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AND THE EXISTENCE OF EQUILIBRIUM:
A NOTE*

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Money, Nonconvex Preferences, and
the Existence of Equilibrium: A Note*

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An important subset of the literature in monetary theory develops models in which nonconvexities are present, usually in transactions technologies.^{1/} These, in turn, imply a potential role for some asset (money) to be held as an inventory for the purpose of exploiting scale economies in transacting. Thus, nonconvexities have played a significant role in traditional monetary theory.

This note explores other roles that might be provided for money by the presence of underlying nonconvexities. In particular, it is demonstrated that for some economies where agents have nonconvex preferences, there exist exact competitive equilibria in economies with a finite number of agents (alive at each date) when fiat money is present, while no competitive equilibrium exists in its absence. In short, for some such economies, there is a natural role for fiat money in supporting an equilibrium.

While it is known that there exist economies which have equilibria in the presence, but not in the absence of fiat money,^{2/} these economies have been relatively complicated in that they require the presence of uncertainty and asymmetrically informed agents. It is demonstrated here that such complications are inessential. In fact, the class of economies considered is quite standard, with the exception that preferences are permitted to be nonconvex. In particular, we study the class of overlapping generations economies examined by Balasko and Shell [2], except that we relax their assumption of strictly quasi-concave utility functions. This class of economies is of interest, as Balasko and Shell, and Balasko, Cass, and Shell [1] have provided general

proofs of existence when preferences are convex, and when money is absent. Not surprisingly, permitting preferences to be nonconvex destroys this existence result. Perhaps surprisingly, some economies with nonconvex preferences have equilibria when fiat money is present, but not when it is absent.

The manner in which this result is obtained is as follows. An overlapping generations model is presented in which the standard Samuelsonian [7] role for money is present. In addition, some agents in the model have nonconvex preferences. In the absence of money, this nonconvexity may preclude existence of an equilibrium. However, the introduction of fiat money can result in existence for the following reason. Introducing money into the economy and focusing on steady states produces a lower bound on rates of return if money is valued. (This bound would not be in effect without money.) Moreover, for an appropriately structured economy, this bound on interest rates can force agents onto a convex portion of their indifference curves. In such a case, nonconvexities in preferences do not affect the relevant portions of excess demand schedules, and existence is restored. Thus, when money is present an equilibrium exists, and moreover, has the property that money must be valued in exchange.

1. The Model

Since general existence results will not be obtainable, we may proceed by considering a relatively special subset of the class of economies examined by Balasko and Shell. We consider, then, an economy in which time is discrete, and indexed by $t = 0, 1, \dots$. At date zero there exist three agents. One of these we

refer to as the initial old generation. This agent lives only in period zero, and cares only about his period zero consumption. The other two agents are young at $t = 0$, old at $t = 1$, and absent at $t = 2$. At $t = 1$, two new young agents appear, and live two periods, etc. The young agents at each date are referred to as agents 1 and 2.

The agents alive at any date operate in a pure exchange economy with a highly limited set of exchange possibilities. In particular, there is only a single nonstorable consumption good at each date, which we select as our numeraire. In addition, fiat money may or may not be present. If it is present, then a fixed quantity of money $M > 0$ circulates $\forall t$, and trades for the consumption good at rate $S(t)$ at t . We refer to an economy with $M > 0$ as a monetary economy. An economy with $M = 0$ is a nonmonetary economy.

Since there is a single good in a nonmonetary economy, any trading which takes place will be intertemporal (trading of consumption-loans). We denote the quantity of consumption-loans made by agent i at t by $x(i,t)$; $i = 1, 2$, $t \geq 0$. $x(i,t) > 0$ means that i is a lender. If agent i makes a loan (borrows) $x(i,t)$ at t , then he receives (repays) $R(t)x(i,t)$ at $t + 1$. We denote consumption of agent i who is young at t by $C(i,t,t)$ when young, and by $C(i,t,t+1)$ when old. If money is present, we denote the quantity of money carried by i into the second period of his life ($t+1$) by $M(i,t)$.

Finally, all type i agents are identical except for their dates of birth. Type 1 agents have endowment stream

(ϕ_1, ϕ_2) , and type 2 agents have endowment stream (w_1, w_2) . Agents who are young have no endowment of money or consumption loans. We restrict their endowments of the good to be nonnegative (but not necessarily positive). The initial old are endowed with all money which circulates (if any). All agents except the initial old are also endowed with a preference ordering over consumption pairs, denoted as follows. $(C(i, t, t), C(i, t, t+1)) P_i(\tilde{C}(i, t, t), \tilde{C}(i, t, t+1))$ means that the first consumption stream is strictly preferred to the second by agent i born at t , and $(C(i, t, t), C(i, t, t+1)) R_i(\tilde{C}(i, t, t), \tilde{C}(i, t, t+1))$ means that the second consumption stream is not preferred to the first. The orderings are defined on nonnegative consumption pairs. Note that for any agent, his level of money holdings is irrelevant in his ordering. Each agent's ordering is assumed to satisfy the standard properties, except that preferences may be nonconvex.

For notational convenience, let $e(i, t)$ and $e(i, t+1)$ also denote the endowment of the consumption good of agent i at t and $t + 1$, respectively. Then define the budget set of agent i (who is young at t) by the correspondence

$$B(i, R(t)) = \{(C(i, t, t), C(i, t, t+1)) : C(i, t, t) + R(t)^{-1} C(i, t, t+1) \leq e(i, t) + R(t)^{-1} e(i, t+1); C(i, t, t), C(i, t, t+1) \geq 0\},$$

for both the monetary and the nonmonetary version of this economy.

We are now prepared to define a (perfect foresight) competitive equilibrium for both monetary and nonmonetary versions of our economy.

Definition. A (nonmonetary) equilibrium is a sequence of vectors $\{\tilde{C}(1,t,t), \tilde{C}(1,t,t+1), \tilde{C}(2,t,t), \tilde{C}(2,t,t+1), \tilde{x}(1,t), \tilde{x}(2,t), R(t)\}_{t=0}^{\infty}$ satisfying

$$(i) \quad (\tilde{C}(i,t,t), \tilde{C}(i,t,t+1)) R_i(C(i,t,t), C(i,t,t+1)) \neq \\ (C(i,t,t), C(i,t,t+1)) \in B(i, R(t)) \quad \forall t > 0; i = 1, 2,$$

$$(ii) \quad (\tilde{C}(i,t,t), \tilde{C}(i,t,t+1)) \in B(i, R(t)); i = 1, 2, t > 0,$$

$$(iii) \quad \tilde{C}(i,t,t) \leq e(i,t) - \tilde{x}(i,t) \\ \tilde{C}(i,t,t+1) \leq e(i,t+1) + R(t)\tilde{x}(i,t); t > 0, i = 1, 2,$$

and

$$(iv) \quad \tilde{x}(1,t) = -\tilde{x}(2,t) \quad \forall t > 0.$$

Definition. A monetary equilibrium is a sequence of vectors $\{\tilde{C}(i,t,t), \tilde{C}(i,t,t+1), \tilde{x}(i,t), \tilde{M}(i,t), S(t), R(t)\}_{t=0}^{\infty}$ satisfying (i), (ii),

$$(iii') \quad \tilde{C}(i,t,t) \leq e(i,t) - \tilde{x}(i,t) - S(t)\tilde{M}(i,t) \\ \tilde{C}(i,t,t+1) \leq e(i,t+1) + R(t)\tilde{x}(i,t) + S(t+1)\tilde{M}(i,t); \\ t > 0, i = 1, 2,$$

(iv), and

$$(v) \quad \tilde{M}(1,t) + \tilde{M}(2,t) = M \quad \forall t > 0; \text{ with } \tilde{M}(i,t) > 0 \quad \forall t.$$

The important point to note about these definitions is that the market structure of the monetary economy does not differ from that of the nonmonetary economy in the following sense: the introduction of money does not alter the fact that intertemporal redistribution of income in any arbitrary feasible fashion is (at least in principle) possible for each young agent. With this in mind, we are now prepared to present our results.

2. Money and Equilibrium

This section contains two propositions. The second is that once nonconvex preferences are permitted, neither a monetary nor a nonmonetary equilibrium need exist. Thus the first proposition will be established by means of an example.

Proposition 1. There exist economies that have competitive equilibria in the presence, but not in the absence of fiat money.

In order to establish the proposition, we present a nonmonetary economy where some agents have nonconvex preferences, and which has no competitive equilibrium. We then demonstrate that it is, in fact, the nonconvexity of preferences which is responsible for nonexistence. Finally, it is shown that the monetary version of the same economy has a competitive equilibrium.

Example 1. The preferences of type 1 agents are representable by the utility function $U_1[C(1,t,t), C(1,t,t+1)] = \ln C(1,t,t) + \ln C(1,t,t+1)$. Agent 1's endowment stream obeys $\phi_2/\phi_1 < 2/3$, and $\phi_1 + \phi_2 < 3$ at each date. Agent 2's preferences are representable by a set of indifference curves which have the slope described below:

$$\text{MRS} = \begin{cases} -1/2; C(2,t,t+1) > 2C(2,t,t) \\ -1; 2C(2,t,t) > C(2,t,t+1) > C(2,t,t) \\ -1/4; C(2,t,t) > C(2,t,t+1) > C(2,t,t)/2 \\ -1/2; C(2,t,t+1) < C(2,t,t)/2. \end{cases}$$

Agent 2 has endowment stream $(w_1, w_2) = (0, 5)$ at each date.

The indifference curve passing through the endowment point for each type 2 agent is shown in Figure 1. It is easily seen that the preferences depicted result in the following demand correspondence for consumption loans by type 2 agents:

$$x = \begin{cases} 0 & ; R > 2/3 \\ 5/(1+R); k \leq R \leq 2/3 \\ 5/R & ; R \leq k. \end{cases}$$

k is a nonnegative constant, and denotes the maximum value of $R(t)$ for which type 2 agents desire to consume only when young. For our purposes, the single important fact about k is that $k < 1/2$. It will be noted that we have dropped arguments for convenience.

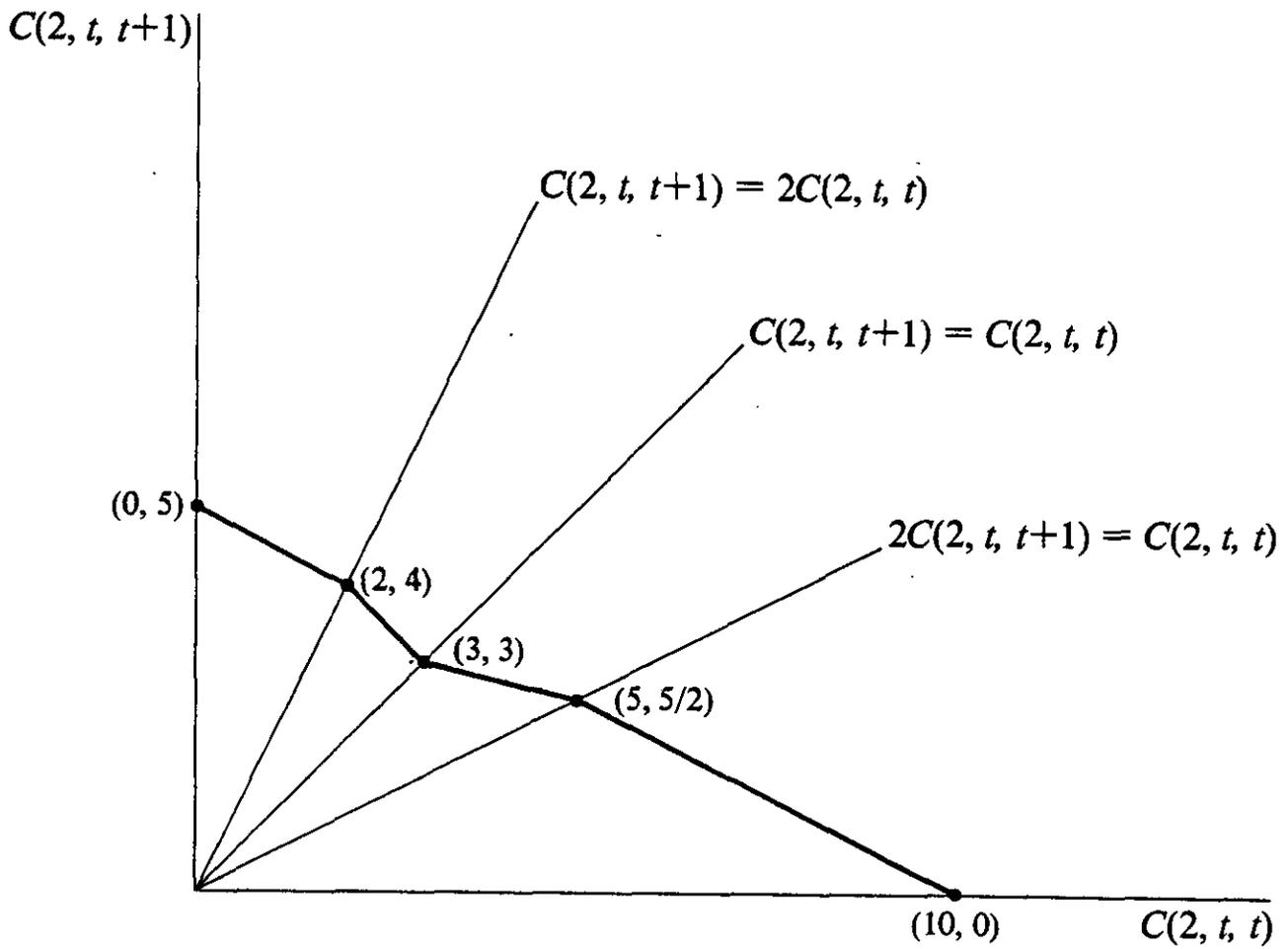
The supply of consumption loans by type 1 agents is

$$x = (R\phi_1 - \phi_2)/2R.$$

We are now ready to demonstrate that this economy has no nonmonetary equilibrium. To see this, note that there are three possible equilibrium configurations of consumption loans; one with $x = 0$, one with $x = 5/R$, and one with $x = 5/(1+R)$. Assume then that the equilibrium has $x = 0$. The form of the supply function of consumption loans requires that for $x = 0$ to hold, $R = \phi_2/\phi_1$. The form of the demand correspondence requires that for $x = 0$ to obtain, $R > 2/3$. However, $\phi_2/\phi_1 < 2/3$ by assumption, so that an equilibrium with $x = 0$ is impossible.

Suppose, then, that an equilibrium has $x = 5/R$. The form of demand implies, then, that $R \leq k < 1/2$, so that $x > 10$. But the total availability of resources at each date is $w_1 + w_2 +$

Figure 1
Preferences of Agent 2



$\phi_1 + \phi_2 < 8$. Thus an equilibrium with $x = 5/R$ would violate feasibility, and is impossible.

Therefore, if an equilibrium exists, it must have $x = 5/(1+R) = (R\phi_1 - \phi_2)/2R$. However, note that the minimum value of loan demand over the range of R values belonging to $[k, 2/3]$ is 3, and that the maximum value of loan supply cannot exceed $\phi_1 + \phi_2 < 3$. This contradicts the assumption that $5/(1 + R) = (R\phi_1 - \phi_2)/2R$ for some $R \in [k, 2/3]$. Thus no equilibrium exists for the nonmonetary version of this economy.

As an aside, it will be noted that the economy of this example violates three of the conditions in Balasko and Shell [2], which provides conditions sufficient for the existence of equilibrium. These are (i) convexity of preferences, (ii) no boundary endowments in R_+^2 , and (iii) no boundary consumption in R_+^2 . We now demonstrate that it is the violation of the first of these which is responsible for nonexistence in our example.

To see this, consider a convexified version of the economy of example 1. This convexified economy is obtained in the natural way, by replacing each of the upper contour sets defined by agent 2's indifference curves for each t with its convex hull. With this being the only change in the economy, it is readily verifiable that an equilibrium exists with $R = 2/3$, and $x = \phi_1/2 - 3\phi_2/4 \neq t > 0$. Thus, it is the nonconvexity of preferences and not the possibility of boundary consumption or boundary endowments which leads to nonexistence of a nonmonetary equilibrium in example 1.

Consider now the monetary version of the economy of example 1. We verify that a monetary equilibrium exists with $x = 0$, $S(t) = (\phi_1 - \phi_2)/2M \forall t$. To see this, note that loan demand is unaltered by the presence of money, and that the behavior of type 1 agents is altered only in so far as the optimal combination of money holdings and consumption loans is given by

$$x(1,t) + S(t)M(1,t) = (R(t)\phi_1 - \phi_2)/2R(t) \forall t.$$

Consider then an equilibrium with $R(t) = 1 \forall t$, and $x = 0$, $S = (\phi_1 - \phi_2)/2M \forall t$. Since $R = 1$, loan demand is zero. The sum of loan supply and holdings of real balances for type 1 agents is $(\phi_1 - \phi_2)/2$. Therefore, if $SM = (\phi_1 - \phi_2)/2 \forall t$, each agent is on his supply/demand schedule at each date, money markets clear, and so do loan markets. Thus, these values of x , S , and R satisfy the definition of a monetary equilibrium, verifying that there exist economies with monetary but no nonmonetary equilibria.

It is clearly the case that neither a monetary, nor a nonmonetary equilibrium need exist for general versions of the economy of section 1 unless preferences are required to be convex. This is

Proposition 2. There exist economies of the form described in section 1 which have neither a monetary, nor a nonmonetary equilibrium.

The importance of this proposition is that, not surprisingly, the introduction of money is not a general panacea for the nonexistence of equilibrium. As this is a fairly obvious point, we do not provide a counterexample to a general existence proposition. Such counterexamples are, of course, easy to construct.

3. Conclusions

It has been established that there exist economies where an equilibrium exists when fiat money is present, but not otherwise. In conclusion, it seems appropriate to place this result in context. It has been widely suggested (although for convex economies) that if an economy had an equilibrium with money, it would also have one without money.^{3/} Put otherwise, if money is present, it is not necessary that it have value. Thus, equilibrium valuation of fiat money has been viewed as something of an "accidental" outcome in theoretical models. However, the demonstration of the previous section indicates that there exist economies where if fiat money is present, the only equilibrium outcome is for it to have value. The presence of nonconvexities, then, is in some cases sufficient to "force" money to be valued in equilibrium.

Footnotes

1/E.g., Baumol [3], Tobin [9], Miller and Orr [6], or
Heller and Starr [4].

2/Smith [8].

3/See, e.g., Kurz [5] and Wallace [10].

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