Macro Theory III
Edward C. Prescott

Lecture 2.

The first welfare theorem holds in great generality. An exception is the Samuelson pure consumption loan economy. Before proceeding with a discussion of this economy, I will discuss the first welfare theorem when there is a countable infinity of people types. Let $i \in \{0,1,2, \dots \}$ denote the type of an individual. There is measure 1 of every type.

Let $\{\{x_i^o\}, \phi\}$ be a valuation equilibrium and let $\{x_i'\}$ be an allocation that is Pareto superior to allocation $\{x_i^o\}$ as in the proof of the First Welfare Theorem in Stokey and Lucas (pp. 453-4). Assuming local non-satiation, as with the finite i case, then

(1) $$\phi(x_i') \geq \phi(x_i^o) \text{ for all } i, \text{ and } \phi(x_i') > \phi(x_i^o) \text{ for at least one } i.$$

The proof goes through if the ϕ is such that the above implies

(2) $$\phi\left(\sum_i x_i'\right) = \sum_i \phi(x_i') > \sum_i \phi(x_i^o) = \phi\left(\sum_i x_i^o\right).$$

Anytime the set I is finite, (1) implies (2) as ϕ is a linear functional. If the set I has an infinite number of elements, in general (1) does not imply (2).

The Samuelson Overlapping Generations Model.

Let t index generations and assume that each generation has measure one. As preferences will be convex, we restrict attention to type-identical allocations. Generation $t > 0$ has endowment 2 when young and 0 when old. Generation t members' preferences are characterized by

$$u(x_t^t) + u(x_{t+1}^t),$$

where $u:[0,\infty)\rightarrow\Re$ is continuous, increasing, differentiable, and strictly concave. The superscript denotes generation. The subscript denotes the consumption good date. The initial old generation has utility function $u(x_1^o)$.

A candidate competitive equilibrium is for all t

$$x_t^t = 2 \quad and \quad x_s^t = 0 \quad for\ s \neq t$$

and price system $p_t = [u'(0)/u'(2)]^t$ for all t.

Question: What is the interest rate?

If the consumption set $X^t = S_+$, then this allocation is consistent with every household maximizing its utility subject to its budget constraint if the date t generation owns the technology

$$Y^t = \{y \in S \mid y_t = 2 \quad and \quad y_{s\neq t} = 0\}.$$

Is this valuation equilibrium? The answer is yes. An appropriate commodity space is $S = \{x \in \Re^{\infty} \mid \|x\| < \infty\}$, where the norm is

$$\|x\| = \sup_t \{|x_t|[u'(0)/u'(t)+1]^t\}.$$

Exercise: Show that the candidate pricing system is a continuous linear functional on this commodity space. Note that the interest rate is negative.

Question: Can the agent operating a technology, borrow $z > 0$ units at date $t+1$ and pay it off next period by borrowing $z\ u'(2)/u'(0)$ next period? In every period enough is borrowed to payoff the debt. Given that $u'(2)/u'(0) < 1$, there is no default and asymptotically debt in current prices goes to zero. This is why some question whether this is a sensible equilibrium concept for this environment when the interest rate is negative.

This equilibrium allocation is not Pareto optimal. A Pareto superior allocation is

$$x_t^t = x_{t+1}^t = 1 \quad all\ t.$$

Every generation is strictly better off with this allocation. Thus, with an infinite number of agents, valuation equilibria are not necessarily Pareto optimal. This is true even though there is local non-satiation.

Reversing the pattern of endowments so that the endowment when young is 2 and when old is 0 results in the interest rate being positive. This in turn suffices to insure that relation (2) holds. Note that for all feasible type-identical allocations, the c_t^i are uniformly bounded in *i* and *t*.

Exercise: Have the endowment pattern be 3 when young and 1 when old and $u(c)$ be $\ln(c)$. Let *x* be excess demand so

$$\ln(3 + x_t^t) + \ln(1 + x_{t+1}^t)$$

is the utility function of generation *t*. Then for all *t*,

$$Y^t = \{0\} \text{ and } X^t = \{x \in S \mid x_t^t \geq -3,\ x_{t+1}^t \geq -1,\ x_s^t \geq 0\ otherwise\}.$$

Find the valuation equilibrium.

Let the prime allocation be $x_t'^s = 1$ if t=s, $x_t'^s = -1$ if t=s+1, and 0 otherwise. Show that this allocation is Pareto superior to the valuation equilibrium allocation. Show

$$\phi\left(\sum_i x_i'\right) \neq \sum_i \phi(x_i').$$

In this case relation (2) does not hold.

Show that if the endowment pattern is reversed, the ϕ is such that relation (2) holds for all feasible allocations. Note that since there are no technologies except the degenerate one $\{0\}$, the resource balance constraint is simply $\sum_i x_i = 0$.

Suppose the definition of competitive equilibrium requires only that

$$\sum_{i \in I} \lambda_i x_i \leq \sum_{j \in J} y_j$$

The commodity space is ℓ_1. The endowments are 3 when young and 1 when old. A competitive equilibrium is price system $p_t = 1$ for all t and allocation that has every generation consume 2 units when young and 2 units when old. The problem with this definition is that the initial generation sells one unit of the consumption good when young and there is nobody . For this reason, I say the weaker definition of competitive equilibrium is deficient.

With the Debreu definition, there is no problem. Autarky is the only equilibrium. See the paper below for an alternative equilibrium concept for this class of economies.

Edward C. Prescott and José-Víctor Ríos-Rull, "On the Equilibrium Concept for Overlapping Generations Organizations," Minneapolis Federal Reserve Bank Staff Report 282.