit of cases subsumed in our analysis in which γ and λ both converge to 1 along the path that keeps $\theta = 0$.

Proof of Proposition 1. To prove this proposition, we guess and verify that the old allocation is an equilibrium allocation under the new policies. From equation (19) in an equilibrium with positive product innovation, the growth rate of incumbent firms g_z is independent of innovation policies as long as $\tau_g = \tau_e$. From equation (11), with $r_t = \tilde{r}_t$, we must have $\tilde{V}_{t+1}/(1 - \tilde{\tau}_e) = \bar{V}_{t+1}/(1 - \tau_e)$. From (10), we see that this condition holds if $\tilde{\tau}_g = \tilde{\tau}_e$ and $\frac{1+\tilde{\tau}_p}{1-\tilde{\tau}_g} = \frac{1+\tau_p}{1-\tau_g}$. Since τ_p , τ_g , and τ_e do not enter into any other equilibrium condition, our conjecture is verified. Note that the equilibrium allocations between these two policies are equal at every date, even away from the BGP. Q.E.D.

Proof of Proposition 2. The problem of the planner is

$$\max_{\{g_{zt}, L_{pt}, K_{t+1}, X_t, N_{t+1}, Z_{t+1}, H_{t+1}\}} \sum_{t=0}^{\infty} \beta^t L_t \log \left(\frac{(N_t Z_t)^{\frac{1}{\rho-1}} (K_t)^\alpha (L_{pt})^{1-\alpha} - X_t + (1 - \delta_k) K_t - K_{t+1}}{L_t} \right)$$

subject to the two following per-period constraints:

$$\mu_t \beta^t L_t : N_{t+1} Z_{t+1} = \left[(1 - \delta_f) \exp(g_{zt}) - \frac{(1 - \delta_f)}{n_e} c(\exp(g_{zt})) \right] N_t Z_t + \frac{(1 - \delta_f)}{n_e} A_t H_t^\gamma (L_t - L_{pt})^\lambda X_t^{1-\lambda}$$

$$\nu_t \beta^t L_t : H_{t+1} = (1 - \delta_r) H_t + A_t H_t^\gamma (L_t - L_{pt})^\lambda X_t^{1-\lambda}$$

with K_0 , $N_0 Z_0$, and H_0 given, and where μ_t and ν_t denote the Lagrange multipliers of each constraint.

We first simplify the problem by showing that g_{zt} is constant over time if there is positive product innovation. To see this, the first-order condition (FOC) w.r.t. g_{zt} is

$$\mu_t \exp(g_{zt}) \left[(1 - \delta_f) - \frac{(1 - \delta_f)}{n_e} c'(\exp(g_{zt})) \right] = 0.$$

If there is positive product innovation, we must have $\mu_t > 0$, so

$$n_e = c'(\exp(g_z)). \quad (\text{A5})$$

To simplify notation in this proof, we define $c = c(\exp(g_z))$.

The FOCs w.r.t. L_{pt} and X_t are, respectively,

$$(1 - \alpha) \frac{1}{C_t} (N_t Z_t)^{\frac{1}{\rho-1}} \left(\frac{K_t}{L_{pt}} \right)^\alpha = \left(\mu_t \frac{(1 - \delta_f)}{n_e} + \nu_t \right) \lambda A_t H_{rt}^\gamma (L_t - L_{pt})^{\lambda-1} X_t^{1-\lambda}, \text{ and}$$

$$\frac{1}{C_t} = \left(\mu_t \frac{(1 - \delta_f)}{n_e} + \nu_t \right) (1 - \lambda) A_t H_{rt}^\gamma (L_t - L_{pt})^\lambda X_t^{-\lambda}.$$

Taking the ratio of these two expressions, we obtain

$$X_t = \frac{(1 - \lambda)(1 - \alpha)}{\lambda} (L_t - L_{pt}) (N_t Z_t)^{\frac{1}{\rho-1}} \left(\frac{K_t}{L_{pt}} \right)^\alpha. \quad (\text{A6})$$

Plugging the solution for X_t into the previous FOC w.r.t. L_{pt} , we rewrite this constraint as

$$(1 - \alpha)^\lambda \frac{1}{C_t} (N_t Z_t)^{\frac{\lambda}{\rho-1}} \left(\frac{K_t}{L_{pt}} \right)^{\alpha\lambda} = \lambda^\lambda (1 - \lambda)^{1-\lambda} \left(\mu_t \frac{(1 - \delta_f)}{n_e} + \nu_t \right) A_t H_{rt}^\gamma. \quad (\text{A7})$$

The FOCs w.r.t. $N_t Z_t$, H_t , and K_t are, respectively,

$$\beta \exp(g_L) \frac{1}{\rho - 1} \frac{(N_{t+1} Z_{t+1})^{\frac{2-\rho}{\rho-1}} \left(\frac{K_{t+1}}{L_{pt+1}} \right)^\alpha L_{pt+1}}{C_{t+1}} - \mu_t + \mu_{t+1} \beta \exp(g_L) (1 - \delta_f) \left(\exp(g) - \frac{c}{n_e} \right) = 0 \quad (\text{A8})$$

$$0 = -\nu_t + \nu_{t+1} \beta \exp(g_L) (1 - \delta_r) + \gamma \beta \exp(g_L) \left(\frac{(1 - \lambda)(1 - \alpha)}{\lambda} \right)^{1-\lambda} \times \quad (\text{A9})$$

$$\left(\mu_{t+1} \frac{(1 - \delta_f)}{n_e} + \nu_{t+1} \right) A_{t+1} H_{t+1}^{\gamma-1} (L_{t+1} - L_{pt+1}) (N_{t+1} Z_{t+1})^{\frac{1-\lambda}{\rho-1}} \left(\frac{K_{t+1}}{L_{pt+1}} \right)^{\alpha(1-\lambda)}$$

$$\frac{1}{C_t} = \frac{\beta \exp(g_L)}{C_{t+1}} \left(\alpha (N_{t+1} Z_{t+1})^{\frac{1}{\rho-1}} \left(\frac{K_{t+1}}{L_{pt+1}} \right)^{\alpha-1} + 1 - \delta_k \right). \quad (\text{A10})$$

We can further simplify (A9) using (A7):

$$-\nu_t + \nu_{t+1} \beta \exp(g_L) (1 - \delta_r) + \beta \exp(g_L) \frac{\gamma(1 - \alpha)(L_{t+1} - L_{pt+1})}{\lambda H_{t+1} C_{t+1}} (N_{t+1} Z_{t+1})^{\frac{1}{\rho-1}} \left(\frac{K_{t+1}}{L_{pt+1}} \right)^\alpha = 0. \quad (\text{A11})$$

We now impose that we are in a BGP with $(N_{t+1} Z_{t+1}) / (N_t Z_t) = H_{t+1} / H_t = \exp(g_N)$. If $\theta > 0$, the growth rate g_N is equal to its level in the equilibrium, given by expression (21) (we discuss the case when $\theta = 0$ below). In the BGP, from (A7), μ_t and ν_t must each grow at the same rate $\exp(g_\mu)$ given by

$$\exp(g_\mu) = \exp(g_N)^{\frac{\lambda-1}{\rho-1}-\gamma} \exp(-g_A - g_L) \exp(g_{K/L})^{\alpha(\lambda-1)} = \exp(-g_N),$$

where we used $g_{K/L} = \frac{g_N}{(\rho-1)(1-\alpha)}$ and (21). From (A10), in the BGP,

$$\frac{K_t}{L_{pt}} = \left[\frac{\alpha}{R_k} (N_t Z_t)^{\frac{1}{\rho-1}} \right]^{\frac{1}{1-\alpha}}, \quad (\text{A12})$$

where $R_k = \beta^{-1} \exp(g_Y - g_L) - 1 + \delta_k$ as in the equilibrium. Combining (A8), (A11), (A12), and the equation (from the law of motions of NZ and H , 7 and 18, respectively)

$$\frac{H_t}{N_t Z_t} = \frac{n_e}{1 - \delta_f} \frac{\exp(g_N) - (1 - \delta_f) \left(\exp(g_z) - \frac{c}{n_e} \right)}{\exp(g_N) - 1 + \delta_r}, \quad (\text{A13})$$

we obtain

$$\frac{L_{pt}}{L_{rt}} = \frac{\mu_t (1 - \delta_f) (\rho - 1) \gamma (1 - \alpha)}{\nu_t n_e \lambda} \times \frac{\left[\exp(g_N) - \beta \exp(g_L) (1 - \delta_f) \left(\exp(g) - \frac{c}{n_e} \right) \right]}{\left[\exp(g_N) - (1 - \delta_f) \left(\exp(g_z) - \frac{c}{n_e} \right) \right]} \frac{[\exp(g_N) - (1 - \delta_r)]}{[\exp(g_N) - \beta \exp(g_L) (1 - \delta_r)]}. \quad (\text{A14})$$

In order to solve for L_{pt}/L_{rt} , we must solve for μ_t/ν_t . Using (A7), (A8), and the law of motion for NZ , we obtain

$$\frac{L_{pt}}{L_{rt}} \frac{\exp(g_L) \beta \lambda n_e}{(1 - \alpha) (\rho - 1) (1 - \delta_f)} \left(\frac{(1 - \delta_f)}{n_e} + \frac{\nu_t}{\mu_t} \right) = \frac{\left[\exp(g_N) - \beta \exp(g_L) (1 - \delta_f) \left(\exp(g) - \frac{c}{n_e} \right) \right]}{\left[\exp(g_N) - (1 - \delta_f) \left(\exp(g) - \frac{c}{n_e} \right) \right]}. \quad (\text{A15})$$

Combining (A14) and (A15), we obtain

$$\frac{\mu_t}{\nu_t} = \frac{n_e}{(1 - \delta_f) \gamma \beta \exp(g_L)} \frac{\exp(g_N) (1 - \gamma \beta \exp(g_L)) - \beta \exp(g_L) (1 - \delta_r) (1 - \gamma)}{\exp(g_N) - 1 + \delta_r}$$

and

$$\begin{aligned} \frac{L_{pt}}{L_{rt}} &= \frac{(\rho - 1) (1 - \alpha)}{\beta \exp(g_L) \lambda} \left[\frac{\exp(g_N) - \beta \exp(g_L) (1 - \delta_f) \left(\exp(g_z) - \frac{c}{n_e} \right)}{\exp(g_N) - (1 - \delta_f) \left(\exp(g_z) - \frac{c}{n_e} \right)} \right] \\ &\times \left[\frac{\exp(g_N) (1 - \gamma \beta \exp(g_L)) - \beta \exp(g_L) (1 - \delta_r) (1 - \gamma)}{[\exp(g_N) - \beta \exp(g_L) (1 - \delta_r)]} \right]. \end{aligned} \quad (\text{A16})$$

We now derive the equilibrium level of L_{pt}/L_{rt} assuming that $\tau_g = \tau_e$. Using (24), (29), and two equations used in the proof of Proposition 6,

$$(1 - \tau_g) P_r n_e M \xi = (1 + \tau_p) \Pi N Z - (1 - \tau_g) P_r c (\exp(g_z)) N Z,$$

and $M/(NZ) = [\exp(g_N) - (1 - \delta_f) \exp(g_z)] / (1 - \delta_f)$, we obtain

$$\frac{L_{pt}}{L_{rt}} = \frac{(\rho - 1) (1 - \alpha)}{\beta \exp(g_L) \lambda} \left(\frac{1 - \tau_g}{1 + \tau_p} \right) \left[\frac{\exp(g_N) - \beta \exp(g_L) (1 - \delta_f) \left(\exp(g_z) - \frac{c}{n_e} \right)}{\exp(g_N) - (1 - \delta_f) \left(\exp(g_z) - \frac{c}{n_e} \right)} \right]. \quad (\text{A17})$$

We are now ready to calculate the policies such that the equilibrium allocations coincide with the optimal allocations in a BGP. To set equation (19) equal to (A5), we need $\tau_g^* = \tau_p^*$. To set equation (A16) equal to (A17), we need

$$\frac{1 - \tau_g^*}{1 + \tau_p^*} = 1 - \gamma \frac{\beta \exp(g_L) [\exp(g_N) - (1 - \delta_r)]}{[\exp(g_N) - \beta \exp(g_L) (1 - \delta_r)]}.$$

To set equation (A6) equal to (14), using the equilibrium wage expression,

$$W_t = \frac{(1 - \alpha) (\rho - 1)}{\rho} (1 + \tau_s) (N_t Z_t)^{\frac{1}{\rho-1}} (K_t / L_{pt})^a,$$

we need $(1 + \tau_s^*) = \rho / (\rho - 1)$. Finally, to set equation (A2) equal to (A12), we set $\tau_k^* = 0$.

Note that in the specification of our model with $\theta = 0$, we cannot use (21) to solve for g_N . In this case, for any given policies, g_N solves equations (11) and (23). The expressions

for optimal policies are the same as those under $\theta > 0$, but g_N needs to be endogenously determined.

Proof of Proposition 3. Result 1 is based on equations (11), (19), and (22). Result 2 is based on equation (23), together with the fact that aggregate growth rates and the interest rate in the BGP are invariant to policy changes. Result 3 is based on equation (24), together with the fact that g_z is unchanged. To calculate the log changes in L_p and L_r individually, we use a first-order approximation of the labor market clearing condition, $\frac{L_p}{L} \Delta \log(L_p/L) + \frac{L_r}{L} \Delta \log(L_r/L) = 0$, and the log change in L_p/L_r , to obtain $\Delta \log(L_p/L) = -(L_r/L) \Delta \log(1 + \tau_p)$ and $\Delta \log(L_r/L) = (L_p/L) \Delta \log(1 + \tau_p)$. To calculate the log change in NZ in result 4, we use equation (25). To calculate the log change in GDP in result 5, we use two intermediate calculations. First, $\Delta \log Y = \frac{1}{(\rho-1)(1-\alpha)} \Delta \log NZ + \Delta \log L_p$, where we are using equations (A1), (A2), and the fact that R_k is constant to calculate the change in the physical capital stock, $\Delta \log K_t = \Delta \log Y_t$. Second, to a first-order approximation, $\Delta \log \frac{Y-X}{Y} = -\frac{X}{Y-X} \Delta \log \left(\frac{L_r}{L_p} \right)$, where we are using that X is proportional to WL_r and that Y is proportional to WL_p . Combining these two calculations, $\Delta \log GDP = \Delta \log Y + \Delta \log \frac{Y-X}{Y}$, we obtain result 5. To calculate the change in s_r in result 6, we use (16). Q.E.D.

Proof of Proposition 4. Result 1 is immediate from (11). In result 2, the change in g_z and Z is obtained by using a first-order approximation of equation (19) and equation (22), respectively. In result 3, the change in Π/P_r is obtained by using a first-order approximation of (23),

$$\begin{aligned} \Delta \log(1 - \tau_e) &= \frac{\Delta \log \Pi/P_r}{1 - s_g} - \frac{s_g}{1 - s_g} \Delta \log(1 - \tau_g) \\ &\quad - \left[\frac{(1 - \tau_g) c'(\exp(g_z))}{(1 + \tau_p) \frac{\Pi}{P_r} - (1 - \tau_g) c(\exp(g_z))} - \frac{(1 - \delta_f)}{1 + r - (1 - \delta_f) \exp(g_z)} \right] \exp(g_z) \Delta g_z, \end{aligned}$$

and noting that the term in square brackets is equal to zero from the first-order condition (19) for g_z , and equations (11) and (23). To obtain the change in L_p/L_r , we use a first-order approximation of equation (24),

$$\Delta \log \frac{L_p}{L_r} = \Delta \log(\Pi/P_r) - \frac{c'(\exp(g_z)) - n_e}{n_e \frac{\exp(g_N) - (1 - \delta_f) \exp(g_z)}{1 - \delta_f} + c(\exp(g_z))} \exp(g_z) \Delta g_z,$$

and note that the second term is zero from equation (19) and the fact that initial policies are

set so that $\tau_g = \tau_e$ (note that this is the only step in which we use the assumption $\tau_g = \tau_e$). The remaining variables are calculated using the same steps as in the proof of Proposition 3. Note that if $\Delta \log(1 - \tau_g) = \Delta \log(1 - \tau_e)$, Proposition 4 is equivalent to Corollary 1. Q.E.D.

Corollary to Proposition 4: If our model has a BGP with semi-endogenous growth and positive product innovation, and policies are initially set so that $\tau_g = \tau_e$, then a change in the subsidy to process innovation of size $\Delta \log(1 - \tau_g)$ and/or a change in the subsidy to product innovation of size $\Delta \log(1 - \tau_e)$ results in the same changes in the constant in firm profits, Π/P_r , the aggregate allocation of labor between production and research, L_p/L_r , the index of total productivity, NZ , aggregate output, GDP , and the research intensity of the economy, s_r , to a first-order approximation, as a change in the subsidy to variable profits of size $\Delta \log(1 + \tau_p) = -[s_g \Delta \log(1 - \tau_g) + (1 - s_g) \Delta \log(1 - \tau_e)]$.

Proof: This follows immediately from Propositions 2 and 4. Q.E.D.

Proof of Proposition 5. Result 1 is immediate from (11). In result 2, the change in g_z and Z is obtained by using a first-order approximation of equation (19) and equation (22), respectively. In result 3, the change in Π/P_r is obtained by using a first-order approximation of (23),

$$\begin{aligned} \Delta \log(1 - \tau_e) &= \frac{\Delta \log \Pi/P_r}{1 - s_g} - \frac{s_g}{1 - s_g} \Delta \log(1 - \tau_g) \\ &\quad - \left[\frac{(1 - \tau_g) c'(\exp(g_z))}{(1 + \tau_p) \frac{\Pi}{P_r} - (1 - \tau_g) c(\exp(g_z))} - \frac{(1 - \delta_f)}{1 + r - (1 - \delta_f) \exp(g_z)} \right] \exp(g_z) \Delta g_z, \end{aligned}$$

and noting that the term in square brackets is equal to zero from the first-order condition (19) for g_z , and equations (11) and (23). To obtain the change in L_p/L_r , we use a first-order approximation of equation (24),

$$\Delta \log \frac{L_p}{L_r} = \Delta \log(\Pi/P_r) - \frac{c'(\exp(g_z)) - n_e}{n_e \frac{\exp(g_N) - (1 - \delta_f) \exp(g_z)}{1 - \delta_f} + c(\exp(g_z))} \exp(g_z) \Delta g_z,$$

and note that the second term is zero from equation (19) and the fact that initial policies are set so that $\tau_g = \tau_e$ (note that this is the only step in which we use the assumption $\tau_g = \tau_e$). The remaining variables are calculated using the same steps as in the proof of Proposition 3. Q.E.D.

The change in Π/P_r in equation (31) follows immediately from Propositions 3, 4, and

5, and the corollary to Proposition 4. The change in E , starting at $\tau_p = \tau_g = \tau_e$, is

$$\begin{aligned}\Delta E &= \Pi NZ \Delta \tau_p + P_r c(\exp(g)) NZ \Delta \tau_g + P_r n_e M \Delta \tau_e \\ &= \Pi NZ \left[\Delta \tau_p + \frac{P_r c(\exp(g)) NZ}{\Pi NZ} \Delta \tau_g + \frac{P_r n_e M}{\Pi NZ} \Delta \tau_e \right].\end{aligned}$$

Equation (32) is obtained from the definition of s_g and expression (28) with $\tau_p = \tau_g = \tau_e$. Q.E.D.

Proof of Proposition 6. Result 1 is derived in the same way as in the proof of Proposition 3. Result 2 that L_p/L_r and s_r are unchanged is obtained from (24) combined with the fact that Π/P_r is constant. The change in GDP in result 3 is derived by first calculating $\Delta \log Y$ using the log-linear approximation of equations (12) and (25) together with the expression $\Delta \log K = \Delta \log Y - \Delta \log(1 + \tau_k)$ from expressions (A1) and (A2). We then have $\Delta \log GDP = \Delta \log Y + \log\left(\frac{Y-X}{Y}\right)$, where $\Delta \log \frac{Y-X}{Y} = 0$ because L_p/L_r is constant. Q.E.D.

Aggregate Effects of Changes in Policies with Endogenous Growth. As we have discussed, with $\theta = 0$, our model has a BGP with fully endogenous growth. Our simple two-step algorithm for computing the impact of a change in policy no longer applies because the BGP aggregate growth rates change with policies. One can show, however, that the change in the BGP growth rate g_N induced by a change in subsidies to process or product innovation individually does not depend, to a first-order approximation, on the responsiveness of firms' investments in process innovation as determined by η_c . Moreover, to a first-order approximation, changes in the subsidy to process or product innovation individually have the same impact on the BGP growth rates as a change in the subsidy to variable profits as long as these policy changes have the same impact on firm profitability, holding fixed firms' process innovation decisions.

Aggregate Implications of Eliminating an Innovation Policy. Consider an economy in which all of the existing policies taken together imply innovation policies τ_p , τ_g , and τ_e (with $\tau_g = \tau_e$). Consider eliminating a specific real-world policy i that is a combination of three abstract innovation policies τ_p^i , τ_g^i , and τ_e^i but that does not involve τ_k or τ_s (note that we are not assuming that τ_k and τ_s are equal to zero, but that they are not part of the real-world policy under consideration). The elimination of this policy i results in new innovation policies $\tau_p - \tau_p^i$, $\tau_g - \tau_g^i$, and $\tau_e - \tau_e^i$.

The current expenditure on this real-world policy i is

$$E^i = \tau_p^i \Pi NZ + P_r \tau_g^i c(\exp(g_z)) NZ + P_r \tau_e^i n_e M.$$

Using an approximation that the change in τ arising from eliminating real-world policy i is $\Delta \log(1 + \tau) = \frac{\tau^i - 0}{1 + \tau}$ for small τ^i , the expenditure on this real-world policy is approximately

$$E^i = (1 + \tau_p) \Pi NZ \left[\Delta \log(1 + \tau_p) + s_g \Delta \log(1 - \tau_g) + \frac{(1 - s_g)}{\xi} \Delta \log(1 - \tau_e) \right].$$

Recall that, according to our corollary above, if initially $\tau_g = \tau_e$, then the change in aggregate variables to changes in innovation policies is, to a first-order approximation, proportional to

$$\Delta \log \frac{\Pi}{P_r} = \Delta \log(1 + \tau_p) + s_g \Delta \log(1 - \tau_g) + (1 - s_g) \Delta \log(1 - \tau_e).$$

Hence, the aggregate impact of eliminating a real-world innovation policy is directly proportional (with the adjustment for ξ) to the current expenditure on that policy.

Appendix B: Sensitivity Analysis of Quantitative Results

We first consider the sensitivity of our results with respect to the parameter λ , indexing the share of research labor in the production of research goods. As discussed above, one common assumption for generating endogenous growth is to set $\lambda = 1$ and $\gamma = 1$. Here we examine the response of GDP in the long run and in transition in our model calibrated with $\lambda = 1$ and a range of values of γ from 0 to 0.95. We recalibrate the other parameters to match our other targets. Note that when $\lambda = 1$, the parameter $\theta = 1$ for all values of $\gamma < 1$. We find again in this case that the long-run response of GDP varies tremendously with γ , with a log change in GDP of essentially 0 when $\gamma = 0$ to a log change in GDP of 0.76 when $\gamma = 0.9$. The welfare gains also vary considerably with γ , becoming quite large as γ gets close to 1. Once again, however, the response of GDP over a 15-year horizon does not differ much with γ , ranging from essentially 0 when $\gamma = 0$ to 0.03 log points when $\gamma = 0.95$.

We next consider the sensitivity of our results to the parameter ρ governing the elasticity of substitution across products. In our calibration, $\rho = 7.15$. This relatively high value of ρ corresponds to our target for the research intensity of the economy of roughly $s_r = 0.15$. We now consider a value of $\rho = 4$, which, after recalibrating our model to match the other targets, results in a research intensity of the economy of $s_r = 0.32$. We choose values of γ ranging from 0 (so that $\theta = 0.51$) to 0.45 (so that $\theta = 0.1$). In the long run, the response of

log GDP ranges from 0.057 when $\gamma = 0$ to 0.77 when $\gamma = 0.45$. The response of log GDP over a 15-year horizon varies substantially less, from 0.034 when $\gamma = 0$ to 0.066 log points when $\gamma = 0.45$.

Finally, we consider the sensitivity of our results to the parameter δ_r governing the speed with which the spillover from accumulated innovation experience decays. Note first that the long-run changes in aggregates do not depend on this parameter. In contrast, this parameter does affect the model's transition dynamics in the standard manner: the higher is the depreciation parameter δ_r , the faster is the transition to the new BGP (and hence, the larger is the change in GDP in the first 15 years of the transition). This effect on the speed of transition is larger, the higher is γ . We find, however, that this effect is not very large quantitatively. For example, when $\gamma = 0.7$ (so that $\theta = 0.195$), increasing δ_r from our baseline value of 0.1 to 0.5 (so that half of accumulated experience with innovation is lost every year) raises the cumulative change in log GDP in the first 15 years from 0.043 to 0.082, which is still a small portion of the long-run change in log GDP of 0.705.

We conclude that, while the parameters λ , ρ , and δ_r affect the absolute magnitude of the response of GDP, the finding that the response of GDP to a 15-year horizon is not particularly sensitive to the parameter γ is robust to our choices of these parameters.

Appendix C: Quality Ladders Model. In this appendix, we describe a discrete time quality ladders model with Schumpeterian innovation by both entering and incumbent firms similar to the model of firm dynamics in Klette and Kortum (2004). In this model, there is a fixed set of products that can be improved through innovation by firms. Firms differ by the number of products that they currently produce. This state variable indexes both the size of an incumbent firm and that firm's capacity for innovation. Entering and incumbent firms compete to capture products produced by other firms by overtaking the incumbent producer of these products through innovation. Firms' innovative effort is not directed to particular products but instead is matched randomly to products through a matching function that captures possible congestion externalities in innovative activity. In contrast to our baseline model, the spillover from innovative activity comes not from cumulative experience from innovation, but instead from cumulative success with innovation as measured by the average quality or productivity of the incumbent firms' fixed set of products. In contrast with Klette and Kortum, who set parameters at the knife-edge condition required for endogenous growth, we allow for both fully and semi-endogenous growth. In this appendix, we show that our main theoretical results apply equally well to this model.

As before, there are two final goods, a consumption and a research good, and a con-

tinuum of mass one of products (intermediate goods) so $N_t = 1$. Associated with each intermediate good is a frontier technology for producing that good indexed by z , and the production function is given by (3). As before, we let $J_t(z)$ denote the distribution of these frontier technologies across products. Production of the final good is given by (2), which can be expressed in the same manner as in our baseline model by (12), where Z_t is defined as before as the average productivity of the frontier technologies. Since the number of products is fixed, Z_t also denotes total productivity.

The production function of the research good is given by

$$Y_{rt} = Z_t^{\gamma-1} A_t L_{rt}^\lambda X_t^{1-\lambda},$$

with $\gamma < 1$. With this specification, the resource cost of innovating on the frontier technology for producing a good rises one for one Z_t when $\gamma = 0$ and in the limit as γ approaches 1, becomes independent of Z_t . Hence, we interpret γ as indexing the extent to which previous innovation embodied in Z spills over to reduce the cost of further innovation. This spillover will be external to any particular firm because it depends on the average frontier technology across all products. Standard specifications of quality ladders models with fully endogenous growth correspond to the case with 100% spillovers ($\gamma = 1$) and $\lambda = 1$. We show below that, just as in our baseline model, this is one set of parameter assumptions that generates endogenous growth. An alternative assumption with incomplete spillovers ($\gamma < 1$) and $\theta = 0$ also generates endogenous growth.

The market structure is as follows. For each product, the frontier technology is owned by an incumbent firm, with exclusive rights to use that technology in production. That firm sells the product at a constant markup over marginal cost. As is standard in quality ladder models, this markup is the minimum of the monopoly markup, $\rho/(\rho - 1)$, and the gap between the marginal cost of production using the frontier technology and the marginal cost associated with previous frontier technology for that product, which we denote by $\Delta > 1$. Below, we assume that the innovation step size Δ is constant, and hence markups are constant at $\mu = \min \left\{ \frac{\rho}{\rho-1}, \Delta \right\}$. Hence, variable profits associated with producing any given product with frontier technology z are given by $(1 + \tau_p) \Pi \exp(z)$, where Π is given by (9) with constant κ_0 adjusted suitably for the markup.²⁶ Calculations of income shares gives that $\Pi_t N_t Z_t = \frac{\mu-1}{\mu} (1 + \tau_s) Y_t$, $W_t L_{pt} = \frac{(1-\alpha)}{\mu} (1 + \tau_s) Y_t$, and hence $W_t L_{pt} = \frac{(1-\alpha)}{(\mu-1)} \Pi_t N_t Z_t$.

²⁶In particular, $\kappa_0 = \mu^{-\rho}(\mu - 1) \left[\alpha^\alpha (1 - \alpha)^{1-\alpha} \right]^{\rho-1}$.

Incumbent firms are indexed by the products for which they own the frontier technologies. Hence, the state variable that characterizes an incumbent firm is a vector of productivities z of length n , where n denotes the number of products owned by this firm. Let $G_t(n)$ denote the measure of incumbent firms with n products at time t . The requirement that each product be owned by some firm implies $\sum_{n=1}^{\infty} nG_t(n) = 1$ for all periods t .

Through the innovation of other firms, an incumbent firm faces a probability δ_{ft} that it loses the frontier technology of any of its n products. This probability is independent of z and independent across products. Then, for a firm with n products, the probability that it loses k of its n products is given by a binomial distribution with

$$Prob_t(-k|n) = \frac{n!}{(n-k)!k!} \delta_{ft}^k (1 - \delta_{ft})^{n-k}.$$

Note that the expectation of this distribution is $n\delta_{ft}$. Adding up across firms gives that the measure of products that switch ownership between any two consecutive periods is given by $\delta_{ft} \sum_{n=1}^{\infty} nG_t(n) = \delta_{ft}$. For standard reasons, in a quality ladders model the owner of a product does not innovate on that product. Hence, the discounted present value of profits of owning a product with productivity index z is described by the Bellman equation

$$V_t \exp(z) = (1 + \tau_p) \Pi_t \exp(z) + \frac{(1 - \delta_{ft})}{1 + \bar{r}_t} V_{t+1} \exp(z).$$

Note that, in contrast to our baseline model, z does not change over time.

Both incumbent firms and entering firms innovate to improve upon the technology frontier of a product. A firm that succeeds in innovating on a product raises the frontier from z to $z + \Delta$ and becomes the sole producer of the product. We assume that these firms use the research good to engage in research effort and that matching of research effort to innovation on specific products is done through random search with a matching function. We will show below that it is optimal for every firm to engage in the same research effort per product it owns, independent of the level of z associated with those products. We impose this result here to simplify the notation. Let each firm of size n invest a total of nq_t units of research effort into trying to obtain new products, and let each entering firm invest a total of one unit of research effort into trying to obtain new products. Let M_t be the measure of entrants. The total research effort is

$$q_t \sum_{n=1}^{\infty} nG_t(n) + M_t = q_t + M_t.$$

Given this research effort, firms are matched at random to successful innovations through a matching function. The total measure of products innovated on is

$$\delta_{ft} = m(1, q_t + M_t),$$

where the first argument denotes the measure of products available to be innovated upon and the second argument denotes the total research effort. The function m is constant returns to scale and increasing and concave in each argument.

These innovations are divided up at random among incumbent and entrant firms. An entrant firm that engages in one unit of research effort obtains a product with probability

$$\eta_t = \frac{1}{q_t + M_t} \delta_{ft}.$$

A firm of size n that invested nq_t obtains products at a Poisson rate of $\phi_t n$, with

$$\phi_t = \frac{q_t}{q_t + M_t} \delta_{ft}.$$

That is, the probability that the firm of size n adds k products is given by

$$Prob_t(k|n) = (\phi_t n)^k \frac{e^{-\phi_t n}}{k!}.$$

Note that the expectation of this distribution is $\phi_t n$. Hence, the total measure of products gained by incumbents of size n is $\phi_t n G_t(n)$. Hence, summing over n gives that the total measure of products gained by incumbents is ϕ_t . Note that, by construction, the total measure of products innovated upon by incumbents plus those innovated upon by entrants is equal to δ_{ft} :

$$\phi_t + \eta_t M_t = \delta_{ft}.$$

These assumptions give us the law of motion for average productivity Z_t :

$$\exp(g_{Zt}) = \frac{Z_{t+1}}{Z_t} = \delta_{ft} \Delta + (1 - \delta_{ft}). \quad (\text{A18})$$

From here on, we will not track the size distribution of firms, $G_t(n)$. One can do so recognizing that firms gain products according to a Poisson distribution and lose them according to a binomial distribution.

Incumbent and entering firms must use the research good to exert research effort. As in Klette and Kortum (2004), the cost to incumbent firms of engaging in research effort falls with the number of products that they own. In particular, a firm with n products must spend $\tilde{c}(qn, n)$ units of the research good to engage in qn units of research effort. We assume that $\tilde{c}(\dots)$ is constant returns to scale, so we can rewrite this total cost as $nc(q)$, where $c(\cdot)$ is increasing and convex. Entering firms must spend n_e units of the research good to engage in one unit of research effort. Summing across firms, using the fact that $\sum_{n=1} nG_t(n) = 1$, gives the resource constraint for the research good,

$$c(q_t) + n_e M_t = Y_{rt}.$$

Firm value in this model is very complicated because it depends on the vector of levels of z for the n products owned by the firm. All that matters for the choice of innovation by an incumbent firm, however, is the expected value of the profits associated with products that it may win. Since the realizations of the number of products it will win and lose each period are all independent, the computation of the expected returns to research effort is relatively simple.

As we have discussed above, the expected discounted present value of variable profits associated with a randomly chosen innovation gained at time $t + 1$ is $V_{t+1}Z_t\Delta$. The firm also gains value from increasing its number of products because that innovative success lowers the cost of further innovation. We denote by $U_t(n)$ the expected present value of profits associated with innovative capacity indexed by n . One can show that this value function can be written as $U_t n$, where U_t is determined by the Bellman equation

$$\begin{aligned} U_t = & \max_{\tilde{q}} - (1 - \tau_g) c(\tilde{q}) P_{rt} + \frac{1}{1 + \bar{r}_t} \frac{\tilde{q}}{q_t + M_t} \delta_{ft} V_{t+1} Z_t \Delta + \\ & + \frac{1}{1 + \bar{r}_t} \left(1 + \frac{\tilde{q}}{q_t + M_t} \delta_{ft} - \delta_{ft} \right) U_{t+1}. \end{aligned}$$

The first term on the right side indicates the cost if exerting q units of research effort per product. The second term indicates the discounted present value of variable profits the firm expects to gain from the innovations that result from this research effort. The third term denotes the expected value of the firm's innovative capacity from next period on, taking into account both the gain in products it expects to obtain from its research effort (i.e., a firm with n products expects to gain $\phi n = \frac{\tilde{q}}{q+M} \delta_f n$ products) and the loss of products it expects as a result of research effort from other firms (i.e., a firm with n products expects to lose $\delta_f n$

products). Note that each firm takes as given the innovative effort of other firms, as given by q_t , M_t , and δ_{ft} .

The first-order condition for \tilde{q}_t is given by

$$(1 - \tau_g) P_{rt} c'(\tilde{q}_t) = \left(\frac{1}{1 + \bar{r}_t} \frac{\delta_{ft}}{q_t + M_t} \right) (V_{t+1} Z_t \Delta + U_{t+1}).$$

If the equilibrium has entering firms, then the zero-profit condition for entry implies

$$(1 - \tau_e) P_{rt} n_e = \left(\frac{1}{1 + \bar{r}_t} \frac{\delta_{ft}}{q_t + M_t} \right) (V_{t+1} Z_t \Delta + U_{t+1}).$$

Hence, just as in our baseline model, we have that in an equilibrium with entering firms,

$$(1 - \tau_g) c'(\tilde{q}_t) = (1 - \tau_e) n_e. \quad (\text{A19})$$

Characterizing a BGP with positive entry

On a BGP, the variables q , M , δ_f , ϕ , η , Y_r (the latter, following from the resource constraint for the research good), $\Pi Z/P_r$, VZ/P_r , U_t/P_r , and \bar{r} are all constant over time. The production functions for the final consumption good and the research good together imply that the growth rate of average productivity is given by $g_Z = \frac{1}{\theta(1-\gamma)}(g_A + g_L)$, where θ is defined in the same manner as in the baseline model. Output per capita therefore grows at the same rate as in our baseline model (but it is driven by growth in average productivity rather than by growth in the number of products). The same parameter value assumptions are required to differentiate between semi-endogenous and fully endogenous growth. The rate at which innovations occur, δ_f , is pinned down from (A18). The matching function then pins down $q + M$. Equation (A19) determines q and hence also M . Note that, in the BGP, policies affect the division of innovative effort between incumbents and entering firms, but not the total research effort. In contrast to our baseline model, total and average productivity here are both equal to Z , so there can be no offset between average productivity and entry. Instead, the offset occurs between the innovative effort of incumbent firms, q , and that of entrants.

As in our baseline model, we can use the firm's Bellman equations to obtain the BGP level of profitability $\Pi Z/P_r$. Manipulating the Bellman equations gives

$$(1 - \tau_e) n_e = \frac{\eta}{r + \delta_f - \phi} \left[\xi_g (1 + \tau_p) \Pi Z/P_r - (1 - \tau_g) c(q) \right], \quad (\text{A20})$$

where $1 + r = (1 + \bar{r}) \frac{P_{rt}}{P_{rt+1}} = \beta^{-1} \exp(-g_L)$, and

$$\xi_g = \frac{r + \delta_f}{r \exp(g_Z) / \Delta + \delta_f} > 1.$$

Finally, we use the macroeconomic structure of the model to determine the BGP levels of Z_t and L_{pt}/L_{rt} for a given level of $\Pi Z/P_r$. To do so, we use the following two equations:

$$\frac{\Pi Z}{P_r} = \kappa_1 (1 + \tau_s)^{\frac{1-\lambda}{1-\alpha}} (1 + \tau_k)^{\alpha(\theta-1)(\rho-1)(1-\gamma)} A_t Z_t^{-(1-\gamma)\theta} L_{pt} \quad (\text{A21})$$

$$\frac{L_p}{L_r} = \frac{(1 - \alpha) \Pi Z / P_r}{(\mu - 1) \lambda [c(q) + n_e M]}, \quad (\text{A22})$$

where κ_1 is adjusted suitably relative to our baseline model.²⁷ These equations are slightly different from (24) and (25) in our baseline model due to the markup μ (which can differ from $\rho/(\rho - 1)$ in our baseline model) and due to the difference in the way we modeled the spillovers in the production of the research good. It is still the case, however, that the value of θ plays a key role in determining how productivity Z moves in response to changes in profitability. Finally, the research intensity of the economy, $s_r = P_r Y_r / (Y - X)$, is given by expression (16) in our baseline model with μ substituting $\rho/(\rho - 1)$.

Aggregate implications of changes in policies

We can use equations (A20), (A21), and (A22) to reproduce the results in our baseline model. The results in Proposition 1 follow immediately from (A20). The optimal innovation subsidy, which can be solved by following steps similar to those in Proposition 2, differs slightly from that in expression (26) because of the slightly different form of spillovers in both models.

Propositions 3, 4, and the corollary to Proposition 4 are unchanged except that the term s_g must be adjusted to take into account the different time profile of the profits associated with successful innovations. In particular, the change (to a first-order approximation) in profitability, $\Pi Z/P_r$, in response to changes in innovation policies τ_p , τ_g , and τ_e is given by

$$\Delta \log(\Pi Z/P_r) = -\Delta \log(1 + \tau_p) + \frac{s_g}{\xi_g} \Delta \log(1 - \tau_g) + \left(1 - \frac{s_g}{\xi_g}\right) \Delta \log(1 - \tau_e),$$

²⁷In particular, $\kappa_1 = \kappa_0 \lambda^\lambda (1 - \lambda)^{1-\lambda} R_k^{-\frac{\alpha(1-\lambda)}{1-\alpha}} \frac{(1-\alpha)^{(1-\alpha)(1-\rho)-\lambda} \alpha^{\frac{\alpha}{1-\alpha} [1+(1-\alpha)(1-\rho)-\lambda]}}{\mu^{\frac{1-\lambda}{1-\alpha}-\rho}}$.

where $s_g = \frac{(1-\tau_g)P_r c(q)}{(1+\tau_p)\Pi Z}$ as in our baseline model. For a given change in $\Pi Z/P_r$, the changes in aggregate output, the aggregate allocation of labor, and the research intensity of the economy are the same as those in our baseline model. The change in the innovation intensity by incumbent firms (which has no first-order effects on aggregates if $\tau_g = \tau_e$ initially) is given by

$$\Delta \log q = \frac{1}{\eta_c} [\Delta \log (1 - \tau_e) - \Delta \log (1 - \tau_g)].$$

The change in aggregate fiscal expenditures, E , in response to changes in innovation policies (starting with $\tau_p = \tau_g = \tau_e = 0$) is given by

$$\Delta E = \Pi Z \left[\Delta \tau_p + s_g \Delta \tau_g + \frac{\xi_g - s_g}{\xi} \Delta \tau_e \right],$$

where

$$\xi = \frac{r + \delta_f - \phi}{\delta_f - \phi} > 1,$$

where we used equation (A20) reexpressed as

$$(1 - \tau_e) n_e P_r M \xi = [\xi_g (1 + \tau_p) \Pi Z - (1 - \tau_g) c(q) P_r].$$

The relation between the change in aggregate fiscal expenditures and the change in profitability (and hence other aggregates) is affected by the time profiles of the profits associated with a successful innovation and the innovation investments of an incumbent firm. Note, however, that as $r \rightarrow 0$ (which is the case as $\beta \exp(g_L) \rightarrow 1$, so that the growth of aggregate GDP approaches the consumption interest rate), then $\xi_g \rightarrow 1$ and $\xi \rightarrow 1$. In this case, the aggregate impact of changes in innovation policies is directly proportional to the fiscal impact of these changes, as in our baseline model. Finally, the aggregate implications of changes in the tax to the use of physical capital (Proposition 6) are equal to those in our baseline model.