Appendix

Facilitation of Transactions in a Chari-Phelan Model

Here, the model is a simplified version of Chari and Phelan (2012b). (See also Aiyagari, Braun and Eckstein 1998.)

Households live forever (dates $t = (0,1,2,...)$). There also exist banks that take deposits and pay an interest rate r on these deposits (where banks take this interest rate as market determined).

At each date, each household receives an endowment of one apple. At the beginning of each day, a household has wealth w (denominated in dollars). It divides this wealth into cash (m), bank deposits (d) (promises to deliver cash upon demand) or bonds (b) (promises to pay a dollar at the beginning of the next period) subject to

$$m + d + q\, b <= w, \tag{1}$$

where q is the price in today's dollars of a promise to deliver a dollar at the beginning of the next period (and thus $(1/q - 1)$ is the nominal rate of interest associated with loans in this market). Implicit in this budget constraint is that a household can freely trade deposits for cash (withdraw cash from the bank) or trade cash for deposits (deposit cash in the bank), one for one. Next, as is common in cash-in-advance models, we assume that households cannot eat their own endowments. Instead, they must use either cash (m) or bank deposits (d) to purchase their consumption at each date. Paying with a bank deposit is akin to paying with a debit card. We further assume that using deposits to make purchases uses up real resources. That is, if the household's consumption $c = c(m) + c(d)$ (where $c(m)$ is the amount of consumption it purchases with cash and $c(d)$ is the amount of consumption it purchases with bank deposits), it faces means-of-payment-in-advance constraints

$$p\, c(m) <= m \tag{2}$$

and

$$p\ (1\text{+phi})\ c(d) <= d. \tag{3}$$

Here, p is the price of apples and phi is the assumed physical cost of employing banking services. If a household wishes to purchase $c(m)$ apples with cash, it must have $p\ c(m)$ dollars on hand to pay for these apples. Likewise, if a household wishes to consume $c(d)$ apples and pay for them with bank deposits, it must actually purchase (1+phi) $c(d)$ apples and have p (1+phi) $c(d)$ dollars on deposit to pay for these apples. (The extra phi $c(d)$ apples are then "used up" in the transaction.) The cost phi is meant to represent the various costs associated with a banking sector.

Assume that deposits pay a market-determined nominal interest rate r at the beginning of the next period. It is this interest rate r that makes it possible that households will be willing to use deposits even though doing so costs them the transaction cost phi. In particular, in Chari and Phelan (2012b), we show that if

$$\text{phi} < q\ (1+\text{phi})\ r, \tag{4}$$

households will use only deposits, and if

$$\text{phi} > q\ (1+\text{phi})\ r, \tag{5}$$

households will use only cash. The idea is to suppose that the household considers buying a little more of the apples it consumes today using deposits (and correspondingly less of the apples it consumes today using cash). To do this, it increases d by (1+phi) dollars for each dollar it decreases m. The cost of increasing d by (1+phi) dollars and decreasing m by one dollar is phi dollars. The benefit is that tomorrow the household receives (1+phi) r dollars as interest, which has a present value of q (1+phi) r dollars. Rearranging (5) has that a household will use only deposits if

$$r > \text{phi}/(q\ (1+\text{phi})). \tag{6}$$

That is, the interest rate on deposits must be sufficiently high to compensate the depositor for having to pay the transaction cost phi. So if phi = .01 (a 1 percent transaction fee) and $q = .96$ (so the interest rate on normal loans $(1/q - 1) = .042$, or 4.2 percent), then households will be willing to pay with deposits if those deposits pay an interest rate r greater than $(.01/(.96 \times 1.01))$, which is approximately 1 percent.

But how is the interest rate on deposits, r, determined? Here, we assume that banks can freely create deposits (by making loans to households at the interest rate $(1/q - 1)$), but doing so obligates a bank to pay interest r on these deposits *and* obligates the bank to hold alpha <= 1 dollars of cash reserves for each dollar of deposits. (We assume the reserve ratio alpha is set by the government.) Assuming the bank doesn't hold excess reserves, if it increases its deposits by one dollar, it gains one dollar (the dollar the depositor gives the bank) but must purchase alpha dollars for reserves and pay r dollars in perpetuity. This has a total present value of $1 - \text{alpha} - (q/(1-q))\ r$. If $1 - \text{alpha} - (q/(1-q))\ r > 0$, the bank can make infinite profits by increasing deposits. Likewise if $1 - \text{alpha} - (q/(1-q))\ r < 0$, the bank would earn negative profits for each dollar of deposits and thus refuse to create any. Given this assumption, free entry into banking requires $1 - \text{alpha} - (q/(1-q))\ r = 0$, or

$$r = (1-\text{alpha})\ (1/q - 1). \tag{7}$$

That is, the interest rate paid on deposits is the interest rate on normal loans $(1/q-1)$ multiplied by one minus the reserve ratio. This interest rate must be lower than the interest rate on normal loans because the bank must be compensated for the fact that for each dollar in deposits, it must hold alpha dollars in noninterest-bearing cash.

Since whether deposits are used depends on the interest rate on deposits (equation (7)) and the interest rate on deposits is determined by the reserve ratio on deposits (equation (8)), in this model economy, whether deposits are used depends on the reserve ratio. In particular, if

$$\text{alpha} < 1 - \text{phi}/((1+\text{phi})(1-q)), \tag{8}$$

households will choose to use deposits for their transactions. For the numbers given above (the transaction cost phi = .01 and $q = .96$), deposits will be used if the reserve ratio alpha is less than .75 (or a rather high 75 percent reserve ratio).

But note that using deposits in this example economy is completely wasteful. In the end, each household eats one apple each day if transactions are conducted using cash and eats 1/(1+phi) apples each day if transactions are conducted using deposits. What is happening is that if all households use cash, each household finds it privately costly to have to tie up wealth in an instrument—cash—that yields no interest. But in the end, the price of apples adjusts so that all households eat the apples there are to eat: one apple per person. Thus, there is no *social* cost to using cash. When the reserve ratio is low enough so that banks provide a high enough interest rate on deposits to induce households to use deposits, each household finds it *privately* optimal to use deposits—the private return to holding deposits exceeds the private loss of resources being used up in the banking sector. But in the end, as when the economy uses cash, the price of apples adjusts so that all households eat the apples there are to eat: one apple per person *minus the apples used up in the banking sector*. While not its intention, in this simple example economy, when a household chooses to use deposits, it is choosing to increase the money supply, since a dollar in a household pocket is one dollar of money that can be used for trade, while a dollar in a bank vault is 1/alpha dollars that can be used for trade. This extra money increases the price of apples, which affects fellow households, an effect no household takes into account when choosing between holding cash and holding deposits for transactions.