International Reserves and Rollover Risk *

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Abstract

This paper provides a theoretical framework for quantitatively investigating the optimal accumulation of international reserves as a hedge against rollover risk. We study a dynamic model of endogenous default in which the government faces a tradeoff between the insurance benefits of reserves and the cost of keeping larger gross debt positions. A calibrated version of our model is able to rationalize large holdings of international reserves, as well as the procyclicality of reserves and gross debt positions. Model simulations are also consistent with spread dynamics and other key macroeconomic variables in emerging economies. The benefits of insurance arrangements and the effects of restricting the use of reserves after default are also analyzed.

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1 Introduction

Over the last 15 years, emerging economies have rapidly increased their holdings of international reserves.[1] A popular view is that behind this build-up of reserves lies an insatiable appetite for safe assets that is intended to weather potential turbulence in financial markets. In line with this "precautionary view," international reserves appeared to play a protective role in emerging markets as sovereign spreads spiked during the global financial crisis.[2] These episodes have reinvigorated debates about the adequate amount of international reserves in emerging as well as developed economies.[3]

Our goal is to propose a quantitative theory to shed light on the optimal management of international reserves in the presence of rollover risk. We study a dynamic model of endogenous default, in which the government borrows by issuing long-duration bonds and saves by investing in a risk-free asset, i.e., reserves. Unlike standard models of sovereign default based on the classic setup of Eaton and Gersovitz (1981), the government manages both gross asset and liability positions to smooth consumption and to hedge against shocks to income and borrowing costs. In our model, a government that wishes to reduce its exposure to adverse shocks can either reduce the stock of gross debt or increase its reserve holdings, or some combination of the two. The fundamental trade-off the government faces is whether to keep reserves that provide a buffer against a future increase in the borrowing cost or to use the reserves to pay down debt and reduce borrowing costs.

For our benchmark calibration, using data for Mexico as a reference, the optimal solution to the portfolio problem is to hold a stock of reserves of 7.5 percent of GDP. This stock is large enough to cover 16 months of coming debt obligations and is one-third higher than

[1] IMF (2001) defines reserves as *"official public sector foreign assets that are readily available to and controlled by monetary authorities for the direct financing of payments imbalances, for indirectly regulating the magnitudes of such imbalances, ... and/or for other purposes."*

[2] See Gourinchas and Obstfeld (2011), Dominguez et al. (2012), Frankel and Saravelos (2012), and Bussiere et al. (2013) for evidence on the protective role of reserves during the global financial crisis and Aizenman and Lee (2007) and Calvo et al. (2012) for systematic empirical evidence supporting the precautionary role of reserve accumulation. In addition, the most frequently cited reason for reserve accumulation in the IMF Survey of Reserve Managers is building a buffer for liquidity needs (80 percent of respondents; IMF, 2011).

[3] For example, in December 2012, the Riksbank requested that the Swedish National Debt Office borrow an equivalent of 2.5 percent of GDP so as to double the stock of international reserves and strengthen the ability to weather financial market turmoil (see Riksgalden, 2013).

the level of reserves prescribed by the celebrated "Greenspan-Guidotti rule" (12 months of future debt obligations), which is often used as a rule of thumb for adequate reserve levels by policymakers. In addition, we show how the amount of reserves should vary with the business cycle and borrowing conditions.

The model in this paper can account for various features of emerging markets. First, governments hold simultaneously large amounts of international reserves and debt. Moreover, reserves are invested in assets that provide a return significantly lower than the rate at which the government borrows (Rodrik, 2006). The difference in returns is reflected in the EMBI plus sovereign spread index, which averaged 4.5 percent between 2000 and 2012. Second, gross government debt and international reserves are both procyclical and collapse during crises.[4] Third, emerging markets face volatile and highly countercyclical interest rates (Neumeyer and Perri, 2005, and Uribe and Yue, 2006). The quantitative literature on sovereign default following Aguiar and Gopinath (2006) and Arellano (2008) has been successful in accounting for the third fact, but accounting for the first two facts on gross flows has remained elusive.

The key feature in the model that contributes to account for these facts is countercyclical default risk, which arises due to the combination of limited commitment and incomplete markets. During good times, sovereign spreads are low –reflecting low incentives to default– inducing the government to borrow more. Since the government anticipates that spreads may increase in the future as incentives to repay become weaker, it finds optimal to accumulate reserves, which it will later use in bad times, when spreads become high. By borrowing and buying reserves, the government brings resources forward and self-insures against future increases in borrowing costs.

It is worth distinguishing between three roles for reserve accumulation that arise in our setup. First, there is a precautionary motive due to rollover risk: Keeping reserves allows the government to have liquid assets available in states of nature where the borrowing cost is high. Second, we show that there is a dynamic efficiency gain from reserve accumulation: Higher reserves allow the government to borrow today at lower spreads by providing commitment

[4]See the empirical evidence by Broner et al. (2013b), Dominguez et al. (2012), Forbes and Warnock (2012), and Jeanne and Ranciere (2011).

to lower future borrowing levels. Third, since the government retains access to reserves upon default, reserves can help to transfer resources to states of the world in which the government defaults.

We perform a battery of exercises in order to shed light on the key determinants of the government's optimal choices for reserves and to help disentangle these three different roles. In order to account for the volatility of net capital flows in the data, we include in our benchmark model a sudden-stop shock, i.e., a shock that prevents new borrowing while imposing an output cost. We show that reserve holdings are increasing with respect to the probability and income cost of sudden stops, as this additional source of rollover risk increases the insurance value of reserves. We also show that our main results are robust to modeling sudden stop episodes as an exogenous increase in the premium demanded on sovereign bonds.

Debt duration plays a key role in our analysis. We show that there is a non-monotonic relationship between debt duration and equilibrium reserve holdings. For a given level of debt, a lower duration increases the vulnerability of the government to rollover risk, as the government needs to roll-over a larger amount of debt each period. This raises the demand for reserves. On the other hand, lower duration also makes spreads more sensitive to gross debt levels, which makes it optimal for the government to use a larger portion of reserves to pay down debt. Overall, we find that for low (high) values of debt duration, reserve holdings are increasing (decreasing) in debt duration. When we parameterize our model to the average duration of government bonds, we find that lowering the duration increases the optimal level of reserves. In particular, if we lower the assumed mean debt duration from 5 years (our benchmark value) to 4 years, the mean level of reserves increases 33 percent. We also show that with one-period debt, the rollover risk disappears and that the levels of debt, reserves, and default risk predicted by the model collapse.

We also study the effects of restricting the use of reserves in default states, in the spirit of Bulow and Rogoff (1989). Because reserves help the government to smooth consumption during default, it is possible that restricting the use of reserves during default can raise the demand for reserves. In fact, we show that the optimal demand for debt and reserves increase respectively by 9 percent and 31 percent when the government is prevented from using reserves during default.

We also analyze the benefits from saving using assets with payments contingent on the sudden-stop shock instead of non-state-contingent assets, as in our baseline model. Allowing for sudden-stop contingent assets instead of reserves results in a consumption volatility decline of 14 percent and a welfare gain equivalent to a permanent increase in consumption of 0.28 percent. Furthermore, the average holding of contingent assets is 50 percent higher than the average holding of reserves. Finally, we extend the baseline model by allowing reserves to affect the probability of a sudden-stop shock and find that this leads to an increase in reserve holdings of up to 64 percent.

Related Literature. We build on the quantitative sovereign default literature that follows Aguiar and Gopinath (2006) and Arellano (2008). They show that predictions of the sovereign default model are consistent with several features of emerging markets, including countercyclical spreads and procyclical borrowing. These papers, however, do not allow indebted governments to accumulate assets. We allow governments to simultaneously accumulate assets and liabilities and show that the model's key predictions are still consistent with features of emerging markets. Furthermore, our model can account for the accumulation of reserves by indebted governments and the procyclicality of reserve holdings.

Our paper is also related to a growing literature on reserves and sudden stops. For example, Durdu, Mendoza and Terrones (2009) study a dynamic precautionary savings model where a higher *net* foreign asset position reduces the frequency and the severity of binding credit constraints. In contrast, we study a setup with endogenous borrowing constraints resulting from default risk and analyze *gross* portfolio positions.

Jeanne and Ranciere (2011) develop an influential simple analytical formula to quantify the optimal amount of reserves. They study a model where reserves are modeled as an Arrow-Debreu security that pays off in a sudden stop. In contrast, reserves in our model take the form on non-state contingent assets, which is consistent with the fact that most reserves are invested in safe government bonds (e.g., US Treasury securities).[5] In a similar vein, Caballero and Panageas (2008) propose a quantitative setup in which the government

[5] Jeanne and Ranciere (2011) also present an implementation of the optimal allocation with non-state-contingent assets, but this implementation still requires insurance contracts in the form of state contingent debt (a perpetuity that yields interest payments until a sudden stop occurs).

issues non-defaultable debt that is indexed to the income growth shock. They find, as we do, that there are significant gains from introducing financial instruments that provide insurance against both the occurrence of sudden stops and changes in the sudden-stop probability. We add to the analysis in these papers by considering an endogenous cost of borrowing due to default risk and shifts the focus from insurance arrangements to safe assets.

Aizenman and Lee (2007) study a Diamond-Dybvig type model where reserves serve as liquidity to reduce output costs during sudden stops. Hur and Kondo (2012) develop a model where reserves reduce the probability of a rollover crisis and show that learning about the sudden-stop process can account for the surge in the reserves-to-debt ratio over the last decade. Overall, a key contribution of our paper is to provide a unified framework for studying the dynamics of reserves, debt, and sovereign spreads. Moreover, our paper points to the importance of debt maturity for a realistic quantification of rollover risk and the need for reserves.

Other studies emphasize other benefits of reserves accumulation. The "mercantilist motive" is investigated in Korinek and Servén (2011) and Benigno and Fornaro (2012). They present models where learning-by-doing externalities in the tradable sector lead the government to accumulate reserves to undervalue the real exchange rate. In Aguiar and Amador (2011), the accumulation of net foreign assets allows the government to credibly commit not to expropriate capital. These studies, however, do not analyze gross debt positions and hence do not address whether governments should use reserves to lower their debt level. We show that rollover risk can go a long way in explaining reserve holdings by indebted governments.

Our work is also closely related to papers that study the optimal maturity structure of government debt in the presence of distortionary taxation. Angeletos (2002) and Buera and Nicolini (2004) study a closed-economy model where the government can issue non-state-contingent bonds of different maturities under perfect commitment. They present examples where the government can replicate complete market allocations by issuing non-defaultable long-term debt and accumulating short-term assets. There are important differences in the mechanism in our paper. In their model, changes in the term structure of interest rates, which contribute to offset shocks to the government budget constraint, arise as a result of fluctuations in the marginal rate of substitution of domestic consumers. Quantitatively, the gross positions that sustain the complete market allocations are on the order of a few hundred times total GDP (Buera and Nicolini, 2004).[6]

In contrast, in our model, fluctuations in the interest rate reflect changes in the default premium that foreign investors demand in order to be compensated for the possibility of government default. This provides not only a fundamentally different reason for the degree of asset spanning, but also lead to an empirically plausible portfolio composition of government assets and liabilities. In addition, reserves affect incentives for debt repayment, a channel absent in this literature. Overall, our paper provides an alternative theory of debt management based on limited commitment, which shifts the focus from minimizing deadweight losses of taxation to default risk.

Arellano and Ramanarayanan (2012) study a model with default risk and highlight the incentive benefits of short-term debt and the hedging benefits of long-term debt. Trade-offs between short-term debt and long-term debt are also analyzed in Niepelt (2008), Broner, Lorenzoni and Schmukler (2013a), Dovis (2013), Aguiar and Amador (2013). However, none of these studies allow the government to accumulate assets for insurance purposes. Alfaro and Kanczuk (2009) study a model with one-period debt where assets are only useful for transferring resources to default states. In contrast, we study the role of reserves in hedging against rollover risk.

Telyukova (2011) and Telyukova and Wright (2008) address the "credit card puzzle," i.e., the fact that households pay high interest rates on credit cards while earning low rates on bank accounts. In these models, the demand for liquid assets arises because of a transaction motive, as credit cards cannot be used to buy some goods. While we also study savings decisions by an indebted agent, we offer a distinct mechanism for the demand of liquid assets based on rollover risk.

The rest of the article proceeds as follows. Section 2 presents the model, section 3 describes the calibration, and section 4 presents the results. Section 5 concludes.

[6]Faraglia et al. (2010) argues that the qualitative predictions of the "complete market approach" to debt management are sensitive to the type of shocks considered due to the high correlation of bonds at different maturities.

2 Model

This section presents a dynamic small open economy model in which the government issues non-state-contingent debt and buys risk-free assets. The government lacks commitment and can default on the debt at any point in time. If the government defaults, it suffers temporary exclusion and direct costs.[7]

The economy's endowment of the single tradable good is denoted by $y \in Y \subset \mathbb{R}_{++}$. The endowment follows a Markov process. Preferences of the government are given by:

$$\mathbb{E}_t \sum_{j=t}^{\infty} \beta^{j-t} u\left(c_j\right),$$

where $\mathbb{E}$ denotes the expectation operator, β denotes the subjective discount factor, and c represents the economy's consumption. The utility function is strictly increasing and concave.

As in Arellano and Ramanarayanan (2012) and Hatchondo and Martinez (2009), we assume that a bond issued in period t promises a deterministic infinite stream of coupons that decreases at an exogenous constant rate δ. In particular, a bond issued in period t promises to pay $(1-\delta)^{j-1}$ units of the tradable good in period $t+j$, for all $j \geq 1$. Hence, debt dynamics can be represented compactly by the following law of motion:

$$b_{t+1} = (1-\delta)b_t + i_t,$$

where b_t is the number of coupons due at the beginning of period t, and i_t is the number of new bonds issued in period t.

[7] In practice reserves are often held by the monetary authority while borrowing is conducted by the fiscal authority, but we treat the government as a consolidated entity, abstracting from possible conflicts of interest between the different branches of the government. Moreover, in many emerging markets, the central bank has little independence from the central government. As anecdotal evidence from Argentina, New York Times (January 7, 2010) reported "President Cristina Fernandez fired Argentina's central bank chief Thursday after he refused to step down in a dispute over whether the country's international reserves should be used to pay debt." The Swedish National Debt Office recently questioned whether a liquidity buffer fund should be at the government's or the Riksbank's disposal (Riksgalden, 2013). Such debate is beyond the scope of this paper.

The government also has access to a one-period risk-free asset, i.e., reserves, that pays $1 + r$ each period.[8] Let $a_t \geq 0$ denote the government's reserve holdings at the beginning of period t. The budget constraint conditional on having access to credit markets is represented as follows:

$$c_t = y_t - b_t + a_t + i_t q_t - \frac{a_{t+1}}{1+r},$$

where q_t is the price of the bond issued by the government, which in equilibrium will depend on the policy pair (b_{t+1}, a_{t+1}) as well as the exogenous shocks.

When the government defaults, it does so on all current and future debt obligations. This is consistent with the observed behavior of defaulting governments and it is a standard assumption in the literature.[9] As in most previous studies, we also assume that the recovery rate for debt in default (i.e., the fraction of the loan lenders recover after a default) is zero.[10]

A default event triggers exclusion from borrowing in credit markets for a stochastic number of periods. The government regains access to debt markets with constant probability $\psi^d \in [0, 1]$. In addition, there is an output loss of $\phi^d(y)$ in every period in which the government is excluded from credit markets because of a default. We assume that this output loss is proportionally higher at higher output levels, which is a property of the endogenous default cost in Mendoza and Yue (2012). Upon default, the government, retains control of their reserves and access to savings.[11] Hence, in case of default, the budget constraint becomes:

$$c_t = y_t - \phi^d(y) + a_t - \frac{a_{t+1}}{1+r}.$$

Sudden-Stop Shock. To captures dislocations to international credit markets that are exogenous to local conditions, we consider a sudden-stop/global shock. Formally during a

[8]Because reserves are a perfectly liquid risk-free asset that pays a constant interest rate each period, the assumed duration of reserves is one period without loss of generality.

[9]Sovereign debt contracts often contain an acceleration clause and a cross-default clause. The first clause allows creditors to call the debt they hold in case the government defaults on a debt payment. The cross-default clause states that a default in any government obligation constitutes a default in the contract containing that clause. These clauses imply that after a default event, future debt obligations become current.

[10]Yue (2010), Bai and Zhang (2012), D'Erasmo (2011) and Benjamin and Wright (2008) are examples of models with endogenous recovery rates.

[11]Notice that with one-period bonds, the portfolio between debt and reserves would be undetermined if all reserves were seized upon default, but gross positions remain determined in case of long-duration bonds. In the quantitative analysis, we analyze the effects of restrictions on savings upon default.

sudden stop, the government cannot issue new debt and suffers an income loss of $\phi^s(y)$ –the government can buy back debt and change its reserve holdings while in a sudden stop. The sudden-stop shock follows a Markov process so that a sudden stop starts with probability $\pi \in [0,1]$ and ends with probability $\psi^s \in [0,1]$.

The sudden-stop shock is in line with a vast empirical literature showing that extreme capital flow episodes are typically driven by global factors (see, for instance, Calvo et al., 1993, Uribe and Yue, 2006, and Forbes and Warnock, 2012). In our model, sudden stops restrict borrowing and produce output losses, and hence can trigger changes in sovereign spreads and default episodes, in the absence of changes to domestic fundamentals. Moreover, we also show that sudden stops are important for the quantitative performance of the model because they contribute to match the volatility of both gross and net capital flows observed in the data. The loss of income triggered by a sudden stop is also consistent with empirical studies and can be rationalized by the adverse effects of these episodes on the economy, which are often associated with a credit crunch and a deep recession (Calvo et al., 1993, Mendoza, 2010). In this way, the model also allows us to capture a double drain scenario, emphasized by Obstfeld et al. (2010).[12]

We would also like to note that there are other modeling approaches for introducing sudden stops or global shocks, as alternatives to our specification that assumes a sudden constraint on new debt issuances. In fact, we show in section 4.5.3 that modelling the sudden stop as increase in the risk premium that foreign investors require to invest in government bonds yield very similar quantitative results. Intuitively, if foreign lenders become sufficiently impatient or risk averse, the interest rate at which they are willing to lend to the government would imply that the government would issue a small amount of debt in equilibrium.[13] Beyond parsimony, an additional advantage of this specification is perhaps that it neutralizes a strategic role for reserve accumulation that would arise if the government could use reserves

[12] To be clear, reserves continue to be optimal even without sudden stop shocks. Below we show how our quantitative results change with the sudden stop process.

[13] There are other approaches for the sudden stop shock as well. For example, an increase in the probability of a self-fulfilling crisis, as in Cole and Kehoe (2000) (see also Roch and Uhlig (2012); Aguiar et al. (2013); Lorenzoni and Werning (2013)); would also reduce debt issuances in equilibrium. More closely related is Borri and Verdelhan (2009) who consider the case of foreign lenders with habit preferences and show that variations in risk aversion are key to account for spread dynamics. Arellano and Bai (2012) considers the case of global shocks triggered by contagion through a common lender.

to repurchase its own debt at "fire sale prices" when investors become more risk-averse or more impatient.

Timing. The timing protocol within each period is as follows. First, the income and sudden-stop shocks are realized. After observing these shocks, the government chooses whether to default on its debt and makes its portfolio decision subject to constraints imposed by the sudden-stop shock and its default decision. Figure 1 summarizes the timing of these events.

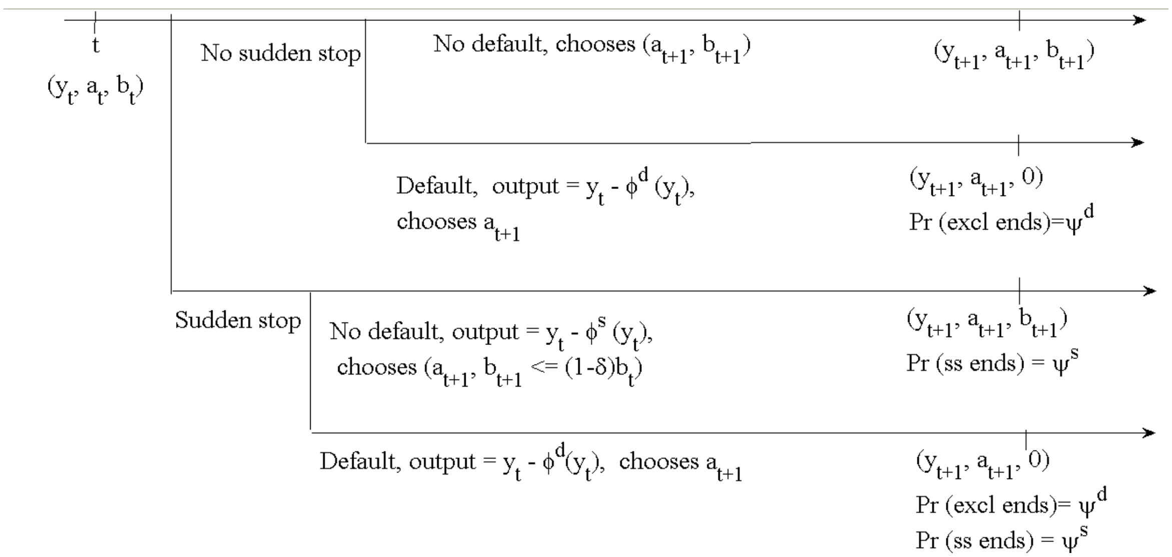

Figure 1: Sequence of events when the government is not in default. The government enters the period with debt b_t and reserves a_t. First, the income and sudden-stop shocks are realized. Second, the government chooses whether to default. Third, the government adjusts its debt and reserves positions. The government can always adjust reserve holdings and buy back debt. It can issue debt only if it did not default and is not in a sudden stop.

2.1 Recursive Formulation

We now describe the recursive formulation of the government's optimization problem. The government cannot commit to future (default, borrowing, and saving) decisions. Thus, one may interpret this environment as a game in which the government making decisions in period t is a player who takes as given the (default, borrowing, and saving) strategies of other players (governments) who will decide after t. We focus on Markov Perfect Equilibrium. That is, we assume that in each period the government's equilibrium default, borrowing, and saving strategies depend only on payoff-relevant state variables.

The sudden-stop shock is denoted by s, with $s = 1$ ($s = 0$) indicating that the economy is (is not) in a sudden-stop. Let V denote the value function of a government that is not currently in default. For any bond price function q, the function V satisfies the following functional equation:

$$V(b, a, y, s) = \max \left\{ V^R(b, a, y, s), V^D(a, y, s) \right\}, \tag{1}$$

where the government's value of repaying is given by

$$V^R(b, a, y, s) = \max_{a' \geq 0, b', c} \left\{ u(c) + \beta \mathbb{E}_{(y',s')|(y,s)} V(b', a', y', s') \right\}, \tag{2}$$

subject to

$$c = y - s\phi^s(y) - b + a + q(b', a', y, s)(b' - (1-\delta)b) - \frac{a'}{1+r},$$

and if $s = 1$, $b' - (1-\delta)b \leq 0$.

The value of defaulting is given by:

$$V^D(a, y, s) = \max_{a' \geq 0, c} u(c) + \beta \mathbb{E}_{(y',s')|(y,s)} \left[(1-\psi^d) V^D(a', y', s') + \psi^d V(0, a', y', s') \right], \tag{3}$$

subject to

$$c = y - \phi^d(y) + a - \frac{a'}{1+r}.$$

The solution to the government's problem yields decision rules for default $\hat{d}(b, a, y, s)$, debt $\hat{b}(b, a, y, s)$, reserves in default $\hat{a}^D(a, y, s)$, reserves when not in default $\hat{a}^R(b, a, y, s)$, consumption in default $\hat{c}^D(a, y, s)$, and consumption when not in default $\hat{c}^R(b, a, y, s)$. The default rule $\hat{d}$ is equal to 1 if the government defaults, and is equal to 0 otherwise. In a rational expectations equilibrium (defined below), investors use these decision rules to price debt contracts.

2.2 Bond Prices

Government bonds and reserves are priced in a competitive market inhabited by a large number of identical risk-neutral international investors. Investors discount future payoffs at the rate $1+r$, i.e., the return on risk-free assets. This implies that in equilibrium the bond-price function solves the following functional equation:

$$q(b', a', y, s)(1+r) = \mathbb{E}_{(y',s')|(y,s)}(1-\hat{d}(b', a', y', s'))(1+(1-\delta)q(b'', a'', y', s')), \tag{4}$$

where

$$\begin{aligned} b'' &= \hat{b}(b', a', y', s') \\ a'' &= \hat{a}^R(b', a', y', s'). \end{aligned}$$

Equation (4) indicates that, in equilibrium, an investor has to be indifferent between selling a government bond today and investing in a risk-free asset and keeping the bond and selling it in the next period. If the investor keeps the bond and the government does not default in the next period, he first receives a coupon payment of one unit and then sells the bond at the market price, which is equal to $(1-\delta)$ times the price of a bond issued in the next period.

Notice that while investors receive on expectation the risk-free rate, the expected cost of borrowing for the government is strictly higher than the risk-free rate. This occurs because while investors receive zero payments in case of default, the government suffers output costs and exclusion after defaulting, raising the average costs of borrowing.

2.3 Recursive Equilibrium

A *Markov Perfect Equilibrium* is defined by

1. a set of value functions V, V^R and V^D,
2. rules for default $\hat{d}$, borrowing $\hat{b}$, reserves $\{\hat{a}^R, \hat{a}^D\}$, and consumption $\{\hat{c}^R, \hat{c}^D\}$,
3. and a bond price function q,

such that:

i. given a bond price function q; the policy functions $\hat{d}$, $\hat{b}$, $\hat{a}^R$, $\hat{c}^R$, $\hat{a}^D$, $\hat{c}^D$, and the value functions V, V^R, V^D solve the Bellman equations (1), (2), and (3).

ii. given government policies, the bond price function q satisfies condition (4).

3 Calibration

The utility function displays a constant coefficient of relative risk aversion, i.e.,

$$u(c) = \frac{c^{1-\gamma} - 1}{1-\gamma}, \text{ with} \gamma \neq 1.$$

The endowment process follows:

$$\log(y_t) = (1-\rho)\,\mu + \rho \log(y_{t-1}) + \varepsilon_t, \quad |\rho| < 1, \quad \varepsilon_t \sim N\left(0, \sigma_\epsilon^2\right).$$

This means that aggregate income follows a stationary process in levels. In Appendix B.1 we present an alternative calibration with deterministic growth in aggregate income. We show that incorporating growth to aggregate income does not significantly affect the level of reserves or the cyclical properties of reserve accumulation. Appendix B also presents additional sensitivity analysis with respect to key parameters of the model including risk aversion and the discount factor.

Following Arellano (2008), we assume an asymmetric cost of default $\phi^d(y)$, so that it is proportionally more costly to default in good times. This is a property of the endogenous default cost in Mendoza and Yue (2012) and, as shown by Chatterjee and Eyigungor (2012), allows the equilibrium default model to match the behavior of the spread in the data. In particular, we assume a quadratic loss function for income during a default episode $\phi^d(y) = max\{0, d_0 y + d_1 y^2\}$, as in Chatterjee and Eyigungor (2012).

We assume that the income loss during a sudden stop is a fraction of the income loss after a default: $\phi^s(y) = \lambda \phi^d(y)$. With this assumption, we have to pin down only one more parameter value in order to determine the cost of sudden stops. Moreover, since both sovereign defaults and sudden stops are associated with disruptions in the availability of private credit, it is natural to assume that the cost of these events is a fraction of the cost of defaulting (see e.g. Gennaioli et al., 2012).

Table 1 presents the benchmark values given to all parameters in the model. A period in the model refers to a quarter, and the risk-free interest rate is set equal to 1 percent, which is a standard value in the quantitative business cycle and in sovereign default studies. As in Mendoza and Yue (2012), we assume an average duration of sovereign default events of three years ($\psi^d = 0.083$), in line with the duration estimated in Dias and Richmond (2007).

We define a sudden stop in the data as an annual fall in net capital inflows of more than 5 percent of GDP, as in Jeanne and Ranciere (2011). Using this definition, the same sample of countries considered by Jeanne and Ranciere (2011), and the IMF's International Financial Statistics annual data from 1970 to 2011, we find one sudden stop every 10 years (as they do). Thus, we set $\pi = 0.025$. It is important to note that with this value for π, we also find that the number of current account reversals in the model is also around 1 every 10 years. Section 4.5 presents comparative statics results on the value of π. The appendix presents the list of sudden stops we identify and the evolution of net capital inflows for each country in our sample.

We set ψ^s to match the duration of sudden stops in the data. We estimate the duration of sudden stops using quarterly data from 1970 to 2011. We define ca_t as the ratio of cumulated net capital inflows over the last four quarters to cumulated GDP over the last four quarters.[14] We identify quarters in which $ca_{t+4} < ca_t - 0.05$. For such quarters, a sudden-stop episode begins the first quarter between t and $t + 4$ in which ca_t falls. This sudden stop ends the first period in which ca_t increases. Following this methodology, we find a mean duration of a sudden stop of 1.12 years, and set accordingly $\psi^s = 0.25$.[15]

We use Mexico as a reference for choosing the parameters that govern the endowment process, the level and duration of debt, and the mean and standard deviation of the spread. Mexico is a common reference for studies on emerging economies because business cycles

[14] Net capital inflows are measured as the deficit in the current account minus the accumulation of reserves and related items.

[15] The duration of sudden stops we estimate is close to the one estimated by Forbes and Warnock (2012) who use gross capital inflows.

in Mexico display the same properties that are observed in other emerging economies (see Aguiar and Gopinath, 2007; Neumeyer and Perri, 2005; and Uribe and Yue, 2006). Furthermore, our sudden-stop parameter values are similar to the ones we would have obtained using only data for Mexico, which has experienced three sudden stops since 1979 with an average duration of 1.4 years. Unless we explain otherwise, we compare simulation results with data from Mexico from the first quarter of 1980 to the fourth quarter of 2011. Therefore, the parameter values that govern the endowment process are chosen so as to mimic the behavior of GDP in Mexico during that period.

We set $\delta = 3.3\%$. With this value, bonds have an average duration of 5 years in the simulations, which is roughly the average debt duration of sovereign bonds in Mexico according to Cruces et al. (2002).[16]

We need to calibrate the value of five other parameters: the discount factor β, the parameters of the income cost of defaulting d_0 and d_1, the parameter determining the relative income cost of a sudden stop compared with a default λ, and the risk aversion γ. Chatterjee and Eyigungor (2012) calibrate the first three parameters to target the mean and standard deviation of the sovereign spread and the mean debt level. We follow their approach but also incorporate as targets the average accumulated income cost of a sudden stop and the ratio of the standard deviations of consumption and income. We choose to make the risk aversion part of the calibration because it is the key parameter determining the government willingness to tolerate rollover risk. The choice of the value for the risk aversion parameter is determined mainly by the consumption-volatility target. We choose to target a volatility of consumption equal to the volatility of income, in line with the findings of Alvarez et al. (2013).[17] The value of the risk aversion parameter that results from the calibration is $\gamma = 4$, which is within the range of values used for macro models of precautionary savings (e.g., Guerrieri and Lorenzoni, 2011).

[16]We use the Macaulay definition of duration that, with the coupon structure in this paper, is given by $D = \frac{1+r^*}{\delta+r^*}$, where r^* denotes the constant per-period yield delivered by the bond. Using a sample of 27 emerging economies, Cruces et al. (2002) find an average duration of foreign sovereign debt in emerging economies—in 2000—of 4.77 years, with a standard deviation of 1.52.

[17]Alvarez et al. (2013) showed that in emerging economies (including Mexico), the volatility of total consumption is higher than the volatility of aggregate income, but the volatility of the consumption of non-durable goods is lower than the volatility of income. As our model does not differentiate between total and non-durable consumption, we choose to target a relative volatility of 1.

Table 1: Parameter Values.

Risk-free rate	r	1%
Probability of reentry after default	ψ^d	0.083
Debt duration	δ	0.033
Probability of entering a SS	π	0.025
Probability of reentry after SS	ψ^s	0.25
Income autocorrelation coefficient	ρ	0.94
Standard deviation of innovations	σ_ϵ	1.5%
Mean log income	μ	(-1/2)σ_ϵ^2
Discount factor	β	0.9745
Income cost of defaulting	d_0	-1.01683
Income cost of defaulting	d_1	1.18961
Income cost of sudden stops	λ	0.5
Risk aversion	γ	4

We target an average accumulated income cost of a sudden stop of 14 percent of annual income, which is at the lower end of the range of estimated values (see Becker and Mauro, 2006; Hutchison and Noy, 2006; and Jeanne and Ranciere, 2011). Finally, we target values for mean debt, average spreads and the volatility of spreads equal to 43, 2.9 and 1.5 respectively.[18] Section 4 presents extensive sensitivity analysis with respect to the value of several parameters in the calibration.

In order to compute the sovereign spread implicit in a bond price, we first compute the yield i, defined as the return an investor would earn if he holds the bond to maturity (forever) and no default is declared. This yield satisfies

$$q_t = \sum_{j=1}^{\infty} \frac{(1-\delta)^{j-1}}{(1+i)^j}.$$

The sovereign spread is then computed as the difference between the yield i and the risk-free rate r. We report the annualized spread

$$r_t^s = \left(\frac{1+i}{1+r}\right)^4 - 1.$$

[18]The time series for the spread is taken from Neumeyer and Perri (2005) for the period 1996-2001 and from the EMBI+ index for the period 2002-2011. The data for public debt is taken from Cowan et al. (2006).

Debt levels in the simulations are calculated as the present value of future payment obligations discounted at the risk-free rate, i.e., $b_t(1+r)(\delta+r)^{-1}$.

3.1 Computation

We solve for the equilibrium of the finite-horizon version of our economy as in Hatchondo et al. (2010). That is, the approximated value and bond price functions correspond to the ones in the first period of a finite-horizon economy with a number of periods large enough that the maximum deviation between the value and bond price functions in the first and second period is no larger than 10^{-6}. The recursive problem is solved using value function iteration. We solve the optimal portfolio allocation in each state by searching over a grid of debt and reserve levels and then using the best portfolio on that grid as an initial guess in a nonlinear optimization routine. The value functions V^D and V^R and the function that indicates the equilibrium bond price function conditional on repayment $q\left(\hat{b}(\cdot), \hat{a}^R(\cdot), \cdot, \cdot\right)$ are approximated using linear interpolation over y and cubic spline interpolation over debt and reserves positions. We use 20 grid points for reserves, 20 grid points for debt, and 25 grid points for income realizations. Expectations are calculated using 50 quadrature points for the income shocks.

4 Quantitative Results

We start the quantitative analysis by showing that the model simulations match the calibration targets and also do a reasonable job at matching other non-targeted moments in the data. In particular, we show that the model generates joint debt and reserve accumulation together with a significant default premium and gross capital flow dynamics consistent with the ones in the data–Appendix A shows analytically in a 3-period model that reserve accumulation can be an optimal response to hedge against rollover risk. We then present the policy functions to analyze the workings of the model. Finally, we present a number of additional experiments to shed further light on the mechanisms of the model. In particular, we show how the key predictions of the model change with different values of the probability and costs of sudden stops, as well as with different values of discount factor and risk aversion.

We analyze the gains from introducing assets with payments contingent on sudden stops and from restricting the use of reserves during default episodes.

4.1 Simulation Results

4.1.1 Long-Run Moments

Business Cycle Statistics— Table 2 reports long-run moments in the data and in the model simulations. The first panel of this table shows that the simulations match the calibration targets reasonably well. The model also does a good job in mimicking other non-targeted moments. Overall, Table 2 shows that the model can account for distinctive features of business cycles in Mexico and other emerging economies, as documented by Aguiar and Gopinath (2007), Neumeyer and Perri (2005), and Uribe and Yue (2006). Previous studies show that the sovereign default model without reserve accumulation can account for these features of the data. We show that this is still the case when we extend the baseline model to allow for the empirically relevant case in which indebted governments can hold reserves and choose to do so.

Reserve Accumulation— Table 2 shows that an indebted government paying a significant spread chooses to hold a significant amount of international reserves. Average reserve holdings in the simulation are about 7.5 percentage points of income, which are about 80 percent of the average reserve holdings for Mexico between 1996 and 2011. Moreover, the maximum value of reserves reached in the simulations is 30 percent of annualized income.

Average reserve holdings in simulation periods without sudden stops represent on average 16 months of future debt obligations. Interestingly, this is quite close to the "Greenspan-Guidotti rule" often targeted by policymakers, which prescribes full short-term debt coverage (12 months of debt obligations).

Our findings suggest that the rollover risk channel investigated in this paper is quantitatively significant as a determinant for reserve accumulation in many emerging economies. It is unlikely, however, to account for the magnitudes of reserve accumulation in China (38

Table 2: Long-Run Statistics

	Model	Data
Mean Debt-to-GDP	46	43
Mean r_s	2.9	2.9
$\sigma(r_s)$	1.6	1.5
Mean sudden stop income cost (% annual income)	14	14
$\sigma(c)/\sigma(y)$	1.0	1.1
$\sigma(tb)$	1.3	1.4
$\rho(c, y)$	0.9	0.9
$\rho(r_s, y)$	-0.4	-0.5
$\rho(r_s, tb)$	0.3	0.6
Mean Reserves-to-GDP	7.5	9.0
$\rho(\Delta a, y)$	0.4	0.4
$\rho(\Delta b, y)$	0.4	0.9
$\rho(\Delta a, r_s)$	-0.3	-0.2

Note: The standard deviation of x is denoted by $\sigma(x)$. The coefficient of correlation between x and z is denoted by $\rho(x, z)$. Changes in debt and reserves levels are denoted by Δa and Δb, respectively. Moments are computed using detrended series. Trends are computed using a linear trend. Moments for the simulations correspond to the mean value of each moment in 250 simulation samples, with each sample including 120 periods (30 years) without a default episode. Default episodes are excluded to improve comparability with the data; our samples start at least five years after a default. Consumption and income are expressed in logs. Due to data availability, debt statistics are at annual frequency.

percent of the GDP at the end of 2012) and some other East Asian economies.[19]

Cyclicality of Reserves and Debt— As the last panel of Table 2 shows, the model reproduces the procyclicality of reserves and debt in the data. That is, in good times, the government increases borrowing and accumulates more reserves. The model also reproduces

[19]China has also a relatively low level of debt-GDP, which may suggest that the rollover risk channel should be weak. However, there is a widespread view that contingent debt associated with state-owned enterprises is several times GDP, which implies that reserves provide important insurance against a full blown financial crisis (see e.g. Aizenman and Lee (2007)). As mentioned above, there are of course other channels that are potentially quantitatively important as well.

the negative correlation between the changes in reserves and spreads. As we will show below, it is the countercyclical default risk that is key to account for these facts.

4.1.2 Sudden-Stop Events

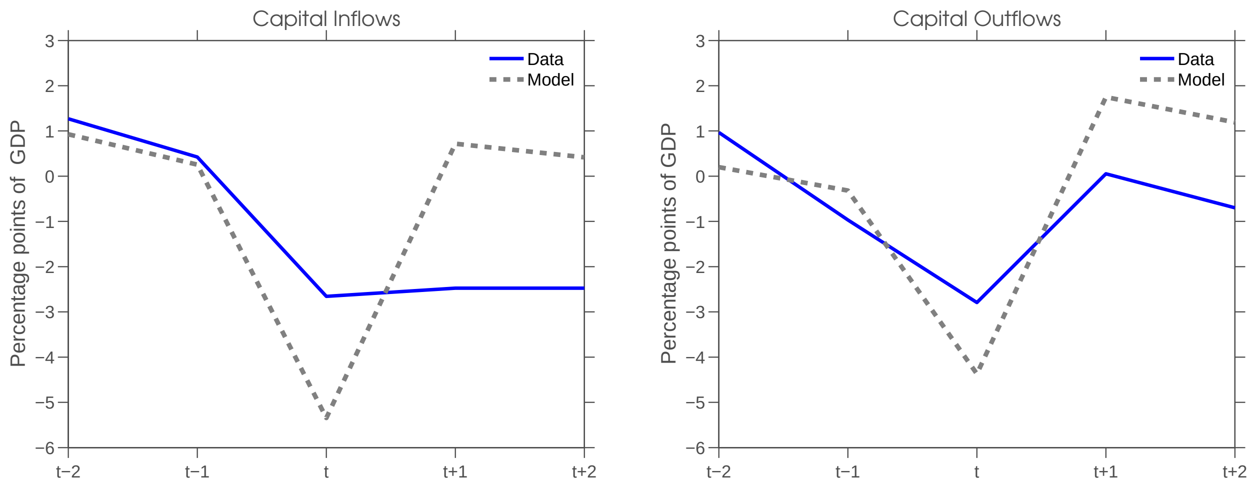

Figure 2: Average gross capital flows as a percentage of trend GDP in the simulations and in the data. The crisis year is denoted by t. In the simulations, we consider only sudden-stop episodes that do not trigger a default (in default episodes changes in the debt level do not correspond to changes in capital inflows). The behavior of flows in the data is the one presented by Broner et al. (2013b).

Figure 2 presents an event analysis of capital flows around sudden stops for the model and the data. To construct the event analysis in the model, we run a long time-series simulation and identify all the periods that are hit by a sudden stop. Then, we construct windows of five years around those episodes. The simulations show that the model predicts a collapse in both capital inflows (sovereign debt) and capital outflows (reserves) during sudden stops. This is consistent with the behavior of debt and reserves documented in Jeanne and Ranciere (2011) and the behavior of gross flows around crises documented by Broner et al. (2013b) and reproduced in Figure 2. That is, purchases of domestic assets by foreign agents and purchases of foreign assets by domestic agents collapse during crises.[20]

[20]Broner et al. (2013b) show that in emerging economies changes in reserves represent about half of the purchases of foreign assets by domestic agents and contract significantly during crisis episodes. In addition, they show that debt inflows play a primary role in accounting for changes in non-resident purchases of domestic assets over the business cycle and during crises.

4.2 Policy Functions and Spreads

We show next that the effect of aggregate income shocks on borrowing conditions is key to explaining the joint accumulation of debt and reserves, the cyclical behavior of debt and reserves, and their dynamics around crises. Figure 3 illustrates how borrowing conditions deteriorate when income falls. This figure plots the menu of spreads for each possible amount of borrowing for two different values of income, conditional on choosing a value of reserves equal to the mean. When income falls, spreads increase for the same value of borrowing, which leads the government to reduce the level of borrowing, as illustrated by the solid dots in Figure 3.

To illustrate the mechanism further, Figure 4 presents the policy functions for debt (left panel) and the changes in reserves (right panel) as a function of current income. The policies correspond to the case in which, at the beginning of the period, the government is not in default and holds an initial level of debt and reserves equal to the mean levels observed in the simulations. The straight (broken) line indicates the demand for reserves when the economy is (is not) in a sudden stop. In all the figures, we express debt and reserves normalized by annualized income so that all expressions can be understood as fractions of GDP.

The vertical dotted lines correspond to the default threshold that separate repayment region and default region. When the government is not hit by a sudden-stop shock, the government repays the debt as long as income does not fall 5 percent or more below its mean. The government defaults otherwise. The fact that the default region is decreasing in the level of income is standard in the literature and reflects the fact that repayment is more costly for low-income levels and that the direct costs from defaulting are lower. Moreover, the default threshold when the economy is in a sudden stop is strictly higher, i.e., the government is more likely to default if it faces a sudden stop. This reflects the fact that default entails less of a punishment during a sudden stop as the government already faces restrictions to credit market and income losses due to the sudden stop.

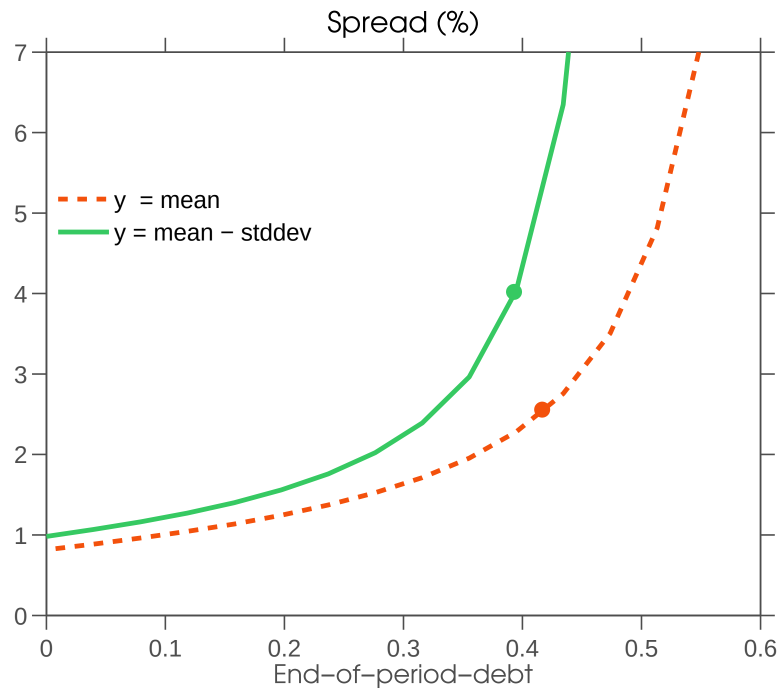

Figure 3: Menus of spread and end-of-period debt levels available to a government that is not facing a sudden stop and chooses a level of reserves equal to the mean in the simulations, i.e., $r^s(b', \bar{a}, y, 0)$, where $\bar{x}$ denotes the sample mean value of variable x. The solid dots present the spread and debt levels chosen by the government when it starts the period with debt and reserves levels equal to the mean levels observed in the simulations (for which it does not default).

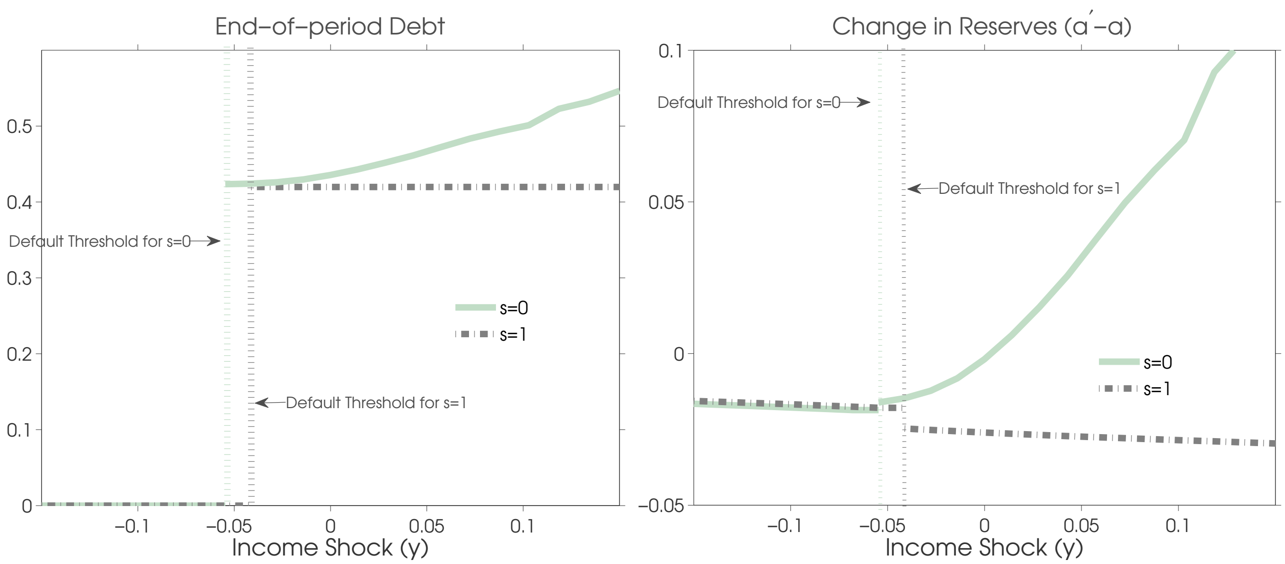

Figure 4: Equilibrium borrowing and reserve accumulation policies for a government that starts the period with levels of reserves and debt equal to the mean levels in the simulations. Debt levels and variations in reserves are presented as a percentage of the mean annualized income (4). That is, the left panel plots $\hat{b}\left(\bar{b}, \bar{a}, y, s\right)/4$ and the right panel plots $\left(\hat{a}\left(\bar{b}, \bar{a}, y, s\right) - \bar{a}\right)/4$.

When the economy is not in a sudden stop and the economy is in the repayment region, both borrowing and changes in reserves are increasing with respect to income. In particular, notice that the government increases its reserve holdings when income is above trend, in line with the permanent income hypothesis. The permanent income hypothesis would also predict that borrowing should be *decreasing* with respect to income. However, because income is persistent, a high current income improves borrowing opportunities (Figure 3) and leads to more borrowing. Moreover, once the government is allowed to accumulate reserves, there is an extra incentive for borrowing more when income is higher: financing reserve accumulation to hedge rollover risk. In the default region, the government sells reserves (and debt levels are equal to zero).

Figure 4 also shows that a sudden stop causes a reduction in borrowing and reserve accumulation. In the repayment region, the government pays the coupons that are due and does not repurchase debt, as illustrated by the flat policy function for borrowing. Notice that changes in reserves are slightly decreasing in the level of income, reflecting the fact that the government expects a low future interest rate when it regains access to credit markets.

4.3 The Effects of Reserves on Spreads

We now analyze how the accumulation of reserves affect spreads. The effects of reserves on spreads is determined by how reserve holdings affect default probabilities in the future. There are two effects of reserves on the decision to default at any given state. First, reserves reduce the cost of repayment and reduces the incentives to default. Second, reserves also reduce the cost of autarchy as reserves allow to smooth the penalty costs from defaulting, and this increases the incentives to default. Furthermore, current spreads reflect not only the next-period default probability but also default probabilities in other future periods. In particular, there is a dynamic effect that makes reserves reduce the level of spreads, as higher reserves reduce the needs of future borrowing to smooth income shocks.

Figure 5 shows how spreads change with the level of reserves as well as the level of debt. The left panel shows how spreads are increasing in the level of borrowing, and how an increase in the choice of reserves from zero to the average value of reserves reduces the level of spreads.

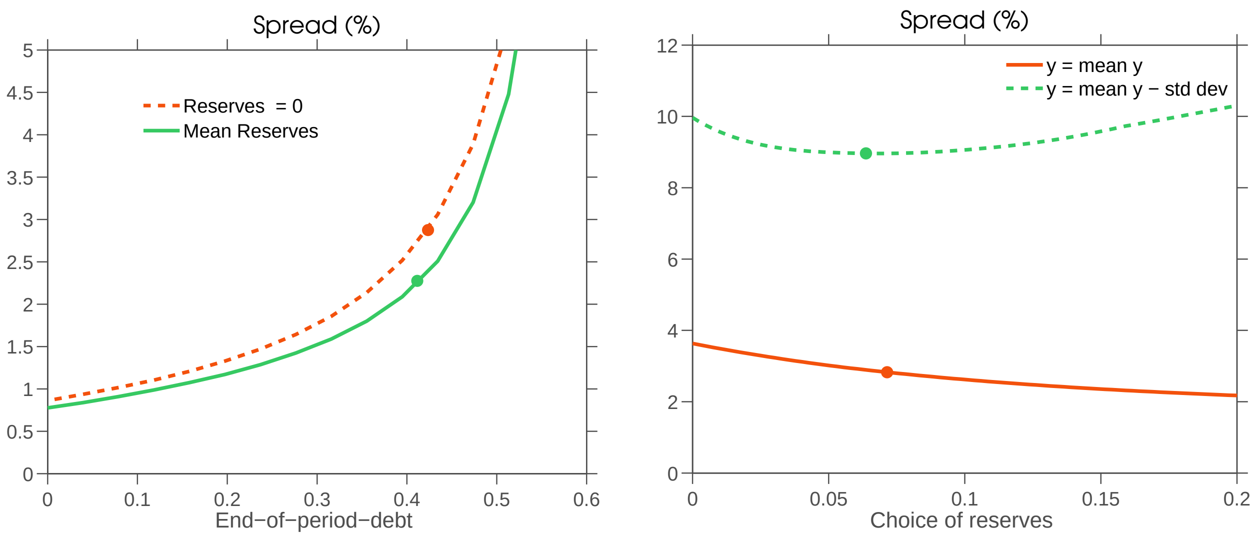

Figure 5: Effect of reserves on credit availability. The left panel presents menus of spread $(r^s(b', a', \bar{y}, 0))$ and end-of-period debt levels (b') available to a government that starts the period with the mean income and that does not face a sudden stop in the current period. Solid dots indicate optimal choices conditional on the assumed value of a'. The right panel presents the spread the government would pay if it chose the optimal borrowing level and different levels of reserves, $r^s(\hat{b}(\bar{b}, \bar{a}, y, 0), a', y, 0)$. Solid dots indicate optimal choices $(\hat{a}(\bar{b}, \bar{a}, y, 0), r^s(\hat{b}(\bar{b}, \bar{a}, y, 0), \hat{a}(\bar{b}, \bar{a}, y, 0)y, 0))$.

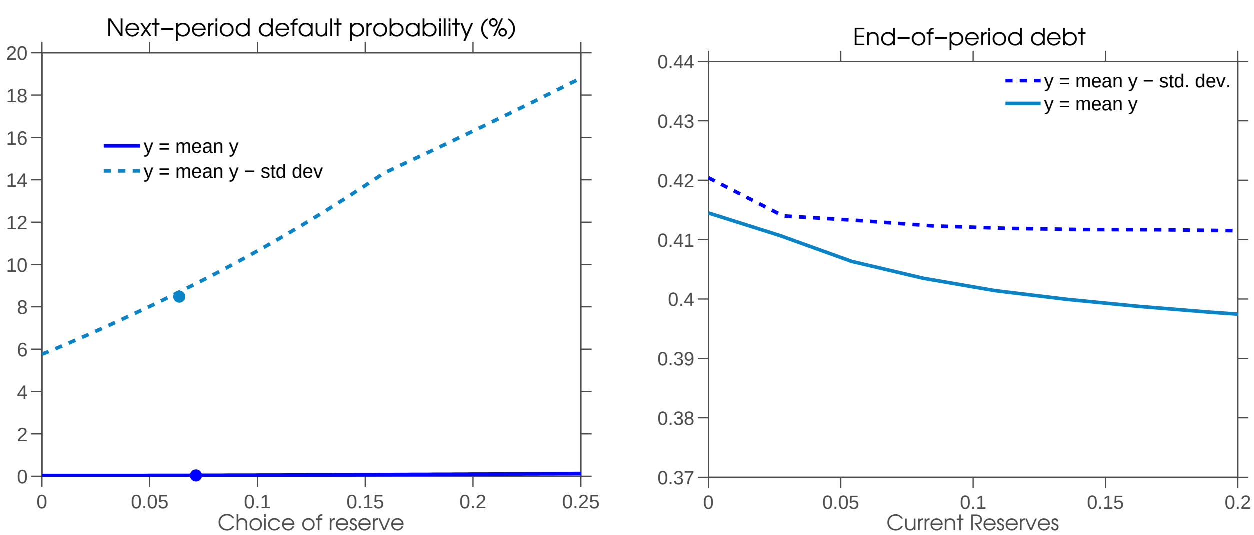

Figure 6: Effect of reserves on next-period default probability and borrowing. The left panel presents the next-period default probability $\left(Pr\left(V^D\left(b', a', y', s'\right) > V^R\left(b', a', y', s'\right) \mid y, s\right)\right)$ as a function of a' when $b' = \hat{b}\left(\bar{b}, \bar{a}, y, 0\right)$. Solid dots mark the optimal choice of reserves when initial debt and reserves levels are equal to the mean levels in the simulations $\left(\hat{a}\left(\bar{b}, \bar{a}, y, 0\right)\right)$. The right panel presents the optimal debt choice $\hat{b}\left(\bar{b}, a, y, 0\right)$ as a function of initial reserve holdings (a), assuming that the initial debt stock equals the mean debt stock in the simulations.

The right panel of Figure 5 shows the relationship between the choice of reserves and spreads conditional on a given borrowing level and different values of income. When income is at the mean value, Figure 5 shows how reserves reduce the levels of spreads. When income is one standard deviation below its mean, however, there is a non-monotonic relationship between the choice of reserves and spreads. As reserves exceed about 10 percentage points of income, spreads become increasing in reserves, reflecting the fact higher reserves makes autarchy relatively more attractive. Notice that even in the region where spreads are reduced for higher level of reserves, the probability of default in the next period may be increasing in the choice of reserves, as illustrated in the left panel of Figure 6. As explained above, this is due to a dynamic effect by which higher reserves reduce the level of future borrowing and hence reduces future default probabilities. This is illustrated in the right panel of Figure 6 that show how the end-of-period debt is decreasing in the current level of reserves.

At the optimal values of reserves, illustrated by the solid dots of Figure 5, reserves tend to lower the spreads. This implies that the costs of pre-funding (i.e., the cost of borrowing to accumulate reserves) is partially offset by the endogenous reduction in spreads.

4.4 Use of Reserves after Default

As mentioned above, a potential role for reserves is to transfer resources to default periods. We show, however, that restricting the use of reserves during defaults actually strengthens incentives to accumulate reserves. This occurs because restricting the use of reserves during default weakens the incentives to default, leading reserve accumulation to cause an even larger drop in spreads. We make this point by comparing our benchmark model with a version of the model in which reserves cannot be reduced when the government is in default. That is, in the modified model, we impose the constraint $a' \geq a$ while the government is in default.

As shown in Table 3, the demand for debt and reserves increase respectively by 9 percent and 31 percent when the government is prevented from using reserves during default. The increase in debt is due to the fact that incentives to default are weakened and this improves borrowing conditions. Reserves increase relatively more, reflecting the fact that accumulating reserves produces a larger drop in spreads. Hence, this section shows that the demand for reserves to insure against rollover risk is actually strengthened when there are limitations on the use of reserves during default.

Table 3: Restrictions on the Use of Reserves during Default Periods

	Benchmark	$a' \geq a$ in default
Mean Debt-to-GDP	46	50
Mean r_s	2.9	3.3
$\sigma(r_s)$	1.6	2.1
Mean sudden stop income cost (% annualized)	13	14
Mean Reserves-to-GDP	7.5	9.8

4.5 Role of Sudden Stops

In this subsection i) we show that introducing sudden stops is important to mimic the volatility of net capital flows observed in the data, ii) we perform a sensitivity analysis with respect to the frequency and severity of sudden stops, and iii) we perform a sensitivity analysis with respect to the modeling of sudden stops: We model sudden stops as an exogenous increase in the premium on sovereign bonds and show that our main results do not change.

4.5.1 Sudden stops and the volatility of capital flows

Figure 7 presents net capital inflows in the model simulations with and without sudden stops for a sample path without defaults. The Figure shows that only in the simulations of the model with sudden stops the volatility of net capital inflows resemble the volatility observed in the data. In particular, notice that drops in net capital inflows never decrease beyond 3 percent, whereas in the data there are much larger drops in net capital flows (see Figures C.1-C.4 in Appendix C).

4.5.2 Frequency and severity of sudden stops

Figure 8 presents simulation results obtained for different sudden-stop frequencies. The figure shows the mean values of debt and reserves for different frequencies and income losses of sudden stops. The left panel shows that higher frequencies of sudden stops generate higher

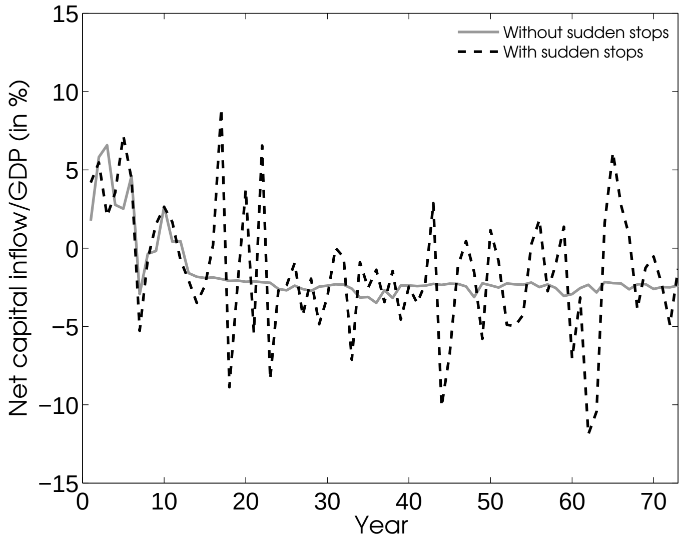

Figure 7: Yearly net capital inflows as a percentage of aggregate income in simulation paths without defaults. The government has no debt or reserves in the initial period. The sequence of income shocks is the same in the economies with and without sudden stops.

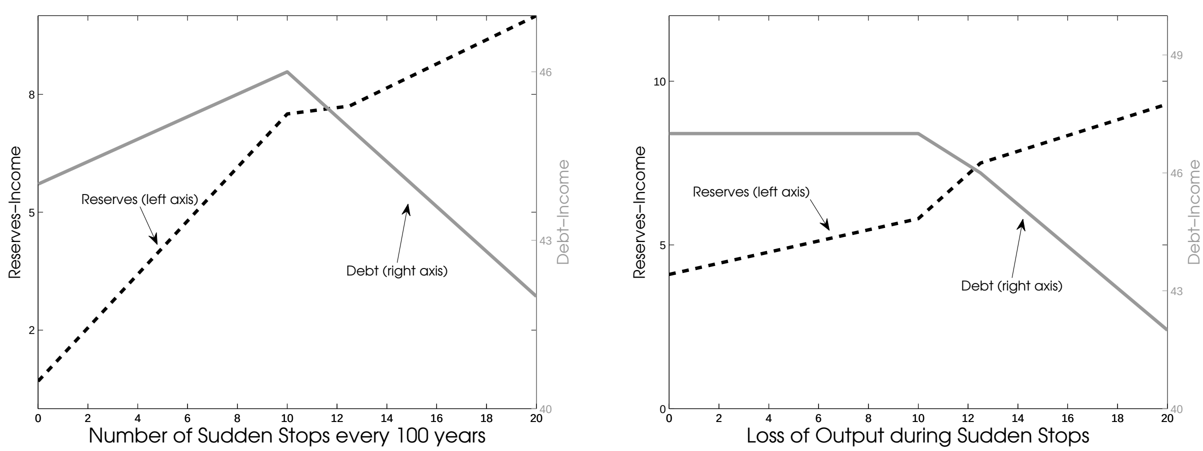

Figure 8: Mean debt and reserves for different sudden stop processes.

reserve holdings. In particular, the figure shows that sudden stops play an important role in accounting for reserve accumulations in our benchmark: Without sudden stops, reserve holdings decrease by more than 6 percentage points of income.[21] The right panel of Figure 8 presents simulation results for different magnitudes of income losses while in sudden stop.

[21]We keep the same values for the rest of the parameters when we change either the frequency or the losses in sudden stops. Notice also that for a risk aversion equal to 8 and no sudden stops, the level of reserves remains the same as in our baseline calibration.

The figure shows that for a higher sudden-stop cost, the government chooses higher reserve holdings and lower debt levels.

It has been argued that the surge of reserve holdings that started after the Asian crisis could be a result of a change in perceptions about financial globalization and the exposure to sudden stops. In particular, there was a view that sudden stops became more likely or more costly than previously perceived (see e.g. Ghosh et al., 2012). Our findings suggest that these forces could have played an important role in the observed surge of reserves.[22]

4.5.3 Alternative modeling of sudden stops

In this section, we model sudden stops as an exogenous shock to the premium on sovereign bonds, rather than as a constraint on new debt issuances. This implies that it is more costly for the government to issue debt in sudden stops periods. Formally, we remove the restriction that the government cannot issue new debt while it is in a sudden stop, and assume that the functional equation for bond prices is given by:

$$q(b',a',y,s)(1+r_b+s\xi) = \mathbb{E}_{(y',s')|(y,s)}(1-\hat{d}(b',a',y',s'))(1+(1-\delta)q(b'',a'',y',s')), \quad (5)$$

where

$$\begin{aligned} b'' &= \hat{b}(b',a',y',s') \\ a'' &= \hat{a}^R(b',a',y',s'). \end{aligned}$$

Equation (5) indicates that bondholders ask for a higher premium for holding bonds during a sudden stop. This is captured by the term $s\xi$, which implies that bondholders discount next-period bond payoffs at a higher rate during a sudden stop (when $s=1$). This is consistent with the observed behavior of sovereign spreads during sudden stops. For instance, during the Global Financial Crisis, JP Morgan EMBI Plus Sovereign Spread increased by 231 basis points from 2007 to 2008-2009, when several economies experienced sudden stops (Figures

[22] In related work, Mendoza, Quadrini and Ríos-Rull (2009) and Maggiori (2011) investigate the hypothesis that increased financial integration is behind the evolution of net foreign assets and "global imbalances" in a model with incomplete markets.

Table 4: Long-Run Statistics

	Baseline	Shocks to premia	Data
Mean Debt-to-GDP	46	43	43
Mean r_s	2.9	2.5	2.9
$\sigma(r_s)$	1.6	1.5	1.5
Mean sudden stop income cost (% annual income)	14	12	14
$\sigma(c)/\sigma(y)$	1.0	1.2	1.1
$\sigma(tb)$	1.3	1.7	1.4
$\rho(c, y)$	0.9	0.9	0.9
$\rho(r_s, y)$	−0.4	−0.8	−0.5
$\rho(r_s, tb)$	0.3	0.2	0.6
Mean Reserves-to-GDP	7.5	6.3	9.0
$\rho(\Delta a, y)$	0.4	0.4	0.4
$\rho(\Delta b, y)$	0.4	0.4	0.9
$\rho(\Delta a, r_s)$	−0.3	−0.1	−0.2

10-13 in the Appendix). This increase in the return premium was also observed in corporate debt and equity markets. The shock to the premium on sovereign bonds (5) captures this empirical feature in a simple fashion, and can be rationalized by fluctuations in the stochastic discount factor of foreign investors.

We assume that the return on reserves does not change during a sudden stop. We solve the model for $r_b = 0.0078$, and $\xi = 0.12$. These two values generate an (endogenous) average increase in the spread of 1.7 percent during sudden stops and do not affect the default-free price of sovereign bonds outside sudden stops.[23] Table 4 shows that our main results are robust to this alternative modeling of sudden stops. The government finds it optimal to

[23]The notional price of a default-free bond in a sudden stop(q^{SS}) satisfies

$$q^{SS} = \frac{1}{1 + r_b + \xi} \left[1 + (1 - \delta) \left(\psi^s q^{NSS} + (1 - \psi^s) q^{SS}\right)\right],$$

simultaneously accumulate debt and reserves, and the cyclical properties of the evolution of both variables is aligned with the data.

4.6 Reserves and Debt Duration

We next analyze the role of debt duration in determining the demand for reserves. For this purpose, we solve the baseline model assuming different values for debt duration while keeping fixed the rest of the parameters.

Figure 9 shows that the relationship between debt duration and reserve accumulation is non-monotonic. This reflects two opposing forces in the effects of debt duration on the demand for reserves. On the one hand, lower duration makes rollover risk more severe as the government needs to roll over a large amount of debt in each period, i.e., if the government does not issue new bonds due to adverse borrowing conditions, the drop in consumption to service the debt is large. As a result, reserves become more valuable as insurance against rollover risk. On the other hand, lower duration makes spreads more sensitive to gross debt levels. Hence, for low duration, it is too costly for the government to issue debt to buy reserves. Overall, we find that for low (high) values of debt duration, reserve holdings are increasing (decreasing) in debt duration.

For the case of one-period debt, the rollover risk motive for reserve accumulation disappears as the entire debt is due next period –Appendix A illustrates this in a 3 period model. There is still, however, the motive to accumulate reserves to transfer resources to default states. But as we show in section 4.4, restricting the use of reserves during default states actually increase the demand for reserves. Indeed, Table 5 shows, for the one-period bond economy both debt and reserves decreases dramatically.($\delta = 1$).[24] ,[25]

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where q^{NSS} denotes the default-free bond price in periods without sudden stops. Recall that ψ^s denotes the probability with which sudden stops end. Likewise, the default-free bond price in periods with no sudden stops satisfies

$$q^{NSS} = \frac{1}{1+r_b}\left[1 + (1-\delta)\left(\pi q^{SS} + (1-\pi)q^{NSS}\right)\right],$$

where π denotes the arrival probability of a sudden stop. We choose a value of r_b such that $q^{NSS} = 1/(r+\delta)$. This implies that the yield of a default-free long-term bond in periods with no sudden stops is the same as in one in our baseline calibration.

[24]This is consistent with the findings of Alfaro and Kanczuk (2009).

[25]Among combinations of reserves and debt levels that command a spread equal to zero, gross asset positions are undetermined: The government only cares about its net position. This is not a problem when

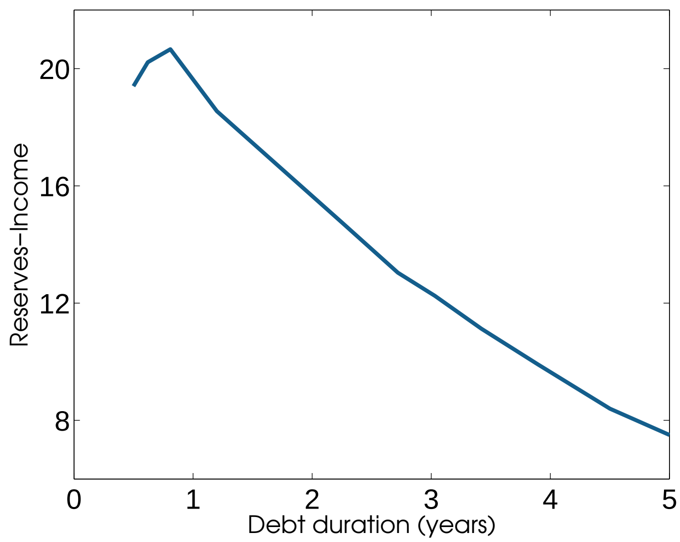

Figure 9: Average reserve-to-income ratio for different debt durations. We vary debt duration by changing the value of δ.

Table 5: Simulation Results

Average debt duration	5 years	One quarter
Mean Debt-to-GDP	46	0.7
Mean Reserves-to-GDP	7.5	0.6
Mean r_s	2.9	0
$\sigma\left(r_s\right)$	1.6	0

We would like to note that while the debt duration is exogenous in the model, this is not significantly different from the ex-ante preferred duration in the model. When we let the government choose once and for all the duration of the government debt at time 0, the government chooses a duration of 3.5, close to the duration of 5 years that we set to mimic the duration of sovereign bonds in Mexico.[26]

solving the model for our benchmark calibration because such combinations of reserves and debt levels are never optimal. However, this becomes a problem when we assume one-period bonds. In order to sidestep this problem, we solve the model with one-period bonds by allowing the government to choose only its net asset position. As indicated by the negligible mean sovereign spread in Table 5, the government chooses net asset positions that command a spread equal to zero in almost all simulation periods.

[26]Ideally, one would like to introduce a short-term bond to allow for a time-varying maturity structure, but this poses highly challenging computational issues due to the introduction of a third endogenous state variable.

4.7 Contingent Asset Payoffs

This section analyzes the benefits from saving using assets with payments contingent on the sudden-stop shock (instead of using non-contingent assets). While the development of markets for these assets may be difficult in practice, this exercise allows us to gauge the potential gains from such markets.

We solve an extension of the baseline model in which the government saves in two Arrow-Debreu securities instead of a non-contingent bond. One Arrow-Debreu security pays off in the next period only when the economy is in a sudden stop while the other security pays off in the next period only if the economy is not in a sudden stop. Formally, the government's value of repayment is now given by

$$V^R(b,a,y,s) = \max_{a'(s')\geq 0, b', c} \left\{ u\left(c\right) + \beta \mathbb{E}_{(y',s')|(y,s)} V(b', a'(s'), y', s') \right\}, \tag{6}$$

subject to

$$c = y - s\phi^s\left(y\right) - b + a + q(b', a', y, s)(b' - (1-\delta)b) - q_A(0)a'(0) - q_A(1)a'(1),$$

and if $s = 1$, $b' - (1-\delta)b \leq 0$,

where $a'(s')$ denotes the choice of securities that only pay off when the next-period sudden-stop shock takes the value s', and $q_A(s') = \frac{Pr(s'|s)}{1+r}$ denotes the equilibrium price of such security. Notice that q is now a function of the choice of the two types of securities, in addition to the choice of debt and the exogenous shocks.

The value of defaulting is given by:

$$V^D(a,y,s) = \max_{a'(s')\geq 0, c} u\left(c\right) + \beta \mathbb{E}_{(y',s')|(y,s)} \left[(1-\psi^d)V^D(a'(s'), y', s') + \psi^d V(0, a'(s'), y', s') \right],$$

subject to

$$c = y - \phi^d(y) + a - q_A(0)a'(0) - q_A(1)a'(1).$$

The bond-price function now solves the following functional equation:

$$q(b', a'(0), a'(1), y, s)(1+r) = \mathbb{E}_{(y',s')|(y,s)} \left[1 - \hat{d}(b', a'(s'), y', s') \right] \left[1 + (1-\delta) q(b'', a''(0), a''(1), y', s') \right],$$

where

$$
\begin{aligned}
b'' &= \hat{b}(b', a'(s'), y', s'), \\
a''(0) &= \hat{a}^R(b', a'(s'), y', s'), \\
a''(1) &= \hat{a}^R(b', a'(s'), y', s')
\end{aligned}
$$

denote the policy functions that solve problem (6).

Table 6 shows that sudden-stop-contingent assets lower the volatility of consumption by 13 percent. Domestic residents would experience an average welfare gain equivalent to a permanent increase in consumption of 0.28 percent if the government could save in assets with payoffs contingent on sudden-stop shocks—the welfare gain was calculated for the average debt and reserve levels observed in the simulations of the benchmark economy. These gains are consistent with the findings in Caballero and Panageas (2008). Table 6 also shows that the demand for reserves is significantly lower than the demand for state-contingent securities, on which previous studies have focused (e.g., Jeanne and Ranciere, 2011).

Table 6: Contingent Asset Payoffs

	Reserves (Benchmark)	Insurance Contracts
Mean Debt-to-GDP	46	41
Mean r_s	2.9	2.4
$\sigma\,(r_s)$	1.6	1.8
$\sigma(c)/\sigma(y)$	1.01	0.87
Welfare gain (%)	na.	0.28
Mean Reserves (% of income)	7.5	na.
Mean Claims contingent on SS (% of income)	na.	11.1
Mean Claims contingent on no SS (% of income)	na.	0.5

4.8 Role of the Endogenous and Countercyclical Spread

We now show that the endogenous and countercyclical sovereign spread plays a key role in generating demand for reserves in our model. To gauge the importance of allowing for an endogenous and countercyclical sovereign spread, we solve a version of the model without the default option. In this case income shocks do not affect the government's borrowing opportunities, which implies that there is no time-varying endogenous rollover risk associated with the possibility of default. The government continues to face sudden stops and pays a constant and exogenous spread for its debt issuances. Because of sudden stops and the presence of long-duration bonds, gross asset positions are relevant despite the lack of default risk. Formally, we solve the following recursive problem:

$$
\begin{aligned}
&W(b,a,y,s) = \max_{a' \geq 0, b', c} \left\{ u\left(c\right) + \beta \mathbb{E}_{(y',s')|(y,s)} W(b',a',y',s') \right\}, \\
&\text{subject to} \\
&c = y - s\phi^{s}\left(y\right) - b + a + q^{*}\left(b' - (1-\delta)b\right) - \frac{a'}{1+r}, \\
&b' \leq \bar{B}, \\
&b' - (1-\delta)b \leq 0 \text{ if } s = 1,
\end{aligned}
$$

where $q^* = \frac{1}{r^*+\delta}$, r^* represents the interest rate demanded by investors to buy sovereign bonds, and $\bar{B}$ is an exogenous debt limit. The values of r^* and $\bar{B}$ are chosen to replicate the mean spread and debt levels in Mexico (also targeted in our benchmark calibration). Remaining parameter values are identical to the ones used in our benchmark calibration.

Table 7 presents simulation results obtained with the no-default model. The table indicates that the endogenous source of rollover risk is important in accounting for reserve accumulation. Simulated reserve holdings decline from 7.5 percent of income in the benchmark to 2.0 percent with an exogenous and constant sovereign spread. Two factors are important for this result. First, rollover risk is lower in the no-default model because borrowing opportunities are independent from the income shock. Second, a model with the spread level observed in the data but without default overstates the financial cost of accumulating reserves financed by borrowing. In a default model, since the government always receives the return from reserve holdings but does not always pay back its debt, the financial cost of accumulating reserves financed by borrowing is lower than in a no-default model with the same spread. Moreover, Table 7 shows that the model with an exogenous constant spread fails to replicate the procyclicality of sovereign debt.

Table 7: Debt and Reserve Levels in a Model without Default and a Constant Spread.

	Benchmark	Constant exogenous spread
Mean Debt-to-GDP	46	46
Mean Reserves-to-GDP	7.5	2.0
$\rho(\Delta a, y)$	0.4	0.3
$\rho(\Delta b, y)$	0.4	-0.1
Mean r_s	2.9	2.9
$\sigma(r_s)$	1.6	0

4.9 Reserve Accumulation for Crisis Prevention

In this subsection we show how the optimal level of reserves increases when we assume reserves are useful for preventing sudden stops. This assumption is consistent with recent evidence showing that international reserves reduce the likelihood of a sudden stop (e.g. Calvo et al., 2012).[27] Following Jeanne and Ranciere (2011), we assume that the probability of a sudden stop is given by:

$$\hat{\pi}\left(\frac{a}{\varrho(b)}\right) = G\left(m - w\frac{a}{\varrho(b)}\right), \tag{7}$$

where $\varrho(b) = b\sum_{t=1}^{t=4}\frac{(1-\delta)^{t-1}}{(1+r)^t}$ denotes the level of short-term debt, i.e., debt obligations maturing within the next year, and G denotes the standard normal cumulative distribution function. Note that our benchmark calibration is a special case of equation (7) with $w = 0$. We assume that m is such that the probability of a sudden stop is 10 percent (our benchmark target) when $w = 0$.

[27]We note that the empirical evidence remains inconclusive about the effects of reserves on the likelihood of sudden stops. The relationship between reserves and the probability of a sudden stop is difficult to estimate: sudden stops are relatively rare events and the relationship between sudden stops and economic fundamentals may differ across countries.

Table 8 presents simulation results for $w \in [0, 0.15]$, which lies within the lower half of values considered by Jeanne and Ranciere (2011). As in the previous sensitivity analysis, all other parameters take the values used in our benchmark calibration. Table 8 shows that as we allow reserves to be more effective in reducing the probability of a sudden stop, optimal reserve holdings increase. In particular, when $w = 0.15$, the level of reserves reach 12.3 percentage points of GDP.

Table 8: Simulation Results when Reserves affect Probability of Sudden Stops

	$w = 0$	$w = 0.05$	$w = 0.10$	$w = 0.15$
Mean Debt-to-GDP	46	45	45	45
Mean Reserves-to-GDP	7.5	9.3	11.1	12.3
Sudden stops per 100 years	10	7.5	6.2	4.8

5 Conclusions

This paper proposes a quantitative equilibrium model to study the optimal accumulation of international reserves as a hedge against rollover risk. In the model, uncertainty about future borrowing conditions leads the government to borrow and accumulate reserves today, so as to have liquid assets in case borrowing conditions worsen. This operation of pre-funding entails costs, as the government keeps a larger gross debt position. Within a calibrated version of our model, we show that the optimal amount of reserves should be large enough to cover 16 months of coming debt obligations.

We show that the predictions of the model are consistent with key empirical regularities. In particular, the model can rationalize why governments hold simultaneously large amount of debt and reserves while paying a significant spread on the debt. Moreover, both debt and reserves are procyclical and collapse during crisis. We have also performed a battery of exercises to test the robustness of our results. Overall, a robust message from our analysis is that in the presence of volatile borrowing conditions, indebted government should hold a sizable stock of international reserves.

Looking forward, our analysis suggests several avenues for further research. Extended to a general equilibrium context, the model can shed some light on the extent to which higher demand for global safe assets can be important for explaining the low interest rate environment since the 2000s. On the normative side, our analysis suggests that it might be optimal to impose rules for the use of reserve holdings, in the spirit of constraints on gross debt and fiscal deficits that are often part of fiscal rules. In addition, the mechanisms studied in this paper could be relevant for understanding the financial decisions of households and corporate borrowers facing rollover risk.

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Appendix: For Online Publication

A Three-Period Model

We present a three-period model that allows us to show analytically the importance of rollover risk and long-duration bonds in accounting for the joint accumulation of debt and reserves. The mechanism is related to the one studied in Angeletos (2002), who shows that the joint accumulation of debt and reserves is an optimal response to smooth out the effects of distortionary taxes in a closed economy model where the government has access to a commitment technology. Here, we show that the joint accumulation of debt and reserves is an optimal response to mitigate the fall in consumption in periods with limited access to debt markets. To simplify the analysis, we consider only exogenous rollover risk in the form of "sudden-stop" shocks and abstract from endogenous rollover risk due to the possibility of default.[28]

Environment. The economy lasts for three periods $t = 0, 1, 2$. The government receives a deterministic sequence of endowments given by $y_0 = 0$, $y_1 > 0$, and $y_2 > 0$. For simplicity, the government only values consumption in period 1. The government maximizes $\mathbb{E}\left[u\left(c_1\right)\right]$, where $\mathbb{E}$ denotes the expectation operator, c_1 represents consumption in period 1, and the utility function u is strictly increasing and strictly concave.

A bond issued in period 0 promises to pay one unit of consumption goods in period 1 and $(1-\delta)$ units in period 2. Note that if $\delta = 1$, this bond becomes a one-period bond. If $\delta < 1$, we say that the government issues long-duration bonds in period 0. The per-period interest rate on borrowing is denoted by r_b. Thus, the price of a bond issued in period 0 and period 1 are respectively $q_0 = 1/(1+r_b) + (1-\delta)/(1+r_b)^2$ and $1/(1+r_b)$.

At $t = 0$, the government can accumulate reserves a that pay a return of r_a. We assume that $r_b \geq r_a$, so that the difference in returns will capture the opportunity cost of holding reserves.

[28] We can derive similar analytical results to the ones derived below with endogenous rollover risk due to the possibility of default, instead of exogenous sudden stops, but at the cost of making the analysis more complex. Intuitively, if an adverse shock today makes default a certain event in the next period, this leads to an endogenous sudden stop. This version of the model is available upon request.

The government is subject to a sudden-stop shock in period 1. When a sudden stop occurs, the government cannot borrow. A sudden stop occurs with probability $\pi \in [0, 1]$.

Let b_{t+1} denote the number of bonds issued by the government in period t. The budget constraints faced by the government for each state are:

$$a \leq q_0 b_1,$$

$$c_1(0) \leq y_1 - b_1 + a(1 + r_a) + \frac{b_2}{1 + r_b},$$

$$c_1(1) \leq y_1 - b_1 + a(1 + r_a),$$

$$b_2 \leq y_2 - (1 - \delta) b_1,$$

where $c_1(0)$ denotes the government's consumption in period 1 when the government is not facing a sudden stop and $c_1(1)$ denotes consumption in period 1 during a sudden stop.

Results Without rollover risk, the government would consume its entire wealth at $t = 1$, i.e., $c_1^\star = y_1 + y_2/(1 + r)$. This can be done by borrowing in period 1 so that the government transfers resources from period 2 to period 1. However, a sudden stop may prevent the government from borrowing in period 1, forcing the government to cut down consumption. The next proposition describes how the government can use reserves and debt to smooth consumption between both period 1 states (with and without a sudden stop).

Proposition 1 (Optimal Reserve Holdings)

i. If there is no rollover risk ($\pi = 0$) and $r_a = r_b$, gross asset positions are undetermined. In particular, the optimal allocation $c^\star$ can be attained without reserves ($a^\star = 0$).

ii. If there is no rollover risk ($\pi = 0$) and $r_a < r_b$, optimal reserves are zero ($a^\star = 0$).

iii. If the government can only issue one-period debt ($\delta = 1$) and $r_a = r_b$, gross asset positions are undetermined. In particular, the optimal allocation can be attained without reserve accumulation ($a^\star = 0$).

iv. If the government can only issue one-period debt ($\delta = 1$) and $r_a < r_b$, optimal reserves are zero ($a^\star = 0$).

v. If $\pi > 0$ and $\delta < 1$, the government accumulates reserves in period 0 ($a^\star > 0$) if and only if

$$\pi \left[q_0(1+r_a) - 1\right] u'(y_1) > (1-\pi)\left[\frac{1-\delta}{1+r_b} + 1 - q_0(1+r_a)\right] u'\left(y_1 + y_2(1+r_b)^{-1}\right), \tag{A.8}$$

Moreover, if $r_a = r_b$, the optimal allocation $c^\star$ can be attained.

Proof. Suppose there is no rollover risk. The optimal allocation is such that $c_1 = y_1 + y_2(1+r_b)^{-1}$. If $r_a = r_b$ (point 1 of Proposition 1), any combination of debt issuances and reserve holdings such that $b_1 q_0 = a$ and $b_2 = y_2 - (1-\delta)b_1$ attain the optimal allocation. In particular, the optimal allocation can be attained without reserve accumulation ($a = b_1 = 0$, and $b_2 = y_2$).

If $r_a < r_b$ and there is no rollover risk (point 2 of Proposition 1), the government can only attain the optimal allocation if it chooses to not accumulate reserves. Let us consider any levels of period-0 savings and borrowing $\hat{a} = \hat{b}_1 q_0 > 0$. It is easy to show that the government can do better choosing $a = b_1 = 0$. Since $r_a < r_b$, $\hat{a}(1+r_a) < \hat{b}_1[1+(1-\delta)(1+r_b)]^{-1}$. Therefore, the level of period-1 consumption is higher with $b_1 = a = 0$ than with $\hat{a} = \hat{b}_1 q_0 > 0$, and $\hat{a} = \hat{b}_1 q_0 > 0$ cannot be part of an equilibrium.

Suppose the government can only issue one-period debt and $r_a = r_b$ (point 3 of Proposition 1). Since $q_0 = (1+r_a)^{-1}$, $c_1 = y_1 + b_2(1+r_b)^{-1}$ for all possible equilibrium borrowing and saving choices satisfying $b_1 q_0 = a$. Then, gross asset positions are undetermined and the optimal allocation can be attained without reserve accumulation ($a = b_1 = 0$).

Suppose now the government can only issue one-period debt and $r_a < r_b$ (point 4 of Proposition 1). Let us consider any levels of period-0 savings and borrowing $\hat{a} = \hat{b}_1 q_0 > 0$. Then, period-1 consumption is given by $c_1 = y_1 + b_2(1+r_b)^{-1} + \hat{b}_1(1+r_b)^{-1}(1+r_a) - \hat{b}_1 < y_1 + b_2(1+r_b)^{-1}$. Therefore, the level of period-1 consumption would be higher if the government chooses $a = b_1 = 0$, and $\hat{a} = \hat{b}_1 q_0 > 0$ cannot be part of an equilibrium.

Next, we show that with $\pi > 0$ and $\delta > 1$, condition (A.8) is necessary and sufficient for reserve accumulation (point 5 of Proposition 1). Since it is optimal for the government to choose $b_1 q_0 = a$, the government's maximization problem can be written as:

$$\max_{b_1} \left\{ \pi u(y_1 + b_1 q_0(1+r_a) - b_1) + (1-\pi) u \left(y_1 + b_1 q_0 (1+r_a) - b_1 + \frac{y_2 - (1-\delta) b_1}{1+r_b} \right) \right\}.$$

Since the objective function maximized by the government is concave, the first-order necessary and sufficient condition of the government's problem is given by:

$$\begin{aligned} &\pi \left[q_0(1+r_a) - 1 \right] u'(y_1 + b_1^\star q_0 (1+r_a) - b_1^\star) \leq \\ &(1-\pi) \left[\frac{1-\delta}{1+r_b} + 1 - q_0(1+r_a) \right] u' \left(y_1 + b_1^\star q_0(1+r_a) - b_1^\star + \frac{y_2 - (1-\delta) b_1^\star}{1+r_b} \right), \end{aligned} \tag{A.9}$$

with equality if $b_1^\star > 0$. Condition (A.8) states that the left-hand side of condition (A.9) is higher than the right-hand side of condition (A.9) when evaluated at $b_1 = 0$. Therefore, if condition (A.8) holds, $a = b_1 = 0$ cannot be part of an equilibrium. Suppose now $b_1^\star > 0$ and, therefore, condition (A.9) holds with equality. Then, the concavity of the objective function maximized by the government implies that condition (A.8) holds.

■

Proposition 1 states that there is a fundamental role for reserves only in the presence of both rollover risk and long-duration bonds. Without rollover risk, there is no need for reserve accumulation: the government can always transfer resources from period 2 to period 1 directly. However, if there is a sudden stop in period 1, the government cannot borrow in that period. Therefore, the government may benefit from issuing long-duration bonds to transfer resources from period 2 to period 0, and then transfer period 2 resources from period 0 to period 1 using reserves. This mechanism is not at work with one-period debt, because the government cannot improve its period 1 net asset position by issuing debt and accumulating reserves in period 0. In fact, if $r_a < r_b$, the government's period 1 net asset position is lower if it issues one-period debt and accumulates reserves in period 0.

With rollover risk and long-duration bonds, the government accumulates reserves if the benefits from hedging against the risk of a sudden stop are high enough to compensate for the financial cost of financing reserve accumulation with debt issuances. The government wants to transfer resources from period 2 to period 1. There are two ways of doing this: (i) borrowing in period 0 to transfer resources from period 2 to period 0 and then accumulating reserves to transfer resources from period 0 to period 1; and (ii) borrowing in period 1. If $r_a < r_b$, option (i) has a financial cost. If there are sudden stops, option (ii) is risky because the government may not be able to borrow in period 1. Another way of thinking about this tradeoff is that option (i) is the best option for transferring resources to the sudden-stop state in period 1 (for which option (ii) is ineffective), while option (ii) is the best option for transferring resources to the no-sudden-stop state in period 1 (for which option (i) is more expensive if $r_a < r_b$). Thus, when accumulating reserves instead of borrowing in period 1, the government increases consumption in the sudden-stop state in period 1 at the expense of lowering consumption in the no-sudden-stop state in period 1. The left-hand side of condition (A.8) represents the expected marginal utility gain from increasing consumption in the sudden-stop state in period 1, while the right-hand side represents the expected marginal utility cost from lowering consumption in the no-sudden-stop state in period 1.

Notice that the financial cost of issuing debt to finance reserve accumulation appears only if $r_a < r_b$. Thus, with $r_a = r_b$, rollover risk, and long-duration bonds, condition (A.8) always holds. With one-period debt ($\delta = 1$), $r_a < r_b$ implies that $q_0(1 + r_a) < 1$. Therefore, the left-hand side of condition (A.8) is always lower than the right-hand side and optimal reserves are zero.

B Additional Robustness Exercises

B.1 Growth and reserve accumulation

In this appendix we incorporate growth to the aggregate income process. Namely, we assume that the aggregate income process is characterized by

$$y_t = \Gamma_t e^{z_t},$$

where z denotes the temporary income shock. The variable z follows a first order autoregressive process:

$$\log(z_t) = (1-\rho)\,\mu + \rho \log(z_{t-1}) + \varepsilon_t.$$

The trend component Γ evolves deterministically:

$$\Gamma_t = \Gamma_{t-1} e^{\mu_g},$$

where μ_g denotes the constant growth rate. We solve the detrended problem. Formally, the value function of a government that is not currently in default satisfies the following functional equation:

$$V(b,a,z,s) = \max\left\{V^R(b,a,z,s), V^D(a,z,s)\right\},$$

where the government's value of repaying is given by

$$V^R(b,a,z,s) = \max_{a' \geq 0, b', c} \left\{u\left(c\right) + \beta e^{\mu_g (1-\gamma)} \mathbb{E}_{(z',s')|(z,s)} V(b',a',z',s')\right\},$$

subject to

$$c = e^z - s\phi^s\left(e^z\right) - b + a + q(b',a',z,s)(b'e^{\mu_g} - (1-\delta)b) - \frac{a'e^{\mu_g}}{1+r},$$

and if $s = 1$, $b'e^{\mu_g} - (1-\delta)b \leq 0$.

Table B.1: Parameter Values.

Autocorrelation coefficient of temporary component	ρ	0.94
Standard deviation of innovations to temporary component	σ_ϵ	1.5%
Mean log of temporary component	μ	$(-1/2)\sigma_\epsilon^2$
Growth rate	μ_g	0.006
Discount factor	β	0.9958
Income cost of defaulting	d_0	-0.92352
Income cost of defaulting	d_1	1.08284
Income cost of sudden stops	λ	0.7

The value of defaulting is given by:

$$V^D(a,z,s) = \max_{a'\geq 0,c} u\left(c\right) + \beta e^{\mu_g(1-\gamma)} \mathbb{E}_{(z',s')|(z,s)} \left[(1-\psi^d)V^D(a',z',s') + \psi^d V(0,a',z',s')\right],$$

subject to

$$c = e^z - \phi^d(e^z) + a - \frac{a' e^{(\mu_g)}}{1+r}.$$

We assume that the cost of defaulting and the output loss during sudden stops are proportional to the trend component, which facilitates removing the trend component. That is, the output level while the economy is in default equals

$$y_t = \Gamma_t \left(e^z - \phi^d(e^z)\right).$$

The model was recalibrated to mimic targets for debt, spread, and the output cost of sudden stops. This implies that the two parameters governing the cost of default (d_0 and d_1), the discount factor β, and the output fall during sudden stops were recalibrated. The growth rate μ_g corresponds to the average growth rate in Mexico between 1980 and 2011. The parameters governing the temporary component of the income process are the same as the parameters governing the income process in the baseline economy. All remaining parameters take the same values that they take in our baseline parameterization.

Table B.2: Long-Run Statistics

	Baseline calibration	Baseline without growth	Calibration with growth	Data
Mean Debt-to-GDP	46	53	43	43
Mean r_s	2.9	3.6	2.9	2.9
$\sigma(r_s)$	1.6	1.9	1.4	1.5
Mean sudden stop income cost	14	14	12	14
$\sigma(c)/\sigma(y)$	1.0	1.1	1.0	1.1
$\sigma(tb)$	1.3	1.5	1.6	1.4
$\rho(c, y)$	0.9	0.9	0.9	0.9
$\rho(r_s, y)$	-0.4	-0.5	-0.5	-0.5
$\rho(r_s, tb)$	0.3	0.2	-0.1	0.6
Mean Reserves-to-GDP	7.5	6.6	7.3	9.0
$\rho(\Delta a, y)$	0.4	0.4	0.4	0.4
$\rho(\Delta b, y)$	0.4	0.4	0.4	0.9
$\rho(\Delta a, r_s)$	-0.3	-0.2	-0.3	-0.2

Note: The standard deviation of x is denoted by $\sigma(x)$. The coefficient of correlation between x and z is denoted by $\rho(x, z)$. Changes in debt and reserves levels are denoted by Δa and Δb, respectively. Moments are computed using detrended series using a linear trend.

Table B.2 shows that the optimal reserve balance in the economy with growth takes a similar value to the one it takes in our baseline calibration.

B.2 Term Premium

We study the robustness of our results to the inclusion of a term premium compensation. We depart from the baseline model and assume that reserves earn a return that is strictly lower than the investors' opportunity cost of lending to the government.

Following Jeanne and Ranciere (2011), we set the term premium equal to 1.5. That is, we set that the return on reserves to 0.63 percent per quarter while the risk-free rate is kept at 1 percent. As Table B.3 shows, the demand for reserves is reduced but remains significant.

Table B.3: Simulation Results with a Lower Return for Reserves.

	Benchmark	Lower return for reserves
Mean Debt-to-GDP	46	44
Mean r_s	2.9	2.5
$\sigma(r_s)$	1.6	1.6
Mean sudden stop income cost (% annualized)	13	13
Mean Reserves-to-GDP	7.5	5.0

B.3 Discount Factor and Risk Aversion

Table B.4 presents additional robustness exercises. First, this table shows that higher degrees of risk aversion are associated with higher debt and reserves. Intuitively, a higher aversion to consumption volatility strengthens the role of reserves as an insurance instrument. Second, increasing the discount factor increases the level of reserves even though it lowers the level of debt. This occurs because a more patient government cares more about the future and hence buy more reserves as insurance.

Table B.4: Additional Robustness Exercises

	Mean Debt-to-GDP	Mean Reserves-to-GDP
Risk Aversion		
$\gamma = 2$	42	2.5
$\gamma = 3$	44	5.0
$\gamma = 4$	46	7.5
$\gamma = 5$	47	9.9
$\gamma = 8$	51	17.1
Discount factor		
$\beta = 0.974$	46	7.5
$\beta = 0.977$	44	9.1
$\beta = 0.98$	42	10.7

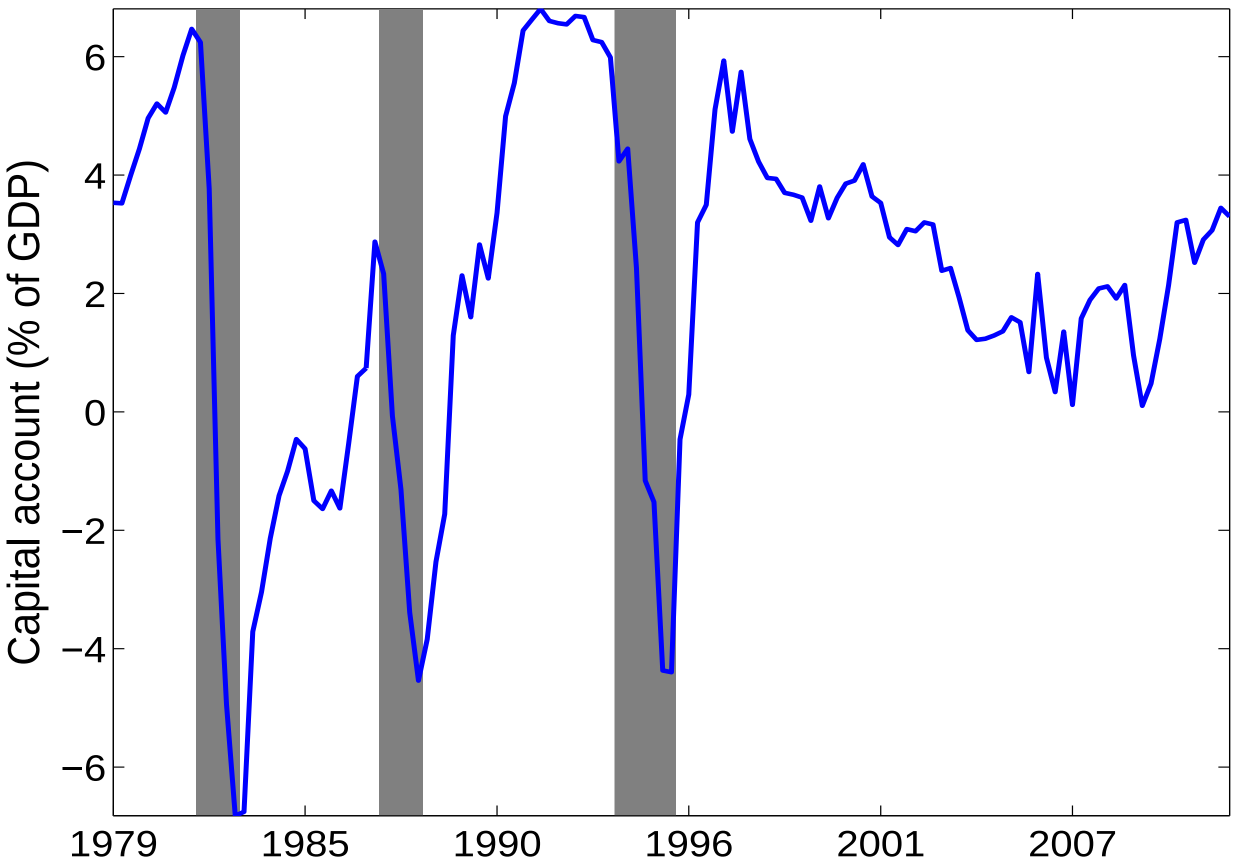
C Sudden Stops identified in the data

Table C.1: Sudden-Stop Episodes.

Argentina	1989, 2001
Bolivia	1980, 1982, 1994
Botswana	1977, 1987, 1991, 1993, 2001, 2003, 2010
Brazil	1983
Bulgaria	1990, 1994, 1996, 1998, 2003, 2008
Chile	1982, 1985, 1991, 1995, 1998, 2009
China, P.R.	
Colombia	
Costa Rica	2009
Czech Republic	1996, 2003
Dominican Republic	1993, 2003
Ecuador	1979, 1986, 1988, 1992, 1999, 2006
Egypt	1990, 1993
El Salvador	1979, 1986, 2005, 2009
Guatemala	
Honduras	2008
Hungary	1994, 1996, 2009
Jamaica	1983, 1985, 1988, 2002, 2009
Jordan	1976, 1979, 1984, 1989, 1992, 1998, 2001, 2008, 2010
Korea, Republic of	1986, 1997, 2008
Malaysia	1984, 1987, 1994, 1999, 2005, 2008
Mexico	1982, 1988, 1995
Morocco	1978, 1995
Paraguay	1988, 1995, 2002
Peru	1983, 1998, 2009
Philippines	1983, 1997, 2000
Poland	1981, 1988, 1990
Romania	1981, 1988, 2008
South Africa	1985
Sri Lanka	
Thailand	1982, 1997, 2009
Tunisia	
Turkey	2001
Uruguay	1982, 2002, 2004, 2007, 2009

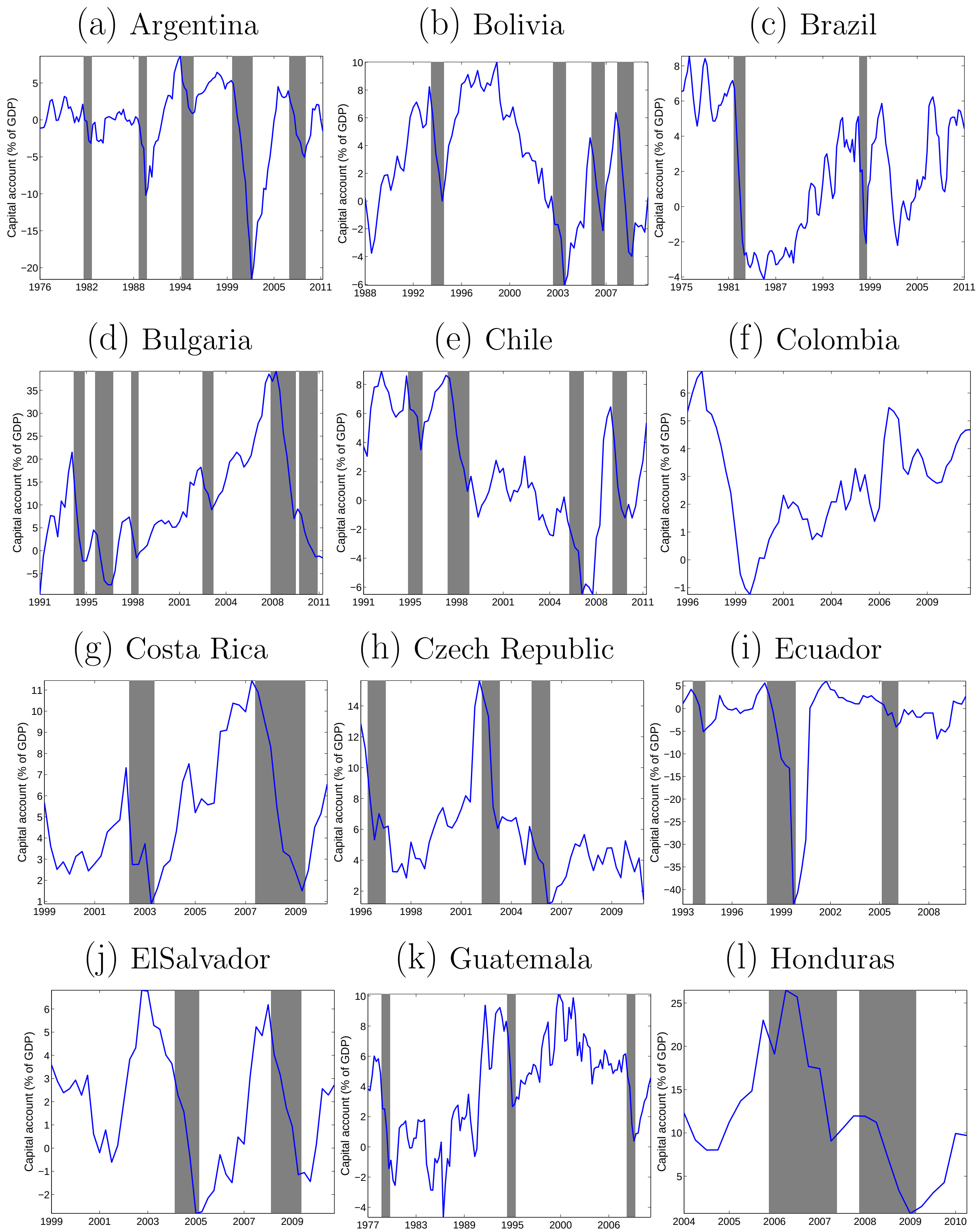
Note: Sudden-stop episodes correspond to years in which the ratio of net capital inflows to GDP falls by more than 5 percentage points. Source: IMF's International Financial Statistics annual data from 1970 to 2011

Figure C.1: Sudden Stops in Mexico



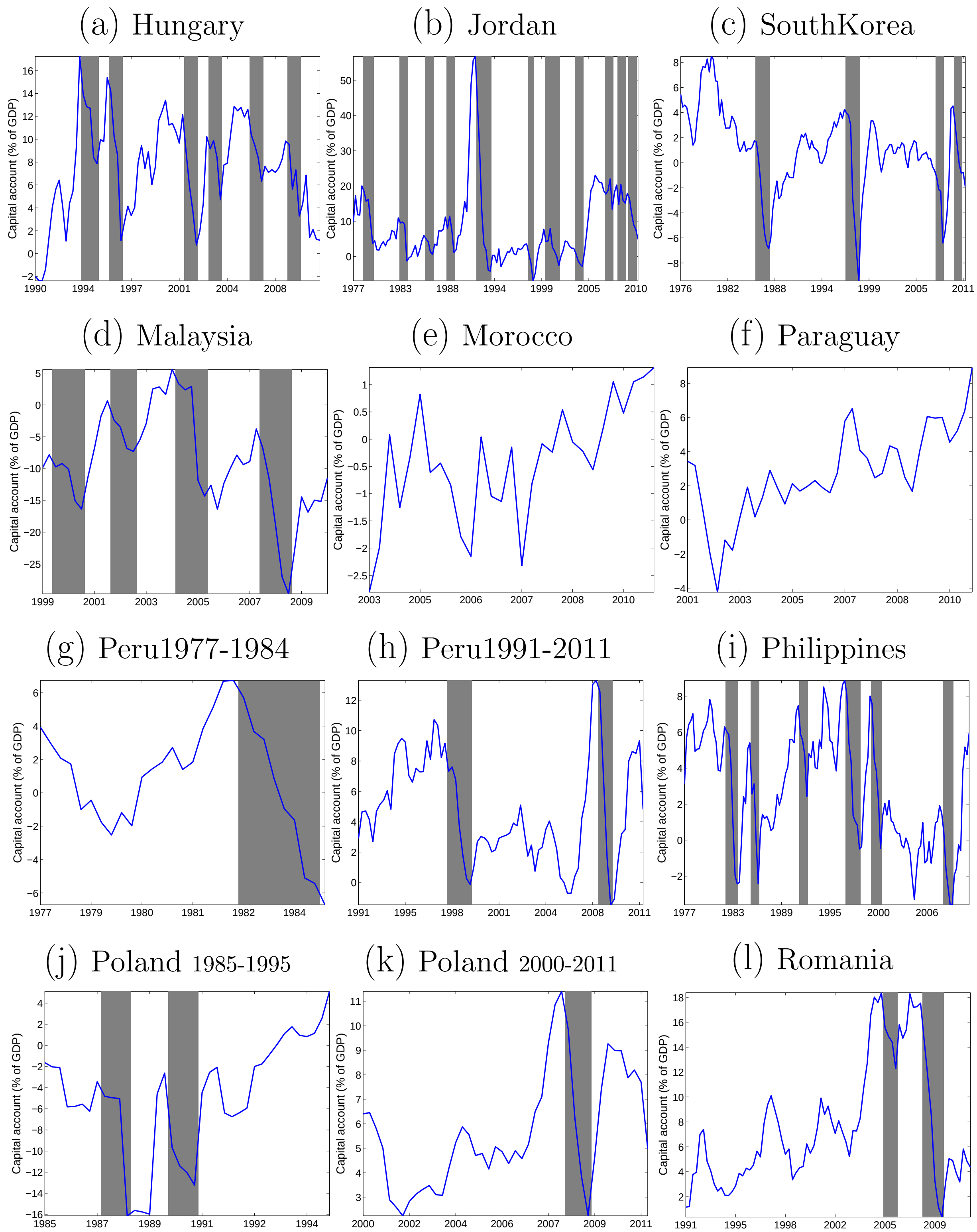
Mexico: Ratio of cumulated net capital inflows over the last four quarters to cumulated GDP over the last four quarters. Shaded areas describe periods of sudden stops.

Figure C.2: Sudden Stops



Ratio of cumulated net capital inflows over the last four quarters to cumulated GDP over the last four quarters. Shaded areas describe periods of sudden stops

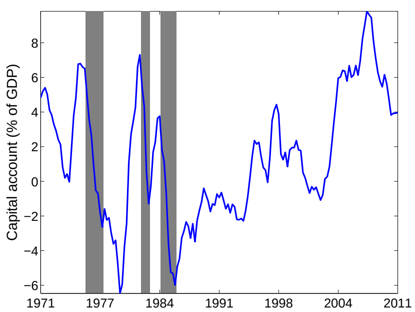
Figure C.3: Sudden Stops



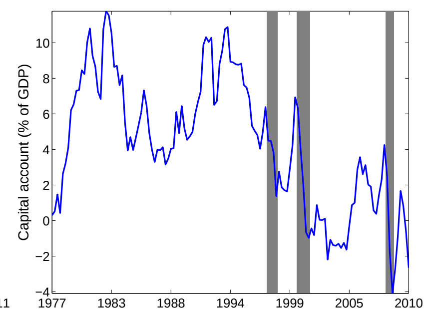
Ratio of cumulated net capital inflows over the last four quarters to cumulated GDP over the last four quarters. Shaded areas describe periods of sudden stops

Figure C.4: Sudden Stops

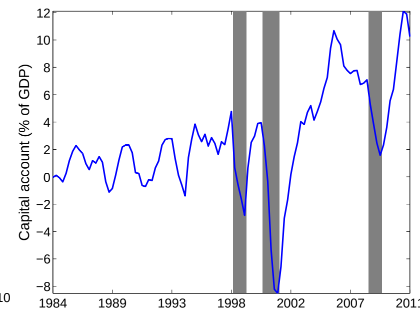
(a) SouthAfrica



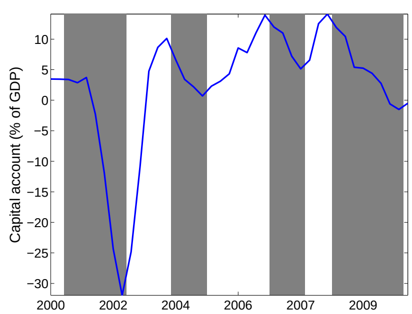
(b) Sri Lanka



(c) Turkey



(d) Uruguay



Ratio of cumulated net capital inflows over the last four quarters to cumulated GDP over the last four quarters. Shaded areas describe periods of sudden stops