Investment-Goods Market Power and Capital Accumulation

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Abstract

We develop a model of capital accumulation in an open economy that imports investment goods from large foreign firms with market power. We model investment-goods producers as a dynamic oligopoly and characterize a Markov Perfect Equilibrium with a Generalized Euler Equation. We use this optimality condition to analyze the joint evolution of investment, prices, and markups. The markup on investment goods decreases as the economy accumulates capital toward its steady state, generating a state-dependent capital adjustment cost. We analyze the role of commitment to future production of investment goods for the dynamics of markups and investment. We use a calibrated version of the model to simulate the effects of shocks to the demand for durable goods and semiconductors during the post-2020 world recovery. The model highlights the separate roles of increasing marginal costs—akin to capacity constraints—and market power.

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1 Introduction

The post-2020 global recovery has been a stark reminder of the dependence of the macroeconomy on the supply of critical inputs that most countries import from highly concentrated industries, such as semiconductors. When demand for durable goods increased during the recovery, prices soared, thereby dampening capital accumulation, contributing to the increase in inflation, and leading to the design of ambitious policy plans to modify the global production structure.¹

Semiconductors are necessary components of equipment goods and it is likely that future economic growth will increasingly rely on them. More in general, many important types of durable inputs are produced by highly concentrated industries. As examples, consider commercial aircraft, commercial ships, electric vehicles, or construction and mining machinery. For all of these investment goods, a relatively small number of large global producers supply the world economy.

What is the role of market power in investment-goods markets for the dynamics of capital accumulation, output, and prices? The goal of this paper is to develop a framework to address this question. To this end, we combine a neoclassical growth model of capital accumulation with a dynamic oligopoly model of investment-goods producers and use it to analyze the aggregate dynamics of investment, prices, and markups.

In the model, a small open economy accumulates capital by importing investment goods according to a standard investment Euler equation. Investment requires an input produced by an oligopolistic industry. Foreign producers maximize the present discounted value of profits, internalizing the effects of their production decisions on prices through the Euler equation. We analyze a Markov Perfect Equilibrium, in which strategies depend on a natural state variable, namely the level of capital in the small open economy.

Because of the durable nature of capital, investment-goods producers effectively compete with the undepreciated stock of capital—equivalently, the secondary market for investment goods—, as well as among themselves, and choose the level of production trading off current and future profits. By focusing on differentiable policy functions, we characterize the optimal trade-off with a Generalized Euler Equation, which relates the markup to the derivatives of the equilibrium policy functions. We then leverage this characterization to

¹In 2021, the two largest semiconductor manufacturers—TSMC and Samsung—jointly accounted for approximately 70% of global sales. In the US, the CHIPS and Science Act of 2022 aimed at generating hundreds of billions of dollars of investment in semiconductor manufacturing to rebalance the global patterns of production of semiconductors, which is concentrated in Asia.
understand the evolution of markups along the equilibrium path of capital accumulation.

We calibrate the model interpreting the foreign oligopoly as the semiconductor industry and perform a quantitative exploration of the role of market power for the dynamics of investment. When the level of capital in the small open economy is low, the price of investment and the markup are high because there is high demand for investment goods. Then, as the small open economy accumulates capital toward its steady state, prices and markups decline over time.

This mechanism generates a state-dependent capital adjustment cost. Endogenous markups contribute to slow convergence in the small open economy. Forward-looking investment-goods producers anticipate future demand conditions along the transition path and internalize the competition with the future capital stock. This feature of our model reinforces the endogenous capital-accumulation friction.

We contrast these findings with a version of the model in which investment-goods producers commit to future production plans. In this case, the internalization of competition with past undepreciated production leads to markups that are higher in levels and do not decrease as the small open economy grows. This comparison sheds light on the nature of time inconsistency in our model and its macroeconomic implications.

Our analysis of the transitional dynamics is useful to understand the response of the economy to aggregate shocks that shift the optimal level of capital in the small open economy. Specifically, we perform several experiments in the calibrated model to reproduce salient features of the post-2020 global recovery, which featured strong demand for durable goods.

We first simulate an increase in Total Factor Productivity (TFP) in the small open economy, which drives a rise in demand for investment goods. We contrast two scenarios, with different slope of marginal costs with respect to production levels. We find that markups increase sharply in response to the shock and then decrease over time, consistent with empirical evidence on the profitability of semiconductor producers in the recent recovery. We also find that in the scenario with a steeper marginal cost—akin to a capacity constraint—the contribution of markups to the equilibrium price hike is significantly smaller. We then extend our model to stochastic, persistent productivity shocks and perform simulations that confirm the main insights of our parsimonious baseline model in a richer business-cycle framework.

The experience of the recent recovery has motivated several policy interventions that
may reduce the concentration of some critical sectors, such as semiconductors. We use our model to simulate the effects of entry of one additional large producer. Marginal costs decrease because the production of investment goods is spread across more units, and so do markups because of enhanced competition pressure. Nevertheless, prices adjust only gradually because demand for investment is initially high. This policy implication confirms the relevance of analyzing market power and capital accumulation in a dynamic equilibrium framework.

The rest of the paper is organized as follows. Section 2 discusses our contributions to the literature. Section 3 presents the model environment. Section 4 characterizes the dynamic oligopoly in investment goods. Section 5 presents the quantitative analysis of the role of market power for capital accumulation. Section 6 discusses the effects of aggregate shocks. Section 7 concludes.

2 Related Literature

This paper contributes to several strands of the literature. A growing body of work in macroeconomics analyzes the aggregate effects of producer market power. De Loecker, Eeckhout, and Unger (2020) studies the evolution of markups over time in the US economy. Edmond, Midrigan, and Xu (2023) provide a quantitative analysis of the social cost of markups. While many studies focus on imperfect competition and price dynamics in output markets (e.g., Mongey, 2021; Wang and Werning, 2022), several recent paper focus on market power and firm granularity in input markets, such as the labor market (e.g., Berger, Herkenhoff, and Mongey, 2022; Jarosch, Nimczik, and Sorkin, 2023), and the credit market. Our contribution is to focus on market power in the production of dynamic inputs such as investment goods. We develop a framework to analyze the effects of market power on capital accumulation.

The literature on investment dynamics typically focuses on frictions on the demand side of the market for investment goods, such as adjustment costs at the firm level (e.g., Cooper and Haltiwanger, 2006; Khan and Thomas, 2008; Baley and Blanco, 2021; Winberry, 2021) or financing constraints (e.g., Buera and Shin, 2013; Moll, 2014; Lanteri and Rampini, 2023), as well as on the role of firm heterogeneity. We explore a complementary approach and analyze distortions stemming from the supply side of investment goods—namely, market power of producers. To gain tractability of the Markov Perfect Equilibrium, we abstract
from firm heterogeneity, but our analysis can be extended to the case of heterogeneous firms in future work. Fiori (2012) analyzes the role of fixed adjustment costs on the supply side of investment goods in a model with heterogeneous firms. Our focus on competition on the production side of investment-goods markets builds on the work Bertolotti and Lanteri (2024), which models endogenous product innovation, but abstracts from strategic interactions.

This paper also contributes to the large literature on international trade and macroeconomic dynamics (e.g. Ghironi and Melitz, 2005; Atkeson and Burstein, 2008). Several papers analyze the role of investment-goods trade and prices in open economies. Since the work of Eaton and Kortum (2001), the literature has emphasized the high degree of geographic concentration in the global production of investment goods. Restuccia and Urrutia (2001) and Hsieh and Klenow (2007) study the effects of investment prices on investment rates and growth across countries. Engel and Wang (2011) emphasizes the critical role of trade in durable goods for the comovement between aggregate activity and trade flows. Burstein, Cravino, and Vogel (2013) focuses on the effects of investment-goods imports on wages. Lanteri, Medina, and Tan (2023) analyzes the effects of trade shocks on capital reallocation in a small open economy. Our paper contributes to this body of work by analyzing market power in investment-goods markets as a source of friction in capital accumulation. Our application on demand for investment goods and capacity constraints during the recent recovery is related to the analyses of Comin, Johnson, and Jones (2023), Fornaro and Romei (2023) and Darmouni and Sutherland (2024).

Our methodology combines a neoclassical growth model with a model of dynamic oligopoly in durable-goods markets and we analyze a Markov Perfect Equilibrium (Maskin and Tirole, 2001). A large theoretical literature in industrial organization investigates monopoly pricing for durable goods with and without commitment (e.g., Coase, 1972; Stokey, 1981; Kahn, 1986; Suslow, 1986) and several papers leverage its insights to provide quantitative analyses of durable-good oligopolies (e.g., Esteban and Shum, 2007; Goettler and Gordon, 2011). We build on this literature to analyze the aggregate capital-accumulation effects of market power, in particular in response to shocks to the demand for investment goods. Following the approach of Villa (2023), we characterize the equilibrium dynamics with an interpretable Generalized Euler Equation, a tool introduced in the literature on optimal fiscal policy (Klein, Krusell, and Ríos-Rull, 2008). We also consider the case of commitment to future production, which we solve recursively using the
multiplier on the investment Euler equation as a state variable (Marcet and Marimon, 2019).

3 Model

In this section, we present our model of a small open economy that accumulates capital by importing investment goods from a finite number of large producers. We then characterize the efficient allocation. We focus on a deterministic model to make the analysis clearer and then extend the model to stochastic shocks in our quantitative analysis.

3.1 Small Open Economy and Investment Demand

We begin by describing the demand side of the market for investment goods. A deterministic small open economy is populated by a representative household with utility function

$$\sum_{t=0}^{\infty} \beta^t u(C_t),$$

where $\beta \in (0, 1)$ denotes the discount factor, $C_t$ is aggregate consumption, and $u_c > 0$, $u_{cc} \leq 0$, where subscripts denote first and second derivative respectively.

The budget constraint of the household reads

$$C_t + P_t^I I_t + B_t = W_t L + R^K_{t} K_{t-1} + RB_{t-1} + D_t,$$

where $P_t^I$ is the price of investment $I_t$, $B_t$ are bonds that offer the world gross interest rate $R$, $W_t$ is the wage, $L$ is a constant endowment of labor, $R^K_t$ denotes the rental rate of capital $K_{t-1}$, and $D_t$ are profits obtained from ownership of domestic firms. We assume that the household is only subject to the natural debt limit.

Investment adds to the capital stock, which depreciates at rate $\delta$:

$$K_t = (1 - \delta) K_{t-1} + I_t.$$

We assume that investment has to be non-negative and restrict attention to a region of the parameter space where this constraint is not binding.

The first-order conditions of the utility maximization problem with respect to bonds
and investment are

\[ 1 = \beta \frac{u_c(C_{t+1})}{u_c(C_t)} R \]  \hspace{1cm} (2)

\[ P_t^I = \beta \frac{u_c(C_{t+1})}{u_c(C_t)} \left( R_{t+1}^K + (1 - \delta) P_{t+1}^I \right). \]  \hspace{1cm} (3)

A representative firm rents capital from the representative household and hires labor to produce output with a constant-returns to scale production function:

\[ Y_t = F(K_{t-1}, L). \]  \hspace{1cm} (4)

The first-order conditions of the profit maximization problem are

\[ F_K(K_{t-1}, L) = R_{t}^K \]
\[ F_L(K_{t-1}, L) = W_t. \]  \hspace{1cm} (5)

For notational convenience, we define \( f(K_{t-1}) \equiv F(K_{t-1}, L) \). Because of constant returns to scale, the representative firm makes zero profits in equilibrium—i.e., \( D_t = 0 \).

We assume that the interest rate satisfies \( R = \beta^{-1} \). By combining the household and firm optimality conditions (2), (3), and (5), we obtain the following investment Euler equation that describes optimal capital accumulation in the small open economy:

\[ P_t^I = R^{-1} \left( f_k(K_t) + (1 - \delta) P_{t+1}^I \right). \]  \hspace{1cm} (6)

Equation (6) implicitly expresses the demand for investment goods as a function of the capital stock \( K_{t-1} \) as well as current and future investment prices \( P_t \) and \( P_{t+1} \).

We stress that our assumptions on ownership of the capital stock are immaterial and we can equivalently derive this condition assuming that firms accumulate capital instead of households.

We also highlight that the open economy is “small” in the sense that the interest rate is exogenous. We make this assumption to focus on the determination of the endogenous price of investment goods and abstract from the possible internalization of interest-rate changes in the decisions of investment-goods producers, which is less likely to be a channel of first-order importance.
3.2 Investment-Goods Production

We now describe the supply side of the market for investment goods.

**Assembly of investment.** A perfectly competitive representative firm combines an amount $Q_t$ of imported investment goods and an amount $X_t$ of output good to assemble domestic investment with a Leontief production function:

$$I_t = \min \left\{ \frac{Q_t}{\theta}, \frac{X_t}{1-\theta} \right\},$$

where $\theta \in [0, 1]$ denotes the share of imported investment goods, which trade at price $P_t$. Profit maximization implies $\frac{Q_t}{\theta} = \frac{X_t}{1-\theta}$ and the equilibrium investment price must satisfy

$$P_t^I = \theta P_t + 1 - \theta,$$  \hspace{1cm} (7)

which implies that the investment assembling firm makes zero profits. It is thus immaterial whether this technology is owned by domestic or foreign investors. Notice that our model nests a standard small-open-economy neoclassical growth model when $\theta = 0$.

**Production of imported investment goods.** We assume that there is an integer number $N \geq 1$ of identical producers of a homogeneous good, which we refer to as “investment-goods producers.” Equivalently, there is a fixed cost of entering the industry and the level of this cost is such that entry is profitable for $N$ firms, but would yield negative profits with a larger number of entrants. These firms are owned by foreign investors.

The production of investment requires output goods. Specifically, each investment-good producer has a cost function $c(q_t)$, where $q_t$ is the quantity produced at date $t$ and we assume $c_q > 0$ and $c_{qq} \geq 0$. Hence, static profits at date $t$ are given by $\pi_t \equiv P_t q_t - c(q_t)$.

We will consider several alternative assumptions on competition and strategic interactions. Across all of these assumptions, we maintain that the objective of investment-goods producers is to maximize the present discounted value of profits:

$$\sum_{t=0}^{\infty} R^{-t} \pi_t.$$  \hspace{1cm} (8)

Our analysis can be extended to domestic investment-goods producers owned by the representative household. However, in this case the objective function (8) would not coincide
with the objective of the firm owner when firms do not take prices as given.²

3.3 First Best

Before analyzing the effects of market power, we briefly introduce the competitive benchmark, which coincides with the solution to the problem of a planner who maximizes welfare in the small open economy taking as given the cost function to produce investment goods. We formulate this problem explicitly in Appendix A.1.

In a competitive equilibrium without market power, investment-goods producers choose a sequence of production levels \( \{q_t\}_{t=0}^{\infty} \) to maximize (8) *taking as given* the sequence of prices \( \{P_t\}_{t=0}^{\infty} \). Thus, the equilibrium price satisfies \( P_t = c_q \left( \frac{\theta I_t}{N} \right) \) and optimal capital accumulation satisfies

\[
\theta c_q \left( \frac{\theta I_t}{N} \right) + 1 - \theta = R^{-1} \left( f_k(K_t) + (1 - \delta) \left( \theta c_q \left( \frac{\theta I_{t+1}}{N} \right) + 1 - \theta \right) \right).
\]

Notice that if the cost function \( c \) is convex, it acts as a capital adjustment cost for the small open economy. Furthermore, convexity implies that it is efficient to produce the same amount in all of the investment-goods firms, which motivates our focus on symmetric equilibria in the remainder of the paper.

4 Dynamic Oligopoly

We now analyze the case of investment-goods producers that act as oligopolists and internalize the residual demand for investment. We describe the Markov Perfect Equilibrium and derive the optimality conditions of the investment-goods producers. We then use these optimality conditions to relate markups and capital accumulation. Finally, we contrast this problem with the case of commitment to future production.

4.1 Markov Perfect Equilibrium and Generalized Euler Equation

To focus on time-consistent decisions in the absence of commitment to future production levels, we analyze a symmetric Markov Perfect Equilibrium with Cournot competition, in which quantities produced are functions of a single natural state variable, the capital

²For an analysis of common ownership in oligopoly in general equilibrium models, see Azar and Vives (2021).
stock in the small open economy. To obtain a sharper characterization, we further restrict attention to differentiable decision rules.

Combining equations (6) and (7) and using recursive notation, we can express the investment Euler equation—i.e., the demand curve for investment goods—as follows:

\[ P = R^{-1} \left( \theta^{-1} f_k(K') + (1 - \delta) P(K') \right) - \kappa, \quad (10) \]

where \( \kappa \equiv \theta^{-1} (1 - \theta) (1 - R^{-1} (1 - \delta)) \).

For a generic investment-goods producer, we denote by \( q_-(K) \) the quantity produced by each other producer as a function of the capital stock \( K \). Furthermore, investment-good producers anticipate the equilibrium price function \( P(K') \) and the continuation value function \( V(K') \), encoding the present discounted value of profits (8). Each producer solves the following problem:

\[
\max_{P,q,K'} P q - c(q) + R^{-1} V(K'),
\]

subject to the Euler equation (10) and the law of motion for capital

\[ K' = (1 - \delta) K + \theta^{-1} ((N - 1) q_-(K) + q), \quad (11) \]

where we used the market-clearing condition \((N - 1) q_-(K) + q = Q = \theta I\) to express aggregate production of the investment good. This formulation of the capital accumulation equation clarifies that each firm effectively competes with the other \( N - 1 \) as well as the existing stock of undepreciated capital.

The optimality condition for the production level can be represented as the following Generalized Euler Equation (GEE):

\[
\theta P - \theta c_q(q) + q R^{-1} \left( \theta^{-1} f_{kk}(K') + (1 - \delta) P_k(K') \right) + R^{-1} V_k(K') = 0.
\quad (12)
\]

This is a functional equation that involves the derivative of the future price with respect to the capital stock, reflecting the fact that investment-good producers cannot commit to future actions, but internalize the effect of current production on future equilibrium outcomes.

In a symmetric equilibrium, the maximum value of this problem coincides with \( V(K) \).
Thus, the envelope condition reads:

$$V_k(K) = -\theta \left( 1 - \delta + \left( \frac{N - 1}{N} \right) I_k(K) \right) \left( P - c_q \left( \frac{\theta I(K)}{N} \right) \right),$$

(13)

where $I(K)$ denotes aggregate investment in the small open economy and we have used the fact that in a symmetric equilibrium each firm produces a fraction $N$ of the total amount of imported investment goods—i.e., $q(K) = q_-(K) = \frac{\theta I(K)}{N}$. The term $I_k(K)$ encodes the strategic interactions among oligopolistic firms, which, in a Markov Perfect Equilibrium, are mediated by changes in the state variable: Each firm internalizes the effect of its current production on future competitors’ production through changes in the level of capital in the small open economy.

To gain intuition on the GEE (12), consider a marginal increase in the quantity produced $q$ (and an associated increase in future capital $K'$). This increase in production has three effects on the present discounted value of profits. First, it yields additional profits equal to the current markup $P - c_q(q)$.

Second, it moves the equilibrium of the market for investment goods along the demand curve, reducing the market-clearing price. The effect of this price change on profits is encoded in the term $qR^{-1} (\theta^{-1} f_{kk}(K') + (1 - \delta)P_k(K'))$.

Third, it leads to a higher future level of capital in the small open economy, which in turn shifts downward the future residual demand curve, with an effect on future profits given by $R^{-1} V_k(K')$, which the envelope condition (13) relates to the future markup. This last term highlights that oligopolistic firms producing a durable good internalize that their future production will compete with the undepreciated fraction of the current production, as well as with their competitors.

### 4.2 Dynamic Markup Rule and Static Markup

We now use the GEE to express the price in terms of the marginal cost and a markup rate. To this end, we first rewrite equation (12) as follows:

$$P \left( 1 + \frac{\theta^{-1} q}{P} \cdot R^{-1} \left( \theta^{-1} f_{kk}(K') + (1 - \delta)P_k(K') \right) \right) = c_q(q) - R^{-1} \theta^{-1} V_k(K').$$
We then observe that \( \frac{dP}{dQ} = \frac{dP}{dK'} \frac{dK'}{dQ} = \theta^{-1} \frac{dP}{dK}, \) as one additional unit of output of the oligopolistic industry translates into \( \theta^{-1} \) additional unit of future capital. Thus, defining the inverse price elasticity of demand

\[
\eta \equiv -\frac{Q}{P} \frac{dP}{dQ} = -\frac{Q}{P} \theta^{-1} R^{-1} \left( \theta^{-1} f_{kk}(K') + (1-\delta)P_k(K') \right),
\]

and using \( q = \frac{Q}{N} \) we get

\[
P = \frac{N}{N - \eta} \cdot \left( c_q(q) - R^{-1} \theta^{-1} V_k(K') \right). \tag{15}
\]

Equation (15) expresses the price as a *dynamic markup* rule. Notice that the appropriate notion of marginal cost is composed of two terms. First, we have the “static” marginal cost \( c_q(q) \), which is the cost of producing one additional unit at the current date. Second, because of the dynamic nature of the oligopolist’s problem, we have the discounted marginal value, which encodes the loss in future profit due to the fact that one additional unit will shift residual demand in the future.

We define the dynamic markup rate as a share of the marginal cost as \( \mu^D \equiv \frac{\eta}{N - \eta} \), where the superscript \( D \) stands for “dynamic.” In equilibrium, the inverse elasticity \( \eta \) varies with the level of aggregate capital \( K \), and so does the markup rate \( \mu^D \).

Using the envelope condition (13), we can also express the static markup rate \( \mu^S \), over the static marginal cost \( c_q(q) \), as follows:

\[
\mu^S \equiv \frac{P - c_q(q)}{c_q(q)} = \mu^D \left( 1 - \frac{NR^{-1} \theta^{-1} V_k(K')}{\eta c_q(q)} \right). \tag{16}
\]

The term in parenthesis on the right-hand side of equation (16) adjusts the dynamic markup to account for the effect of future competition on the overall marginal cost.

### 4.3 Prices and Markups Around Steady State

To gain further insight into the effect of the level of capital on the equilibrium price, let us define the equilibrium law of motion of capital, \( g(K) \equiv K(1-\delta) + I(K) \). We proceed under the regularity condition that a stable steady-state level of capital exists and capital converges to it monotonically from below (at least locally). We will verify this condition numerically. In a neighborhood of the steady state, we then have \( 0 \leq g_k(K) < 1 \). A
steady-state level of capital and price satisfy

$$(\theta P + 1 - \theta)(R - 1 + \delta) = f_k(K).$$

Differentiating the Euler equation (6) with respect to $K$, we obtain

$$P_k(K_{t-1}) = \left( R^{-1}\theta^{-1}f_{kk}(K_t) + R^{-1}(1 - \delta)P_k(K_t) \right) g_k(K_{t-1})$$

$$= \sum_{s=0}^{\infty} R^{-s-1}(1 - \delta)^s \left( \Pi_{\tau=t-1}^{t-s} g_k(K_{\tau}) \right) \theta^{-1}f_{kk}(K_{t+s}) ,$$

which expresses the slope of the equilibrium price function as a present discounted value of the second derivatives of the production function moving forward in time along the equilibrium capital accumulation path.

In steady state, equation (17) becomes

$$P_k(K) = \frac{R^{-1}\theta^{-1}f_{kk}(K)g_k(K)}{1 - R^{-1}g_k(K)}.$$

The numerator of (18) is negative by concavity of the production function. The denominator is positive. Hence the equilibrium price is decreasing in the level of capital, $P_k < 0$, in a neighborhood of a steady state. This result, together with $f_{kk} < 0$, ensures that the inverse elasticity $\eta$ is positive in a neighborhood of a steady state.

Furthermore, in steady state we can use the envelope condition (13) together with equation (16) to express the static markup rate as follows:

$$\mu^S = \frac{\mu^D}{1 - \frac{N}{N-\eta} R^{-1}(1 - \delta + \left( \frac{N-1}{N} \right) I_k(K))}.$$

### 4.4 Capital Level and Price Elasticity of Investment

We now investigate the relation between the level of capital and the price elasticity of investment, which is a key determinant of the markup on new investment goods. Whereas it is necessary to examine this relation numerically in our model, we can make analytical progress in a simplified setting.

Consider the limiting case of full depreciation, $\delta = 1$, and assume there is a monopoly, i.e. $N = 1$ and that $\theta = 1$. Moreover, assume the economy has an endowment of capital $K_0$ that is not purchased from the monopolist. This endowed capital acts as stand-in for
undepreciated capital from the past in our model with partial depreciation and shifts the demand for investment.

In this case, taking logs of the investment Euler equation, we can write

$$\log(P) = -\log(R) + \log \left(f_k(K_0 + I)\right).$$

Thus, the inverse price elasticity is

$$\eta = -\frac{f_{kk}(K_0 + I)}{f_k(K_0 + I)}.$$  

Assume further that the production function is Cobb-Douglas, $f(K) = AK^\alpha$ with $\alpha \in (0, 1)$, as we will maintain in our quantitative analysis. Then,

$$\eta = (1 - \alpha)\frac{I}{K_0 + I},$$

which is decreasing in $K_0$ for a given level of quantity demanded $I$. Hence, investment demand is less elastic with respect to the price for low $K_0$ and the optimal markup is decreasing in $K_0$.

More in general, the sign of the derivative of the inverse elasticity with respect to $K_0$ depends on the the first three derivatives of the production function:

$$\frac{\partial \eta}{\partial K_0} = I \left(\frac{(f_{kk})^2 - f_{kkk} f_k}{(f_k)^2}\right),$$

and is negative when $f_{kk}^2 - f_{kkk} f_k < 0$. Intuitively, the first derivative of the production function appears in the Euler equation, which is the demand schedule for investment goods. Thus, the second derivative determines the price elasticity. Finally, the third derivative is a determinant of the slope of the elasticity with respect to the predetermined level of capital.

### 4.5 Commitment to Future Production

We now analyze the role of commitment to a future production plan. We consider the following game. At $t = 0$, each investment-good producer commits to an infinite sequence of production levels $\{q_t\}_{t=0}^\infty$ taking as given a sequence of competitors’ production levels $\{q_{-t}\}_{t=0}^\infty$. We then impose symmetry across investment-goods producers in equilibrium.

We interpret this setup as the limiting case of a world with long-lived managers that
formulate production plans and face high costs of deviating from them, for instance because of large costs of changing the production capacity.

In this formulation, we assume that investment-goods producers cannot collude because of coordination costs that we do not explicitly model. In Appendix A.2 we consider the case of collusion with commitment, in which case the objective is to maximize the present discounted value of total profits. The two problems coincide if $N = 1$.

The oligopolist’s maximization problem is

$$\max_{\{P_t, q_t, K_t\}} \sum_{t=0}^{\infty} R^{-t} (P_t q_t - c(q_t))$$

subject to the demand schedule (or, using the language of Ramsey-optimal policy, “implementability constraint”)

$$P_t = R^{-1} \left( \theta^{-1} f_k(K_t) + (1 - \delta)P_{t+1} \right) - \kappa$$

for $t = 0, 1, \ldots$, with multiplier $R^{-t} \gamma_t$, and the law of motion

$$K_t = (1 - \delta)K_{t-1} + \theta^{-1} ((N - 1)q_{-t} + q_t).$$

The first-order conditions of this problem are:

$$q_t - \gamma_t + \gamma_{t-1}(1 - \delta) = 0 \quad (19)$$

$$\theta P_t - \theta c_q(q_t) + \gamma_t R^{-1} \theta^{-1} f_{kk}(K_t) - R^{-1} \theta (1 - \delta) (P_{t+1} - c_q(q_{t+1})) = 0, \quad (20)$$

with initial condition on the multiplier $\gamma_{-1} = 0$. These optimality conditions trade off present and future profits, similar to the GEE (12). However, we highlight two important differences between the dynamics under commitment and the ones we obtained in a Markov Perfect Equilibrium.

First, equation (19) reveals the nature of the time inconsistency of the optimal production plan under commitment. A higher price at $t$ relaxes the past implementability constraint allowing a higher price at $t - 1$. However, at $t = 0$, the producer is not bound by any past commitment. Then, over time, past commitments, encoded in the multiplier $\gamma_t$, accumulate, thereby making it increasingly costly to reduce prices. In contrast, in a Markov Perfect Equilibrium, firms always disregard the competition with their past selves.
and only internalize future equilibrium decision rules.

Second, because under commitment we assume that firms take as given the whole path of competitors’ decisions, they do not internalize the effect of their production levels on future competitors’ production, which accounts for the term $I_k(K’)$, which is present in the envelope condition (13) but absent in equation (20).

As in the no-commitment case, we can use the optimality conditions (19) and (20) to express the price in terms of the marginal cost and a markup rate. To this end, we first rewrite equation (20) as follows:

$$P_t \left(1 + \frac{\theta^{-1} (q_t + (1 - \delta) \gamma_{t-1})}{P_t} \cdot \frac{dP_t}{dK_t} \right) = c_q(q_t) + R^{-1} (1 - \delta) (P_{t+1} - c_q(q_{t+1})).$$

We then observe that $\frac{dP_t}{dQ_t} = \theta^{-1} \frac{dP_t}{dK_t}$. Thus, defining the inverse price elasticity of demand

$$\eta^{FC} \equiv -\frac{Q}{P} \frac{dP_t}{dQ_t} = -\frac{Q}{P} R^{-1} \theta^{-2} f_{kk}(K_t),$$

we get

$$P_t = \frac{N}{N - \left(1 + \frac{N(1-\delta)\gamma_{t-1}}{Q_t}\right) \eta^{FC}} \cdot \left(\frac{c_q(q_t) + R^{-1} (1 - \delta) (P_{t+1} - c_q(q_{t+1}))}{\text{Marginal Cost}}\right). \quad (21)$$

Equation (21) expresses the price as a dynamic markup rule that is both forward looking and backward looking. In particular, the commitment problem features the backward-looking term $\frac{N(1-\delta)\gamma_{t-1}}{Q_t}$ that was not present in the Markov Perfect Equilibrium. This term captures the fact that the firm internalizes that a marginal increase in price at time $t$ has an effect on the demand schedule at time $t - 1$. Notice also that the appropriate notion of marginal cost is composed of two terms. First, we have the “static” marginal cost $c_q(q_t)$, which is the cost of producing one additional unit at the current date. Second, because of the dynamic nature of the oligopolist’s problem, we have the discounted future markup. Similarly to the no-commitment case, we can also define the dynamic markup rate under commitment as a fraction of the marginal cost.
5 Quantitative Analysis

In this section, we calibrate the model and solve it numerically to explore the implications of market power in investment-goods markets for capital accumulation. We focus on the dynamics of markups along the transition path to steady state in the small open economy.

5.1 Solution Method

We begin this section by briefly discussing our global solution method.

**Markov Perfect Equilibrium.** We solve the Markov Perfect Equilibrium using a version of the time-iteration algorithm to approximate the policy functions $I(K)$ and $P(K)$. Specifically, we guess a polynomial approximation for $I(K)$. Given this candidate policy function, we obtain an associated guess for $P(K)$ by doing time iteration on equation (6), recursively solving for the left-hand side on a grid for $K$ and then plugging the obtained price function in the right-hand side. Once we obtain a converged price function, we use it to numerically approximate the derivative $P_k(K)$. Then, to update $I(K)$, we apply time iteration to the GEE (12) substituting in it the envelope condition (13) with an approximation of the derivative $I_k(K)$. We repeat these steps until all policy functions converge.

**Commitment.** In this case, we solve the model recursively by adding the multiplier on the past investment Euler equation as a state variable. We then solve the equilibrium under commitment using a time-iteration algorithm on equations (19) and (20) to approximate the policy functions $I(K, \gamma)$ and $\gamma'(K, \gamma)$ with polynomials.

5.2 Calibration

We proceed to describe our choices of functional forms and parameter values, which we report in Table 1. The length of a period is one year. We assume the production function in the small open economy is Cobb-Douglas: $F(K_{t-1}, L) \equiv AK_{t-1}^\alpha L^{1-\alpha}$ and normalize the labor endowment $L = 1$. We interpret capital as the stock of equipment, which is approximately one third of the total capital stock, and calibrate the capital share and depreciation rate accordingly.

We calibrate the share of imported investment goods in total investment using US data on investment-goods prices as follows. We first deflate the Producer Price Index of semiconductors and the Producer Price Index of machinery and equipment using the GDP.
Table 1: Parameters Values

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Symbol</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Investment Demand</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Discount Factor</td>
<td>$\beta$</td>
<td>0.96</td>
</tr>
<tr>
<td>Depreciation</td>
<td>$\delta$</td>
<td>0.12</td>
</tr>
<tr>
<td>Capital Share</td>
<td>$\alpha$</td>
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<tr>
<td>Oligopolistic Capital Share</td>
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<tr>
<td>Total Factor Productivity</td>
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<tr>
<td>Investment Supply</td>
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<td></td>
</tr>
<tr>
<td>Number of Producers</td>
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<td>Marginal Cost (Intercept)</td>
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</tr>
<tr>
<td>Marginal Cost (Slope)</td>
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<tr>
<td>TFP Stochastic Process</td>
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<td></td>
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<tr>
<td>Persistence</td>
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</tr>
<tr>
<td>Standard Deviation</td>
<td>$\sigma_\epsilon$</td>
<td>0.014</td>
</tr>
</tbody>
</table>

Notes: The table reports the parameter values used in the quantitative analysis.

deflator. We fit a linear trend in both series during 2012-2019. We then match the pass-through of the cumulative increase in the real price of semiconductors to the real price of machinery and equipment during 2019-2023. Relative to trend, we observe a 20% increase in the real price of semiconductors and a 7% increase in the real price of machinery and equipment.

We set the number of foreign investment-goods producers to closely resemble the highly concentrated market structure in semiconductor manufacturing. We then experiment with a change in market structure in Section 6.3. We assume that the cost function to produce investment goods is quadratic: $c(q) = c_1q + \frac{c_2}{2}q^2$. Given a calibrated value for the slope of the marginal cost $c_2$, we set the intercept $c_1$ to normalize the marginal cost of investment to one in the first-best steady state. We calibrate $c_2$ so that the ratio of profits to sales in steady state closely matches the ratio of operating income to sales in balance-sheet data for the major semiconductor manufacturers. Specifically, using ORBIS data on TSMC and Samsung, we obtain a ratio of approximately 30%.

5.3 Capital Accumulation, Prices, and Markups

Figure 1 illustrates some key properties of the Markov Perfect Equilibrium. The left panel portrays the law of motion of aggregate capital, comparing the oligopoly outcome (solid line) with the first-best allocation (dashed line). The right panel portrays the equilibrium price (solid line) and the marginal cost (dashed line) as functions of the capital level in the
small open economy.

In the Markov Perfect Equilibrium, the steady-state level of capital is lower than in first best because of the presence of a markup. Moreover, as the small open economy grows toward its steady state, the price of investment declines faster than the marginal cost, which implies that the static markup is decreasing in the level of capital. As a consequence, capital accumulation is slower in the presence of market power than in the first-best allocation.

Figure 1: Markov Perfect Equilibrium: Capital Accumulation, Investment Price, and Marginal Cost

Notes: The figure displays capital accumulation and prices in the Markov Perfect Equilibrium. Panel (a) illustrates the law of motion of capital. The solid line refers to the dynamic oligopoly and the dashed line to the competitive equilibrium. The intersections of the two lines with the 45-degree line identify the steady-state equilibrium in the two settings. Panel (b) displays the price (solid line) and marginal cost (dashed line) as functions of aggregate capital stock of the small open economy.

Next we investigate the dynamics of markups, which Figure 2 displays. We distinguish between the static markup $\mu^S_t$ (solid line) and the dynamic markup $\mu^D_t$, which we defined in Section 4.2. The static markup is larger than the dynamic markup because it has to cover the part of marginal cost due to competition with the future undepreciated capital stock. Both markup rates decline as aggregate capital increases. When the level of capital is low, the price elasticity of investment is low, consistent with the analytical insights of Section 4.4 in a simplified setting. This feature accounts for the negative slope of $\mu^D_t$.

Furthermore, a low level of capital, combined with low elasticity, implies that investment-goods producers can extract rents from the small open economy for a relatively long time,
while capital accumulates toward the steady state. This anticipation of future markups accounts for the decreasing gap between \( \mu^S_t \) and \( \mu^D_t \) in the figure. Overall, both the price elasticity and the anticipation of future markups contribute to generate a larger distortion for lower levels of capital.

Quantitatively, our results imply that when the level of capital is approximately half of its steady-state target, the price of investment is approximately 35% higher than in steady state and the static markup rate is 40% larger than steady state.

**Figure 2: Static and Dynamic Markup**

![Graph showing static and dynamic markup](image)

*Notes:* The figure illustrates the static markup rate \( \mu^S \) (solid line) and the dynamic markup rate \( \mu^D \) in the Markov Perfect Equilibrium.

### 5.4 Role of Commitment

We now investigate the difference between the Markov Perfect Equilibrium and the case of full commitment to future production (Section 4.5). Figure 3 displays the dynamics of aggregate capital, multiplier on the investment Euler equation \( (\gamma_t) \), price of investment, and static markup. The figure compares the Markov Perfect Equilibrium (solid lines) with the case of commitment (dashed lines).

First, we notice that in the presence of commitment the price of investment and the markup are substantially higher than in the Markov Perfect Equilibrium. As a result, capital in the small open economy converges to a lower steady-state level. In steady state, the static markup rate is approximately 120% with commitment and 15% in the Markov
Perfect Equilibrium.

Second, by comparing the transition dynamics in the two regimes, we uncover the source of time inconsistency of the commitment plan. Under full commitment, at the beginning of the transition, when the multiplier is zero, each oligopolist has an incentive to set a relatively high level of production and, accordingly, a lower price than in the long run. As a consequence the small open economy experiences an investment boom and overshoots its long-run level of capital. Over time, as the promise-keeping multiplier accumulates, prices and markups grow and the small open economy reverts to its steady-state level of capital.

These dynamics display a sharp contrast with the outcome in the absence of commitment, which, as we have seen, features decreasing price and markup as capital accumulates to the steady state.

5.5 Inspecting the Mechanism: Markup Decomposition

We now provide a decomposition of markups along the equilibrium capital-accumulation path. This decomposition highlights the main forces at play in the evolution of markups, namely shifts in the demand for investment goods and changes in the slope of the demand curve.

In the Markov Perfect Equilibrium, we can reformulate the GEE (12) along the transition path in terms of future sequences of three objects: quantities produced, derivatives of the demand function 

\[ \frac{dP}{dq} \equiv \theta^{-1} R^{-1} (\theta^{-1} f_{kk}(K_t) + (1 - \delta) P_k(K_t)) \]

and an endogenous discount factor, which we define recursively as follows:

\[ B_{t,t} = 1, \quad B_{t,t+1} = R^{-1} (1 - \delta + \left(\frac{N-1}{N}\right) I_k(K_t)), \quad B_{t,t+s} = B_{t,t+s-1} R^{-1} (1 - \delta + \left(\frac{N-1}{N}\right) I_k(K_{t+s-1})) \]

We express the difference between price and marginal cost as follows:

\[ P_t - c_q(q_t) = -\sum_{s=0}^{\infty} B_{t,t+s} \gamma_{t+s} \frac{dP_{t+s}}{dQ_{t+s}}. \]

To quantify the role of each factor for the dynamics of markups, we then compute counterfactual markups using steady-state values for two of the three determinants and letting the third one vary according to the equilibrium path.

Similarly, we can reformulate the commitment first-order condition (20) as follows:

\[ P_t - c_q(q_t) = -\sum_{s=0}^{\infty} R^{-s} (1 - \delta)^s \gamma_{t+s} \left( \frac{dP_{t+s}}{dQ_{t+s}} \right) \]
Figure 3: Role of Commitment

Notes: The figure compares the transition of the economy to the steady-state equilibrium without commitment (Markov Perfect Equilibrium, solid lines) and with full commitment (dashed lines). In both settings, we assume that the initial level of capital equals half of its steady-state value. Panels (a), (b), (c), and (d) plot the transitions of aggregate capital $K_t$, demand schedule multiplier $\gamma_t$, price $P_t$, and static markup rate $\mu^S_t$ respectively.

with $\frac{dP_t}{dQ_t} = R^{-1} \theta^{-2} f_{kk}(K_t)$ and decompose the roles of quantities and slopes of the demand curve along the equilibrium path.

Figure 4 illustrates this decomposition for the Markov Perfect Equilibrium (left panel) and the case of commitment (right panel). In the absence of commitment, quantities
decline over time because investment in the small open economy is initially high and then decreases as the economy approaches steady state. This path contributes to declining markups over time. Furthermore, rotations in the demand curve amplify the effect of the decline in investment, leading to a steeper decline in markups. In contrast, in the presence of commitment, the multiplier $\gamma_t$ accumulates as capital in the small open economy accumulates, leading to an increasing gap between price and marginal cost.

Figure 4: Markup Decomposition in the Markov Perfect Equilibrium vs. Commitment

![Figure 4](image_url)

Notes: The figure displays a decomposition of the evolution of the difference between price and marginal cost, $P_t - c_{q,t}$, over the transition of the economy to steady state. Panel (a) refers to the Markov Perfect Equilibrium. It uses equation (22) to decompose the markup (solid line) into variations driven by: quantities $q_{t+s}$ produced by each oligopolist (dashed line); derivative of inverse demand with respect to quantities $dP_{t+s}/dQ_{t+s}$ (dash-dotted line); implicit discounting $B_{t,t+s}$ (dotted line). Panel (b) refers to the case of full commitment. It uses equation (23) to decompose the markup (solid line) into variation driven by: the demand multiplier $\gamma_{t+s}$ (dashed line); derivative of inverse demand with respect to the quantity produced $dP_{t+s}/dQ_{t+s}$ (dash-dotted line).

6 Aggregate Shocks

In this section, we analyze the effects of aggregate shocks. We simulate an increase in the demand for investment goods, similar to the one experienced in the post-2020 recovery. We highlight the roles of increasing marginal costs—akin to capacity constraints—and market power for the dynamics of investment and prices. We also consider a stochastic version
of our model with persistent business-cycle shocks. Finally, we simulate the effects of a change in the number of investment-goods producers.

6.1 Investment-Demand Shock: Marginal Cost vs Markup

We now simulate a positive unexpected shock to the demand for investment goods. We calibrate a permanent increase in the level of TFP in the small open economy to match a cumulative 20% increase in the price of semiconductors during 2019-2023.

We compare two scenarios, one with our baseline calibration for the marginal cost of producing investment goods and a counterfactual one with a smaller slope of the marginal cost relative to the quantity produced ($c_2 = 10$).

In so doing, we leverage the model to gain insight into the dynamics of the post-2020 recovery, when a rise in demand for durable goods (and thus for semiconductors) led to a dramatic increase in the price of equipment. Two factors likely contributed to this pattern. First, producers of semiconductors as well as other manufacturers overall experienced tight capacity constraints, which we interpret as steeply increasing marginal costs in our parsimonious model. Second, these producers could exert market power and extract profits from the period of high demand. The calibrated model allows us to decompose these channels.

Figure 5 displays the response of economy to a permanent 20% increase in TFP, contrasting two scenarios, one with baseline marginal cost of producing investment goods ($c_2 = 40$) and the other with flatter marginal cost ($c_2 = 10$). In both cases, investment increases and so do the marginal cost and the equilibrium price of investment. In the case of steep marginal cost, this price response is significantly more pronounced. However, whereas the higher value of $c_2$ leads to a change in marginal cost that is more than twice as large, it only leads to a change in the equilibrium price that is 80% higher.

The difference in these responses is due to endogenous markups. Figure 6 portrays the response of static and dynamic markup rates. The static markup rate increases in response to the shock and then decrease as the economy adjusts to the new steady state. By comparing the two slopes of the marginal cost, we find that a steeper marginal cost leads to a smaller response of markups on impact. Thus, marginal costs account for a larger fraction of the overall increase in the investment price. In contrast, with a flatter marginal cost, markups are more responsive to the shock and account for approximately 30% of the equilibrium price jump.
Figure 5: Permanent Productivity Increase: Capital Accumulation and Prices

Notes: The figure illustrates the aggregate response of the economy to an unanticipated and permanent 10% increase in TFP in the Markov Perfect Equilibrium. Panel (a) plots the exogenous change in TFP. Panel (b) plots the transition of aggregate capital to the new steady state in the small open economy. Panels (c) and (d) illustrate the transition of price of investment and marginal cost to the new steady state. In panels (b), (c), and (d), the solid vs. dashed lines compare the response of markups when the increase in producer’s marginal cost as quantity produced grows is equal to our baseline calibration ($c_2 = 40$) vs. flatter ($c_2 = 10$), respectively. We assume that the shock occurs at $t = 0$, that the economy is in the initial steady state at $t = -1$, and that agents have perfect foresight of the evolution of all variables after the unexpected shock occurs.
Notes: The figure illustrates the response of static markup rate $\mu^S_t$ (panel a) and dynamic markup rate $\mu^D_t$ (panel b) to an unanticipated and permanent 20% increase in TFP in the Markov Perfect Equilibrium. The solid and dashed lines compare the response of markups when the increase in producer’s marginal cost as quantity produced grows is equal to our baseline calibration ($c_2 = 40$) vs. flatter ($c_2 = 20$), respectively. We assume that the shock occurs at $t = 0$, that the economy is in the initial steady state at $t = -1$, and that agents have perfect foresight of the evolution of all variables after the unexpected shock occurs.

6.2 Stochastic Productivity

We now extend our model to include stochastic productivity shocks in the small open economy. To this end, we assume that the production function is $Y_t = A_tK_{t-1}L$ and that productivity follows an AR(1) process in logs: $\log(A_t) = \rho \log(A_{t-1}) + \varepsilon_t$. We parameterize the autocorrelation and standard deviation of innovations following the calibration of TFP shocks for the US economy in Khan and Thomas (2013)—i.e., $\rho = 0.909$ and $\sigma_\varepsilon = 0.014$. We provide all derivations of the stochastic model in Appendix A.3. In the presence of stochastic shocks, the GEE of a generic investment-goods producer becomes:

$$\theta P - \theta c_q(q) + qR^{-1}\mathbb{E} \left[ \theta^{-1} f_{kk}(A', K') + (1 - \delta)P_k(K', s')\mid s \right] + R^{-1}\mathbb{E} [V_k(K', s') \mid s] = 0,$$
Table 2: Stochastic Productivity: Business Cycle Moments

<table>
<thead>
<tr>
<th></th>
<th>FB</th>
<th>MPE</th>
<th>FC</th>
</tr>
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<tbody>
<tr>
<td>Mean I</td>
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<td>0.079</td>
<td>0.059</td>
</tr>
<tr>
<td>Mean P</td>
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<td>1.960</td>
</tr>
<tr>
<td>Mean Markup</td>
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<td>1.222</td>
</tr>
<tr>
<td>St. Dev. I/St. Dev. Y</td>
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<td>1.350</td>
<td>1.541</td>
</tr>
<tr>
<td>St. Dev. P</td>
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<td>0.015</td>
<td>0.005</td>
</tr>
<tr>
<td>St. Dev. Markup</td>
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<td>0.022</td>
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<td>-0.942</td>
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Notes: The table reports several moments related to investment, the price of investment goods, and the static markup rate, from a long simulation of the model with stochastic productivity in the small open economy. The first column refers to the first-best allocation, the second column to the Markov Perfect Equilibrium, and the third column to the case of full commitment. Standard deviations and correlation are computed for the logarithm of the variables, except for the markup rate.

whereas the optimality conditions with commitment become:

\[ q_t - \gamma_t + \gamma_{t-1}(1 - \delta) = 0 \]

\[ \theta P_t - \theta c_q(q_t) + \gamma_t R^{-1} \mathbb{E}_t \left[ \theta^{-1} f_{kk}(A_{t+1}, K_t) \right] - R^{-1} \theta (1 - \delta) \mathbb{E}_t \left[ (P_{t+1} - c_q(q_{t+1})) \right] = 0, \]

Table 2 reports several business-cycle moments from a long simulation of the stochastic model. The stochastic model confirms the main insights that we have highlighted in the previous section. Prices and markups are higher on average in the presence of commitment. Intriguingly, while the price of investment is positively correlated with output, the static markup rate is negatively correlated with output. This is consistent with the basic mechanism captured by Figure 1: Higher productivity induces higher investment, which in turn leads to a higher output, hence lower markups. Moreover, both these measures are less correlated with output in the presence of commitment.

Overall, this analysis confirms that endogenous movements in the price of investment generate an aggregate capital adjustment cost in the small open economy.

6.3 Change in Market Structure

Our findings on the implications of market power for capital accumulation motivate us to analyze the effects of a change in the number of producers. We now use our model to
shed light on the possible effects of policies that affect the production capacity and market structure for dynamic inputs.

To this end, we consider an increase in the number of competitors from $N = 3$ to $N = 4$ and compute the equilibrium transitional dynamics after this regime change. Figure 7 represents the transition of capital stock (panel a) as well as price and marginal cost of investment (panel b) in the Markov Perfect Equilibrium.

As the number of capital producers increases, total capacity expands and competition rises. Given any level of aggregate investment, a larger production capacity reduces individual quantities, thus reducing the marginal cost. This contributes to a decline in the price, inducing more capital accumulation in the small open economy. Over time, higher competition gradually depresses markups, as implied by the larger decline in capital price than in marginal costs, which further stimulates investment.

Figure 7: Increase in the Number of Investment-Goods Producers

Notes: The figure illustrates the response of the Markov Perfect Equilibrium economy to an unanticipated and permanent increase in the number of investment-goods producers, from $N = 3$ to $N = 4$. Panel (a) plots the transition of the small open economy’s aggregate capital to the new steady state. Panels (b) illustrate the evolution of the investment price $P_t$ (solid line) and marginal cost $c_{q,t}$ (dashed line) to the new steady state. We assume that the shock occurs at $t = 0$, that the economy is in the initial steady state at $t = -1$, and that agents have perfect foresight of the evolution of all variables after the unexpected shock occurs.
7 Conclusion

We have developed an open-economy model with market power in the global production of investment goods. In so doing, we were motivated by the post-2020 global recovery, which featured a large increase in demand for inputs produced by a highly concentrated industries, such as semiconductors.

In our framework, the price of investment goods equals the sum of a marginal cost—which can be affected by capacity constraints—and an endogenous markup, which depends critically on the level of demand for investment goods. When investment-goods producers behave as oligopolists without commitment, the markup rises in response to positive shocks to investment demand, thereby generating a microfounded aggregate capital adjustment cost.

Our model provides a laboratory to analyze policy interventions that may increase the productive capacity in the global semiconductors industry. In future versions of the paper, we will analyze efficiency and policy interventions. Furthermore, we will enrich the stochastic version of the model to analyze aggregate shocks to both demand and supply of investment goods.
References


**APPENDIX**

A.1 First-Best Planning Problem

The social planner chooses sequences \( \{C_t, B_t, q_{jt}, K_t\} \) for \( j = 1, \ldots, N \) and \( t = 0, \ldots, \infty \) to maximize household utility (1) subject to the resource constraint

\[
C_t + \sum_{j=1}^{N} c(q_{jt}) + X_t + B_t = f(K_{t-1}) + \beta^{-1}B_{t-1},
\]

with multiplier \( \beta^t \lambda_t \), where \( X_t = \theta^{-1}(1 - \theta) \sum_{j=1}^{N} q_{jt} \) and where we used \( R = \beta^{-1} \), as well as the capital accumulation equation

\[
K_t = \theta^{-1} \sum_{j=1}^{N} q_{jt} + (1 - \delta)K_{t-1},
\]

with multiplier \( \beta^t \nu_t \).

The optimality conditions are

\[
\begin{align*}
 u_c(C_t) & = \lambda_t \\
 \lambda_t & = \lambda_{t+1} \\
 \lambda_t \left( c_q(q_{jt}) + \theta^{-1}(1 - \theta) \right) & = \theta^{-1} \nu_t \\
 \nu_t & = \beta \left( \lambda_{t+1} f_k(K_{t-1}) + (1 - \delta) \nu_{t+1} \right),
\end{align*}
\]

which imply symmetric production \( q_{jt} = q_t = \frac{\theta I_t}{N} \) for all \( j \) if \( c_{qq} > 0 \), and can be combined to obtain equation (9):

\[
\theta c_q \left( \frac{\theta I_t}{N} \right) + 1 - \theta = R^{-1} \left( f_k(K_t) + (1 - \delta) \left( \theta c_q \left( \frac{\theta I_{t+1}}{N} \right) + 1 - \theta \right) \right).
\]

A.2 Commitment with Collusion

A planner chooses sequences of prices and quantities for all \( N \) producers, \( \{P_t, q_{jt}\} \), for \( t = 0, \ldots, \infty \) and \( j = 1, \ldots, N \) to maximize

\[
\sum_{t=0}^{\infty} R^{-t} \left( P_t \sum_{j=1}^{N} q_{jt} - \sum_{j=1}^{N} c(q_{jt}) \right), \tag{A1}
\]
subject to

$$P_t = R^{-1} \left( \theta^{-1} f_k(K_t) + (1 - \delta) P_{t+1} \right) - \kappa,$$

for \( t = 0, 1, \ldots \), with multiplier \( R^{-t} \Gamma_t \), and

$$K_t = (1 - \delta) K_{t-1} + \theta^{-1} \sum_{j=1}^{N} q_{jt},$$

with multiplier \( R^{-t} \nu_t \).

The first-order conditions with respect to \( P_t, q_{jt}, \) and \( K_t \) are:

$$\sum_{j=1}^{N} q_{jt} - \Gamma_t + (1 - \delta) \Gamma_{t-1} = 0$$

$$P_t - c_q(q_{jt}) - \theta^{-1} \nu_t = 0$$

$$\Gamma_t R^{-1} \theta^{-1} f_{kk}(K_t) + \nu_t - R^{-1} (1 - \delta) \nu_{t+1} = 0,$$

which imply \( q_{jt} = q_t \) for all \( j \) (as long as \( c_{qq} > 0 \)) and

$$\theta P_t - \theta c_q \left( \frac{\theta I_t}{N} \right) = -\Gamma_t R^{-1} \theta^{-1} f_{kk}(K_t) + R^{-1} \theta (1 - \delta) \left( P_{t+1} - c_q \left( \frac{\theta I_{t+1}}{N} \right) \right). \quad \text{(A2)}$$

Notice the similarity between equation (A2) and equation (20). The key difference between these two optimality conditions is given by the multiplier on the investment Euler equation, which under collusion accounts for the aggregate capital accumulation path.

### A.3 Stochastic Model

Let \( s_t \) be a vector of shocks. Given \( s_0 \), and history of shocks \( s^t = \{s^{t-1}, s_t\} \), a stochastic small open economy is populated by a representative household with utility function

$$\sum_{t=0}^{\infty} \sum_{s^t} \beta^{t} u(C(s^t)) Pr(s^t), \quad \text{(A3)}$$

where \( \beta \in (0, 1) \) denotes the discount factor, \( C_t = C(s^t) \) is aggregate consumption, and \( u_c > 0, u_{cc} \leq 0 \), where subscripts denote first and second derivative respectively.

We assume the household has access to state contingent bonds. Given \( s^t \), the budget
constraint of the household at time $t$ reads

$$C(s^t) + P^I(s^t)I(s^t) + \sum_{s_{t+1}} B(s_{t+1}|s^t) = W(s^t)L + R^K(s^t)K(s^{t-1}) + R^b(s_t|s^{t-1})B(s_t|s^{t-1}) + D(s^t),$$

where $P^I(s^t) = P^I_t$ is the price of investment goods $I(s^t) = I_t$, $B_t = B(s_{t+1}|s^t)$ are state-contingent bonds that pays $R^b(s_t|s^{t-1})$, $W_t = W(s^t)$ is the wage, $L$ is a constant endowment of labor, $R^K_t = R^K(s^t)$ denotes the rental rate of capital $K_{t-1} = K(s^{t-1})$, and $D_t = D(s^t)$ are profits obtained from ownership of domestic firms. We assume that the household is only subject to the natural debt limit. For ease of notation, we drop the dependency from the history of shocks and simply indicates all variables with their corresponding time subscript.

Investment adds to the capital stock, which depreciates at rate $\delta$:

$$K_t = (1 - \delta)K_{t-1} + I_t.$$  \hspace{1cm} (A5)

As in the deterministic model, we assume that investment has to be non-negative and restrict attention to a region of the parameter space where this constraint is not binding.

The first-order conditions of the utility maximization problem with respect to bonds and investment are

$$\forall s^{t+1}: \quad 1 = \beta \frac{u_c(C(s^{t+1}))}{u_c(C(s^t))} R^b(s_{t+1}|s^t)Pr(s^{t+1})$$

$$P^I_t = \mathbb{E}_t \left[ \beta \frac{u_c(C_{t+1})}{u_c(C_t)} \left( P^K_{t+1} + (1 - \delta)P^I_{t+1} \right) \right].$$ \hspace{1cm} (A6) (A7)

A representative firm rents capital from the representative household and hires labor to produce output with a constant-returns to scale production function:

$$Y_t = F(A_t, K_{t-1}, L).$$ \hspace{1cm} (A8)

The first-order conditions of the profit maximization problem are

$$F_K(A_t, K_{t-1}, L) = R^K_t$$

$$F_L(A_t, K_{t-1}, L) = W_t.$$  \hspace{1cm} (A9)
For notational convenience, we define \( f(A_t, K_{t-1}) \equiv F(A_t, K_{t-1}, L) \). Because of constant-returns to scale, the representative firm makes zero profits in equilibrium—i.e., \( D_t = 0 \).

We assume that the risk-free interest rate satisfies \( R = \beta^{-1} \) and that \( R^b(s_{t+1} | s^t) P_t(s^{t+1}) = R \). Given our choice of \( R \), equation (A6) implies that \( \forall s^{t+1} : \frac{u_t(C(s^{t+1}))}{u_t(C(s^t))} = 1 \). Hence, by combining the household and firm optimality conditions (A6), (A7), and (A9), we obtain the following investment Euler equation that describes optimal capital accumulation in the stochastic version of the small open economy:

\[
P^I_t = R^{-1}E_t \left[ f_k(A_{t+1}, K_t) + (1 - \delta)P^I_{t+1} \right]. \tag{A10}
\]

Equation (6) implicitly expresses the demand for investment goods as a function of the capital stock \( K_{t-1} \) as well as current and future investment prices \( (P^I_t, P^I_{t+1}) \) and future shocks.

As in the deterministic case, as long as markets are complete, our assumptions on ownership of the capital stock are immaterial and we can equivalently derive this condition assuming that firms accumulate capital instead of households.

### A.3.1 Investment-Goods Producers

We now describe the supply side of the market for investment goods. We assume that there is an integer number \( N \geq 1 \) of identical investment-goods producers owned by foreign investors. The objective of investment-goods producers is to maximize the present discounted value of profits:

\[
\sum_{t=0}^{\infty} \sum_{s^t} R^{-t} \pi_t(s^t) P(s^t). \tag{A11}
\]

Similarly to the deterministic case, we assume that a perfectly competitive representative firm combines an amount \( Q_t \) and an amount \( X_t \) of output good to assemble domestic investment with a Leontief production function. Hence, \( P^I_t = \theta P_t + 1 - \theta \), as in equation (7). Equation (A10) becomes:

\[
\theta P_t + 1 - \theta = R^{-1}E_t \left[ f_k(A_{t+1}, K_t) + (1 - \delta)(\theta P_{t+1} + 1 - \theta) \right]. \tag{A12}
\]

Divide everything by \( \theta \) and factor out the constant to get:

\[
P_t = R^{-1}E_t \left[ \theta^{-1} f_k(A_{t+1}, K_t) + (1 - \delta)P_{t+1} \right] - \theta^{-1}(1 - \theta)(1 - R^{-1}(1 - \delta)) \equiv \kappa. \tag{A13}
\]
A.3.2 First Best

Before analyzing the effects of market power, we briefly introduce the competitive benchmark, which coincides with the solution to the problem of a planner who maximizes welfare in the small open economy taking as given the cost function to produce investment goods.

In a competitive equilibrium without market power, investment-goods producers choose a plan of production levels \( \{q(s^t)\}_{t=0}^{\infty} \) to maximize (A11) taking as given the sequence of prices schedules \( \{P(s^t)\}_{t=0}^{\infty} \). Thus, the equilibrium price satisfies \( P_t = c_q \left( \frac{L_t}{N} \right) \) and optimal capital accumulation satisfies

\[
\theta c_q \left( \frac{\theta I_t}{N} \right) + 1 - \theta = R^{-1} E_t \left[ f_k(A_{t+1}, K_t) + (1 - \delta) \theta c_q \left( \frac{\theta I_{t+1}}{N} \right) + 1 - \theta \right]. \tag{A14}
\]

A.3.3 Markov Perfect Equilibrium and Generalized Euler Equation

A generic investment-goods producer solves the following problem:

\[
\max_{P,K',q} P q - c(q) + R^{-1} E[V(K', s')|s] \tag{A15}
\]

subject to the demand schedule

\[
P = R^{-1} E \left[ \theta^{-1} f_k(A', K') + (1 - \delta) P(K', s')|s \right] - \kappa,
\]

the market-clearing condition

\[
(N - 1)q_-(K) + q = Q = \theta I,
\]

and the law of motion for capital

\[
K' = (1 - \delta)K + I.
\]

First, substitute investment \( I \) from the market-clearing condition in the law of motion for capital. Second, use the derived equation to substitute \( q \) in the objective function. Third, substitute \( P \) in the objective function using the demand schedule. Hence, take the first-order condition with respect to \( K' \) to get the following Generalized Euler Equation...
(GEE): 
\[ \theta P - \theta c_q(q) + qR^{-1}\mathbb{E} [\theta^{-1} f_{kk}(A', K') + (1 - \delta)P_k(K', s')|s] + R^{-1}\mathbb{E} [V_k(K', s')|s] = 0, \]  
(A16)

which involves the derivative of the future price with respect to capital in every possible future realization of shocks.

### A.3.4 Commitment to Future Production

Given \( K_{-1} \), the oligopolist’s problem involves finding sequences \( \{P(s^t), K(s^t)\}_{t=0}^{\infty} \) such that

\[
\sum_{t=0}^{\infty} \sum_{s^t} R^{-t} \left( P_t (\theta(K_t - (1 - \delta)K_{t-1}) - (N - 1)q_{-t,t}) - c(\theta(K_t - (1 - \delta)K_{t-1}) - (N - 1)q_{-t,t}) \right) P_r(s^t)
\]

is maximized subject to the demand schedule (or, using the language of Ramsey-optimal policy, “implementability constraint”)

\[
P_t = R^{-1}\mathbb{E}_t [\theta^{-1} f_k(A_{t+1}, K_t) + (1 - \delta)P_{t+1}] - \kappa
\]

for \( t = 0, 1, \ldots \), with multiplier \( R^{-t}\gamma_t \). The first-order conditions with respect to price \( P_t = P(s^t) \) and capital level \( K_t = K(s^t) \) are:

\[
q_t - \gamma_t + \gamma_{t-1}(1 - \delta) = 0
\]

\[
\theta P_t - \theta c_q(q_t) + \gamma_t R^{-1}\mathbb{E}_t [\theta^{-1} f_{kk}(A_{t+1}, K_t)] - R^{-1}\theta(1 - \delta)\mathbb{E}_t [(P_{t+1} - c_q(q_{t+1}))] = 0,
\]

with initial condition on the multiplier \( \gamma_{-1} = 0 \).