Federal Reserve Bank of Minneapolis
Research Department Working Paper

"CAN THE CYCLE BE RECONCILED WITH A CONSISTENT THEORY OF EXPECTATIONS" OR A PROGRESS REPORT ON BUSINESS CYCLE THEORY

Edward C. Prescott
with collaboration of
Ann Guenther, Patrick Kehoe, and Rody Manuelli

Working Paper 239
PACS File 3200

June 1983

Prepared for IMSSS Summer Workshop July 28, 1983.

A puzzle that has long confronted economists is, why have virtually all industrial market economies been subject to recurrent fluctuations in output and employment? These fluctuations vary in both amplitude and duration but, as repeatedly emphasized by Lucas (1981), all the peacetime cycles exhibit about the same pattern of co-movements among the variables. The principal regularities are (1) aggregate output deviates by as much as five percent from trend in the post-war period, even more in earlier periods; (2) hours of employment varies almost proportionately to output while the average product of labor varies less and is not as strongly procyclical; (3) production of producer and consumer durables exhibit much greater percentage amplitude of flucttutations than does the production of nondurables; and (4) monetary aggregates and velocity measures are procyclical. In Section 2, these and other business cycle observations are documented for the U.S. post-war economy. Section 3 documents that all of the seven other industrial economies examined have had in the post-war period fluctuations in aggregate output similar to those in the United States. This supports Lucas's surmise (1981, p. 218) that "cycles are regularities common to all decentralized economies."

If there were large procyclical movements in the marginal product of labor, there would be no business cycle puzzle. When the relative price of leisure (nonmarket produced goods) is high relative to consumption (market produced goods), the household would respond by choosing less leisure. Movements in measured real wages, however, may not be closely related to movements in the marginal product of labor. To obtain estimates of the real wage compensation paid and hours of employment in a given time period are measured and their ratio used as the estimate. Competitive equilibrium theory, however, does not require that payments be contemporaneous with delivery of goods. The restriction imposed is that the value of the bundle of event-time contingent commodities received equals the value of the event-time contingentment payments made (see for example Hall and Lillien (1979)).

For this reason, I focus on the average product of labor, which is the ratio of two measured quantities. Provided that the average and marginal products move together the average should be a good proxy for the marginal. I think there are reasons why they should. First, the work week of capital moves with the cycle so the capital used by a worker does not change much over the cycle. If an individual works 48 rather than 40 hours using the same machines, output should increase by about 20 percent. Similarly, if a plant is operated two shifts rather than one and the labor input per shift is not changed, again output should double. These observations suggest that increases in hours of employment per household should increase output proportionately. To summarize the small cyclical variations in the average product of labor along with the large cyclical variations in hours of employment is the key puzzle confronting any expectationally consistent competitive theory of the cycle.

In Section 4 certain methodological issues are addressed. The question is whether an explicit artificial economy can be constructed which both exhibits business cycles and incorporates assumptions not inconsistent with other findings in economics. That the models do not explain unemployment is not bothersome, for they are designed to explain the allocation of time between market and nonmarket activity and not the division of the nonmarket time between search and other activities. This is not to suggest that the latter allocation is not an important question, but rather that the former is important and puzzling.

The first class of models abstract from monetary factors and use the stochastic growth model structure. It is well known that the solution to the optimal growth problem is the competitive equilibrium allocation of the Arrow-Debreu contingent claim variety for that economy. Given the homogeneity of the households, the allocation results if there are only spot markets, agents have rational expectations, and markets clear. In this sense then, these models are expectationally consistent theories.

The second class of equilibrium models reviewed rely upon monetary shocks. The only ones with a precise and complete specification of the economic environment are the ones developed by Lucas (1972) and the nominal wage contracting model developed in Section 7. The former abstracts from features that would provide a propogation mechanism for the monetary shock. The latter relies upon capital accumulation for propogation. The papers of Kydland (1983a) and Townsend (1983) attempt to remedy this. Kydland introduces capital accumulation along with introducing real cash balances in the utility functions. He explores whether in this framework monetary shocks can produce fluctuations of the magnitude observed. Like Lucas, he uses the Phelps (1970) island construct. Townsend explores the implication of lags in aggregate information becoming available for the propogation of shocks. This requires agents to have expectationally consistent expectations of the expectations of others. A one period delay in the information becoming available radically alters the impulse response functions from those obtained by Lucas (1975), producing much greater persistence.

The nature of the empirical discipline of the micro equilibrium approach to the business cycle, I hope, will become apparent in this review. The models examined have few parameters and most of these are not free. The value of some parameters are dictated by micro observations on individuals and firms. The value of other parameters are implied by the requirement that the averages of labor share, capital-output ratio, interest rate, and the like, for the artificial economy be consistent with the historical averages for the actual economy being modelled. The paucity of free parameters impose great discipline upon the analysis.

One conclusion of this report is that the stochastic growth structure economy in which technological change is random displays business cycle behavior remarkably similar to that experienced by the U.S. economy in the post-war period. With respect to the monetary shock models, the models are not sufficiently well developed, particularly with respect to their propogation mechanisms, to subject them to empirical tests of this variety. Progress in this respect is being made as will be apparent in the Kydland and Townsend reviews.

Section 2

Here the business cycle is defined to be deviation of aggregate output from a smoothly varying path. Operationally, a smooth function of time is fit to the logarithm of real output with the deviations being defined to be the cycle. There are many ways a curve can be fitted through the data and, provided that the curve is a smooth function of time, which method is used matters only a little.[1/] Here our measure of smoothness is the average squared second difference of the function. Consider series x_t for $t = 1 \ldots, T$ and let s_t be the smoothed value and $d_t = x_t - s_t$ the deviation. The values of $\{s_t\}$ solve

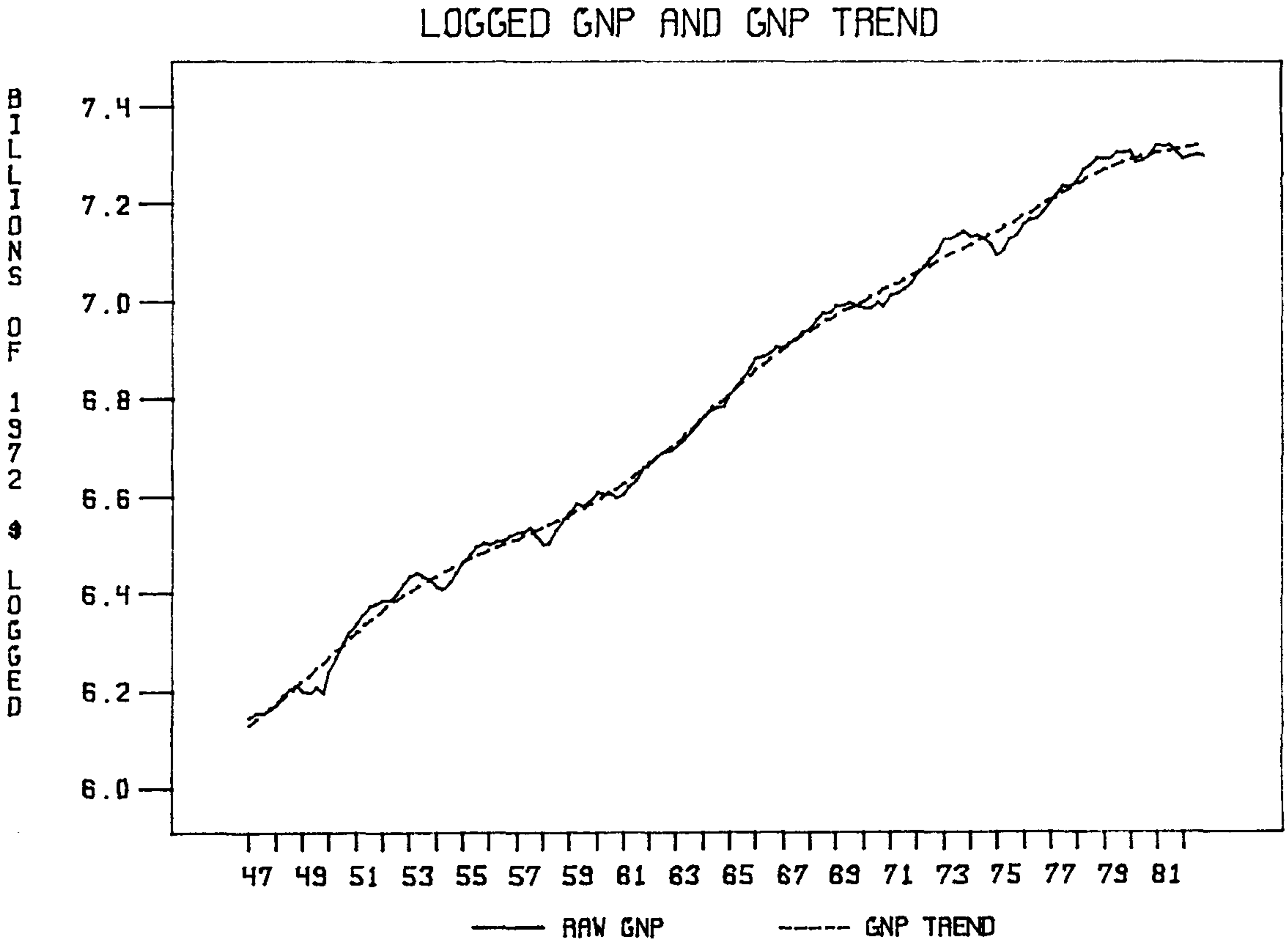
$$(*) \qquad \min \left\{\frac{1}{T} \sum_{t=1}^{T} (x_t - s_t)^2 + \frac{\lambda}{T} \sum_{t=2}^{T-1} [(s_{t+1} - s_t) - (s_t - s_{t-1})]^2\right\}$$

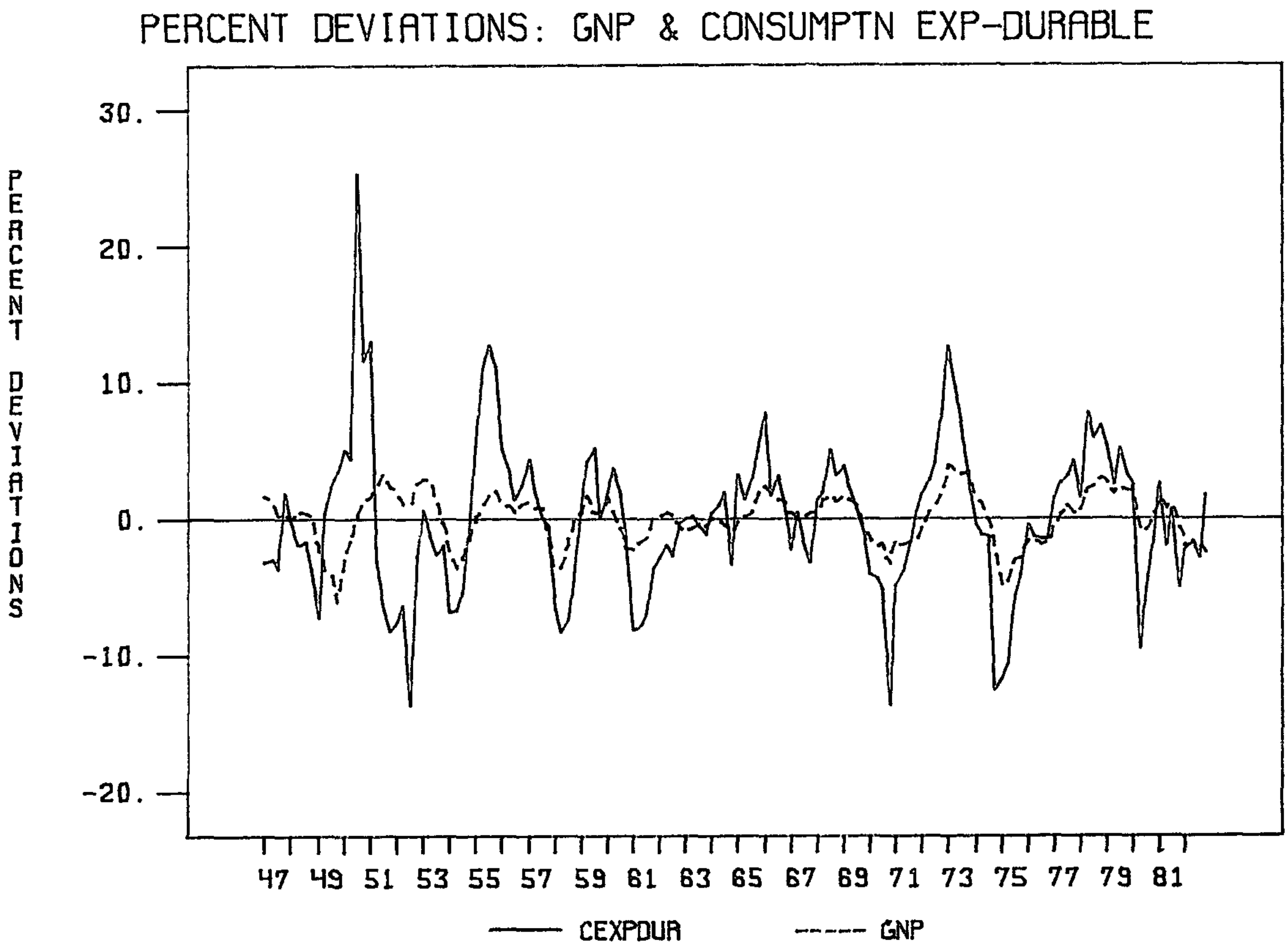
where $\lambda > 0$ is the penalty on variation as measured by the average squared second difference.

The use of this measure of smoothness is standard in numerical analysis. Cubic splines minimize this quantity over all differentiable functions of time; that is, the curve $f: [0,T] \to R$ is found which minimizes this measure subject to $f(t) = x(t)$ for $t \in \{t_1, \ldots, t_n\} \subset [0,T]$. The method we employ has been used for 60 years in the actuarial science and by many distinguished scientists including John von Neuman. The solution is easily calculated, as the first order conditions for program (*) are linear. By choosing λ appropriately, the resulting smooth curve s_t is essentially the one which would result if one simply fit the data by drawing a smooth curve. It dominates the judgmental procedure, for modern high speed computers can compute it at almost zero cost and others can easily replicate it.

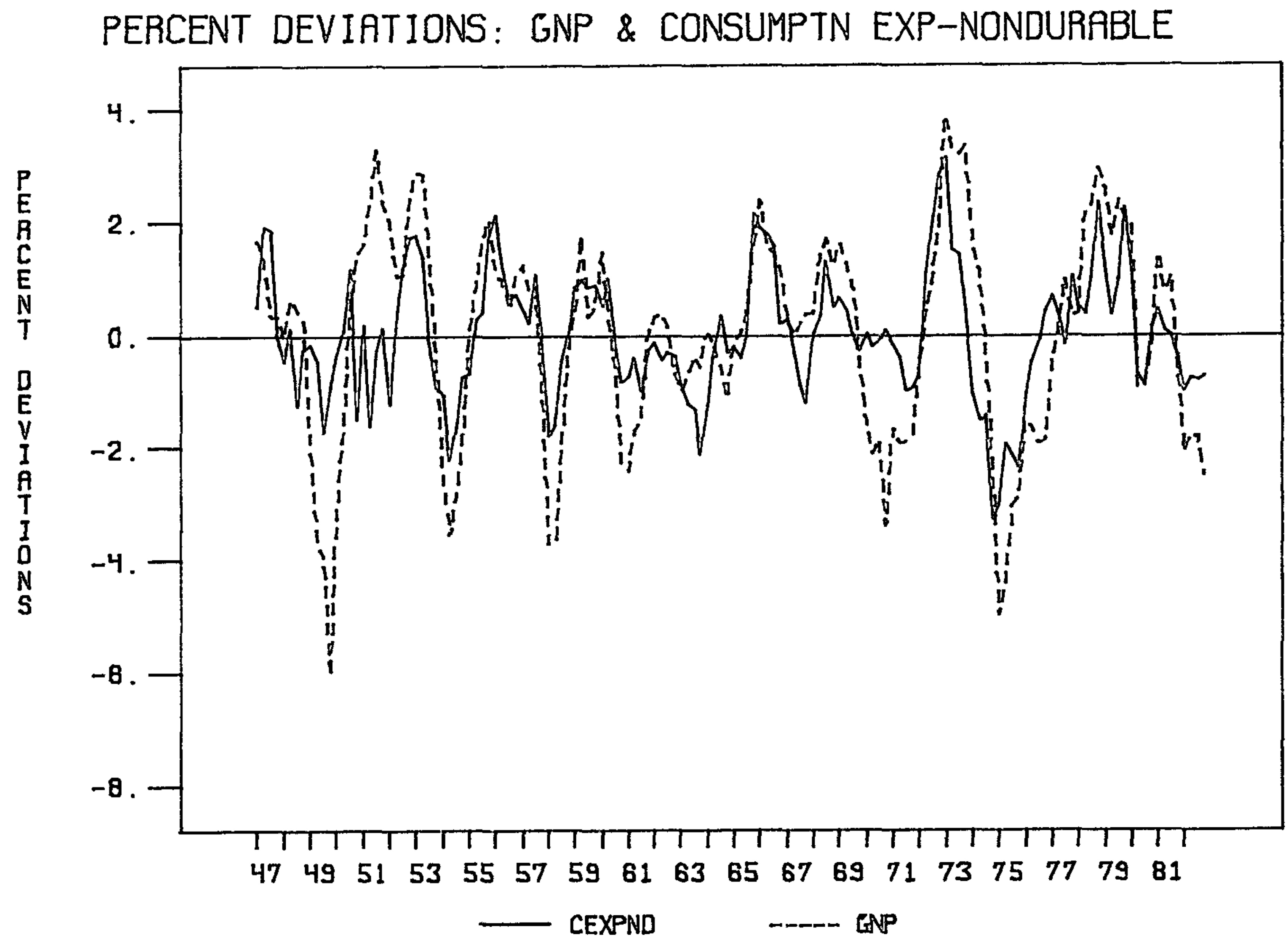
One question is what value of λ to choose. If $\lambda = \infty$, the fitted curve must be perfectly smooth which implies that the s_t will be of the form $s_t = \beta_0 + \beta_1 s_t$ for only then is $\Delta^2 s_t = 0$. We found that $\lambda = 1600$ produced curves with about the right degree of smoothness, when fit to the logarithm of the real GNP series. This value is used for the subsequent empirical analysis.

Figure 1 depicts the smooth path and the actual path of the logarithm of real GNP for the U.S. economy. As the logarithms of the series are used, deviations correspond to percentage deviations from the smooth curve. Figures 2-14 plot deviations of selected series jointly with the real GNP deviations, which is our empirical definition of the cycle. Figures 2, 3 and 4 show clearly that nondurables fluctuate less and durables more than GNP. The close relation between output and hours can be seen in Figure 5 and the weak positive association between productivity and real output is shown in Figure 6. That money and its velocity are procyclical variables is documented in Figure 8 and 9 and the lack of relation between real government purchases and output in Figure 10. Tables 1 to 4 present standard deviations of the deviations of several economic time series and their cross serial correlations with and real output. Variables with highest cross correlation for some $k < 0$ tend to peak before the cycle peaks and those for which the correlation is highest for some $k > 0$ peak later. For example, inventory stocks peak later than GNP while the average work week peaks before GNP.

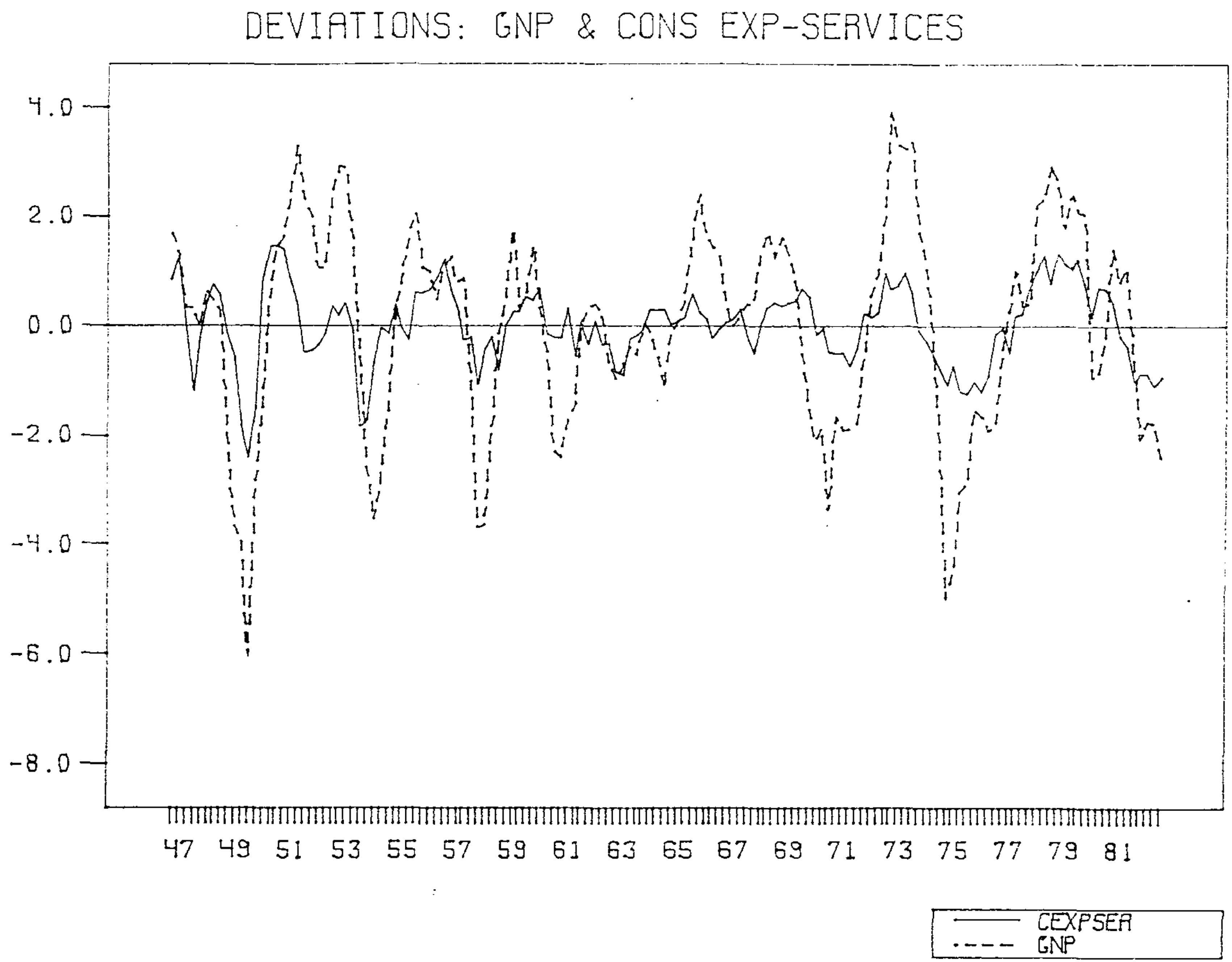

LOGGED GNP AND GNP TREND
BILLIONS OF 1972 $ LOGGED
7.4
7.2
7.0
6.8
6.6
6.4
6.2
6.0
47 49 51 53 55 57 59 61 63 65 67 69 71 73 75 77 79 81
RAW GNP
GNP TREND



PERCENT DEVIATIONS: GNP & CONSUMPTN EXP-DURABLE
30.
20.
10.
0.
-10.
-20.
PERCENT DEVIATIONS
47 49 51 53 55 57 59 61 63 65 67 69 71 73 75 77 79 81
CEXPDUR
GNP

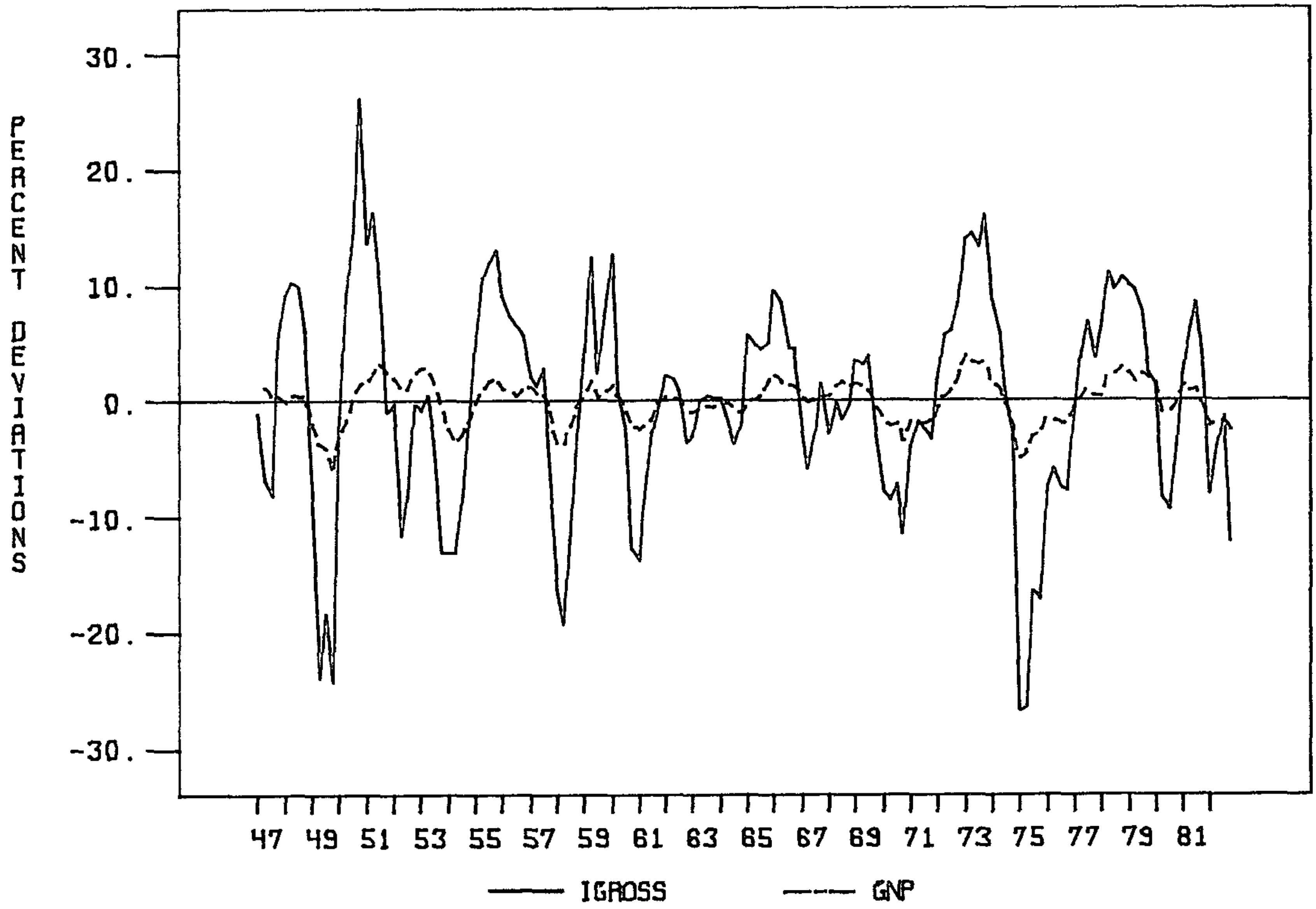


PERCENT DEVIATIONS: GNP & CONSUMPTN EXP-NONDURABLE
PERCENT DEVIATIONS
4.
2.
0.
-2.
-4.
-6.
-8.
47 49 51 53 55 57 59 61 63 65 67 69 71 73 75 77 79 81
CEXPND
GNP

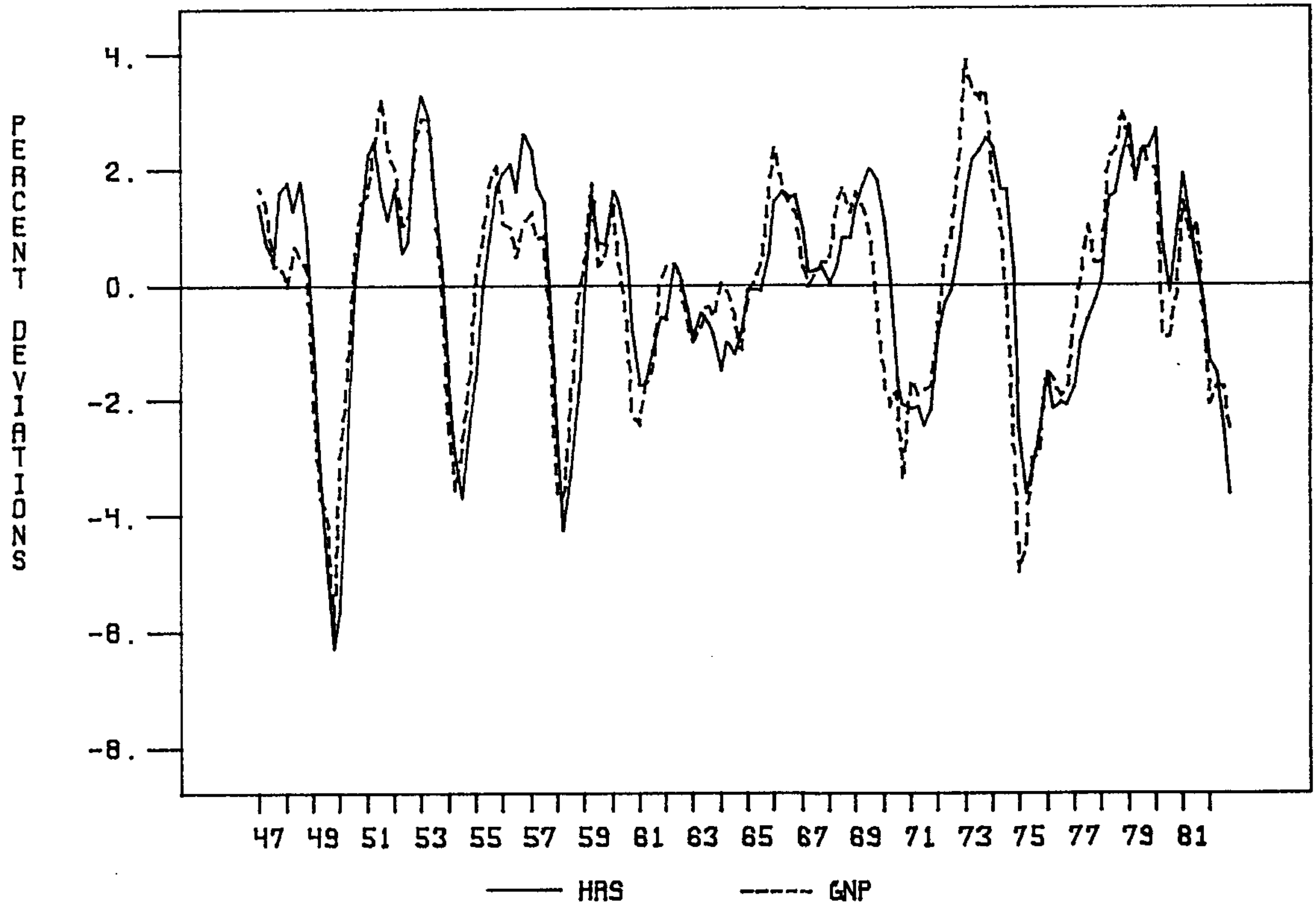


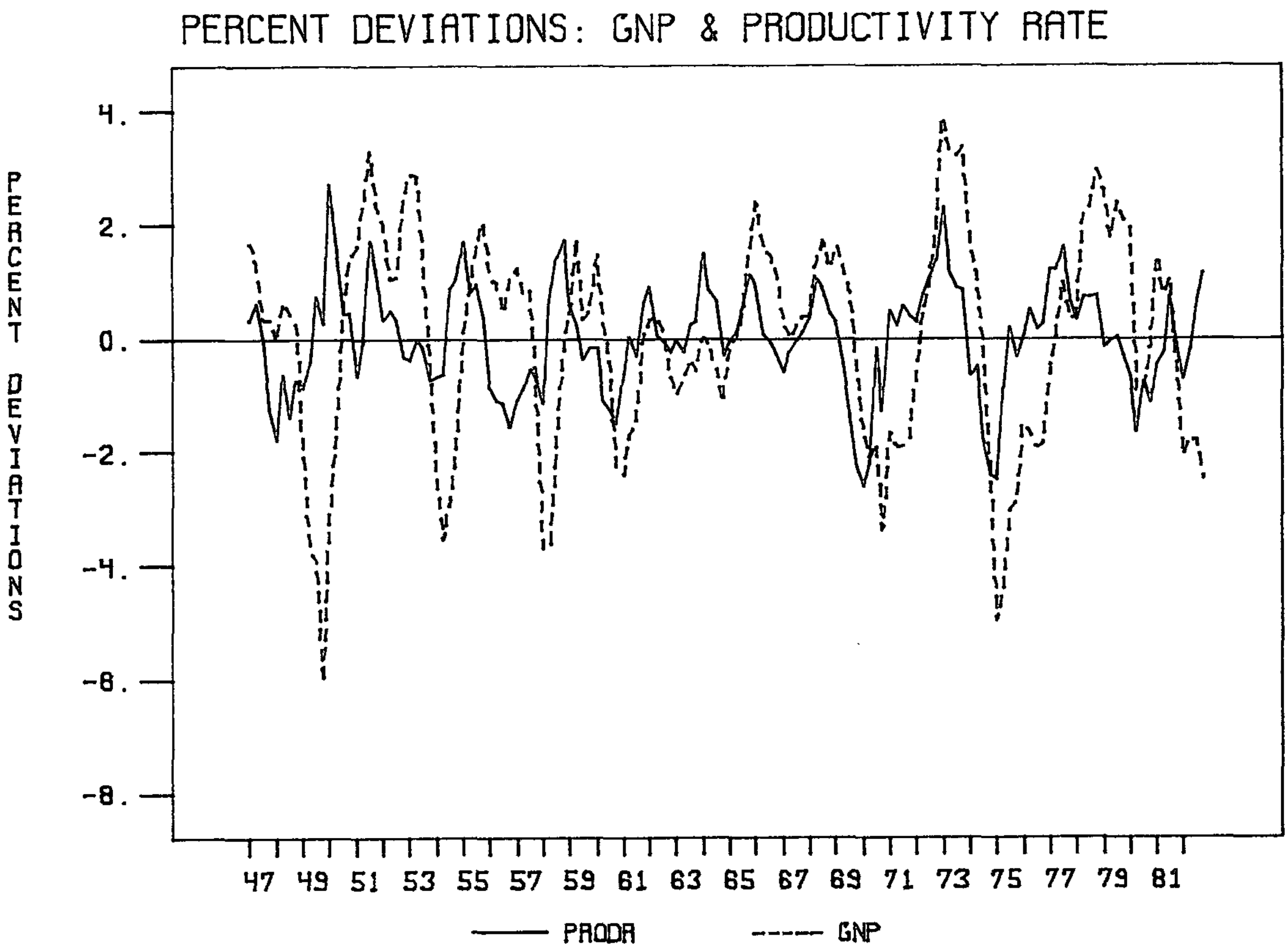
DEVIATIONS: GNP & CONS EXP-SERVICES
4.0
2.0
0.0
-2.0
-4.0
-6.0
-8.0
47
49
51
53
55
57
59
61
63
65
67
69
71
73
75
77
79
81
CEXPSER
GNP

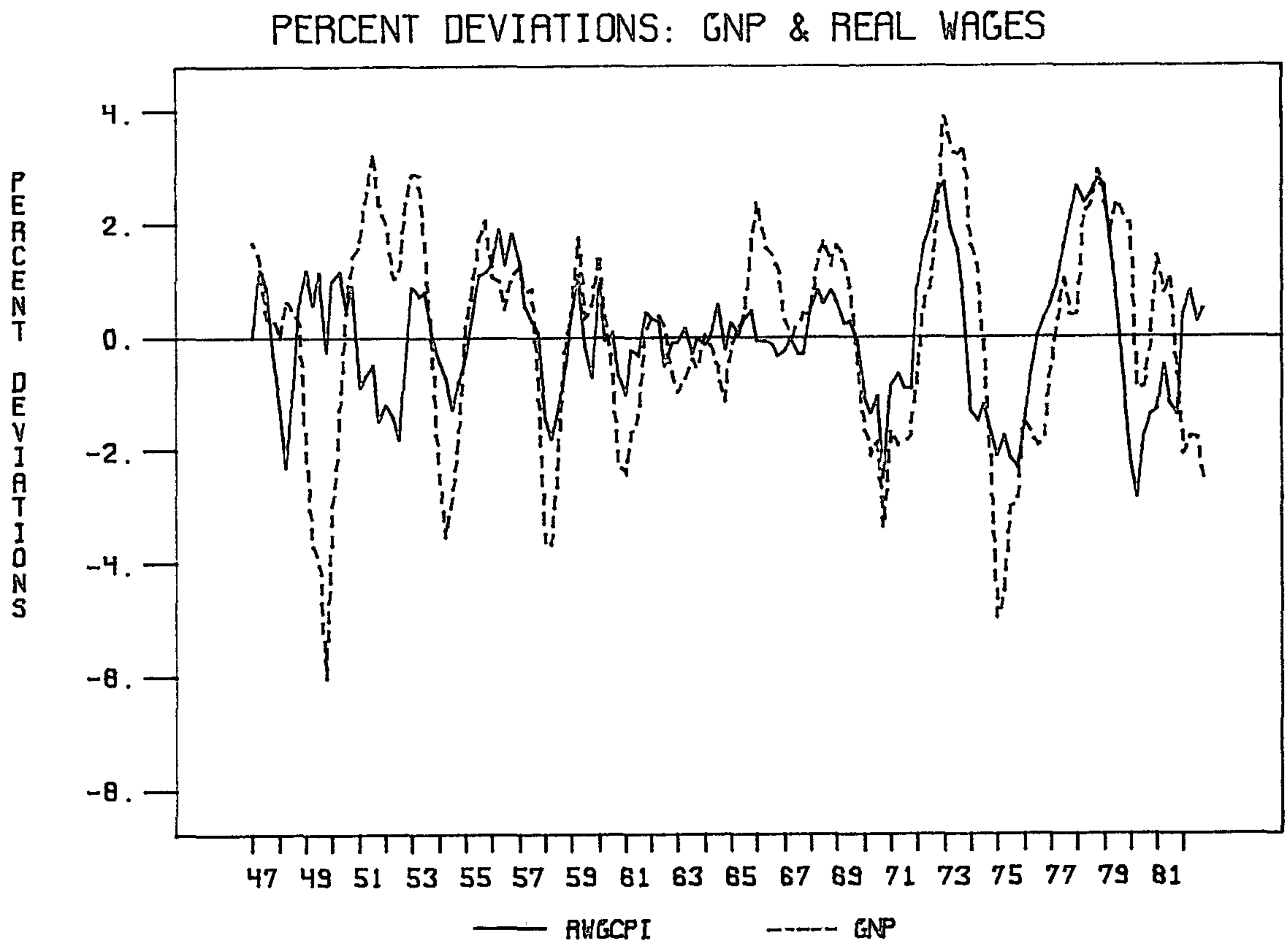
PERCENT DEVIATIONS: GNP & GROSS INVESTMENT

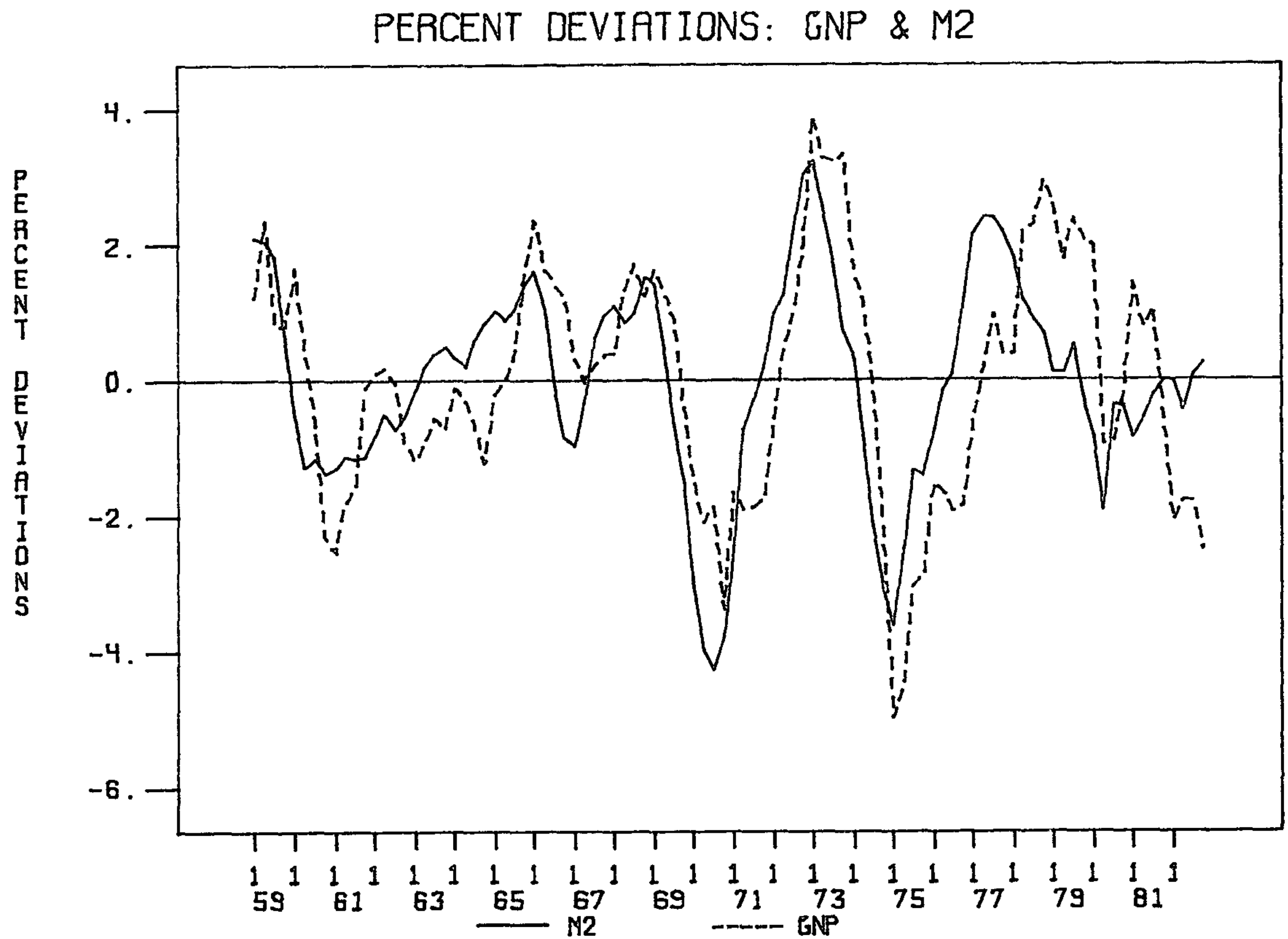


PERCENT DEVIATIONS: GNP & NONAG EMPLOYEE HOURS

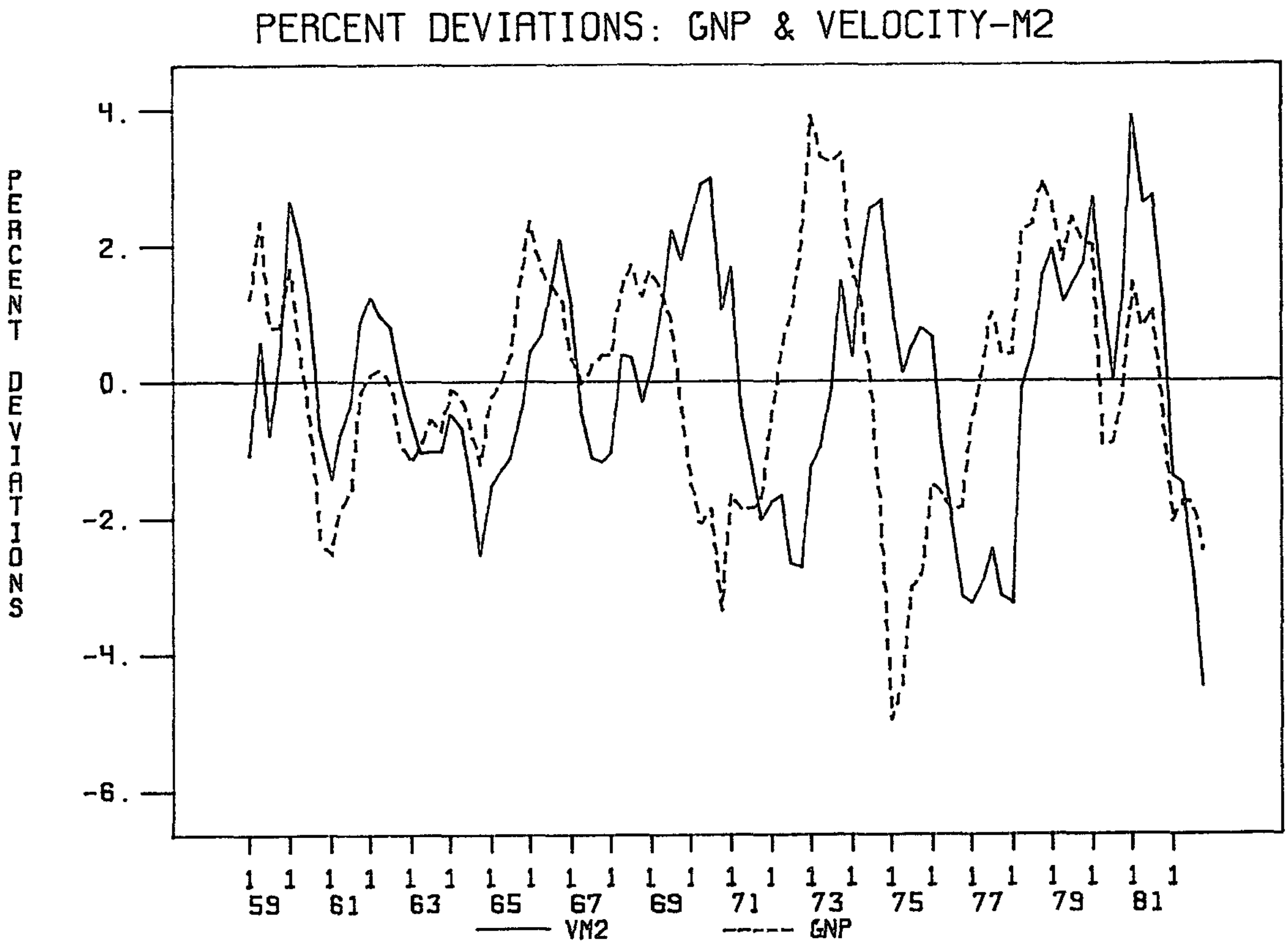


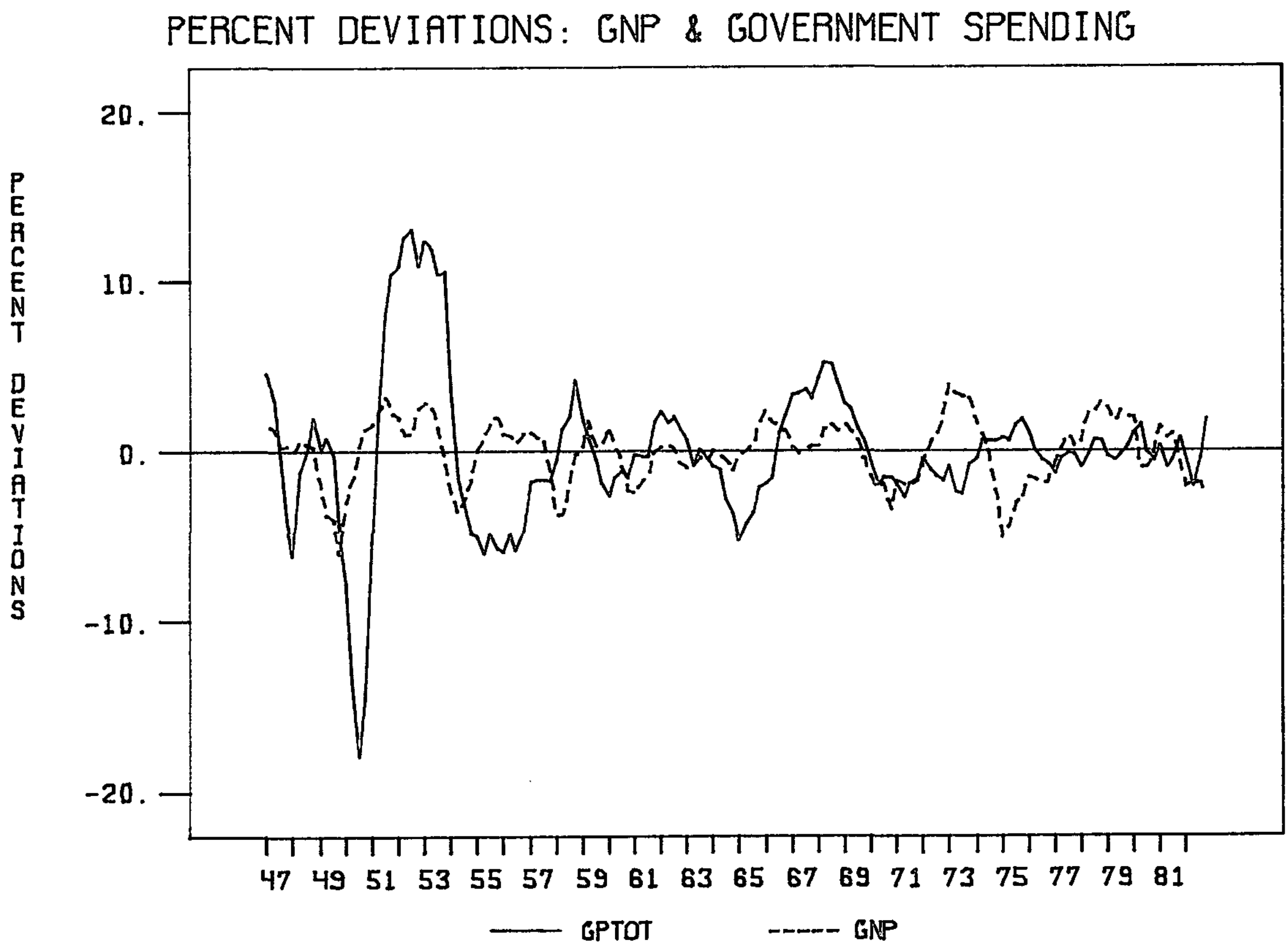

PERCENT DEVIATIONS: GNP & PRODUCTIVITY RATE
PERCENT DEVIATIONS
4.
2.
0.
-2.
-4.
-6.
-8.
47 49 51 53 55 57 59 61 63 65 67 69 71 73 75 77 79 81
PRODA
GNP

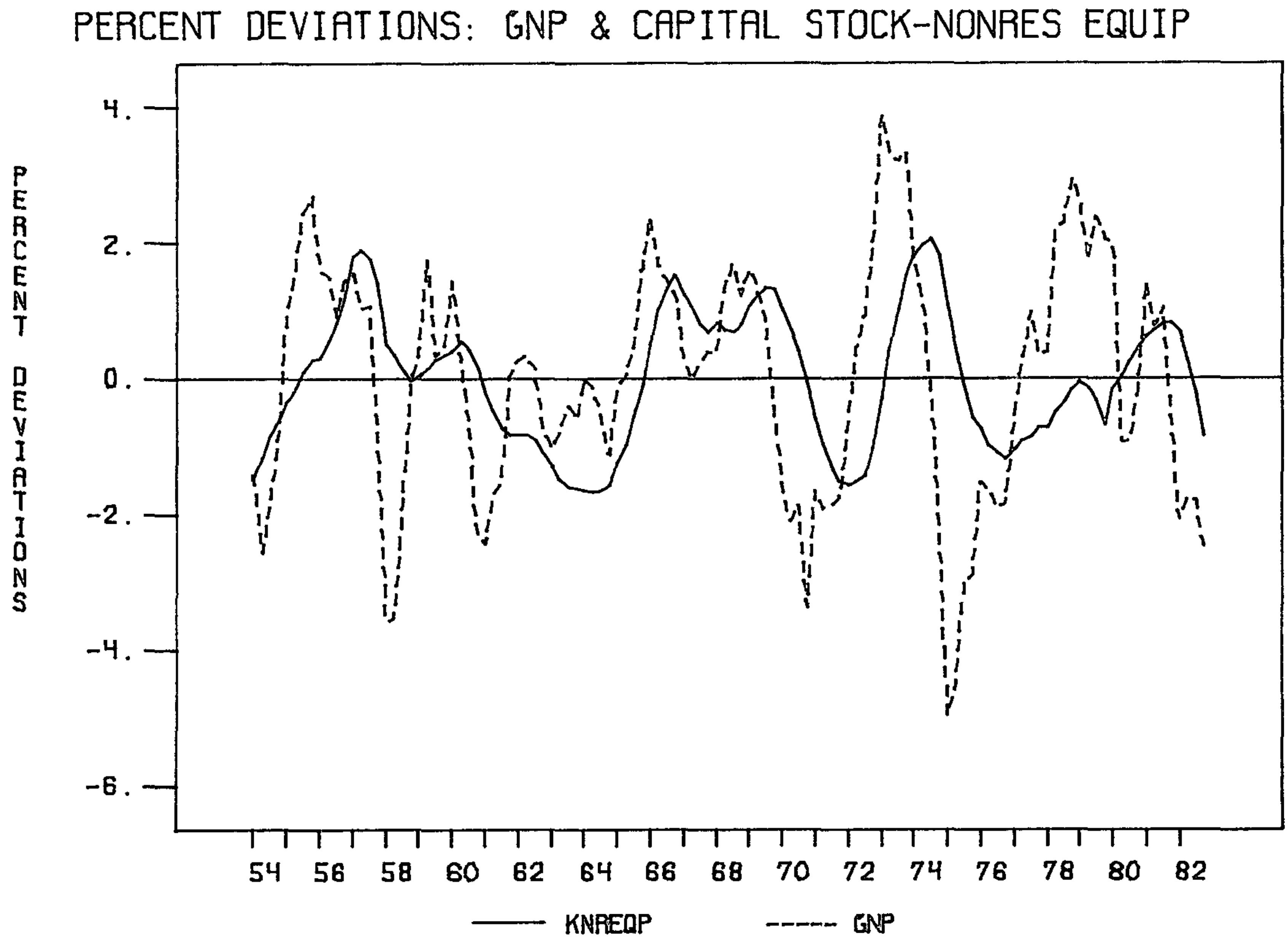

PERCENT DEVIATIONS: GNP & REAL WAGES
PERCENT DEVIATIONS
4.
2.
0.
-2.
-4.
-6.
-8.
47 49 51 53 55 57 59 61 63 65 67 69 71 73 75 77 79 81
RWGCPI
GNP



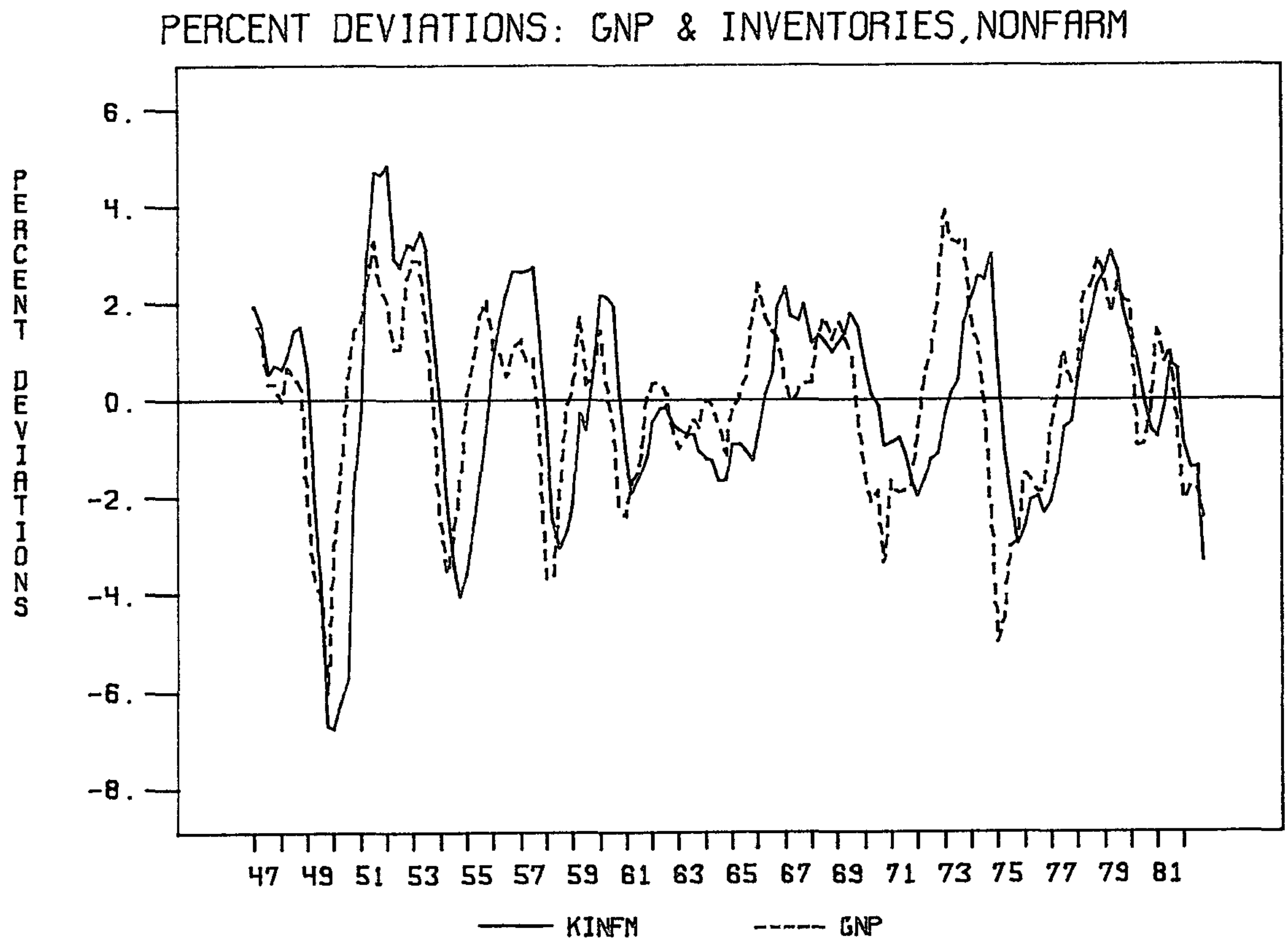
PERCENT DEVIATIONS: GNP & M2
PERCENT DEVIATIONS
4.
2.
0.
-2.
-4.
-6.
1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1
59 61 63 65 67 69 71 73 75 77 79 81
M2
GNP

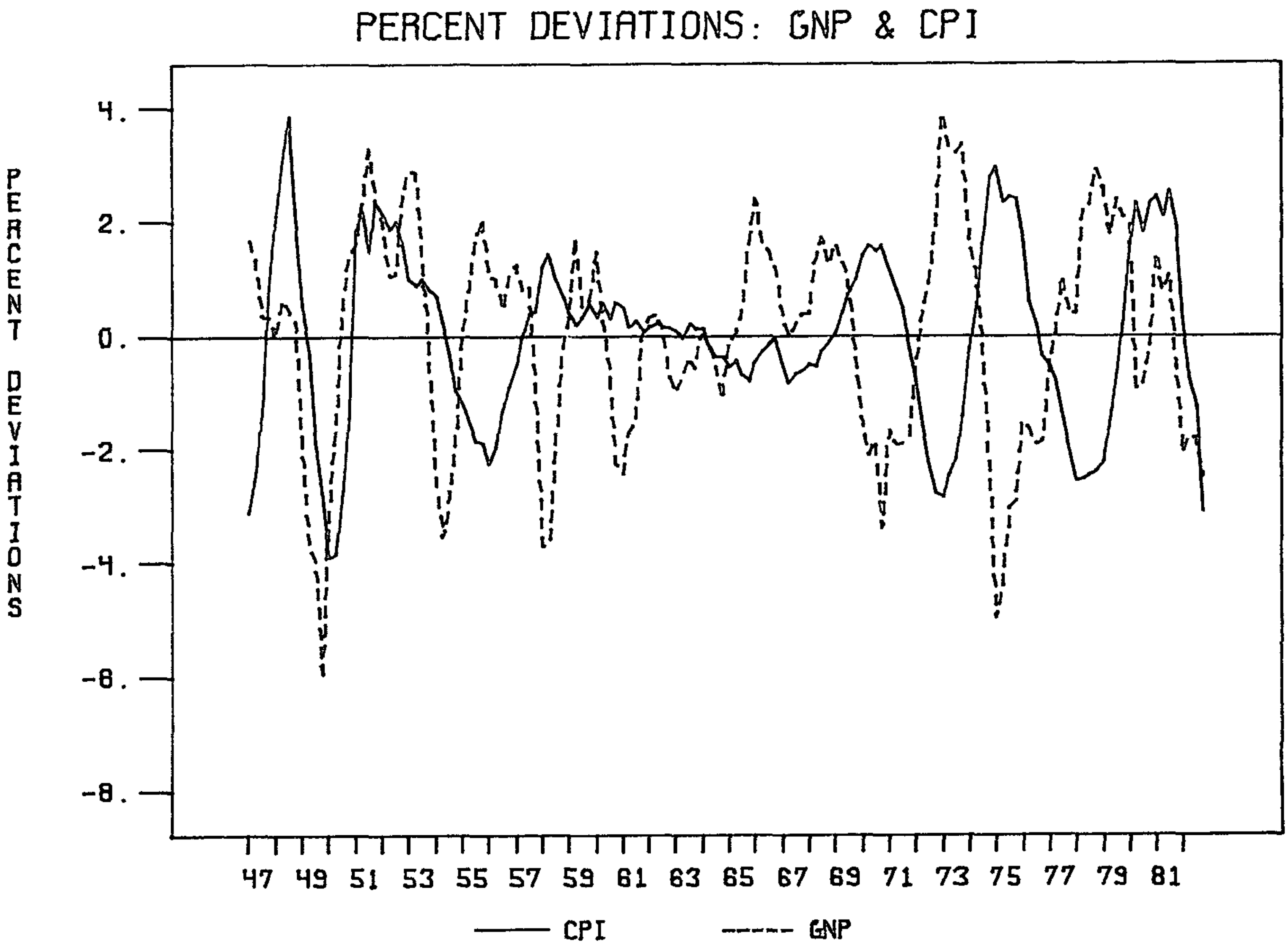

PERCENT DEVIATIONS: GNP & VELOCITY-M2
PERCENT DEVIATIONS
4.
2.
0.
-2.
-4.
-6.
1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1
59 61 63 65 67 69 71 73 75 77 79 81
VM2
GNP


PERCENT DEVIATIONS: GNP & GOVERNMENT SPENDING
20.
10.
0.
-10.
-20.
PERCENT DEVIATIONS
47 49 51 53 55 57 59 61 63 65 67 69 71 73 75 77 79 81
GPTOT
GNP



PERCENT DEVIATIONS: GNP & CAPITAL STOCK-NONRES EQUIP
PERCENT DEVIATIONS
4.
2.
0.
-2.
-4.
-6.
54
56
58
60
62
64
66
68
70
72
74
76
78
80
82
KNREQP
GNP


PERCENT DEVIATIONS: GNP & INVENTORIES, NONFARM
PERCENT DEVIATIONS
6.
4.
2.
0.
-2.
-4.
-6.
-8.
47 49 51 53 55 57 59 61 63 65 67 69 71 73 75 77 79 81
KINFM
GNP


PERCENT DEVIATIONS: GNP & CPI
PERCENT DEVIATIONS
4.
2.
0.
-2.
-4.
-6.
-8.
47
49
51
53
55
57
59
61
63
65
67
69
71
73
75
77
79
81
CPI
GNP

PERCENT DEVIATIONS: GNP & INFLATION

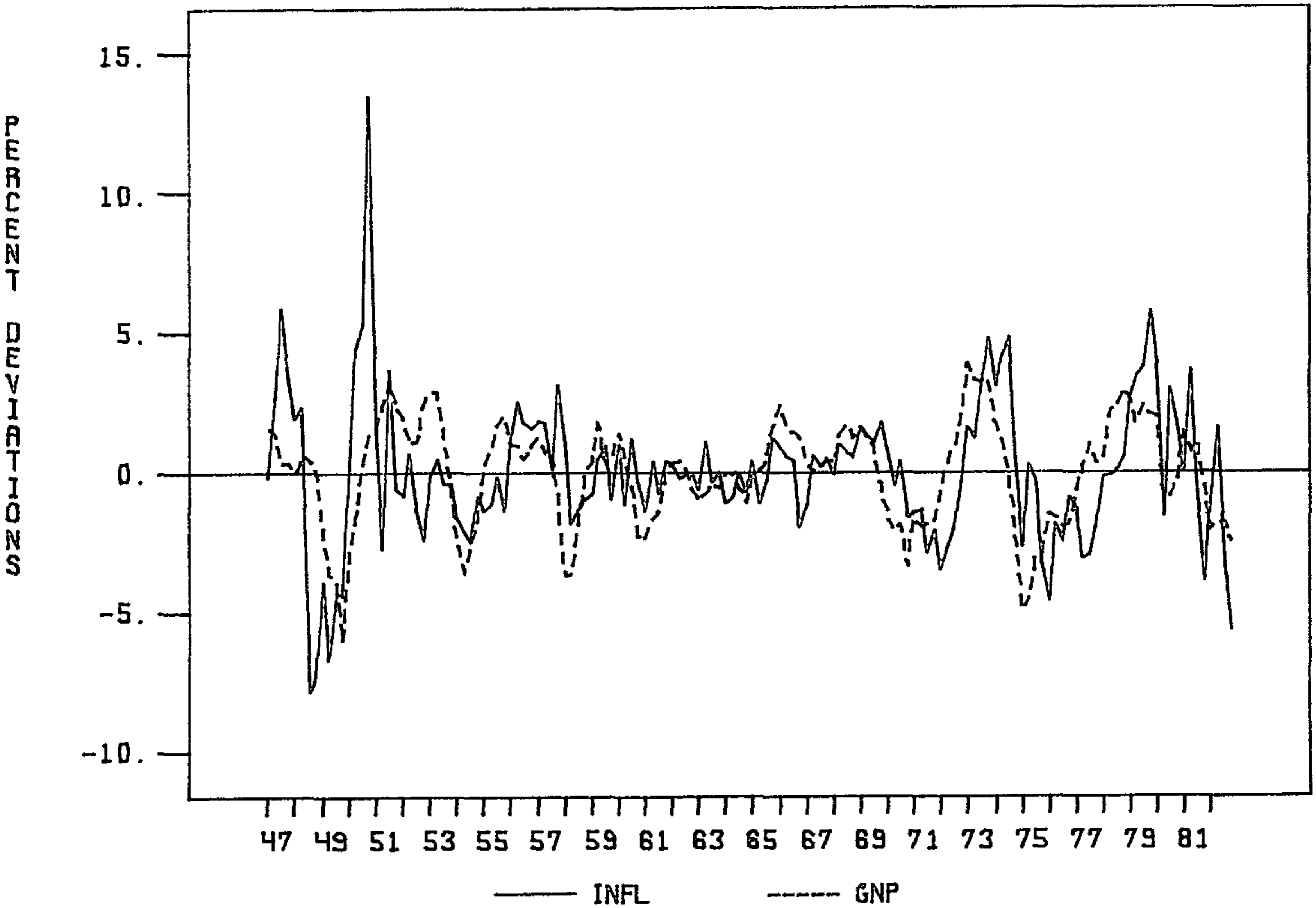


Table 1

Cyclical Behavior of Real GNP Components
Deviations from Trend

	Root Mean Squared Percentage Deviation	Cross Correlations with GNP -5	-4	-3	-2	-1	0	1	2	3	4	5
GNP	1.8	-.036	.143	.336	.566	.824	1.000	.824	.566	.336	.143	-.036
Consumption Expenditures	1.3	.155	.341	.484	.648	.811	.875	.672	.421	.197	-.019	-.222
Services	.6	-.040	.173	.365	.532	.664	.717	.606	.523	.454	.318	.110
Nondurables	1.2	.183	.339	.435	.546	.711	.763	.590	.349	.123	-.089	-.247
Durables	5.0	.156	.309	.434	.600	.745	.813	.606	.335	.105	-.097	-.280
Investment Expenditures	8.4	.021	.174	.354	.537	.761	.910	.726	.455	.207	-.011	-.198
Fixed	5.3	.046	.222	.392	.594	.778	.888	.777	.564	.341	.117	-.100
Nonresidential	5.2	-.234	-.101	.051	.279	.543	.794	.859	.793	.657	.484	.272
Structures	4.6	-.215	-.088	.061	.231	.421	.615	.702	.678	.580	.433	.249
Producers' Durable Equipment	6.0	-.237	-.114	.028	.271	.556	.819	.869	.784	.629	.455	.249
Residential	10.7	.348	.485	.587	.682	.705	.600	.344	.059	-.159	-.332	-.466
Nonfarm Structures	11.2	.352	.490	.593	.687	.706	.596	.340	.055	-.163	-.336	-.470
Farm Structures	21.6	.005	-.076	-.105	-.089	.017	.115	.081	.057	.000	-.002	.008
Producers' Durable Equipment	8.8	-.056	.093	.226	.351	.447	.471	.287	.115	.021	-.068	-.088
Government	2.4	-.052	-.085	-.109	-.097	-.067	-.049	.002	-.020	-.024	.035	.133
Federal	4.2	-.010	-.047	-.075	-.097	-.087	-.071	.011	.003	-.021	-.002	.058
State and Local	1.1	-.045	-.009	.030	.105	.121	.102	-.005	-.074	.021	.189	.342
Net Exports	6.4	-.401	-.474	-.465	-.398	-.310	-.172	-.075	.123	.310	.487	.576
Exports	5.9	-.489	-.479	-.383	-.185	.077	.359	.486	.531	.564	.602	.538
Imports	4.7	-.032	.086	.189	.328	.525	.689	.697	.469	.250	.050	-.157

Table 2

Cyclical Behavior of Factor Inputs and Labor Productivity
Deviations From Trend

	Root Mean Squared Percentage Deviation	Cross Correlations with GNP										
		-5	-4	-3	-2	-1	0	1	2	3	4	5
GNP	1.8	-.036	.143	.336	.566	.824	1.000	.824	.566	.336	.143	-.036
Labor Input												
Employment (Nonagricultural)	1.5	-.343	-.235	-.080	.156	.480	.783	.879	.801	.683	.538	.362
Average Weekly Hours (Mfg)	1.0	.013	.204	.357	.544	.757	.848	.606	.249	.004	-.186	-.286
Hours (Nonagricultural)	1.7	-.297	-.158	.008	.249	.572	.852	.885	.750	.605	.455	.279
Capital Stocks												
Total Nonfarm Inventories	1.7	-.392	-.368	-.283	-.127	.149	.481	.684	.740	.723	.652	.524
Stock of Nonresidential Equipment	1.0	-.561	-.516	-.422	-.259	-.032	.233	.406	.540	.622	.644	.627
Stock of Nonresidential Structures	.4	-.567	-.539	-.474	-.360	-.201	-.030	.163	.322	.424	.477	.498
Productivity (GNP/HRS)	1.0	.440	.521	.590	.598	.514	.335	-.041	-.275	-.426	-.510	-.541

Table 3

Cyclical Behavior of Labor Market Variables
Deviations From Trend

	Root Mean Squared Percentage Deviation	Cross Correlations with GNP										
		-5	-4	-3	-2	-1	0	1	2	3	4	5
Nominal Compensation Per Hour (Mfg)	.7	-.269	-.248	-.296	-.281	-.160	.047	.019	-.056	-.106	-.098	.004
Real Compensation Per Hour (Mfg)	1.2	.230	.425	.560	.642	.667	.634	.471	.264	.037	-.181	-.356
Civilian Labor Force	.5	-.087	-.000	.004	.069	.192	.344	.428	.443	.468	.527	.464
Unemployment	.8	.248	.144	-.013	-.252	-.586	-.875	-.903	-.758	-.576	-.363	-.160
Unemployment Duration	.1	.409	.399	.336	.196	-.052	-.359	-.649	-.803	-.813	-.749	-.605
Total Accession Rate (Mfg)	.4	-.134	.099	.371	.591	.772	.792	.531	.312	.131	-.041	-.189
Quit Rate	.3	-.300	-.128	.090	.360	.665	.893	.871	.711	.511	.309	.086
Separation Rate (Mfg)	.3	-.364	-.373	-.347	-.328	-.183	.046	.423	.607	.612	.599	.490
Layoff Rate (Mfg)	.4	-.003	-.164	-.347	-.590	-.763	-.780	-.462	-.146	.052	.221	.319

Table 4

Cyclical Behavior of Price Levels, Interest Rates and Money
Deviations From Trend

	Root Mean Squared Percentage Deviation	Cross Correlations with GNP -5	-4	-3	-2	-1	0	1	2	3	4	5
Prices												
GNP Deflator	.9	-.374	-.520	-.651	-.694	-.631	-.457	-.364	-.282	-.142	.027	.241
CPI	1.4	-.331	-.502	-.653	-.728	-.693	-.548	-.425	-.271	-.084	.122	.331
CPI Less Food	1.3	-.164	-.372	-.586	-.724	-.783	-.754	-.634	-.494	-.304	-.128	.063
PPI	2.4	-.513	-.608	-.673	-.657	-.544	-.355	-.170	.008	.196	.356	.458
PPI Less Farm Products	2.4	-.407	-.544	-.667	-.703	-.624	-.457	-.284	-.109	.083	.266	.412
Interest Rates												
Treasury Bill Rate	1.3	-.562	-.519	-.438	-.247	.071	.387	.471	.471	.464	.491	.482
Long Term Interest Rate	.5	-.489	-.503	-.518	-.493	-.293	.012	.032	.068	.124	.213	.334
Real Treasury Bill Rate*	1.4	-.123	-.174	-.260	-.247	-.215	-.029	.039	-.048	-.104	-.114	-.060
Monetary Aggregates												
M0	.8	-.121	-.067	.018	.199	.334	.441	.442	.413	.424	.445	.457
M1	.9	-.169	.003	.200	.478	.633	.691	.607	.477	.383	.305	.187
M2	1.5	.444	.596	.717	.789	.750	.624	.405	.160	-.068	-.283	-.462
Velocities												
M0	1.2	-.158	-.035	.095	.267	.509	.745	.616	.419	.226	.096	.006
M1	1.1	-.112	-.087	-.055	.014	.246	.535	.468	.337	.205	.135	.148
M2	1.7	-.569	-.597	-.582	-.453	-.184	.168	.251	.318	.389	.502	.616

*Nominal Rate Less
Inflation Rate

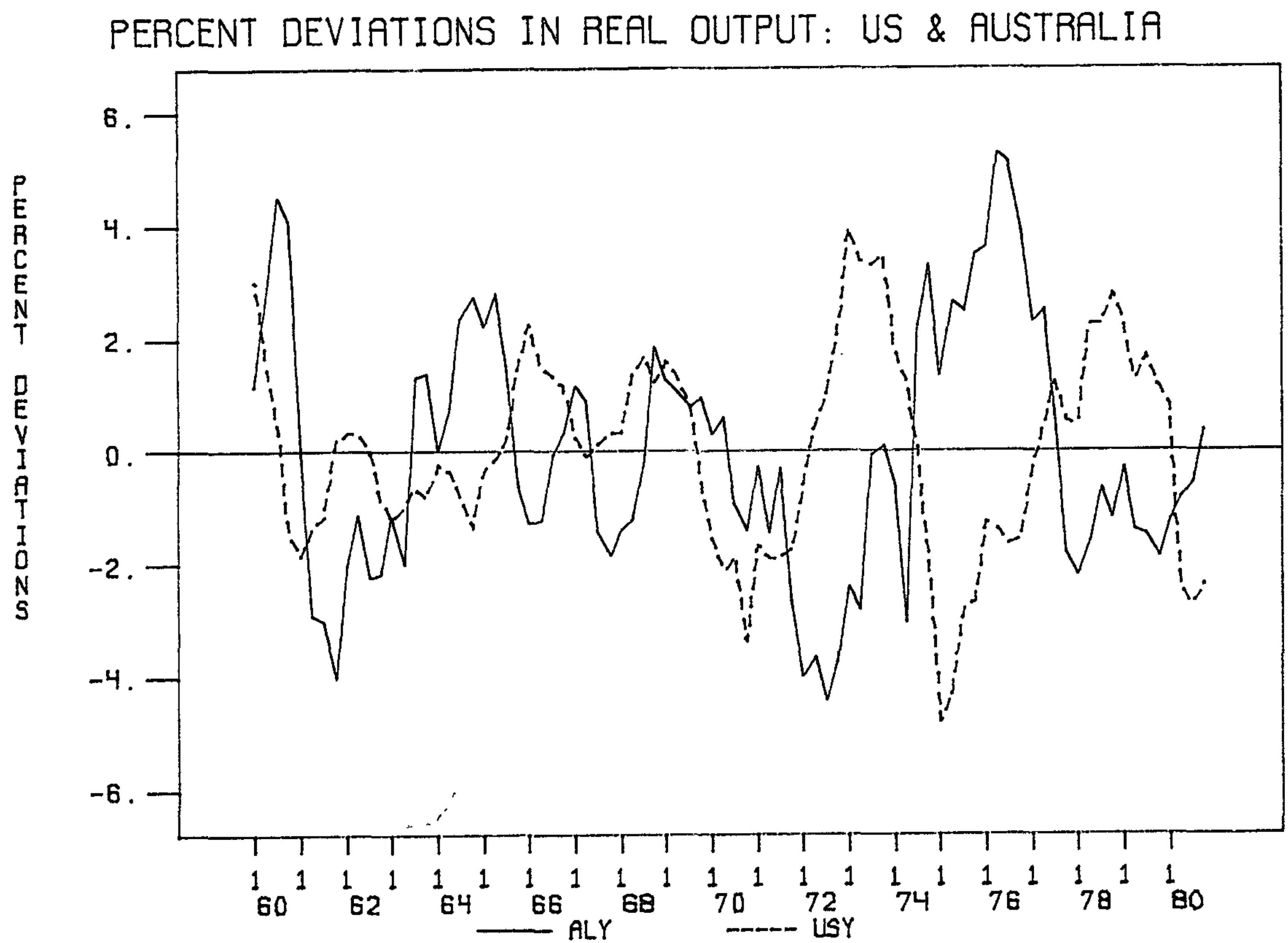
Table 5

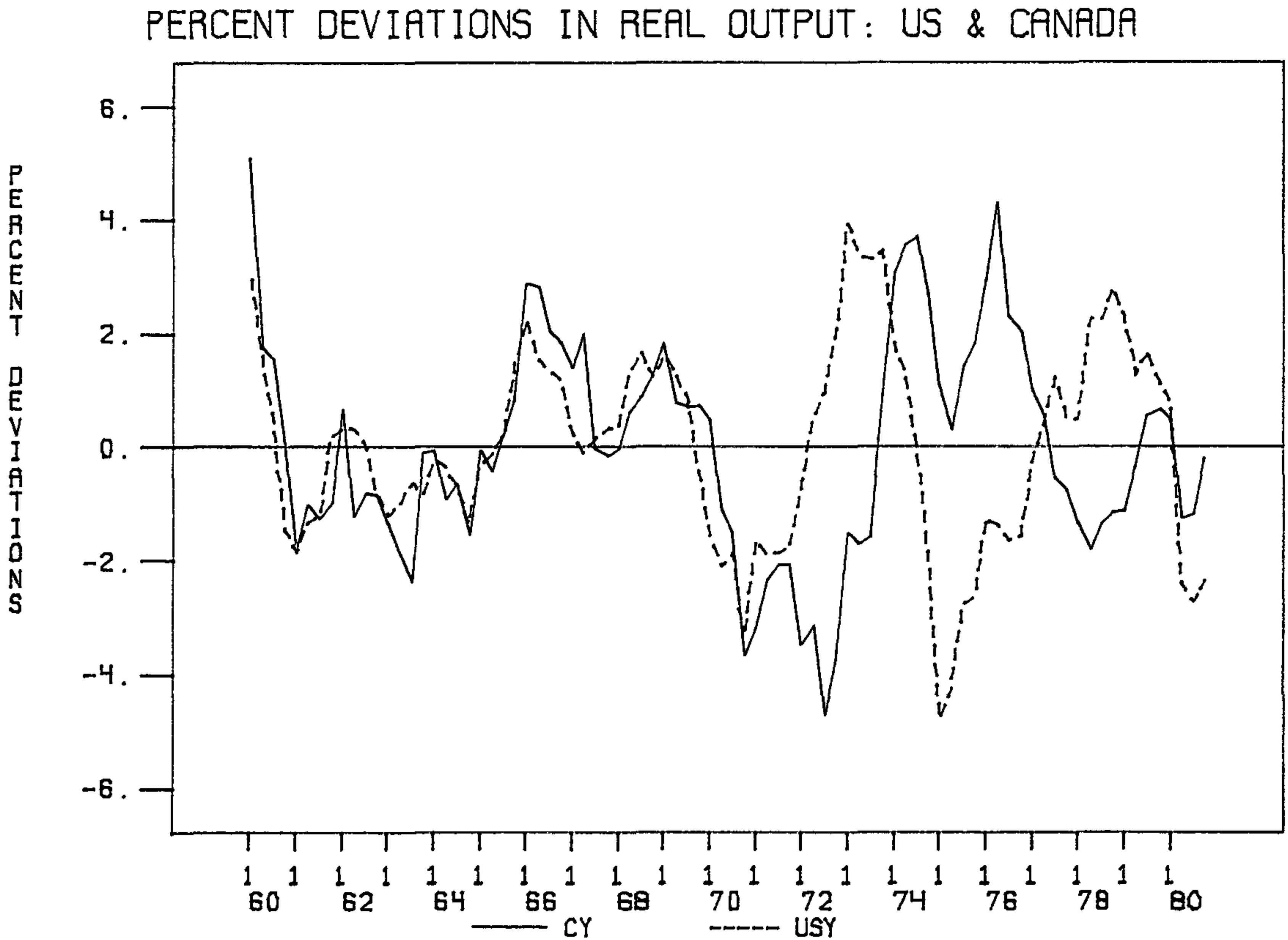
Comovements Between the U.S. Cycle and the Cycle of Selected Industrial Market Economies
Deviations From Trend

	Root Mean Squared Percentage Deviation	Cross Correlations with U.S.										
		-5	-4	-3	-2	-1	0	1	2	3	4	5
United States	1.8	-.025	.211	.460	.651	.836	1.000	.836	.651	.460	.211	-.025
Germany	2.0	-.049	-.054	.011	-.004	.071	.133	.139	.074	-.001	-.102	-.207
Korea	5.2	.023	-.100	-.156	-.182	-.141	-.055	-.050	-.013	-.002	.016	.056
United Kingdom	2.1	.368	.291	.145	-.062	-.176	-.313	-.473	-.531	-.534	-.496	-.371
Japan	2.3	-.582	-.588	-.524	-.387	-.239	-.118	-.021	.098	.196	.353	.422
Canada	1.9	-.391	-.457	-.431	-.333	-.160	.118	.172	.205	.259	.242	.229
Australia	2.2	-.147	-.246	-.328	-.421	-.407	-.300	-.227	-.205	-.161	-.109	-.035
South Africa	3.2	-.580	-.593	-.567	-.491	-.397	-.223	-.012	.196	.337	.444	.489

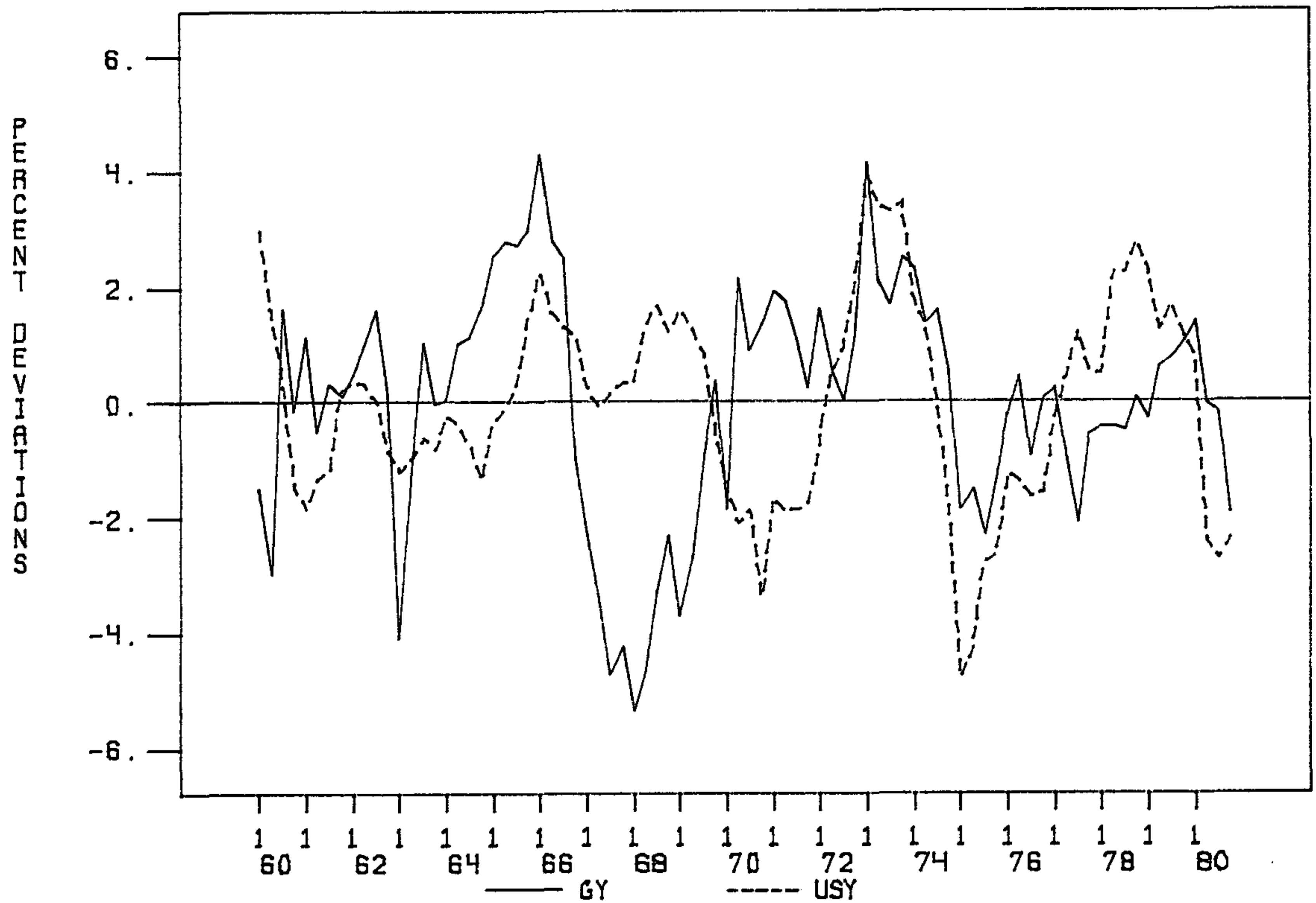
Section 3

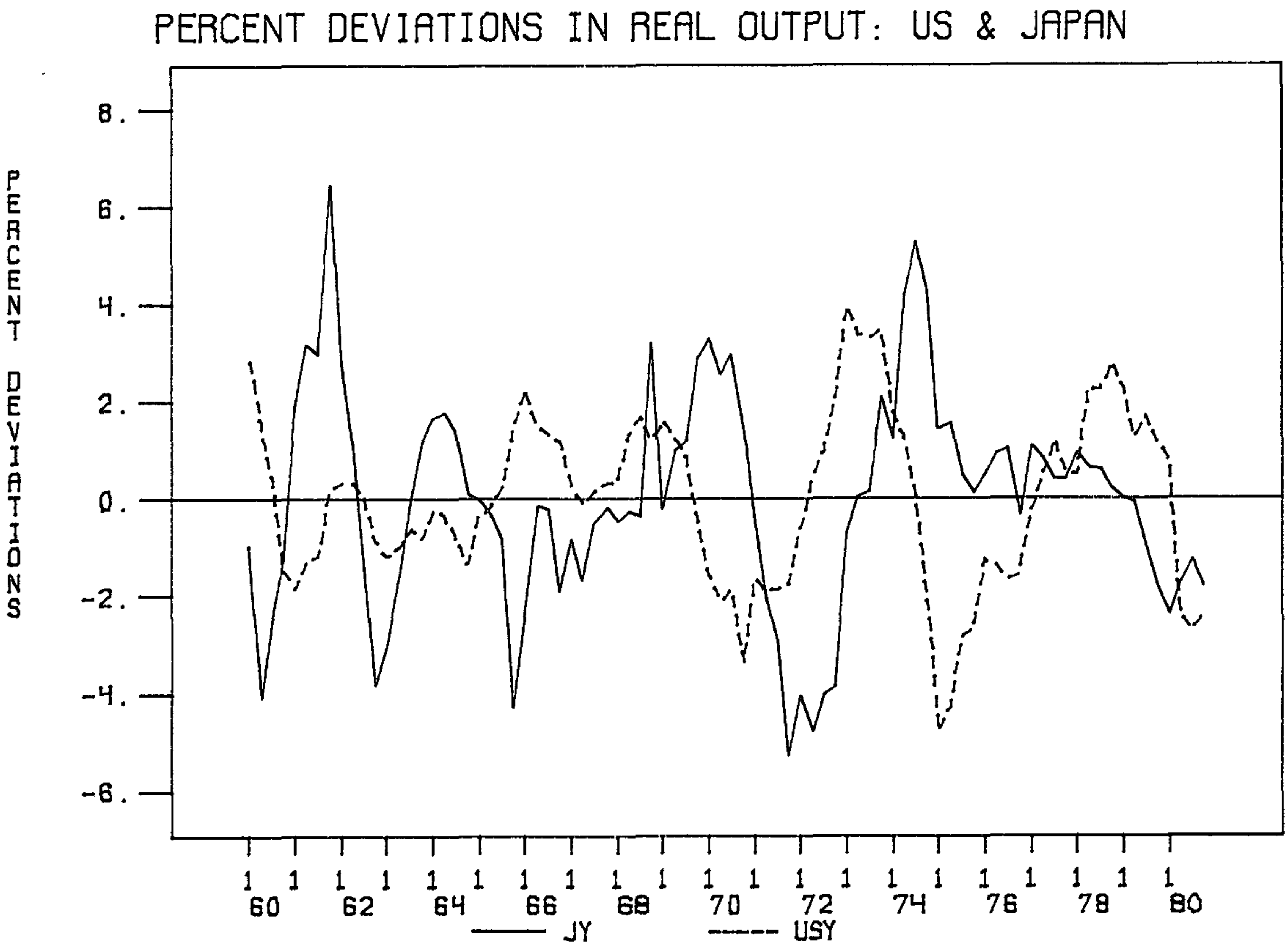
In Figure 15-21 the cycle for seven industrial market economies are plotted along with the U.S. cycle. As can be seen all these countries display fluctuations that are similar in nature to those for the United States. Table 5 presents the cross serial correlations between each of these countries' cycles and the U.S. cycle. The associations are remarkably weak, suggesting that fluctuations are, in significant part, specific to a country. A notable exception is the co-movement of the U.S. and Canada cycle prior to 1970. All major industrial market economies for which at least 20 years of quarterly data were readily available were included in the study.2/ The analysis strongly support Lucas's surmise that all industrial market economies are subject to business cycles.

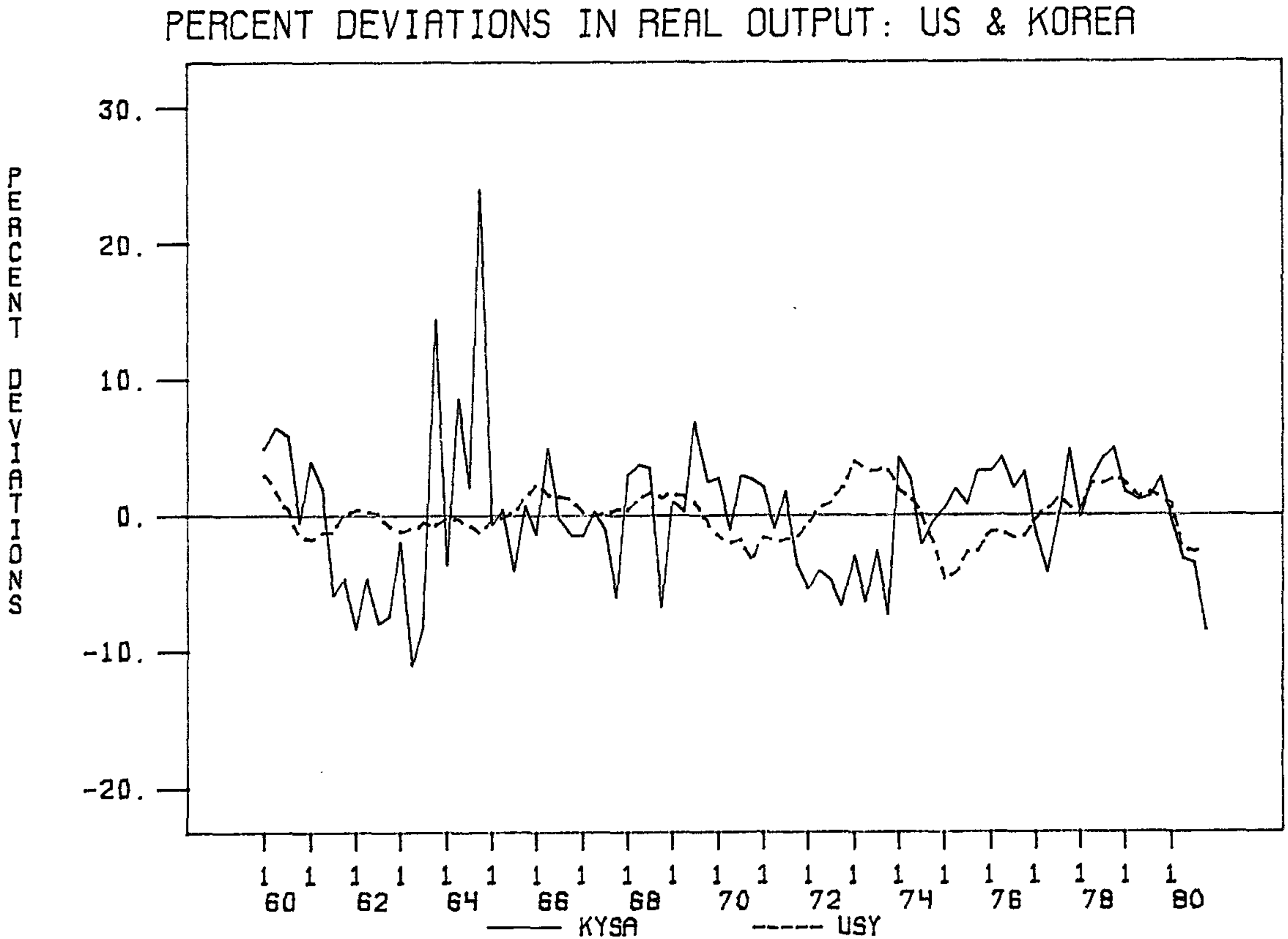

PERCENT DEVIATIONS IN REAL OUTPUT: US & AUSTRALIA
PERCENT DEVIATIONS
6.
4.
2.
0.
-2.
-4.
-6.
60
62
64
66
68
70
72
74
76
78
80
ALY
USY


PERCENT DEVIATIONS IN REAL OUTPUT: US & CANADA
PERCENT DEVIATIONS
6.
4.
2.
0.
-2.
-4.
-6.
60
62
64
66
68
70
72
74
76
78
80
CY
USY

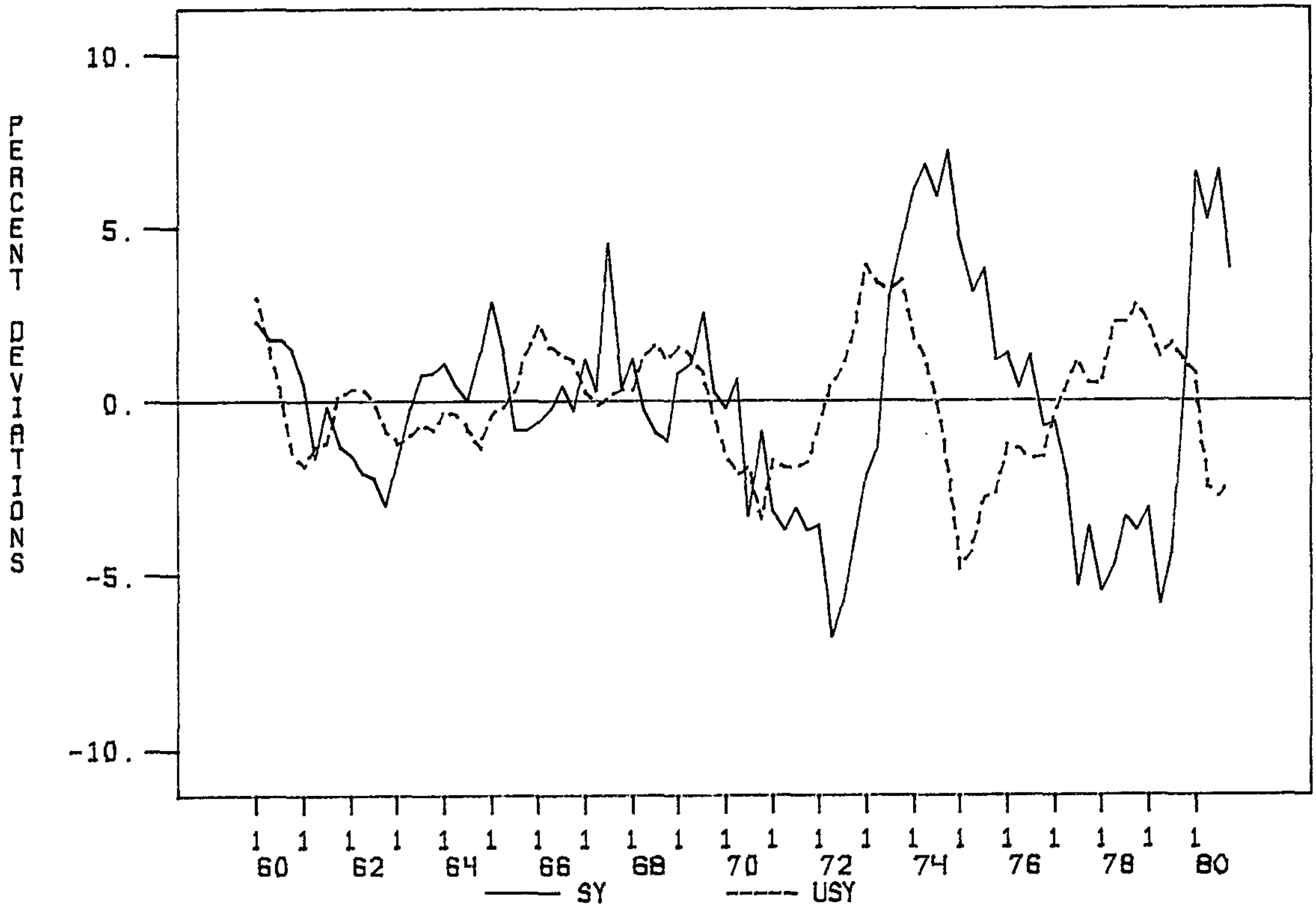
PERCENT DEVIATIONS IN REAL OUTPUT: US & GERMANY



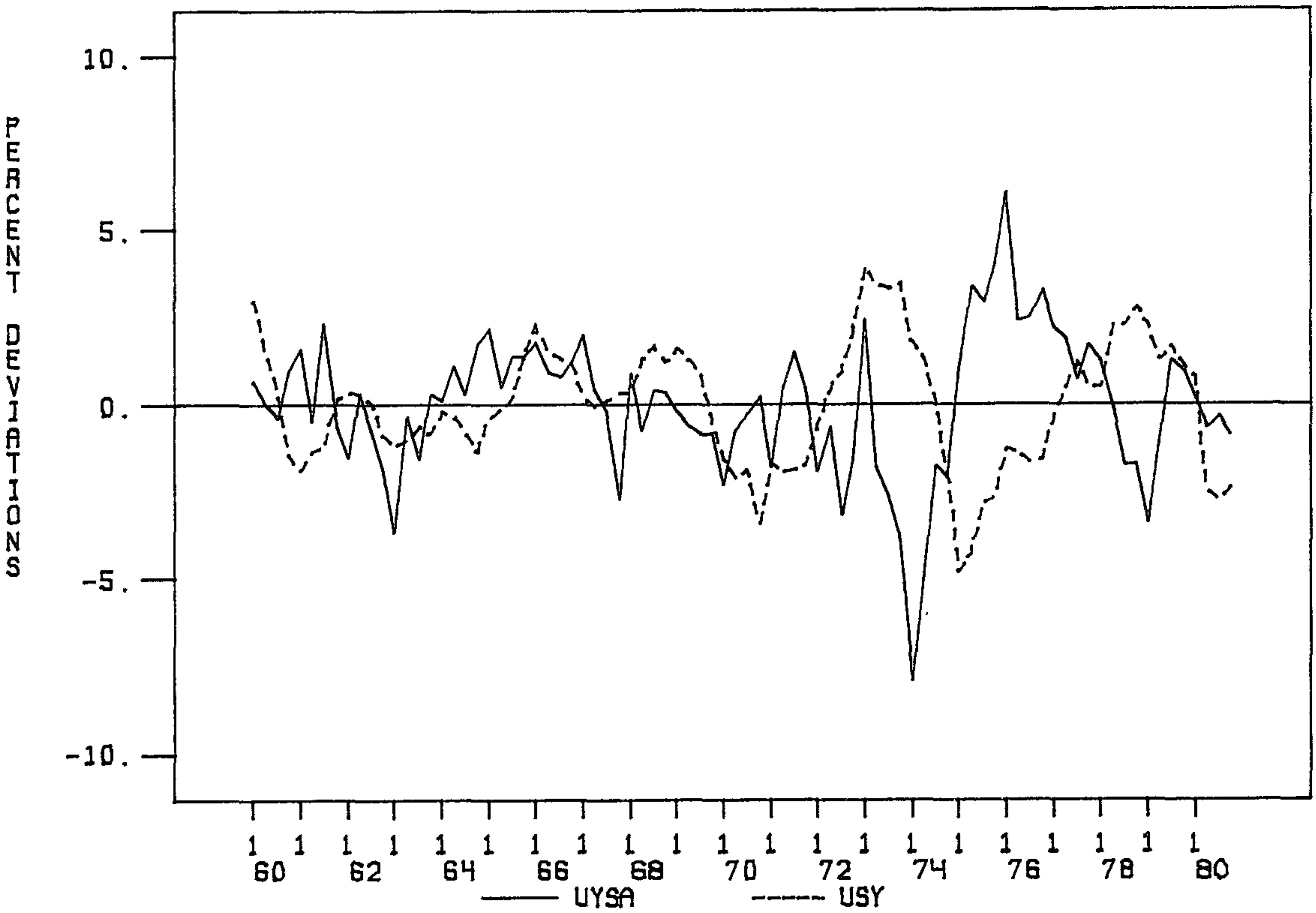

PERCENT DEVIATIONS IN REAL OUTPUT: US & JAPAN
PERCENT DEVIATIONS
8.
6.
4.
2.
0.
-2.
-4.
-6.
1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1
60 62 64 66 68 70 72 74 76 78 80
JY
USY


PERCENT DEVIATIONS IN REAL OUTPUT: US & KOREA
30.
20.
10.
0.
-10.
-20.
PERCENT DEVIATIONS
60
62
64
66
68
70
72
74
76
78
80
KYSA
USY

PERCENT DEVIATIONS IN REAL OUTPUT: US & SOUTH AFRICA



PERCENT DEVIATIONS IN REAL OUTPUT: US & UK



Section 4: Methodology

The methodology employed is that of constructing explicit, artificial economies which display business cycles of the type industrial market economies experienced in the post-war period. Necessarily, the economies studied are highly abstract and clearly unrealistic and, as Lucas (1981, p. 271) emphasizes, "they are artificial." One rather severe requirement for a model is that its equilibrium be computable at reasonable cost given current technology. In insisting upon computability, this approach is in the tradition of Scarf (1973), who fostered the development of algorithms to solve systems of excess demand functions. Since then and, I think, because of that development, general equilibrium theory has been playing an increasingly important role in substantive economics, particularly in the public finance field. Only if a model's equilibrium can be computed can the quantitative implications of the model be deduced. All too often, without restrictions upon the parameters defining the artificial economy, about the only implication of equilibrium theory is that anything can happen, which is not a very useful result.

The Scarf approach is of little use in computing equilibria of models designed to explain the business cycle. Business cycles are recurrent random events, so the commodity point is necessarily infinite dimensional, being indexed by the history of shocks and the time period, the number of which is infinite. The alternative approach, reviewed here, is to assume a representative or stand-in household with well-defined preferences and a production technology, and then use the competitive construct to determine equilibrium behavior for the artificial economy being studied.

The equilibrium for a model is a set of stochastic difference equations

$$x_{t+1} = F(x_t, \varepsilon_t, \omega)$$

where the x_t are the state, the ε_t i.i.d. shocks and ω the parameter defining the preferences and technology structure. Given ω, stochastic simulations produce equilibrium path realizations for the artificial economy, say $x = \{x_t\}_{t=0}^{T} \varepsilon X$. Any statistics (that is an $s: X \rightarrow R^n$) sampling distribution can be determined cheaply using Monte Carlo techniques.

The traditional econometric approach is to specify some set of economies, Ω, typically a subset of a finite dimensional Euclidean space, and to select an estimator $\hat{\omega}: X \rightarrow \Omega$ which has "good" statistical properties. One estimator commonly used is the one which maximizes the likelihood function over Ω. With the traditional econometric approach, the domain of the likelihood function is expanded to $X \times \Omega^*$ where $\Omega \subseteq \Omega^*$. For example, if Ω is a small subset of the linear vector autoregressive (VAR) processes, as it typically is for the structures that can be analyzed, Ω^* would be a weakly restricted class of VAR models (see Sims (1980)). The null hypothesis that $\omega \varepsilon \Omega$ is tested versus the less restricted alternative $\omega \varepsilon \Omega^*$ using a likelihood ratio test, and almost inevitably the theory is rejected. This leads to the introduction of arbitrary processes on unobserved shocks to technology or preferences until a theory is developed that places very weak restrictions upon the data and passes the test. The view expressed here is that this is not good inference practice. A model may mimic the cycle well but not perfectly.

An alternative econometric approach is briefly reviewed here and applied to the technology-shock equilibrium model in Section 5. With it, a requirement is that nearly all of the parameters that define the artificial economy be determined by noncyclical observations and findings in other economic studies. For the model in the next section, the fact that the household has been observed to allocate about one-third of its nonsleeping time to market activities restricts a key parameter of preferences to be near one-third. The fact that labor share has been approximately constant at two-thirds of output (which includes services of consumer durables) determines another. Unlike the statistical approach, there are few free parameters and explaining the cycle is challenging. It is important to emphasize that the artificial economy must not only qualitatively mimic the data but also must quantitatively mimic it and, mimic it to a "reasonable" degree of accuracy. What is reasonable depends upon the accuracy and reliability of the data relative to the theoretical model and the nature of the abstractions. Only statistics whose sampling distributions are not sensitive to "whimsical" assumptions (to quote Leamer (1983)), that is assumptions of convenience, should be used to test the theory. If it is a theory of the cycle, these statistics should include the degree and persistence of the fluctuations as well as the statistics summarizing some of the key comovements. Inconsistency with <u>any</u> of the key cycle facts is grounds for rejecting the model.

Criticism of the Approach

The major skeptics to this approach are the experts in the application of the statistical discipline to inference in economics. The response to their criticisms is that the paucity of free parameters provides the discipline. Assumptions concerning preferences and technology that are crucial to explaining the cycle typically can be subjected to test by examining some naturally occurring experiment that is well suited to testing the validity of the assumption. The experiment might be a negative income tax experiment conducted by the government, a set of cross country observations, a panel survey study or some other cheaply obtainable data set.

Some other criticisms are summarized by the following questions: (a) How can equilibrium theory be used to explain aggregate fluctuations when there currently is ten percent unemployment in the advanced market economies? (b) How can a theory claim to explain the business cycle if it cannot explain the Great Depression? (c) How can a model with a representative household that abstracts from contractual elements be a model of an economy involving hundreds of millions of people each with unique problems? (d) Given there are externalities, how can an artificial economy with none serve as a theory of the cycle?

That approximately 60 percent of the people surveyed report that they are employed and another six that they are not employed but have sought employment in the previous four weeks is a fact. The successes in labor economics to date have not been in explaining the time allocated to searching and waiting for employment but rather in explaining the number of nonsleeping hours allocated to market activity. Modern contract theory provides a possible reason for this. Small changes in the underlying environment sometimes result in large changes in unemployment but typically result in only small changes in average hours of employment. I am confident that advances in economic theory will result someday in models which explain the large secular movements and cross sectional variations in the unemployment rate. But now, even regional variations in the association between the cyclical unemployment and aggregate output are unexplained. Why does unemployment in the Ninth Federal Reserve District vary half as much as unemployment nationally yet employment and output fluctuate the same amount? I have no answer. There is a regularity in employment variation which is both important and puzzling and warrants study.

The answer to question (b) is simply that competitive equilibrium theory is not suited to modelling economic fluctuations in periods of great political and financial institution instability. The inability of either the equilibrium monetary or the technology shock theories to explain the Great American Depression is evidence of the discipline of the methodology. If any observations can be rationalized with some approach, then that approach is not scientific.

Perhaps contractual elements are the essence of the cycle. The nominal wage equilibrium contract model developed in

Section 7 relies upon it, though good theoretical justification for contracting in nominal terms has yet to be developed.[3] Quite likely when the contracting element is properly introduced (that is, introduced without adding free parameters such as contract length) the model will resemble Lucas's (1972) competitive monetary shock theory which has prices imperfectly conveying information.

Modern contract theory does provide some justification for abstracting from contractual elements when studying labor supply. Bruce Smith (1983) has constructed a simple equilibrium model in which there is an adverse selection problem. If the econometrician abstracts from contractual elements and assumes the household chooses hours optimally given the wage, as is done by both labor and macro economists, only the macro economist correctly identifies preferences. For his example the elasticity of response to temporary changes in the real wage are estimated to be three using aggregate data and zero when using micro panel data. This is an important point, for to explain the cycle using either a monetary or technology shock competitive theory, the elasticity of response to temporary changes in the real wage must be large. Micro panel studies find the elasticity near one when it must be two or three times that big to rationalize aggregate data using competitive theory (see Heckman and MaCurdy (1980), MaCurdy (1981), and Lucas and Rapping (1969)).

On the other hand, the work of Holmstrom and Weiss (1983) suggests that contractual elements may increase the magnitude of the effect of a common productivity shock when there is private information concerning individual-specific shocks. Their contracts are indexed by both the common and individual shocks and are optimal *given the resource and incentive feasible* con-straints. The econometrics of private information economies, however, is at an embryotic stage and it is difficult to assess the quantitative importance of the contractual element for aggregate observation.

The final criticism is that for at least some of the models, fluctuations are optimal. In order to simplify the analysis, the theories abstract from externalities if they are not essential to the explanation. In fact there are externalities and some may be aggravated by fluctuations, say externalities associated with searching and matching of workers to jobs.[4/] Insofar as this is the case, there is a potential role for stabilization policy. A closely related issue is that the models abstract from issues in public finance that arise as the result of agent heterogeneity. Optimal taxation policy may result in cyclically varying tax rates and transfers. At this time, however, our knowledge is far too limited to construct an artificial economies that provide good estimates of the costs and benefits and the distributional consequences of alternative stabilization policy rules.

For the study of some phenomena, abstracting from externalities is inappropriate. Competitive theory with externalities or endogenous policy is more difficult and at a less advanced stage of development. Kydland (1983a) and Whiteman (1983) have made some progress in this regard though the parametric class of economies that they analyze is limited. Paul Romer (1982), who has developed a deterministic competitive model with endogenous growth, relies upon an externality to obtain simultaneously increasing returns at the aggregate level and decreasing returns at the individual level. His is a growth model but, possibly, that externality may be important to understanding the cycle.

Section 5: Shocks to Technology Theory

This theory and its variants build upon the neo-classical growth economy. That model of Solow (1956) and Swan (1956) is, in the language of Lucas (1981, p. 281), "a fully articulated, explicit, artificial economy" that can be used to generate economic time series of a number of important economic aggregates. The model assumes a constant returns to scale aggregate production function with inputs labor n and capital k and an output which can be allocated either to current consumption c or investment x. Letting t denote the date, $f:R^2 \to R$ the production function, n the labor input, and k the capital stock, the production constraint is

$$x_t + c_t \leq f(k_t, n_t)$$

where x_t, c_t, k_t, $n_t \geq 0$. It is further assumed that the services provided by a unit of capital depreciate exponentially at rate $0 < \delta < 1$, so

$$k_{t+1} = (1-\delta)k_t + x_t \; .$$

Solow completed the specification of his economy by hypothesizing that the fraction $0 < \sigma < 1$ of output is invested and the fraction $1 - \sigma$ consumed along with n_t being a constant, say $\bar{n}$, for all t.

This model is poorly suited for the study of the cycle for a number of reasons, the foremost being that the time series generated by the model are deterministic. The economy converges monotonically to its rest point and there are no oscillations. The natural place to introduce uncertainty is in the technology.

Let z_t be positive and independently distributed random variables with bounded support. The technology constraint can be made stochastic by assuming the z_t are multiplicative shocks to the production function:

$$c_t + x_t \leq z_t\ f(k_t, n_t).$$

Now the law of motion of capital,

$$k_{t+1} = (1-\delta)k_t + \sigma z_t f(k_t, \bar{n}),$$

is a stochastic difference economy. The economy does not settle down but rather fluctuates.

The structure is still far from adequate for the study of the cycle because neither employment nor the savings rate varies. By being explicit about the economy, the question of what determines these variables, which are central to the cycle, naturally arises.

This leads to the introduction of a stand-in household with some explicit preferences. Abstracting from the labor supply and uncertainty, the standard form of the utility function is

$$\sum_{t=0}^{\infty} \beta^t u(c_t) \qquad 0 < \beta < 1 .$$

Here β is the subjective time discount factor. The function $u: R_+ \rightarrow R$ is twice differentiable and concave. The commodity space for the deterministic version of this model is ℓ_∞, infinite sequences of uniformly bounded consumptions $\{c_t\}_{t=0}^{\infty}$.

One could apply the theorems of Bewley (1972) to establish existence of a competitive equilibrium for this ℓ_∞ economy even with heterogeneity of agents. That existence argument, however, does not provide an algorithm for computing the equilibria. The alternative approach adopted is to use the welfare theorems of Debreu (1954), in which the commodity space is only restricted to be a linear topological space. Given local non-saturation, competitive equilibria are Pareto optima and, with some additional conditions that are satisfied for this economy, any Pareto optimum can be supported as a competitive equilibrium with redistributions. Given a single agent and the convexity, there is a unique optimum which is therefore the unique competitive equilibrium allocation. The advantage of this approach is that algorithms for computing solutions to concave programming problems can be used to determine the competitive equilibrium allocation for this economy. Cass (1965) and Koopmans (1965) analyze this deterministic optimal growth problem.

In supporting the optima by a competitive equilibrium, Debreu's theorem does not guarantee that the separating hyperplane belongs to ℓ_1; that is, the value of $x \in \ell_\infty$ is not necessarily representable as

$$\sum_{t=0}^{\infty} p_i x_i$$

for some $p \in \ell_1$ (since the dual space of ℓ_∞ is larger than ℓ_1, a point made by Radner (1967)). With discounting, however, if a separating hyperplane exists, say $v: \ell_\infty \to R$, then element $p \in \ell_1$ also separates, where p_i is defined to be $v(e^i)$ where $e^i_j = 0$ for $j \neq i$ and $e^i_i = 1$. See Bewley (1972), Prescott and Lucas (1972) and Brown and Lewis (1981) for further details.

This endogenizes the savings decisions but, as Cass and Koopmans show, the economy still converges monotonically to its steady state without cycling. With uncertainty, the same results hold if the objective is the expected discounted utility. The commodity vector is now indexed by the history of the shocks; that is $\{c_t(z_1,\ldots,z_t)\}_{t=0}^{\infty}$ is the commodity point. As is shown by Brock and Mirman (1972), an optimum to the social planner's problem exists and the optimum is a stationary stochastic process $k_{t+1} = g(k_t,z_t)$. Rather than settling down, the competitive equilibrium for this artificial economy fluctuates.

The optimal allocation can be supported with spot markets only provided that agents are homogenous and have rational expectations. Spot prices are stationary functions of the economy's state, that is $p_t = p(k_t,z_t)$. With this approach it is necessary to distinguish between capital owned by a single individual and capital owned by other people. The optimal law of motion of an individual's capital is $a_{t+1} = h(k_t,z_t,a_t)$. Note that the individual's decision has no effect upon future prices which are forecasted using equilibrium functions $k_{t+1} = g(k_t,z_t)$ and $p_t = p(k_t,z_t)$. For consistency or in equilibrium the representative household must be representative, or,

$$k_{t+1} = h(k_t,z_t,k_t) = g(k_t,z_t)$$

for all (k_t,z_t). With this formulation, unlike the state-contingent formulation, expectations of future prices are crucial.[5/] Thus, any cycle theory within this framework is expectationally consistent.

As fluctations in hours of employment per household is the key to the business cycle, the preferences must be modified to include leisure as well as consumption. This is what Finn Kydland and I (1982) did. The objective of the stand-in household is

$$E\{\sum_{t=0}^{\infty} \beta^t u(c_t, \ell_t)\}$$

where ℓ_t is leisure and $\ell_t + n_t = 1$ if the per period time endowment is normalized to be one. Letting ρ be the subjective discount factor, discount factor β is $1/(1+\rho)$.

Micro observations and secular patterns severly restrict possible utility functions. The facts that leisure has been nearly constant, that real wages have increased by a few hundred percent, and that there has been no trend in the real interest rate over the last forty years requires

$$u(c_t, \ell_t) = \frac{(c_t^{\phi}\ \ell_t^{(1-\phi)})^{\gamma} - 1}{\gamma}$$

where $-\infty < \gamma \leq 1$ and $0 < \phi < 1$. As average ℓ_t is approximately 2/3 (see Ghez and Becker (1975)), $1 - \phi$ must be approximately 2/3 or $\phi = 1/3$. Low average real interest rates along with growing consumption constrain β to be near one. The appropriate value of β depends upon the time period, which we took to be a quarter of a year. We used $\beta = .99$ which corresponds to the subjective time discount rate being 1 percent per quarter, that is 4 percent per year. Only γ is a free parameter.

This structure does not admit to sufficient intertemporal substitution of leisure to account for the data. One way to introduce greater intertemporal substitution without altering the steady state properties of the growth model is to assume the utility function u is a function of c_t and a distributed lag of past leisure: that is to assume an objective function

$$(1) \qquad E \sum_{t=0}^{\infty} \beta^t \, u(c_t, \sum_{i=0}^{\infty} \alpha_i \ell_{t-i}).$$

The a_i are constrained to sum to one and α_{i+1}/α_i is some constant η for all $i > 1$. This implies

$$(2) \qquad \sum_{i=o}^{\infty} \alpha_i \ell_{t-i} = \alpha_0(1-n_t) + (1-\alpha_0)(1-\eta)(1-a_t)$$

where

$$a_{t+1} = \eta \, a_t + n_t \; .$$

This structure can be justified by assuming an unobserved capital stock in the household sector (see Kydland, 1983a) with household production being distributed between consumption and investment of the nonmarket produced good.

The production side assumes unit elasticity of substitution between capital and labor: namely

$$f(k_t, n_t) = n_t^{\theta} k_t^{(1-\theta)}$$

where $0 < \theta < 1$. As inventory stocks are a strongly procyclical variable, inventories y_t are needed and are introduced as follows.

$$f(k_t, n_t, y_t) = u_t^{\theta} \left(\sigma k_t^{\gamma} + (1-\sigma) y_t^{\gamma}\right)^{\frac{1-\theta}{\gamma}}$$

where $-\infty < \gamma < 1$. Parameter θ is determined by the average labor share while the average inventory-capital ratio restricts either σ or γ. The stochastic technology constraint is

(3) $$c_t + i_t + (y_{t+1} - y_t) \leq z_t \, f(k_t, n_t, y_t).$$

Unlike Brock and Mirman (1972), independent shocks $\{z_t\}$ to technology do not suffice. We assume that technology shocks are unobserved and are the sum of unobserved shocks x_{1t} and x_{2t},

$$z_t = x_{1t} + x_{2t} .$$

It is further assumed that the $\{x_{1t}\}$ process is first order autoregressive with an autoregressive in parameter near one:

(4) $$x_{1,t+1} = .95 \, x_{1t} + \varepsilon_{1t} .$$

The second component x_{2t} is independent of x_{it} and transitory:

(5) $$x_{2,t+1} = \varepsilon_{2t} .$$

Finally, a noisy indicator $\tilde{z}_t$ of z_t is observed:

(6) $$\tilde{z}_t = x_{1t} + x_{2t} + \varepsilon_{3t} .$$

The shocks ε_{it} are all independent and sufficiently near to being normal so that inference rules based upon normality introduce an error that is too small to be of concern. With this structure, expectation of $x_t = (x_{1t}, x_{2t})$ conditioned upon past z_t is a sufficient statistic for forecasting future z_t. This is the standard Kalman filter result.

A reason for persistence in shocks is that technological change may be permanent. A new invention which increases the production possibility set today increases it in the future as well. We found that the artificial economy produces cycles similar to those experienced by the U.S. economy in the post-war period only if the variance of the permanent shocks ε_{1t} are large relative to the variance of the temporary shocks ε_{2t}.

The final modification was to introduce multi-stage production of new capacity with a period required for each stage. In particular, four quarters are required to build a new unit of capacity k_t with one quarter of the value put in place at each stage. This expands the number of capital goods from two to five with there being completed capital k_t, three-quarters completed capital s_{1t}, half-completed capital s_{2t}, one-quarter completed capital s_{3t} and inventories y_t.

At the beginning of period t the indicator $\tilde{z}_t$ of the productivity shock z_t is observed. The employment decision n_t and new capital project decision s_{4t} are made. Note

(7)
$$k_{t+1} = (1-\delta)k_t + s_{1t}$$
$$s_{t+1,j} = s_{t,j+1} \qquad j = 1,2,3$$

are linear laws of motion. Investment is

(8) $$i_t = 1/4[s_{1t} + s_{2t} + s_{3t} + s_{4t}] \, .$$

Output, and therefore z_t, is subsequently observed and then the remaining output is allocated between consumption c_t and carry over inventories y_{t+1}.

The state variables for this economy are the five capital stocks, the conditional expectations of the two technology shocks, and the distributed lag of past employment decisions a_t. The decision variables are employment n_t, new investment s_{4t}, *consumption* c_t *and carry over inventories* y_{t+1}. The social planner's problem is to maximize (1) subject to (2) - (8) and the nonnegativity constraints.

In practice the social optimum for a quadratic approximation to this economy about its steady state is computed. First, the rest point for the deterministic version of the economy is determined. Constraint (3), which must be binding since consumption is a good, is used to eliminate c_t in (1). Then the objective function depends only upon current decision and state variables and the constraints are linear. The quadratic approximation about the rest point for the deterministic version of the model then is made. The resulting problem is the standard linear-quadratic regulator problem whose equilibrium decision rules, which specify decisions as a function of the state variables, are linear and easily computed (see Sargent (1981)). The equilibrium is a linear vector autoregressive system that can be simulated to determine whether the behavior of the artificial economy resembles the behavior of the actual economy.

I found it remarkable that this simple structure mimicked the post-war business cycle so well. The relative variability of consumption and investment matched, as did the serial correlation properties of the model's cycles. Further, hours moved strongly procyclical with more than half of the variability in output being accounted for by variability in this input. This is remarkable giving that this structure was the <u>only</u> one considered and the only parameters that were free and affected equilibrium behavior in an important way were the variance of the productivity shocks and the parameters defining the distributed lag of leisure in the utility function. The standard deviation of the highly persistent productivity was 0.9 percent per quarter. This number seems large as intuitively shocks to technology should average out over the economy. Its magnitude, is consistent with the magnitude of residuals in estimated aggregate production functions, but I would have much more confidence in the theory if micro observations restricted this parameter to be near this value.

The model's fit is far from perfect, which is not surprising given the abstract nature of this artificial economy. The three principal discrepancies are (1) that inventories are not as strongly procyclical as in the data; (2) that the cyclical aplitude of hours variation is a third too small; and (3) cyclically the average product of labor and hours are highly correlated. The first discrepancy is easily remedied by modifying the production function. A better technology would use the nested CES production function with labor and capital having unit elasticity of substitution and with inventories being a poor substitute for the composit labor-capital input. The second and third discrepancies would be ameliorated by treating the hours a machine is employed as being endogenous. This would result in the magnitude of hours variation relative to productivity variation over the cycle to increase. This change, along with small errors in measuring labor input, would make the fit almost perfect.

An advantage of being explicit about the artificial economy is that deviations from the theory suggest specification changes which not only improve the fit but also match better with micro observations. Another advantage of this approach is that it focuses attention on key feature such as the high intertemporal substitution of leisure. If some micro observation prove to be inconsistent with the assumption, this theory of the cycle fails.

Finally, introducing multiple industries as is done in Long and Plosser (1983) appears promising. This does not introduce new free parameters, for input-output tables restrict these new parameters. They find that much less persistence of shocks is needed to produce persistence in fluctuations when the industrial structure is incorporated into the model in this way.

7: An Economy With Nominal Wage Contracting Giving Rise to the Cycle

The purpose of this section is to describe an environment, i.e., preferences, endowments, and technology, such that when households and firms agree on a contract that specifies the nominal wage one period in advance, unexpected monetary shocks have effects on output and employment. The structure of the economy is extremely simple building on the work of Fischer (1977). It uses the basic framework of the optimal growth model, supplemented with the restriction that cash is necessary to make every transaction, i.e., goods are exchanged for money and money is exchanged for goods, but goods cannot be exchanged for goods.

We analyze the dynamics of this economy under a specific notion of equilibrium. It is basically the standard recursive competitive equilibrium concept, modified to require that the representative household and the firm agree on the nominal wage before the current period monetary shock and, consequently, before the current period's price level, is known. The other goods, consumption, capital and investment, are exchanged in spot markets. The money supply is exogenous to the model. Its rate of growth is a random variable. Changes in the stock of money are brought about through lump sum transfers to the households.

Schematically, the economy evolves in the following way: At the end of a given period, the household finds itself with holdings of cash balances and real capital. Households, in their role of owner of capital stock, pool their capital to form a firm. They are the residual claimant to the surplus generated by that firm. Subsequently, each household, in its role of worker, agrees with some firm (not necessary the same firm it owns) on a labor contract for next period. The contracts that are analyzed in this section specify the nominal wage before the monetary shock is observed, and the number of hours the representative worker will sell to the firm, possibly contingent on the realization of the shock. After this is reazlied and the market clearing spot prices of new capital and consumption are announced, the household uses the cash balances it holds to buy these two goods. At the end of the period, the household receives cash from the firm for which it worked and also dividends from the firm it owned. It is assumed that in forming their expectations, economic agents efficiently use all the information available to them.

Given this brief description, it is possible to analyze the effect of, say, an increase in the money supply that is larger than expected: As the nominal wage is fixed, the positive monetary shock increases the price level and, consequently, decreases the ex post real wage. As the capital stock is also fixed (it is chosen one period in advance), and profit maximization by the firm requires that marginal product of labor be equal to the ex post real wage, this generates an increase in employment and output, which in turn allows for an increase in consumption and investment. The larger-than-average investment is the channel through which monetary shocks have "persistent" effects, as high investment today results is larger than average levels of output in the future.

The analysis clearly points out some of the limitations of the structure we use to explain some of the regularities listed in the introduction and Section 2. On the one hand, profit maximization on the part of the firm under full information results in employment and the real wage being negatively correlated. Therefore, if anything, this model predicts that hours of employment and marginal product are negatively correlated. On the other hand, the way money is introduced into the model, through the cash-in-advance constraint, severely limits the possibility of producing an economy with procyclical velocity measures. In particular if, as is the case with our example, the cash-in-advance constraint is binding--agents do not want to hold cash balances in excess of those needed to make the desired transactions--then a strong form of the quantity theory obtains where the income velocity of money is identically one over the cycle.

As is the case with almost every structure, the results we obtain depend substantially on the equilibrium concept that is being used, which should properly be regarded as another assumption. However, the specification that labor contracts which set the nominal wage one period in advance may exceed the limits of "reasonable" assumptions. This is so because it severely restricts the class of labor contracts that can be analyzed and, in particular, leaves out the indexed contracts--i.e., contracting in real terms--that turns out to be a type of contract that achieves the Pareto optimal allocation. Therefore, this structure should be considered as an intermediate step between those models that do not specify the environment and that use supply and demand curves as primitive concepts, and the models--still to be developed--where ideally both the length of the contract and the fact that it is written in nominal terms emerge endogenously. However, this hybrid structure is an inexpensive way of exploring if these models have any potential to explain the regularities associated with the cycle. If they do, then the investment of more resources to understand nominal contracting seems justified. If they do not, it suggests resources might better be allocated to searching for some other explanation of the business cycle.

Denote by K the per household stock of capital at the beginning of the period, and let M be also the per household stock of currency before the current period, per household, monetary injection, Mx. Therefore, per capita stock of money follows

$$M' = M(1+x)$$

where x is a positive random variable and M' next period's per household stock of money. It is assumed that the x_t are identically and independently distributed random variables with $E[x_t] = \bar{x} > 0$. We further assume that the distribution of x is known to all agents in this economy, and that the realizations of the process are publicly and freely observed. We search for an equilibrium for which the spot price of the consumption and the investment good[2] at time t is a time invariant function of the

[2] The technology will be assumed to display a constant marginal rate of transformation between the consumption good and the investment good equal to one. Therefore, if both are produced, their relative price must be one and, in fact, they are the same good.

economy wide average variables and the monetary shock

$$p = P^*(M,K,x).$$

It is also required of an equilibrium that the rate of return on capital be the same regardless of which firm the household works, and that it depends on the same variables as the aggregate price level. It is given by

$$u = U^*(M,K,x).$$

Notice that although this period's per capita money stock is $M' = M(1+x)$, M and x are introduced as separate arguments in the pricing function. This is so because money will not be totally neutral in this economy. In particular, the same value of M' can be attained by infinitely many combinations of M and (1+x), and each is associated with a different value of the shock. Given the nominal wage contract, this unpredictable component has real effects, while it is possible to argue that different M's should have none.

The representative household is assumed to have preferences defined over, possibly stochastic, sequences of consumption and leisure given by the utility functional

$$(7.1) \qquad E\{\sum_{t=0}^{\infty} \beta^t u(c_t, 1-h_t)\}$$

where $0 < \beta < 1$ is the discount factor and $u(\cdot)$ is concave, twice differentiable and increasing in its two arguments: consumption (c_t) and leisure $(1-h_t)$. The endowment of time is one unit in every period and labor supply (h_t) is restricted to be between zero and one.

Consider now the problem faced by a household that at the beginning of period t has m units of cash and k units of capital. At this point, and before x is observed, the household and a firm agree on a contract that specifies this period's nominal wage and capital per worker. Given the contract, and after observing x, the market clearing price, p, and the rate of return on capital, u, are observed. The household honors the contract and buys this period's consumption, c, and next period's capital, k', with the post-transfer cash balances that it holds. Finally, next period's cash balances are equal to unspent cash balances, wage income and capital income.

Competition by firms for workers results in the wage rate being set optimally from the point of view of the household. If it is set too high, a slightly lower wage increases both the return on capital and the expected utility of the worker. On the other hand, if w is set below its optimal value, a firm by offering less capital per worker or equivalently hiring more workers, given its stock of capital, achieves the same result. Thus, the equilibrium nominal wage must be set at the optimal value subject, of course, to the nominal wage contracting requirement. This result is assumed in the subsequent analysis.

Notation conventions that are upper case denotes a per capita value or function while lower case denotes an individual's value or function. The star denotes a real-valued function. Functions are defined for nonnegative values of m, M, k, and K and for values of x which are at least one. Prime denotes the next period value of a variable.

Equilibrium Definition

We are now ready to define a recursive equilibrium for this economy. The equilibrium elements are three pricing functions

$U^*(K,M,x)$ nominal return on capital

$W^*(K,M)$ nominal wage rate

$P^*(K,M,x)$ nominal price level

and three allocation functions

$C^*(K,M,x)$ per capita consumption

$K^*(K,M,x)$ per capita investment

$H^*(K,M,x)$ per capital hours of employment

Before we specify the equilibrium conditions, first it is necessary to consider the household's problem.

Household's Problem

Let $v^*(k,m,K,M)$ be the discounted expected utility of a household given it behaves optimally given equilibrium pricing functions P^* and U^* and equilibrium investment function K^*. By Bellman's optimality principle, it must satisfy

$$v^*(k,m,K,M) = \max_{w} E_x\{ \max_{h,c,k',m'\geq 0} [u(c,h)+\beta v^*(k',m',K',M')]\}$$

subject to

(i) the nominal wage contracting constraint $P^*(K,M,x)\, f_2(K,h) = w$;

(ii) the cash-in-advance constraint $P^*(K,M,x)\,(c+k') \leq m + Mx$;

(iii) the equilibrium law of motion of the per capita capital stock $K' = K^*(K,M,x)$;

(iv) the law of motion of per capita money supply $M' = M(1+x)$;

(v) the budget constraint $m' = m + Mx - P^*(K,M,x)\,(c+k') + wh + U^*(K,M,x)\,k$.

A stationary solution to the above is a set of five functions

$$w = w^*(k,m,K,M)$$

$$c = c^*(k,m,K,M,x)$$

$$k' = k^*(k,m,K,M,x)$$

$$m' = m^*(k,m,K,M,x)$$

$$h = h^*(k,m,K,M,x)$$

Equilibrium requires that for all values of (K,M,x) that the following conditions are satisfied:

1. Goods market clear $c^*(K,M,K,M,x) + k^*(K,M,K,M,x) = f[K,h^*(K,M,K,M,x)]$.

2. The representative individual is representative

$$C^*(K,M,x) = c^*(K,M,K,M,x)$$

$$H^*(K,M,x) = h^*(K,M,K,M,x)$$

$$K^*(K,M,x) = k^*(K,M,K,M,x)$$

$$W^*(K,M) = w^*(K,M,K,M)$$

$$M^*(1+x) = m^*(K,M,K,M,x)$$

This completes the definition of equilibrium.

An Example

In this subsection, an example is worked out explicitly with closed forms for the equilibrium pricing and allocation functions. The utility function is

$$u(c,h) = \ln c + 2 \ln (1-h)$$

and the per capita production function

$$f(k,h) = k^{\alpha}h^{(1-\alpha)} \qquad 0 < \alpha < 1.$$

We restrict our search for an equilibrium to ones for which the three pricing functions are homogenous of degree one in M. This is just the restriction that the unit of account does not effect real allocations. The only real effect of the money supply is through the shocks x.

To simplify notation in specifying the equilibrium, let

$$\theta \equiv \frac{1 - \alpha}{[(1-\alpha)+2(1-\beta\alpha)](1+\bar{x})}$$

where $\bar{x} = E_x\, x$. An equilibrium for this economy is

$$H^*(K,M,x) = \theta(1+x)$$

$$K^*(K,M,x) = \beta\alpha K^{\alpha}\ [\theta(1+x)]^{(1-\alpha)}$$

$$C^*(K,M,x) = (1-\beta\alpha)\ K^{\alpha}\ [\theta(1+x)]^{(1-\alpha)}$$

$$P^*(K,M,x) = M\ K^{-\alpha}\ [\theta(1+x)]^{(1-\alpha)}\ (1+x)$$

$$U^*(K,M,x) = M\ (1+x)\ \alpha\ K^{-1}$$

$$W^*(K,M) = [2(1-\beta\alpha)+(1-\alpha)]\ M(1+\bar{x}).$$

This allocation is a restricted, equal weight Pareto optimum. The restriction is that there be no indexing of labor contracts.

Nonoptimality of the Nominal Wage Contract for the Example

The previous example shows that the "optimal" nominal contract cannot fully insulate the economy from random monetary shocks. The question arises then whether it is possible to find another contract that dominates nominal wage contracts.

For this structure, it is a relatively simple exercise to show that the "equal weight," Pareto optimal allocation; i.e., the solution to the planner's problem, satisfies

$$C_t = (1-\beta\alpha)\ K_t^{\alpha}\ H_t^{1-\alpha}$$

$$K_{th} = \beta\alpha\ K_t^{\alpha}\ H_t^{1-\alpha}$$

$$H_t = \frac{1-\alpha}{2(1-\beta\alpha) + 1-\alpha}$$

Then it is not hard to specify a contract that dominates the optimal nominal wage contract, in the sense that it achieves equivalent Pareto optimal allocation. This contract specifies the real wage given the capital stock; in other words, it requires full indexing of the nominal wage to the price level.

For this example it is to set w_t/p_t equal to the marginal product of labor, given K_t and H_t at its optimal value

$$\frac{w_t}{p_t} = (1-\alpha)\ K_t^{\alpha}\ H_t^{-\alpha} \text{ and } H_t = \frac{1-\alpha}{2(1-\beta\alpha) + 1-\alpha}.$$

Then to justify looking at nominal wage contracting to generate business cycle type observations it is necessary either to abandon in some sense the rationality principle or to introduce asymmetric or private information to preclude writing contracts contingent on p_t. Such an extension does not appear to be straightforward because equilibrium would require cash balances and capital stocks to vary across individuals. This greatly complicates the analysis for the aggregate state variable must describe this distribution which results in a high-dimensional state variable.

Summary

At this point, it seems appropriate to summarize how the model economy that we analyzed works. The "key" feature that accounts for the real effects of monetary shocks is the presence of "long-term" nominal wage contracts. This introduces some degree of stickiness in the short-run that explains why purely monetary shocks that affect real wages turn out to influence output even under rationale expectations. For our model economy, independent and identically distributed monetary shocks give rise to positively correlated levels of output (and consequently of consumption and investment). This "persistence" is due to the presence of capital in our economy.

Finally, the cash-in-advance constraint serves only the purpose of guaranteeing the existence of a well-defined demand for money. In its absence, money would not be held in equilibrium. In the example, money as a store of value is dominated; therefore, monetary shocks do not have portfolio effects. Their "real" effects are truly the result of the presence of nominal contracts. Hence, it is important to ask whether the specification of the environment; i.e., preferences, endowments, and technology, implies that economic agents will choose to write contracts in nominal terms. The answer, at least for the example, is in the negative. We argued that even within the class of "contract equilibria" (loosely defined to include every possible equilibrium where labor is traded using contracts as opposed to spot markets) it is possible to find a contract that dominates the "optimal" nominal contract; namely, full indexation of the nominal wage to the price level. Our environment is not rich enough to preclude endogenously contracting in real terms.

Although this nonoptimality of the nominal contract is a serious criticism to the model of this section, we find it nevertheless useful for two reasons: having worked out the equilibrium behavior from basic assumptions about preferences and technology allowed us to pose the question about the optimality of the nominal contract equilibrium. We believe that only models that can potentially give an answer to this question can be useful in understanding and evaluating the consequences of the cycle. The second reason is that our model economy highlights some of the difficulties of specifying environments that give rise to nominal contracting as the type of contractual arrangement rational agents will choose. In order to specify a model economy where this happens, several modifications could be made. First, we could introduce some cost of making contracts in real terms. Secondly, we could introduce asymmetric information between firms and workers, although it is not obvious how this could be done within our framework. Finally, we could introduce noisy observations of the price level. One way of doing this would be to have spatially separated markets as in Lucas (1972). This, however, introduces heterogeneity among agents and the problems of computing equilibrium with heterogeniety among agents.

Footnotes

1/We experimented with spectral techniques that filtered out the low frequency movements and obtained very similar relations and pictures. The advantage of our approach is that it is readily reproduced. The first-order conditions of the minimization are linear and can be solved using standard matrix inversion routines. The spectral techniques are not so simple to calculate.

2/By readily available we mean the data are on the IMF tapes.

3/Most of the labor contract models entail contracting in real terms (e.g., Azariadis (1983), Chari (1983), Green and Kahn (1983), Grossman and Hart (1983), Hart (1983) and Stiglitz (1983)).

4/The equilibrium search models of Diamond (1982) suggests the natural rate may not be optimal while the work of Mortensen (1982) finds it depends upon the compensation arrangement and the model of Lucas and Prescott (1979) find no market failure. None of these equilibrium search models have aggregate fluctuations. One such model with aggregate fluctuations is Jovanovic (1983), whose conclusions are closer to Diamond.

5/In Lucas and Prescott (1971), the optimum is supported as a competitive equilibrium with a stage-contingent commodity point and as a competitive equilibrium only with spot markets and rational expectations for the basic growth model structure.

Labor Economic Readings Related to the Cycle

Azariadis, C. "Employment with Asymmetric Information," Quarterly Journal of Economics, forthcoming (1983).

Chari, V. "Involuntary Unemployment and Implicit Contracts," Quarterly Journal of Economics, forthcoming (1983).

Diamond, P. "Aggregate Demand Management in Search of Equilibrium," Journal of Political Economy 90 (October 1982): 881-894.

Ghez G. R. and G. S. Becker. The Allocation of Time and Goods Over the life Cycle, New York: National Bureau of Economic Research 1975.

Green, J., and Kahn, C. "Wage Employment Contracts," Quarterly Journal of Economics, forthcoming.

Hall, R. F. "Labor Supply and Aggregate Fluctuations" in On the State of Macro-Economics, edited by K. Brunner and A. H. Meltzer. Amsterdam: North-Holland 1980.

Hall, R. F., and Lillien, D. "Efficient Wage Bargains Under Uncertain Supply and Demand," American Economic Review 69 (December 1979): 868-79.

Hart, O. "Optimal Labor Contracts Under Asymmetric Information: An Introduction," Review of Economics and Statistics (1983).

Heckman, J. J., and MaCurdy, T. E. "A Life Cycle Model of Female Labor Supply," Review of Economic Studies 47 (1980): 47-74.

Holmstrom, B., and Weiss, L. "Managerial Incentives, Investment and Aggregate Implications," Technical Report No. 397, Institute for Mathematic Studies in the Social Sciences, Stanford University (January 1983).

Kydland, F. E. "Nonseparable Utility and Labor Supply," Hoover Institute Stanford University (May 1983b). Presented at NBER Conference on Macroeconomics Cambridge, Massachusetts, July 7 and 8, 1983.

Jovanovic, B. "Turnover, Unemployment and the Cycle," Bell Telephone Laboratories working paper, (January 1983).

______________. "Job Matching and the Theory of Turnover," Journal of Political Economy 87 (October 1979): 972-990.

Lillard, L. A., and Willis, R. J. "Dynamic Aspects of Earning Mobility," Econometrica 46 (September 1978): 985-1012.

Lucas, R. E., and Prescott, E. C. "Equilibrium Search and Unemployment," Journal of Economic Theory 2 (February 1974): 188-209.

MaCurdy, T. E. "An Empirical Model of Labor Supply in a Life-Cycle Setting," Journal of Political Economy 89 (1981): 1059-1085.

Mortensen, D. T. "Property Rights and Efficiency in Rating, Racing and Related Games," American Economic Review 72 (December 1982): 968-979.

Smith, B. D. "Reconciling Individual and Aggregate Differences in Labor Markets: An Adverse Selection Approach," Minneapolis Federal Reserve Bank Working Paper 232, (January 1983).

Growth, Capital Theory Readings Related to the Cycle

Allen, B. "Generic Existence of Completely-Revealing Equilibria for Economies With Uncertainty When Prices Convey Information," Econometrica 49 (September 1981).

Arrow, K. J. "The Economic Implications of Learning by Doing," Review of Economic Studies 80 (June 1962): 155-173.

Arrow, K. J., and Kurz, M. Public Investment, the Rate of Return, and Optimal Fiscal Policy, Baltimore: The John Hopkins University Press, (1970).

Becker, R. A. "The Existence of Perfect Foresight Competitive Equilibrium in a Simple Dynamic Model of Capital Accumulation with Heterogenous Households," Indiana University working paper (1978).

Bewley, T. "Existence of Equilibrium in Economies with Infinitely Many Commodities," Journal of Economic Theory 4 (June 1972): 514-540.

____________. T. "Thoughts on Tests of the Intertemporal Asset Pricing Model," Working Paper, Northwestern University (July 1982).

Bewley, T., and Radner, R. "Stationary Monetary Equilibrium with a Continuum of Independently Fluctuating Consumers," Northwestern University, (October 1980).

Brock, W. A. "Asset Prices in a Production Economy," in The Economics of Information and Uncertainty, ed. by J. J. McCall, Chicago: University of Chicago Press, (1982): 1-46.

Brock, W. A., and Mirman, L. "Optimal Economic Growth Under Uncertainty: The Discounted Case," Journal of Economic Theory 4 (June 1972): 479-513.

Brown, D. J., and Lewis, L. M. "Myopic Economic Agents," Econometrica 49 (March 1981): 359-368.

Burmeister, E., and Dobell, A. R. Mathematical Theories of Economic Growth, London: The MacMillan Company, (1970): 405-406.

Cass, D. "Optimal Growth in an Aggregative Model of Capital Accumulation," Review of Economic Studies 32 (1965): 233-240.

Christiano, L. J. "On the Accuracy of Linear Quadratic Approximations: An Example," University of Chicago Graduate School of Business (November 1982).

Danthine, J. P., and Donaldson, J. "Stochastic Properties of Fast vs. Slow Growing Economies," Econometrica 79 (July 1981): 1007-1033.

Debreu, G. "Valuation Equilibrium and Pareto Optimum," Proceedings of the National Academy of Science 70 (1954): 588-592.

Donaldson, J., and Mehra, R. "Stochastic Growth with Correlated Production Shocks," Journal of Economic Theory 29 (1983): 288-312.

Green, J. "The Non-Existence of Informational Equilibria," Review of Economic Studies 44 (1977): 451-463.

Heller, W. P. "Disequilibrium Dynamics of Competitive Growth Paths," Review of Economic Studies 116 (October 1971): 385-400.

__________. "Tâtonnement Stability of Infinite Horizon Models with Saddle-Point Instability," Econometrica 43 (January 1975): 65-80.

Jones, L. "Existence of Equilibria with Infinitely Many Consumers and Infinitely Many Commodities: A Theorem Based on Models of Commodity Differentiation," Northwestern University, (July 1982).

Jordan, J. S., and Radner, R. "The Nonexistence of Rational Expectations Models: A Robust Example," Department of Economics, University of Minnesota, (1979).

Kanodia, C. "Effects of Stockholder Information on Corporate Decisions and Capital Market Equilibrium," Econometrica 48 (May 1980): 923-953.

Koopmans, T. C. "On the Concept of Optimal Economic Growth," The Econometric Approach to Development Planning, Chicago: Rand-McNally, (1965).

Kreps, D. M. "A Note on Fulfilled Expectations Equilibria," Journal of Economic Theory 14 (1977): 23-43

Kydland, F. E., and Prescott, E. C. "Rules Rather than Discretion: The Time Inconsistency of Optimal Plans," Journal of Political Economy 85 (June 1977): 473-492.

Leamer, E. "Let's Take the Con Out of Econometrics," American Economic Review 73 (March 1983): 31-44.

Lucas, R. E., Jr., and Stokey, N. L. "Optimal Growth with Many Consumers," Northwestern University, the Center of Mathematical Studies in Economics and Management Science, Discussion Paper No. 518, (March 1982).

Lucas, R. E., Jr., and Prescott, E. C. "Investment Under Uncertainty," Econometrica 39 (September 1971): 659-681.

Prescott, E.C., and Lucas R. E., Jr. "A Note on Prices in Infinite Dimensional Space," International Economic Review 13 (June 1972): 416-422.

Prescott, E. C., and Mehra, R. "Recursive Competitive Equilibrium: The Case of Homogenous Households," Econometrica 48 (September 1980): 365-379.

Radner, R. "Optimal Growth in a Linear-Logarithmic Economy," International Economic Review 7 (January 1966): 1-33.

__________. "Efficiency Prices for Infinite Horizon Production Programs," Review of Economic Studies 34 (January 1967): 51-66.

__________. "Rational Expectations Equilibrium: Generic Existence and the Information Revealed by Prices," Econometrica 47 (1979): 655-678.

Romer, P. "Notes on Existence of Dynamic Competitive Equilibria: Externalities, Increasing Returns and Unbounded Growth," University of Rochester, (May 1982).

Sargent, T. J. "Lecture Notes on Filtering and Control," Minneapolis Federal Reserve Bank Working Papers 1981.

Scarf, H. in collaboration with T. Hansen. The Computation of Economic Equilibria. New Haven: Yale University Press (1973).

Solow, R. "A Contribution to the Theory of Economic Growth," Quarterly Journal of Economics 70 (February 1956): 65-94.

Whiteman, C. H. Linear Rational Expectations Models a User's Guide Minneapolis: University of Minnesota Press 1983.

Business Cycle References

Barro, R. J. "A Capital Market in an Equilibrium Business Cycle Model," Econometrica 48 (September 1980): 1393-1417.

__________. "Rational Expectations and the Role of Monetary Policy," Journal of Monetary Economics 2, (January 1976): 1-32.

Becketti, Sean. "The Persistence of Nominal Shocks in a Particular Equilibrium Model," Working Paper, University of California, Los Angeles, (January 1983).

Black, F. "General Equilibrium and Business Cycles," manuscript, MIT Sloan School of Management (1977).

Fischer, S. "Long-Term Contracts, Rational Expectations, and the Optimal Money Supply Rule," Journal of Political Economy 85 (February 1977): 191-206.

Futia, C. A. "Stochastic Business Cycles," manuscript, Bell Telephone Laboratories, (1980).

Grossman, S. J., and Weiss, L. "A Transaction Based Model of the Monetary Transmission Model: Part I," NBER Working Paper 973, (July 1982).

Grossman, S. J. "A Transactions Based Model of the Monetary Transmission Mechanism: Part II," NBER Working Paper No. 974, (September 1982).

__________. "Heterogenous Information and the Theory of the Business Cycle," Journal of Political Economy 90 (August 1982): 699-727.

Grossman S. J., Hart, O., and Maskin, E. "Unemployment with Observable Aggregate Shocks," NBER Working Paper No. 975 (September 1982).

King, R. G., and Plosser, C. I. "The Behavior of Money, Credit, and Prices in a Real Business Cycle," University of Rochester Working Paper No. GPB 81-8, (revised November 1982).

Kydland, F. E. "The Role of Money in a Competitive Theory of Fluctuations," Hoover Institution Working Paper, (1983a).

Kydland, F. E., and Prescott, E. C. "Time to Build and Aggregate Fluctuations, Econometrica 50 (November 1982): 1345-1370.

Lillien, D. "Sectoral Shifts in Cyclical Unemployment," Journal of Political Economy 90 (August 1982): 777-793.

Long, J. B., Jr., and Plosser, C. I. "Real Business Cycles," Journal of Political Economy 91 (February 1983): 39-69.

Lucas, R. E., Jr. "Capacity Overtime and Empirical Production Functions," American Economic Review 60, papers and proceedings (May 1970): 23-27.

________. "Understanding Business Cycles," Journal of Monetary Economics (supplemental 1977): 7-29.

________. "An Equilibrium Model of the Business Cycle," Journal of Political Economy 83 (December 1975): 1113-1144.

________. Business Cycle Theory, Cambridge: MIT Press (1981).

________. "Expectations and the Neutrality of Money," Journal of Economic Theory 4 (April 1972): 103-124.

Phelps, E. S. "Introduction: The New Microeconomics in Employment and Inflation Theory," in Microeconomic Foundations of Employment and Inflation Theory, Norton, New York, (1970).

Phelps, E. S., and Taylor, J. B. "Stabilizing Powers of Monetary Policy Under Rational Expectations," Journal of Political Economy 85 (February 1977): 163-190.

Sargent, T. J. "A Classical Macroeconomic Model for the United States," Journal of Political Economy 84 (April 1976): 207-237.

____________. Macroeconomic Theory, Academic Press, New York, (1979), Chapter 16.

__________. "Beyond Demand and Supply Curves in Macroeconomics," American Economic Review 72, Paper and Proceedings (May 1972): 382-389

Sims, C. A. "Macroeconomics and Reality," Econometrica 48 (January 1980): 1-48.

Stutzky, E. "The Summation of Random Causes as the Source of Cyclical Process," Econometrica 5 (April 1977): 105-146.

Taylor, J. G. "Estimation and Control of a Macroeconomic Model with Rational Expectations," Econometrica 47 (September 1979): 1267-1286.

_______________. "Aggregate Dynamics and Staggered Contracts," Journal of Political Economy 88 (February 1980): 1-23.

Townsend, R. M. "Forecasting the Forecasts of Others," Journal of Political Economy, forthcoming.