Intro	Model	Equilibrium	Cashless limit	Quantitative	CIA/MIU	Conclusion

The Limits of onetary Economics

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Is the medium-of-exchange role of money

relevant for Monetary Economics?



Current wisdom: Monetary Economics without M

Medium-of-exchange considerations are irrelevant for monetary transmission in modern high-velocity credit economies.

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Current wisdom: Monetary Economics without M

Medium-of-exchange considerations are irrelevant for monetary transmission in modern high-velocity credit economies.

Based on two results:

Monetary equilibrium is continuous under a certain "cashless limit"

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2 Real balances play small quantitative role in calibrations

Current wisdom: Monetary Economics without M

Medium-of-exchange considerations are irrelevant for monetary transmission in modern high-velocity credit economies.

Based on two results:

Monetary equilibrium is continuous under a certain "cashless limit"

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2 Real balances play small quantitative role in calibrations

Both rely on a class of reduced-form monetary models (CIA/MIU)



Develop a model

• explicit about the roles of money and credit in exchange

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- can exhibit any transaction velocity of money
- allows for market power in credit intermediation

Study monetary policy as velocity becomes very high

Intro	Model	Equilibrium	Cashless limit	Quantitative	CIA/MIU	Conclusion
Main	finding					

Medium-of-exchange considerations are resilient and significant.





Given financial frictions (credit intermediaries with *some* market power):

As the cash-and-credit economy converges to a pure-credit economy, the monetary equilibrium does not converge to the equilibrium of the economy without money.

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Given financial frictions (credit intermediaries with *some* market power):

- As the cash-and-credit economy converges to a pure-credit economy, the monetary equilibrium does not converge to the equilibrium of the economy without money.
- Effects of monetary policy remain large even as aggregate real money balances vanish along the cashless limit.

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Given financial frictions (credit intermediaries with *some* market power):

- As the cash-and-credit economy converges to a pure-credit economy, the monetary equilibrium does not converge to the equilibrium of the economy without money.
- Effects of monetary policy remain large even as aggregate real money balances vanish along the cashless limit.

 \Rightarrow Cashless economies are not good approximations to monetary economies—even high-velocity economies

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Intro	Model	Equilibrium	Cashless limit	Quantitative	CIA/MIU	Conclusion
Intuiti	on					

Money affects prices in transactions that do not involve money.



Money affects prices in transactions that do not involve money.

• The *option* to engage in monetary trade disciplines the market power of financial intermediaries.

• *Off-equilibrium* small volume of monetary trades feeds back into the prices negotiated in all pure-credit nonmonetary transactions.

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Leverage and asset prices Kiyotaki and Moore (1997, 2005)

Strategic bargaining advantage from holding money Zhu and Wallace (2007), Rocheteau, Wright and Zhang (2018)

Trade option as disciplining device for market power and mark-ups Bhagwati (1965), Markusen (1981), Baumol (1982), Holmes et al. (2014)

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Value of resale options

Harrison and Kreps (1978)

Monetary policy in cashless and near-cashless economies Woodford (1998, 2003), Galí (2008)

Intro	Model	Equilibrium	Cashless limit	Quantitative	CIA/MIU	Conclusion
Prev	iew					



Period t



- Time. Discrete, infinite horizon, two subperiods per period
- Population. [0, 1] investors, [0, 1] brokers
- Commodities. Two divisible, nonstorable consumption goods:

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- dividend good
- general good

	Model	Equilibrium	Cashless limit	Quantitative	CIA/MIU	Conclusion
Prefe	erences					

Brokers:
$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t (c_t - h_t)$$

Investors:
$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left(\varepsilon_t y_t + c_t - h_t \right)$$

• ε_t : valuation shock, i.i.d. over time, cdf $G(\cdot)$ on $[\varepsilon_L, \varepsilon_H]$

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Endowments and production technology

First subperiod

- A^s productive units (*trees*)
- Each unit yields y_t dividend goods at the end of the first subperiod

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Second subperiod

• Linear technology to transform effort into general goods

	Model	Equilibrium	Cashless limit	Quantitative	CIA/MIU	Conclusion
Finar	ncial as	sets				

Money

• $A^m_{t+1} = \mu A^m_t$, $\mu \in \mathbb{R}_{++}$ (implemented with lump-sum taxes)

Equity

• A^s equity shares

Bond

- issued by investors in first subperiod of t, repaid next subperiod
- 1 bond = claim to 1 unit of general good
- no commitment; if issuer defaults, bond holder appropriates fraction
 λ of the issuer's equity shares

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 Market structure
 first subperiod:
 OTC trade

Two contemporaneous competitive markets

- bond market (bonds and money)
- equity market (equity and money)

Brokers

always access bond market

Investors

- prob. α : access equity market only
- prob. $\alpha_c \equiv 1 \alpha$: access equity market and contact a bond broker

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Bilateral terms of trade between investor and broker

• Nash bargaining (investor bargaining power θ)



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- Walrasian trade between all brokers and investors
- equity, general good, money

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Timeline and market structure



Period t

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access



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 OTC trade
 valued money
 with credit access
 Valued
 Valued

$$\max_{(\bar{a}^m,\bar{a}^{\rm s},\bar{a}^{\rm b},k)\in\mathbb{R}^2_+\times\mathbb{R}\times\mathbb{R}_+}\Gamma_t(\textit{a},\varepsilon)^{\theta}k^{1-\theta}$$

$$\overline{a}^m + p_t \overline{a}^s + q_t \overline{a}^b \le a^m + p_t a^s$$

$$-\lambda \phi_t^s \overline{a}^s \leq \overline{a}^b$$

 $0 \leq \Gamma_t(\boldsymbol{a}, \varepsilon)$

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$$\Gamma_t(\boldsymbol{a},\varepsilon) \equiv \varepsilon y_t \overline{a}^s + W_t(\overline{a}^m, \overline{a}^s, \overline{a}^b, k) - [\varepsilon y_t \hat{a}_t^s(\boldsymbol{a},\varepsilon) + W_t(\hat{a}_t^m(\boldsymbol{a},\varepsilon), \hat{a}_t^s(\boldsymbol{a},\varepsilon), 0, 0)]$$

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Euler equation

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$$\begin{split} \phi_{t}^{s} &= \beta \mathbb{E}_{t} \Biggl\{ \bar{\varepsilon} y_{t+1} + \phi_{t+1}^{s} \\ &+ \alpha_{c} \theta \frac{\lambda \phi_{t+1}^{s}}{\bar{\phi}_{t+1}^{s} - \lambda \phi_{t+1}^{s}} \int_{\varepsilon_{t+1}^{**}}^{\varepsilon_{H}} \left(\varepsilon y_{t+1} + \phi_{t+1}^{s} - \bar{\phi}_{t+1}^{s} \right) dG\left(\varepsilon \right) \\ &+ \alpha_{c} \theta \int_{\varepsilon_{L}}^{\varepsilon_{t+1}^{**}} \left[\bar{\phi}_{t+1}^{s} - \left(\varepsilon y_{t+1} + \phi_{t+1}^{s} \right) \right] dG\left(\varepsilon \right) \end{split}$$

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$$\begin{split} \phi_{t}^{s} &= \beta \mathbb{E}_{t} \left\{ \bar{\varepsilon} y_{t+1} + \phi_{t+1}^{s} \right. \\ &+ \alpha_{c} \theta \frac{\lambda \phi_{t+1}^{s}}{\bar{\phi}_{t+1}^{s} - \lambda \phi_{t+1}^{s}} \int_{\varepsilon_{t+1}^{**}}^{\varepsilon_{H}} \left(\varepsilon y_{t+1} + \phi_{t+1}^{s} - \bar{\phi}_{t+1}^{s} \right) dG\left(\varepsilon\right) \\ &+ \alpha_{c} \theta \int_{\varepsilon_{L}}^{\varepsilon_{t+1}^{**}} \left[\bar{\phi}_{t+1}^{s} - \left(\varepsilon y_{t+1} + \phi_{t+1}^{s} \right) \right] dG\left(\varepsilon\right) \\ &+ \left[\alpha + \alpha_{c} \left(1 - \theta \right) \right] \int_{\varepsilon_{L}}^{\varepsilon_{t+1}^{*}} \left[p_{t+1} \phi_{t+1}^{m} - \left(\varepsilon y_{t+1} + \phi_{t+1}^{s} \right) \right] dG\left(\varepsilon\right) \right\} \end{split}$$

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Model	Equilibrium	Cashless limit	Quantitative	CIA/MIU	Conclusion

Nonmonetary equilibrium

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Nonmonetary equilibrium

Proposition

There exists a unique RNE:

$$\varphi^{n} = \bar{\varepsilon} + \alpha_{c}\theta \left[\int_{\varepsilon_{L}}^{\varepsilon^{n}} (\varepsilon^{n} - \varepsilon) \, dG(\varepsilon) + \frac{\lambda}{1 - \lambda} \int_{\varepsilon^{n}}^{\varepsilon_{H}} (\varepsilon - \varepsilon^{n}) \, dG(\varepsilon) \right]$$

 $\varepsilon^n \in [\varepsilon_L, \varepsilon_H]$ is the unique solution to

$$G(\varepsilon^n) = \lambda$$

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Model	Equilibrium	Cashless limit	Quantitative	CIA/MIU	Conclusion

Monetary equilibrium

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 Monetary equilibrium
 existence
 high policy rate

Proposition

If $\hat{\iota}(\lambda) < \iota < \bar{\iota}(\lambda)$, there exists a unique RME:

$$\varphi = \varphi^{n} + \left[\alpha + \alpha_{c} \left(1 - \theta\right)\right] \int_{\varepsilon_{l}}^{\varepsilon^{*}} \left(\varepsilon^{*} - \varepsilon\right) dG\left(\varepsilon\right)$$

 $\varepsilon^{**}=\varepsilon^n$ and $\varepsilon^*\in(\varepsilon_L,\varepsilon^n)$ is the unique solution to

$$\frac{\left[\alpha+\alpha_{c}(1-\theta)\right]\int_{\varepsilon^{*}}^{\varepsilon}H(\varepsilon-\varepsilon^{*})dG(\varepsilon)+\alpha_{c}\theta\left[\varepsilon^{n}-\varepsilon^{*}+\frac{1}{1-\lambda}\int_{\varepsilon^{n}}^{\varepsilon}H(\varepsilon-\varepsilon^{n})dG(\varepsilon)\right]}{\varphi^{n}+\left[\alpha+\alpha_{c}(1-\theta)\right]\int_{\varepsilon_{L}}^{\varepsilon^{*}}(\varepsilon^{*}-\varepsilon)dG(\varepsilon)} = h$$

Model	Equilibrium	Cashless limit	Quantitative	CIA/MIU	Conclusion

Cashless limit

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- - $\alpha = 1 \alpha_c \in [0, 1]$: prob. of *not* accessing credit

• As $\alpha \rightarrow 0$, the equity price in the RNE converges to:

$$\lim_{\alpha \to 0} \varphi^n = \bar{\varepsilon} + \theta \left[\int_{\varepsilon_L}^{\varepsilon^n} \left(\varepsilon^n - \varepsilon \right) dG(\varepsilon) + \frac{\lambda}{1 - \lambda} \int_{\varepsilon^n}^{\varepsilon_H} \left(\varepsilon - \varepsilon^n \right) dG(\varepsilon) \right]$$

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Intro Model Equilibrium Cashless limit Quantitative CIA/MIU Conclusion

What happens to monetary equilibrium as $\alpha \rightarrow 0$?



Limit as fraction of cash-only trades goes to zero

Proposition (high policy rate) If $\hat{\zeta} < \iota < \bar{\zeta}$ (positive inside rate), then as $\alpha \to 0$, $\mathcal{Z} \to 0$ $\mathcal{V} \to \infty$ $\varphi \to \lim_{\alpha \to 0} \varphi^{n} + (1 - \theta) \int_{\epsilon}^{\epsilon^{*}} (\epsilon^{*} - \epsilon) \, dG(\epsilon)$ where $\varepsilon^* \in (\varepsilon_L, \varepsilon^n)$ is the unique solution to $\frac{(1-\theta)\int_{\varepsilon^*}^{\varepsilon} (\varepsilon-\varepsilon^*) dG(\varepsilon) + \theta \left[\varepsilon^n - \varepsilon^* + \frac{1}{1-\lambda}\int_{\varepsilon^n}^{\varepsilon} (\varepsilon-\varepsilon^n) dG(\varepsilon)\right]}{\overline{\varepsilon} + (1-\theta)\int_{\varepsilon_I}^{\varepsilon^*} (\varepsilon^n - \varepsilon) dG(\varepsilon) + \theta \left[\int_{\varepsilon_I}^{\varepsilon^n} (\varepsilon^n - \varepsilon) dG(\varepsilon) + \frac{\lambda}{1-\lambda}\int_{\varepsilon^n}^{\varepsilon} (\varepsilon-\varepsilon^n) dG(\varepsilon)\right]} = \iota$

Limit as fraction of cash-only trades goes to zero

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Limit as fraction of cash-only trades goes to zero

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Model	Equilibrium	Cashless limit	Quantitative	CIA/MIU	Conclusion

Intuition

Intro Model Equilibrium Cashless limit Quantitative CIA/MIU Conclusion M/bx + bo discontinuity - 2c + 4 + 2 + 02

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Why the discontinuity as $\alpha \rightarrow 0$?

$$\lim_{\alpha \to 0} \frac{\mathcal{Z}}{\varphi} = \lim_{\alpha \to 0} \frac{1}{\mathcal{V}} = 0 < \lim_{\alpha \to 0} \left(\varphi - \varphi^n \right)$$

Intro Model Equilibrium Cashless limit Quantitative CIA/MIU Conclusion Why the discontinuity as $\alpha \to 0$?

$$\lim_{\alpha \to 0} \frac{\mathcal{Z}}{\varphi} = \lim_{\alpha \to 0} \frac{1}{\mathcal{V}} = 0 < \lim_{\alpha \to 0} \left(\varphi - \varphi^n\right) = \left(1 - \theta\right) \int_{\varepsilon_L}^{\varepsilon^*} \left(\varepsilon^* - \varepsilon\right) dG\left(\varepsilon\right)$$

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Intro Model Equilibrium Cashless limit Quantitative CIA/MIU Conclusion Why the discontinuity as $\alpha \to 0$?

$$\lim_{\alpha \to 0} \frac{\mathcal{Z}}{\varphi} = \lim_{\alpha \to 0} \frac{1}{\mathcal{V}} = 0 < \lim_{\alpha \to 0} \left(\varphi - \varphi^n\right) = (1 - \theta) \int_{\varepsilon_L}^{\varepsilon^*} \left(\varepsilon^* - \varepsilon\right) dG\left(\varepsilon\right)$$

$$\lim_{\alpha \to 0} \frac{\mathcal{Z}/\varphi}{\alpha} = \lim_{\alpha \to 0} \frac{G(\varepsilon^*)}{1 - G(\varepsilon^*)\alpha} = \lim_{\alpha \to 0} G(\varepsilon^*) > 0$$

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Intro Model Equilibrium Cashless limit Quantitative CIA/MIU Conclusion Why the discontinuity as $\alpha \to 0$?

$$\lim_{\alpha \to 0} \frac{\mathcal{Z}}{\varphi} = \lim_{\alpha \to 0} \frac{1}{\mathcal{V}} = 0 < \lim_{\alpha \to 0} \left(\varphi - \varphi^n\right) = \left(1 - \theta\right) \int_{\varepsilon_L}^{\varepsilon^*} \left(\varepsilon^* - \varepsilon\right) dG\left(\varepsilon\right)$$

$$\lim_{\alpha \to 0} \frac{\mathcal{Z}/\varphi}{\alpha} = \lim_{\alpha \to 0} \frac{G(\varepsilon^*)}{1 - G(\varepsilon^*)\alpha} = \lim_{\alpha \to 0} G(\varepsilon^*) > 0$$

$$\lim_{\iota \to \bar{\iota}(\lambda)} \frac{\mathcal{Z}/\varphi}{\alpha} = \lim_{\iota \to \bar{\iota}(\lambda)} \frac{G(\varepsilon^*)}{\left[1 - G(\varepsilon^*)\right]\alpha + \alpha_c} = 0$$

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Intro	Model	Equilibrium	Cashless limit	Quantitative	CIA/MIU	Conclusion

Quantitative analysis

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- The monetary equilibrium is not continuous under the cashless limit; is the discontinuity *quantitatively relevant*?
- Are these monetary frictions important for monetary policy transmission in modern high-velocity credit economies?

 \rightarrow Study monetary transmission to asset prices (well documented empirically)

	Model	Equilibrium	Cashless limit	Quantitative	CIA/MIU	Conclusion
Cali	bration					
		variable		value	targe	t
	dividend process		$y_{t+1} = e^{x_{t+1}} y_t$	g = .04	Ludvigcon Lot	-+ (OE)
	dividend process		$x_{t+1} \sim \mathcal{N}\left(extbf{g}, \Sigma^2 ight)$	Σ^{2}) $\Sigma = .12$	Luavigson-Le	.tau (05)
	asset depreciation		δ	.075	risk pro	ху
	nominal policy rat	e	ρ^{p}	.0447	3-M ED futur	e (94-08)
	inflation rate		$\pi - g$.0269	CPI inflation	(94-08)
	real risk-free rate		r	.0178	$\rho^p-(\pi$	- g)
	margin		$1-\lambda$.25	Rule 4210 (1	FINRA)
	fraction with no c	redit	α	.04	$\mathcal{V}=25$ daily	(CHIPS)
	broker market pov	ver	1- heta	.84	2.3% margir	spread
	idiosyncratic shoc	ks	$\ln arepsilon \sim \mathcal{N}\left(-rac{1}{2}\Sigma_arepsilon^2$, Σ	Σ_{ε}^{2}) 2.08	$\mathcal{S} \equiv \left \frac{d\phi^s / \phi^s}{d\rho^p} \right $	$\left = 11 \right $

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Intro Model Equilibrium Cashless limit Quantitative CIA/MIU Conclusion
Quantitative exercises

- Compute asset price responses to increases in ρ^{p} for all $(\alpha, \lambda, \theta)$
- Since response is negative, report the absolute value of the semi-elasticity of the asset price to the policy rate, i.e.,

$$\mathcal{S} = \left| \frac{d\phi^s / \phi^s}{d\rho^p} \right|$$

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Intro Model Equilibrium Cashless limit Quantitative CIA/MIU Conclus

$\lim_{\alpha \to 0} \mathcal{S}$ as a function of λ and θ



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Model	Equilibrium	Cashless limit	Quantitative	CIA/MIU	Conclusion

Reduced-form models of money demand

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Intro Model Equilibrium Cashless limit Quantitative CIA/MIU Conclusion
Reduced-form money

The recursive equilibrium conditions of our model can be obtained from the following representation:

$$\max_{\{c_t, h_t, \boldsymbol{a}_{t+1}\}} \mathbb{E} \sum_{t=0}^{\infty} \beta^t \left[U(c_{1t}, c_{2t}) + c_t - h_t \right]$$

s.t.
$$c_t + \phi_t^s a_{t+1}^s + \phi_t^m a_{t+1}^m = h_t + (\bar{\epsilon}y_t + \phi_t^s) a_t^s + \phi_t^m a_t^m + T_t$$

 $c_{1t} = \frac{a_t^m}{\rho_t} y_t$
 $c_{2t} = a_t^s y_t$

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Intro Model Equilibrium Cashless limit Quantitative CIA/MIU Conclusion
Reduced-form money

The recursive equilibrium conditions of our model can be obtained from the following representation:

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s.t.
$$c_t + \phi_t^s a_{t+1}^s + \phi_t^m a_{t+1}^m = h_t + (\bar{\epsilon}y_t + \phi_t^s) a_t^s + \phi_t^m a_t^m + T_t$$

 $c_{1t} = \frac{a_t^m}{\rho_t} y_t$
 $c_{2t} = a_t^s y_t$

with

$$U(c_{1t},c_{2t}) \equiv u^z c_{1t} + u^s c_{2t}$$

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Reduced-form money

The recursive equilibrium conditions of our model can be obtained from the following representation:

$$\max_{\{c_t, h_t, \boldsymbol{a}_{t+1}\}} \mathbb{E} \sum_{t=0}^{\infty} \beta^t \left[U(c_{1t}, c_{2t}) + c_t - h_t \right]$$

s.t.
$$c_t + \phi_t^s a_{t+1}^s + \phi_t^m a_{t+1}^m = h_t + (\bar{\epsilon}y_t + \phi_t^s) a_t^s + \phi_t^m a_t^m + T_t$$

 $c_{1t} = \frac{a_t^m}{\rho_t} y_t$
 $c_{2t} = a_t^s y_t$

with

$$U(c_{1t},c_{2t}) \equiv \mathbf{u}^{\mathbf{z}}c_{1t} + \mathbf{u}^{\mathbf{s}}c_{2t}$$

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$$\varphi = \bar{\varepsilon} + u^{s}$$

$$\iota \geq rac{u^z}{arphi}$$
, with " = " if $0 < \mathcal{Z}$

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• Since u^s and u^z are treated as "deep" parameters:

• φ determined independently of ι and money

•
$$\mathcal{Z} > 0$$
 only if $\iota = \frac{u^z}{\varphi}$, and $\mathcal{Z} = 0$ if $\iota > \frac{u^z}{\varphi}$

 $\rightarrow\,$ Monetary considerations are irrelevant

$$\varphi = \bar{\varepsilon} + u^s$$

$$\iota \geq rac{u^z}{arphi}$$
 , with " = " if 0 $< \mathcal{Z}$

• But u^s and u^z are not "deep" parameters...

$$u^{s} = \left[\alpha + \alpha_{c} (1 - \theta)\right] \int_{\varepsilon_{L}}^{\varepsilon^{*}} \left(\varepsilon^{*} - \varepsilon\right) dG(\varepsilon) + \alpha_{c} \theta \left[\int_{\varepsilon_{L}}^{\varepsilon^{**}} \left(\varepsilon^{**} - \varepsilon\right) dG(\varepsilon) + \frac{\lambda}{1 - \lambda} \int_{\varepsilon^{**}}^{\varepsilon_{H}} \left(\varepsilon - \varepsilon^{**}\right) dG(\varepsilon)\right]$$

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$$\varphi = \bar{\varepsilon} + u^{s}$$

$$u \geq rac{u^z}{arphi}$$
, with " = " if $0 < \mathcal{Z}$

- But u^s and u^z are not "deep" parameters...
- The utility function *itself* changes with monetary policy

$$u^{s} = u^{s}(\iota)$$

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$$\varphi = \bar{\varepsilon} + u^{s}$$

$$u \geq rac{u^z}{arphi}$$
, with " = " if $0 < \mathcal{Z}$

- But u^s and u^z are not "deep" parameters...
- The *shape of the utility function* depends on: policy, credit conditions, and mark-ups in financial markets

$$U\left(c,\frac{M}{p};\iota,\alpha,\lambda,\theta\right)$$

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Intro	Model	Equilibrium	Cashless limit	Quantitative	CIA/MIU	Conclusion
Cond	clusion					

Medium-of-exchange considerations are important for monetary transmission—even in near-cashless economies where credit supports a large volume of transactions with arbitrarily small real balances.

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Model	Equilibrium	Cashless limit	Quantitative	CIA/MIU	Conclusion

Thank you all

for Minnesota Economics

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