What assets should banks be allowed to hold?

Appendix A-1

Maturity transformation in the Diamond-Dybvig model

In their stylized model, Diamond and Dybvig (1983) suppose that a large group of risk-averse households lives for three periods (dates $t = \{0,1,2\}$). At date $t=0$, each household has one apple. If this apple is planted at date $t=0$ (that is, invested) and left untouched for two periods, it becomes $R>1$ apples in period $t=2$. However, if the apple is "uprooted" in $t=1$, it remains one apple.

Households can be one of two types: patient or impatient. A patient type cares only about the total quantity of apples it eats over periods $t=1$ and $t=2$. Assume that this household's payoff is $u(c(1)+c(2))$, where $c(1)$ is consumption at period $t=1$ and $c(2)$ is consumption at period $t=2$.

An impatient type is identical except that it cares only about the total quantity of apples it eats in period $t=1$. (So this household's payoff is $u(c(1))$.) Importantly, while at date $t=0$ all households know what fraction of the households will be patient (let this fraction be denoted p), no one knows which households will be patient or impatient. That is, each household sees itself as being in the patient group of households with probability p and being in the impatient group of households with probability $1-p$.

Since the two-period return dominates the one-period return ($R > 1$), if all households were known to be patient (instead of only fraction p of them), the efficient social arrangement for this model society would be for all apples to be untouched until

period $t=2$. Further, households wouldn't need to interact with each other in any way. Each could, on its own, "plant" its own supply of apples and harvest at period $t=2$.

But, if $p < 1$ (or fraction $1-p$ of the households are impatient), households can all benefit by interacting with each other. If a household cannot interact with other households, it will plant its apple and then eat one apple at $t=1$ if it is impatient and eat R apples at $t=2$ if it is patient.

Instead, suppose that households get together in large groups and agree that impatient households will eat $c(1)$ and patient households will eat $c(2)$. The only constraint such a group would face (assuming exactly p fraction will be patient) is that the group can afford the consumption plan $(c(1), c(2))$, or

$$(1-p)\, c(1) + p\, c(2)/R = 1.$$

If households were risk neutral, they would all agree to set $c(1) = 0$ and $c(2) = R/p$. This maximizes expected consumption (which, by definition, is all a risk-neutral household cares about) because none of the apples are harvested early. However, if households are infinitely risk averse (or care only about the minimum of $c(1)$ and $c(2)$), they will set $c(1) = c(2)$, which along with equation (1) solves for

$$c(1) = c(2) = R/(p + (1-p)R).$$

So, for instance, if $R=2$ and $p = 3/4$, then each household is promised 1.6 apples regardless of whether it is patient or impatient. In period 1, when the identities of the one-quarter of impatient households is determined, the group harvests 40 percent of the apples it planted (since $1.6 \times (1/4) = .4$). Then the remaining 60 percent of the apples are held to maturity, yielding exactly enough apples so that the remaining three-quarters of the households also consumes 1.6 apples (since $1.6 \times (3/4) = .6 \times 2$). Note that for an

infinitely risk-averse household, this arrangement (eating 1.6 apples regardless of whether the household is impatient or patient) is preferable to what it could have achieved on its own (eating one apple if impatient and two apples if patient). In fact, infinite risk aversion is not necessary. This arrangement is preferable to what a household could have achieved on its own as long as households are sufficiently (but not necessarily infinitely) risk averse.

Bank runs in the Diamond-Dybvig model

Like actual banks, the banks in this model can have runs. Suppose that when households ask for consumption in periods $t=1$ or $t=2$, they are randomly ordered, as if in a queue, and further, government policy requires that these "banks" deliver the promised $c(1)$ to any household that demands it as long as the household still has the resources (apples in the ground) to do so. Then two things can happen. First, if all the patient households believe the other patient households will ask for their consumption at $t=2$, they will be willing to go along and ask for their consumption at $t=2$ as well. Then the arrangement works exactly as described above: One-quarter of the households (the impatient ones) demand their apples at $t=1$.

But suppose instead that each patient household believes the other patient households will ask for their consumption at $t=1$. Then each patient household will foresee that there won't be any apples left at $t=2$. If all patient as well as impatient households ask for their apples at $t=1$, once the first $1/1.6 = 5/8$ of the group demands their 1.6 apples, there will be none left. Any patient household that waits until $t=2$ to demand its apples will get none. But a patient household that asks for its apples at $t=1$

will get 1.6 apples with probability 5/8 (the probability of being sufficiently close to the front of the line that apples will still be left when the household gets served). It's better to get a 5/8 chance of some apples than a zero chance, so a patient household believing the other patient households will run on the bank will run as well.

Diamond and Dybvig argue that such logic justifies deposit insurance such as that provided by the Federal Deposit Insurance Corp. The rationale: If an outside entity ensures that each patient household will receive its 1.6 apples in period $t=2$ regardless of the behavior of the other households, such households have no incentive to demand early payment.