Indeterminacy in Sovereign Debt Markets: an Empirical Investigation*

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Abstract

How important was non-fundamental risk in driving interest rate spreads during the euro-area sovereign debt crisis? To answer this question, we consider a model of sovereign borrowing with three key ingredients: multiple debt maturities, risk averse lenders and coordination failures *á la* Cole and Kehoe (2000). In this environment, lenders' expectations of a default can be self-fulfilling, and market sentiments contribute to variation in interest rate spreads along with economic fundamentals. We show that the joint distribution of interest rate spreads and debt duration provides information to distinguish between fundamental and non-fundamental sources of default risk. We calibrate the model to match the empirical distribution of Italian sovereign spreads and debt duration. The process for the lenders' stochastic discount factor, a key imput in our analysis, is estimated using state of the art asset pricing techniques. Our preliminary results indicate that the rise in Italian interest rate spreads over the 2011-2012 period was mostly the result of high risk premia and bad economic fundamentals, with a limited role played by non-fundamental uncertainty. We show how this information is critical to understand the implications of the OMT program announced by the ECB.

1 Introduction

The summer of 2012 marked one of the most spectacular developments of the Eurozone sovereign debt crisis. After a period of sharp increases, in August 2012, interest rate spreads in the euro area periphery declined to almost their pre-crisis level. These declines have been attributed to the establishment of the Outright Monetary Transaction (OMT) program, a framework that gives the ECB powers to purchase sovereign bonds in order to prop up their prices. One reading of these events is that the establishment of the OMT program was successful in dealing with coordination failures among bondholders and decreased the likelihood of self-fulfilling debt crises. By promising to act as a lender of last resort, the argument goes, the ECB reduced the scope for these confidence driven fluctuations, bringing back bond prices to the value justified by economic fundamentals.

This is not, however, the only interpretation. Indeed, the high interest rate spreads observed in Europe could have purely been the results of poor economic conditions. A credible announcement by the ECB to sustain prices in secondary bond markets would still produce a decline in interest rate spreads, because it would eliminate downside risk for bondholders. However, it would also generate inefficient moral hazard. Unlike the coordination failure view, this moral hazard view implies excessive borrowing by governments and the potential of balance sheet risk for the ECB. Therefore, any assessment of these interventions needs first to address a basic question: were interest rate spreads in the euro-area periphery the result of confidence driven fluctuations, or were they due to bad economic fundamentals? This paper takes a first step toward answering this question by bringing a benchmark model of sovereign borrowing with self-fulfilling debt crisis to the data and applying it to the debt crisis in the euro-area.

We consider the canonical model of strategic sovereign borrowing in the tradition of Eaton and Gersovitz (1981) and Arellano (2008). In our environment, a government issues debt of different maturities in order to smooth out endowment risk. We follow Cole and Kehoe (2000) and assume that the government cannot commit to repaying its debt within the period. This opens the door to self-fulfilling debt crises: if lenders expect the government to default and do not buy new bonds, the government may find it too costly to service the entire stock of debt coming due, thus validating the expectations of lenders. This can happen despite the fact that a default would not be triggered if lenders held more optimistic expectations about the government's willingness to repay. These rollover crises can arise in the model when the stock of debt coming due is sufficiently large and/or economic fundamentals are sufficiently weak.

As commonly done in the literature, we assume that this indeterminacy in the *crisis*

zone is resolved by the realization of a coordination device.[1] Our selection rule consists of a Markov process for the probability that lenders coordinate on the bad equilibrium when the economy falls in the crisis zone. Conditional on this selection rule, the equilibrium is unique. In this set up, default risk varies over time because of "fundamental" and "non-fundamental" uncertainty. More specifically, default risk may be high because lenders expect the government to be insolvent in the near future- they expect that the government will default irrespective of their behavior. Or, it may be high because of the expectation of a future inefficient rollover crisis. The goal of our exercise is to distinguish these different sources of default risk.

The first contribution of this paper is to point out that the joint behavior of interest rate spreads and debt duration provides key information to accomplish this goal. Our argument builds on two key properties of canonical sovereign default models. First, if default risk reflects the expectation of a future rollover crisis, then the government has incentives to *lengthen* the maturity of its debt: by doing so, it reduces the amount of debt that needs to be serviced in the near future and it decreases the prospect of a self-fulfilling debt crisis. Second, if default risk is purely the result of bad economic fundamentals, the government has incentives to *shorten* the maturity of its debt. This is due to the combination of two effects. On the one hand, as emphasized by Arellano and Ramanarayanan (2012) and Aguiar and Amador (2014), short term debt is less prone to dilution: by shortening the duration of his debt, the government gains some commitment, obtaining better terms from lenders when issuing debt. On the other hand, Dovis (2014) shows that the need to hold long term debt for insurance reasons falls when default risk increases. These two properties imply that fundamental and non-fundamental sources of default risk predict a different comovement pattern between interest rate spreads and debt duration: the two variables are positively associated when rollover risk is an important driver of interest rate spreads, they are negatively correlated otherwise. Their joint behavior during a debt crisis is therefore very informative about the underlying sources of default risk.

The second contribution of our paper is to make this insight operational. Indeed, a key problem in using these identifying restrictions is that the relationship between interest rate spreads and debt duration is not only a product of government's incentives, but also depends on the lenders' attitude toward risk. Broner et al. (2013) show that sovereign debt crisis are typically accompanied by significant increase in term premia. Neglecting these shifts could undermine our identification strategy: rollover risk could be an important driving force for interest rate spreads and yet we could observe a shortening of debt

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[1]We use the term *crisis zone* to indicate the region of the state space where self-fulfilling crises are possible.

duration simply because lenders demand high compensation to hold long term risky bonds. We deviate from most of the quantitative literature on sovereign debt and allow for time-varying risk premia for the lenders. We build on the work of Borri and Verdhelan (2013) who study a sovereign debt model where lenders have time-varying risk aversion *á la* Campbell and Cochrane (1999). However, we consider a more flexible specification of the external habit model that allows for time-variation in term premia (Wachter, 2006; Bakaert et al., 2009).

We apply our framework to the recent sovereign debt crisis in Italy. First, we estimate the parameters of the lenders' stochastic discount factor using the term structure of German zero coupon bonds, stock prices and consumption data in the euro area. Implicit in our approach is the assumption that financial markets in euro-area are sufficiently integrated, and that our lenders price other assets beside Italian government securities. Conditional on this empirically plausible pricing kernel, we then calibrate the parameters of the government decision problem by matching moments of the joint distribution of output, interest rate spreads and debt duration for the Italian economy.

We next measure the importance of non-fundamental risk in driving Italian spreads during the recent sovereign debt crises. Using a preliminary calibration, we apply a filter to our model and we estimate the path of the model's state variable over our sample. Given this path, we are able to decompose observed interest rate spreads into a component reflecting the expectation of a future rollover crisis, a component reflecting the expectation of a future solvency crisis and a risk premium component. We document that rollover risk was an important driver of Italian sovereign spread during their run up in the summer of 2011, explaining roughly 30% of their variation. However, its role became negligible during the first half of 2012: a combination of high risk premia and bad domestic fundamentals can account for the bulk of interest rate spreads at the height of the debt crisis.

Finally, we show how our results can be used to understand the implications of the OMT announcements. We model OMT as a price floor schedule implemented by the ECB. These interventions can eliminate the possibility of rollover crises and they do not require the ECB to ever intervene in bond markets along the equilibrium path, resulting in a Pareto improvement. Our question is whether the ECB followed this benchmark. To test for this hypothesis, we use the model to construct a fundamental value for the Italian spread- the value that would prevail in the absence of coordination failures- and we compare it with the actual spread observed after the ECB announcement. We document that this counterfactual fundamental spread is roughly 150 basis points higher than the actual interest rate spread observed in the second half of 2012. This result indicates that

the sharp decline in interest rate spreads observed after these announcements to a large extent reflects a prospective subsidy offered by the ECB to the peripheral countries.

Literature review (to be completed). This paper contributes to the literature on multiplicity of equilibria in sovereign debt model. Previous works in this area like Alesina et al. (1989), Cole and Kehoe (2000), Calvo (1988), and Lorenzoni and Werning (2013) have been qualitative in nature. More recently, Conesa and Kehoe (2012), Roch and Uhlig (2014), Stangebye (2014) and Navarro et al. (2015) considered more quantitative models with multiplicity. The main contribution of our paper is to conduct a formal assessment of the importance of rollover risk in accounting for the interest rate spreads during the recent Eurozone crisis.[2] The main innovation relative to the existing literature is our identification strategy based on the comovement between duration of debt and interest rate spreads.

This paper contributes to the quantitative literature on sovereign debt. Papers that are related to our work include Arellano and Ramanarayanan (2012), Chatterjee and Eyigungor (2013), Hatchondo and Martinez (2009), Bianchi et al. (2014) and Borri and Verdhelan (2013). Relative to the existing literature, this is the first paper to jointly consider a model with rollover risk, endogenous maturity choice and risk aversion on the side of the lenders. These three ingredients are necessary for our purposes (explain).

Our analysis of OMT-like policies is related to Roch and Uhlig (2014) and Corsetti and Dedola (2014). These papers show that such policies can eliminate self-fulfilling debt crisis when appropriately designed. We contribute to this literature by using our calibrated model to test whether the drop in interest rates observed after the announcement of OMT is consistent with the implementation of such policy or whether it signals a prospective subsidy paid by the ECB.

Finally, our paper is related to the literature on the empirical analysis of indeterminacy in macroeconomic models. Discuss Jovanovic (1989), Farmer and Guo (1995) Lubik and Schorfheide (2004). The closest in methodology is Aruoba et al. (2014). Explain relation to Passadore and Xandri (2014).

Layout. The paper is organized as follows. Section 2 presents the model. Section 3 discusses our identifying restrictions within the context of a simple three period model. Section 4 presents the calibration of our model and discusses some of the shortcomings of our current implementation. Section 5 presents our filtering exercise and reports our

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[2]There is also a reduced form literature that addresses this issue, see for instance De Grauwe and Ji (2013).

decomposition of Italian spreads. Section 6 analyzes the OMT program. Section 7 concludes.

2 Model

2.1 Environment

Preferences and Endowments: Time is discrete, $t \in \{0, 1, 2, ...\}$. The exogenous state of the world is $s_t \in S$. We assume that s_t follows a Markov process with transition matrix $\mu(\cdot|s_{t-1})$. The exogenous state has two types of variables: *fundamental*, $s_{1,t}$, and *non-fundamental*, $s_{2,t}$. The fundamental states are stochastic shifters of endowments and preferences while the non-fundamental states are random variables on which agents can coordinate. These coordination devices are orthogonal to fundamentals.

The economy is populated by lenders and a domestic government. The lenders value flows according to the stochastic discount factor $M(s_t, s_{t+1})$. Hence the value of a stochastic stream of payments $\{d\}_{t=0}^{\infty}$ from time zero perspective is given by

$$\mathbb{E}_0 \sum_{t=0}^{\infty} M_{0,t} d_t, \tag{1}$$

where $M_{0,t} = \prod_{r=0}^{t} M_{r-1,r}$.

The government receives an endowment (tax revenues) $Y_t = Y(s_t)$ every period and decides the path of spending G_t. The government values a stochastic stream of spending $\{G_t\}_{t=0}^{\infty}$ according to

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t U(G_t), \tag{2}$$

where the period utility function U is strictly increasing, concave, and it satisfies the usual assumptions.

Market Structure: The government can issue non-contingent defaultable bonds to lenders in order to smooth fluctuations in G_t. For simplicity, we assume that he can issue bonds of only two maturities: a short term bond, $B_{S,t}$, and a long term bond, $B_{L,t}$. We will denote the portfolio of debt outstanding by $B_t = (B_{S,t}, B_{L,t})$. For the long term bond, we follow Leland and Toft (1996) and consider a security that pays $\lambda^t \iota$ unit of the numeraire t periods ahead if the government has not defaulted in the meantime. The parameter $\lambda \in [0, 1]$ captures the duration of the security: if $\lambda = 0$ the security is a one period zero coupon bonds, while it is a consol when $\lambda = 1$. We denote the price of the

short and long term bond by $q_{S,t}$ and $q_{L,t}$ respectively and we let $q_t = (q_{S,t}, q_{L,t})$.

The timing of events within the period follows Cole and Kehoe (2000): the government issues a new amount of debt, lenders submit their pricing schedule, and finally the government decides to default or not, $\delta_t = 0$ or $\delta_t = 1$ respectively. Differently from the timing in Eaton and Gersovitz (1981), the government does not have the ability to commit not to default within the current period. As we will see, this opens the door to self-fulfilling debt crisis.

The budget constraint for the government when he does not default is

$$G_t + B_{S,t} + \iota B_{L,t} \leq Y_t + q_t \cdot x_t, \tag{3}$$

where x_t is the newly issued debt and can be expressed as

$$x_t = (B_{S,t+1}, B_{L,t+1} - (1 - \lambda)B_{L,t}).$$

We assume that if the government defaults, he is permanently excluded from financial markets and he suffers losses in output. We denote by $\underline{V}(s_{1,t})$ the value for the government conditional on a default. Lenders that hold inherited debt and the new debt just issued do not receive any repayment.[3]

2.2 Recursive Equilibrium

2.2.1 Definition

We now consider a recursive formulation of the equilibrium. Let $\mathbf{S} = (B, s)$ be the state today and $\mathbf{S}'$ the state tomorrow. The problem for a government that has not defaulted yet is

$$V(\mathbf{S}) = \max_{\delta \in \{0,1\}, B', G} \delta \left\{ U(G) + \beta \mathbb{E}[V(\mathbf{S}') | \mathbf{S}] \right\} + (1 - \delta)\underline{V}(s_1) \tag{4}$$

subject to

$$G + B_S + \iota B_L \leq Y(s) + q(\mathbf{S}, B') \cdot x.$$

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[3]This is a small departure from Cole and Kehoe (2000), since they assume that the government can use the funds raised in the issuance stage. Our formulation simplifies the problem and it should not change its qualitative features. The same formulation has been adopted in other works, for instance Aguiar and Amador (2014).

The lender's no-arbitrage condition requires that

$$q_S\left(\mathbf{S}, B'\right) = \delta\left(\mathbf{S}\right) \mathbb{E}\left\{M\left(s, s'\right) \delta\left(\mathbf{S}'\right) | \mathbf{S}\right\}, \tag{5}$$

$$q_L\left(\mathbf{S}, B'\right) = \delta\left(\mathbf{S}\right) \mathbb{E}\left\{M\left(s, s'\right) \delta\left(\mathbf{S}'\right) \left[\iota + (1-\lambda)q\left(\mathbf{S}', B'(B', \mathbf{S}')\right)\right] | \mathbf{S}\right\}. \tag{6}$$

The presence of $\delta\left(\mathbf{S}\right)$ in equations (5)-(6) means that new lenders receive a payout of zero in the event of a default today.

A *recursive equilibrium* is value function for the borrower V, associated decision rules δ, B', G and a pricing function q such that V, δ, B', G are a solution of the government problem (4) and the pricing functions satisfy the no-arbitrage conditions (5) and (6).

2.2.2 Multiplicity of equilibria

We now show that there are multiple recursive equilibria in this model. It is convenient first to define the *fundamental equilibrium outcome*, $\mathbf{y}^* = \{G_t, B_{t+1}, \delta_t, q_t\}$ as the optimal choice associated with the solution to the following functional equation that attains the higher value:[4]

$$V^*\left(B_0, s\right) = \max_{\{G_t, B_{t+1}, \delta_t, q_t\}} \mathbb{E}_0 \sum_{t=0}^{\infty} \left(\delta_t \beta^t U\left(G_t\right) + (1-\delta_t)\underline{V}_t\right) \tag{7}$$

subject to

$$G_t + B_{S,t} + \iota B_{L,t} \leq Y_t + q_t \cdot x_t$$

$$q_{S,t} = \mathbb{E}_t\left\{M_{t,t+1}\delta_{t+1}\right\}$$
$$q_{L,t} = \mathbb{E}_t\left\{M_{t,t+1}\delta_{t+1}\left[\iota + (1-\lambda)q_{t+1}\right]\right\}$$

and if $\delta_t = 1$

$$\mathbb{E}_t \sum_{j=0}^{\infty} \left(\delta_{t+j}\beta^{t+j}U\left(G_{t+j}\right) + (1-\delta_{t+j})\underline{V}_{t+j}\right) \geq V^*\left(B_t, s_t\right)$$

It is clear that such outcome can be implemented as a recursive competitive equilibrium outcome. However it is not the unique equilibrium outcome. Following Cole and Kehoe (2000), when inherited debt is sufficiently high a coordination problem can generate a

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[4]It is not obvious that the operator defined by (7) has a unique fixed point. Auclert and Rognlie (2014) show that if the government can only issue one period debtand it is allowed to save,then the fundamental equilibrium is indeed unique. With long-term debt there is no guarantee that this is the case. Our fundamental equilibrium outcome is the "best" among all the fundamental equilibria. That is, the one that attains the highest vale for the borrower.

"run" on debt, whereby it is optimal for an atomistic investor not to lend to the government whenever the other investors decide not to buy the bonds. This can happen despite the fact that the atomistic investor would lend to the government if the other lenders would. This form of strategic complementarity gives rise to a multiplicity of equilibria.

In fact, suppose that lenders expect the government to default today so that the price of government debt is $q = 0$. The expectation of the lenders is validated in equilibrium if default is an optimal choice of the government. This can happen in this economy if[5]

$$U\left(Y - B_S - \iota B_L\right) + \beta \mathbb{E}\left[V\left([0, (1-\lambda)B_L], s'\right) | \mathbf{S}\right] < \underline{V}(s_1). \tag{8}$$

From condition (8) it is clear that there exists a level of inherited debt $B = (B_S, B_L)$ sufficiently high such that the condition is satisfied. Condition (8) depends on V, an equilibrium object. We can however find sufficient conditions on the primitives of the model for (8) to be satisfied, thus establishing the presence of multiple equilibria. In fact, let $B_{\text{crisis}}(s)$ be the set of (B_S, B_L) such that

$$U\left(Y - B_S - \iota B_L\right) + \beta \mathbb{E}\left[V^*\left([0, (1-\lambda)B_L], s'\right) | \mathbf{S}\right] < \underline{V}(s_1) \tag{9}$$

Hence, since $V^* \geq V$, we can conclude that condition (9) implies (8).

Note that such $B_{\text{crisis}}(s)$ is smaller than the default cut-off in state s, denote it by $B_\delta(s)$ defined as

$$U\left(Y - B_S - \iota B_L + q^*\left(s, B^{*\prime}(B, s)\right) \cdot x^*(B, s)\right) + \beta \mathbb{E}\left[V^*\left(B^{*\prime}(B, s), s'\right) | \mathbf{S}\right] = \underline{V}(s_1) \tag{10}$$

This is because choosing $x^* = (0, 0)$ is a feasible choice and it is not optimal and so the left hand side of the expression above is higher than the left hand side of (9) if $B = B_\delta(s) = B_{\text{crisis}}(s)$.

We can then summarize the discussion above with the following proposition. For such debt levels we have that the outcomes are indeterminate. In particular we can prove the following proposition:

Proposition 1. *There exists at least two recursive equilibria in "pure strategies" that differ for all (B, s) such that $B \in [B_{crisis}(s), B_\delta(s))$:*

  1. The fundamental equilibrium with $q(\mathbf{S}, B') > 0$ for all $B' \leq B'(B, s)$;

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[5]If condition (8) is not satisfied, instead, no coordination problem among lenders can arise. This is because if lenders decide to run, and so $q = 0$, it is still optimal for the government to repay his debt. Thus, lenders have no incentive to run: it is optimal for an individual lender to lend at a positive price even if other lenders do not and so $q = 0$ cannot be an equilibrium price.

2. *An equilibrium in which there is always a run whenever $B > B_{crisis}$ and so $q(\mathbf{S}, B') = 0$ for all B' when $B > B_{crisis}$.*

2.2.3 Markov selection

So far we have shown that debt crisis may be self-fulfilling for sufficiently high level of debt: lenders may lend to the sovereign and there will be no default, or the lenders may not roll-over government debt, in which case the sovereign would find it optimal to default. Therefore, the outcomes are *indeterminate* in this region of the state space. We now propose a parametric mechanism that selects among these possible outcomes. Conditional on this selection rule, the equilibrium will be unique.

Let the exogenous state be $s = (Y, z, p, \xi)$ where Y are tax receipts, z is a (possibly vector valued) factor that affects the stochastic discount factor for the lenders, and (p, ξ) are coordination devices. In terms of our previous partition of the state space, we have that $s_1 = (Y, z)$ is the vector collecting the "fundamental" exogenous state variables, while $s_2 = (p, \xi)$ collects the "non-fundamental" components.

It is useful to partition the state space in the three region. Following the terminology in Cole and Kehoe (2000), we say that the borrower is in the *safe zone*, $\mathcal{S}^{\text{safe}}$, if the government does not default even if lenders do not rollover his debt. That is,

$$\mathcal{S}^{\text{safe}} = \left\{ \mathbf{S} : U(Y - B_S - \iota B_L) + \beta \mathbb{E} V((0, (1 - \lambda)B_L), s') \geq \underline{V}(s_1) \right\}.$$

We say that the borrower is in the *crisis zone*, $\mathcal{S}^{\text{crisis}}$, if the initial state is such that it is not optimal to repay debt during a rollover crisis but the government finds it optimal to repay whenever lenders roll-over their debt. That is,

$$
\begin{aligned}
\mathcal{S}^{\text{crisis}} = \Big\{ \mathbf{S} : \; & U(Y - B_S - \iota B_L) + \beta \mathbb{E} V((0, (1 - \lambda)B_L), s') < \underline{V}(s_1) \text{ and} \\
& U(Y - B_S - \iota B_L + q(s, B'(B, s)) \cdot x(B, s)) + \beta \mathbb{E}\left[V(B'(B, s), s') | \mathbf{S}\right] \geq \underline{V}(s_1) \Big\}.
\end{aligned}
$$

Finally, the residual region of the state space, the *default zone*, $\mathcal{S}^{\text{default}}$ is the region of the state space in which the government defaults on his debt regardless of lenders' behavior. That is,

$$
\begin{aligned}
\mathcal{S}^{\text{default}} = \Big\{ \mathbf{S} : \; & U(Y - B_S - \iota B_L + q(s, B'(B, s)) \cdot x(B, s)) \\
& + \beta \mathbb{E}\left[V(B'(B, s), s') | \mathbf{S}\right] < \underline{V}(s_1) \Big\}.
\end{aligned}
$$

Indeterminacy in outcomes arises only in the crisis zone.

The selection mechanism works as follows. Whenever the economy is in the crisis zone, lenders roll-over the debt if $\xi \geq p$. In this case, there are no run on debt and $\delta(\mathbf{S}) = 1$ by our definition of crisis zone. If $\xi < p$, instead, the lenders do not roll-over the government debt. We will assume that ξ is an i.i.d. uniform on the unit interval while p follows a first order Markov process, $p' \sim \mu_p(.|p)$. Given these restrictions, we can interpret p as the probability of having a rollover crises this period conditional on the economy being in the crisis zone.

Note that the outcome of the debt auctions are unique in the crisis zone once we adopt this selection rule. However, even with this selection, we cannot assure that the equilibrium value function, decision rules and pricing functions are unique as the operator that implicitly defines a recursive equilibrium may have multiple fixed points. In order to overcome these issues, we restrict our attention to the limit of the finite horizon version of the model. Under our selection rule, the finite horizon model features a unique equilibrium and so does its limit. The equilibrium outcome is a stochastic process

$$\mathbf{y} = \{G(s^t, B_0), B(s^t, B_0), \delta(s^t, B_0), q(s^t, B_0)\}_{t=0}^{\infty}$$

naturally induced by the recursive equilibrium objects. The outcome path depends on properties of the selection, i.e. the process for $\{p_t\}$. For example, if $\{p_t\}$ is identically equal to zero then equilibrium outcome coincides with the fundamental one, $\mathbf{y}^*$. The goal of our exercise is to understand how properties of the process for $\{p_t\}$ affect the equilibrium outcome path and then estimate such process.

3   Interest Rate Spreads, Debt Duration and Default Risk

In order to explain the objectives and challenges of our empirical analysis, it is convenient to express interest rate spreads on short term debt as

$$
\begin{aligned}
\frac{r_{S,t} - r_t^*}{r_{S,t}} &= \Pr_t\{\mathbf{S}_{t+1} \in \mathcal{S}^{\text{default}}\} + \Pr_t\{\mathbf{S}_{t+1} \in \mathcal{S}^{\text{crisis}}\}\mathbb{E}_t[p_{t+1}] \\
&\quad - \text{Cov}_t\left(\frac{M_{t,t+1}}{\mathbb{E}_t[M_{t,t+1}]}, \delta_{t+1}\right),
\end{aligned}
$$

(11)

where $r_{S,t}$ are the yield on a short term bond issued by the sovereign and r_t^* is the return on a risk free bond maturity next period.

The equation tells us that interest rate spreads are a function of three components. The first two components represent the different sources of default risk in the model, and they add up to the conditional probability that the government defaults at $t+1$. As we have seen earlier, this can happen because of two events. First, if $\mathbf{S}_{t+1} \in \mathcal{S}^{\text{default}}$, the sovereign finds it optimal to default irrespective of the behavior of lenders. Second, the sovereign tomorrow may be in the crisis zone, in which case the outcome is indeterminate and it will be pinned down by the realizations of the two coordination devices, (ξ_{t+1}, p_{t+1}). The conditional probability of observing a default in this region of the state space is $\mathbb{E}_t[p_{t+1}]$. The third component, $\text{Cov}_t\left(\frac{M_{t,t+1}}{\mathbb{E}_t[M_{t,t+1}]}, \delta_{t+1}\right)$, reflects a premium required by the lenders to hold risky government securities.

Our objective is to obtain an empirical counterpart to $\text{Pr}_t\{\mathbf{S}_{t+1} \in \mathcal{S}^{\text{crisis}}\}\mathbb{E}_t[p_{t+1}]$, and to measure its importance in driving interest rate spreads during the sovereign debt crises in Europe. Given a parametrization of the model, this can be achieved by applying standard filtering techniques and using observed time series to measure this object of interest.

Clearly, the challenge for this type of exercise concerns the choice of the structural parameters and of the data series used to infer the shocks. The literature gives little guidance on the set of variables that can provide information on the $\{p_t\}$ process. Previous quantitative studies have showed that economic fundamentals alone can replicate key features of interest rate spreads once we allow for sufficient flexibility on the output costs of default.[6] Absent direct observations on these output costs, it is unlikely that the parameters of the $\{p_t\}$ process can be separately identified by looking at interest rate spreads only.

In the empirical analysis, we will use a key insight from the model to inform the parametrization of the $\{p_t\}$ process. We are going to show that the joint distribution of interest rate spreads and debt duration provides information that is useful to distinguish between fundamental and non-fundamental sources of default risk. As in Cole and Kehoe (2000), the sovereign in our model has an incentive to "exit" from the crisis zone whenever he expects rollover risk to be high in the future. To achieve this objective, he may lengthen the maturity of his debt since long term debt is less susceptible to runs. If rollover risk is a major driver of default risk in the model, we should expect the duration of the debt to lengthen when interest rate spreads increase. On the contrary, previous research - for instance Arellano and Ramanarayanan (2012), Aguiar and Amador (2014) and Dovis (2014) - has shown that a shortening of maturity may be an optimal response of the sovereign when facing a default crises driven by fundamental shocks: a negative comovement between spreads and duration would then indicate a more limited role for

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[6]See for example Arellano (2008) and Chatterjee and Eyigungor (2013) for emerging markets or Salomao (2014) for a recent application to Greece.

rollover risk. We now illustrate these insights within a simplified version of our model.

3.1 A Three Period Model

We consider a three period economy with risk neutral lenders, $M(s_0, s_1) = m_{01}$ and $M(s_1, s_2) = m_{12}$ for all s_0, s_1 and s_2. We further assume that at $t = 0$ the government can issue two type of securities: a zero coupon bond maturing in period 1, b_{01}, and a zero coupon bond maturing in period 2, b_{02}. In period 1, the government issues only a zero coupon bond maturing in period 2, b_{12}. It is convenient to present the model starting from the last period. At $t = 2$, the government does not issue new debt and his only choice is whether to default on the previously issued debt ($\delta_2 = 0$),

$$V_2 (b_{02} + b_{12}, Y_2) = \max \left\{ U (Y_2 - b_{02} - b_{12}) ; \underline{V}_2 \right\}.$$

At $t = 1$, the government issues b_{12} and he decides whether to default ($\delta_1 = 0$). The price of debt issued at $t = 1$ out of the crisis zone, or when $\xi_1 \geq p$, is then given by

$$q_{12}(s_1, b_{02} + b_{12}) = \mathbb{E}_1 [m_{12} \delta_2],$$

and it equals zero otherwise. The decision problem of the sovereign at $t = 1$ is

$$V_1 (b_{01}, b_{02}, s_1) = \max_{\delta_1, G_1, b_{12}} \delta_1 \left\{ U (G_1) + \beta \mathbb{E}_1 [V_2 (b_{02} + b_{12}, Y_2)] \right\} + (1 - \delta_1) \underline{V}_1$$

subject to

$$G_1 + b_{01} \leq Y_1 + q_{12} (s_1, b_{02} + b_{12}) b_{12}$$

Finally at $t = 0$ the government issues both short and long term debt to solve

$$V_0 (s_1) = \max_{G_0, b_{01}, b_{02}} U (G_0) + \beta \mathbb{E}_0 [V_1 (s_1, b_{01}, b_{02})]$$

subject to

$$G_0 + \Delta_0 \leq Y_0 + q_{01} (s_0, b_{01}, b_{02}) b_{01} + q_{02} (s_0, b_{01}, b_{02}) b_{02},$$

with Δ_0 being the debt inherited from the past. To avoid problems associated with dilution of legacy debt, we assume that the government does not inherit long-term debt. We further assume that Δ_0 is sufficiently small that the government does not default at $t = 0$.

We now consider the optimal maturity structure of government debt in this simple set up. We start from the case in which rollover risk is absent, $p = 0$. Previous works

on incomplete market models without commitment have emphasized two channels as the main determinants of the maturity composition of debt in the face of default risk: insurance and incentives not to dilute outstanding debt.

The *insurance channel* refers to the fact that long term debt is a better asset than short term debt to provide the government with insurance against shocks. More specifically, capital gains and losses imposed on holders of long term debt can approximate wealth transfers associated with state contingent securities, and this gives an incentive for the government to issue bonds of longer duration. In order to understand this point, suppose that at $t = 0$ the government implements a marginal variation that lengthen the duration of his debt keeping constant the debt issued at $t = 0$ and the debt maturing at $t = 2$. That is, the government decreases b_{01} by ε, increases b_{02} by $\varepsilon \frac{q_{01}}{q_{02}}$ to keep market value of issuance at $t = 0$ constant, and finally decreases b_{12} by $\varepsilon \frac{q_{01}}{q_{02}}$ so to keep the amount of debt to be repaid in the last period constant. Under this variation, the change in government consumption at $t = 1$ in state s_1 is

$$\Delta G_1 (s_1) = \left[1 - \frac{q_{01}}{q_{02}} q_{12}(s_1) \right] \varepsilon = \left[1 - \frac{q_{12}(s_1)}{\mathbb{E}[q_{12}|\delta_1 = 1]} \right] \varepsilon.$$

The equation shows that this variation leads to a decline in G_1 when the price of newly issued debt is above average at $t = 1$, while it leads to an increase in G_1 in states of the world in which the price is below its average. Since q_{12} is high in "good" states of the world, then lengthening the duration of debt at $t = 0$ provides more insurance for the government: lower consumption when marginal utility is relatively low and higher consumption when the marginal utility is relatively high.

While insurance generates a motive for the government to issue long term debt, *incentives not to dilute outstanding debt* pushes the government to issue relatively more short term debt. When agents follow Markovian strategies,[7] short term debt is relatively more attractive because it is not prone to be diluted. That is, the government cannot reduce ex-post the value of short term debt absent a default, while he can dilute long term debt by increasing issuance relative to the ex-ante expectations of lenders. This capital loss realized by lenders is tantamount to a partial default on existing debt. Since the government cannot commit to future issuance of debt, lenders will demand a compensation for this dilution risk. This makes long term debt more expensive relative to short term debt, implying that the latter asset is a better instrument for the government to raise resources from creditors. In particular, we can prove that in absence of insurance motives and in

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[7] In infinite horizon, the debt-dilution problem is not present if we consider the best SPE (which is history dependent).

absence of rollover risk, the government never issues long term debt in this economy:[8]

Proposition 2. *In the three period example, if (i) there is no rollover risk, (ii) output is deterministic in $t = 1$, and (iii) the distribution of output in $t = 2$ does not depend on s_1 then $b_{02} = 0$ if β/m_{01} is sufficiently small (i.e. government wants to borrow a lot in period $t = 0$).*

With rollover risk, there is an additional motive governing the composition of government debt. When $p > 0$, in fact, rollover crisis can occur with positive probability if the economy happens to be in the crisis zone at $t = 1$. Since the government dislikes those outcomes, he has an incentives to alter (b_{01}, b_{02}) in order to lower the probability of being in the crisis zone next period. As emphasized in Cole and Kehoe (2000), this can be achieved by lengthening the maturity of issued debt. The logic of why lengthening the maturity of debt at $t = 0$ helps avoiding the crisis zone at $t = 1$ can be best understood by looking at the condition defining the crisis zone,

$$U(Y_1 - b_{01}) + \beta \mathbb{E}_2[V_2(b_{02}, Y_2)] < \underline{V}_1. \tag{12}$$

The government has a desire to issue debt when facing a rollover crises: if borrowing was not optimal, the above inequality would not hold and the government would not be in the crisis zone.[9] This means that repaying the stock of short term debt coming due at $t = 1$ is particularly costly from the perspective of the government. By issuing relatively more long term securities at $t = 0$, the government reduces the burden of debt coming due at $t = 1$, and this implies an increase in the left hand side of (12).

Figure 1 illustrates this point. We plot the crisis zone (light gray area) and the default zone (dark gray area) at $t = 1$ for fixed Y_1 and for different combinations of (b_{01}, b_{02}). The vertical axis represents total issuance of debt at $t = 0$ while the horizontal axis refers to the fraction of long term debt over the total amount issued: picking a point on the vertical axis and moving horizontally means that the government issue the same amount of debt at $t = 0$ but with a longer duration. For a given level of issuance, lengthening the maturity of debt shrinks the crisis zone.

This discussion suggests that the government has an incentive to lengthen the duration of his debt when rollover risk is sizable. In the extreme case where $p > 0$ and it is always optimal to repay the debt absent a rollover crisis, the government will issue only long term debt in this economy.

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[8]The same proposition can be proved if output is deterministic but $\underline{U}_t$ is stochastic as in Aguiar and Amador (2014)

[9]In the case in which borrowing is not optimal we have $U(Y_1 - b_{01}) + \beta \mathbb{E}_2[V_2(b_{02}, Y_2)] = V_1(b_{01}, b_{02}, Y_1) \geq \underline{V}_1$.

Figure 1: **Maturity composition of debt and crisis zone**
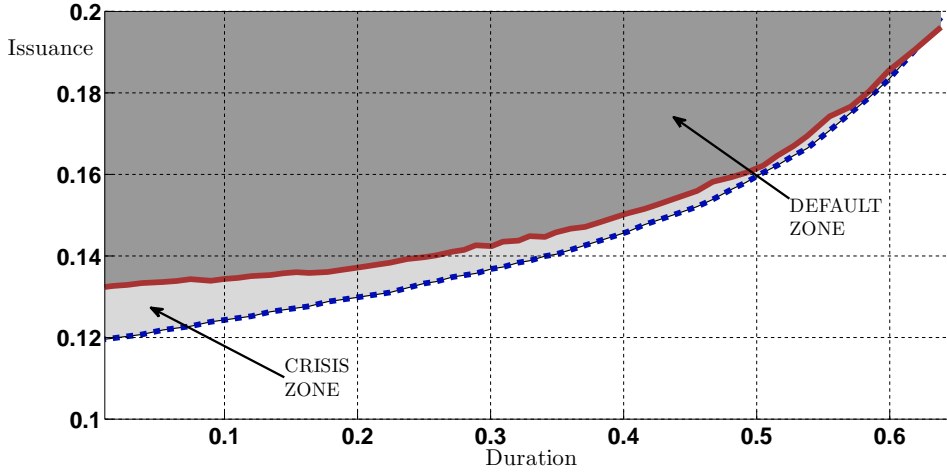
Notes: The dotted line represents the combinations of (b_{01}, b_{02}) such that the relation in (12) holds as an equality. The solid lines represents the combination of (b_{01}, b_{02}) such that the government is indifferent between defaulting or repaying his debt. The light grey area represents the crisis zone while the dark grey area the default set. The figure is drawn for a fixed Y_1.

Proposition 3. *In the three period example, if there is only rollover risk and no fundamental shock at $t = 1, 2$ then $b_{01} = 0$ and all debt is long term.*

Having described the key motives governing the optimal maturity structure of government debt, we now consider the equilibrium relations between debt duration and interest rate spreads. First, consider the case in which default risk is driven purely by economic fundamentals, $p = 0$. In this scenario, shocks that push the economy closer to the default zone typically lead to a *decline* in the duration of debt. We illustrate this in Figure 2 using a parametrized version of our example, but the point is more general and it holds for typical calibrations of quantitative sovereign default models, see Arellano and Ramanarayanan (2012). The solid line in the figure plots annualized interest rate spreads on short term debt (left panel) and an indicator of debt duration, $q_{02}b_{02}/(q_{01}b_{01} + q_{02}b_{02})$, as a function of Y_0. As Y_0 declines, the probability of a default in the next period increases and so does the interest rate spread. Our proxy for debt duration, instead, decreases.

This shortening of debt duration in the face of "fundamental" default risk reflects two phenomena. First, incentives not to dilute outstanding debt are stronger the higher is the risk of default. Indeed, in low Y_0 states, the government would like to issue more debt in order to smooth out consumption. With dilution and no rollover risk the value of issuance is maximized for all new debt being short term, since short term debt allows the government to commit not to issue too much debt in the future. This helps to keep the price of debt high today. See Aguiar and Amador (2014) for a similar argument. Second,

Figure 2: **Interest rate spreads and debt duration as a function of Y_0**
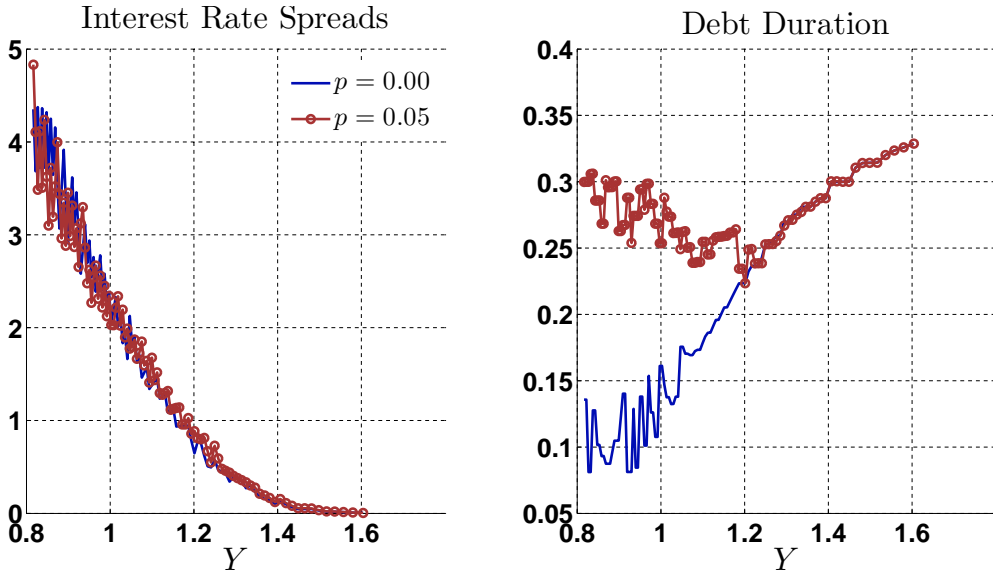
Notes: *The left panel plots the annualized interest rate spread on short term debt as a function of Y_0 for $p = 0.00$ (blue solid line) and for $p = 0.05$ (red circled line). The right panel plots the same information for our indicator of debt duration.*

the need to hold long term debt for insurance reasons falls when default risk increases. As discussed in Dovis (2014), this happens because pricing functions become more sensitive to shocks when the economy is approaching the default region. Hence the same amount of long term debt provides more insurance.[10]

The circled line in Figure 2 reports the same policy functions when $p = 0.05$. In this case, higher interest rate spreads on short term debt are associated to a lengthening of the duration of government debt. Differently from the previous scenario, default risk in this example partly reflects the anticipation of a rollover crisis at $t = 1$. The lengthening of debt duration in the face of "non-fundamental" default risk arises because of the government's efforts to avoid the crisis zone at $t = 1$. Indeed, as Y_0 declines, the government places a higher likelihood of falling in the crisis zone next period, and this generates an incentive to lengthen the duration of debt. When rollover risk is sizable, this motive counteracts the ones described earlier and it may lead to an increase in the duration of debt.

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[10]Suppose at $t = 1$ there are only two states: s_L and s_H. The value of debt for the lenders in each state (assuming no default at $t = 1$) is $B_1(s) \equiv b_{01} + q_{12}(s) b_{02}$. Then the amount of insurance is (suppose s_H is the good state) $B_1(s_H) - B_1(s_L) = [q_{12}(s_H) - q_{12}(s_L)] b_{02}$. The claim is that $[q_{12}(s_H) - q_{12}(s_L)]$ is larger the larger is default risk.

3.2 Insights from Three Period Model

To summarize, the structure of a typical sovereign default model implies two important properties. First, interest rate spreads increase and the duration of debt declines when the sources of default risk are fundamental: short term debt provides more incentives for the government to repay in the future, and these incentives are very valuable when a country is facing a solvency crises. Second, interest rate spreads and debt duration both increase when the underlying source of default risk is not fundamental: a government can reduce the probability of a future rollover crisis by lengthening the maturity of his debt.

These properties imply restrictions on the joint distribution of debt duration and interest rate spreads that can be used to assess the relevance of extrinsic uncertainty in driving fluctuations in interest rate spreads. In order to understand this point, we simulate the three period model for many periods under different values for p.[11] For each of these samples, we compute two statistics: the sample mean of $\frac{\Pr_t\{\mathbf{S}_{t+1} \in \mathcal{S}^{\text{crisis}}\}}{\text{spread}_t}$ and the correlation between our indicator of debt duration and the spread. Figure 3 show how these statistics vary with p.

Figure 3: **Interest rate spreads, debt duration and rollover risk**
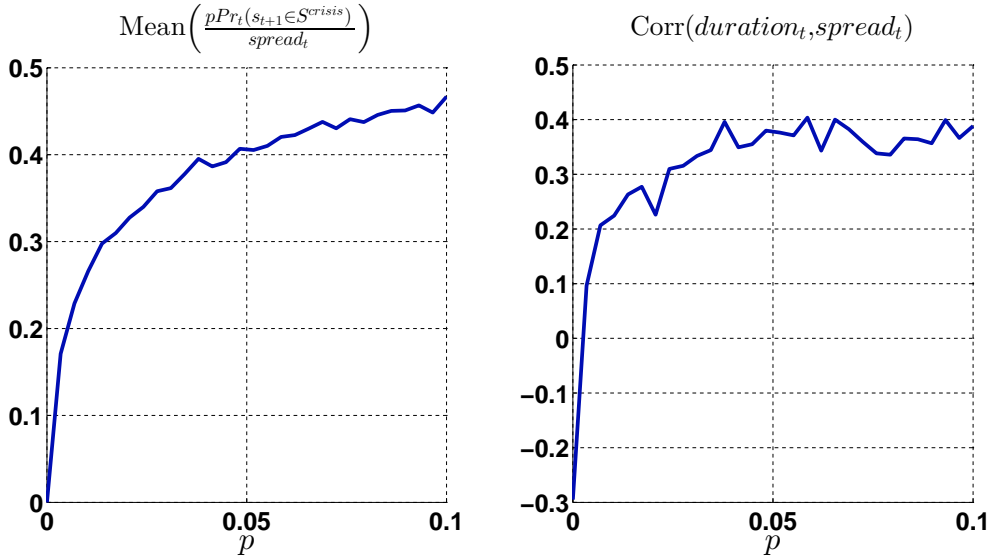
Notes: For each value of p, we simulate the model for $T = 10000$ periods as described in footnote 11. For each of these simulations, we compute the sample mean of $\frac{\Pr_t\{\mathbf{S}_{t+1} \in \mathcal{S}^{\text{crisis}}\}}{\text{spread}_t}$ and the sample correlation between duration and interest rate spreads on short term debt. The panels plot how these statistics varies with p.

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[11]We simulate the model as follows. We draw a sequence of innovations to the endowment process. Next, we feed the policies (b_{01}, b_{02}) and (q_{01}, q_{02}) with this innovation, updating at each point in time the issued stock of debt. Our sample consists for debt issuance and bond prices corresponds to these repeated simulation of period 0 choices of the government.

When $p \approx 0$, rollover risk is a negligible driver of interest rate spreads, and the model predicts that sovereign debt crisis are associated to a shortening of the duration of debt, $\text{Corr}(\text{duration}_t, \text{spread}_t) < 0$. As p increases, so does the relative importance of rollover risk. When this latter is sufficiently important, we should expect on average a positive association between interest rate spreads and the duration of debt. This example suggests that the joint behavior of interest rate spreads and debt duration is very informative for learning about the sources of default risk. These restrictions will play a key role in our empirical analysis, to which we now turn.

4   Empirical Analysis

This section and the ones that follow are preliminary. We consider a version of the model with only one period debt. We are currently working on implementing the full model so that we can use information on the joint distribution of interest rate spreads and debt duration to distinguish between fundamental and non-fundamental sources of default risk using the insights of the previous section.

We now apply our framework to Italian data. This section proceeds in four steps. Section 4.1 describes the parametrization of the model and our empirical strategy. Section 4.2 describes the data. Section 4.3 reports the results of our calibration. Section 4.4 discusses some pitfalls in the identification of rollover risk in our procedure.

4.1   Parametrization and Empirical Strategy

4.1.1   Government Preferences and Endowment

A period in our model is a quarter. The government period utility function is CRRA

$$U^{\text{gov}}(G_t) = \frac{G_t^{1-\sigma} - 1}{1 - \sigma},$$

with σ being the coefficient of relative risk aversion. The government discounts future flow utility at the rate β. If the government enters a default state, he is excluded from international capital markets and he suffers an output loss τ_t. These costs of default are a function of the country's income, and they are parametrized following Chatterjee and Eyigungor (2013),

$$\tau_t = \max\{0, d_0 e^{y_t} + d_1 e^{2y_t}\}.$$

If $d_1 > 0$, then the output losses are larger when income realizations are above average.[12] We also assume that, while in autarky, the sovereign has a probability of reentering capital markets ψ. If the government reenters capital markets, he pays the default costs and he starts his decision problem with zero debt.

The endowment shock follows an autoregressive process in logs,

$$y_{t+1} = \rho_y y_t + \sigma_y \varepsilon_{y,t},$$

while the probability of lenders not rolling over the debt in the crises zone follows the stochastic process $p_t = \frac{\exp\{\tilde{p}_t\}}{1+\exp\{\tilde{p}_t\}}$, with $\tilde{p}_t$ given by

$$\tilde{p}_{t+1} = (1 - \rho_p)p^* + \rho_p \tilde{p}_t + \sigma_p \varepsilon_{p,t}.$$

We let $\theta^{\text{def}} = [\sigma, \beta, d_0, d_1, \psi, \lambda, \iota, \rho_y, \sigma_y, p^*, \rho_p, \sigma_p]$ denote the parameters associated to the government decision problem. The innovations $\{\varepsilon_{y,t}, \varepsilon_{p,t}\}$ are i.i.d. standard normal random variables.

4.1.2 Lenders Stochastic Discount Factor

It is common practice in the sovereign debt literature to assume risk neutrality on the lenders' side. This specification, however, is not desirable given our objectives. First, several authors have argued that risk premia are quantitatively important to account for the volatility of sovereign spreads (Borri and Verdhelan, 2013; Longstaff et al., 2011). Assuming risk neutrality would imply that other unobserved factors in the model, for instance p_t, would need to absorb the variations in this component of the spread. Second, sovereign debt crisis are typically accompanied by a significant increase in term premia (Broner et al., 2013). Neglecting these shifts could undermine our identification strategy: rollover risk could be an important driving force for interest rate spreads of peripheral countries in the euro area and yet we could observe a shortening in the duration of debt simply because high term premia made short term borrowing cheaper during the crises.

Therefore, we deviate from the existing literature and we endow the lenders with preferences that are sufficiently flexible to capture the behavior of risk premia and term premia over our sample. We use a variant of the Campbell and Cochrane (1999) external habit model. Bakaert et al. (2009) and Wachter (2006) have shown that empirical version of this

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[12]This feature makes it easier for the model to match the empirical observation that sovereign spreads are countercyclical. With convex output costs, in fact, the sovereign has more incentives to default in presence of bad income realization, see Arellano (2008).

model can successfully fit the behavior of risky and riskless assets for the U.S. economy, while preserving an economic interpretation of the factors driving asset prices.

The lenders' utility function is

$$U^{\text{lend}}(C_t, X_t) = \frac{(C_t - X_t)^{1-\gamma} - 1}{1 - \gamma},$$

with C_t being lenders' consumption and X_t an "habit" level. The lenders discount future utility at the rate $\overline{\beta}$. The habit level is implicitly defined by the surplus function $\exp(\chi_t) = \frac{C_t - X_t}{C_t}$, with χ_t following the stochastic process,

$$\chi_{t+1} = (1 - \phi)\chi^* + \phi\chi_t + \sigma_{\chi,c}\{\Delta c_{t+1} - \mathbb{E}_t[\Delta c_{t+1}]\} + \lambda(\chi_t)\varepsilon_{\chi,t}.$$

In our notation, Δc_t stands for the first difference of log consumption. We assume this latter variable is a white noise process with time-varying volatility,

$$\Delta c_t = g + \exp\{v_{t-1}\}\sigma_c\varepsilon_{c,t},$$

with the volatility component v_t being drawn from the Gaussian autoregressive model

$$v_t = \rho_v v_{t-1} + \sigma_v \varepsilon_{v,t}.$$

We assume that $\varepsilon_{\chi,t}$, $\varepsilon_{c,t}$ and $\varepsilon_{v,t}$ are i.i.d standard normal random variables. The "sensitivity function" $\lambda(.)$ is given by[13]

$$\lambda(\chi_t) = \frac{1}{\overline{S}}\sqrt{1 - 2(\chi_t - \chi^*)}, \qquad \overline{S} = \sigma_\chi\sqrt{\frac{\gamma^2}{(1-\phi)\gamma - b}}. \tag{13}$$

The vector $\theta^{\text{sdf}} = [\gamma, g, \overline{\beta}, b, \chi^*, \phi, \sigma_c, \sigma_{c,s}, \rho_v, \sigma_v]$ collects the parameters governing the preferences and the endowment of the lenders. This formulation implies that the prices of bonds and of claims over the aggregate endowment are a function of the state variables $\{\chi_t, v_t\}$. We will refer to χ_t as "risk aversion", since the coefficient of relative risk aversion in the model equals

$$\text{RRA}(\chi_t) = \frac{-U_{cc}^{\text{lend}}(C_t, X_t)C_t}{U_c^{\text{lend}}(C_t, X_t)} = \frac{\gamma}{\exp\{\chi_t\}}. \tag{14}$$

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[13]In order to guarantee that the quantity within the square roots remains positive, we will set $\lambda(.)$ to 0 whenever $\chi_t > \chi_{\max}$, with $\chi_{\max} = \chi^* + \frac{1}{2}(1 - \overline{S}^2)$.

We will refer to the second factor, v_t, as "volatility". Note that the stochastic process $\{M_{t,t+1}\}$ can be used to express asset prices as a function of the state variables $[\chi_t, v_t]'$ and of the parameters θ^{sdf}.[14]

Broadly speaking, our empirical strategy consists in choosing $\theta = [\theta^{\mathrm{def}}, \theta^{\mathrm{sdf}}]$ in two steps. In the first step, we estimate the parameters of the lenders' stochastic discount factor using information from the term structure of German zero coupon bonds and the stock price-consumption ratio for the euro-area. This step will make sure that our model is sensible in pricing risky and riskless assets at different maturities. Implicit in our approach is the assumption that the lenders are "marginal" for pricing other financial assets in the euro-area beside Italian government securities and that the stochastic discount factor is exogenous to the issuance of government debt and to default decisions. We will discuss this last assumption in Section 4.4. In the second step, and conditional on the parameters governing the lenders' behavior, we calibrate θ^{def} by matching some basic facts about Italian public finances. In view of our previous discussion, we will place empirical discipline on the $\{p_t\}$ process by making sure that the calibrated model replicates the joint behavior of interest rate spreads and the duration of debt for the Italian economy.

4.2 Data

Our sample starts in 1999:Q1 and ends in 2012:Q4. We collect quarterly data on private consumption expenditures for members of the euro area are from the ECB-SDW database.[15] We construct a quarterly series for the stock-price consumption ratio in the euro area by scaling the Dow Jones Euro Stoxx 50 with our consumption series. Nominal bond yields for Germany (1 year and 5 year maturity) at a monthly frequency are from Bundesbank, while monthly data on CPI inflation in the euro-area from Eurostat. We convert monthly series at a quarterly frequency using simple averages. These data series will be used in the first step of our procedure to estimate θ^{sdf}.

The endowment process y_t will be mapped to linearly detrended log real Italian GDP. The quarterly GDP series is obtained from OECD. The interest rate spread series is the annualized difference between yields on Italian government debt of a 1 year maturity and

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[14]More specifically, the price of an asset x that pays the stochastic dividend stream $\{d_t^x\}$ is given by

$$P_t^x = \mathbb{E}_t\left[\sum_{j=0}^{\infty} M_{t,t+j} d_{t+j}^x\right] = f^x(\chi_t, v_t; \theta^{\mathrm{sdf}}).$$

[15]In what follows, the euro-area is defined as the 18 members of the monetary union as of December 2013. CPI inflation is calculated by Eurostat as a weighted average of CPI inflation in these countries (changing composition).

the yields on German bonds of the same duration. We will map this data series to the interest rate spread on short term debt in our model. We use [explain data on duration]. These data series will be used in the second step of our procedure to estimate θ^{def}.

4.3 Results

The results are organized in two sections. First, we describe the estimation of our pricing model. Then, we discuss the calibration of the parameters governing the government decision problem.

4.3.1 The lenders' stochastic discount factor

We fit our pricing model to the yield curve for German government securities and our time series on the price-consumption ratio in the euro area. Before describing the details of our estimation and the results, it is useful to describe some basic asset pricing properties of our model, with the objective of highlighting what feature of the data helps in identifying the model parameters.

The price of risk free real zero coupon bonds (ZCB) maturing in n periods can be expressed recursively as

$$P_{n,t} = \mathbb{E}_t\left[M_{t,t+1} P_{n-1,t}\right] = f_n^{\text{zcb}}(\chi_t, v_t; \theta^{\text{sdf}}),\tag{15}$$

with initial condition $P_{0,t} = 1$. These equations can be solved numerically using the initial condition and quadrature integration. Given this price, log yields are defined as $r_{n,t} = -\frac{1}{n}\log(P_{n,t})$.

We can use the above equation, along with the expression for the stochastic discount factor and the normality of innovations, to express the short term risk free rate as[16]

$$r_{1,t} = r^* + b(\chi^* - \chi_t) - \frac{\gamma^2[\sigma_c + \sigma_{\chi,c}]^2}{2}\exp\{2v_t\}.\tag{16}$$

This equation clarifies how changes in risk aversion and in volatility affect the *level* of the yield curve. First, a decline in χ_t has ambiguous effects on the risk free rate. On the one hand, investors whose consumption is close to their subsistence level (low χ_t) would like to borrow more in order to smooth these low marginal utility states: the increase in the demand for savings puts upward pressure on the risk free rate. On the

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[16]In the expression, $r^* = -\log(\bar{\beta}) + \gamma g - \frac{\gamma(1-\phi)-b}{2}$.

other hand, equation (13) shows that low χ_t states are associated with a higher $\lambda(\chi_t)$ and a higher sensitivity of the pricing kernel to shocks. Because of that, the investor has a precautionary motive to save, and this increase in the supply of savings puts downward pressure on the risk free rate. The parameter b governs the relative strength of these two opposing forces: if $b > 0$, the intertemporal smoothing effect dominates, and low χ_t states are associated with high real short rates. The opposite happens if $b < 0$. Second, an increase in consumption volatility is unambiguously associated to a decline in the risk free rate: when the volatility of consumption growth is high, precautionary motives induces investors to save more, and this increase in the supply of savings depresses the rate of returns on safe assets.

In order to gain insights on the model's implications for the *slope* of the real yield curve, we can approximate the return differentials of bonds with two periods and one period maturity as follows

$$[r_{2,t} - r_{1,t}] \approx \frac{1}{2}\mathbb{E}_t[r_{1,t+1} - r_{1,t}] + \frac{1}{2}\text{cov}_t[m_{t,t+1}, r_{1,t+1}]. \tag{17}$$

The above equation decomposes the spread $[r_{2,t} - r_{1,t}]$ into two pieces: a component related to the expectation hypothesis and a risk premium. Substituting in the above expression the risk-free rate from equation (16) and the stochastic discount factor, we can write these two components as follows

$$\begin{aligned}[r_{2,t} - r_{1,t}] &\approx \frac{1}{2}\left\{ b(\phi - 1)(\chi^* - \chi_t) - \frac{\gamma^2[\sigma_c + \sigma_{\chi,c}]^2}{2}\mathbb{E}_t[\exp\{2v_{t+1}\} - \exp\{2v_t\}]\right\} \\ &+ \frac{\gamma b}{2}\left\{\lambda(\chi_t)^2\sigma_s^2 + \sigma_c\sigma_{\chi,c}[1 + \sigma_c\sigma_{\chi,c}]\exp\{2v_t\}\right\}.\end{aligned} \tag{18}$$

There are two important things to notice. First, the yield curve on real bonds slopes up on average only when $b > 0$. When $b > 0$, low χ_t states are associated with a low price (high returns) for risk free securities. Thus, investors demand a compensation for holding long term debt because of the possibility of getting low holding period returns when their marginal utility of consumption is high. In estimation, we will find that the model requires a positive b to fit the average slope of the yield curve of nominal ZCB, even after accounting for inflation risk premia, a result that mirrors previous findings for the U.S. economy.

Second, and conditional on $b > 0$, we can see that an increase in volatility unambiguously leads to an increase in the slope of the yield curve. When v_t increases, the short

rate falls and $\mathbb{E}_t[r_{1,t+1} - r_{1,t}]$ goes up because of mean-reversion. Moreover, an increase in volatility raises the risk premium component. Thus, both components of the spread $r_{2,t} - r_{1,t}$ increase in response to a volatility shock. An increase in risk aversion, instead, has in principle ambiguous effects on the slope of the yield curve. On the one hand, when χ_t declines, the short term rate increases, and mean reversion implies that $\mathbb{E}_t[r_{1,t+1} - r_{1,t}]$ becomes negative. On the other hand, higher risk aversion increases the risk premia on long term ZCB, thus pushing up the second component in equation (18). Some algebra shows that the first effect dominates in the model, and the slope of the yield curve decreases conditional on an increase in risk aversion.[17]

Finally, we can price a claim that pays the realization of aggregate consumption in every period, P_t^e. The stock price-consumption ratio, $pc_t = \frac{P_t^e}{C_t}$ solves

$$pc_t = \mathbb{E}_t \left\{ M_{t,t+1} \left[1 + \frac{C_{t+1}}{C_t} pc_{t+1} \right] \right\} = f^{\text{pc}}(\chi_t, v_t; \theta^{\text{sdf}}), \qquad (19)$$

When $b > 0$, an increase in risk aversion is unambiguously associated with a decline in the price-consumption ratio. Indeed, higher χ_t implies more aggressive discounting of future payouts and an increase in the required compensation for holding risky assets, factors that contribute to a decline in pc_t. An increase in the volatility of consumption growth, on the other hand, has ambiguous effects on the price-consumption ratio. While higher volatility leads to higher premia on risky assets, it also implies a decline in the risk-free rate, which puts upward pressure on the price-consumption ratio, see Barksy (1989) for a discussion of these two effects. For typical parametrizations of the model, changes in volatility of consumption growth have very little effects on this variable.

Since we use nominal yields data in our application, we need to specify the process governing inflation. We follow Wachter (2006) and assume that the joint process for consumption growth and inflation is

$$
\begin{aligned}
\Delta c_t &= g + \sigma_c \mathbf{e}_t, \\
\pi_t &= \pi + \gamma \mathbf{Z}_t + \sigma_\pi \mathbf{e}_t \\
\mathbf{Z}_t &= \mathbf{\Phi} \mathbf{Z}_t + \mathbf{e}_t.
\end{aligned}
\qquad (20)
$$

Notice that we are allowing for correlation between consumption growth and inflation via the innovations $\mathbf{e}_t$. This makes inflation a priced-factor for nominal ZCB, a feature that other studies have found to be empirically relevant for the U.S. economy, see for example

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[17]The effect of a one percent decline in χ_t on the left hand side of equation (18) equals $-\frac{b^2}{\gamma}$.

Piazzesi and Schneider (2006). Moreover, this formulation implies that yields of nominal ZCB are linear in $\mathbf{Z}_t$, a feature that simplifies substantially the numerical simulations of the model. Following common practice in the literature, we first estimate the consumption-inflation process given by the system (20). The top panel of Table 1 reports maximum likelihood estimates of these parameters and their associated standard errors.

Table 1: **Estimates of Model Parameters**

Consumption-Inflation Process							
g	$\sigma_{c,1}$	$\sigma_{c,2}$	π	η_1	η_2	$\sigma_{\pi,1}$	$\sigma_{\pi,2}$
.0021	.0022	.0026	.0051	.0008	-.0007	- .0008	.0006
(.0006)	(.0006)	(.0006)	(.00007)	(.0002)	(.0001)	(.0001)	(.0002)
$\phi_{1,1}$	$\phi_{1,2}$	$\phi_{2,1}$	$\phi_{2,2}$				
0.54	0.45	- 0.48	1.14				
(0.16)	(0.12)	(0.18)	(0.07)				

Stochastic Discount Factor Parameters								
χ^*	γ	r^*	b	ϕ	σ_χ	$\sigma_{\chi,c}$	ρ_v	σ_v
0.0000	1.1132	0.0026	0.0029	0.9853	0.0000	0.3947	0.9870	0.2413
fixed	( )	( )	( )	( )	( )	( )	( )	( )

Notes: The top panel reports MLE estimates of the parameters in the linear state space system of 20. The log-likelihood function is computed using the Kalman filter. Standard errors are computed using the inverse hessian of the log-likelihood function at the point estimates. The bottom panel reports SMM estimates of the remaining parameters. Model implied moments are calculated via a long (N = 20000) simulation. Standard errors are computed as .

Fixing these parameters, we estimate $[\chi^*, \gamma, r^*, b, \phi, \sigma_\chi, \sigma_{\chi,c}, \rho_v, \sigma_v]$ using the method of simulated moments. We normalize $\chi^* = 0$, so that γ represents the coefficient of relative risk aversion in a deterministic steady state. The vector of moments to match include the sample mean, standard deviation, autocorrelation and cross-correlation matrix for the yields on German government securities and on the euro-area price-consumption ratio. The corresponding model implied statistics are computed on a long simulation ($N = 20000$).[18] The bottom panel of Table 1 reports the value of the estimated parameters. As we have anticipated earlier, the model requires a positive b in order to match the average slope of the yield curve observed in the data. Indeed, inflation and consumption growth are only moderately correlated in our sample (-0.24), this implying a modest role for inflation risk premia in accounting for the upward sloping yield curve observed in the data. It is also important to point out that we are not directly using consumption data to

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[18]Details on the weighting matrix.

estimate v_t.[19] With such a short sample it is problematic to estimate time-varying second moment for consumption growth. The volatility process in our procedure is indirectly inferred from the behavior of yields and of the log price-consumption ratio.

Table 2: **The Fit of the Pricing Model**

	$\mu(r^\$_{4,t})$	$\mu(r^\$_{20,t})$	$\sigma(r^\$_{4,t})$	$\sigma(r^\$_{20,t})$	$\sigma(\text{pc}_t)$
Data	2.50	3.28	1.47	1.24	0.33
Model	2.68	2.95	2.12	1.18	0.20
	$\text{Acorr}(r^\$_{4,t})$	$\text{Acorr}(r^\$_{20,t})$	$\text{Acorr}(\text{pc}_t)$	$\rho(r^\$_{4,t}, r^\$_{20,t})$	$\rho(r^\$_{4,t}, \text{pc}_t)$
Data	0.95	0.95	0.96	0.94	0.87
Model	0.94	0.97	0.98	0.90	-0.34

Notes: $\mu(.)$ represents the mean, $\sigma(.)$ the standard deviation, Acorr(.) the first order autocorrelation and $\rho(.)$ the correlation.

The $ upperscript means that the variable is measured in nominal terms.

Table 2 reports the in-sample fit of the model. The "data" rows report sample moments while the "model" rows report model implied moments at the estimated parameters. We can see that the model does a fairly good job in matching the joint behavior of yields and price-consumption ratio in Europe. More specifically, the model matches the level and volatility of the nominal yields observed in the data. The log price-consumption ratio is fairly volatile, with a standard deviation of 0.20, although not as volatile as in the data. The model implied yields and log price-consumption ratios are very persistent processes, as in the data. The model fails in reproducing the correlation between the price-consumption ratio and nominal yields: this correlation in the data is 0.87, while the model implied one is -0.34.

4.3.2 The government decision problem (in progress)

We next turn to the calibration of $\theta^{\text{def}} = [\sigma, \beta, d_0, d_1, \psi, \lambda, \iota, \rho_y, \sigma_y, p^*, \rho_p, \sigma_p]$. We fix σ to 2, a conventional value in the literature. We set $\psi = 0.0492$, a value that implies an average exclusion from capital markets of 5.1 years following a sovereign default, in line with the evidence in Cruces and Trebesch (2013). The endowment process is calibrated by fitting an AR(1) on our linearly detrended output series. This yields $\rho_y = 0.95$ and $\sigma_y = 0.007$. The coupon parameter on long term debt, ι, is chosen so that long term debt is traded on average at par.

In a future draft we will choose the remaining parameters $[p^*, \rho_p, \sigma_p, \lambda, \beta, d_0, d_1]$ to

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[19]For this reason, it is more appropriate to refer to the estimate of the inflation-consumption process as quasi-MLE since the distribution of the error term is misspecified in estimation.

match basic facts about the price, quantity and duration of Italian public debt along the lines illustrated in Section 3. For the moment, we fix those parameters at the value in Table 3

Table 3: **Calibration of $\theta^{\mathbf{def}}$**

	Numerical Value	Source
σ	2.000	
ψ	0.049	Cruces and Trembesh (2011)
ρ_y	0.950	AR(1) on linearly detrended real GDP
σ_y	0.007	AR(1) on linearly detrended real GDP
ι	1.000	
λ	0.000	
β	0.800	
d_0	-0.304	
d_1	0.329	
$\frac{\exp\{p^*\}}{1+\exp\{p^*\}}$	0.005	
ρ_p	0.990	
σ_p	0.200	

4.4 Pitfalls in the identification of p_t (to be completed)

For tractability, we have assumed so far that lenders' stochastic discount factor is an exogenous process, independent on the other disturbances in the economy. This might be a very strong assumption: it is natural to think that an Italian default would have very adverse consequences on investors, and that the prospect of this event may alter their attitude toward risk.[20] Therefore, one may think that our procedure underestimates the importance of rollover risk in driving Italian spreads: by making a sovereign default more likely, an increase in the probability of a rollover crisis could lead to an increase in the risk aversion of lenders, and impact interest rate spreads through risk premia.

However, this is not likely to be the case in our application. First, it is important to stress that the quantitative importance of rollover risk is identified in the model from the joint behavior of debt duration and interest rate spreads, more specifically their comovement. We can verify this claim by looking at how the objective function in our calibration varies with the parameters of the $\{p_t\}$ when we include and we exclude targets related to debt duration. [perform the experiment]. [...].

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[20]For example, this prediction would arise in a set up where lenders are exposed to Italian debt and they face occasionally binding constraints on their funding ability, see Bocola (2014) and Lizarazo (2013).

Second, endogeneizing the behavior of the lenders' stochastic discount factor would not affect qualitatively the relation between debt duration and interest rate spreads in our model. To explain why, we consider a slight modification of the three period example studied in Section 3. In particular, let's assume that the prospect of a government default makes the stochastic discount factor more volatile,

$$
M\left(s,s'\right) = \begin{cases} \frac{\mu(s'|s)}{1+r^*} \frac{1}{\mathbb{E}[(1+m)(1-\delta(s'))]} & \text{if } \delta\left(s'\right) = 1 \\ \frac{\mu(s'|s)}{1+r^*} \frac{(1+m)}{\mathbb{E}[(1+m)(1-\delta(s'))]} & \text{if } \delta\left(s'\right) = 0 \end{cases} , \quad m > 0. \tag{21}
$$

From (21) we have that the risk free rate is constant and equal to $1 + r^*$ and the risk premium is increasing in the probability of default. Formally, if $\mathbb{E}[1 - \delta]$ increases then $M\left(s, \cdot\right)$ increases in SOSD sense.

Note that Proposition 3 still holds in this environment: when default risk is driven only by extrinsic uncertainty, the government does not issue short term debt in this economy. If anything, in this set up the government has an extra motive to lengthen the duration of his debt in the face of rollover risk because this makes M more volatile, raising the welfare costs of extrinsic uncertainty. This last fact indicates that our calibration would assign a more limited role to rollover risk if we were to incorporate this type of feedbacks in the model.

5 Decomposing Italian spreads

We now use the calibrated model to measure the importance of non-fundamental risk in driving Italian spreads during the recent sovereign debt crises. We proceed in two steps. In the first step, discussed in Section 5.1, we use our calibrated model along with the data presented in Section 4 to estimate a time series for the model state variables, $\{\mathbf{S}_t\}_{t=1999:Q1}^{2012:Q2}$. In the second step, discussed in Section 5.2, we use the filtered state variables and the model equilibrium conditions to measure the three components of interest rate spreads defined in equation (12): i) the risk of a rollover crises; ii) the risk of a solvency crises; iii) the compensation required by lenders to hold Italian sovereign risk.

5.1 Filtering the unobserved states

Our model defines the nonlinear state space system

$$\mathbf{Y}_t = \mathbf{g}(\mathbf{S}_t; \theta) + \eta_t$$

$$\mathbf{S}_t = \mathbf{f}(\mathbf{S}_{t-1}, \varepsilon_t; \theta),$$

(22)

with $\mathbf{Y}_t$ being a vector of measurements, η_t classical measurement errors, the state vector is $\mathbf{S}_t = [b_t, y_t, \chi_t, v_t, p_t]$ and ε_t are innovations to structural shocks. The first part of the system collects measurement equations, describing the behavior of observable variables while the second part of the system collects transition equations, regulating the law of motion for the potentially unobserved states.[21] We obtain estimates for the model state variables by applying the particle filter to the above system. The set of measurements $\mathbf{Y}_t$ includes German nominal bond yields (1 and 5 years), the log price-consumption ratio, linearly detrended Italian real GDP and our interest rate spread series. The sample period is 1999:Q1-2012:Q2.

The solid lines in Figure 4 plot the data used in this exercise. The top panel reports the level (left) and slope (center) of the nominal yield curve for German government securities and the log price-consumption ratio (right) for the euro-area over the 2005-2012 period. Starting with the U.S. financial crisis in 2007-2008, we have seen a sharp decline in the level of the yield curve, an increase in its slope and a drastic decline in stock prices. The bottom panel reports domestic variables for Italy: the performance of the Italian economy after 2008 was very weak, with output being substantially below trend. Interest rate spreads were stable at roughly 30 basis points until 2008. From that point on, and more markedly from the second half of 2011, yields differential between Italian and German bonds increased sharply. The dotted lines in the figure are the model implied (filtered) time series. These differ from the actual data because of the presence of measurement errors in (22).

Figure 5 plots our estimates for the structural shocks. The yield curve and the price-consumption ratio are mostly informative about the shocks driving the pricing kernel of the lenders, $\{\chi_t, v_t\}$. Our pricing model interprets the behavior of financial markets in the euro-area as the result of an increase in lenders' risk aversion and an increase in the volatility of their endowment process. It is clear from Figure 4 and 5 that movements in risk aversion are mainly responsible for the behavior of the price-consumption

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[21]Note that $\mathbf{Y}_t$ may include some of the state variables.

Figure 4: **Variables in the Measurement Equations**
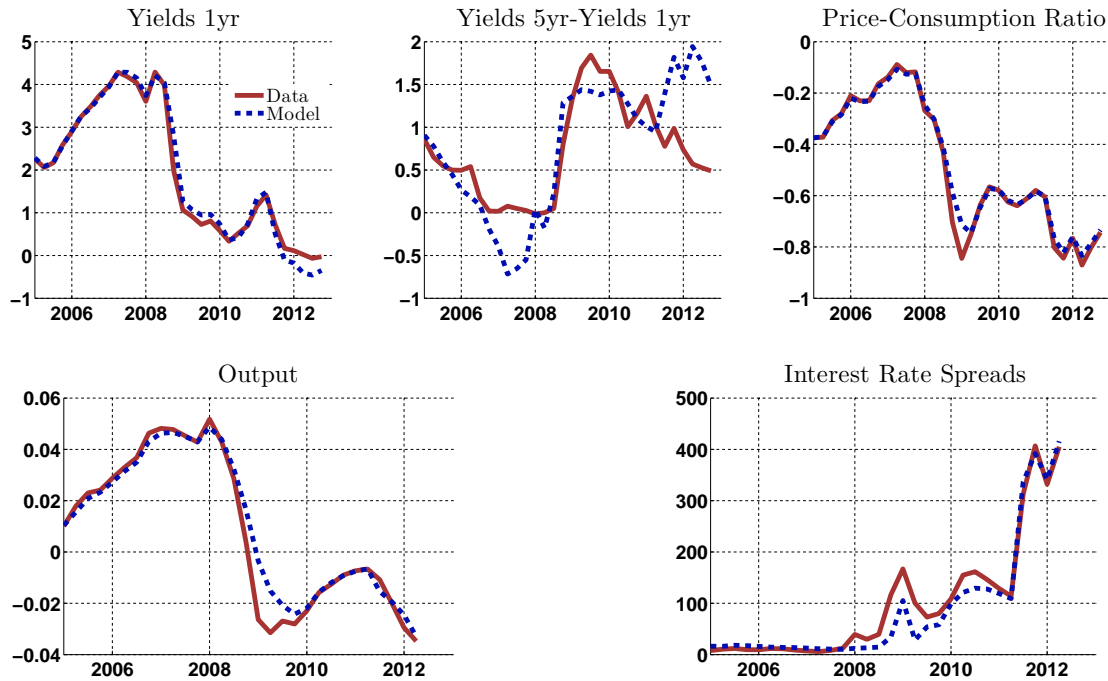
Notes: The top panel plots, respectively, 1 year nominal yields on German government securities, the difference between the 5 years and 1 years yields and the log demeaned price-consumption ratio for the euro area. The bottom panel reports linearly detrended real GDP and the interest rate differential between Italian and German bonds (1 year). Solid line reports the data while dotted line represents the filtered series from the model.

ratio while aggregate volatility drives the level and the slope of the yield curve. This separation reflects the inability of our pricing model to fit, with only one factor, the movements in the nominal yield curve and in the price-consumption ratio in our sample. As described earlier, the increase in risk aversion required to match the behavior of the price-consumption ratio would imply a counterfactual rise in the level and a decline in the slope of the yield curve. Aggregate volatility, on the other hand, does have very little effects on the price-consumption ratio: a model with only this factor would not be able to match the large fluctuations in this variable observed in our sample. The endowment shock y_t tracks our detrended output series for Italy very closely. This discussion suggests that the shocks $\{\chi_t, v_t, y_t\}$ are essentially identified by the German yield curve, the euro-area price-consumption ratio and the Italian detrended output. The bottom-right panel of the figure plots the estimated time series for $\{p_t\}$. At the moment, this object is identified mainly as a residual from the component of the spreads that is not accounted by movements in economic fundamentals.

Figure 5: **Filtered Shocks**
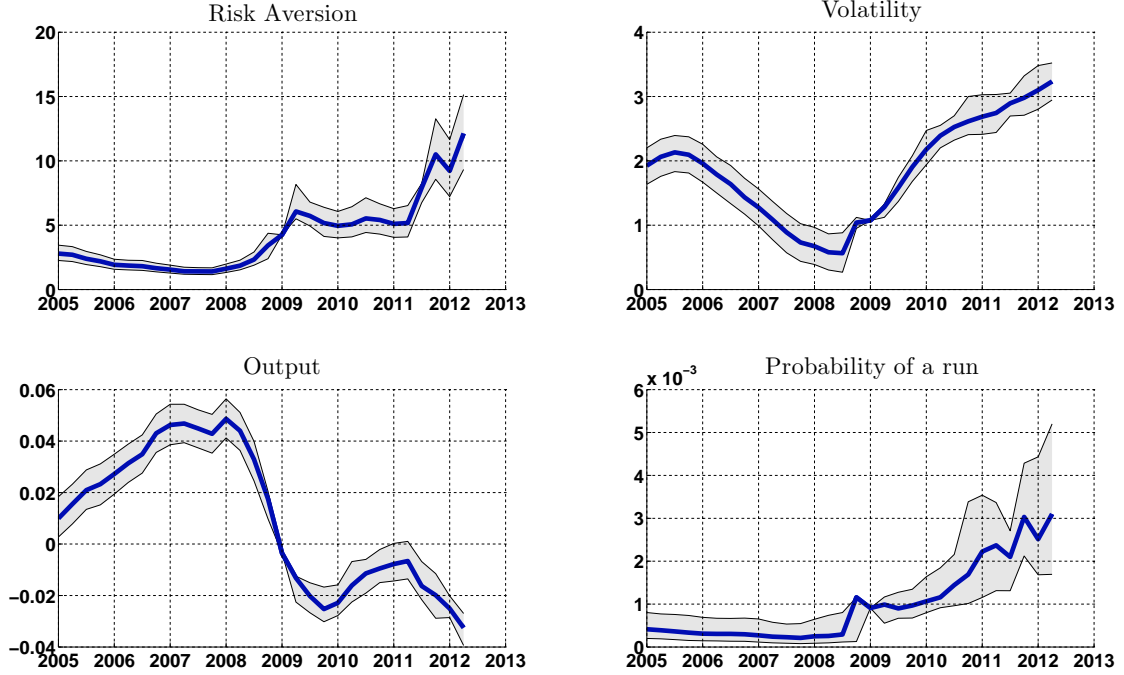
Notes: Solid lines report the conditional expectations of the structural shocks obtained by applying the particle filter to the state space system (22). Risk aversion is defined as $\frac{\gamma}{\exp\{\chi_t\}}$, while the probability of a run is $\frac{\exp\{p_t\}}{1+\exp\{p_t\}}$. The shaded area reports the 90% pointwise confidence interval.

5.2 Measuring the drivers of Italian spreads

We now use the estimated path for the state variables $\{\mathbf{S}_t\}_{t=1999:Q1}^{2012:Q2}$ and the equilibrium conditions of the model to measure the importance of rollover risk during the sovereign debt crisis in Italy. Remember that interest rate spreads on short term debt can be decomposed as follows

$$
\frac{r_{S,t} - r_t^*}{r_{S,t}} = \Pr_t\{\mathbf{S}_{t+1} \in \mathcal{S}^{\text{default}}\} + \Pr_t\{\mathbf{S}_{t+1} \in \mathcal{S}^{\text{crisis}}\}\mathbb{E}_t[p_{t+1}]
$$

$$
- \operatorname{Cov}_t\left(\frac{M_{t,t+1}}{\mathbb{E}_t[M_{t,t+1}]}, \delta_{t+1}\right).
$$

(23)

We can obtain estimates for these three components by feeding our model with $\{\mathbf{S}_t\}_{t=1999:Q1}^{2012:Q2}$. Figure 6 reports the filtered series for Italian sovereign spreads along with the decomposition of equation (24).

From the red shaded area we can see that a sizable fraction of the movements in the spreads over our sample reflects pure compensation for holding Italian risk. In 2012:Q2,

Figure 6: **Decomposition of Italian Spreads**
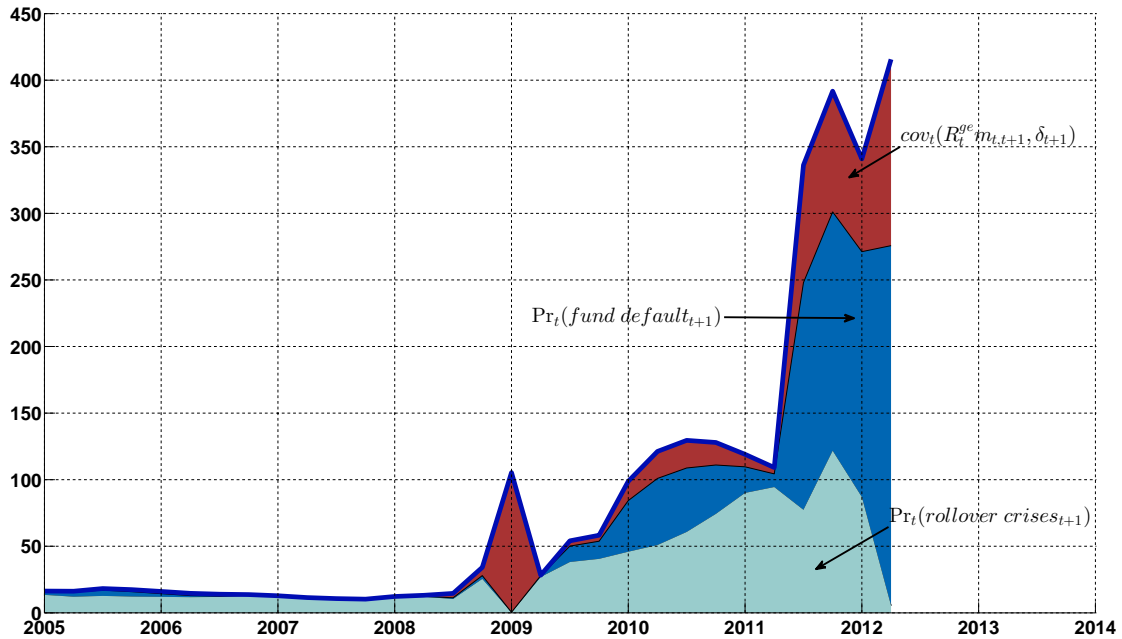
for example, the risk premium accounts for 170 basis points, roughly 35% of the observed spread. This is mainly driven by the behavior of the lenders' risk aversion estimated earlier: risk premia were sizable for most securities at the time, and this had adverse implications on the price of Italian bonds.

Movements in default risk account for the remaining fraction of spreads. Rollover risk (green shaded area) represented a sizable component of the default probability in our sample. In 2011:Q3, it was responsible for 120 basis points, roughly 30% of the yields differential between Italian and German bonds at the time. From that point on, though, it gradually declined and became negligible toward the end of the sample. Note that this decline in rollover risk is not coming from a reduction in $\{p_t\}$. Rather, it is the result of a decline in the probability of falling into the crisis zone in the future generated by a decline in the debt issuance of the government. Alongside rollover risk, the probability of a fundamental default (blue shaded area) is the key contributor to the spreads in our sample.

Overall, our findings suggest that expectations of future coordination failures among bondholders were an important determinant of Italian spreads during the second half of 2011. Their role, however, became negligible toward the end of our sample. Our calculations imply that bad domestic fundamentals and high risk aversion in European financial markets accounted for the bulk of the observed variation in Italian spreads at the

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end of 2012:Q2, with rollover risk playing only a negligible role.

6   Evaluating OMT Announcements

As a response to soaring interest rate spreads in the euro-area periphery, the Governing Council of the European Central Bank (ECB) announced during the summer of 2012 that it would consider outright transactions in secondary, sovereign bond markets. The technical framework of these operations was formulated on September 6 of the same year. The Outright Monetary Transaction (OMT) program replaced the Security Market Program as a mean through which the ECB could intervene in sovereign bond markets.

OMTs consist in direct purchases of sovereign bonds of members of the euro-area in secondary markets.[22] These operations are considered by the ECB once a member state asks for financial assistance, and upon the fulfillment of a set of conditions.[23] The Governing Council decides on the start, continuation and suspension of OMTs in full discretion and acting in accordance with its monetary policy mandate. There are two important characteristics of these purchases. First, *no ex ante quantitative limits* are set on their size. Second, the ECB accepts the same (*pari passu*) treatment as private or other creditors with respect to bonds issued by euro area countries and purchased through OMTs.

Even though the ECB has not yet implemented OMTs, the mere announcement of the program had significant effects on interest rate spreads of peripheral countries. Altavilla et al. (2014) estimate that OMT announcements decreased the Italian and Spanish 2 years government bonds by 200 basis points, magnitudes in line with Figure **??** discussed in the Introduction. This decline in interest rate spreads was widely interpreted by economists and policy makers as a reflection of the success of this program in reducing non-fundamental inefficient fluctuations in sovereign bond markets of euro-area peripheral countries. Accordingly, OMT has been regarded thus far as a very successful program. The aim of this section is to put this interpretation under scrutiny.

We introduce OMTs in our model as a price floor schedule implemented by a Central Bank. Section 6.1 shows that an appropriate design of this schedule i) can eliminate the bad equilibria in our model, and ii) it does not require the Central Bank to ever intervene in bond markets. Therefore, along the equilibrium path the Central Bank can

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[22]Transactions are focused on the shorter part of the yield curve, and in particular on sovereign bonds with a maturity of between one and three years. The liquidity created through OMTs is fully sterilized.

[23]A necessary condition for OMTs is a *conditionality* attached to a European Financial Stability Facility/European Stability Mechanism (EFSF/ESM) macroeconomic adjustment or precautory programs. For a country to be eligible for OMTs, these programs should include the possibility of EFSF/ESM primary market purchases.

achieve a Pareto improvement without taking risk for their balance sheet. However, we also show that alternative formulations of the price floor may induce the sovereign to ask for assistance in the face of bad fundamental shocks. *Ex-ante*, this option leads the sovereign to overborrow. Under both of these scenarios, interest rate spreads decline once the Central Bank announces the price floor schedule: in the first scenario, the reduction in interest rate spreads is due the elimination of rollover risk. In the second scenario, this reduction reflects the option for bondholders to resell the security to the Central Bank whenever the sovereign is approaching a solvency crises. Section 6.2 proposes a simple procedure to test which of these two hypothesis better characterizes the observed behavior of Italian spreads after the announcements of the OMT program.

6.1 Modeling OMT

We model OMT as follows. At the beginning of each period, after all uncertainty is realized, the government can ask for assistance. In such case the Central Bank (CB) commits to buy government bonds in secondary markets at a price $q_{CB}(\mathbf{S}, B')$ that may depend on the state of the economy, $\mathbf{S}$, and on the quantity of debt issued, B'. We assume that assistance is conditional on the fact that total debt issued is below a cap $\bar{B}_{CB} < \infty$ also set by the CB. This limit captures the conditionality of the assistance in the secondary markets. Moreover, it rules out Ponzi-scheme on the central bank. Hence OMT is fully characterized by a policy rule

$$\left(q_{CB}(\mathbf{S}, B'), \bar{B}_{CB}(\mathbf{S})\right) \in \mathbb{R}^2 \times \mathbb{R}^2.$$

We assume that the CB finances such transactions with a lump sum tax levied on the lenders. We further assume that such transfers are small enough that they do not affect the stochastic discount factor $M_{t,t+1}$.

The problem for the government described in (4) changes as follows. We let $\chi \in \{0,1\}$ be the decision to request CB assistance, with $\chi = 1$ for the case in which assistance is requested. Then we have:

$$V(\mathbf{S}) = \max_{\delta, B', G, \chi} \delta\left[U(G) + \beta\mathbb{E}V(\mathbf{S}')\right] + (1-\delta)\underline{V}(Y) \tag{24}$$

subject to

$$G + B_S + \iota B_L \leq Y + q(\mathbf{S}, \chi, B') \cdot x,$$

$$B' \leq \bar{B}_{CB}(\mathbf{S}) \quad \text{if } \chi = 1.$$

The lenders have the option to resell government bonds to the CB at the price q_{CB} in case the government asks for assistance. Then the no-arbitrage conditions for the lenders (5) and (6) are modified as follows:

$$q_S\left(\mathbf{S},\chi,B'\right) = \max\{\delta\left(\mathbf{S}\right)\mathbb{E}\left\{M\left(s,s'\right)\delta\left(\mathbf{S}'\right)|\mathbf{S}\right\};\chi q_{CB,S}(\mathbf{S},B')\} \tag{25}$$

$$q_L\left(\mathbf{S},\chi,B'\right) = \max\{\delta\left(\mathbf{S}\right)\mathbb{E}\{M\left(s,s'\right)\delta\left(\mathbf{S}'\right)[\iota$$
$$+(1-\lambda)q\left(\mathbf{S}',B'(B',\mathbf{S}')\right)]|\mathbf{S}\};\chi q_{CB,L}(\mathbf{S},B')\}. \tag{26}$$

Note that the bonds prices now depend also on the decision of the government to activate assistance because only in that situation the CB stands ready to buy the bonds.

Given a policy rule $(q_{CB},\bar{B}_{CB})$, a *recursive competitive equilibrium with OMT* is value function for the borrower V, associated decision rules δ, B', G and a pricing function q such that V, δ, B', G are a solution of the government problem (24) and the pricing function satisfies the no-arbitrage conditions (25) and (26).

We now turn to show that an appropriately designed policy rule can uniquely implement the fundamental equilibrium outcome defined in (7), our normative benchmark.[24]

Proposition 4. *The OMT rule can be chosen such that the fundamental equilibrium is the unique equilibrium and assistance is never activated along the equilibrium path. In such case, OMT is a weak Pareto improvement relative to the equilibrium without OMT (strict if the private equilibrium does not coincide with the fundamental equilibrium).*

Proof. An obvious way to uniquely implement the fundamental equilibrium outcome is to set $q_{CB}(\mathbf{S},B') = q^*\left(\mathbf{S},B^{*\prime}\left(\mathbf{S}\right)\right)$ and $\bar{B}_{CB}\left(\mathbf{S}\right) = B^{*\prime}\left(\mathbf{S}\right)$ where recall that a star denotes fundamental equilibrium objects. Such construction is not necessary. A less extreme alternative is to design policies such that for all $\mathbf{S}$ for which there is no default in the fundamental equilibrium, $\delta^*\left(\mathbf{S}\right) = 1$, there exists at least one $B' \leq \bar{B}_{CB}\left(\mathbf{S}\right)$ with associated new issuance x such that

$$U\left(Y - B_S - \iota B_L + q_{CB}(\mathbf{S},B')\cdot x\right) + \beta\mathbb{E}V^*\left(B',s'\right) \geq \underline{V}(s), \tag{27}$$

$$\bar{B}_{CB}\left(\mathbf{S}\right) \leq B^{*\prime}\left(\mathbf{S}\right). \tag{28}$$

Under (27) and (28), it is clear that no self-fulfilling run is possible and there is no over-

borrowing. Hence (27) and (28) are set of sufficient conditions to eliminate runs and to uniquely implement the fundamental equilibrium outcome. □

Note that quantity limits (conditionality) are necessary to uniquely implement the fundamental equilibrium. In absence of $\bar{B}_{CB}$, because the CB guarantees a price q_{CB} then borrower act as a price taker and so it will issue new debt such that $\partial \mathbb{E} V (s', B') / \partial B' = q_{CB} U'(G)$. That is, under assistance it is optimal for the government to choose a B' that is larger than the one in the fundamental equilibrium,

$$\frac{\partial \mathbb{E} V (s', B')}{\partial B'} = q^* U'(G) + \frac{\partial q^* (s, B')}{\partial B'} U(G) < q^* U'(G) = q_{CB} U'(G).$$

So a limit to B' when the government asks for assistance is needed to prevent overborrowing while uniquely implement the desired outcome.

Proposition 4 gives us the most benevolent interpretation of the drop in Italian spreads after OMT was announced. If OMT follows the rule described in the proof of Proposition 4 and it uniquely implements the fundamental equilibrium outcome. In this case the observed drop in spreads is due to the fact that lenders anticipate that no run can happen along the equilibrium path and resulting in lower default probability and hence lower spreads.

However, the central bank does not want to support bond prices if they are low because of fundamental reasons. This entails a subsidy from the lenders to the borrower, reducing welfare for the lenders relative to the equilibrium without OMT (assuming lenders are the ones that have to pay for the losses of the bailout authority). Even in this scenario, bond prices may decline. To see this, suppose that in a given state the fundamental price for long-ten debt is $q_L^{*'}$. Suppose now that the ECB sets an assistance price $q_{CB,L}' > q_L^{'*}$. It is clear from (26) that the price today increases (the spread drops) relative to a counterfactual world without OMT.

Thus, decline in price not informative on whether ECB is following the benchmark rule, or whether it is providing some subsidy to peripheral countries. Next we use the model to test between these two alternatives.

6.2 A Robust Test

We now test for the hypothesis that the ECB did follow the policy described in Proposition 4. The logic of our approach goes as follows. Suppose that the Central Bank credibly commits to our normative benchmark. The announcement of this intervention would eliminate all extrinsic uncertainty, and the spreads today would jump to their "funda-

mental" value, i.e. the value that would arise if rollover crisis were not conceivable from that point onward. This fundamental level of the interest rate spread represents a lower bound on the post-OMT spread under the null hypothesis that the program was directed exclusively to prevent runs on Italian debt. Our test consists in comparing the spreads observed after the OMT announcements to their fundamental value: if the latter is higher than the observed ones, it would be evidence against the null hypothesis that the ECB followed the policy described in Proposition 4.

We perform this test using our calibrated model. Our procedure consists in three steps:

1. Obtain decision rules from the "fundamental" equilibrium defined in (7).

2. Feed these decision rules with our estimates for the fundamental shocks $\{\chi_t, v_t, y_t\}$. Obtain counterfactual post-OMT spreads justified purely by economic fundamentals.[25]

3. Compare post-OMT spreads with the counterfactual ones.

Table 4 shows the result. The first column reports the Italian spreads observed after the OMT announcements, while the second column presents the counterfactual spreads constructed with the help of our model. We can verify that the observed spreads lie below the one justified by economic fundamentals under the most "optimistic" interpretation of OMT. In 2012:Q4, the observed interest rate spread on Italian debt was 222.25 basis points, while our model suggests that the spread should have been 394.43 basis points if the program was exclusively eliminating rollover risk. Therefore, our model suggests that the decline in the spreads observed after the OMT announcements partly reflects the anticipation of a future intervention of the ECB in secondary sovereign debt markets. This is not surprising given our result in Section 5: since rollover risk was almost negligible in 2012:Q2, the observed drastic reduction in the spreads should partly reflect the value of an implicit put option for holders of Italian debt guaranteed by the ECB.

Table 4: **Actual and Fundamental Sovereign Interest Rate Spreads in Italy**

	Actual value	**Fundamental Value**
2012:Q3	348.24	422.41
2012:Q4	222.25	394.43

[25]The estimates of the state vector ends in 2012:Q2. For the 2012:Q3-2012:Q4 period, we set y_t equal to linearly detrended Italian output and we filter out $\{\chi_t, v_t\}$ using our pricing model along with the data on the German yield curve and the euro-area price-consumption ratio.

Clearly, it would be interesting to use our model to dig deeper into the implications of the OMT program. For example, we could try to measure the put option implicit in this intervention, to calculate the amount of resources that the ECB is implicitly committing under this policy or we could assess the moral hazard implications associated to this policy. This would not be an uncontroversial task, as it would require us to i) specify the policy rule followed by the ECB and to ii) specify how the selection mechanism responds to the policy intervention. The test we have described in this section is robust to these caveats, and we regard it as a first step for the evaluation of this type of interventions in sovereign debt models with multiple equilibria.

7   Conclusion

In this paper, we studied how important was non-fundamental risk in driving interest rate spreads during the euro-area sovereign debt crisis. We show that the joint behavior of interest rate spreads and debt duration provides key information for this purpose. Our preliminary results indicate that non-fundamental risk accounted for a modest fraction of the increase in interest rate spreads during the recent sovereign debt crisis.

Our analysis is limited to non-fundamental risk that arises from rollover risk as introduced in Cole and Kehoe (2000). We did not consider the type of multiplicity emphasized in Calvo (1988) and recently revived by Lorenzoni and Werning (2013) and Navarro et al. (2015). In future work we plan to investigate which features of the data can provide information about the relevance of this type of multiplicity.

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A Appendix

A.1 Proofs for the Three Period Example

Maturity and rollover risk: In the three period example, if there is only rollover risk and no fundamental shock at $t = 1, 2$ then $b_{01} = 0$ and all debt is long term.

Proof. Suppose not, i.e. $b_{01} > 0$. Consider then the following variation: increase b_{02} by $\varepsilon/q_{02} > 0$, and decrease b_{01} by $\varepsilon/q_{01} > 0$ so that G_0 in unchanged. Moreover, notice that - under the assumption that there is no fundamental default risk - the optimal allocation that can be achieved at $t = 1$ starting from (b_{01}, b_{02}, s) can be achieved with $(b_{01} - \varepsilon/q_{01}, b_{02} + \varepsilon/q_{02})$. In fact, since there are no shocks at $t = 1, 2$ we have that $q_{12} = m_{12}$ and at the original allocation the following Euler equation is satisfied:

$$m_{12}U'(G_1) = \beta U'(G_2) \tag{29}$$

and so it is clear that achieving same G_1, G_2 is budget feasible and optimal.

We next turn to show that the proposed variation reduces the crisis zone. In fact, at $t = 1$ there can be a rollover crisis only if

$$U(Y - b_{01}) + \beta EV_2(b_{02}, Y_2) \leq \underline{V}_1 \tag{30}$$

$$U(Y - b_{01} + \varepsilon/q_{01}) + \beta EV_2(b_{02} + \varepsilon/q_{02}, Y_2) \leq \underline{V}_1 \tag{31}$$

The fact that (29) holds at the original allocation implies that if $b_{12} > 0$ then

$$q_{12}U'(Y - b_{01}) > \beta EV_2'(b_{02} + b_{12}) \iff \frac{1}{q_{01}}U'(Y - b_{01}) > \frac{1}{q_{02}}\beta EV_2'(b_{02} + b_{12})$$

So we have that

$$
\begin{aligned}
U(Y - b_{01} + \varepsilon/q_{01}) + \beta EV_2(b_{02} + \varepsilon/q_{02}, Y_2) &\approx [U(Y - b_{01}) + \beta EV_2(b_{02}, Y_2)] \\
&\quad + \left[\frac{1}{q_{01}}U'(Y - b_{01}) + \frac{1}{q_{02}}\beta EV_2'(b_{02}, Y_2)\right]\varepsilon \\
&> U(Y - b_{01}) + \beta EV_2(b_{02}, Y_2)
\end{aligned}
$$

Hence if the second inequality is satisfied so it is the first but not vicersa. Hence the variation reduces the probability of default (rollover crisis) at $t = 1$ because (31) is less likely to hold than (30). Then we have that consumption in the first period is larger and

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so the variation increases utility, a contradiction. *Q.E.D.*

The content of the proposition is the following: in a deterministic economy, if there is rollover risk then no short term debt is issued. It is always preferable to reduce the issuance of short-term debt (and therefore the issuance of debt on path at $t = 1$ conditional on a crisis not happening) and increase the amount of long-term debt issued. So doing the amount of debt that must be raised at $t = 1$ declines and so the crisis zone is reduced. And hence rollover risk is reduced, allowing the government to issue debt more cheaply.

Commitment not to dilute: *In the three period example, if (i) there is no rollover risk, (ii) output is deterministic in $t = 1$, and (iii) the distribution of output in $t = 2$ does not depend on s_1 then $b_{02} = 0$ if β / m_{01} is sufficiently small (i.e. government wants to borrow a lot in period $t = 0$).*

Proof. It is helpful to use a "primal approach" to solve for the equilibrium outcome. Without rollover risk and uncertainty at $t = 0$, we can consider the following programming problem:

$$\max_{b_{01}, b_{02}, b_{12}, \delta_1, \delta_2} U(G_0) + \beta \mathbb{E}_0 \left\{ [\delta_1 U(G_1) + (1 - \delta_1)\underline{U}_1] + \beta [\delta_2 U(G_2) + (1 - \delta_2)\underline{U}_2] \right\} \quad \text{(P)}$$

subject to budget constraints

$$G_0 + \Delta_0 \leq q_{01} b_{01} + q_{02} b_{02} + Y_0$$
$$G_1 + b_{01} \leq q_{12} b_{12} + Y_1$$
$$G_2 + (b_{02} + b_{12}) \leq Y_2$$

the restriction that if $\delta_1 = 0$ then $\delta_2 - 0$, pricing equations

$$q_{01} = \mathbb{E}_0 [m_{01} \delta_1] = m_{01}$$
$$q_{12} = \mathbb{E}_1 [m_{12} \delta_2]$$
$$q_{02} = \mathbb{E}_0 [m_{01} m_{12} \delta_2] = \mathbb{E}_0 [m_{01} q_{12}]$$

the "default" constraint:

$$U(G_1) + \beta \mathbb{E}_1 U(G_2) \geq \underline{V}_1 = \underline{U}_1 + \beta \mathbb{E}_1 \underline{U}_2$$
$$U(G_2) \geq \underline{U}_2$$

and the "no-dilution" constraint

$$U(G_1) + \beta \mathbb{E}_1 U(G_2) \geq V_1(b_{01}, b_{02}) \tag{33}$$

It is clear that a fundamental equilibrium outcome solves the above problem and the converse is also true.

First, consider first a relaxed version of (P) in which we drop the no-dilution constraint (33). Notice that such relaxed problem has a continuum of solutions indexed by ε:

$$\left\{ b_{01}^* + \frac{\varepsilon}{q_{01}^*}, b_{02}^* - \frac{\varepsilon}{q_{02}^*}, b_{12}^* + \frac{\varepsilon}{q_{02}^*}, \delta_1^*, \delta_2^* \right\}$$

For all ε prices and government consumption associated with the debt issuance are the same. To see this just notice that since $delta_1^* = 1$ we have that $q_{02} = q_{01}q_{12}$ because $b_{12}^*(s_1) = b_{12}^*$.

Second, we argue that the no-dilution constraint (33) is more relaxed when $b_{02} = 0$. To this end, compare two allocations, that achieves the maximum in (P): $\left\{ b_{01}^*, b_{02}^*, b_{12}^*, \delta_1^*, \delta_2^* \right\}$ and

$$\left\{ \hat{b}_{01}, \hat{b}_{02}, \hat{b}_{12}, \hat{\delta}_1, \hat{\delta}_2^* \right\} = \left\{ b_{01}^* + \frac{\varepsilon}{q_{01}^*}, b_{02}^* - \frac{\varepsilon}{q_{02}^*}, b_{12}^* + \frac{\varepsilon}{q_{02}^*}, \delta_1^*, \delta_2^* \right\}$$

Under our assumptions:

$$V_1(b_{01}^*, b_{02}^*) = \max_{b_{12}} U(G_1) + \beta \int \max \left\{ U(Y_2 - b_{02}^* - b_{12}); \underline{U}_2 \right\} d\mu$$

subject to

$$G_1 + b_{01}^* \leq m_{12}\mu_{Y2} \left(\frac{b_{02}^* + b_{12}}{\tau} \right) b_{12} + Y_1$$

and

$$V_1\left(\hat{b}_{01}, \hat{b}_{02}\right) = \max_{b_{12}} U(G_1) + \beta \int \max \left\{ U\left(Y_2 - \left(b_{02}^* + \frac{\varepsilon}{q_{02}^*}\right) - b_{12}\right); \underline{U}_2 \right\} d\mu$$

subject to

$$G_1 + \left(b_{01}^* + \frac{\varepsilon}{q_{01}^*} \right) \leq m_{12}\mu_{Y2} \left(\frac{b_{02}^* - \frac{\varepsilon}{q_{02}^*} + b_{12}}{\tau} \right) b_{12} + Y_1$$

If the government starting from starting from $\left(b_{01}^*, b_{02}^*\right)$ chooses

$$b_{12} = b_{12}^* + \Delta > b_{12}^*$$

then such allocation cannot be replicated starting from $\left(\hat{b}_{01}, \hat{b}_{02} \right)$. In fact, to replicate the same allocation in period $t = 2$ the government would have to borrow

$$\tilde{b}_{12} = b_{12}^* + \frac{\varepsilon}{q_{02}^*} + \Delta$$

Consider now the G_1 that such deviation can attain starting from the two alternative states:

$$\tilde{G}_1^* + b_{01}^* = m_{12}\mu_{Y2} \left(\frac{b_{02}^* + b_{12}^* + \Delta}{\tau} \right) [b_{12}^* + \Delta] + Y_1$$

$$\tilde{G}_1 + \left(b_{01}^* + \frac{\varepsilon}{q_{01}^*} \right) = m_{12}\mu_{Y2} \left(\frac{b_{02}^* + b_{12}^* + \Delta}{\tau} \right) \left[b_{12}^* + \frac{\varepsilon}{q_{02}^*} + \Delta \right] + Y_1$$

Then

$$\begin{aligned}
\tilde{G}_1^* - \tilde{G}_1 &= \left[\frac{1}{q_{01}^*} - m_{12}\mu_{Y2} \left(\frac{b_{02}^* + b_{12}^* + \Delta}{\tau} \right) \frac{1}{q_{02}^*} \right] \varepsilon \\
&> \left[\frac{1}{q_{01}^*} - m_{12}\mu_{Y2} \left(\frac{b_{02}^* + b_{12}^*}{\tau} \right) \frac{1}{q_{01}^* q_{12}^*} \right] \varepsilon \\
&= \left[\frac{1}{q_{01}^*} - q_{12}^* \frac{1}{q_{01}^* q_{12}^*} \right] \varepsilon = 0.
\end{aligned}$$

Since borrowing more is the relevant deviation (as borrowers are borrowing constrained with default risk) we have that

$$V_1 \left(\hat{b}_{01}, \hat{b}_{02} \right) = V_1 \left(b_{01}^* + \frac{\varepsilon}{q_{01}^*}, b_{02}^* - \frac{\varepsilon}{q_{02}^*}, s_1 \right) < V_1 \left(b_{01}^*, b_{02}^* \right) \tag{34}$$

Hence, the no-dilution constraint (33) is more relaxed for the hat allocation.

Combining these two observations, we have that whenever the no-dilution constraint is binding it must be that $b_{02} = 0$. A sufficient condition for the no-dilution constraint to be binding is that β/m_{01} is sufficiently small. [Can we show that no-dilution always binding whenever $\Pr(\delta_2 = 0) > 0$?] Q.E.D.

In Markov environment short term debt is more attractive because it is not prone to be diluted. Mechanically, short term debt relaxes the "no-dilution" constraint (33).

So, the incentive motive calls for having lots of short-term debt. Now, why is the incentive motive stronger when there is higher default risk? Suppose the country enters with lots of debt – and so high default risk – then it is crucial to raise a lot of resources today to avoid default. It can be shown that with dilution and no rollover risk the value of issuance is maximized for all new debt being short term (it is just a variant of the

argument given above). Then when need to raise lots of resources – high debt – use short term debt more at the cost of sacrificing insurance (but not much because as argued above need less long term debt to achieve same amount of insurance).