Macro Theory III
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Lecture 1.

Definition of a Debreu Economy:

People: A list of the measures of the types of people. Typically, the number of different types will be finite. Often there will be only one type. The measure of type $\boldsymbol{i} \in \boldsymbol{I}$ is denoted by λ_i.

Comment: The reason for a measure is that people are infinitesimal and the price taking assumption is literally true. The mathematical economists have provided justification for dealing with the limiting case.

Commodities: The things traded.

Commodity space: Topological vector space $\boldsymbol{S}$. An element of this space, $\boldsymbol{x} \in \boldsymbol{S}$, is a list of the quantities of every commodity. Elements of $\boldsymbol{S}$ are commodity vectors.

Consumption set for a type *i* person: $X_i \subseteq S$.

Preference ordering: Typically characterized by $u_i : X_i \to \Re$ for $\boldsymbol{i} \in \boldsymbol{I}$.

Comment 1: A person of type $\boldsymbol{i}$ has a preference ordering on the set $\boldsymbol{X_i}$. If certain axioms are satisfied, a preference ordering can be characterized by a utility function. Any increasing transformation of this utility function also characterizes preferences.

Comment 2: As technologies exist to create uncertainty, given certain reasonable assumption, people maximize expected utility. Further, this function is defined up to a positive affine transformation.

Technologies: $Y_j \subset S$ for $\boldsymbol{j} \in \boldsymbol{J}$. Here the set of technologies $\boldsymbol{J}$ is assumed countable.

Comment 3: Often we deal only with the aggregate production set $Y = \sum_{j \in J} Y_j$. Sometimes people own endowments. In these cases, there are three options. The first option is to give them ownership to a technology characterized by the set $\{y \in S \mid y = e_i\}$ where $\boldsymbol{e_i}$ is the endowment an $\boldsymbol{i}$ type person. The second option is deal with net trades and then $x_i + e_i$ is the argument of a type i utility function. The third option is to keep the endowment separate and to modify the resource balance constraint below.

Definition of a (type-identical) allocation: $\{\{x_i\}_{i \in I}, \{y_j\}_{j \in J}\}$, where $\boldsymbol{x_i}, \boldsymbol{y_j} \in \boldsymbol{S}$ for all $\boldsymbol{i}$ and $\boldsymbol{j}$, is an allocation.

Comment 4: If preferences are convex, restricting attention to type-identical allocation makes sense. This will be established formally.

Feasible (type-identical) allocation: An allocation is feasible if

(a) $x_i \in X_i$ for all $\boldsymbol{i} \in \boldsymbol{I}$;

(b) $y_j \in Y_j$ for all $\boldsymbol{j} \in \boldsymbol{J}$;

(c) and the resource balance constraint,

$$\sum_{i \in I} \lambda_i x_i = \sum_{j \in J} y_j$$

is satisfied.

Comment 5:

The requirement that the resource balance constraint holds with equality is the way Debreu does it, but many mathematical economists only require

$$\sum_{i \in I} \lambda_i x_i \leq \sum_{j \in J} y_j .$$

This alternative definition is equivalent to Debreu's definition if, for example, there is a finite number of people, free disposal, and more is preferred to less. As we will see, this alternative definition is deficient in important classes of economies that arise in macroeconomics and more generally in applied general equilibrium.

Definition of Pareto optimal allocation: An allocation is Pareto optimal if it is feasible and no other feasible allocations yields as high utility for almost all people and higher utility for a positive measure of people.

Comment 6: Note that nothing has been said about competitive equilibrium. To do this a price system is needed.

Price systems: Let ϕ be a continuous linear function mapping ***S*** into the reals. Here is where we are using the topology on ***S*** as we said the function is continuous. Typically ***S*** will be a normed vector space and the topology will be the one induced by this norm. If ***S*** is finite dimensional then ϕ is a finite dimensional vector of the same dimension.

The case of a countable infinity of commodities, $\{x_t\}_{t=0,1,2,\ldots}$: The ℓ_p norms (with $1 \leq p \leq \infty$) are the common norms on such spaces. In macro $p = \infty$ is heavily used. The norm in this case is

$$||x||_\infty = \sup_t |x_t| .$$

More generally,

$$||x||_p = \left[\sum_{t=o}^{\infty} |x_t|^p \right]^{1/p} .$$

Note if p = 2, this is just the infinite dimensional version of the Euclidean norm. This norm is heavily used in the finance literature, as they are interested in variances. If p = 1, this is just the sum of the absolute values. If p = ∞, only the biggest $|x_t|$ matters. The sup (which is shorthand for supremum) is used to deal with the case that there is not a biggest $|x_t|$ but there is a finite least upper bound.

Exercise: For the case that $S = \Re^2$, graph the unit neighborhood of the zero point for p = 1, 2, and ∞.

If there is a continuum of differentiated goods and commodities that are "close" substitutes, the natural commodity space is the space of signed measures. The natural topology is the one called the weak topology in probability theory and called the weak* topology in functional analysis.

Often in macro economics there is a continuum of commodities. Workweeks of different lengths are different commodities and there is a continuum of possible workweek length. In this course, we will not be dealing with this continuum case in a formal way.