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HUMAN CAPITAL INVESTMENT, AND THE INEFFICIENCY
OF COMPENSATION BASED ON MARGINAL PRODUCTIVITY:
THE STATIC CASE

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ABSTRACT

A simple extension of the traditional analysis of human capital accumulation is considered in a general equilibrium context. When real wages are equated to marginal products in the presence of human capital investment, resulting equilibria are almost never efficient even by very weak criteria. This is true even though labor is not a quasi-fixed factor, and informational asymmetries are excluded from the model. It is shown that human capital investment generates externalities, and has associated with it a "free-rider problem." This, in turn, explains the common practice of employers requiring minimum levels of human capital accumulation for some employees, and refusing to hire "overqualified" workers for other positions.

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A large literature exists which considers inefficiencies that arise when workers are compensated according to their marginal products. There are two basic strands of this literature. One considers dynamic inefficiencies which arise due to the accumulation of "specific" human capital. A second considers in various forms problems of asymmetric information, and the resulting inefficiencies associated with marginal product-based compensation. Within this second body of literature, there is a rich array of theories concerning the shortcomings of such compensation. Included in this are inefficiencies which arise due to the necessity of "signalling" marginal products, on the one hand, and inefficiencies which arise due to agency problems, on the other.^{1/} This paper argues that neither dynamic considerations, nor the existence of informational asymmetries are required for compensation on the basis marginal productivity to result in inefficiency. In fact, in the presence of "human capital" investments, such inefficiencies are the norm. Moreover, human capital need not be "firm specific" for this result; to emphasize this we consider a static economy with a single firm. To indicate that informational asymmetries are not required for the result, we consider an economy in which there is no uncertainty. In spite of the simplicity of the economy considered, however, the equation of real wages to the marginal products of labor is inefficient.

Throughout the analysis, human capital is defined as follows. Human capital is a factor of production, which may not be supplied independently of the labor of its "owner." In short, human capital may not be independently sold or rented. This feature of the economy is sufficient to generate the result that it is generally inefficient to pay wages equal to marginal products. This should come as no surprise, since in an economy with an incomplete set of markets there is no presumption that an equilibrium with competitive

firms is a Pareto optimum. However, the result presented here is stronger than this. We demonstrate, using an extremely weak optimality concept, that the equilibrium of our economy is suboptimal. The optimality concept employed is one designed to fail only in the presence of externalities, given the existing market structure. Hence we demonstrate that there are economically important externalities associated with the existence of human capital. In addition, we will demonstrate that the necessary conditions for optimality in our economy are closely related to the necessary conditions for optimality in an economy with public goods. We thus provide an economically meaningful basis for the commonly-made statement that education (considered as an investment in human capital) is a public good.

Finally, our analysis indicates a sense in which labor markets with competitive firms are innately distorted. We then suggest some arrangements which serve to support an optimal (in our sense) allocation of resources. These include commonly-observed arrangements in which firms behave in one of the following ways: they hire only agents with some minimal level of human capital accumulation for certain positions, and/or they refuse to hire agents who are "overqualified" for other positions. Thus the analysis provides a rationale for commonly-observed, but heretofore unexplained, behavior in labor markets.

The scheme of the paper is as follows. Section 1 sets out the model, and considers its equilibria. Section 2 defines the notion of optimality employed, and establishes the general inefficiency of the Section 1 equilibria. Section 3 considers the externalities associated with the existence of human capital investment. Section 4 considers arrangements to support an optimum in the presence of these externalities. Section 5 concludes.

1. An Economy with Human Capital

In order to focus on the single issue of the inefficiency of marginal product compensation, we present as simple a model as possible. Moreover, to demonstrate that neither dynamic aspects, nor the presence of uncertainty are required to establish this inefficiency, we eliminate any such considerations from the model. Finally, to demonstrate that the issue has nothing to do with specific versus general human capital we equate the two by considering an economy with a single firm. With this in mind, we present the economy under consideration.

A. The Model

Our economy consists of a single firm, and I agents (worker-consumers) indexed by $i = 1, \dots, I$. The firm uses the labor of (possibly a subset of) these agents, and their accumulated human capital to produce a single consumption good. Each agent is endowed with a single unit of labor, none of the consumption good, and no human capital. In addition, each agent is endowed with some fractional share in the firm. We begin by describing firm behavior, as this is essential to the behavior of other agents in the model.

Denote the quantity of labor of agent i employed by the firm by L_i , with the I vector $L \equiv (L_1, \dots, L_I)$. Let the quantity of human capital owned by agent i be denoted z_i , with the I vector $z \equiv (z_1, \dots, z_I)$. $(L, z) \in R_+^{2I}$ by assumption. As will be seen, under certain assumptions each agent supplies his holding of human capital inelastically to the firm. Hence if the firm hires agent i for any portion of his single unit of time, it receives in addition the input of his entire stock of human capital in the production process. (This is inessential, and serves only to simplify the analysis.) Then if the firm has employment vector L , with associated vector of human

capital holdings z , its production of the consumption good is $F(L,z)$. F has the following properties: $F:R_+^{2I} \rightarrow R_+$, $F \in C^3$, $D_j F(L,z) > 0 \forall j$, where D_j denotes the partial derivative with respect to the j^{th} argument, $F(L,z)$ is strictly concave, and $D_{I+j} D_j F(L,z) > 0 \forall j = 1, \dots, I$.

It should be noted that the definition of human capital as a factor of production which cannot be supplied independently of the labor of its owner means that $L_i = 0$ implies that the input of z_i to the firm is zero. Moreover, the assumption that no purchase or rental market exists in human capital implies that there is no price established for it.

If the firm hires agent i , that person receives a (real) wage rate of w_i units of the consumption good, which is our numeraire, per unit of labor supplied. Firm profits, then, are

$$\pi = F(L,z) - wL,$$

where $w \equiv (w_1, \dots, w_I) \in R_+^I$. These are distributed to the I agents according to the share of agent i , $e_i \in [0,1]$, and the profits of the firm are taken by each agent as a parameter. In order to focus on the inefficiency of marginal productivity based compensation, we assume that $\forall i = 1, \dots, I$,

$$(1) \quad w_i = D_i F(L,z).$$

The behavior of workers in the model is more complicated. Each agent has a utility function $U_i(C_i, 1-L_i)$, where C_i is consumption of the good by agent i , and $1 - L_i$ is our definition of leisure. $U_i \in C^2 \forall i$, with $D_1 U_i(\) > 0 \forall i$, and $D_2 U_i > 0 \forall i$. In addition, $U_i(\)$ is quasi-concave $\forall i$. As indicated previously, each agent has an endowment of e_i shares in the firm, one unit of labor (time), and nothing else.

Given that agents are not initially endowed with human capital, they obtain it, if at all, by converting real resources into human capital. In order to acquire z_i units of human capital, agent i foregoes $G_i(z_i)$ units of the consumption good. In other words, $G_i(\cdot)$ is agent i 's inverse production function for human capital, expressing in units of goods the input required to obtain an output of z_i units of human capital. While we treat z_i as a scalar, this is not necessary to the analysis. The properties of G_i are $G_i \in C^2$, $G_i', G_i'' > 0 \forall i = 1, \dots, I$, and $G_i(0) = 0$.

We are now prepared to discuss the economic problem faced by each agent. For all $i = 1, \dots, I$, economic behavior is described by the solution to the following two-part optimization problem:

$$(2) \quad \max U_i(C_i, 1-L_i) \text{ by choice of } (C_i, L_i) > 0 \text{ subject to } C_i \leq w_i L_i - G_i(z_i^*) + \theta_i \pi; z_i^*, w_i \text{ parametric,}$$

where z_i^* is the solution to (3), and

$$(3) \quad \max w_i L_i^* + G_i(z_i) \text{ by choice of } z_i > 0 \text{ subject to (1),}$$

where L_i^* is the solution to (2) obtained by setting

$$w_i = D_i F(L^*, z^*).$$

The reason for considering this two-part optimization problem is to abstract from distortions which would result from the monopolistic labor supply behavior associated with the more straightforward problem

$$(4) \quad \max U_i(C_i, 1-L_i) \text{ by choice of } (C_i, L_i, z_i) > 0 \text{ subject to (1) and } C_i \leq w_i L_i - G_i(z_i) + \theta_i \pi.$$

Our results would be unaltered, however, were we to adopt the formulation in (4).

Since (1) expresses w_i as a function of the 2I vector (L, z) , in order to complete our specification of agent i 's problem we must elaborate on i 's beliefs about the way in which other agents react to his choices. The assumption made is that each agent takes the optimal choices of others as parameters. It will be convenient to have a notation for the choices of all agents except i . Thus we define $z^{-i} = (z_1, \dots, z_{i-1}, z_{i+1}, \dots, z_I)$, and $L^{-i} = (L_1, \dots, L_{i-1}, L_{i+1}, \dots, L_I)$.

The first order conditions for (2) and (3) are

$$(5) \quad \frac{D_2 U_i(\cdot)}{D_1 U_i(\cdot)} = w_i$$

and

$$(6) \quad L_i * D_{I+i} D_i F(L^*, z^*) = G'_i(z_i^*),$$

where we have assumed an interior optimum. Under the assumption that $D_{I+i} D_{I+i} D_i F(L, z) < 0$ globally, (5), (6), and (1) define a continuous function $\phi_i: R_+^{2(I-1)} \rightarrow R_+^2$;

$$(7) \quad (L_i^*, z_i^*) = \phi_i(L^{-i}, z^{-i}).$$

(7) defines the optimal labor supply and stock of human capital of agent i . It will be noted that, since $D_{I+j} D_j F(L, z) > 0 \forall j$, it is never optimal for any agent to withhold a portion of his stock of human capital on the job, regardless of L_j^* . This confirms our previous statement to this effect.

Finally, prior to defining an equilibrium for this economy, we conclude this section by defining for each i the value function

$$V_i(L^*, z^*) = U_i(w_i^* L_i^* + \theta_i \pi - G_i(z_i^*), 1 - L_i^*); w_i^* = D_i F(L^*, z^*),$$

which defines the payoff received by agent i for any given vector of choices (L^*, z^*) of the I agents, including, of course, his own optimal choices. We are now prepared to define an equilibrium for our economy.

B. Equilibrium

To begin, define $\phi: R_+^{2I} \rightarrow R_+^{2I}$ as follows:

$$(x, y) = \phi(L, z) \equiv (\phi_1(L^{-1}, z^{-1}), \dots, \phi_I(L^{-I}, z^{-I})).$$

Then we have the following definition.

Definition. An equilibrium for this economy is a nonnegative $3I$ vector (L^*, z^*, w^*) satisfying

$$(i) \quad (L^*, z^*) = \phi(L^*, z^*)$$

and

$$(ii) \quad w_i^* = D_i F(L^*, z^*); i = 1, \dots, I.$$

Thus an equilibrium is a set of choices (L_i^*, z_i^*) which solve the two-part optimization problem (2) and (3) given that $(L^{-i}, z^{-i}) = (L^{*-i}, z^{*-i})$ for each i , which are consistent with each other, and with a vector w^* which equals the vector of marginal products given input levels (L^*, z^*) .

To establish existence of an equilibrium, we make the following additional assumption: there exists some $K > 0$ such that

$$D_{I+j} D_j F(L, z_1, \dots, z_{j-1} K, z_{j+1}, \dots, z_I) < G_j'(K) \quad \forall j = 1, \dots, I \quad \forall (L, z^{-j}) \in$$

$\prod_{i=1}^I [0, 1] \times \prod_{i=1}^{I-1} [0, K]$, where "X" denotes the I fold Cartesian product. Under

this assumption, $\phi: \prod_{i=1}^I [0, 1] \times \prod_{i=1}^I [0, K] \rightarrow \prod_{i=1}^I [0, 1] \times \prod_{i=1}^I [0, K]$, so that the

proof of existence of an equilibrium is a trivial application of Brouwer's fixed point theorem. Thus, an equilibrium exists for our economy, and we may now proceed to examine its optimality properties or lack thereof.

Prior to defining our notion of optimality, however, it may be useful to review the functioning of our economy. Agents work, and are paid their marginal product for their time. In addition, they may invest in human capital. This may be either some intangible, such as "skill," or some form of physical capital. Its distinguishing feature is that it cannot be sold or rented separately from the labor of its owner, so that markets are incomplete. The only reason for agents to invest in human capital, then, is because they are paid their marginal products, which can be augmented by human capital investment. It is this fact, that human capital investment occurs only to augment marginal products, which is the source of suboptimality in this economy.

2. Optimality of Equilibrium

A. Restricted Nash Optimality

Given the equilibrium concept employed, and the incompleteness of markets which gives rise to it, it should come as no surprise that the equilibrium defined above is not generally a Pareto optimum. Therefore, we focus on a weaker concept of optimality, which we refer to as Restricted Nash Optimality. The notion underlying Restricted Nash Optimality is roughly as follows. An allocation is a Restricted Nash Optimum if it is impossible to change some element of the vector (L,z) , leaving all other elements unaffected, and make lump-sum transfers in such a way that no agent is made worse off, and some agent is made better off. Formally,

Definition. An allocation (L^*, z^*, C^*) , with $C_i^* = D_i F(L^*, z^*) L_i^* - G_i(z_i^*) + e_i \pi$, is a Restricted Nash Optimum (RNO) if there is no alternate allocation $(\tilde{L}, \tilde{z}, \tilde{C}) \neq (L^*, z^*, C^*)$ such that

(iii) $U_i(\tilde{C}, 1 - \tilde{L}_i) > V_i(L^*, z^*) \forall i = 1, \dots, I$, with strict inequality for j only.

(iv)
$$\sum_{i=1}^I \tilde{C}_i = F(\tilde{L}, \tilde{z}) - \sum_{i=1}^I G_i(\tilde{z}_i)$$

(v) either $(\tilde{L}^{-j}, \tilde{z}) = (L^{*-j}, z^*)$, or $(\tilde{L}, \tilde{z}^{-j}) = (L^*, z^{*-j})$

(vi) $\tilde{L}_i \in [0, 1] \forall i = 1, \dots, I$.

In other words, for an allocation to be an RNO, it must be impossible for some agent j to change any of his choice variables and arrange a set of lump-sum transfers in a way that makes him better off, and no one else worse off. The motivation for using this optimality concept is twofold. First, it asks whether some agent can be made better off while making no one else worse off by some change in behavior if any externalities induced by this change are internalized. Second, the notion of changing only one agent's actions reflects the Nash behavioral assumption. It will be the case that a Nash equilibrium fails to be an RNO only when externalities are present, by the same argument which implies that a competitive equilibrium is Pareto optimal in the absence of externalities. Thus, if the equilibrium of Section 1 is not an RNO, it will be because of externalities associated with human capital accumulation. We now consider the optimality of the Section 1 equilibrium on the basis of this criterion.

B. Optimality Conditions

We consider whether or not the equilibrium vector (L^*, z^*, C^*) is an RNO. Consideration of RNO allocations implies that we wish to ask whether there exists some agent, j , who can change an element of his decision vector in a welfare improving way, taking the actions of other agents as given. In order to answer this question, we note that in any alternate allocation to the equilibrium proposed by j , $(L^{-j}, z^{-j}) = (L^{*-j}, z^{*-j})$. Therefore, (iii) requires $C^{-j} = C^{*-j}$ as well. Then, in any alternate arrangement proposed by j , his consumption will be

$$C_j = F(L_1^*, \dots, L_{j-1}^*, L_j, L_{j+1}^*, \dots, z_{j-1}^*, z_j, z_{j+1}^*, \dots, z_I^*) - \sum_{i \neq j} (C_i^* + G_i(z_i^*)) - G_j(z_j) \equiv H_j(L_j, z_j, L^{*-j}, z^{*-j}).$$

Thus, if the equilibrium is RNO, it solves

$$(8) \quad \max U_j(H_j(L_j, z_j, L^{*-j}, z^{*-j}), 1-L_j) \quad \forall j = 1, \dots, I.$$

Given the concavity conditions imposed, a necessary and sufficient condition for the equilibrium values (L^*, z^*) to solve (8) is

$$(9a) \quad D_{I+j} F(L^*, z^*) = G'_j(z_j^*) \quad \forall j.$$

$$(9b) \quad D_2 U_j(\) / D_1 U_j(\) = D_j F(L^*, z^*).$$

It is clear under what conditions the equilibrium will be an RNO. Since (6) holds for any equilibrium values, an equilibrium is RNO iff

$$(10) \quad D_{I+j} F(L^*, z^*) = D_{I+j} D_j F(L^*, z^*) L_j^* \quad \forall j.$$

Thus if and only if the production function of an economy satisfies the system of partial differential equations (10) in equilibrium will that equilibrium be

an RNO. While (10) is satisfied by equilibrium values for some economies, as will be seen below, it will not be satisfied in general. Therefore, the equilibria of Section 1 are not, in general, Restricted Nash Optima, which is an indication that externalities are present. We now turn our attention to the nature of these externalities.

3. Human Capital as a Public Good

It is often suggested that education, or "knowledge" has significant public goods aspects. We now suggest an economic sense in which this is true of human capital accumulation in this form.

Consider the impact of a "small" change in agent j 's accumulation of human capital, holding all other choice variables fixed. Denoting partial differentiation with respect to z_j by D_{z_j} , the impact of such a change on agent i 's labor income is

$$L^*_i D_{z_j} w_i(L^*, z^*); i \neq j, \text{ and } L^*_j D_{z_j} w_j(\cdot) - G'_j(z_j) \text{ for } j$$

where $w_i(\cdot)$ is given by (1). The impact on profits is

$$D_{I+j} F(L^*, z^*) - \sum_{i=1}^I L^*_i D_{z_j} w_i(L^*, z^*),$$

so the total impact on i 's income is

$$L^*_i D_{z_j} w_i(\cdot) + \theta_i (D_{I+j} F(\cdot) - \sum_{i=1}^I L^*_i D_{z_j} w_i(\cdot)); i \neq j.$$

Summing over all i , and noting that firm shares sum to one, we obtain the total impact of the change in z_j on consumption, which is just $D_{I+j} F(L^*, z^*) - G'_j(z_j)$. However, it will be noted that the perceived change in agent j 's income from this change, since π is taken as parametric, is $D_{I+j} D_j F(L^*, z^*) -$

$G'_j(z_j)$. This differs (in general) from the social impact of the change in z_j , thus generating an externality. In addition, it will be noted that, except for production functions which obey (10), the effect on i 's income from a change in j 's human capital is not limited to the pecuniary externalities generated by the impact on w_i .^{3/} Thus all agents share in j 's investment in human capital in a way not confined to the pecuniary externalities generated, so that j 's human capital investment has public good aspects.

It is useful to contrast the effect of a change in z_j with that of a change in L_j . As before, the impact on i 's labor income is

$$L_i^* D_{L_j} w_i(\cdot); \quad i \neq j$$

and the impact on profits is

$$D_j F(L^*, z^*) - \sum_{i=1}^I L_i^* D_{L_j} w_i(\cdot) - w_j = - \sum_{i=1}^I L_i^* D_{L_j} w_i(\cdot),$$

so that the effect on i 's total income is

$$L_i^* D_{L_j} w_i(\cdot) - \theta_i \left(\sum_{i=1}^I D_{L_j} w_i(\cdot) L_i^* \right); \quad i \neq j$$

and

$$L_j^* D_{L_j} w_j(\cdot) - \theta_j \left(\sum_{i=1}^I L_i^* D_{L_j} w_i(\cdot) \right) + w_j$$

for j . Summing over all agents, the total effect on consumption is $D_j(L^*, z^*) = w_j$, so that the total impact of the change in consumption is internalized in j 's decision-making. Hence changes in quantities supplied of factors which receive their marginal products generate pecuniary externalities, but there is no sense in which (potentially) all other agents in the economy can free ride on agent j 's change in labor supply. Human capital investment, on the other hand, generates more than pecuniary externalities, so that in the aggregate,

the rest of the economy can free ride on any agent's human capital accumulation.

In what practical sense is human capital a public good? In our model human capital investment affects both firm profits, and the marginal products of (in general) all agents. It is not difficult to think of several ways in which human capital accumulation, broadly enough interpreted, plays this role. First, the accumulation of skills which augment marginal productivity may be passed to others on the job in both direct and indirect fashion.^{4/} Thus, one agent's human capital accumulation may augment the productivity of other workers. Second, it is not difficult to conceive of production processes in which the increased productivity of some workers enhances (at least potentially) the marginal products of other workers. Assembly line processes come to mind in this regard. Finally, the knowledge of one agent may directly benefit co-workers. In this regard academic seminars come to mind, especially as we permit negative as well as positive externalities. In short, there seem to be a large number of ways in which human capital accumulation has associated with it externalities in the work place.

In closing this section, it may be helpful to provide an example which illustrates the public goods aspect of human capital investment. To this end, we present

Example 1. $I = 2$. $U_1(C_1, 1-L_1) = C_1$, and $U_2(C_2, 1-L_2) = C_2 + (1-L_2)$. $F(L, z) = L_1^a z_1^{1-a}$, $G_1(z_1) = z_1^{2-a}$; $a < 1$, and $e_2 = 1$. Since $L_1 = 1$, agent 1's problem is to maximize, by choice of z_1 ,

$$D_1 F(L, z) - G_1(z_1) = a z_1^{1-a} - z_1^{2-a}.$$

The solution to this problem is to set $z_1^* = a(1-a)/(2-a)$. In contrast, the RNO level of human capital accumulation for 1 is $\tilde{z}_1 = (1-a)/(2-a)$, which for

this example is also the Pareto optimal level of human capital accumulation. Thus the equilibrium has $z^*_1 = a \tilde{z}$. It is clear that for $a = 1$, the equilibrium is also optimal, as in this case human capital is unproductive. As "a" falls below unity, the distortion discussed in the text occurs, and the equilibrium level of human capital accumulation becomes suboptimal. It is also clear that, in the example, agent 2 free rides on agent 1's investment in human capital, and it is this public good aspect of human capital which leads to inefficiency.

Given that, in general, an equilibrium with human capital investment where agents receive their marginal product of labor is inefficient, we turn now to a consideration of arrangements which induce an optimal allocation of resources.

4. The Support of an Optimum

There are a large number of arrangements which could potentially induce equilibrium allocations to be RNO. However, the most straightforward arrangements, which serve essentially to complete the set of markets, are the most difficult to implement. The first of these is to permit firms either to rent, or to purchase outright, the human capital of workers. While from an analytical standpoint rental markets in human capital solve the problem of equilibria being inefficient, it is difficult to see how, in practice, such markets can operate without some form of indenture of workers to their employers. A potentially equivalent scheme analytically is to permit workers to buy and sell shares in the human capital of other workers. However, this scheme is equivalent only if an equilibrium exists under it. The assumptions made thus far are insufficient to guarantee existence of an equilibrium under this scheme, essentially because there may be nonconvexities associated with the externalities generated by human capital.^{5/} Thus simple completion of the

set of markets, without permitting some form of indenture, will not always rectify the problems raised in the previous sections.

Optimal arrangements, then, must generally take other forms. One possibility is for some governmental agency to levy taxes which induce an optimum, while balancing its budget by lump-sum side payments. Since human capital affects only production possibilities in our model, discussion of optimal taxes follows Starrett (1972). Again, however, a caveat must be issued regarding such taxes. This is that while every optimal allocation has an associated supporting tax vector, for any given tax vector there may be multiple equilibria, not all of which are optimal. Thus there are potential problems associated with the use of taxes to induce optimal allocations as well.

Two simple arrangements remain which can be used to support optimal levels of human capital accumulation. These are to impose minimum hiring requirements, on the one hand, or maximum hiring requirements on the other, which are prerequisites for any agent being hired by some firm. What we have in mind is that in the first case, no agent would be hired without, say, a high school education. In the second case, firms might refuse to hire workers who are by background "overqualified" for some position. Both types of arrangements are widely observed, and if they are sufficiently flexible, can be used to induce optimal investment in human capital. To indicate how this is the case, we present an example.

Example 2. The list of agents, endowments, and preferences is the same as for example 1. $F(L,z) = a_0L_1 + a_1L_1z_1 - (a_2/2)L_1^2 + b_1z_1 - (b_2/2)z_1^2$, with $a_0 = a_2$, and $a_2b_2 > a_1^2$. Finally, $G_1(z_1) = z_1^2$. Since $L_1 = 1$, agent 1's problem is to maximize

$$D_1F(L,z) - G_1(z_1) = a_1z_1 - z_1^2$$

by choice of z_1 . In an equilibrium, $z^*_1 = a_1/2$. At an RNO (or a Pareto optimum), however,

$$\tilde{z}_1 = (a_1 + b_1)/(2 + b_2).$$

There are now three cases to consider.

(i) $a_1 b_2 = 2b_1$. In this case, $z^*_1 = \tilde{z}_1$, and the equilibrium is an optimum. This verifies that there do exist economies which satisfy (10) in equilibrium, and in which human capital is productive.

(ii) $a_1 b_2 < 2b_1$. In this case, $z^*_1 < \tilde{z}_1$. If a requirement is imposed which states that no one will be hired without a minimum human capital accumulation of \tilde{z}_1 units, however, the resulting equilibrium will generally be an optimum. Thus under certain circumstances, minimum standards for hiring support optimal resource allocations.

(iii) $a_1 b_2 > 2b_1$. In this case, $z^*_1 > \tilde{z}_1$. The equilibrium is associated with overinvestment in human capital, and an optimal arrangement is for the firm to refuse to hire agents with $z_1 > \tilde{z}_1$, or in other words, for the firm not to hire "overqualified" workers.

There are, however, at least two caveats which should be issued with regard to "hiring standards." The first is that, as example 2 indicates, whether upper or lower bounds on human capital accumulation are called for can depend entirely on values of a small number of the list of parameters affecting technology. It thus may be difficult or impossible for outsiders to determine whether or not firm hiring policies induce optimal resource allocations, or are intended for some other, perhaps discriminatory, purpose. The second caveat is that when there are a large number of workers, unless firms can discriminate among them with regard to hiring standards, such arrangements

cannot generally induce an RNO allocation of resources. This is not to say they may not be welfare improving, however.

In short, there are several arrangements which may be used to support optimal levels of investment in human capital. In particular, widely-observed hiring practices that amount to quantity constraints on human capital accumulation may play this role. However, in a world where such constraints may not be applied in a sufficiently "discriminatory" fashion, one would expect to observe a mix of such constraints and taxes-cum-subsidies on human capital accumulation. Such a prediction would seem to accord closely with actual observation.

5. Conclusions

The normal treatment of investment in human capital proceeds as follows.^{6/} There is assumed to exist for any agent an "earnings function," which depends on the level of investment in human capital. In addition, there is a cost function for acquiring human capital. Agents invest in human capital, taking these functions as given, in order to maximize some quantity such as the discounted present value of earnings net of costs. Comparison of the above analysis with this approach will indicate that we have not deviated from the normal treatment of investment in human capital in any significant way. The sole departure above from the standard analysis of human capital investment was the equation of the earnings function to the marginal product of labor, and the subsequent consideration of general rather than partial equilibrium issues. However, despite its simplicity, this approach is quite powerful, as it permits one to demonstrate that compensation based on marginal product is generally inefficient. It also permits a simple explanation of commonly observed hiring practices.

It will be noted that the mathematics of the suboptimality of equilibrium differ only moderately from the mathematics of the agency problem. However, the interpretation of this suboptimality is substantially different. Rather than basing the inefficiency of marginal product compensation on limited asymmetric information, our approach derives this inefficiency in a world where all agents are fully informed. This indicates that the suboptimality of marginal productivity based compensation is all pervasive, given the presence of human capital investment, and not resolvable merely by improving observability or discovering some analog of a "demand-revealing mechanism." Thus, in a world which has dealt with this inefficiency, either marginal products set equal to real wages, or hiring practices which are totally unrestricted, should be the exception rather than the norm. In such a case, the issues considered in (for instance) the agency literature should not be the primary focus of analysis.

There are several issues which are of substantial interest that have not been addressed here. The first is related to dynamic inefficiencies associated with human capital accumulation.^{7/} In a dynamic economy where agents face a sequence of budget constraints (i.e., where agents operate in imperfect capital markets), the inefficiency which we have investigated will generally be coupled with an inefficiency arising from the presence of such constraints. Interestingly, from a macroeconomic perspective, these inefficiencies can sometimes be dealt with through the use of what is commonly considered monetary policy. This is the subject of future research.

A second issue which is of interest concerns an interpretation of human capital as a local public good. In particular, we have indicated a sense in which there are public goods aspects to human capital. In practice, however, those who benefit from an agent's investment in human capital are

generally those who are associated with the same firm as the agent. Thus only agents in the same "locality" (firm) can free ride on each others' human capital investments. In a world with free mobility of workers, and firms which offer differing compensation vectors (packages of remuneration including, e.g., working conditions, etc.), it would be of substantial interest to consider both the optimality of equilibrium, and arrangements which support an optimum.

In short, there are a number of issues related to the (in)efficiency of equilibrium in the presence of human capital investment which are of interest. By extending the normal analysis of human capital investment to a general equilibrium setting, these issues can be analyzed fairly easily, and arrangements which support optimal resource allocations derived. Given the resources devoted to human capital investment, this would appear to be a useful area of research.

FOOTNOTES

1/Oi (1961) is the classic work on firm specific human capital. Spence (1974) and Riley (1975) discuss signalling inefficiencies, and Lazear (1981) discusses the agency problem.

2/In fact, an additional restriction is needed to guarantee that $\phi_i()$ is continuous. Since this involves second and third derivatives of $U_i()$ and $F()$, and is devoid of economic content, it is not discussed here.

3/This is reflected in the appearance of terms other than those arising from changes in wage rates in the expression for the change in i 's income.

4/See, e.g., Piore (1972).

5/See, e.g., Starrett (1972).

6/For a presentation of this approach, see, e.g., Riley (1976), Rosen (1977), or Weiss (1971).

7/It may seem strange that we have considered human capital accumulation in a static setting. However, by redefining marginal products so that they are discounted, and by appropriately redefining our cost function for acquiring human capital, our analysis is easily translated into a dynamic setting.

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