A Theory of Non-Coasean Labor Markets

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This paper: First step toward a unifying framework

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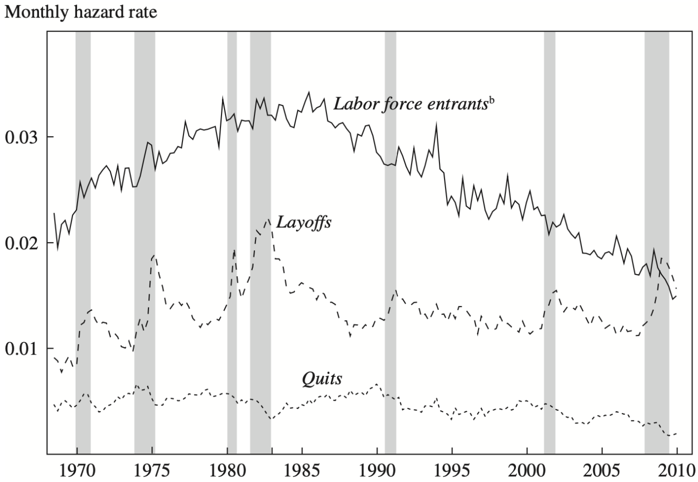
3. Survey evidence on sticky wages $\Longrightarrow$ layoffs (Bertheau et al. '23; Davis & Krolikowski '23)

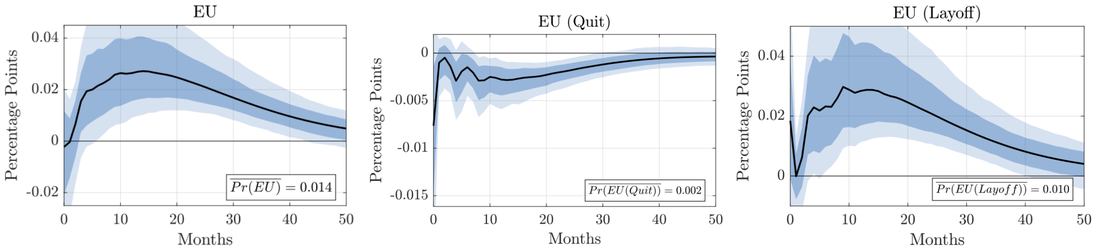
Empirical Reason 1: Quits $\neq$ Layoffs (Elsby et al. '10)

Unemployment Inflows by Reason for Unemployment



Empirical Reason 2: Labor Market Non-Neutrality (Graves et al. ’23)

Impulse Responses to a Contractionary Monetary Policy Shock



Empirical Reason 3: Sticky Wages $\Longrightarrow$ Layoffs (Davis & Krolikowski '23)

Percent of UI Recipients Who Would Accept a Pay Cut to Save Lost Job

Size of proposed pay cut	5%	10%	15%	20%	25%
Permanent layoffs	60.6	52.3	43.7	38.4	32.4
	(2.4)	(2.5)	(2.5)	(2.4)	(2.3)
	404	413	410	419	423
Temporary layoffs	54.5	42.9	35.8	34.3	37.4
	(5.0)	(5.0)	(4.9)	(4.7)	(4.9)
	101	98	95	102	99

For permanent layoffs: *"Would you have been willing to stay at your last job for another 12 months at a pay cut of X percent?"*
For temporary layoffs: *"Suppose your employer offered a temporary pay cut of X percent as an alternative to the temporary layoff. Would you have been willing to accept the temporary pay cut to avoid the layoff?"*

A Theory of Non-Coasean Labor Markets

Overview of Our Theory

We develop a theory of Non-Coasean labor markets featuring:

1. Directed search
2. Nominally rigid wages within jobs
3. Idiosyncratic productivity and aggregate monetary shocks
4. Two-sided lack of commitment

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Implications:

1. $\Longrightarrow$ Costly to find and fill jobs

2.–3. $\Longrightarrow$ Real wages do not track productivity

4. $\Longrightarrow$ Workers and firms play a game in Markov strategies

1.–4. $\Longrightarrow$ Inefficient separations through unilateral worker quits and firm layoffs

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- Worker's state:
  - Employment status s_t, either employed (h) or unemployed (u)
  - Log productivity z_t follows $\mathrm{d}z_t = \gamma \, \mathrm{d}t + \sigma \, \mathrm{d}\mathcal{W}_t^z$ [today: $\gamma = 0$, in paper: $\gamma \neq 0$]

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- Production: $y(s_t, z_t) = \begin{cases} e^{z_t} & \text{for } s_t = h \\ \tilde{B} e^{z_t} < e^{z_t} & \text{for } s_t = u \end{cases}$

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  - Market tightness: $\theta(w, z) = \mathcal{V}(w, z)/\mathcal{U}(w, z)$
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- **Key friction:** wages are rigid after match formation [in paper: Calvo hazard]

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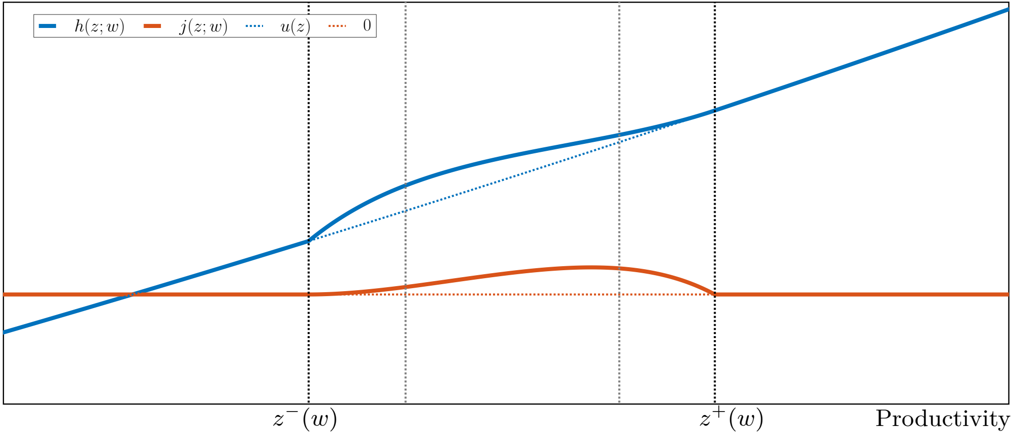
- **Match duration:** $\tau^m = \min\{\tau^\delta, \tau^h, \tau^j\}$

Characterizing Equilibrium Quits and Layoffs

Suff. Conditions



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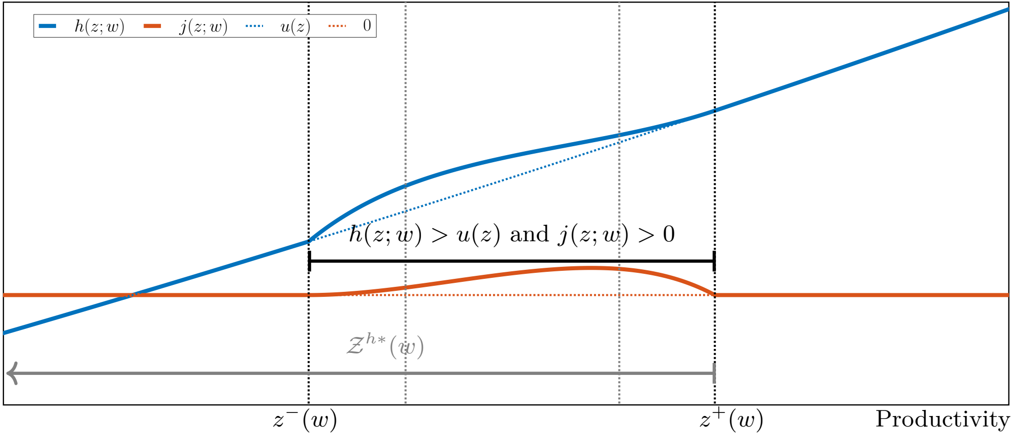

Value matching condition: $z \notin \mathcal{Z}^{h*}(w) \implies j(z;w) = 0$

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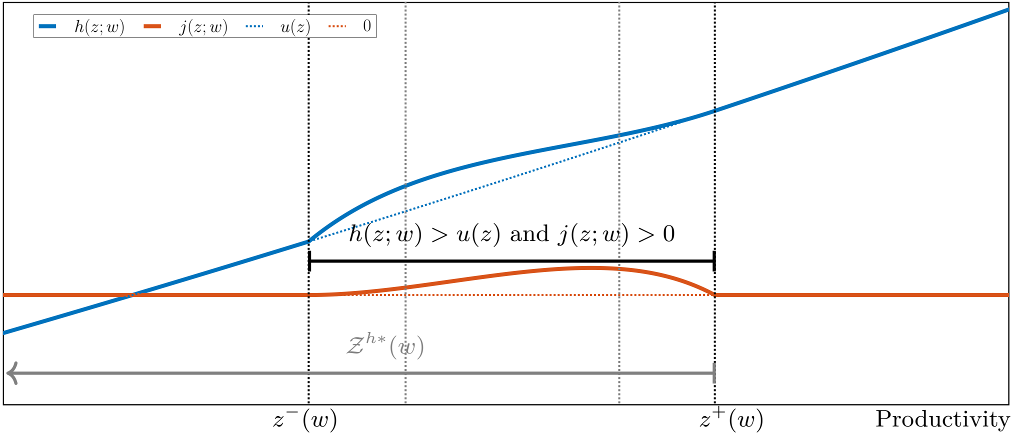

Firms' value under optimal policy: $(\rho+\delta)j(z;w) = \max\left\{e^z - e^w + \frac{\sigma^2}{2} j_{zz}(z;w), 0\right\}, \ \forall z \in \mathcal{Z}^{h*}(w)$

Smooth pasting condition: $j(\cdot;w) \in \mathbb{C}^1(\mathcal{Z}^{h*}(w)) \cap \mathbb{C}(\mathbb{R})$

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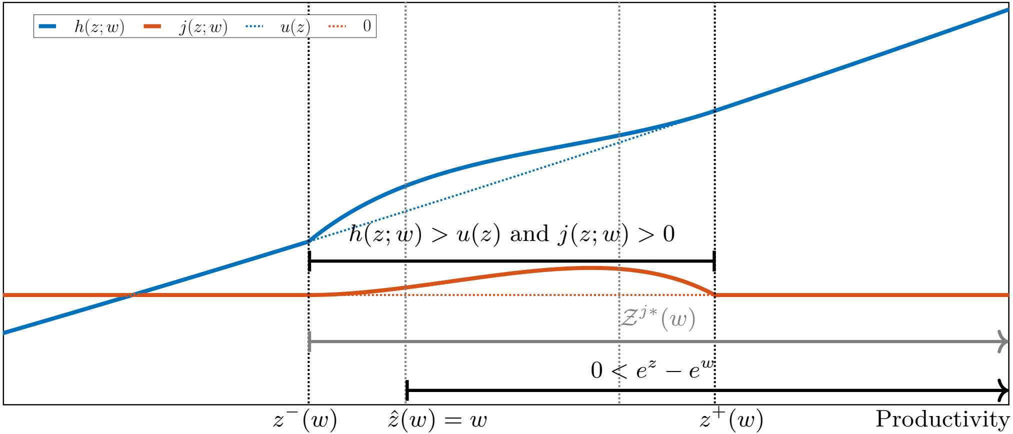


Firm's continuation set: $\mathcal{Z}^{j*}(w) = int\,\{z \in \mathbb{R} : e^z - e^w > 0 \text{ or } j(z;w) > 0\}$

$$\tau^{j*}(z;w) = \inf\left\{t \geq 0 : z_t \notin \mathcal{Z}^{j*}(w),\ z_0 = z\right\}$$

Equilibrium Existence and Uniqueness

Proposition 1 (Equilibrium Existence and Uniqueness).

There **exists** a **unique** block-recursive equilibrium.

- Extends Menzio & Shi ('10) to case of two-sided limited commitment
- Leverages continuous-time methods from stochastic diff. games literature
- Specifically, application of the Birkhoff-Tartar fixed point theorem

Details

Equilibrium Characterization

Characterizing the Game's Equilibrium

- State reduces to wage-productivity ratio $\hat{w} \equiv w - z$
  - Normalized values: $\hat{W}(\hat{w}) \equiv (h(z;w) - u(z))/e^z$, $\hat{J}(\hat{w}) \equiv j(z;w)/e^z$, $\hat{U} \equiv u(z)/e^z$
  - Normalized discount rate $\hat{\rho} \equiv \rho - \gamma - \sigma^2/2$
- Unemployed workers' policy: $\hat{w}^* \equiv w^*(z) - z$
- Optimal continuation region: (S, s) band in wage-productivity ratio $\hat{w} \in (\hat{w}^-, \hat{w}^+)$
- Match surplus $\hat{S}(\hat{w}) \equiv \hat{J}(\hat{w}) + \hat{W}(\hat{w})$ and worker's surplus share $\eta(\hat{w}) \equiv \hat{W}(\hat{w})/\hat{S}(\hat{w})$

Match Surplus $\hat{S}(\hat{w})$, Entry Wage $\hat{w}^*$, Job Finding Rate $f(\hat{w}^*)$

Proposition 2 (Surplus and Entry Wage).

1. Match surplus: $\hat{S}(\hat{w}) = (1-\hat{\rho}\hat{U})\mathcal{T}(\hat{w},\hat{\rho})$ with $\mathcal{T}(\hat{w},\hat{\rho}) \equiv \mathbb{E}^{\hat{w}}\left[\int_0^{\tau^{m*}} e^{-\hat{\rho}t}\,\mathrm{d}t\right]$, $\hat{\rho}\hat{U} \in (\tilde{B},1)$
2. The entry wage solves a "Nash bargaining problem":

$$\hat{w}^* = \arg\max_{\hat{w}} \left\{\hat{J}(\hat{w})^{1-\alpha}\hat{W}(\hat{w})^{\alpha}\right\} = \arg\max_{\hat{w}} \left\{(1-\eta(\hat{w}))^{1-\alpha}\eta(\hat{w})^{\alpha}\mathcal{T}(\hat{w},\hat{\rho})\right\}$$

with FOC

$$\eta'(\hat{w}^*)\underbrace{\left(\frac{\alpha}{\eta(\hat{w}^*)} - \frac{1-\alpha}{1-\eta(\hat{w}^*)}\right)}_{\text{Share channel}} = -\underbrace{\frac{\mathcal{T}'_{\hat{w}}(\hat{w}^*,\hat{\rho})}{\mathcal{T}(\hat{w}^*,\hat{\rho})}}_{\text{Surplus channel}}$$

3. Job finding rate: $f(\hat{w}^*) = \left[(1-\eta(\hat{w}^*))(1-\hat{\rho}\hat{U})\mathcal{T}(\hat{w}^*,\hat{\rho})/\tilde{K}\right]^{\frac{1-\alpha}{\alpha}}$

- Share channel: higher wage vs. higher job-finding rate [as in Moen ('97)!]
- Surplus channel: entry wage $\Rightarrow$ match duration $\Rightarrow$ surplus $\Rightarrow$ job-finding rate [novel!]

Aggregate Shocks in a Non-Coasean Labor Market

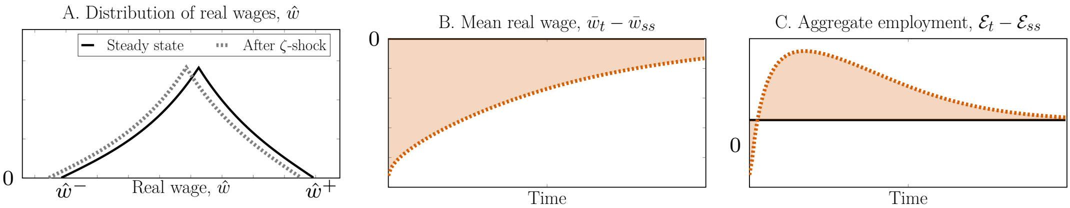
Aggregate Fluctuations with Sticky Wages

- Monetary economy summarized by distribution of real wage-to-productivity ratios:

$$\hat{w} = w - z - p$$

- Consider unanticipated one-off price level increase by ζ from steady state: $p_t = p_{t^-} + \zeta$
- **Employed workers'** nominal wages are sticky, so real wages become $\hat{w}_t = \hat{w}_{t^-} - \zeta$
- **New hires'** nominal wages fully flexible [in paper: sticky entry wages!]

Effects of an Inflationary Shock: An Illustration



- Cumulative Impulse Response (CIR) summarizes on-impact size and persistence of Impulse Response Function (IRF) of aggregate employment ($\mathcal{E}_t$) to monetary shock ζ:

$$CIR_{\mathcal{E}}(\zeta) = \int_0^\infty IRF_{\mathcal{E}}(\zeta, t)\, dt$$

Inflation Greases the Wheels of the Labor Market

Proposition 3 (CIR under Flexible Entry Wages).

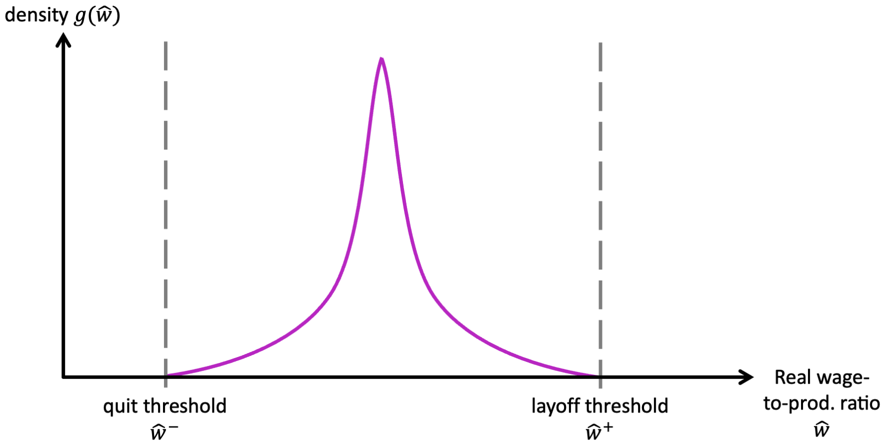
Sufficient statistic for CIR based on observed wage changes across jobs:

$$\frac{\mathrm{CIR}_{\mathcal{E}}(\zeta)}{\zeta} = \frac{1}{3} \times \underbrace{\frac{1}{f(\hat{w}^*)}}_{\text{exit rate } u} \times \underbrace{\frac{1}{\mathbb{V}ar_{\mathcal{D}}[\Delta w]}}_{\text{wage-change dispersion}} \times \underbrace{\mathbb{E}_{\mathcal{D}}\left[\Delta w \frac{\Delta w^2}{\mathbb{E}_{\mathcal{D}}\left[\Delta w^2\right]}\right]}_{\text{wage-change "skewness"}} + o(\zeta)$$

- Statistic captures marginal quits and layoffs on-impact + future dynamics
- Inflationary shock reduces real wages of employed workers
  - $f(\hat{w}^*)$ measures exit rate from unemployment
  - $\mathbb{V}ar_{\mathcal{D}}[\Delta w]$ measures total incidence of quit vs. layoff risk
  - "skewness" measures asymmetric incidence of quit vs. layoff risk

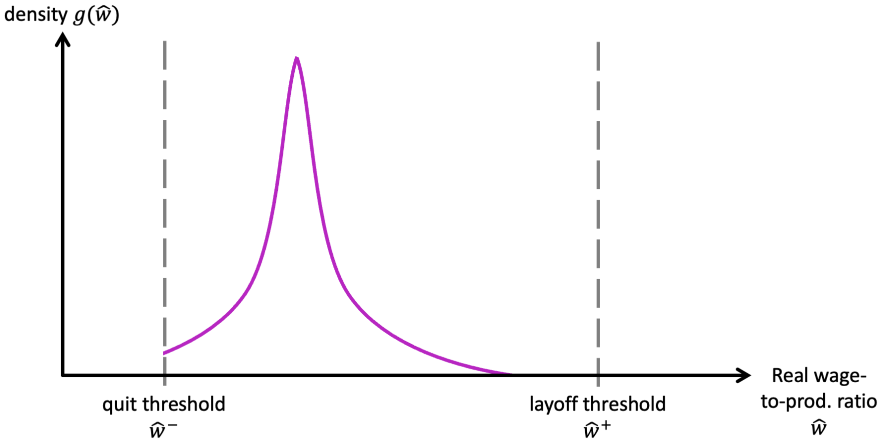
Graphical Intuition: Symmetric Steady State

Zero skewness of real-wage-to-productivity ratios
$\Longrightarrow$ Zero employment effects of inflation



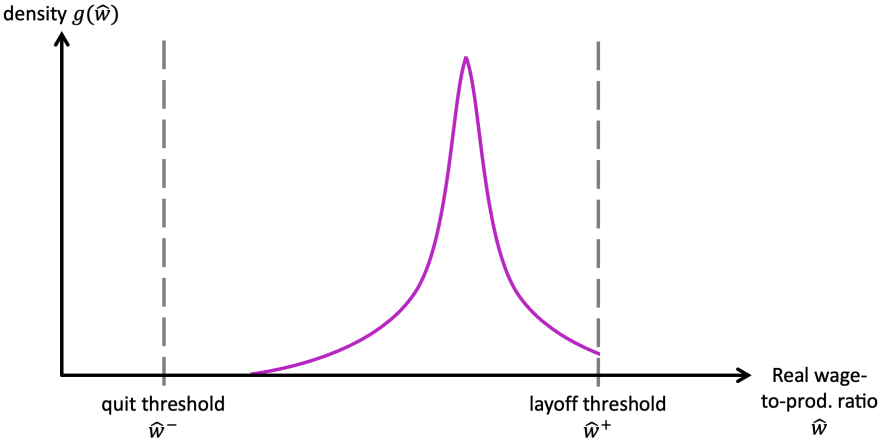
Graphical Intuition: Expansion

Negative skewness of real-wage-to-productivity ratios
$\Longrightarrow$ Negative employment effects of inflation



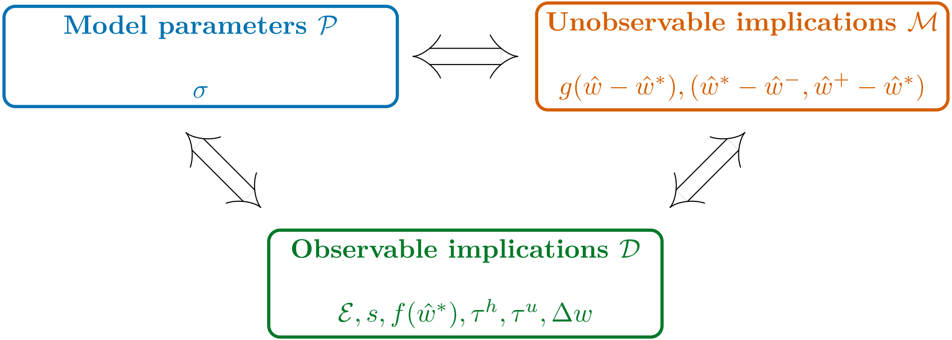
Graphical Intuition: Recession

Positive skewness of real-wage-to-productivity ratios
$\Longrightarrow$ Positive employment effects of inflation



Empirical Application

Overview of Model Identification



We provide a closed-form mapping between $\mathcal{P}$, $\mathcal{M}$ and $\mathcal{D}$ 3 steps

U.S. Labor Market Data

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Survey of Income and Program Participation (SIPP)

- Monthly employment and income (wage, salary) information
- Standard selection criteria and cleaning procedures (Barattieri et al. '14)
- Wage filter to correct measurement error (Blanco et al. '22)
- Two periods: hot labor market 1996–2000 and cold labor market 2008–2012

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Future work:

1. Low to medium inflation in U.S. (LEHD) from 2001–2022
2. High inflation in Brazil (RAIS) from 2015–2018
3. Hyperinflation in Argentina (SIPA) from 2001–2002

Expansionary Monetary Policy in Hot and Cold Labor Markets

According to our model estimates, an expansionary monetary policy shock:

1. leads to net decrease in employment in hot labor market (U.S. 1996–2000)
   - Increase in quits > decrease in layoffs
2. leads to net increase in employment in cold labor market (U.S. 2008–2012)
   - Increase in quits < decrease in layoffs

Period	CIR	$=$	$1/f$	$\times$	$1/Var(\Delta w)$	$\times$	"$Skewness$"
U.S. 1996–2000 (hot)	−0.073		5.170		0.019		−0.710
U.S. 2008–2012 (cold)	0.100		7.450		0.021		0.640

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$\Longrightarrow$ **inflation greases the wheels of the labor market (Tobin '72)**

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- We developed a theory of **Non-Coasean labor markets**
- Novel characterization of inefficient separations in the form of layoffs vs. quits
- First step towards quantitative analysis:

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$$\text{labor market heterogeneity} \iff \text{monetary policy}$$

Future work:

- More heterogeneity: match-specific shocks, firm heterogenity
- Additional channels: on-the-job search, costly renegotiations
- Applications: optimal monetary policy, severance pay, wage indexation
  $\longrightarrow$ Low to medium (U.S.), high (Brazil), and hyperinflation (Argentina)

Backup

Equilibrium Refinement Back

Assumption: one-period game. $e^z - e^w > 0$, $e^w \,\mathrm{d}t + \mathbb{E}_z[e^{-\rho \,\mathrm{d}t} U(z')] > U(z)$

	Worker stops	**Worker continues**
Firm stops	$(\underline{0}, \underline{U(z)})$	$(0, U(z))$
Firm continues	$(0, \overline{U(z)})$	$(\underline{(e^z - e^w)\,\mathrm{d}t}, \underline{e^w \,\mathrm{d}t + \mathbb{E}_z[e^{-\rho \,\mathrm{d}t} U(z')]})$

Necessary and Sufficient Conditions Back

1. Lower bounds for values [GLB]:

$$\underbrace{(h(z;w), j(z;w))}_{\text{value of match}} \geq \underbrace{(u(z), 0)}_{\substack{\text{value of} \\ \text{outside option}}}$$

2. Firm's and worker's continuation sets [GCS]:

$$\mathcal{C}_w^j := int \{z \in \mathbb{R} : j(z;w) > 0 \ or \ 0 < e^z - e^w\}$$

$$\mathcal{C}_w^h := int \left\{ z \in \mathbb{R} : h(z;w) > u(z) \ or \ 0 < e^w - \rho u(z) + \gamma \frac{\partial u(z)}{\partial z} + \frac{\sigma^2}{2} \frac{\partial^2 u(z)}{\partial z^2} \right\}$$

with stopping times given by

$$\tau^{j*}(w, z) = \inf \left\{ t \geq 0 : z_t^z \notin \mathcal{C}_w^j \right\}$$

$$\tau^{h*}(w, z) = \inf \left\{ t \geq 0 : z_t^z \notin \mathcal{C}_w^h \right\}$$

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$$(h(z;w), j(z;w)) \geq (u(z), 0)$$

2. Firm's and worker's continuation sets [GCS]:

$$\mathcal{C}_w^j := int\,\{z \in \mathbb{R} : j(z;w) > 0 \; or \; 0 < e^z - e^w\}$$
$$\mathcal{C}_w^h := int\left\{z \in \mathbb{R} : h(z;w) > u(z) \; or \; 0 < e^w - \rho u(z) + \gamma \frac{\partial u(z)}{\partial z} + \frac{\sigma^2}{2}\frac{\partial^2 u(z)}{\partial z^2}\right\}$$

with stopping times given by

$$\tau^{j*}(w,z) = \inf\left\{t \geq 0 : z_t^z \notin \mathcal{C}_w^j\right\}$$
$$\tau^{h*}(w,z) = \inf\left\{t \geq 0 : z_t^z \notin \mathcal{C}_w^h\right\}$$

Necessary and Sufficient Conditions Back

3. Game's value matching conditions [GVM]:

$$z \notin \mathcal{C}_j^w \implies h(z;w) = u(z), \quad z \notin \mathcal{C}_h^w \implies j(z;w) = 0$$

4. Firm's and workers optimal layoff and quit policies [GOP]:

$$\text{if } z \in \mathcal{C}_w^h \quad \rho j(z;w) = \max\left\{e^z - e^w + \gamma \frac{\partial j(z;w)}{\partial z} + \frac{\sigma^2}{2}\frac{\partial^2 j(z;w)}{\partial z^2} - \delta j(z;w), 0\right\}$$

$$j(\cdot;w) \in \mathbb{C}^1(\mathcal{C}_w^h) \cap \mathbb{C}(\mathbb{R})$$

$$\text{if } z \in \mathcal{C}_w^j \quad \rho h(z;w) = \max\left\{e^w + \gamma \frac{\partial h(z;w)}{\partial z} + \frac{\sigma^2}{2}\frac{\partial^2 h(z;w)}{\partial z^2} + \delta\left(u(z) - h(z;w)\right), \rho u(z)\right\}$$

$$h(\cdot;w) \in \mathbb{C}^1(\mathcal{C}_w^j) \cap \mathbb{C}(\mathbb{R})$$

Necessary and Sufficient Conditions Back

5. Free entry condition [FEC]:

$$(K(e^z) - q(w,z)j(z;w))\theta(w,z) = 0, \quad K(e^z) - q(w,z)j(z;w) \geq 0, \quad \theta(w,z) \geq 0 \;\; \forall (w,z)$$

6. Firm's and workers optimal layoff and quit policies [HBJ U]:

$$\rho u(z) = \tilde{B}e^z + \gamma \frac{\partial u(z)}{\partial z} + \frac{\sigma^2}{2}\frac{\partial^2 u(z)}{\partial z^2} + \max_w f(w,z)[h(z;w) - u(z)],$$

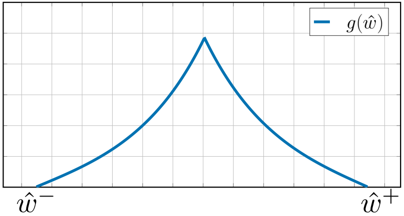
Cross-sectional Distribution of $\hat{w}$ Back

- Kolmogorov Forward Equation:

$$\delta g(\hat{w}) = \gamma g'(\hat{w}) + \frac{\sigma^2}{2} g''(\hat{w}) \quad \text{for all } \hat{w} \in (\hat{w}^-, \hat{w}^+)/\{\hat{w}^*\}$$

- Border conditions:

$$g(\hat{w}^-) = g(\hat{w}^+) = 0, \quad 1 = \int_{\hat{w}^-}^{\hat{w}^+} g(\hat{w})\, \mathrm{d}\hat{w}, \quad g(\hat{w}) \in \mathbb{C}$$



Cross-sectional Distributions of Δz Back

- KFE for employed workers

$$\delta g^h(\Delta z) = \gamma (g^h)'(\Delta z) + \frac{\sigma^2}{2}(g^h)''(\Delta z) \quad \text{for all } \Delta z \in (-\Delta^-, \Delta^+)/\{0\}$$

$$g^h(\Delta z) = 0, \ \text{for all } \Delta z \notin (-\Delta^-, \Delta^+)$$

- KFE for unemployed workers

$$f(\hat{w}^*)g^u(\Delta z) = \gamma (g^u)'(\Delta z) + \frac{\sigma^2}{2}(g^u)''(\Delta z) \quad \text{for all } \Delta z \in (-\infty, \infty)/\{0\}$$

$$\lim_{\Delta z \to -\infty} g^u(\Delta z) = \lim_{\Delta z \to \infty} g^u(\Delta z) = 0$$

- Measure 1 of worker, constant employment, and regularity conditions

$$1 = \int_{-\infty}^{\infty} g^u(\Delta z)\, \mathrm{d}\Delta z + \int_{-\Delta^-}^{\Delta^+} g^h(\Delta z)\, \mathrm{d}\Delta z$$

$$f(\hat{w}^*)(1-\mathcal{E}) = \delta \mathcal{E} + \frac{\sigma^2}{2}\left[\lim_{\Delta z \downarrow -\Delta^-}(g^h)'(\Delta z) - \lim_{\Delta z \uparrow \Delta^+}(g^h)'(\Delta z)\right]$$

$$g^h(\Delta z), g^u(\Delta z) \in \mathbb{C}$$

From $\mathcal{D}$ata to $\mathcal{M}$odel Back

- Step 1: Recover model parameters $\mathcal{P}$

$$\gamma = \frac{\mathbb{E}_{\mathcal{D}}[\Delta w]}{\mathbb{E}_{\mathcal{D}}[\tau]} \quad \sigma^2 = \frac{\mathbb{E}_{\mathcal{D}}[(\Delta w - \gamma\tau)^2]}{\mathbb{E}_{\mathcal{D}}[\tau]} \text{ where } \tau = \tau^m + \tau^u$$

Intuition:

- average Δw must compensate for productivity trend between jobs
- dispersion of (detrended) Δw captures dispersion of cumulative shocks during h-u-h

From $\mathcal{D}$ata to $\mathcal{M}$odel Back

- Step 1: Recover model parameters ✓
- Step 2: Recover distribution of Δz conditional on a h-u transition

Intuition:

$$\begin{aligned}
\Delta w &= w_{t_0+\tau^m+\tau^u} - w_{t_0} \\
&= \underbrace{w_{t_0+\tau^m+\tau^u} - z_{t_0+\tau^m+\tau^u}}_{=\hat{w}^*} - \underbrace{\left(w_{t_0} - z_{t_0}\right)}_{=\hat{w}^*} + z_{t_0+\tau^m+\tau^u} - z_{t_0} \\
&= z_{t_0+\tau^m+\tau^u} - z_{t_0+\tau^m} + z_{t_0+\tau^m} - z_{t_0} \\
&= -(\Delta z | h\text{-}u \text{ transition} + \Delta z | u\text{-}h \text{ transition at } z_{t_0+\tau^m}) \\
&= -(\Delta z | h\text{-}u \text{ transition} + \underbrace{\Delta z | u\text{-}h \text{ transition}}_{\text{independent and } \textbf{known}})
\end{aligned}$$

From $\mathcal{D}$ata to $\mathcal{M}$odel Back

- Step 1: Recover model parameters ✓
- Step 2: Recover distribution of Δz conditional on a h-u transition ✓
- Step 3: Recover unconditional distribution of Δz

  Intuition:

  - conditional distribution + model during inaction ⇒ unconditional distribution

From $\mathcal{D}$ata to $\mathcal{M}$odel Back

- Step 1: Recover model parameters ✓
- Step 2: Recover distribution of Δz conditional on a h-u transition ✓
- Step 3: Recover unconditional distribution of Δz ✓

Environment Back

- **Money supply:** $\mathrm{d}\log(M_t) = \pi\,\mathrm{d}t + \zeta\,\mathrm{d}W_t^m$, W_t^m is a Wiener process
- **Preferences:** $\mathbb{E}_0\left[\int_0^\infty e^{-\rho t}\left(C_{it} + \mu\log\left(\hat{M}_{it}/P_t\right)\right)\mathrm{d}t\right]$
- **Complete financial markets**
- **Budget constraint:** $\mathbb{E}_0\left[\int_0^\infty Q_t\left(P_tC_{it} + i_t\hat{M}_{it} - Y_{it}(lm^{it}) - T_{it}\right)\mathrm{d}t\right] = \hat{M}_0$
  - Q_t : time-zero Arrow-Debreu price
  - $Y_{it}(lm^{it})$: nominal labor income
  - lm^{it} : labor market strategy
- **Market clearing:**

$$\int_0^1 \hat{M}_{it}\,\mathrm{d}i = M_t$$

$$\int_0^1 \left(C_{it} + \theta_{it}\mathbb{1}_{\{e_{it}=u\}}K(Z_{it})\right)\mathrm{d}i = \int_0^1 \left(e^{z_{it}}\mathbb{1}_{\{e_{it}=h\}} + B(Z_{it})\mathbb{1}_{\{e_{it}=u\}}\right)\mathrm{d}i$$

Equilibrium Dynamics and Worker's Problem Back

Lemma. (Equilibrium Prices and Workers' Problem)

Let $Q_0 = 1$ be the numéraire and $\mu = \rho + \pi + \frac{\zeta^2}{2}$. Then, $P_t = M_t$. Define $V_0(z, M_t)$ as the worker's optimal value, then

$$V_0(z, M) = \max_{\{lm_{it}\}} \mathbb{E}_0 \left[\int_0^\infty e^{-\rho t} \frac{Y_{it}(z_{it}, lm^{it})}{M_t} \, \mathrm{d}t \right] + \text{terms independent of policy}$$

- In equilibrium, monetary shocks translate one-to-one to prices
- Since worker is risk neutral in consumption

$$\text{maximizing consumption} = \text{maximizing PDV of real income}$$

- New state for the worker: $\hat{w} = w - z - m$